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Multi-stage diffusion dynamics in multiple generation high technology products

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ABSTRACT

In marketing literature, traditional innovation diffusion models have been used for measuring new product sales growth with mixed results. This is primarily because such models fail to identify the difference between the diffusion of awareness about a new product and the actual adoption by consumers. This aspect is extremely important for manufacturers of high technology products because there is a definite lag between the time of reception of information about a new product and the time when the final purchase decision is made by a consumer. In this paper a new diffusion model has been proposed for products with multiple technological generations. The proposed model treats sales as a consequence of the spread in awareness about new products, and models awareness diffusion by explicitly incorporating the effects of unfavorable information along with the more traditional positive feedback effects. Our framework also incorporates the effect of prices, thereby addressing one of the major limitations of the existing diffusion models. The proposed model has been validated using data on world-wide DRAM shipments.

1. Introduction

High technology markets are characterized as a complex system that exists under rapidly changing technological conditions which lead to shorter product life cycles. The importance of speed in such markets is driven by increasing competition and continually evolving expectations of customers. Apple is the one of the pioneers in technological investments, and has a very loyal set of customers. Due to strong customer loyalty Apple not only attracts new customers, but also retains them by continuously offering new technology innovations. To remain a leader in the industry, Apple has often introduced substitutable products that can satisfy a range of customer need. But the biggest threat to IT companies such as Apple comes from its highly established competitors.

High technology industries periodically introduce new products with better value and added features. But such technological advancements and feature additions do not essentially imply that previous generation products are immediately withdrawn from the market (Bayus, 1994; Jaakkola, Gabbouj, & Neuvo, 1998; Chanda & Bardhan, 2008). For personal computers Bayus (1998) observed that the rate of introduction of a new product is much higher than the rate of withdrawal of existing products from the market. As a result, very often more than one generation of products compete in the same market at a given time.

Rogers (1983) proposed that diffusion should be considered as the transmission of messages related to new ideas that lead to subsequent innovations (products, processes, technology, etc.), As a result diffusion generates expectations of change in receptor behavior which is evident from the adoption or the rejection of the innovation (Bonus, 1973; Lin & Burt, 1975; Weenig & Midden, 1991; Zaltman & Stiff, 1973).

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Thus, it is important to study a multi-stage empirical model that can distinguish the two separate aspects of diffusion—*awareness diffusion* and *adoption* by identifying different factors that influences consumer decision making process. Zaltman and Stiff (1973) reviewed several multi-staged models and observed that there is not a single model that considered the intergenerational diffusion aspect, even though it is crucial from the point of view of high technology industries.

In addition, high technology industries are dependent on the customer feedback effects; word-of-mouth behaviors have strong financial (positive and negative) impact on customer loyalties that ultimately affect growth of a company. Word-of-mouth effect is related to that aspect of consumer behavior where they feel encouraged to talk about a product or company to friends and neighbors. This sets off a chain of communication that could spread throughout the market and in the process creating strong, positive brand images and beliefs. Positive feedback effects can encourage existing customers to spend more on average and encourage potential customers to generate new business, whereas negative word of mouth may decrease the purchasing value (perhaps even defection), as well as increase the potential loss of new business. Thus, it is important to understand the sales growth pattern of innovations introduced in the market, as well as the different aspects that are affected by it.

Unfortunately diffusion modeling of products with technology generations has received little attention (Bayus, 1992a). The objective of this study is to propose an adoption–diffusion model for a multiple-generations product without violating the existing theories of individual behavior. The framework we suggest here is similar to that of Kalish (1985), where the awareness information of a product spreads in an epidemic-like manner, and the actual adoption depends on an individual's expenditure capacity. The model proposed here is a multi-stage mixed influence model for technological generations that relies on explicit assumptions regarding the adoption process.

2. Literature review

Several models have been proposed to study the pattern of sales growth of a new product. The Bass (1969) model is one of the most cited publications in this area. The model proposes the division of potential buyers into two groups—innovators and imitators—and models the adoption of a new product to be dependent upon the behavior of these two groups. Despite its good fit to historical data, the Bass model has been criticized for its oversimplifying conclusions regarding the adopter's decision making process without fully accounting for market heterogeneity, as well as, completely ignoring the role of marketing-mix (Bayus, 1992b).

Several attempts have been made to overcome these limitations through analytical modifications (Kalish & Lilien, 1983; Parker, 1991), multi-stage structures (Kalish, 1985), multi-innovations model (Fisher & Pry, 1971; Norton & Bass, 1987, 1992; Mahajan & Muller, 1996; Islam & Meade, 1997; Danaher, Hardie, & William, 2001; Chanda & Bardhan, 2008), individual level parameters (Roberts & Urban, 1988; Chatterjee & Eliashberg, 1990; Lattin & Roberts, 2000; Adner & Levinthal, 2001) or dynamic potential markets (Kalish, 1985; Mahajan & Peterson, 1978; Mahajan & Peterson, 1982; Milling, 1996; Weil & Utterback, 2005).

Furthermore, neither the basic diffusion model nor its extensions explicitly consider the impact of an innovation's characteristics or its perception among potential adopters—they all tend to consider that every innovation is equal. By ignoring the effect of an innovation's perceived attributes on its adoption rate, these models fail to reconcile that there is sufficient evidence to confirm that an adopter's perceptions of an innovation's attributes conditions the rate of adoption (Rogers, 1983).

The market success of a given innovation can also be aided by another product (multi-product interactions) or by product generations (successive generations). Shocker, Bayus, and Kim (2004) point out the relative lack of attention that multi-product growth models have received, compared to other research topics involving diffusion. Norton and Bass (1987, 1992) provided one of the earliest diffusion models that tried to describe the growth in sales for multiple generations competing in the same market. Later on several extensions of Bass model had been proposed for multiple-generation diffusion (Speece & MacLachlan, 1995; Mahajan & Muller, 1996; Islam & Meade, 1997; Jun & Park, 1999; Danaher et al., 2001; Chanda & Bardhan, 2008). But, they all failed to account for market heterogeneity and varied individual sensitivity associated with a new innovation.

Zaltman and Stiff (1973) hypothesized that adoption and diffusion of innovations is the outcome of a decision process. Most of the earlier research distinguishes two separate stages in the decision to adopt—awareness stage and evaluation stage (Bonus, 1973; Hauser & Urban, 1977; Kalish, 1985; Lin & Burt, 1975; Weenig & Midden, 1991). Lieberson (2000) has argued that what separates adoption from awareness is whether the object of adoption has appeal to the potential adopter.

Thus it becomes strategically important to explicitly distinguish between awareness and adoption to identify the different variables that influence the decision process. Van Den Bulte and Lilien (2001) has identified two such factors—marketing efforts and word of mouth—and suggested that the initial awareness of a new innovation occurs mainly through commercial sources such as salespeople and direct mailings, whereas personal contacts with colleagues gain importance in later stages (Coleman, Katz, & Menzel, 1966; Peay & Peay, 1984). Van Den Bulte and Lilien (2001) argued that without distinguishing the awareness stage and evaluation stage it is not possible to compare the effect of mass media and commercial efforts with the effect of word of mouth and other social transmission processes.

3. Modeling framework

Success of a new product largely depends on the product's goodwill (consumer perceived utility) and price over time. Therefore, while modeling the diffusion process it becomes imperative to get a clear distinction between the spread of product information and the consumer purchase dynamics (Kalish, 1985). The effective dissemination of initial product-information is very important to increase the awareness and that can be done primarily through advertising. Interpersonal communication and word-of-mouth also play important roles. On the other hand understanding of product attributes (e.g., consistency, robustness, performance) can be

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encouraged through actual users only. The adopters once they have used the product may become opinion leaders and influence the position (goodwill) of the product in the market.

In this paper a multistage awareness-adoption model for two-generation products has been proposed. The model assumes that in the awareness stage an individual will develop a certain attitude (favorable or unfavorable) for the product-generations. In the next stage, a positively aware individual will become a potential adopter of any of the competing technologies after evaluating the price. The model also assumes that for second and subsequent generations there are two groups of buyers: (a) new purchasers, who are first-time adopters of the technology and (b) repeat buyers, who had adopted a previous generation product and upgrade with the latest one. The relationship between the repeat purchasers and the new purchasers has been shown in the overall diffusion of a new technology over multiple generations, by separately identifying the two types of adopters.

