The Relationship Between Diffusion Rates, Experience Curves, and Demand Elasticities for Consumer Durable Technological Innovations

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PRODUCT POLICY

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The Relationship between Diffusion Rates, Experience Curves, and Demand Elasticities for Consumer Durable Technological Innovations*

Innovations are not adopted by a population simultaneously. Rather, the timing of adoption is distributed in some fashion over the population. The literature on the adoption and diffusion of new ideas or new products by a social system is vast (see, e.g., Rogers 1977). Bass (1969) developed a mathematical model of the timing of adoption of new products and applied it in forecasting demand growth for new consumer durable products. The model has been widely adopted, extended, and employed for forecasting purposes (Nevers 1972; Dodds 1973; Robinson and Lakhani 1975; Dodson and Muller 1978). Several companies, including Eastman Kodak, RCA, IBM, Sears, and Hewlett-Packard, have employed the model and variations of it for forecasting purposes. Of the many papers which the model has spawned, the most general and comprehensive extension is provided by Dodson and Muller (1978). Although the model (Bass 1969) has certain nice properties for forecasting purposes and is adequate for these purposes, it is incomplete in that the premises on which it is

In this paper, experience-curve theory is merged with the theory of adoption of innovations on the basis of the profit-maximizing principle to form a dynamic theory of demand and price for consumer durable technological innovations. As a result of learning, costs and prices for new durables will fall continuously, while demand will grow at first and then decline as the market becomes saturated. Empirical estimates are made of learning rates, contagion rates, and demand elasticities for several consumer durable innovations.

based deal only with the social and behavioral influences on the timing of adoption. Economic forces such as price are ignored. From a forecasting viewpoint models with omitted variables are often superior to complete models in the sense that a forecast need not be required of exogenous variables in developing a forecast of an endogenous variable. Nevertheless, if the purpose of modeling is to enhance understanding of the relationship between social and economic forces, a model which integrates these elements is required. A theoretical framework will be developed here which provides such an integration with respect to the timing of adoption (or demand growth rates) for technological innovations sold to consumers. In addition, empirical data for several consumer durable innovations will be analyzed.

For every innovation a probability density function will describe the time of adoption of that innovation by a population. The density function will depend on the manner in which elements in the population interact with one another in learning about and in responding to the innovation, and it will depend on the rate at which the price of the innovation changes. Falling costs and prices are the rule for technological innovations, a result of learning and accumulating experience in the methods of producing a new product. Empirical evidence concerning the well-known "experience curve" indicates that the inverse relationship between unit cost and accumulated output (experience) generalizes across a wide range of technological innovations. These costs and prices depend on the adoption rate and vice versa. The objective of this work will be to investigate the nature of these dependencies.

The Experience Curve

The concept of declining costs and prices resulting from learning as expressed in the accumulated experience of a firm has been extensively developed and applied by the Boston Consulting Group (1968). Apparently the earliest identification of the particular form of the experience curve was found in the study of learning curves for airframes. Arrow (1962) utilized this form in his pioneering paper, "The Economic Implications of Learning by Doing," and the same form has been employed by the Boston Consulting Group.

The marginal cost function, called the experience curve, is

$$MC\ [E(t)] = C_1[E(t)]^{-\lambda}, \tag{1}$$

where

$$MC\ [E(t)] = \text{the marginal cost of producing the } E\text{th unit of output;}$$
$$E(t) = \text{accumulative output at time } t;$$

C_1 = a scaling parameter, sometimes referred to as the cost of producing the first unit; and

λ = a learning-rate parameter, $\lambda > 0$.

The current marginal cost depends not only on current output but also on earlier output or experience. If time is measured in discrete units the cost function of the firm for producing the quantity q_t in period t will be $C_t(q_1, q_2, \ldots, q_t)$. The experience of the firm at the beginning of time t will be

$$\sum_{j=1}^{t-1} q_j = E_{t-1}$$

and at the end of the period its experience will be $E_{t-1} + q_t = E_t$. If marginal cost at the end of period $t - 1$ is $C_1 E_{t-1}^{-\lambda}$, the total cost of the output q_t will be

$$C_t(q_1, q_2, \ldots, q_t) = C_1 \int_0^{q_t} (E_{t-1} + Z)^{-\lambda} \, dZ$$
$$= C_1/(1 - \lambda)(E_t^{1-\lambda} - E_{t-1}^{1-\lambda}). \quad (2)$$

Thus the marginal cost of producing the q_tth unit at time t will be $\partial C_t / \partial q_t = C_1 E_t^{-\lambda}$. If the firm determines price p_t in a way which will maximize current period profit it will $\max_{p_t} \Pi(p_t) = p_t q_t - C_t$. The well-known result is

$$p_t = [\eta_t/(\eta_t - 1)]C_1 E_t^{-\lambda}, \quad (3)$$

where η_t is the elasticity of demand at time t associated with the optimal pricing strategy. In the subsequent development it will be assumed that the elasticity is constant. The assumption of constant elasticity is not as restrictive as it may seem. In the dynamic framework developed in the next section, the assumption does not imply that the demand response to price is constant through time. The demand function is shifted in time. The assumption merely implies that the slope of the demand function, in a comparative static sense, remains constant through time.

