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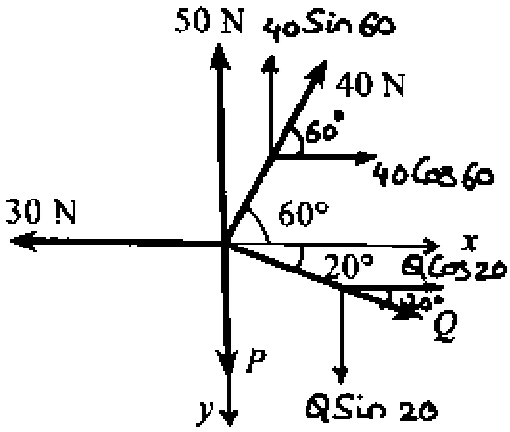
Engineering Mechanics D'23 Solution

1a. Find forces P and Q such that resultant of given system in fig is zero. [M'13,1a,4M] [D'09], D'23, 1a, 5M 5M



(i) $R_x = \Sigma F_x$

$\because R = 0, \quad R_x = R_y = 0$

$\Sigma F_x = 0$

$+40 \cos 60 - 30 + Q \cos 20 = 0$

$Q = 10.64 \text{ N}$

$\Sigma F_y = 0$

$+40 \sin 60 + 50 - \overset{?}{P} - 10.64 \sin 20 = 0$ Coplanar, Concurrent

$\underline{P = 81 \text{ N}} \downarrow$

1b **Define ICR and write the properties of an ICR. [Dec'23, 1b, 5M]** 5M

ICR

- In GPM, which is combined motion of rotation & translation, it is possible to locate an imaginary point about which a body may be assumed to have pure rotational motion without performing any motion of translation.
- The imaginary point about which the body is assumed to have pure rotational motion is called as ICR which is denoted by I.

Properties of ICR

- Linear velocity of ICR is zero hence it is also called 'Instantaneous centre of zero velocity'.
- ICR is an imaginary point.
- ICR may lie inside or outside body.
- Position of ICR is changing at every instant & it is not a fixed point.
- The locus of ICR during the motion is known as centrode.

1 C 5 M

A particle, starting from rest, moves in a straight line and its acceleration is given by $a = 50 - 36t^2$ m/s^2. Determine the velocity of the particle when it has travelled 52 m and the time taken by it before it comes to rest again.

Solution

(i) $a = 50 - 36t^2$ ..... (I)

$$\frac{dv}{dt} = 50 - 36t^2 \Rightarrow dv = (50 - 36t^2)\,dt$$

Integrating both sides, we get

$$\int dv = \int (50 - 36t^2)\,dt$$

$$v = 50t - \frac{36t^3}{3} + c_1$$

At $t = 0,\ v = 0 \quad \therefore c_1 = 0$

$$v = 50t - 12t^3 \qquad \text{..... (II)}$$

$$\frac{dx}{dt} = 50t - 12t^3 \Rightarrow dx = (50t - 12t^3)\,dt$$

Integrating both sides, we get

$$\int dx = \int (50t - 12t^3)\,dt$$

$$x = \frac{50t^2}{2} - \frac{12t^4}{4} + c_2$$

Assuming particle starts from origin,

At $t = 0,\ x = 0 \quad \therefore c_2 = 0$

$$x = 25t^2 - 3t^4 \qquad \text{..... (III)}$$

(ii) Putting $x = 52$ m in Eq. (III),

$$52 = t^2(25 - 3t^2)$$

$$3(t^2)^2 - 25t^2 + 52 = 0$$

$t^2 = 4$ s and $t^2 = 4.33$ s

$\therefore t = 2$ s or $t = 2.08$ s

To check whether particle changes its direction of motion, put $v = 0$ in Eq. (II)

$\therefore t = 2.04$ s

The particle is moving in same direction till $t = 2.04$ s

$\therefore$ Displacement = Distance travelled

After $t > 2.04$ s, distance travelled is more than magnitude of displacement

$\because x = 52$ m is the distance travelled

$\therefore t = 2$ s ($t \neq 2.08$ s)

From Eq. (II), $v = 4$ m/s.

1d **Define angle of repose and prove that angle of friction = angle of repose. [Dec'23,1d,5M]** 5M

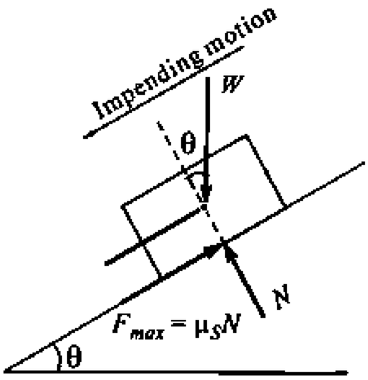
Angle of repose θ

It is minimum angle of inclination of a plane with the horizontal at which the body kept will just slide down on it without the application of any external force (due to self weight).

Consider F.B.D. with θ being angle of repose,

For limiting equilibrium condition we have,

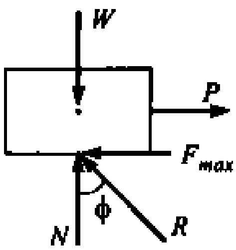
$$\Sigma F_x = 0$$
$$\mu_s N - W \sin\theta = 0$$
$$W \sin\theta = \mu_s N \text{.............}(1)$$
$$\Sigma F_y = 0$$
$$N - W \cos\theta = 0$$
$$W \cos\theta = N \text{................}(2)$$
$$\textit{Dividing eq } (1) \textit{ by } (2)$$
$$\tan\theta = \mu_s$$
$$\theta = \tan^{-1} \mu_s$$



Angle of friction ϕ

It is the angle made by the resultant of the limiting frictional force Fmax and the normal reaction N with the normal reactions.

$$F_{max} = R \sin\phi$$
$$\mu_s N = R \sin\phi \text{..........}(1)$$
$$N = R \cos\phi \text{.............}(2)$$
$$\mu_s R \cos\phi = R \sin\phi$$
$$\tan\phi = \mu_s$$
$$\phi = \tan^{-1} \mu_s$$

