`

Math Exam August 29th, 2019

Linear Algebra and Probabilities

STUDENT NAME: ....

TIME: 90 minutes

PROBLEM 1

Given any constants a,b,c where $a \neq 0$, find all values of x such that the matrix A is invertible

$$A = \begin{bmatrix} 1 & 0 & c \\ 0 & a & -b \\ -1/a & x & x^2 \end{bmatrix}$$

a. $x = 1$
b. $x \neq \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
c. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
d. $x \neq \frac{-b - \sqrt{b^2 - 4ac}}{2a}$, $x \neq \frac{b + \sqrt{b^2 - 4ac}}{2a}$
e. $x = 0$

PROBLEM 2

A three-digit number has two properties. The tens-digit and the ones-digit add up to 5. If the number is written with the digits in the reverse order, and then subtracted from the original number, the result is 792. Use a system of equations to find all of the three-digit numbers with these properties.

a. if c < 1 then $a = 9, b = 9$
b. if c = 0 then $a = 7, b = 4$, resulting $abc = 740$
c. if c = 0 then $a = 8, b = 5, resulting\, abc = 850$, or $c = 1$ then $a = 9, b = 4$ resulting $abc = 941$
d. none
e. if c = 0 then $a = 6, b = 3$, resulting $abc = 630$

PROBLEM 3

Let A be the matrix given by

$$A = \begin{bmatrix} -2 & 0 & 1 \\ -5 & 3 & a \\ 4 & -2 & -1 \end{bmatrix}$$

for some variable a . Find all values of a which will guarantee that A has eigenvalues $0, 3$ and -3 .

a. a = 1

b. a = 2
c. a = 3
d. a = 24
e. none

PROBLEM 4

Solve the given vector equation for x

$$2\begin{bmatrix}1 & 2 & 3\\0 & 4 & 2\end{bmatrix}-3\begin{bmatrix}1 & 1 & 2\\0 & 1 & x\end{bmatrix}=\begin{bmatrix}-1 & 1 & 0\\0 & 5 & -2\end{bmatrix}$$

a. a = 1
b. a = 2
c. a = 3
d. a = 4
e. none

PROBLEM 5

Compute the product of AB of the two matrices below using the definition of the matrix-vector product:

$$A=\begin{bmatrix}3 & 1\\2 & 4\\6 & 5\\1 & 2\end{bmatrix} \text{ and } B=\begin{bmatrix}-3 & 1\\4 & 2\end{bmatrix}$$

a. $AB=\begin{bmatrix}-5 & 4\\10 & 10\\2 & 16\\5 & 5\end{bmatrix}$

b. $AB=\begin{bmatrix}-5 & 5\\10 & 10\\1 & 16\\5 & 5\end{bmatrix}$

c. $AB=\begin{bmatrix}-5 & 5\\10 & 10\\2 & 16\\5 & 5\end{bmatrix}$

d. none

PROBLEM 6

For matrix $A=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$, find A^2, A^3, A^4. The general formula for A^n for any positive integer n, is:

a. $A^n=\begin{bmatrix}-n & 1\\ 0 & 1\end{bmatrix}$
b. $A^n=\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$
c. $A^n=\begin{bmatrix}0 & 1\\ 1 & n\end{bmatrix}$
d. $A^n=\begin{bmatrix}1 & -n\\ 0 & 1\end{bmatrix}$
e. none

PROBLEM 7

Let $S=\{1,2,3\}$. Write all possible partitions of S.

a. {1}, {2}, {3} and {1, 2}, {3} and {1, 3}, {2} and {2, 3}, {1} and {1, 2, 3}
b. {1}, {2}, {3} and {1, 2}, {1, 3}, {2, 3} and {1, 2, 3}
c. {1, 2}, {1, 3}, {2, 3} and {2, 3}, {1} and {1, 2, 3}
d. none
e. {1}, {2}, {3}

PROBLEM 8

A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?

a. 240/1000
b. 240/1001
c. 24/1001
d. 2/5
e. 3/2

PROBLEM 9

If n people are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year? How large need n be so that this probability is less than $1/2$?

a. probability of birthday on the same day of the year 1/366; n = 252
b. probability of birthday on the same day of the year 1/365; n = 252
c. probability of birthday on the same day of the year 1/366; n = 253
d. probability of birthday on the same day of the year 1/365; n = 253
e. none

PROBLEM 10

A coin is flipped twice. Assuming that all four points in the sample space S = {(h, h), (h, t),(t, h),(t, t)} are equally likely, what is the conditional probability that both flips land on heads, given that (a) the first flip lands on heads? (b) at least one flip lands on heads?

a. for (a) 1/2, for (b) 1/3
b. for (a) 1/3, for (b) 1/2
c. for (a) 1, for (b) 1
d. for (a) 1/2, for (b) 1/2
e. none