Fig. 1 presents the diffusion process of a new technology (first generation) as proposed in this paper. The framework can be visualized as a three-stage diffusion process—a favorable target population that gets motivated and forms an attitude towards buying the product after evaluating the price. There is a finite time lag between awareness and actual purchase. The framework depicted in Fig. 1 is extended to a situation where a second generation of the technology is introduced in the market before the first generation is withdrawn. An overview of the key model components is presented in Fig. 2.

The basic assumptions behind the models are as follows:

1. Once adopters adopt a new technology, they do not revert to a technology of an earlier generation.
2. Sales of a second or succeeding generation products come from two sources:
   a. *New Purchasers* (*First Time Buyers*)—those who have adopted the product for the first time.
   b. *Repeat Purchasers*—adopters who had bought an earlier generation and are now upgrading to latest generation.
3. Each adopter can purchase exactly one unit of the product and makes no further purchases of the same generation.
4. An adopter's choice during each generation is independent of her/his choice in previous generations.
5. An existing user of an earlier generation, who has complete information about all the available generations, can become a potential adopter of any one of the latest generations, depending on his/her expenditure capability.

3.1. Adoption model for single generation

The Bass (1969) model for diffusion of innovation assumes that the potential adopter of a new product can either be an innovator or an imitator depending on how he/she makes the purchase decision. The model has been applied to variety of new product types during last four decades. In spite of its various limitations, including non-inclusion of explanatory variables like price etc., most of

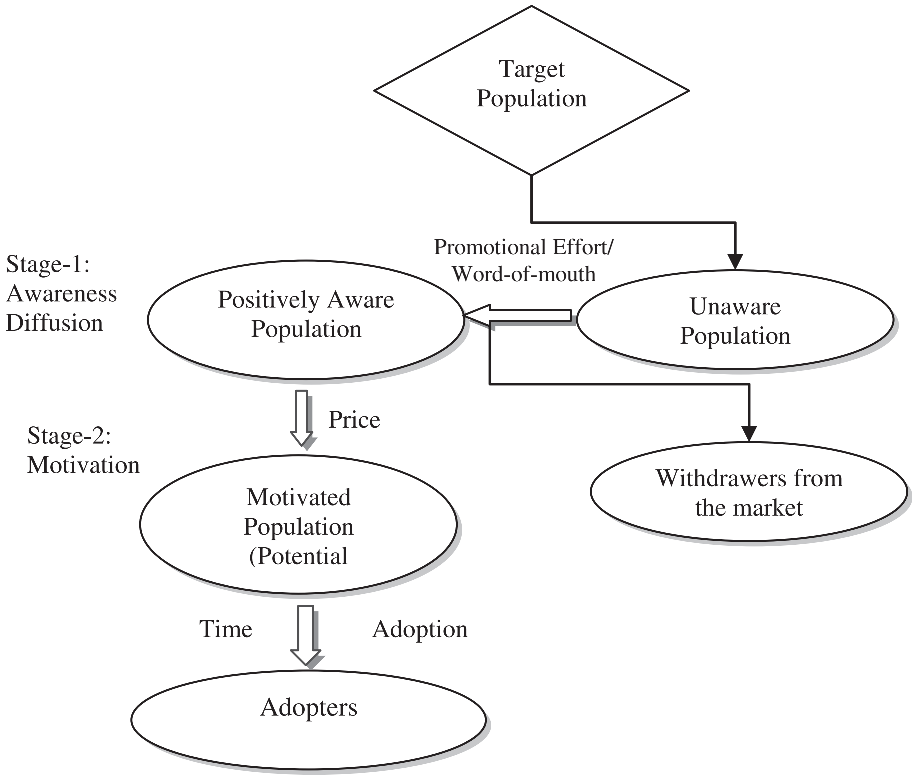

Fig. 1. Various stages in diffusion of a technological innovation.

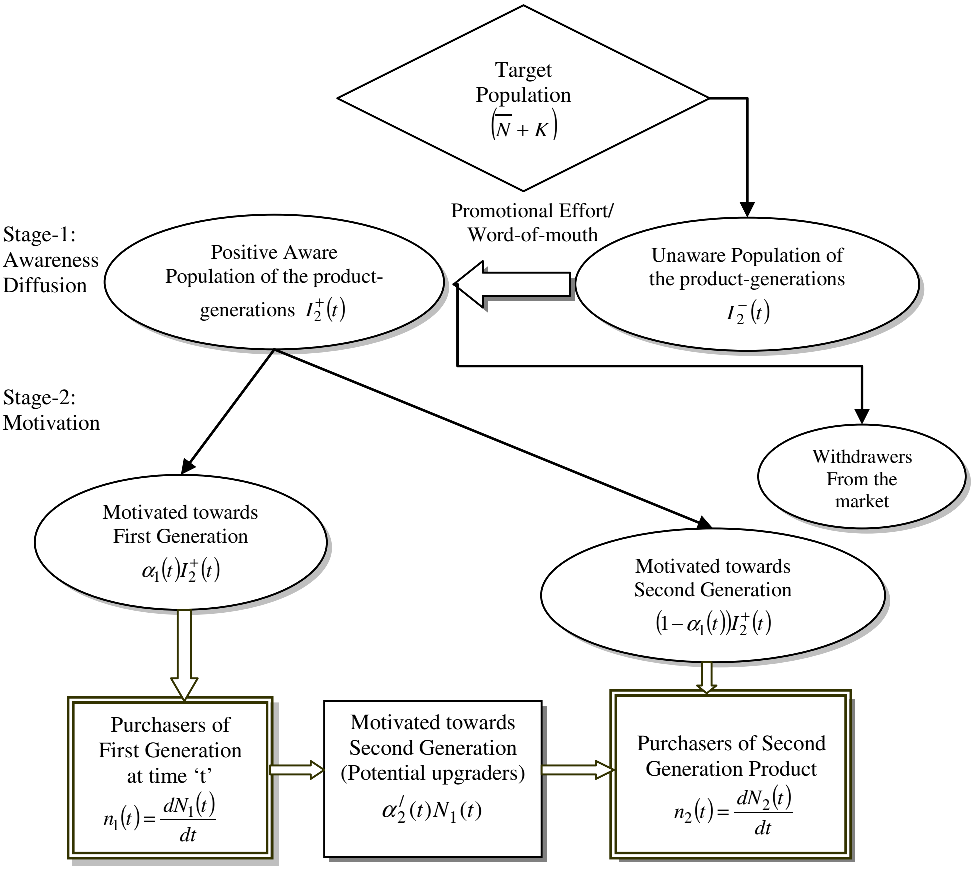

Fig. 2. Diffusion of two generations of a technological innovation.

the applications of the model have been successful (Mahajan, Muller, & Bass, 1990; Bass, Krishnan, & Jain, 1994). One explanation could be that most of the products considered are single-generation products, with no substitutions available within the period of study. In this paper we have used the framework similar to that of Kalish (1985) to incorporate the effect of unfavorable information into the awareness diffusion process.

3.1.1. Diffusion of awareness

Customer interactions through word-of-mouth can have a major impact on consumer awareness of a product and the accompanying promotional strategies (Arndt, 1967; Bayus, 1985; Danaher & Rust, 1996; Herr, Kardes, & Kim, 1991). Word-of-mouth can reduce the uncertainty when the product in question is a new innovation, and this has been found to be especially effective in driving the diffusion of new products (Rogers, 1983; Mahajan, Muller, & Kerin, 1984). Anderson (1998) observed that the impact of word-of-mouth largely depends on the satisfaction level of the customers with the existing products, and very often there is resistance to change because people are reluctant to dump the familiar to adopt the unfamiliar. Hogan, Lemon, and Libai (2004) used a customer lifetime value approach to demonstrate that the word-of-mouth generated after an ad-induced purchase can represent a considerable portion of the economic worth of a promotion. In the present paper, it has been assumed that:

- The spread of product information happens due to promotional efforts by the firm and also by inter-personal communications.
- An aware individual will go to the next level of purchase decision if he/she is satisfied by the available product information; otherwise he/she will leave the system.

Thus the likelihood of becoming aware is proportional to the effectiveness of the promotional strategy, and to the number of transmitters who are influenced by the word-of-mouth (Fig. 3).