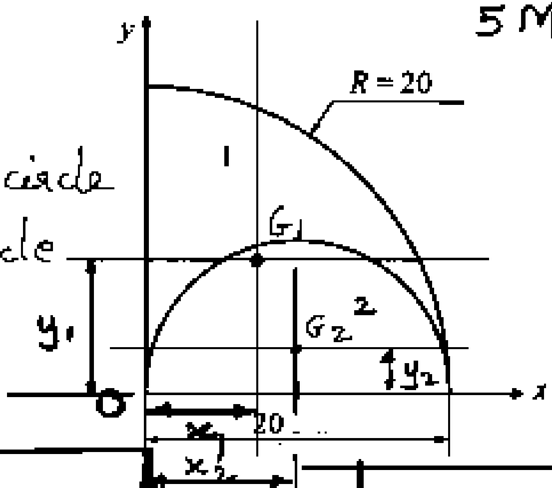


Angle of friction ϕ = angle of repose θ

e Find Centroid of Shaded area

5 M

$R_1 = 20$ units for quarter circle
$R_2 = 10$ units for semi circle



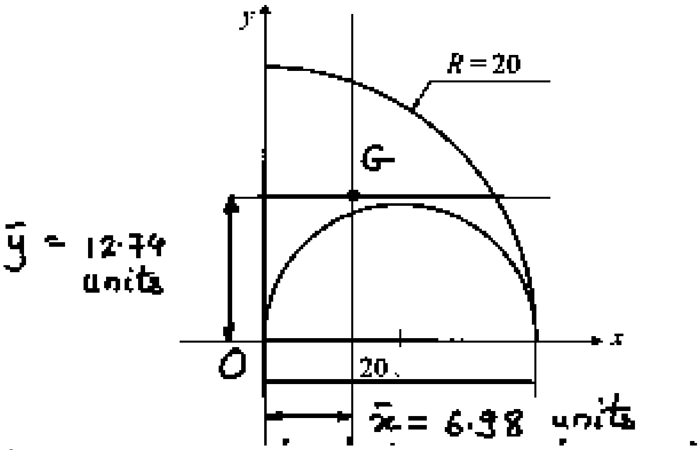
shape	Area A_i	x_i	$A_i x_i$	y_i	$A_i y_i$
1.	$\frac{\pi \times 20^2}{4} = 100\pi$	$\frac{4 \times 20}{3\pi}$	2667.22	$\frac{4 \times 20}{3\pi}$	2667.22
2. cut	$-\frac{\pi \times 10^2}{2}$	10	-500π	$\frac{4 \times 10}{3\pi}$	666.67
	$\Sigma A_i = 50\pi$		$\Sigma A_i x_i = 1096.42$		$\Sigma A_i y_i = 2001.2$

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{1096.42}{50\pi}$$

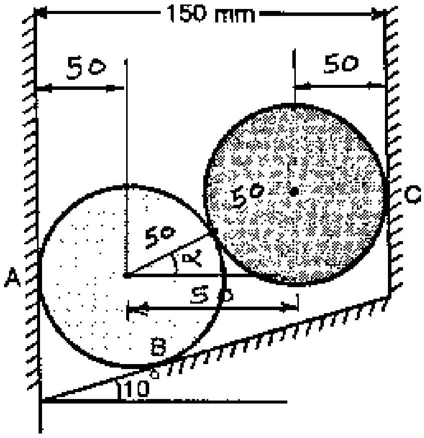
$$\boxed{\bar{x} = 6.98 \text{ units}}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{2001.2}{50\pi}$$

$$\boxed{\bar{y} = 12.74 \text{ units}}$$

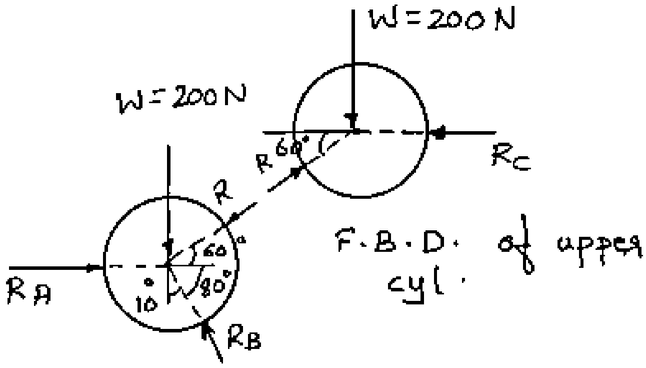


2a Two identical cylinders of diameter 100 mm and each weighing 200 N are placed as shown in fig. All the contact surfaces are smooth. Find the reactions at A, B and C. 8M



$\alpha = \cos^{-1}\left(\frac{50}{100}\right) = 60^\circ$

Diag in qt not drawn to scale



F.B.D. of lower cyl.

Consider F.B.D. of Upper cylinder,
Apply Lami's Theorem,

$$\frac{200}{\sin 120} = \frac{R_c}{\sin 150} = \frac{R}{\sin 90}$$

$R_c = 115.47\ N\ (\leftarrow)$

$R = 230.94\ N$

Consider F.B.D. of lower cylinder,

$\Sigma F_y = 0$

$-200 - 230.94 \sin 60 + R_B \cos 10 = 0$

$R_B = 406.17\ N$ 80°

$\Sigma F_x = 0$

$+R_A - 230.94 \cos 60 - 406.17 \sin 10 = 0$

$R_A = 186\ N\ (\rightarrow)$

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2. (b) Replace the system of forces and couples shown in Fig. 2(a) by a single force and locate the point on the *x*-axis through which the line of action of the resultant passes. **[06]**

