Radio Astronomy with a Satellite Dish

Michael Gaylard
Hartebeesthoek Radio Astronomy Observatory

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# 1 Radio Waves

Radio waves have wavelengths greater than a centimetre (Fig. 1). For example, commercial FM radio operates in the frequency range from 88 to 108 MHz, corresponding to a wavelength of about 3 metres. Cellphones operate at a frequency of 900 MHz, i.e. a wavelength of 33 cm.

The microwave band is the short wavelength part of the radio band and covers 1 to 30 cm wavelength. Microwave ovens operate at 12 cm (2.4 GHz). DSTV satellites transmit at 2.5 cm (12 GHz).

# 2 Radio Telescope Antennas

A "classic" radio telescope comprises a circular parabolic reflector with a feed horn at the focus to collect the incoming microwaves and pass them to transistor amplifiers in a receiver. A DSTV satellite dish also works in this way. It can be used as a mini-radio telescope by replacing the DSTV decoder with a radiometer for measuring the signal strength.

To understand how a reflector antenna responds to radiation coming from different angles, consider what happens when a plane wave of wavelength $\lambda$ arrives at a circular aperture of diameter $D$ (Fig. 2). Constructive and destructive interference produces a circularly symmetric diffraction pattern, with a central maximum and concentric rings of decreasing strength (Fig. 3). The first minimum or null occurs at a radius of $1.22\lambda/D$ radians. This same pattern describes the response of a circular antenna to plane waves coming from different angles, and it is then called an antenna beam pattern.

An "ideal" antenna would produce a beam that captures 100% of the incoming energy in the centre of the main beam and would have no sidelobes. This antenna would have an "aperture efficiency" $\epsilon_{ap}$ of 1.0. It is not possible to actually achieve this, and it usually lies between 0.4 and 0.7.

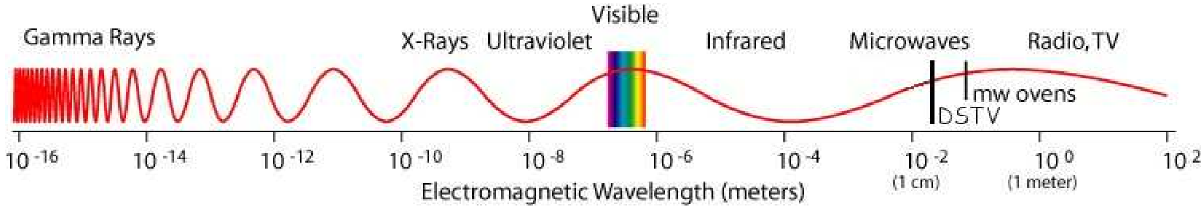


Figure 1: *The electromagnetic spectrum.*

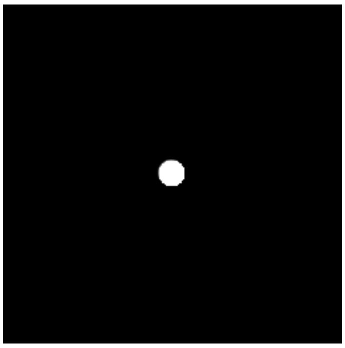

Figure 2: *A focusing lens or reflector with a circular aperture.*

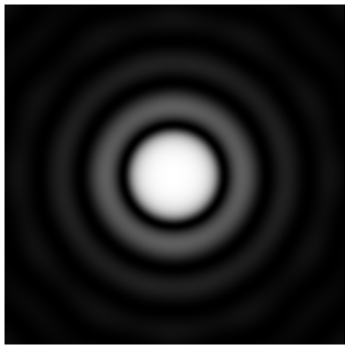

Figure 3: *The diffraction pattern produced by a circular focusing lens or reflector.*

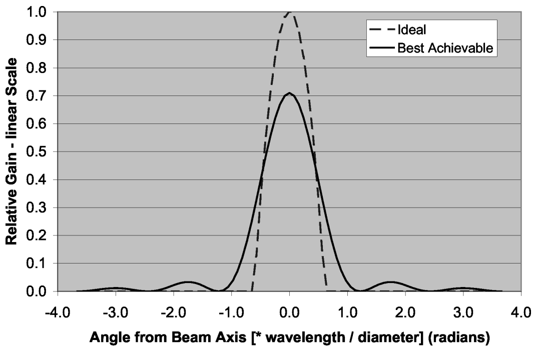


Figure 4: *Cross-sections through the beam pattern of an ideal antenna and a practically realizable antenna, shown with a linear vertical scale.*

