Advection eq: $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ in $\Omega = (0, 1)$

Multiplying with Test func. $v \in H^1(\Omega)$ & then integrating we get,

$$\int_\Omega v \frac{\partial u}{\partial t} dx + \int_\Omega v \frac{\partial u}{\partial x} dx = 0$$

$$\Rightarrow \int_\Omega v \frac{u^{n+1} - u^n}{\Delta t} dx + \int_\Omega v \frac{\partial u}{\partial x} dx = 0$$

$$\Rightarrow \int_\Omega v\, u^{n+1} dx + \Delta t \int_\Omega v \frac{\partial u}{\partial x} dx = \int_\Omega v\, u^n dx$$

$$\Rightarrow \underset{\text{(bilinear)}}{a} = \underset{\text{(linear)}}{L}$$

FEniCS:

```
a = inner(u, v)*dx
    + dt * inner(grad(u)[0], v) * dx
L = inner(u_n, v) * dx
```

(n+1) is current time step [u, v]

(n) is previous " " [u_n]