Let $\overline{N}$ be the initial market size.

$I^-(t)$	Unaware population i.e., those who are uninformed about the existence of the new product
$I^+(t)$	Aware and favorable population i.e., those who are well informed about the existence of the new product
p	Coefficient of innovation, q: Word-of-mouth influence.
ε	Fraction of the aware population who left the market on being unsatisfied with the product information.

The specification of the above flow diagram (Fig. 3) can be expressed in Table 1.

We can define $I^+(t)$ and $I^-(t)$ as:

$$\dot{I}^+(t) = \frac{dI^+(t)}{dt} = pI^-(t) + q\frac{I^+(t)I^-(t)}{\overline{N}} - \varepsilon I^+(t) \tag{1}$$

$$\dot{I}^-(t) = \frac{dI^-(t)}{dt} = -pI^-(t) - q\frac{I^+(t)I^-(t)}{\overline{N}}. \tag{2}$$

$$\text{Considering } A_1(t) = \frac{I^+(t)}{\overline{N}} \text{ and } A_2(t) = \frac{I^-(t)}{\overline{N}}; \text{ where } A_1(t) + A_2(t) = 1, \tag{3}$$

then, $\dot{A}_1(t) = \frac{dA_1(t)}{dt} = \frac{1}{\overline{N}}\frac{dI^+(t)}{dt} = \frac{1}{\overline{N}}\left[pI^-(t) + q\frac{I^+(t)I^-(t)}{\overline{N}} - \varepsilon I^+(t)\right]$.

Dropping the time notation, we have

$$\begin{aligned} &\Rightarrow \dot{A}_1 = pA_2 + qA_1A_2 - \varepsilon A_1 \\ &= p(1-A_1) + qA_1(1-A_1) - \varepsilon A_1 \\ &= p + (q-p-\varepsilon)A_1 - qA_1^2 = a + bA_1 - cA_1^2 \\ &\text{where}, p = a;\ (q-p-\varepsilon) = b \text{ and } q = c. \end{aligned} \tag{4}$$

Now integrating Eq. (4) with the initial condition $A(0) = 0$; we have

$$A_1 = \frac{\eta + b}{2c}\left[\frac{1-e^{-\eta t}}{1+\varphi e^{-\eta t}}\right] \tag{5}$$

where, $\eta = \sqrt{b^2 + 4ac}$ and can be assumed as the total effect of promotion and word-of-mouth on adopters, and $\varphi = \frac{\eta+b}{\eta-b}$.

Now, from Eqs. (3) and (5) we have,

$$I^+ = \overline{N}\left(\frac{\eta + b}{2c}\right)\left[\frac{1-e^{-\eta t}}{1+\varphi e^{-\eta t}}\right]. \tag{6}$$

The model specified in Eq. (6) is capable of addressing the market in aggregate terms. Furthermore, the parameters used are competent enough to represent the two modes of communication—mass media and word of mouth (both positive and negative).

3.1.2. Market potential

Customers buy a product in accordance with their varied needs and expenditure capacity. Thus, price plays a major role in the overall success of the product. An aware individual would become a potential purchaser after satisfying himself/herself of the utility of the product vis-à-vis the unit price. Thus an informed target customer's motivation towards a new product will depend on its price.

If $\alpha(t)$ is the pricing function at time t, then the cumulative number of potential adopters $\overline{N}(t)$ at time t can be written as:

$$\overline{N}(t) = f\left(I^+(t), \alpha(t)\right) = I^+(t)\alpha(t). \tag{7}$$

3.1.3. Actual adoption

Several researchers have shown that the diffusion process can be used to estimate the number of potential adopters that might make a purchase for a single generation, which has been studied in details by several researchers. As Kalish (1985) suggested, a potential purchaser can take some time or delay the purchase depending on his/her convenience (e.g., due to personal problem, shortages of money, etc.).

In other words, if k is the rate parameter, then the likelihood of a potential adopter will adopt the new innovation in the time interval d t is k d t. Thus, when k is large, a potential adopter will not take much time for purchasing the product, and vice versa.

Table 1
Validation of the flow diagram.

Flow of customers	Model
Innovation effect + word-of-mouth	The unaware population decreases due exit for the external, and internal influence: $-pI^- - q\frac{I^+I^-}{\overline{N}}$
Innovation effect + word-of-mouth	Aware population increases due to dissemination of product information to the unaware population: $pI^- + q\frac{I^+I^-}{\overline{N}}$
Disappointment factor	A fraction of the aware population leave the market due to unfavorable information: $-\ \varepsilon I^+$

The adoption model for a single generation can now be specified as:

$$\begin{aligned}&\dot{N}_1(t)=\frac{d}{dt}N_1(t)=k\left[\left(I^+(t)\alpha(t)\right)-N_1(t)\right]\\&\alpha(t)=e^{-d_1\varphi_1(t)};\ d_1\ \text{is the price coefficient.}\end{aligned}\tag{8}$$

3.2. Adoption model for two generations

After the second generation product is introduced in the market, the two generations compete in the market. As a result, an individual who is confident of both the products can make a choice. In such situation an adopter's purchasing decision is influenced by comparative price.

Let $(\overline{N}+K)$ be the size of the target market, an increase of K over the initial market size $\overline{N}$.

$I_2^+(t)$ be the aware and favorable population, i.e., those who are well informed about the existence of the product generations till time 't'.

$I_2^-(t)$ be the unaware population, i.e., those who are unaware about the existence of the product generations till time 't'.

Now following Eqs. (1)–(6), we have

$$I_2^+=(\overline{N}+K)\left(\frac{\eta+b}{2c}\right)\left[\frac{1-e^{-\eta t}}{1+\varphi e^{-\eta t}}\right].\tag{9}$$

To keep the model simple and to save on estimation efforts without much loss of generality here in Eq. (9) we have considered the same coefficients p, q and ε as in Eq. (6). Once positive awareness about the two-generation product spreads in the market, price becomes one of the most important motivating factors for selecting a particular generation.

Let us assume that due to the pricing factor a fraction $\alpha_1(t)$ of $I_2^+(t)$ would be motivated towards the first generation and $\alpha_2(t)$ towards the second generation $\left(\sum_{i=1}^{2}\alpha_i(t)=1\right)$. $\alpha_i(t)$ can be modeled as function of prices of both the generations.

When there are two competing generations in the market, the potential growth function of the second generation depends on first time adopters of the newer generation, as well as, on upgrade/repeat purchasers (first generation adopters who switch to adopt the second generation). In this paper it is assumed that repeat purchasing is allowed only in future generations.[1] Thus the cumulative number of adopters of the 2nd generation product can be given as the sum of the first time and the upgrade adopters (this adoption process has been depicted in Fig. 2).

$$\text{Total Purchasers}_{2\text{nd}}(t)=(\text{first time purchasers})_{2\text{nd}}(t)+(\text{Upgrade purchasers})_{2\text{nd}}(t)\tag{10}$$

Now, to model the potential upgraders of second-generation product we have assumed that a newer generation product is a continuous innovation of the preceding generation, and can provide substantially higher customer benefits (e.g., DRAM-chipsets, Personal Computers, Television).

Using all these assumptions, we can argue that for a continuous innovation, the decision of the potential repeat purchasers to upgrade will depend on their own judgment about the earlier purchase (Chanda & Bardhan, 2008). If none of the new purchasers in a generation drops out of the market for the later generation, then the number of upgrade adopters, who would consider buying the latest technology, can be expressed as follows:

$$[\text{Potential Repeat Purchasers}]_{2\text{nd}}(t)=f(N_1(t),\varphi_2(t))=[N_1(t_2)]\alpha_2'(t_2)\tag{11}$$

$\alpha_2'(t_2)$ is the function of second generation price.

Where, $\alpha_2'(t_2)$ is the pricing function for upgrade purchasers.

Define the dummy-variable $\omega(t)$ as

$$\omega(t)=\begin{cases}0, & 0<t\le\tau\\1, & \tau<t\end{cases}.$$

The equation for the rate of adoption of the first generation product can be expressed as:

$$\dot{N}_1(t)=\frac{d}{dt}N_1(t)=k_1\left[\left\{(1-\omega(t))I^+(t)\alpha(t)+\omega(t)I_2^+(t)\alpha_1(t)\right\}-N_1(t)\right].\tag{12}$$

[1] In this paper the words 'repeat' and 'upgrade' have been used interchangeably.