Demand and Price as Functions of Time

The objective of the theoretical development of this section will be to derive demand and price as functions of time. The analysis will be facilitated if time is measured continuously. Thus $q(T)$, $p(T)$, and $E(T)$ are continuous functions of time. It is useful to note that $E(T) = \int_0^T q(t) \, dt$ and that $[dE(T)]/dT = q(T)$.

Because elasticity is assumed to be constant the demand function must have the form

$$q(T) = f(T) C[p(T)]^{-\eta}. \quad (4)$$

The function $f(T)$ serves to shift the demand function in time. We deal with durable products, and the demand during the early years of production will consist entirely of initial purchase demand, replacement demand being nil. Thus (4) is the initial purchase demand function. The market will therefore become increasingly saturated as time proceeds. Hence it is important that the function f serve to capture the saturation effect, and at the same time it is desirable for f to be defined in the most general way. Because of saturation f will be a bounded function. It is conceptually useful to think of f as a density function of time to purchase if price is fixed. Therefore if price is unchanging, $f(T)$ will describe the time behavior of demand and $Cp^{-\eta}$ will be the size of the market that will ultimately purchase the product. Price, however, is not constant but equal to

$$p(T) = [\eta/(\eta - 1)]C_1 [E(T)]^{-\lambda}. \tag{5}$$

Substituting (5) into (4) we have

$$q(T) = f(T)C \{[\eta/(\eta - 1)]C_1\}^{-\eta}[E(T)]^{\lambda\eta} \tag{6}$$

or

$$\frac{dE(T)}{dT} = q(T) = f(T) K[E(T)]^{\lambda\eta}, \tag{7}$$

where $K = C\{[\eta/(\eta - 1)]C_1\}^{-\eta}$. The demand function may be separated as follows: $E^{-\lambda\eta} dE = K f(T)dT$. Therefore $\int E^{-\lambda\eta} dE = K\int f(T) dT$ or $E^{1-\lambda\eta} = K(1 - \lambda\eta) F(T)$. It then follows that

$$E(T) = [K(1 - \lambda\eta)]^{\frac{1}{(1-\lambda\eta)}} F(T)^{\frac{1}{1-\lambda\eta}}. \tag{8}$$

It is apparent that the positiveness of E requires that $\lambda\eta < 1$. Note that

$$\lim_{T\to\infty} E(T) = [K(1 - \lambda\eta)]^{\frac{1}{(1-\lambda\eta)}} = m$$

is the size of the market that will ultimately purchase the product. Hence

$$E(T) = m F(T)^{\frac{1}{(1-\lambda\eta)}} \tag{9}$$

and

$$\frac{dE(T)}{dT} = q(T) = \frac{m}{(1 - \lambda\eta)}f(T) F(T)^{\frac{\lambda\eta}{(1-\lambda\eta)}}. \tag{10}$$

This last equation will define the time behavior of demand. Under fairly general conditions $q(T)$ will increase at first and later decline.

Price is a function of experience, and thus

$$p(T) = [\eta/(\eta - 1)] \, C_1 \, [E(T)]^{-\lambda}$$

$$= [\eta/(\eta - 1)] C_1 \, K^{\frac{-\lambda}{(1-\lambda\eta)}} \, (1 - \lambda\eta)^{\frac{-\lambda}{(1-\lambda\eta)}} \, F(T)^{\frac{-\lambda}{(1-\lambda\eta)}}, \qquad (11)$$

or, substituting m,

$$p(T) = [\eta/(\eta - 1)] \, C_1 \, m^{-\lambda} \, F(T)^{\frac{-\lambda}{(1-\lambda\eta)}}. \qquad (12)$$

We now have both demand and price expressed as closed-form expressions of time. The results are general and the assumptions not very restrictive. Specification of the form of the density function f permits not only a study of the time behavior of demand and price but also serves to define the basis for the estimation of λ and η.

In studying the time behavior of price we note that

$$\frac{dp(T)}{dT} = - \, \lambda[\eta/(\eta - 1)] \, C_1 \, \frac{dE}{dT} E^{-\lambda-1} = - \, \lambda \, \frac{q(T) \, p(T)}{E(T)}. \qquad (13)$$

Hence price will decrease monotonically with time. Therefore, although price will decrease monotonically with time, demand will ordinarily increase at first and later decline. A period of falling price and increasing demand will be followed by a period of falling price and decreasing demand, a result that is entirely consistent with the empirical evidence for consumer durable technological innovations.