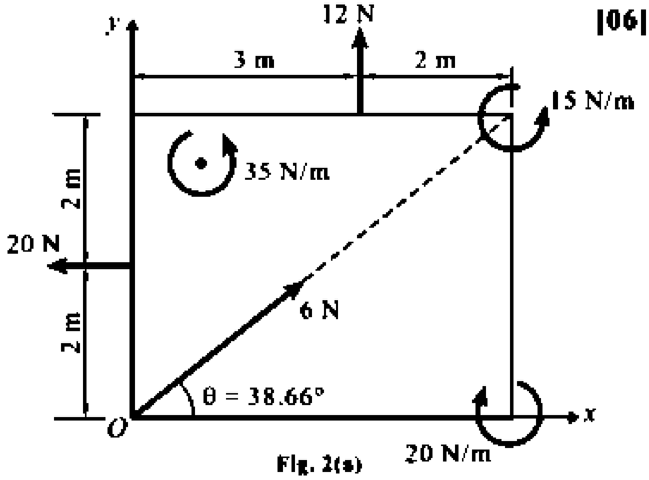


Fig. 2(a)

Solution

$$\theta = \tan^{-1}\left(\frac{4}{5}\right) \quad \therefore \ \theta = 38.66^{\circ}$$

(i) $\Sigma F_x = -20 + 6\cos 38.66^{\circ}$

$= -15.31 \text{ N} = 15.31 \text{ N} (\leftarrow)$

(ii) $\Sigma F_y = 12 + 6\sin 38.66^{\circ}$

$= 15.74 \text{ N} (\uparrow)$

(iii) $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(15.31)^2 + (15.74)^2} = 21.95 \text{ N}$ ***Ans.***

(iv) $\phi = \tan^{-1}\left(\frac{15.74}{15.31}\right) = 45.79^{\circ}$ ***Ans.***

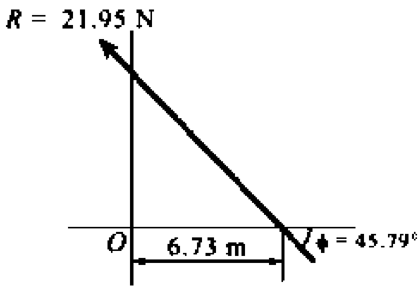
(v) $\Sigma M_O = -20 + 15 + 35 + 20 \times 2 + 12 \times 3$

$= 106 \text{ N.m} (\circlearrowleft)$

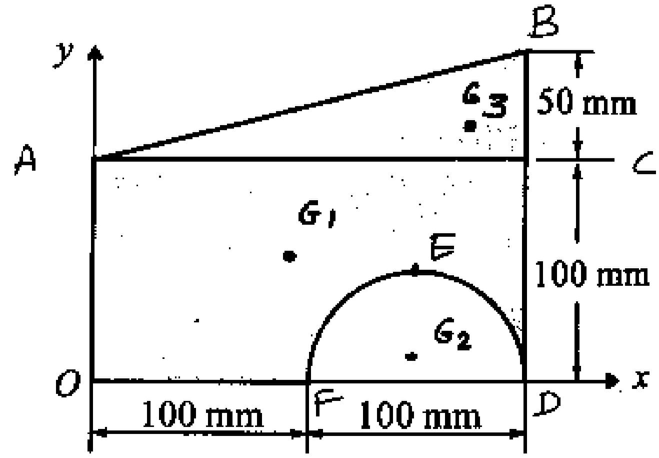
(vi) By Varignon's theorem, we have

$$x = \left(\frac{\Sigma M_O}{\Sigma F_y}\right) = \frac{106}{15.74} = 6.73 \text{ m} \quad \textbf{\textit{Ans.}}$$

(vii) Position of R w.r.t. point O



D'23, 2C, 6M



Shape	Area (mm²)	x_i (mm)	$A_i \cdot x_i$	y_i (mm)	$A_i \cdot y_i$
OACD	200×100	100	2×10^6	50	1×10^6
ABC	$\frac{1}{2} \times 200 \times 50 = 5000$	$\frac{2}{3} \times 200$ $= 133.33$	666666.66	$100 + \frac{50}{3}$ $= 116.67$	583333.33
DEF cut	$-\frac{\pi \times 50^2}{2} = -1250\pi$	$100 + 50$ $= 150$	-187500π	$= \frac{4 \times 50}{3\pi}$	-83333.33
Σ	21073.009		2077618.037		1.5×10^6

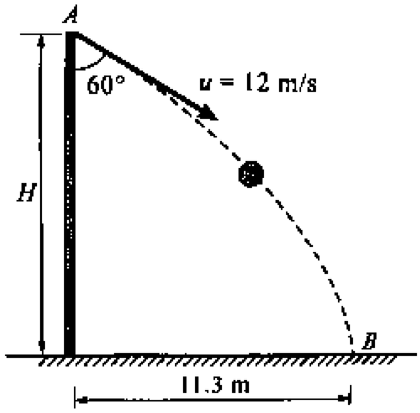
$$\bar{x} = \frac{\Sigma A_i \cdot x_i}{\Sigma A_i} = \frac{2077618.037}{21073.009}$$

$$\bar{x} = 98.59 \text{ mm}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{1.5 \times 10^6}{21073.009}$$

$$\bar{y} = 71.18 \text{ mm}$$

3 a A ball thrown with a speed of 12 m/s at an angle of 60° with a building strikes the ground 11.3 m horizontally from the foot of the building as shown in fig. Determine the height of the building. **[D12, 4(b), 6M] [Feb'23, 3c, 4M]** 8 M



By general equation of projectile motion,

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta) = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$-H = 11.3\tan(-30) - \frac{9.81\times 11.3^2}{2\times 12^2}\left(1+\tan^2(-30)\right)$$

$$H = 12.3233 \text{ m}$$

OR

$u_x = 12\sin 60 = 10.39$ m/s

$$u_x = \frac{x}{t}$$

$$10.39 = \frac{11.3}{t} \Rightarrow t = 1.087 \text{ sec}$$

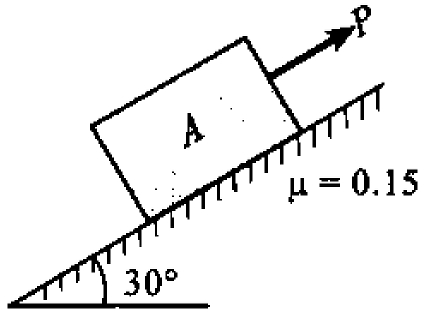
$u_y = -12\cos 60$, $s = ut + \frac{1}{2}at^2$