Similarly, the expressions for the potential number of adopters of the second-generation product due to first time purchasers and repeat purchasers can be given as $(1-\alpha_1(t_2))I_2^+(t) + N_1(t_2)\alpha_2'(t_2)$ respectively. Suppose, k_2 is the adoption parameter of the second generation product then the rate of adoption of the second generation product at time t can be given as:

$$\dot{N}_2(t) = \frac{d}{dt}N_2(t) = k_2\left[\omega(t)\left\{\left(I_2^+(t)\alpha_2(t) + N_1(t)\alpha_2'(t)\right) - N_2(t)\right\}\right] \tag{13}$$

where,

$$\alpha_1(t) = \alpha_1(\varphi_1(t), \varphi_2(t)) = \frac{e^{-d_1(\varphi_1(t))}}{e^{-d_1(\varphi_1(t))} + e^{-d_2(\varphi_2(t))}};$$
$$\alpha_2(t) = \alpha_2(\varphi_1(t), \varphi_2(t)) = \frac{e^{-d_2(\varphi_2(t))}}{e^{-d_1(\varphi_1(t))} + e^{-d_2(\varphi_2(t))}}$$

(where, $\alpha_1(t) + \alpha_2(t) = 1$) and $\alpha_2'(t_2) = \alpha_2'(\varphi_2(t)) = e^{-d_2\varphi_2(t_2)}$; where $\varphi_1(t)$ and $\varphi_2(t)$ are the quoted price of the first and second generation products respectively and d_1, d_2 are the price parameter. $\frac{\delta\alpha_i}{\delta\varphi_i} \le 0$, $\frac{\delta\alpha_i}{\delta\varphi_j} \ge 0$; $(i \ne j = 1, 2)$.

$t_2 = t - \tau$, τ is the introduction time of second generation product.

4. Policy implications

Previous literature has shown that price declines can have big impact on potential market and can stimulate the potential adopters to adopt the new product. Robinson and Lakhani (1979), Dolan and Jeuland (1981), Kalish and Lilien (1983) and Horsky (1990) have all concluded that the phenomenon is due to the learning curve effect which suggest that as production experience increases the cost of manufacturing decreases. Krishnan et al. (1999) had observed that there are two key catalysts which influence the pricing policies:

- Diffusion Pattern: optimal pricing policy should be based on the product *diffusion pattern* (Kalish & Lilien, 1983; Robinson & Lakhani, 1979).
- Discount Rate—when the discount rate is high, then a monotonically declining pricing policy is optimal (Kalish, 1985).

Danaher et al. (2001) showed that with decrease in price of second-generation increases its demand and at the same time it decrease the demand for first-generation. Padmanabhan and Bass (1993) discussed pricing strategy for inter-firm dependency. The authors compared the optimal pricing policies between the generational products manufactured by the same firm, with those produced by different firms. Their analysis shows that in competitive markets, the price of the first generation is higher than the price when they are produced by a single firm. In both cases, the price of the second generation is the same.

In this paper, we suggest optimal dynamic pricing policies for two generational products by considering intergenerational diffusion effect on prices under a finite time-horizon. We observed that when there are two technology generations of a product in the market, the relative adoption decision can become more price sensitive. While devising the pricing strategies, therefore it becomes imperative that the stages of diffusion as discussed above are imbibed into the analysis.

Consider a monopolist who has two generations of a high-technology product in the market. The firm intends to control its price over a finite planning horizon. Let 't' denote the time such that $0 \le t \le T$. The length of the planning period, T, is fixed.

The general diffusion model for the two generations can be given by Eqs. (12) and (13), where $N_1(0) = N_{10} \ge 0 =$ a constant and $N_2(0) = N_{20} \ge 0 =$ a constant.

We assume that the firm wants to maximize its total present value of net revenue discounted at a fixed interest rate r, over a finite planning horizon. The mathematical statement of the problem is (Sethi & Thompson, 2005):

$$\underset{p_1,p_2}{Max}\, J = \int_0^T e^{-rt}\left[[\varphi_1(t) - C_1]\,\dot{N}_1(t) + [\varphi_2(t) - C_2]\,\dot{N}_2(t)\right]dt \tag{14}$$

where, C_i are the constant marginal cost of production for product generation i $(i = 1, 2)$, r is the discount rate and T is the length of planning horizon such that

$$\begin{aligned} x_1(t) = \dot{N}_1(t) &= \frac{d}{dt}N_1(t) = g_1\left(I^+(t), N_1(t), N_2(t), \varphi_1(t), \varphi_2(t)\right) \\ &= k_1\left[\left\{(1-\omega(t))I^+(t)\alpha(t) + \omega(t)I_2^+(t)\alpha_1(t)\right\} - N_1(t)\right] \end{aligned} \tag{15}$$

$$\begin{aligned} x_2(t) = \dot{N}_2(t) &= \frac{d}{dt}N_2(t) = g_2\left(\overline{N}(t), N_1(t), N_2(t), \varphi_1(t), \varphi_2(t)\right) \\ &= k_2\left[\omega(t)\left\{\left(I_2^+(t)\alpha_2(t) + N_1(t)\alpha_2'(t)\right) - N_2(t)\right\}\right]. \end{aligned} \tag{16}$$

Functions (15) and (16) are twice differentiable, and $g_{1\varphi_1}, g_{2\varphi_2} > 0$; $g_{1\varphi_1}, g_{2\varphi_2\varphi_2} < 0$ and $g_{1\varphi_1\varphi_2} = g_{1\varphi_2\varphi}$; $g_{2\varphi_1\varphi_2} = g_{2\varphi_2\varphi_1}$.

Here, cost decline due to experience curve effect (Pegels, 1969) is assumed to be constant, to keep the derivations simple. The control variable $\varphi_i \ (i = 1, 2) = \varphi_i(t)$ are twice differentiable in 't' and satisfy $\varphi_i(t) \geq 0$, for $t \geq 0$.

4.1. Optimal pricing strategies

In this section the optimal pricing strategies for the two-product-generations monopolist is derived using dynamic optimal control theory. First the Hamiltonian is formulated and then the necessary conditions for optimality are formulated. The solution to the optimization problem is obtained by Pontryagin Maximum principle (Sethi & Thompson, 2005). Dropping the time notation, the current value Hamiltonian can be written as:

$$H = (\varphi_1 - C_1 + \lambda) \ \dot{N}_1(t) + (\varphi_2 - C_2 + \mu) \ \dot{N}_2(t) \tag{17}$$

where $\lambda(t)$ and $\mu(t)$ are the current value adjoint variables (i.e., shadow price of $\dot{N}_1 \ (t)$ and $\dot{N}_2 \ (t)$). Pontryagin maximum principle gives the following necessary conditions:

$$\begin{aligned} &\frac{d}{dt}\lambda = \dot{\lambda} = r\lambda - \frac{dH}{dN_1}, \quad \lambda(T) = 0 \\ &= r\lambda - (\varphi_1 - C_1 + \lambda)\frac{\delta x_1}{\delta N_1} - (\varphi_2 - C_2 + \mu)\frac{\delta x_2}{\delta N_1} \end{aligned} \tag{18}$$

$$\begin{aligned} &\frac{d}{dt}\mu = \dot{\mu} = r\mu - \frac{dH}{dN_2}, \quad \mu(T) = 0 \\ &= r\mu - (\varphi_1 - C_1 + \lambda)\frac{\delta x_1}{\delta N_2} - (\varphi_2 - C_2 + \mu)\frac{\delta x_2}{\delta N_2}. \end{aligned} \tag{19}$$

The physical interpretation of the current value Hamiltonian H can be given as follows: $\lambda(t)$ and $\mu(t)$ stand for the future benefits from first and second generation (at time 't') of having one more units produced. Thus the current value Hamiltonian is the sum of current profit $\left[(p_1 - C_1)\dot{N}_1 \ + (p_2 - C_2)\dot{N}_2\right]$ and the future benefit $\left[\lambda \dot{N}_1 \ + \mu \dot{N}_2\right]$. In short H represents the instantaneous total profit of the firm at time 't'. Other necessary conditions include $H_{\varphi_1} = H_{\varphi_2} = 0$,

$$\text{i.e. } \frac{dH}{d\varphi_1} = x_1 + (\varphi_1 - C_1 + \lambda)x_{1\varphi} + (\varphi_2 - C_2 + \mu)x_{2\varphi_1} = 0 \tag{20}$$

$$\text{and, } \frac{dH}{d\varphi_2} = (\varphi_1 - C_1 + \lambda)x_{1\varphi_2} + x_2 + (\varphi_2 - C_2 + \mu)x_{2\varphi_2} = 0. \tag{21}$$