Theory of the Timing of Adoption of Innovations

In the preceding section the function $f(T)$ was described as a density function of time to purchase of an innovation if the price of the innovation is fixed. The function $q(T)$ shown in (10) is therefore determined on the basis of the specification of the form of $f(T)$. Thus a theory of the timing of adoption is needed in order to complete the theoretical development of the previous section of demand and price variations in time. Such a theory already exists. It is the model of adoption mentioned briefly in the introductory section of this paper. In this section the theory of adoption timing will be reviewed.

Rogers (1962) has studied the extensive social science literature on the adoption and diffusion of innovations. The literature is primarily literary with a focus on identification and description of various classes of adopters. The following classification scheme is based on the timing of adoption of the innovation by various groups: (1) innovators, (2) early adopters, (3) early majority, (4) later majority, and (5) laggards. In
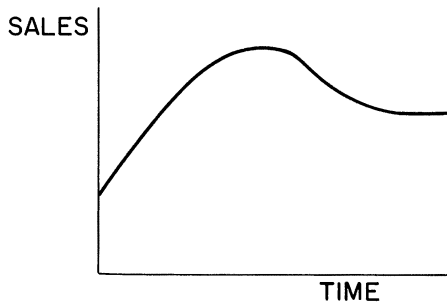
SALES

TIME

FIG. 1.—Growth of a new product

the mathematical formulation of the Bass model (1969) the groups 2 through 5 are aggregated and defined as "imitators," consistent with the characterization of the behavior of these groups in the social science literature. Imitators, unlike innovators, are influenced in the timing of adoption by the decisions of other members of the social system. The mathematical version of the behavioral findings is similar in certain respects to contagion models which have found widespread application in epidemiology (Bartlett 1960). The basic premise of the model is that the probability that an initial purchase (adoption) will be made at T given that no purchase has yet been made is a linear function of the number of previous buyers. Thus $\Pr(T) = \alpha + \beta/m\ Y(T)$ where α and β/m are constants and $Y(T)$ is the number of previous buyers. Since $Y(0) = 0$, the constant α is the probability of an initial purchase at $T = 0$ and its magnitude reflects the importance of innovators in the social system. The product β/m times $Y(T)$ reflects the pressure operating on imitators as the number of buyers increases.

The demand model characterizes the social contagion of the adoption process. Figure 1 shows the typical early demand growth of a consumer durable innovation. Sales consist of initial purchase demand. The demand grows to a peak and then declines as the early buyer segment approaches saturation. After the peak has been reached, replacement demand becomes an increasingly important component of demand, and sales of the product will ultimately start to grow again as replacement demand is added to the demand from population growth and from the deeper penetration of the innovation into additional market segments. Dodson and Muller (1978) discuss a framework for the inclusion of repeat purchases in the model. It is the early pattern of demand growth that is important in the industry development process, and it is the early pattern of demand growth that is the focus of this study.

The following fundamental assumptions characterize the model: (a) Over the period of interest there will be m initial purchases of the product. (b) The likelihood of purchase at time T given that no pur-

chase has yet been made is $f(T)/[1 - F(T)] = \alpha + \beta F(T)$, where $f(T)$ is the likelihood of purchase at T and $F(T) = \int_0^T f(t) \, dt$, and $F(0) = 0$. Therefore sales at $T = S(T) = mf(T) = \{\alpha + \beta \int_0^T [S(t)/m] \, dt\}$ $[m - \int_0^T S(t) \, dt]$. The behavioral rationales for these assumptions are summarized as follows.

1. Initial purchases of the product are made by "innovators" and "imitators," the important distinction between an innovator and an imitator being the buying motive. Innovators are not influenced in the timing of their initial purchase by the number of people who have already bought the product, while imitators are influenced by the number of previous buyers. Imitators "learn," in some sense, from those who have already bought.

2. The importance of innovators will be greater at first but will diminish monotonically with time.

3. The parameter α has been referred to as the coefficient of innovation and the parameter β as the coefficient of imitation. Since

$$
\begin{aligned}
f(T) &= [\alpha + \beta F(T)][1 - F(T)] \\
&= \alpha + (\beta - \alpha) F(T) - \beta [F(T)]^2,
\end{aligned}
\tag{14}
$$

to find $F(T)$, the solution to the following nonlinear differential equation must be found: $dT = dF/[\alpha = (\beta - \alpha)F - \beta F^2]$. The solution is:

$$
F = \frac{[\beta + \alpha e^{-(T+C)(\alpha+\beta)}]}{\beta[1 + e^{-(T+C)(\alpha+\beta)}]}.
\tag{15}
$$

Since $F(0) = 0$, the integration constant may be evaluated: $-C = 1/(\alpha - \beta) \ln (\beta/\alpha)$ and

$$
F(T) = \frac{[1 - e^{-(\alpha+\beta)T}]}{[\beta/\alpha e^{-(\alpha+\beta)T} + 1]}.
\tag{16}
$$