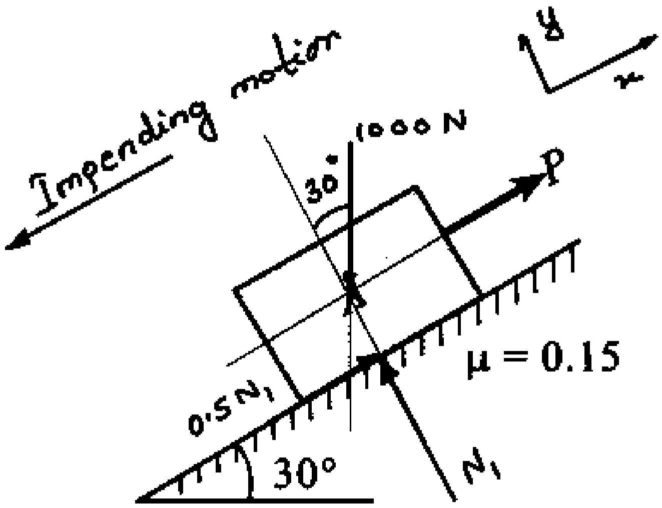
$$-H = -12\cos 60\times 1.087 - \frac{1}{2}\times 9.81\times 1.087^2$$

$$H = 12.32 \text{ m}$$

3b 6M

A block of weight 1000 N is kept on an inclined surface (30 degrees to horizontal). Determine the force required to prevent the siding of the block down the plane if the coefficient of friction between the block and the surface is 0.15. **[D'23,3b,6M]**





$\Sigma F_y = 0$

$+ N_1 - 1000 \cos 30 = 0$

$N_1 = 866.0254\ N$

$\Sigma F_x = 0$

$+ 0.15 \times N_1 + P - 1000 \sin 30 = 0$

$\underline{P = 370.09\ N}$

3 c 6 M

A particle moves along a track which has a parabolic shape with a constant speed of 10 m/sec. The curve is given by $y = 5 + 0.3x^2$. Find the components of velocity and normal acceleration when x = 2 m. **[D'14,6c,4M] [D'23,3c,6M]**

D'14
6c
4m

$y = 5 + 0.3x^2$, $v = 10$ m/s (constant)

$\therefore a_t = 0$, $a_n = a = \frac{v^2}{\rho}$

$\frac{dy}{dx} = 0.6x$ $\quad \left(\frac{dy}{dx}\right)_{x=2} = 1.2$

$\frac{d^2y}{dx^2} = 0.6$

$\tan\theta = \frac{dy}{dx} = 1.2 \quad \therefore \theta = 50.19°$

$v_x = v\cos\theta = 10\cos 50.19 = \underline{\underline{6.4 \text{ m/s}}}$

$v_y = v\sin\theta = 10\sin 50.19 = \underline{\underline{7.68 \text{ m/s}}}$

$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right|$$

$$= \frac{\left[1 + (1.2)^2\right]^{3/2}}{0.6}$$

$\rho = 6.352$ m

$a_n = \frac{v^2}{\rho} = \frac{10^2}{6.352}$

$a_n = a = \underline{\underline{15.74 \text{ m/s}^2}}$

4 a A 20 N block is released from rest. It slides down the inclined having $\mu = 0.2$ as shown in Fig. 14.16(a). Determine the maximum compression of the spring and the distance moved by the block when the energy is released from compressed spring. Springs constant $k = 1000$ N/m. 8M

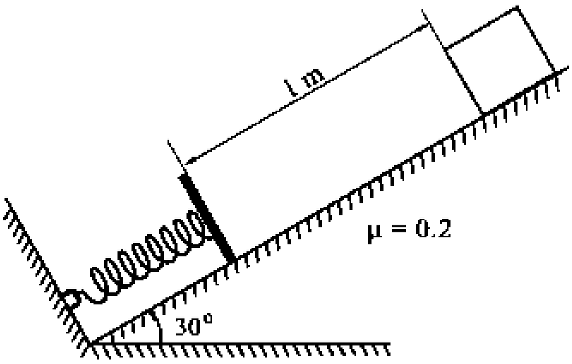


Fig. 14.16(a)

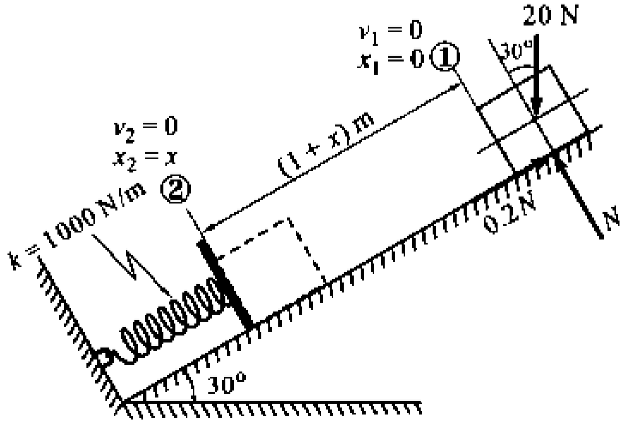


Fig. 14.16(b)

Solution

Part (i) Maximum compression of the spring

Let x be the maximum deformation of spring at position ② where the block comes to rest ($v_2 = 0$).

