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Lab 4 : LINEAR REGRESSION USING GRADIENT DESCENT

Preamble

In this lab assignment, first, we will learn about the linear regression and loss function. Second, check how to optimizes loss function for the given linear regression problem using Gradient Descent and implement it from the scratch in Scilab/Python.

Linear Regression:

It is a supervised machine learning algorithm which learns mapping between input X and output Y i.e., it predicts Y for the given X. With simple linear regression the data is modelled as follows:

$$\mathbf{H:= Y = aX + b}$$

where **'a'** is the slope of the line and **'b'** is the intercept. The goal of linear regression is to fit a line to a set of points.

Exercise I Consider X = [1, 2, 3, 4, 5] and Y = [10, 40, 50, 78, 83] , where X is the object size and Y is the corresponding price. Determine the value of **a** and **b**, such that the line corresponding to those values is the **best fit**. Plot the best fit line for the data points.

Loss Function:

The loss function quantifies our unhappiness with the predictor. It quantifies the error in predicted value of **a** and **b**. The goal is to minimize this error and obtain the most accurate value of **a** and **b**. Most commonly, Mean Squared Error loss function is used to measure loss for linear regression problem.

$$E = \frac{1}{2m}\sum_{i=1}^{m}(H(x_i) - y_i)^2$$

where, x_i is the input, y_i is the ground truth label, $H(x_i)$ is the Hypothesis.

Exercise II Evaluate the minimum value of cost function manually for the linear model $H_i = aX_i + 0$ (i.e., with different values of **a**). Plot the cost function with respect to a.

In Exercise II, we manually find the minimum cost function,. However, we need an optimization algorithm to do it automatically.

Gradient Descent Algorithm:

Gradient descent is an iterative optimization algorithm to find the minimum of a loss function. The equation of gradient descent is as given below:

$$W_{n+1} = W_n - \eta \frac{\partial E}{\partial W}; W = [a\ b]';$$

where, η is the learning rate.

Exercise III Evaluate the minimum value of cost function for the linear model $H_i = aX_i + b$ using Gradient Descent Algorithm. Plot the cost function.

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