Then

$$
f(T) = \frac{(\alpha + \beta)^2}{\alpha} \frac{e^{-(\alpha+\beta)T}}{[\beta/\alpha e^{-(\alpha+\beta)T} + 1]^2},
\tag{17}
$$

and

$$
S(T) = \frac{m(\alpha + \beta)^2}{\alpha} \frac{e^{-(\alpha+\beta)T}}{[\beta/\alpha e^{-(\alpha+\beta)T} + 1]^2}.
\tag{18}
$$

To find the time at which the sales rate reaches its peak, differentiate S,

$$
S' = \frac{m/\alpha \, (\alpha + \beta)^3 \, e^{-(\alpha+\beta)T} \, [\beta/\alpha e^{-(\alpha+\beta)T} - 1]}{[\beta/\alpha e^{-(\alpha+\beta)T} + 1]^3}.
$$

Thus $T^* = -[1/(\alpha + \beta)] \ln (\alpha/\beta) = [1/(\alpha + \beta)] \ln (\beta/\alpha)$, and if an interior maximum exists, $\beta > \alpha$. The social contagion theory of the timing of adoption results in $f(T)$ and $F(T)$ shown in (17) and (16). These functions may be substituted into (10) to complete the theoretical development of the previous section.

Estimation Methods

Ordinarily demand and price data for consumer durable technological innovations are available only at the industry level and are typically reported in the form of industry output and average price. Therefore, aggregation issues must be addressed if the theoretical developments of the previous sections of this paper are to be employed in the pursuit of empirical estimation of parameters of demand and price functions. Moreover, the functions are highly nonlinear and thus nonlinear estimation methods are required. In this section aggregation and nonlinear estimation will be discussed.

Aggregation

Using equation (5) the relationship between the price of the ith firm and industry experience is

$$p_i(T) = [\eta_i/(\eta_i - 1)] \, C_{i1} \, [S_i(T)]^{-\lambda_i} \, [E(T)]^{-\lambda_i} \quad i = 1, 2, \ldots, n, \quad (19)$$

where $S_i(T)$ is the ith firm's share of industry experience, $E(T)$, and λ_i is that firm's learning rate parameter. The geometric mean of industry prices will then be

$$\left[\prod_{i=1}^{n} p_i(T)\right]^{\frac{1}{n}} = p_G(T) = A \, [E(T)]^{-\bar{\lambda}}, \quad (20)$$

where

$$\bar{\lambda} = \sum_{i=1}^{n} \lambda_i/n$$

is the average learning rate parameter for the industry and assuming

$$A = \left[\prod_{i=1}^{n} \eta_i/(\eta_i - 1) \, C_{i1} \, S_i(T)^{-\lambda_i}\right]^{\frac{1}{n}}$$

is unchanging. The variables in (20) are linear in the logarithms. Therefore, linear methods may be employed in conjunction with industry price and output time-series statistics in the estimation of the average learning-rate parameter for the industry.

Nonlinear Estimation

Assuming that industry demand depends on $p_G(T)$ in the manner indicated by equation (4), we have from (6)

$$q(T) = f(T) \, K[E(T)]^{\bar{\lambda}\eta}, \quad (21)$$

where $K = CA^{-\eta}$. Thus equations (7) through (13), with $\bar{\lambda}$ replacing λ, apply with respect to industry demand and price relationships.

Using equation (9) and (10) we can write

$$q(T)/E(T) = [1/(1 - \bar{\lambda}\eta)] \, f(T) \, F(T)^{\bar{\lambda}\eta - 1/1 - \bar{\lambda}\eta}$$
$$= [1/(1 - \bar{\lambda}\eta)] \, f(T)/F(T). \quad (22)$$

Substituting (14) into (22) we have

$$q/E = [\alpha/(1 - \bar{\lambda}\eta)]/F + (\beta - \alpha)/(1 - \bar{\lambda}\eta) - \beta F/(1 - \bar{\lambda}\eta). \quad (23)$$

Using from equation (9) the equality $F = m^{-(1-\bar{\lambda}\eta)} E^{(1-\bar{\lambda}\eta)}$ and substituting into (23) we find

$$q/E = [\alpha/(1 - \bar{\lambda}\eta)] \, m^{(1-\bar{\lambda}\eta)} E^{-(1-\bar{\lambda}\eta)} + (\beta - \alpha)/(1 - \bar{\lambda}\eta)$$
$$- \beta m^{-(1-\bar{\lambda}\eta)} E^{1-\bar{\lambda}\eta}/(1 - \bar{\lambda}\eta). \qquad (24)$$

Therefore

$$q = [\alpha/(1 - \bar{\lambda}\eta)] \, m^{(1-\bar{\lambda}\eta)} E^{\bar{\lambda}\eta} + [(\beta - \alpha)/(1 - \bar{\lambda}\eta)] \, E$$
$$- \beta m^{\bar{\lambda}\eta-1} E^{2-\bar{\lambda}\eta}/(1 - \lambda\eta). \qquad (25)$$

