CSE474 Offline: Channel Equalization

Problem Description

When a sequence of bits is transmitted through a channel, they may get distorted because of intersymbol interference (ISI) & noise. The task of channel equalization is to regenerate originally transmitted bit sequence from the received, possibly distorted values.

Assume, the k-th transmitted bit is I_k & received sample is x_k. If we assume ISI spans over n successive bits, x_k can be expressed as follows.

$$x_k = f(I_k, I_{k-1}, I_{k-2}, ..., I_{k-(n+1)}) + \eta_k \tag{1}$$

Here $f(.)$ represents the action of the channel & denotes η_k noise for k-th bit. We assume $f(.)$ to be a linear function represented as:

$$f(I_k, I_{k-1}, I_{k-2}, ..., I_{k-(n+1)}) = \sum_{j=0}^{n-1} w_j I_{k-j} \tag{2}$$

We can also assume noise to follow a normal distribution with mean = 0 & variance = σ^2.

The task of equalizer is to predict k-th transmitted bit from previous l successively received samples.

$$\hat{I}_k = g(x_k, x_{k-1}, x_{k-2}, ..., x_{k-(l+1)}) \tag{3}$$

In the above equation, $g(.)$ is the equalizer method. There are several ways for implementing it such as using markov chain model or cluster based approach. In this assignment you have to implement the markov chain model with proper selection of states & Viterbi algorithm for predicting transmitted bit sequence.

Input

Read the parameters from a file named "config.txt". The description of this file is as follows.

- The first line contains two positive integers n and l.
- Next line contains n space separated real numbers denoting w_0, w_1, ... , w_{n-1}
- Next line contains a single real number denoting variance of noise, σ^2

From another file named "train.txt", read a single bit string consisting of 0's and 1's, lets call it $trainBits$.

Finally, read another bit string from "test.txt" file, lets call it $testBits$.

Sample

config.txt

3 2
0.7 0.5 0.1
0.225

train.txt

00000101001110010111011111010011100101101001110010111011 1

test.txt

0110100011

Here, $n = 3$, $l = 2$, $w_0 = 0.7$, $w_1 = 0.5$, $w_2 = 0.1$, $\sigma^2 = 0.225$

$trainBits =$ 00000101001110010111011111010011100101101001110010111011 1

$testBits =$ 0110100011

Tasks

1. Construct a markov chain model for which the following is required.
   - Define all possible states based on value of n.
   - Calculate prior probabilities of all states from $trainBits$. Note that each consecutive n bits in $trainBits$ refer to a state. [Hint: Prior probability of a state is related to the number of appearances of the state in $trainBits$]
   - Calcuate transition probabilities from $trainBits$. [Hint: Transistion probability from state A to state B is related to the count of state transition from A to B in $trainBits$.]
   - Assume observation probability follows normal distribution & calculate the means of observations.
2. Transmit $testBits$, calculate x_k's and use Viterbi algorithm on your markov model to reconstruct originally transmitted bits.
3. Calculate accuracy by comparing originally transmitted and predicted bit sequence.

Marks Distribution

Task	Mark
Calculate & show prior probabilities	1
Calculate & show transition probabilities	2
Calculate & show observation means	1
Implement Viterbi algorithm for equalization	5
Calculate & show accuracy	1
Total	10