From Eqs. (20) and (21), we have

$$\varphi_1^* = \begin{cases} C_1 - \lambda - \dfrac{x_1}{x_{1\varphi_1}}; & t < \tau \\ C_1 - \lambda - \dfrac{x_1 x_{2\varphi_2} - x_2 x_{2\varphi_1}}{x_{1\varphi_1} x_{2\varphi_2} - x_{2\varphi_1} x_{1\varphi_2}}; & t \geq \tau \end{cases} \tag{22}$$

and, the expression for the optimal price path for the second generational product can be given as

$$\varphi_2^* = \begin{cases} C_2 - \mu - \dfrac{x_2}{x_{2\varphi_2}} + \dfrac{x_{1\varphi_2}}{x_{2\varphi_2}}\left[\dfrac{x_1 x_{2\varphi_2} - x_2 x_{2\varphi_1}}{x_{1\varphi_1} x_{2\varphi_2} - x_{2\varphi_1} x_{1\varphi_2}}\right]; & t \geq \tau \\ C_2 - \mu - \dfrac{x_2}{x_{2\varphi_2}}; & \text{when the first generation product is withdrawn from the market.} \end{cases} \tag{23}$$

Other optimal conditions are:

$$\frac{\delta^2 H}{\delta\varphi_1 \delta\varphi_1} < 0; \ \frac{\delta^2 H}{\delta\varphi_2 \delta\varphi_2} < 0 \text{ and} \begin{vmatrix} H_{\varphi_1\varphi_1} & H_{\varphi_1\varphi_2} \\ H_{\varphi_2\varphi_1} & H_{\varphi_2\varphi_2} \end{vmatrix} > 0. \tag{24}$$

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Integrating Eqs. (18) and (19) with the transversality conditions, the future benefit of having one more unit produced for the respective generations can be written as:

$$\lambda(t) = \int_t^T \left[(\varphi_1 - C_1 + \lambda) x_{1N_1} + (\varphi_2 - C_2 + \mu) x_{2N_1} \right] e^{-rs} ds \tag{25}$$

$$\mu(t) = \int_t^T \left[(\varphi_1 - C_1 + \lambda) x_{1N_2} + (\varphi_2 - C_2 + \mu) x_{2N_2} \right] e^{-rs} ds. \tag{26}$$

Using the functional form of $\dot{N}_1(t)$ and $\dot{N}_2(t)$, the pricing strategies of first and second generation products can be given as:

$$\varphi_1^* = \begin{cases} C_1 - \lambda + \dfrac{x_1}{k_1 d_1 \overline{N}_1^{//}}; & t < \tau \\ C_1 - \lambda + \dfrac{1}{k_1} \left[\dfrac{1}{d_1} \left(\dfrac{x_1}{\overline{N}_1^{/} \alpha_2} \right) \left(\dfrac{\alpha_2 \overline{N}_1^{/} + \overline{R}_2^{/}}{\overline{R}_2^{/}} \right) + \dfrac{1}{d_2} \left(\dfrac{x_2}{\overline{R}_2^{/}} \right) \right]; & t \geq \tau \end{cases} \tag{27}$$

$$\varphi_2^* = \begin{cases} C_2 - \mu - \dfrac{1}{k_2 \overline{R}_2^{/}} \left[\dfrac{x_1}{d_1} + \left(\dfrac{x_2}{d_2} \right) \left(\dfrac{\alpha_2 \overline{N}_1^{/}}{\alpha_2 \overline{N}_1^{/} \alpha_2 + \overline{R}_2^{/}} \right) \right]; & t \geq \tau \\ C_2 - \mu + \dfrac{x_2}{d_2 k_2 \left[\overline{N}_1^{/} \alpha_2 + \overline{R}_2^{/} \right]}; & \textit{when the first genation product is withdrawn from the market} \end{cases} \tag{28}$$

where,

$\overline{N}_1^{//}(t) = I^+(t)\alpha(t)$ Potential first time purchasers of first generation product till the second generation is introduced.
$\overline{N}_i^{/}(t) = I_2^+(t)\alpha_i(t)$ Potential first time purchasers of ith $(i = 1, 2)$ generation product.
$\overline{R}_2^{/}(t) = N_1(t)\alpha_2'(t)$ Potential repeat purchasers of second generation product.

If the planning horizon is long enough, the following two general pricing strategies can be followed:

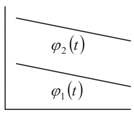
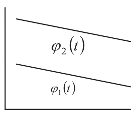
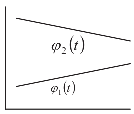
Policy 1. When the replacement rate of second generation product is high and if the potential upgraders from the first generation are inclined to purchase the second generation, then the optimal price paths of both the generations monotonically decrease over time.

The result indicates that the dynamic pricing pattern of first generation product depends on the behavior of upgraders, as well as, on the effectiveness of generating awareness for the product. Whereas, under full product information the pricing pattern of the second generation product depends on the replacement behavior of the first generation adopters, as well as, on the reduction of uncertainty of early adopters.

4.2. A subclass of the special functional form

If the planning horizon is long enough and also the discount rate is positive, then depending on the sign of relative future benefits for the adopters of the first generation and the proportion of potential up-graders for the second-generation, we have the following general pricing strategies:

Policy 2. If $\rho_1 > 0$ and $x = x(\varphi_1, \varphi_2)$, and the potential number of upgraders are very high, then:

	Conditions	Results	
Case 1	$H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right) H_{\varphi_1\varphi_1} > 0$	$\dot{\varphi}_1 < 0$ and $\dot{\varphi}_2 < 0$	
Case 2	$H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right) H_{\varphi_1\varphi_2} > 0$	$\dot{\varphi}_1 < 0$ and $\dot{\varphi}_2 < 0$	
Case 3	$H_{\varphi_1\varphi_2} = H_{\varphi_2\varphi_1} < 0$	$\varphi^{\bullet}{}_1 > 0$ and $\varphi^{\bullet}{}_2 < 0$	

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Proof. See Appendix A.

Case 1 and Case 2 suggest that when $H_{\varphi_2\varphi_1}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1}>0$ or $H_{\varphi_2\varphi_2}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2}>0 \Rightarrow H_{\varphi_1\varphi_2}=H_{\varphi_2\varphi_1}>0$ i.e., when the potential number of upgraders to second-generation is high, then decreasing the prices of both the generations simultaneously can optimize the total profit H at any time 't'. This is quite logical since the future benefit of selling a unit of any of the generations is always positive, and therefore, the sales growth rate of both the product generations may decrease the price. When the volume of potential upgraders is high and $H_{\varphi_2\varphi_1}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1}>0$, then variation in first-generation prices will have a significant effect on the overall profitability and sales. The optimal policy in such situation is that the first-generation price is initially high and then goes down and the optimal price of the second-generation product follows the same trend (because $H_{\varphi_2\varphi_1}>0$).

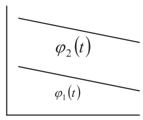
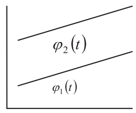
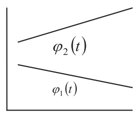
Similarly, when $H_{\varphi_2\varphi_2}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2}>0$ and the volume of potential repeat purchasers are high, variation in second-generation price will have a significant effect on the sales and profitability. The optimal policy in such situation is that the second-generation price is initially high and then goes down and the optimal price of the first-generation product will follow the similar trend.

$$\text{Case 3} \Rightarrow H_{\varphi_1\varphi_2}=H_{\varphi_2\varphi_1}<0 \Rightarrow H_{\varphi_2\varphi_1}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1}<0 \text{ and } H_{\varphi_2\varphi_2}+\left(\frac{\alpha_{2\varphi}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2}<0.$$

This indicates the firm should adopt a policy where prices of the two generations will go in opposite directions. So, in a situation where the rate of upgrades rate is high, the optimal strategy is to increase the price of first generation product and decrease the price of second-generation product over time.