The discrete analog of (25) is

$$q_T = a[E_{T-1}^{\bar{\lambda}\eta}/(1 - \bar{\lambda}\eta)] + b[E_{T-1}/(1 - \bar{\lambda}\eta)] + c[E_{T-1}^{2-\bar{\lambda}\eta}/(1 - \bar{\lambda}\eta)], \quad (26)$$

where

$$E_{t-1} = \sum_{t=1}^{T-1} q_T,$$

$a = \alpha m^{1-\bar{\lambda}\eta}$, $b = \beta - \alpha$, and $c = \beta m^{\bar{\lambda}\eta-1}$. If $\bar{\lambda}\eta$ is fixed q_T is a linear function of the terms in the brackets in equation (26). Therefore with $\bar{\lambda}\eta$ fixed, a, b, and c may be estimated by means of linear regression. Furthermore, since $\bar{\lambda}\eta$ lies between zero and one, it is possible to search over this interval for the value of $\bar{\lambda}\eta$ and the corresponding values of a, b, and c which produce the best fit of time-series data for equation (26) and its equivalent, equation (10). Fortunately the parameters α, β, and m are uniquely identified by a, b, and c. Thus, since $-m^{1-\bar{\lambda}\eta} c = \beta$, $a/m^{1-\bar{\lambda}\eta} = \alpha$, and $(\beta - \alpha) = -m^{1-\bar{\lambda}\eta} c - a/m^{1-\bar{\lambda}\eta} = b$, $cm^{2(1-\bar{\lambda}\eta)} + bm^{(1-\bar{\lambda}\eta)} + a = 0$. Hence

$$m = [(-b - \sqrt{b^2 - 4ca})/2c]^{1/1-\bar{\lambda}\eta} , \qquad (27)$$

and the three parameters are uniquely identified.

Empirical Estimates of Learning Rates and Demand Elasticities

Learning-Rate Estimates

Time-series data for major appliance industry sales and average prices are published by *Merchandising* (March 1975). Data for several major consumer durable innovations including electric refrigerators, room air conditioners, automatic dishwashers, black-and-white television, electric clothes dryers, and color television have been employed in estimating the values of the parameters of the demand and price functions. Because published data are available only for average price, it has been necessary to substitute this figure for the geometric mean

TABLE 1   Estimates of the Learning-Rate Parameter for Consumer Durables

	Regression Statistics		
Product and Years	$\bar{\lambda}$	σ_λ	R^2
Electric refrigerator:			
1922–40	.0987	.0109	.83
Room air conditioners:			
1946–61	.1211	.0115	.89
1946–74	.1879	.0138	.87
Dishwashers:			
1947–68	.1498	.0194	.75
1947–74	.1832	.0153	.85
Black-and-white television:			
1948–60	.1959	.0310	.78
1948–74	.3641	.0441	.73
Electric clothes dryers:			
1950–61	.0896	.0194	.68
1950–74	.1930	.0184	.83
Color television:			
1961–70	.0667	.0089	.88
1961–74	.0972	.0150	.78

called for in the theoretical development. The price data have been deflated on the basis of the consumer price index. Equation (20) provides the basis for estimation of the learning rate parameter $\bar{\lambda}$.

Shown in table 1 are estimates of the learning rate parameter for several consumer durables. The estimates have been developed, in most instances, for two different time spans. The first of these covers the early growth stage of demand when sales are comprised primarily of initial purchases of the product and the second encompasses a longer time span. In every instance the learning rate is greater when estimated over the longer time period. One possible explanation for this result may be that firms with lower learning rates are eliminated as time progresses so that the average industry learning rate increases over time.

Although the learning rates for the consumer durables studied here are substantially lower than those reported by the Boston Consulting Group (1968) for industrial technological innovations, they are large enough to have resulted in very substantial price reductions. Shown in table 2 are percentage price reductions over time and the percentage price reduction associated with a doubling of experience based on the estimated learning-rate parameters. The figures in table 2 are even more impressive when one takes into account the fact that the price data have not been adjusted for the significant improvements in quality of the products which have occurred over the time periods studied. In figures 2 and 3 are shown graphs of the time series of actual prices and those predicted by the fitted model for room air conditioners and for color television sets.

TABLE 2   Price Reductions over Time and Related to Experience for Consumer Durables (%)

Product and Years	Price Reduction	Average Annual Price Reduction	Price Reduction with Doubled Experience
Electric refrigerators:			
1922–40	50.2	2.6	7.0
Room air conditioners:			
1946–61	48.0	3.0	8.0
1946–74	73.0	2.5	12.0
Dishwashers:			
1947–68	42.9	2.0	10.0
1947–74	55.7	2.0	11.0
Black-and-white television:			
1948–60	52.1	4.0	13.0
1948–74	82.0	3.0	22.0
Electric clothes dryers:			
1950–61	27.6	2.3	6.0
1950–74	54.4	2.2	12.0
Color television:			
1961–70	33.5	3.4	5.0
1961–74	49.9	3.6	7.0

Demand Elasticity Estimates

Using the estimation method described in the Nonlinear Estimation section above, and requiring the best fit of equation (10) to actual demand data, the parameter estimates shown in table 3 have been developed. The time-series observations employed in the analysis were restricted to the early years of sales growth when sales were comprised largely of initial purchase demand as opposed to replacement demand.