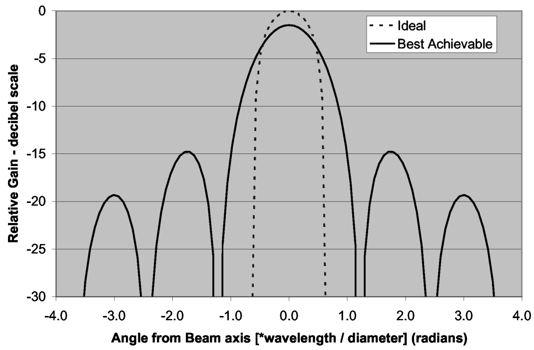


Figure 5: *Beam cross-sections of ideal and practically realizable antennas, with a logarithmic vertical scale to show the sidelobe structure.*

Figs. 4 and 5 show an ideal and an actual beam pattern in cross-section on linear and logarithmic scales. The "ideal" pattern has been modelled here with a parabolic shape, while the mathematical form of the real pattern is a $(\sin X/X)^2$ function. This describes the diffraction pattern where nothing obstructs the path of the waves, i.e. has an "unblocked aperture". The first minimum or null in the pattern occurs at a radius of about $1.2\lambda/D$ radians, so the beamwidth to first nulls is

$$BWFN \sim 2.4\lambda/D \quad \text{[radians]}. \tag{1}$$

The beamwidth at the half-power points (HPBW), also called the Full Width at Half Maximum (FWHM) is about half this, as shown in figs. 4 and 5.

$$HPBW = FWHM \sim 1.1\lambda/D \quad \text{[radians]}. \tag{2}$$

## 3 Brightness Temperature and Antenna Temperature

Radio astronomy appears as a blend of astronomy and basic electric circuit theory. This derives from the fact that the radio telescope can be considered as an electric circuit, and the object being observed can be considered as a resistor at a particular temperature connected to the first amplifier in the receiver (by radio waves rather than by wires). For some astronomical objects the temperature of a radio source that is measured using a radio telescope is meaningful as a physical temperature, for example a rocky planet or moon. For others it is not, depending on the radiation mechanism involved. However we call the apparent temperature of these emitters their "brightness temperature" $T_B$, at a particular frequency, as though they were black body radiators.

Fig. 6 shows the brightness as a function of frequency for several black body radiators modelled as having equal size but different temperatures. The frequencies of satellite TV transmission and visible light are