When the number of repeat purchasers is very low then for the positive discount rate, we have the following pricing strategy:

Policy 3. If $\rho_1>0$ and $x=x(\varphi_1,\varphi_2)$, and the potential numbers of upgraders are low, then:

	Conditions	Results	
Case 1	$H_{\varphi_2\varphi_1}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1}>0$	$\dot{\varphi}_1<0$ and $\dot{\varphi}_2<0$	
Case 2	$H_{\varphi_2\varphi_2}+\left(\frac{\alpha_{2\varphi_2}}{\alpha\varphi_1}\right)H_{\varphi_1\varphi_2}>0$	$\dot{\varphi}_1>0$ and $\dot{\varphi}_2>0$	
Case 3	$H_{\varphi_1\varphi_2}=H_{\kappa_2\varphi_1}<0$	$\dot{\varphi}_1<0$ and $\dot{\varphi}_2>0$	

Proof. See Appendix A.

For slow rate of upgrades, Case 1 and Case 2 suggests that $H_{\varphi_1\varphi_2}=H_{\varphi_2\varphi_1}>0$. This implies that we can optimize the total profit H by simultaneously increasing (or simultaneously decreasing) the prices of both the generations over time. When second-generation product is introduced in the initial stages, firms can introduce reduced-price benefits for customers of both generations to increase the sales. The amount of the benefit depends on the discount rate—when it is zero the additional benefit is maximized during the initial stages of production. As the discount rate increases the firm reduces the level of assistance by increasing the price of first or second generation, depending on which one is more efficient to increase sales and overall profitability. This in turn depends on $H_{\varphi_1\varphi_2}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1}>0$ and $H_{\varphi_2\varphi_2}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2}>0$.

Similar to Policy 2 we can interpret the results of Policy 3, when $H_{\varphi_2\varphi_1}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1}>0$. In this situation fluctuation in first-generation prices has a major effect on the overall profitability and sales. Thus for slow rates of upgrades the optimal policy is that the first-generation price is initially high and then goes down and the optimal price of the second-generation product follows the same trend, as $H_{\varphi_2\varphi_1}>0$.

Similarly, for $H_{\varphi_2\varphi_2}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2}>0$ the optimal policy is that the second-generation price is initially low and then rises, and the optimal price of the first-generation product follows the same pattern.

$$\text{Case 3} \Rightarrow H_{\varphi_1\varphi_2}=H_{\varphi_2\varphi_1}<0 \Rightarrow H_{\varphi_2\varphi_1}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1x_1}}\right)H_{\varphi_1\varphi_1}<0 \text{ and } H_{\varphi_2\varphi_2}+\left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2}<0.$$

This indicates that the firm should adopt a policy where prices of both the generations will go up or down in the opposite direction. So, in a situation where rate of upgrades is slow the optimal strategy is to increase the price of second-generation product and decrease the price of first-generation product over time.

Results shown above are consistent with previous works in the area. We have shown that the pricing pattern of earlier generation product depends on the diffusion rate of the latest generation. Here, we proposed a model, which can optimize price for two-generation product. The results are also consistent with the conjecture made based on numerical optimization of price by Bayus (1992). He proposed a model for consumer sales of a new durable by incorporating the replacement behavior of previous generation product and proposed pricing strategies using numerical methods.

5. Parameter estimation and comparison

DRAMs are the highest volume commodity semiconductors built today, with about 11% of the total semiconductor market. It has shown clear discrete innovations in its product characteristics, especially in memory density, making it a typical multi-generation product. The DRAM, not only allows for almost perfect substitution among successive generations it also enhances competition, and causes product innovation while maximizing the number of generations to skim the high margins associated with early introduction. Also due to the PC boom and the growing need for memory in all information appliances, the DRAM sector became the lead product in the overall integrated circuit (IC) market by 1990 (Victor & Ausubel, 2001).

In this section the parameter estimation on a sales data set is discussed. The proposed model has been validated on Dynamic Random Access Memory (DRAM) computer chips data[2] and compared with two alternative models. Here, we have considered two-generation (64k and 256k) datasets of DRAM-family having 15 yearly observations of worldwide shipments from 1978 to 1992. The 64k DRAM-chipsets were introduced in the year 1978 and the 256k in the year 1982 (Islam & Meade, 1997; Norton & Bass, 1987; Versluis, 2002; Victor & Ausubel, 2001).

The proposed models for estimation are as follows:

Case 1. Single generation product in the market

$$\frac{d}{dt}N_1(t)=k\left[\left(I^+(t)\alpha(t)\right)-N_1(t)\right].$$

Case 2. Two-generation products are in the market

$$\frac{d}{dt}N_1(t)=k_1\left[\left\{(1-\omega(t))I^+(t)\alpha(t)+\omega(t)I_2^+(t)\alpha_1(t)\right\}-N_1(t)\right]$$
$$\frac{d}{dt}N_2(t)=k_2\left[\omega(t)\left\{\left(I_2^+(t)\alpha_2(t)+N_1(t)\alpha_2^{/}(t)\right)-N_2(t)\right\}\right].$$

The joint estimates of the parameter were obtained from a system of simultaneous nonlinear equations for three models—proposed by Norton and Bass (1987) and Islam and Meade (1997). The fit of these three models is summarized in Table 2. Table 2 gives the SSE values for each of the models along with the R-square values. From the table it is clear that the SSE of the proposed model is least in comparison to the other two models.

Also, as expected the estimated values of d_1 and d_2 are highly significant (Table 3). The absolute value of d_2, for newer generations is less than those for older generations, implying that the price sensitivity of newer generations of DRAM is lesser than that for older generations. This may be due to the fact that in the early stages the initial product generation may face a lot of restrictions in the market due lack of awareness. As a result the market initially becomes more price-sensitive, but as the positive word-of-mouth spreads in the market people become more aware of the product generations leading to a reduction of price sensitivity for the second or higher product generations.

The empirical results are encouraging for the estimation of the awareness. In Fig. 5, the estimated cumulative aware populations (favorable) of the product-generations and the eventual adopters of 64k and 256k have been plotted. The figure gives us a clear

[2] The data are available online at http://phe.rockefeller.edu/LogletLab/DRAM.

Table 2
Summary of the estimation for the proposed model, Norton–Bass and Islam–Meade model.

Model	Number of parameters	Is price significant		SSE		Adjusted R^2 64k/256k
		64k	256k	64k	256k	
Norton–Bass	4	–	–	189,260	585,574	0.773/0.632
Islam–Meade	6	–	–	51,024	507,185	0.934/0.654
Proposed	7	Yes[a]	Yes[a]	35,700	495,109	0.959/0.722

[a]Denotes significance at the 99% level.

Table 3
Parameter estimates of the proposed model.

Parameter	Estimate	Approximate standard errors
m	605,446.6	5587.6
K	1200	1790.7
p	0.00542	0.0019
q	0.91548	0.022
ε	0.38498	0.09
d_1	0.093	0.0015
d_2	0.076	0.0026
k_1	0.164	0.092
k_2	0.067	0.019
Adjusted R^2 (64k)		0.959
Adjusted R^2 (256k)		0.722

picture of transformation of favorably aware individuals to ultimate adopters. From the figure it can also be seen that in the initial stages the approval rate for the product generations are very low but in the later stages it gets momentum. This is probably due to the fact that at the initial stages people are uncertain about the product's perceived quality and, hence, the product faces the resistance in the market but at the later stages when more and more people started purchasing the product the uncertainty regarding the quality reduces substantially.

In Fig. 6, the overall potential market sizes of 64k and 256k have been plotted. The figure shows that there is substantial variation in the estimated potential markets over time. At the initial period, price of 256k DRAM was higher than that of 64k chipsets which dampen its sales and on the contrary boost the market size of 64k-chipsets. But, when the price of 256k falls below that of 64k (from the year 1984 onwards) the market potential for 256k shoots-up.

From Table 2 and Fig. 4, it can be concluded that the proposed model works better for the two generation DRAM-chipsets. Apart from goodness of fit criteria, the parameters of the proposed model provide important information. Fig. 7 separates the potential repeat purchasers from the potential new purchasers of 256k DRAM-chips. From the figure, it can be observed that at the initial stage major contribution to the market size of the chipset are coming from the first time purchasers, though the contribution from the repeat purchasers are increasing at the later stages.