By work - energy principle, we have

Work done = Change in kinetic energy

$$\frac{1}{2} \times 1000(0^2 - x^2) + 20 \sin 30^\circ (1 + x) - 0.2 \times 20 \cos 30^\circ (1 + x) = 0 - 0$$

$\therefore\ x = 0.121$ m

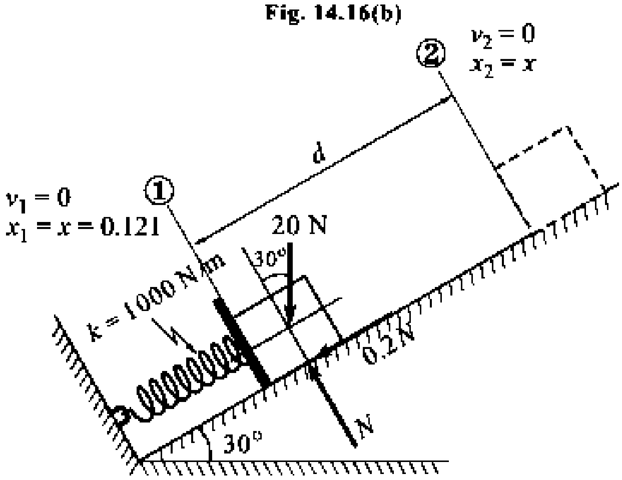


Fig. 14.16(c)

Part (ii) Distance moved by the block

By work - energy principle, we have

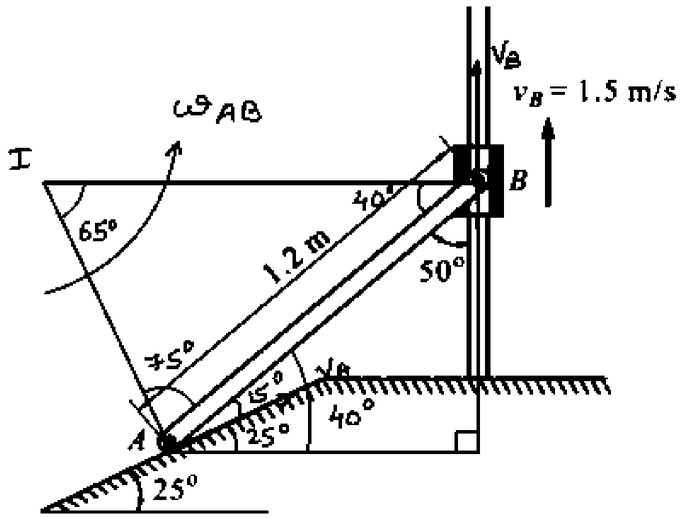
Work done = Change in kinetic energy

$$\frac{1}{2} \times 1000(0.121^2 - 0^2) - 20 \sin 30^\circ \times d - 0.2 \times 20 \cos 30^\circ \times d = 0 - 0$$

$\therefore\ d = 0.5437$ m

4b 6 M

For the system shown in fig. if the collar is moving upwards with a velocity of 1.5 m/s. Locate the ICR for the instant shown. Determine the angular velocity of rod AB, Velocity of A



In ΔIAB, using Sine rule,

$$\frac{1.2}{\sin 65} = \frac{IA}{\sin 40} = \frac{IB}{\sin 75}$$

$IA = 0.851$ m

$IB = 1.2789$ m

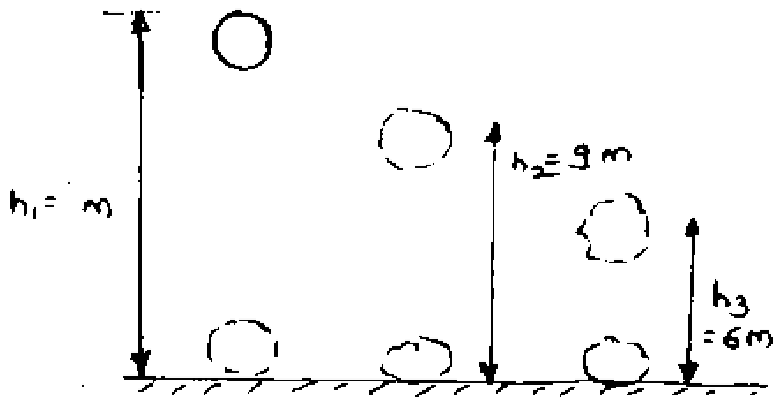
$V_B = IB \times \omega_{AB}$

$\omega_{AB} = \frac{1.5}{1.2789}$ $\therefore$ $\omega_{AB} = 1.1728$ rad/s ↺

$V_A = IA \times \omega_{AB} = 0.851 \times 1.1728$ $\therefore$ $V_A = 0.998$ m/s $\measuredangle 25°$

4C 6M

A glass ball is dropped onto a smooth horizontal floor from which it bounces to a height of 9 m. On the second bounce it rises to a height of 6 m. From what height the ball was dropped and what is the coefficient of restitution between the glass and the floor. **[D'16,1e,4M] [D'23,4c,6M]**



$$e = \sqrt{\frac{h_2}{h_1}} \quad \therefore e = \sqrt{\frac{9}{h_1}}$$

$$e^2 = \frac{9}{h_1}$$

$$e = \sqrt{\frac{h_3}{h_2}} = \sqrt{\frac{6}{9}}$$

$$\boxed{e = 0.8164}$$

$$e^2 = \frac{9}{h_1} \quad \therefore h_1 = \frac{9}{0.8164^2}$$

$$\boxed{h_1 = 13.5\ m}$$

5a 8M

The a-t curve is shown for a particle moving in a straight line. Show v-t and s-t diagram for 0-4 sec, if particle has started from rest from origin. **[D'23,5a,8M]**

