SWEs : $\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$ —— (1) & $\frac{\partial q}{\partial t} + gh\frac{\partial (h+z)}{\partial x} + gn^2 \frac{|q|\,q}{h^{7/3}} = 0$ —— (2)

where $q = hu$.

$\therefore$ (1) $\rightarrow$ $\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) = 0 \Rightarrow \frac{\partial h}{\partial t} + h\frac{\partial u}{\partial x} + u\frac{\partial h}{\partial x} = 0$

W. Form : $\int \frac{h^{(n+1)} - h^{(n)}}{dt}\, v\,dx + \int h\frac{\partial u}{\partial x} v\,dx + \int u\frac{\partial h}{\partial x} v\,dx = 0$

$$\boxed{\begin{aligned} &\Rightarrow \int h^{(n+1)} v\,dx + dt\int h\frac{\partial u}{\partial x} v\,dx + dt\int u\frac{\partial h}{\partial x} v\,dx = \int h^{(n)} v\,dx \\ &\Rightarrow \underset{(\text{bilinear})}{a} = L \rightarrow (\text{linear}) \quad \text{—— (3)} \end{aligned}}$$

If u is const. then $\frac{\partial u}{\partial x} = 0$ [Taken in exp-1 of SWE paper]

$\therefore$ (3) $\rightarrow$ $\int h^{(n+1)} v\,dx + dt\, u\int \frac{\partial h}{\partial x} v\,dx = \int h^{(n)} v\,dx$

$\Rightarrow a_{new} = L_{new}$ —— (4)

(2) $\rightarrow$ $\frac{\partial}{\partial t}(hu) + gh\left[\frac{\partial h}{\partial x} + \frac{\partial z}{\partial x}\right] + gn^2\frac{|hu|\,hu}{h^{7/3}} = 0$

$\Rightarrow u\frac{\partial h}{\partial t} + h\frac{\partial u}{\partial t} + gh\left[\frac{\partial h}{\partial x} + \frac{\partial z}{\partial x}\right] + gn^2\frac{h^2|u|\,u}{h^{7/3}} = 0$

$\Rightarrow -hu\frac{\partial u}{\partial x} - u^2\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial t} + gh\left(\frac{\partial h}{\partial x} + \frac{\partial z}{\partial x}\right) + gn^2\frac{|u|\,u}{h^{1/3}} = 0$

$\Rightarrow \frac{\partial u}{\partial t} - u\frac{\partial u}{\partial x} - \frac{u^2}{h}\frac{\partial h}{\partial x} + g\left(\frac{\partial h}{\partial x} + \frac{\partial z}{\partial x}\right) + gn^2\frac{|u|\,u}{h^{4/3}} = 0$ —— (5)

$$\boxed{\begin{aligned} &\Rightarrow \text{W. Form:} \\ &\int \frac{u^{(n+1)} - u^{(n)}}{dt} v\,dx - \int u\frac{\partial u}{\partial x} v\,dx - \int \frac{u^2}{h}\frac{\partial h}{\partial x} v\,dx \\ &\quad + g\int \frac{\partial h}{\partial x} v\,dx + g\int\left(\frac{\partial z}{\partial x}\right) v\,dx + gn^2\int \frac{|u|\,u}{h^{4/3}} v\,dx = 0 \quad \text{—— (6)} \end{aligned}}$$

If u is const. (Taken on SWE paper)

(5) $\rightarrow$ $-u^2\int \frac{1}{h}\frac{\partial h}{\partial x} v\,dx + g\int \frac{\partial h}{\partial x} v\,dx + g\int \frac{\partial z}{\partial x} v\,dx$
$+ gn^2u^2\int \frac{v}{h^{4/3}}\,dx = 0$

—— (6a)

SWE : Exp : 1 $\quad u = \text{Const.}$ & $-u\frac{\partial u}{\partial x} - \frac{u^2}{h}\frac{\partial h}{\partial x} \approx u\frac{\partial u}{\partial x}$.

Eqs:

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} = 0 \quad \text{— (7)}$$

$$\& \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\left(\frac{\partial h}{\partial x} + \frac{\partial z}{\partial x}\right) + g\frac{n^2u^2}{h^{4/3}} = 0$$

$$\Rightarrow \frac{\partial h}{\partial x} = -\frac{\partial z}{\partial x} + \frac{n^2u^2}{h^{4/3}}. \quad \text{— (8)}$$

$$(7) \rightarrow h(x,t) = h(x-ut, 0)$$

$$(8) \rightarrow \quad h = \left[-\frac{7}{3}\left(n^2u^2x + c\right)\right]^{3/7}$$

BC : $h(ut, t) = 0 \Rightarrow c = -(n^2u^2 \cdot ut)$ (Moving BC)

$$\therefore h(x,t) = \left[-\frac{7}{3}\left\{n^2u^2(x-ut)\right\}\right]^{3/7}$$

So $h(0,t) = \left(\frac{7}{3}n^2u^3t\right)^{3/7}$