6. Conclusions

In this paper we proposed a diffusion model, consistent with dynamics of the basic innovation–diffusion model, for the sales growth of a product with successive technology generations. The proposed model embeds a broader theoretical framework

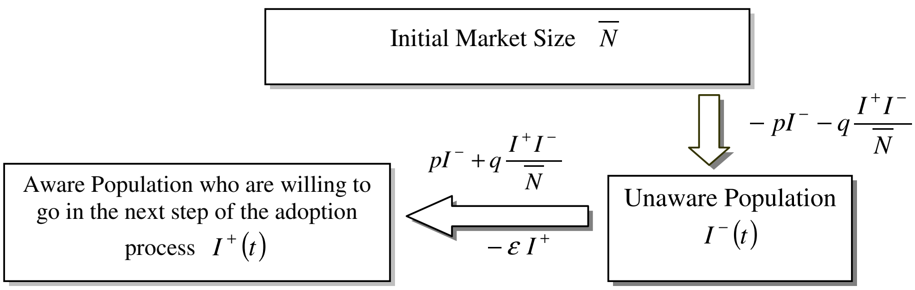


Fig. 3. Customer flow diagram.

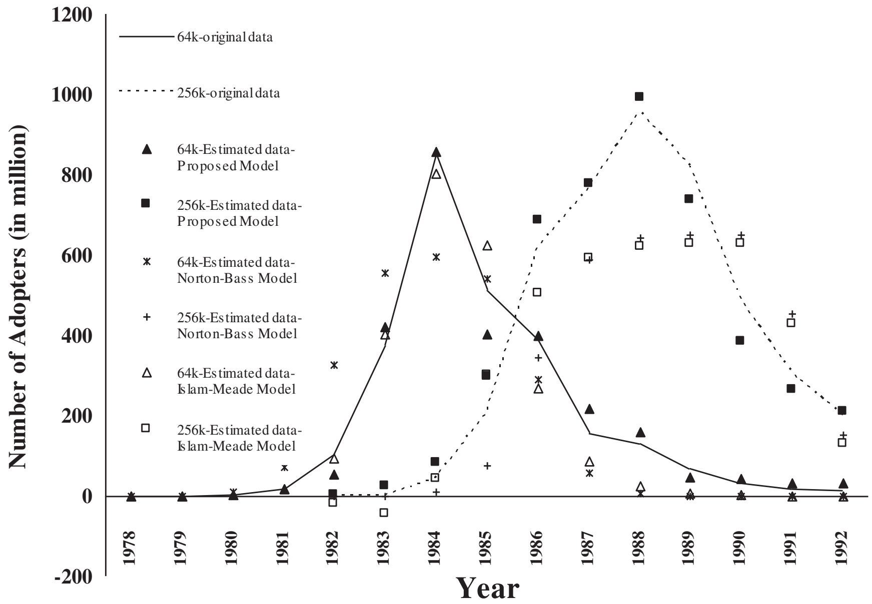


Fig. 4. Actual and estimated adopters (in million) of two-generation DRAM shipment.

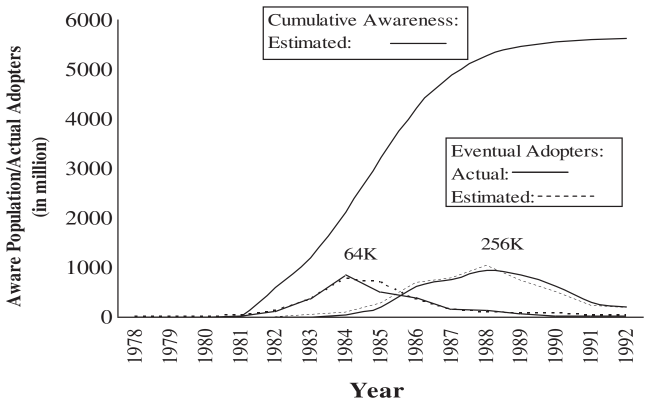


Fig. 5. Estimated cumulative awareness and actual and estimated adoption of 64k and 256k DRAM chipsets.

which accounts for the interactions between technological evolutions, market adoption. The proposed model also considers the heterogeneity of the market population, the innovation decision process based on expected utility, the reduction of perceived risk of adoption through collecting information, plus upgrading. The link between technological evolution and market dynamics

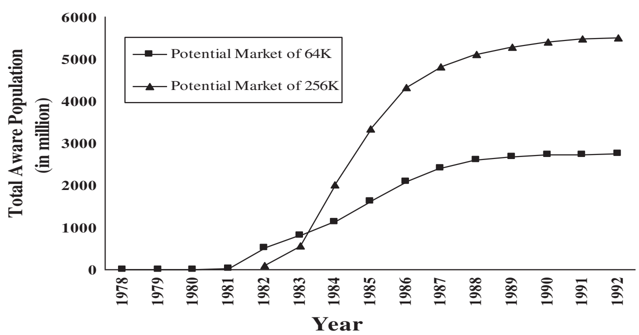


Fig. 6. Dynamics of estimated potential market size of 64k and 256k DRAM chipsets (in million).

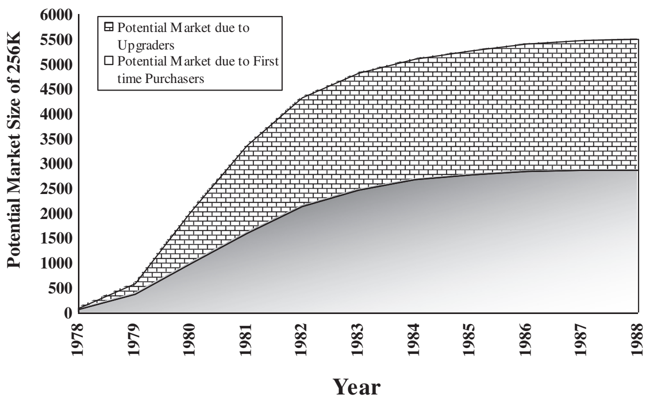


Fig. 7. Segmentation of estimated potential markets of 256k DRAM chipset (in million).

has been allowed by modeling the diffusion process in several stages. For many technology-generations especially for high technology market word-of-mouth can have a major impact on consumer awareness to a product and the associated promotional strategy. It can play an important role to reduce the product-uncertainty and can play an effective role in the acceptance of the new product. But none of the technological-generation models discuss the effect of dropouts in driving the diffusion of new products. The proposed model incorporates the effects of unfavorable information, as well as, the positive word-of-mouth in the awareness diffusion process. The proposed model produces reliable and better parameter estimates. We also discussed situations where optimal new product prices decrease and increase. The optimal timing strategy for introduction of second generation product was discussed briefly.

Extending the model to third and fourth generations is straightforward. Though the experience during statistical estimation has been encouraging it would be interesting to observe the marketing mix interaction for a variety of products. Also, the proposed model is based on the traditional communication channels and doesn't explicitly include network effects. For multiple technology generations network effects can influence the takeoff or success of a new product heavily. In this model, we have considered the effect of continuous innovation. It will be interesting to see in the presence of other dimensions (e.g., when the new product is substantially different from the core technology).

To validate our model, data on successful consumer durable have been used. It would be interesting to see whether the proposed model works well for other product categories and new-products that have failed. As the proposed model is based on traditional diffusion model which are sensitive to the number of observations, substantial data is required for parameter estimation and validation. For many consumer durables the life-span of the product is not long enough and to forecast the growth of such products during early stages can be more productive to managers. Thus research on some alternative approach to predict the sales growth for very limited data is an important area to explore. Incorporating advertisement as one of the decision variable in the model can be another likely extension as it can reduce search costs substantially on the consumer side. Modeling and optimizing the influence of social networks on success of technological innovations is another upcoming and challenging area for future research.