$t=0,\ s=0$ (given)

v-t :

change in velocity = area under a-t diag

$t = 2\,sec,\quad v_2 - \cancel{v_0}^{\,0} = \frac{1}{2} \times 2 \times 3$

$v_2 = 3\ m/s$

$t = 3\,sec,\quad v_3 - v_2 = \frac{1}{2} \times 1 \times 1$

$v_3 = 0.5 + 3 = 3.5\ m/s$

$t = 4\,sec,\quad v_4 - v_3 = 1 \times 1$

$v_4 = 4.5\ m/s$

s-t :

change in displacement = area under v-t diagram

at $t = 2\,sec,\quad s_2 - \cancel{s_0}^{\,0} = \frac{2}{3} \times 2 \times 3$

$s_2 = 4\ m$

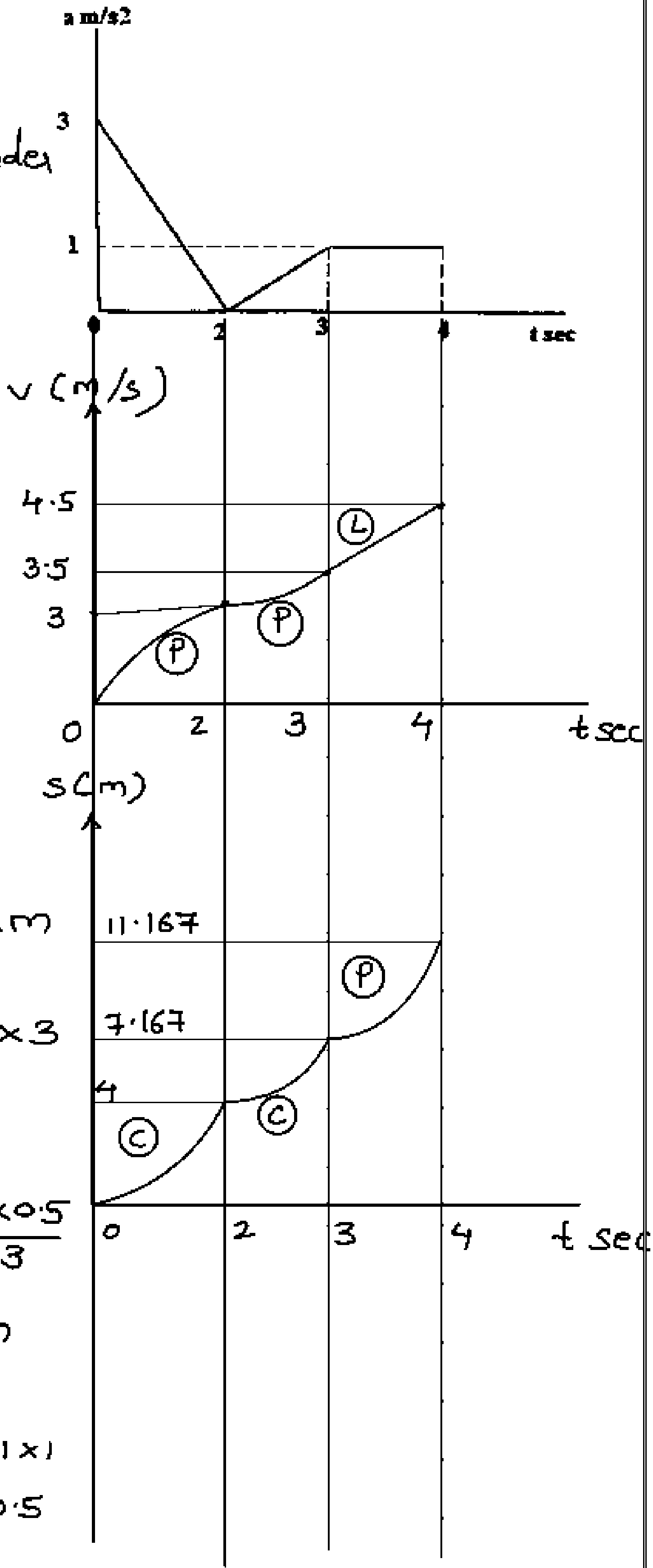
at $t = 3\,sec,\quad s_3 - s_2 = 1 \times 3 + \frac{1 \times 0.5}{3}$

$s_3 = 4 + 3 + \frac{0.5}{3} = 7.167\ m$

at $t = 4\,sec,\quad s_4 - s_3 = 1 \times 3.5 + \frac{1}{2} \times 1 \times 1$

$s_4 = 7.167 + 3.5 + 0.5$

$s_4 = 11.167\ m$



5b **Block *A* and *B* of mass 6 kg and 12 kg respectively are connected by a string passing over a smooth pulley. Neglect mass of pulley. If coefficient of kinetic friction between the block *A* and the inclined surface is 0.2, determine the acceleration of block *A* and block *B*.** 6M

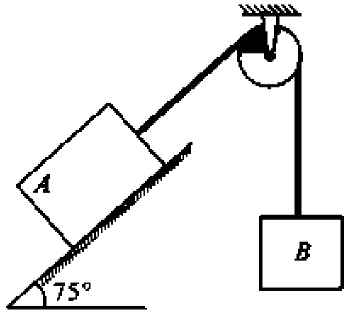


Fig. 6(d)

Solution

(i) Consider the F.B.D. of block *A*

$\Sigma F_y = ma_y = 0$

$N - 6 \times 9.81 \cos 75^\circ = 0$

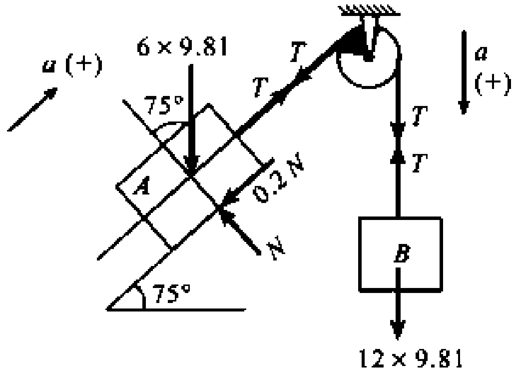
$N = 15.234$ N

By Newton's second law

$\Sigma F_x = ma_x$

$T - 0.2\,N - 6 \times 9.81 \sin 75^\circ = 6a$

$T - 6a = 59.90 \qquad \ldots \text{(i)}$



(ii) Consider the F.B.D. of block *B*

$\Sigma F_y = ma_y$

$12 \times 9.81 - T = 12a$

$-T - 12a = -117.72 \qquad \ldots \text{(ii)}$

Solving equations (i) and (ii)