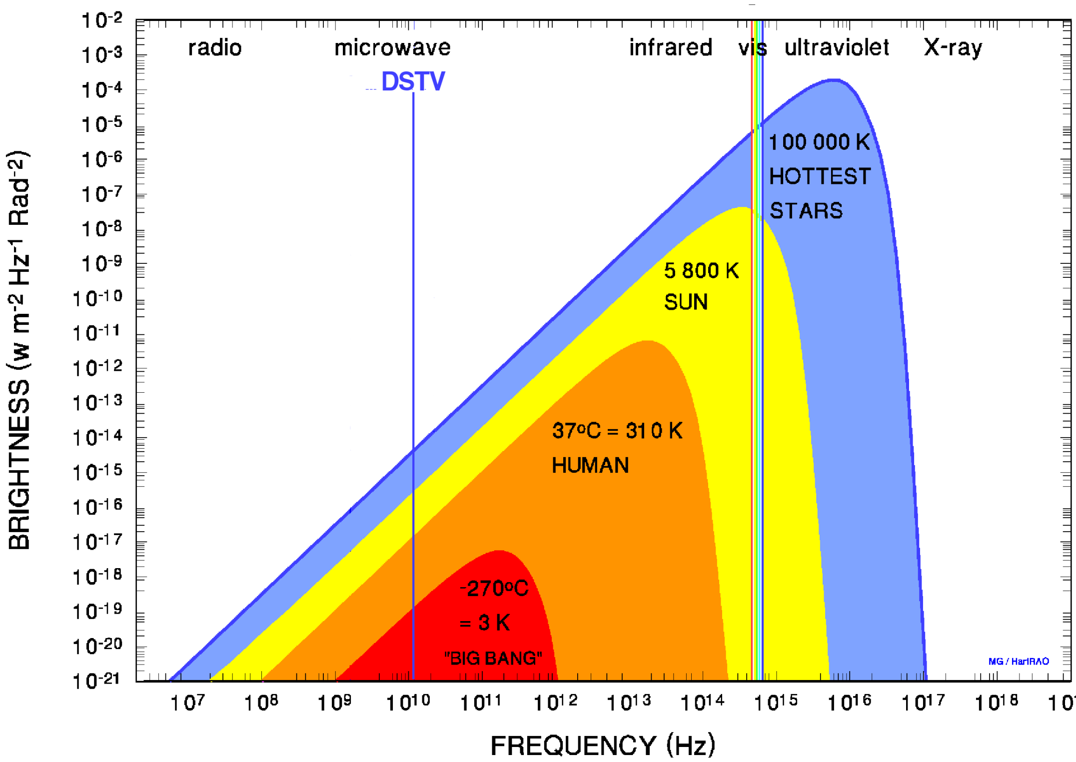


Figure 6: *Blackbody radiation from solid objects of the same angular size, at different temperatures*

marked. Clearly hotter objects produce more radiation than cooler ones, and the brightness maximum occurs at a higher frequency. The wavelength or frequency at which the intensity peaks is given by the well-known Wien displacement law.

From Fig. 6 we can see that for all objects with temperatures more than a couple of degrees above absolute zero, the brightness peak occurs well above the operating range of radio telescopes. Hence we are working in the range where $h\nu << kT$, so the Rayleigh-Jeans law applies and the brightness $B$ is proportional to the temperature:

$$B = \frac{2kT}{\lambda^2} \quad [\mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}}] \tag{3}$$

where k = Boltzmann's constant = $1.38 \times 10^{-23}$ [J K$^{-1}$].

The power $w$ per unit bandwidth received at the terminals of a resistor is proportional to its temperature $T$:

$$w = kT \quad [\mathrm{W\ Hz^{-1}}] \tag{4}$$

We can regard the radio source observed by the telescope as being equivalent to a resistive load with a given temperature on the input to the first amplifier in the receiver system on the telescope. We call this temperature the oberved "antenna temperature" $T_A$ produced by the radio source in the beam. Note that "antenna temperature" has nothing to do with the physical temperature of the antenna.

In summary, the brightness temperature at a given frequency is a property of the object emitting the radio waves. The antenna temperature is what is measured by a given radio telescope pointing at the object. A radio telescope is a remote-sensing thermometer.

To obtain the brightness temperature $T_B$ of the emitter from its measured antenna temperature $T_A$, we measure the angular sizes of the emitter and the telescope beam and correct for these.

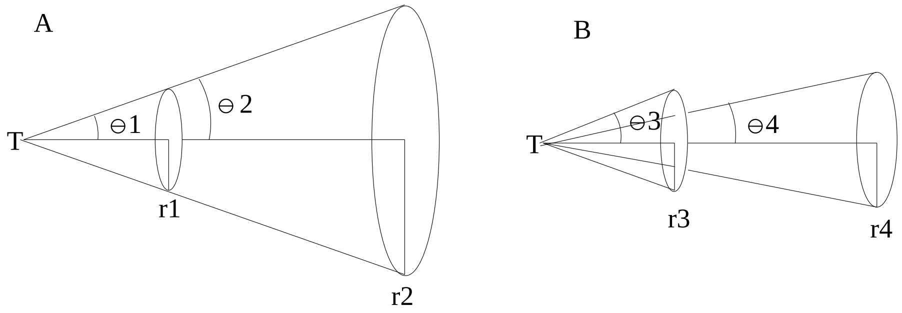


Figure 7: *Comparison of physical radius $r$ and angular radius $\theta$. In diagram A the two objects have different physical radii: $r2 < r1$, but the same angular radii as seen by telescope $T$: $\theta 1 = \theta 2$. This applies to the Moon and Sun as seen from Earth. By contrast, in diagram B, $r4 > r3$, but $\theta 4 < \theta 3$.*