With one exception the estimated demand equations provide good descriptions of the empirical observations. The estimates of demand
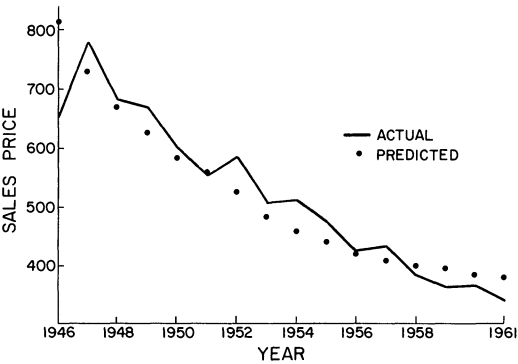

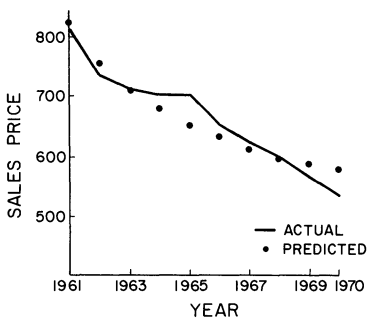FIG. 2.—Actual and predicted sales price of room air conditioners

FIG. 3.—Actual and predicted sales price of color television sets

elasticity, η, have been determined by dividing the estimates of $\bar{\lambda}\eta$ by the estimates of $\bar{\lambda}$ shown in table 1. For two of the products η is undefined. There is a logical necessity for η to exceed unity, and for dishwashers and black-and-white television the estimates of $\bar{\lambda}\eta$ are too small to produce elasticity estimates which satisfy this constraint. For these two products the demand equation is probably misspecified. Dishwasher demand is strongly geared to construction activity and adoption of black-and-white television appears to have been strongly influenced by the timing of the geographic expansion of broadcasting facilities. The demand function consists of a social contagion compo- nent and a price-effects component, and when exogenous forces influence demand growth the model is misspecified.

For the other products the demand function appears to work rea- sonably well. The price influence modifies the social contagion process but does not change its fundamental character. In every instance the coefficient of imitation substantially exceeds the coefficient of innova- tion. In the absence of price changes a consumer durable technological innovation will follow a diffusion path that reflects the rate of social contagion for that innovation. When prices fall in accordance with experience-curve theory the total market for the product is expanded and the rate of adoption is speeded up.

TABLE 3   Parameter Estimates for the Demand Equation

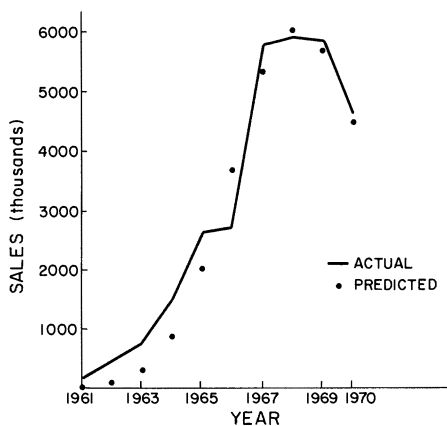|	Parameter Estimates					
Product	α	β	$m(10^3)$	$\bar{\lambda}\eta$	η	R^2
Electric refrigerators	.0236	.0565	101,880	.4978	5.04	.84
Room air conditioners	.0489	.2326	19,433	.5124	4.23	.94
Dishwashers	.0004	.1678	127,751	0	...*	.98
Black-and-white television	.0590	.1315	106,608	.1026	...*	.10
Electric clothes dryers	.0689	.1778	11,738	.3870	4.32	.84
Color television	.0141	.5298	37,110	.5347	8.02	.95

*Undefined.

FIG. 4.—Actual and predicted unit sales of color television sets

Demand and Price Patterns

Shown in figures 4 and 5 are the theoretical and actual sales of color television sets and electric clothes dryers. Demand grows to a peak and then declines as the market becomes increasingly saturated. The scenario of generic demand growth for consumer durable innovations is well established. Demand grows to a peak and then declines rapidly. The peaking of demand has often not been well anticipated by the industry with the result that substantial excess capacity has often been built. Declining demand accompanied by continuously falling price ordinarily results in an industry shake-out. Because the pattern is general it would appear that an opportunity exists through improved understanding of the process to reduce social costs associated with industry development. Shown in figures 6 and 7 are the theoretical
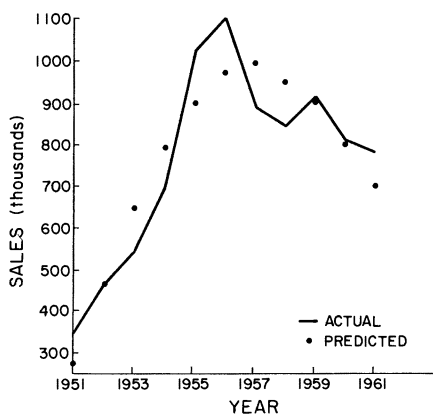