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Appendix A

Optimal pricing

Optimal dynamic price paths of $\varphi_1(t)$ and $\varphi_2(t)$ can be characterized by determining the sign of the time derivatives of the two price functions. Taking the time derivative of Eqs. (11) and (12) we have

$$\left[2x_{1\varphi_1} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_1\varphi_1} + (\varphi_2 - C_2 + \mu)x_{2\varphi_1\varphi_1}\right]\dot{\varphi}_1 + \left[x_{1\varphi_2} + x_{2\varphi_1} + (\iota_1 - C_1 + \lambda)x_{1\varphi_1\varphi_2} + (\varphi_2 - C_2 + \mu)x_{2\varphi_1\varphi_2}\right]\dot{\varphi}_2 = \rho_1 \quad (A.1)$$

$$\left[x_{1\varphi_2} + x_{2\varphi_1} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_2\varphi_1} + (\varphi_2 - C_2 + \mu)x_{2\varphi_2\varphi_1}\right]\dot{\varphi}_1 + \left[2x_{2\varphi_2} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_2\varphi_2} + (\varphi_2 - C_2 + \mu)x_{2\varphi_2\varphi_2}\right]\dot{\varphi}_2 = \rho_2 \quad (A.2)$$

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where

$$\rho_1 = -\dot{\lambda} x_{1\varphi_1} - \dot{\mu} x_{2\varphi_1} - \dot{N}_1\left[x_{1N_1} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_1 N_1} + (\varphi_2 - C_2 + \mu)x_{2\varphi_1 N_1}\right] - \dot{N}_2\left[x_{1N_2} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_1 N_2} + (\varphi_2 - C_2 + \mu)x_{2\varphi_1 N_2}\right] \tag{A.3}$$

$$\rho_2 = -\dot{\lambda} x_{1\varphi_2} - \dot{\mu} x_{2\varphi_2} - \dot{N}_1\left[x_{2N_1} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_2 N_1} + (\varphi_2 - C_2 + \mu)x_{2\varphi_2 N_1}\right] - \dot{N}_2\left[x_{2N_2} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_2 N_2} + (\varphi_2 - C_2 + \mu)x_{2\varphi_2 N_2}\right]. \tag{A.4}$$

Since

$$\begin{aligned}
H_{\varphi_1\varphi_1} &= 2x_{1\varphi_1} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_1\varphi_1} + (\varphi_2 - C_2 + \mu)x_{2\varphi_1\varphi_1};\\
H_{\varphi_2\varphi_2} &= 2x_{2\varphi_2} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_2\varphi_2} + (\varphi_2 - C_2 + \mu)x_{2\varphi_2\varphi_2};\\
H_{\varphi_1\varphi_2} &= x_{1\varphi_2} + x_{2\varphi_1} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_1\varphi_2} + (\varphi_2 - C_2 + \mu)x_{2\varphi_1\varphi_2};\\
H_{\varphi_2\varphi_1} &= x_{1\varphi_2} + x_{2\varphi_1} + (\varphi_1 - C_1 + \lambda)x_{1\varphi_2\varphi_1} + (\varphi_2 - C_2 + \mu)x_{2\varphi_2\varphi_1}
\end{aligned}$$

we can rewrite Eqs. (A.1) and (A.2) as

$$H_{\varphi_1\varphi_1}\dot{\varphi}_1 + H_{\varphi_1\varphi_2}\dot{\varphi}_2 = \rho_1 \tag{A.5}$$

$$H_{\varphi_2\varphi_1}\dot{\varphi}_1 + H_{\varphi_2\varphi_2}\dot{\varphi}_2 = \rho_2. \tag{A.6}$$

Solving Eqs. (A.5) and (A.6), we have

$$\dot{\varphi}_1 = \frac{1}{\Delta}\left[\rho_1 H_{\varphi_2\varphi_2} - \rho_2 H_{\varphi_1\varphi_2}\right] \text{ and } \dot{\phi}_2 = -\frac{1}{\Delta}\left[\rho_1 H_{\varphi_2\varphi_1} - \rho_2 H_{\varphi_1\varphi_1}\right] \tag{A.7}$$

where $\Delta = \begin{vmatrix} H_{\phi_1\phi_1} & H_{\phi_1\phi_2} \\ H_{\phi_2\phi_1} & H_{\phi_2\phi_2} \end{vmatrix} > 0$ also, by definition $(H_{\phi_1\phi_1}; H_{\phi_2\phi_2}) < 0$.

The R.H.S. of the above two equations will determine the sign of the price path of the respective generations.

When word-of-mouth influence is not important, then from Eqs. (26)–(28) $\rho_1 = rd_1(\lambda k_1 \overline{N}_1' \alpha_2 - \mu k_2 \overline{N}_2' \alpha_1)$ $\rho_2 = rd_2[\mu k_2(\overline{N}_2'\alpha_1 + \overline{R}_2') - \lambda k_1 \overline{N}_1' \alpha_2] = -\rho_1\left(\frac{d_2}{d_1}\right) + rd_2\mu k_2 \overline{R}_2'$. Here, ρ_1 and ρ_2 can be defined as the relative future benefits of first and second generation products, respectively. We can also write $\frac{d_2}{d_1} = \frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}$.

$$(A.7) \Rightarrow \dot{\varphi}_1 = \frac{1}{\Delta}\left[\rho_1\left(H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2}\right) - rd_2\mu k_2 \overline{R}_2' H_{\varphi_1\varphi_2}\right] \tag{A.8}$$

$$\text{and } \dot{\varphi}_2 = -\frac{1}{\Delta}\left[\rho_1\left(H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1}\right) - rd_2\mu k_2 \overline{R}_2' H_{\varphi_1\varphi_1}\right]. \tag{A.9}$$

For the proposed model, we have $H_{\varphi_1\varphi_2} = H_{\varphi_2\varphi_1}$. Also, it is quite clear that when the discount rate $r = 0$, the price path of the both the generation remains the same over the entire planning horizon.

Proposition 1. If $H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} > 0$, then $H_{\varphi_1\varphi_2} > 0$ and $H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1} < 0$.

Proof. We have $(H_{\varphi_1\varphi_1}; H_{\varphi_2\varphi_2}) < 0$. Since $H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} > 0 \Rightarrow H_{\varphi_1\varphi_2} > 0$. Again the determinant

$$\begin{vmatrix} H_{\varphi_1\varphi_1} & H_{\varphi_1\varphi_2} \\ H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1} & H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} \end{vmatrix} = \begin{vmatrix} H_{\varphi_1\varphi_1} & H_{\varphi_1\varphi_2} \\ H_{\varphi_2\varphi_1} & H_{\varphi_2\varphi_2} \end{vmatrix} = \Delta. \text{ Thus, when}$$

$$H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} > 0 \Rightarrow H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1} < 0.$$

Proposition 2. If $H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1} > 0$, then $H_{\varphi_2\varphi_1} > 0$ and $H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} < 0$.

Proof. Similar to Proposition 1.

Proposition 3. If $H_{\varphi_1\varphi_2} = H_{\varphi_2\varphi_1} < 0$ then $H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1} < 0$ and $H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} < 0$.

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Proof. Proof follows from Propositions 1 and 2.

Corollary 1.1. When $\rho_1 < 0$ and $x = x(\varphi_1, \varphi_2)$ and repeat purchasing is high, then

	Conditions	Results
Case 1	$H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1} > 0$	$\dot{\varphi}_1 < 0$ and $\dot{\varphi}_2 < 0$
Case 2	$H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} > 0$	$\dot{\varphi}_1 < 0$ and $\dot{\varphi}_2 < 0$
Case 3	$H_{\varphi_1\varphi_2} = H_{\varphi_2\varphi_1} < 0$	$\dot{\varphi}_1 > 0$ and $\dot{\varphi}_2 < 0$

Corollary 2.1. When $\rho_1 < 0$ and $x = x(p_1, p_2)$ and repeat purchasing is slow, then

	Conditions	Results
Case 1	$H_{\varphi_2\varphi_1} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_1} > 0$	$\dot{\varphi}_1 > 0$ and $\dot{\varphi}_2 > 0$
Case 2	$H_{\varphi_2\varphi_2} + \left(\frac{\alpha_{2\varphi_2}}{\alpha_{1\varphi_1}}\right)H_{\varphi_1\varphi_2} > 0$	$\dot{\varphi}_1 < 0$ and $\dot{\varphi}_2 < 0$
Case 3	$H_{\varphi_1\varphi_2} = H_{\varphi_2\varphi_1} < 0$	$\dot{\varphi}_1 > 0$ and $\dot{\varphi}_2 < 0$

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