# 4 Angular Sizes and the Sun's Brightness Temperature

The difference between physical diameter and angular diameter is shown in Fig. 7. For example, the Sun has a physical diameter of 1.4 million km, while the Moon has a diameter of 3500 km. Yet, as seen from the Earth, the Sun and the Moon appear to be the same size, i.e. they have the same angular diameter. How can this be? The Sun is 400 times bigger than the Moon, but it is also 400 times further away. As the projection of the antenna beam onto the sky is two-dimensional, we shall need to find the angular area that it covers. Angular area is called a "solid angle" and the units are radians$^2$, or steradians (sr). An object with an angular radius $\theta$ radians subtends a solid angle

$$\Omega = 2\pi(1 \; - \; cos \; \theta) \quad [\text{sr}]. \tag{5}$$

For small $\theta$,

$$\Omega = \pi\theta^2 \quad [\text{sr}]. \tag{6}$$

We can use this equation to calculate the solid angle of the Sun, $\Omega_s$.

The beam solid angle $\Omega_A$ of the antenna can be obtained from the HPBW - in units of radians - using

$$\Omega_A \; \sim \; 4/3 \; (HPBW)^2 \quad [sr]. \tag{7}$$

This formula includes an allowance for the beam efficiency of a typical reflector antenna.

The Sun's brightness temperature $T_B$ can then be estimated by scaling its measured antenna temperature $T_A$ by the ratio of the beam solid angle $\Omega_A$ to the Sun's solid angle $\Omega_s$:

$$T_B = \frac{\Omega_A T_A}{\Omega_s} \quad [\text{K}]. \tag{8}$$

# 5 A Simple Radio Telescope

The main parts of a simple radio telescope comprising a satellite dish and radiometer are shown in Fig. 8. The incoming radio waves from natural emitters are weak and noise-like. If the output of the detector is connected to a loudspeaker, the signal sounds like a hiss, as one hears if a radio is tuned off-station. The internal noise produced in the amplifiers is generally larger than the signal from natural radio sources.

The internal noise of the receiver can be characterised as though it were a resistor with a "noise temperature" $T_R$. This is often quoted in the form of a "noise figure" $F$, which is calculated relative to a nominal ambient temperature of 290 K (17°C):