~~*T* = 8.64 N *Ans.*~~ T = 79.173 N

~~$a = 6.54$ m/s² ($a = a_A = a_B$) *Ans.*~~ a = 3.21 m/s²

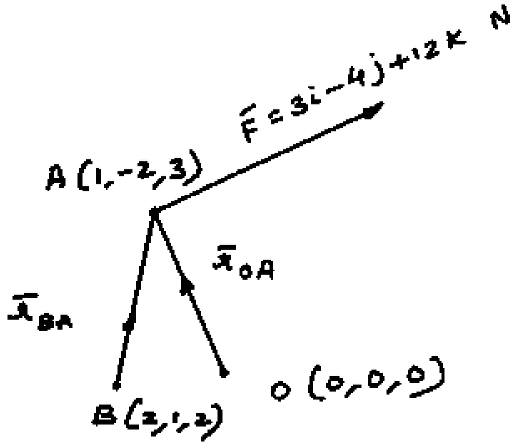
●●●

5C Force $\bar{F} = (3i - 4j + 12k)$ N acts at a point A (1,-2,3). Find: 6 M

(i) Moment of force about origin

(ii) Moment of force about point B (2,1,2) m.

[M'13,6a,4M]



Force vector

$\bar{F} = 3i - 4j + 12\,k$

Position vector

(i) $\bar{r}_{OA} = 1i - 2j + 3k$

(ii) $\bar{r}_{BA} = (1-2)\,i + (-2-1)\,j + (3-2)\,k$

$= -i - 3j + k$

Moment vector

(i) $\bar{M}_O = \bar{r}_{OA} \times \bar{F} = \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 3 & -4 & 12 \end{vmatrix}$

$= (-24+12)\,i - (12-9)\,j + (-4+6)\,k$

$\bar{M}_O = -12i - 3j + 2k$ N·m

(ii) $\bar{M}_B = \bar{r}_{BA} \times \bar{F} = \begin{vmatrix} i & j & k \\ -1 & -3 & 1 \\ 3 & -4 & 12 \end{vmatrix}$

$= +(-36+4)\,i - (-12-3)\,j + (4+9)\,k$

$\bar{M}_B = -32i + 15j + 13k$ N·m

6a **Find the reactions at supports B and F for the beam loaded as shown in Fig. 4(a).** **[08]** 8M

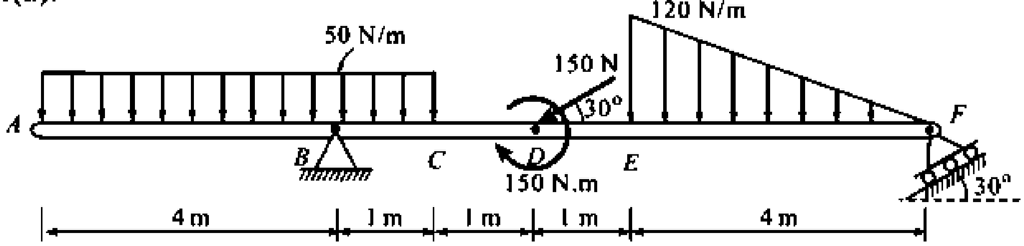
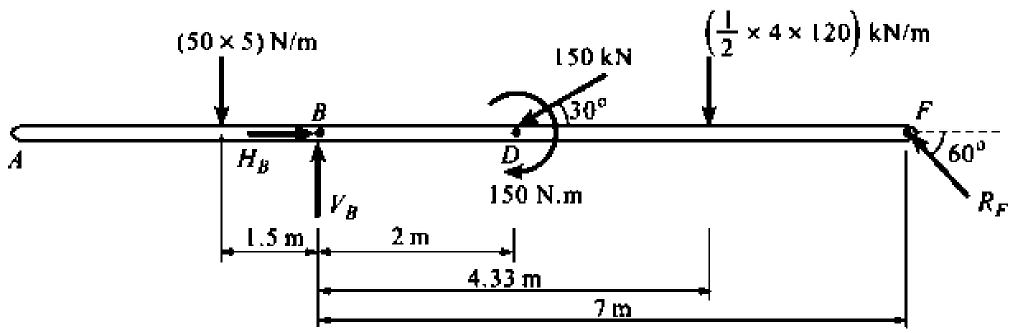


Fig. 4(a)

Solution



F.B.D. of beam ADF

(i) Consider the F.B.D. of the beam ADF with equivalent point load is shown in above figure.

(ii) $\Sigma M_B = 0$

$$50 \times 5 \times 1.5 - 150 - 150 \sin 30^\circ \times 2 - \frac{1}{2} \times 4 \times 120 \times 4.33 + R_F \sin 60^\circ \times 7 = 0$$

$R_F = 159.05$ kN (60°) ***Ans.***

(iii) $\Sigma F_x = 0$

$$H_B - 150 \cos 30^\circ - R_F \cos 60^\circ = 0$$

$H_B = 209.42$ N $(\rightarrow)$ ***Ans.***

(iv) $\Sigma F_y = 0$

$$V_B - 50 \times 5 - 150 \sin 30^\circ - \frac{1}{2} \times 4 \times 120 + R_F \sin 60^\circ = 0$$