PROBLEM 11

Suppose that we have 3 cards that are identical in form, except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

a. 1/2
b. 1/3
c. 6/31
d. 6/13
e. none

PROBLEM 12

A school class of 120 students is driven in 3 buses to a symphonic performance. There are 36 students in one of the buses, 40 in another, and 44 in the third bus. When the buses arrive, one of the 120 students is randomly chosen. Let X denote the number of students on the bus of that randomly chosen student, and find E[X] (expected value).

a. 40.2667
b. 40.2688
c. 40.2666
d. 38.2212
e. 399

PROBLEM 13

Calculate $Var(X)$ if X represents the outcome when a fair die is rolled. $E[X]$ is given as to be equal to 7/2.

a. 35/12
b. 35/24
c. 12/35
d. 24/35
e. none

PROBLEM 14

Find the expected value of the sum obtained when n fair dice are rolled.

a. 3.5
b. 35n

c. 3.5n
d. 35
e. 3n

PROBLEM 15

For three events A, B, and C, we know that: A and C are independent, B and C are independent, A and B are disjoint, P(A∪C)=2/3, P(B∪C)=3/4, P(A∪B∪C)=11/12. Find P(A), P(B) and P (C).

a. P(A) = 1/2, P(B) = 1/2 and P(C) = 1/2
b. P(A) = 1/3, P(B) = 1/3 and P(C) = 1/2
c. P(A) = 1/3, P(B) = 1/2 and P(C) = 1/3
d. P(A) = 1/3, P(B) = 1/2 and P(C) = 1/2
e. none

PROBLEM 16

A disease test is advertised as being 99% accurate: if you have the disease, you will test positive 99% of the time, and if you don't have the disease, you will test negative 99% of the time.

If 1% of all people have this disease and you test positive, what is the probability that you actually have the disease?

a. 1
b. 50
c. 75
d. 99
e. none

PROBLEM 17

Suppose that X and Z are zero-mean jointly normal random variables, such that $\sigma_X^2 = 4$, $\sigma_Z^2 = 17/9$

and E[XZ] = 2. We define a new random variable Y = 2X − 3Z. Determine the PDF of Y , the conditional PDF of X given Y, and the joint PDF of X and Y.

a. $f_y(y)=\frac{1}{\sqrt{2\pi}}e^{-y^2/18}$, $\frac{x \vee y}{f_{X \vee Y}¿}$) = $\frac{3}{\sqrt{2\pi}\sqrt{32}}e^{\frac{-(x-(2y/9))^2}{2\cdot 32/9}}$, $\frac{x \vee y}{f_{X,Y}¿}$) = $\frac{1}{2\pi\sqrt{32}}e^{\frac{-\frac{y^2}{9}+\frac{x^2}{4}-\frac{2}{3}\cdot\frac{xy}{2\cdot 3}}{2(1-(1/9))}}$

b. $f_y(y)=\frac{1}{\sqrt{2\pi}3}e^{-y^2/18}$, $\frac{x \vee y}{f_{X \vee Y}¿}$) = $\frac{3}{\sqrt{2\pi}}e^{\frac{-(x-(2y/9))^2}{2\cdot 32/9}}$, $\frac{x \vee y}{f_{X,Y}¿}$) = $\frac{1}{2\pi\sqrt{32}}e^{\frac{-\frac{y^2}{9}+\frac{x^2}{4}-\frac{2}{3}\cdot\frac{xy}{2\cdot 3}}{2(1-(1/9))}}$

c. $f_y(y)=\frac{1}{\sqrt{2\pi}3}e^{-y^2/18}$, $\frac{x \vee y}{f_{X \vee Y}¿}$) = $\frac{3}{\sqrt{2\pi}\sqrt{32}}e^{\frac{-(x-(2y/9))^2}{2\cdot 32/9}}$, $\frac{x \vee y}{f_{X,Y}¿}$) = $\frac{1}{\sqrt{32}}e^{\frac{-\frac{y^2}{9}+\frac{x^2}{4}-\frac{2}{3}\cdot\frac{xy}{2\cdot 3}}{2(1-(1/9))}}$

d. $f_y(y)=\frac{1}{\sqrt{2\pi}3}e^{-y^2/18}$, $\frac{x \vee y}{f_{X \vee Y}¿}$) = $\frac{3}{\sqrt{2\pi}\sqrt{32}}e^{\frac{-(x-(2y/9))^2}{2\cdot 32/9}}$, $\frac{x \vee y}{f_{X,Y}¿}$) = $\frac{1}{2\pi\sqrt{32}}e^{\frac{-\frac{y^2}{9}+\frac{x^2}{4}-\frac{2}{3}\cdot\frac{xy}{2\cdot 3}}{2(1-(1/9))}}$

e. none

`

Northeastern University

Math Exam
August 29th, 2019

Linear Algebra and Probabilities