$$F = 1 + T_R/290 \tag{9}$$

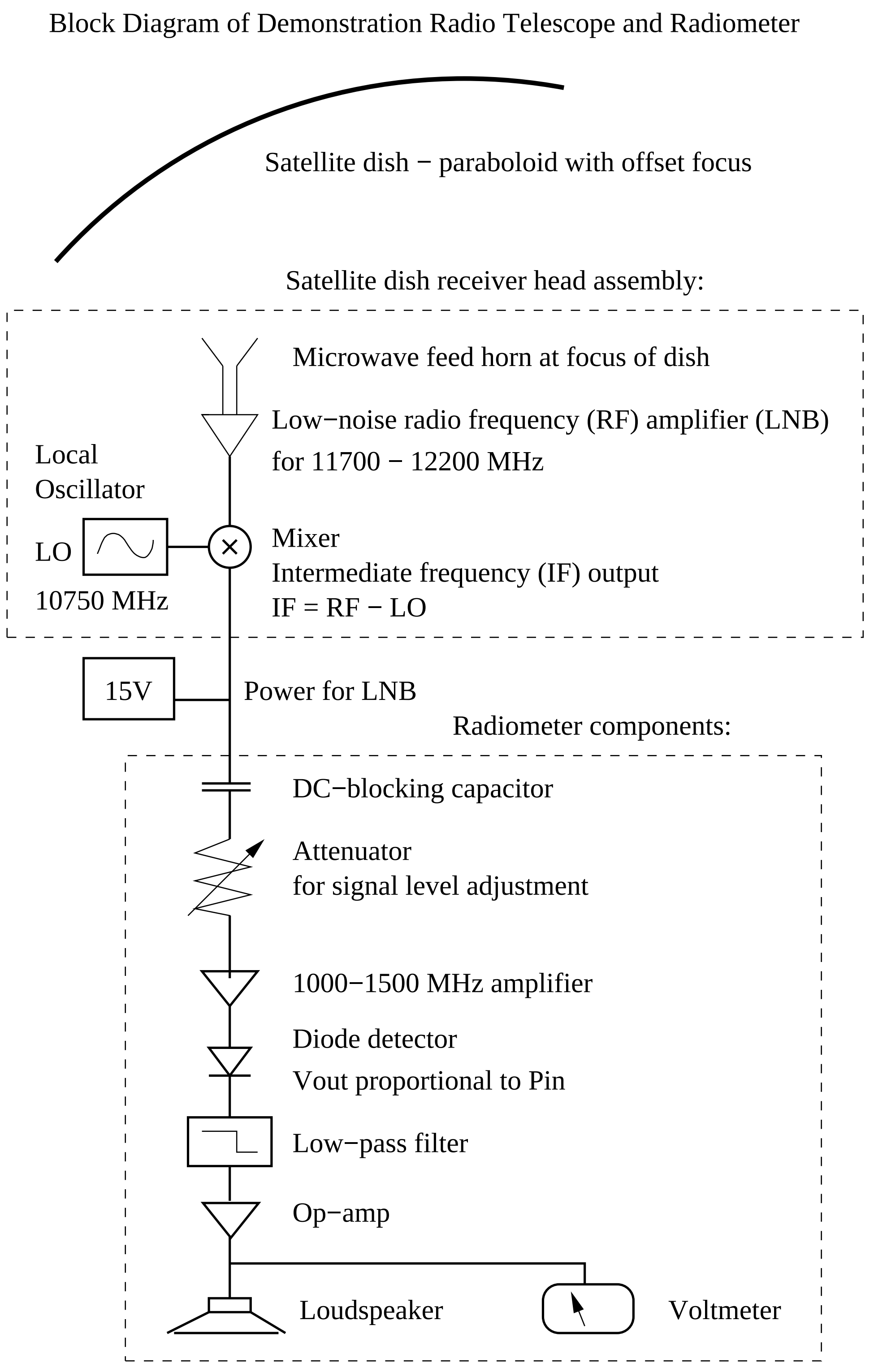


Figure 8: *Main components of a typical satellite dish and radiometer.*

and can also be expressed in decibels:

$$F_{dB} = 10\ log_{10}(F) \quad \text{[dB]} \tag{10}$$

Signal losses in waveguide and co-axial cable are large at microwave frequencies, so mixers are used to convert signals down to lower frequencies that can be passed with little loss through long cables.

# 6 Detecting Radio Emission from Space

When the telescope looks at a radio source in the sky, the receiver output is the sum of the radio waves received from several different sources:

- Behind the radio source whose brightness we want to measure is the cosmic microwave background (CMB) coming from every direction in space. This is the relic radiation left as the first atoms formed 380 000 years after the Big Bang. The black body temperature of the CMB $T_{cmb}$ has now decreased to 2.7 Kelvins, thanks to the expansion of the Universe. This produces a brightness temperature $T_{Bcmb}$ of ~2.7 K at 1.4 or 4 GHz, reducing to 2.5 K at 12 GHz.
- The emission from the radio source we want to measure, which produces the antenna temperature $T_A$.
- Radiation from the dry atmosphere $T_{at}$ and from the water vapour in the atmosphere $T_{wv}$. The dry air adds about 1K, and at 12 GHz water vapour adds 1 - 2 K, depending on the humidity.
- The radiation the feed receives from the ground beyond the edge of the antenna, $T_g$. Lowest value with antenna pointing at zenith could add 3 K. It increases when pointing away from zenith.
- The amplifiers in the receivers generate their own electronic noise and so produce a receiver noise temperature $T_R$.

The sum of these parts is called the "system temperature" $T_{sys}$. Summing from the most distant noise contributor to the nearest we have:

$$T_{sys} = T_{Bcmb} + T_A + T_{at} + T_{wv} + T_g + T_R \quad \text{[K]} \tag{11}$$

# 7 Calibrating the Radio Telescope

The satellite dish produces an output voltage proportional to the sum of the temperature of the object it is pointed at plus its own internal receiver temperature. We need to establish a scale of Kelvins per radiometer output unit. We do this by using the sky at zenith as a "cold load", and the ground as a "hot load". If $V1$ and $V2$ are the two meter readings and $c$ is a constant of proportionality, then:

$$T_R + T_{sky} = cV1 \tag{12}$$

where $T_{sky} = T_{Bcmb} + T_{at} + T_g + T_{wv} \sim\ 7\ -\ 8$ K.

$$T_R + T_{ground} = cV2 \tag{13}$$

$T_{ground}$ must be measured (or estimated).

