DS5020 - Introduction to Linear Algebra and Probability for Data Science

Fall 2017

Assignment 1

(**Due:** Monday, 25 Sept 2017, 11:59pm)

Note 1: Solutions should be submitted on Blackboard. It is **NOT** mandatory to type your solutions; scanned hand-written submissions are fine. You are responsible for making sure your handwriting is readable.
Note 2: You can make multiple submissions for a particular assignment. Your **LAST** submission will be graded.
Note 3: Bold numbers in curly brackets show available points for each problem.

1. **{10 pts}** Suppose $\mathbf{u} = \begin{bmatrix}1\\2\end{bmatrix}$, $\mathbf{v} = \begin{bmatrix}2\\-1\end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix}1\\7\end{bmatrix}$.
   - **a.** Find the linear combination $3\mathbf{u} + 2\mathbf{v}$.
   - **b.** Find the linear combination $\mathbf{u} - \mathbf{v}$.
   - **c.** Find two scalars c and d that satisfy $c\mathbf{u} + d\mathbf{v} = \mathbf{w}$.
   - **d.** Describe geometrically (line, plane etc.) all linear combinations of $\mathbf{u}$ and $\mathbf{v}$.

2. **{10 pts}** Suppose $\mathbf{u} = \begin{bmatrix}2\\1\end{bmatrix}$, $\mathbf{v} = \begin{bmatrix}-1\\1\end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix}0\\3\end{bmatrix}$.
   - **a.** Draw vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ on a plane where each axis represents one component of the vectors.
   - **b.** Find two scalars c and d that satisfy $c\mathbf{u} + d\mathbf{v} = \mathbf{w}$.
   - **c.** Draw $\mathbf{u} + 2\mathbf{v}$ on the same plane you drew on part **a**. Show that this linear combination of $\mathbf{u}$ and $\mathbf{v}$ is equal to $\mathbf{w}$.

3. **{20 pts}** Suppose $\mathbf{u} = \begin{bmatrix}2\\-1\end{bmatrix}$, $\mathbf{v} = \begin{bmatrix}1\\2\end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix}2\\4\end{bmatrix}$.
   - **a.** Find the dot product of vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \cdot \mathbf{v}$.
   - **b.** Are vectors $\mathbf{u}$ and $\mathbf{v}$ perpendicular? What is the angle between them?
   - **c.** Find the dot product of vectors $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{v} \cdot \mathbf{w}$.
   - **d.** Using the cosine formula, find the angle between vectors $\mathbf{v}$ and $\mathbf{w}$.
   - **e.** Find scalar c that satisfies $c\mathbf{v} = \mathbf{w}$.
   - **f.** Draw vectors $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ on a plane.

4. **{15 pts}** Suppose $\mathbf{A} = \begin{bmatrix}-1 & 2\\0 & 1\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}1\\2\end{bmatrix}$.
   - **a.** Describe the columns and rows of A.
   - **b.** Find vector $\mathbf{x} = \begin{bmatrix}x_1\\x_2\end{bmatrix}$ that satisfies $\mathbf{Ax} = \mathbf{b}$.
   - **c.** Write down the system of equations that the matrix form $\mathbf{Ax} = \mathbf{b}$ represents.

5. **{25 pts}** Suppose $\mathbf{E} = \begin{bmatrix}0 & 1 & 0\\-1 & 1 & 0\\0 & 0 & 1\end{bmatrix}$, $\mathbf{P} = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$, $\mathbf{A} = \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$, $\mathbf{x} = \begin{bmatrix}0\\1\\2\end{bmatrix}$, and $\mathbf{y} = [1\ 1\ 0]$.
   - **a.** Compute $\mathbf{yA}$. Describe in words what the result is in terms of rows of $\mathbf{A}$.
   - **b.** Compute $\mathbf{Ax}$. Describe in words what the result is in terms of columns of $\mathbf{A}$.
   - **c.** Compute $\mathbf{EA}$. Describe in words what this operation does to rows of $\mathbf{A}$.
   - **d.** Compute $\mathbf{PA}$. Describe in words what this operation does to rows of $\mathbf{A}$.
   - **e.** Compute $\mathbf{AP}$. Describe in words what this operation does to columns of $\mathbf{A}$. Is the result the same as the result in part **d**?

6. **{20 pts}** Suppose $\mathbf{A} = \begin{bmatrix}1 & 2 & 1\\2 & 5 & 4\\1 & 5 & 3\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}0\\1\\-1\end{bmatrix}$.
   - **a.** Using elimination, convert matrix $\mathbf{A} = \begin{bmatrix}1 & 2 & 1\\2 & 5 & 4\\1 & 5 & 3\end{bmatrix}$ to upper triangular matrix $\mathbf{U} = \begin{bmatrix}? & ? & ?\\0 & ? & ?\\0 & 0 & ?\end{bmatrix}$.
   - **b.** Find vector $\mathbf{x} = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ such that $\mathbf{Ax} = \mathbf{b}$.