$V_B = 427.25$ N $(\uparrow)$ ***Ans.***

(v) $\theta = \tan^{-1}\left(\frac{427.25}{209.42}\right)$

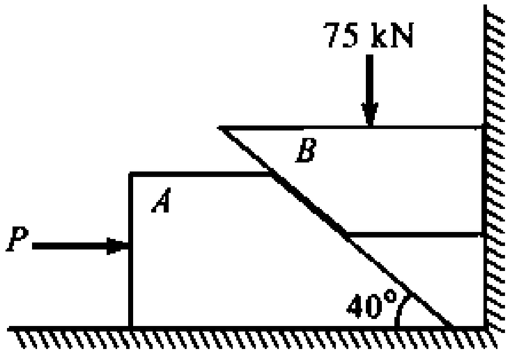
$\theta = 63.88^\circ$ ***Ans.***

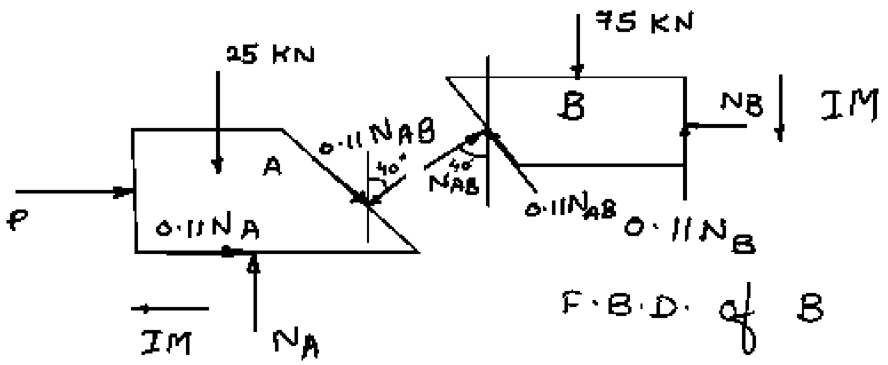
(vi) $R_B = \sqrt{(209.42)^2 + (427.25)^2}$

$R_B = 475.81$ N (63.88°) ***Ans.***

6b 6M

Block A weighs 25 kN and block B weighs 75 kN. Coefficient of friction for all contact surfaces is 0.11. Determine the value of "P" for holding the system in equilibrium. **[D'23,6b,6M]**





F.B.D. of A

Consider F.B.D. of B,

$\Sigma F_y = 0$

$0.11\, N_B - 75 + N_{AB} \cos 40$

$+ 0.11\, N_{AB} \cos 50 = 0$

$0.11\, N_B + 0.8367\, N_{AB} = 75 \quad -①$

$\Sigma F_x = 0$

$-N_B + N_{AB} \sin 40 - 0.11\, N_{AB} \sin 50$

$= 0$

$-N_B + 0.5585\, N_{AB} = 0 \quad -②$

Solving ① & ②,

$N_B = 46.64$ KN

$N_{AB} = 83.5064$ kN

Consider F.B.D. of A,

$\Sigma F_y = 0$

$+ N_A - 25 - N_{AB} \cos 40$

$- 0.11\, N_{AB} \cos 50 = 0$

$N_A = 94.874$ kN

$\Sigma F_x = 0$

$+ P + 0.11 \times 94.874$

$- N_{AB} \sin 40 + 0.11\, N_{AB}$

$\sin 50 = 0$

$P = 36.2$ KN

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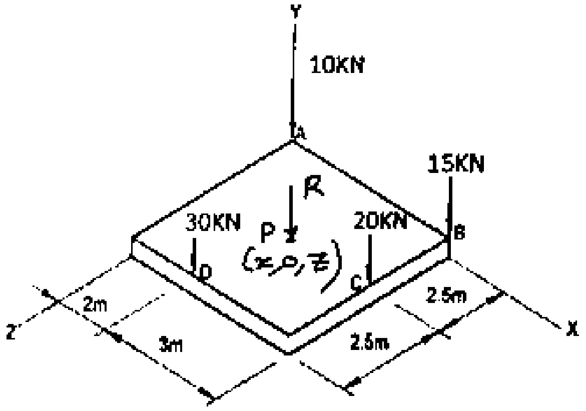
6 M

6 C Determine the resultant of the system of parallel forces and determine the position on X-Z plane. **[D'23,6c,6M]**



Co-ordinates

A(0,0,0) , B(5,0,0) , C(5,0,2.5) , D(2,0,5)

Force vectors

$\bar{F}_A = -10j$, $\bar{F}_B = -15j$, $\bar{F}_C = -20j$, $\bar{F}_D = -30j$

Position vector

$\bar{r}_A = 0$, $\bar{r}_B = \overline{AB} = 5i$, $\bar{r}_C = \overline{AC} = 5i + 2.5k$, $\bar{r}_D = \overline{AD} = 2i+5k$

Moment vector

$\bar{M}_A = 0$, $\bar{M}_B = \bar{r}_B \times \bar{F}_B = \begin{vmatrix} i & j & k \\ 5 & 0 & 0 \\ 0 & -15 & 0 \end{vmatrix} = -75k$

$\bar{M}_C = \bar{r}_C \times \bar{F}_C = \begin{vmatrix} i & j & k \\ 5 & 0 & 2.5 \\ 0 & -20 & 0 \end{vmatrix} = 50i - 100k$

$\bar{M}_D = \bar{r}_D \times \bar{F}_D = \begin{vmatrix} i & j & k \\ 2 & 0 & 5 \\ 0 & -30 & 0 \end{vmatrix} = 150i - 60k$

Resultant force vector

$$\bar{R} = \bar{F}_A + \bar{F}_B + \bar{F}_C + \bar{F}_D = -10j - 15j - 20j - 30j$$

$$\bar{R} = -75j \text{ kN}$$

Resultant moment vector

$$\Sigma \bar{M}_A = \bar{M}_A + \bar{M}_B + \bar{M}_C + \bar{M}_D = -75k + 50i - 100k + 150i - 60k$$

$$\Sigma \bar{M}_A = 200i - 235k \text{ kN.m}$$

$$\bar{M}_R = \bar{r}_{AP} \times \bar{R} = \begin{vmatrix} i & j & k \\ x & 0 & z \\ 0 & -75 & 0 \end{vmatrix} = 75z\,i - 75x\,k$$

Applying Varignon's Theorem,

$$\Sigma \bar{M}_A = \bar{M}_R$$

$$200i - 235k = 75z\,i - 75x\,k$$

Equating coefficients

$75z = 200$, $-75x = -235$

$z = 2.67$ m , $x = 3.13$ m

$\bar{R} = -75j$ kN acts at point P (3.13, 0, 2.67)