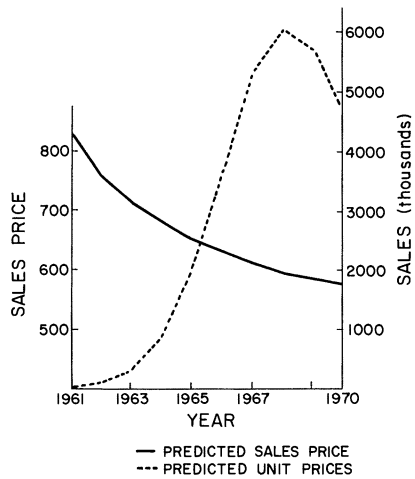FIG. 5.—Actual and predicted unit sales of electric clothes dryer

FIG. 6.—Predicted sales price and unit sales of color television sets

demand and price projections for color television and for electric clothes dryers, patterns which are typical of the dynamics of demand and price in the early years of a consumer durable innovation.

Concluding Comments

The demand and price patterns discussed here have been observed frequently enough that they may be termed "empirical generalizations." Whatever the limitations of the theoretical developments of this work may be, the patterns do exist and they do have significant implications for management strategy. In those industries with high
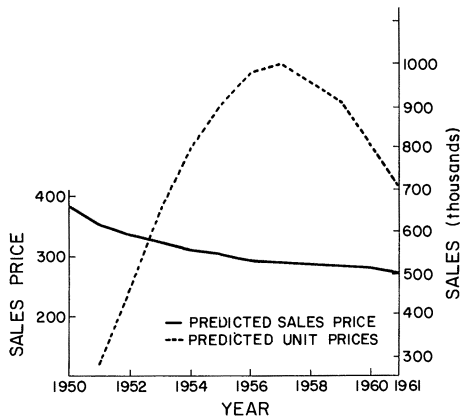

FIG. 7.—Predicted sales price and unit sales of electric clothes dryers

levels of innovative activity, "pricing according to the learning curve" is a common phrase. Apparently, however, the meaning of the phrase varies from one firm to another since one firm will price with future costs in mind while another will consider current costs. Pricing strategy can be strongly influenced by the length of the planning horizon. Robinson and Lakhani (1975) have developed a framework for analyzing pricing strategy taking into account multiple period planning horizons. Although their approach does not yield analytic price solutions, there are strong results from their analysis indicating the sensitivity of strategy to the planning horizon.

In the theoretical development presented earlier it was assumed that firms maximized current period profit. When policies chosen for the current period impinge on policies chosen in the future, it is clear that single-period planning horizons will not result in a globally optimal policy. Under these conditions a globally optimal policy will be such that if the firm has a planning horizon of T periods it will seek a pricing strategy $(p_1, p_2, \ldots, p_T)$ which will maximize the present value of current and future profits. Thus

$$\pi(p_1, p_2, \ldots, p_T) = \sum_{t=1}^{T} (p_t \, q_t - C_t) \, \rho^{t-1},$$

where the discount factor $\rho = 1/(1 + i)$ and i is the cost of capital. Differentiating π:

$$\frac{\partial \pi}{\partial p_1} = \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - \frac{\partial C_1}{\partial q_1} \frac{\partial q_1}{\partial p_1} \right)$$

$$+ \left(p_2 \frac{\partial q_2}{\partial q_1} \frac{\partial q_1}{\partial p_1} - \frac{\partial C_2}{\partial q_1} \frac{\partial q_1}{\partial p_1} \right) \rho$$

$$+ \ldots + \left(p_T \frac{\partial q_T}{\partial q_1} \frac{\partial q_1}{\partial p_1} - \frac{\partial C_T}{\partial q_1} \frac{\partial q_1}{\partial p_1} \right) \rho^{T-1},$$

$$\frac{\partial \pi}{\partial p_2} = \left(q_2 + p_2 \frac{\partial q_2}{\partial p_2} - \frac{\partial C_2}{\partial q_2} \frac{\partial q_2}{\partial p_2} \right) \rho$$

$$+ \left(p_3 \frac{\partial q_3}{\partial q_2} \frac{\partial q_2}{\partial p_2} - \frac{\partial C_3}{\partial q_2} \frac{\partial q_2}{\partial p_2} \right) \rho^2$$

$$+ \ldots + \left(p_T \frac{\partial q_T}{\partial q_2} \frac{\partial q_2}{\partial p_2} - \frac{\partial C_T}{\partial q_2} \frac{\partial q_2}{\partial p_2} \right) \rho^{T-1},$$

$$\vdots$$

$$\frac{\partial \pi}{\partial p_{T-2}} = \left(q_{T-2} + p_{T-2} \frac{\partial p_{T-2}}{\partial p_{T-2}} - \frac{\partial C_{t-2}}{\partial q_{T-2}} \frac{\partial q_{T-2}}{\partial p_{T-2}} \right) \rho^{T-3}$$

$$+ \left(p_{T-1} \frac{\partial q_{T-1}}{\partial q_{T-2}} \frac{\partial q_{T-2}}{\partial p_{T-2}} - \frac{\partial C_{T-1}}{\partial q_{T-2}} \frac{\partial q_{T-2}}{\partial p_{T-2}} \right) \rho^{T-2}$$

$$+ \left(p_T \frac{\partial q_T}{\partial q_{T-2}} \frac{\partial q_{T-2}}{\partial p_{T-2}} - \frac{\partial C_T}{\partial q_{T-2}} \frac{\partial q_{T-2}}{\partial p_{T-2}} \right) \rho^{T-1},$$

$$\frac{\partial \pi}{\partial p_{T-1}} = \left(q_{T-1} + p_{T-1} \frac{\partial q_{T-1}}{\partial p_{T-1}} - \frac{\partial C_{T-1}}{\partial q_{T-1}} \frac{\partial q_{T-1}}{\partial p_{T-1}} \right) \rho^{T-2}$$

$$+ \left(p_T \frac{\partial q_T}{\partial q_{T-1}} \frac{\partial q_{T-1}}{\partial p_{T-1}} - \frac{\partial C_T}{\partial q_{T-1}} \frac{\partial q_{T-1}}{\partial p_{T-1}} \right) \rho^{T-1},$$

$$\frac{\partial \pi}{\partial p_T} = \left(q_T + p_T \frac{\partial q_T}{\partial p_T} - \frac{\partial C_T}{\partial q_T} \frac{\partial q_T}{\partial p_T} \right) \rho^{T-1}.$$

Setting the partials equal to zero, starting with the last and solving recursively in dynamic programming fashion, the optimal solution may be found. The last partial will be equated to zero when $p_T = \eta_T/(\eta_T - 1) C_1 E_T^{-\lambda}$. Therefore, if the planning horizon is only for a single period, prices should fall along with falling marginal cost provided that the demand elasticity, η_T, does not change in a manner that will offset the gains from experience.

The solution to the two-period planning-horizon problem is achieved by equating the second to last partial to zero. Thus $p_{T-1} = \eta_{T-1}/(\eta_{T-1} - 1) [C_1 E_T^{-\lambda}_{-1} (1 - \rho) + C_1 E_T^{-\lambda} \rho - (\partial q_T/\partial q_{T-1}) P_T \rho]$. The price in the first period is proportional to a convex combination of the marginal costs for the two periods less the present value of revenue benefits (or plus the present value of revenue losses)[1] in the second period of an increase in demand in the first period. Similarly, the price in the initial period of a three-period planning horizon is

$$p_{T-2} = \eta_{T-2}/(\eta_{T-2} - 1) \left\{ [C_1 E_T^{-\lambda}_{-2} (1 - \rho) \right.$$
$$+ C_1 E_T^{-\lambda}_{-1} \rho (1 - \rho) + C_1 E_T^{-\lambda} \rho^2]$$
$$\left. - \left[p_{T-1} \frac{\partial q_{T-1}}{\partial q_{T-2}} \rho + p_T \frac{\partial q_T}{\partial q_{T-2}} \rho^2 \right] \right\}.$$

The pattern of the determinants of optimal initial period price for a planning horizon of arbitrary length is clear. The price will be proportional to a convex combination of marginal costs of each of the periods in the planning horizon less the present value of future revenue

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1. The effect of shifting demand forward on demand in later periods could be either positive or negative. On one hand, it would build the firm's competitive position in the market and thus tend to have a positive effect; on the other, it would tend to deplete a market more rapidly and thus possibly have a negative effect.

benefits (or plus the present value of revenue losses) from an increase in demand in the initial period. If the firm updates the planning horizon at the end of each period and maintains a planning horizon of length T, then for each period the optimal solution is found by equating a weighted combination of prices with a convex combination of marginal costs. Assuming that there exists precise knowledge of the demand function so that the elasticities and partial derivatives are known, it is possible, in principle, to find the globally optimal policy by means of dynamic programming. However, it is unlikely that in a competitive market this information would be known. Therefore it is not surprising that there is variance in the meaning of the phrase, "pricing according to the learning curve." It is clear that, with learning, marginal costs will fall and along with them prices will decline. Therefore although the assumption that firms will determine prices in a way that current period marginal revenue is equated with current period marginal cost may not be entirely valid, the assumption does capture, however crudely, the inevitable result that prices will decline as experience accumulates.

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