Now we can use Eqns. 12 and 13 to solve for the two unknowns, $c$ and $T_R$.

Then, using Eqns. 9 and 10, what is the noise figure of the receiver in dB?

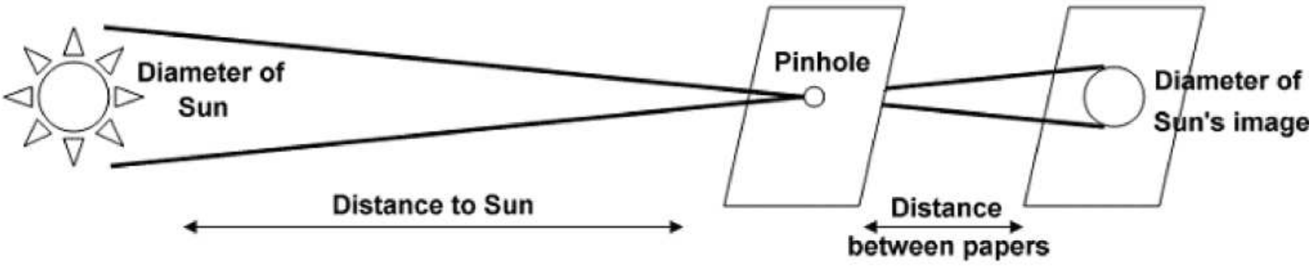


Figure 9: *Sun projection.*

# 8 Measuring the Antenna Temperature of the Sun

Aiming the telescope at the Sun, we get a new meter reading $V3$:

$$T_R + T_{sky} + T_{Asun} = cV3. \quad (14)$$

As $c$, $T_R$ and $T_{sky}$ are known, we can immediately get the antenna temperature of the Sun as measured with this telescope.

To convert this to a brightness temperature we need the angular diameters of the Sun and the telescope beam to get their solid angles.

# 9 Measuring the Angular Diameter of the Sun

The usual sources of information will give the angular diameter of the Sun. However it is instructive to actually measure the diameter of the photosphere - the light-emitting surface of the Sun.

Use pinhole projection to measure the angular diameter of the Sun. You will need two pieces of paper or card, one pierced with a small hole ($\sim$2 mm), a ruler and a tape measure. The hole need not be circular, and indeed it is interesting to make it an equilateral triangle - it is also then easy to cut with a sharp knife or blade. The card with the hole is used to project an image of the Sun onto the second card (Fig. 9). The two cards need to be separated by a distance $D$, measured by tape measure. The linear diameter $d$ of the projected Sun is measured with the ruler. The angular diameter of the Sun is d/D radians. What range of $D$ will give an accurate result? If you were given a small mirror instead of a card with a hole, how would you use it? Don't forget to use the radius in calculating the solid angle of the Sun using Eqn. 6.

# 10 Measuring the Half-Power Beamwidth

The halfpower beam width of the antenna can be estimated using Eqn. 2.

However, if the satellite dish is mounted so that its azimuth and elevation can be set and measured to an accuracy better than a degree, then it is possible to set it to carry out a "drift scan" across the Sun. To do this a prediction of the Sun's position is needed - use a program such as Starcalc (free download) to predict its position in the near future. Recall that the Sun moves 1 degree in four minutes, so use the estimated separation of the first minima of the beam (Section 2) to calculate how far ahead you must position the antenna.

The drift scan will give a cross-section of the antenna beam pattern, as in Fig. 4. The $HPBW$ is the width between the points at which the signal is half its maximum value. Units of time are converted to angle by noting that the Sun moves through 1 degree in 4 minutes / cos(declination). The Sun's declination is how far north or south of the equator it is at the time of observation. Its declination is -23.44° on about Dec 21, 0° on March 21 and September 21, and +23.44° on June 21. It follows a sinusoidal path between those dates.

Then calculate the beam solid angle $\Omega_A$ from the half-power beamwidth HPBW (in radians) using Eqn. 7.

## 11 Calculating the Brightness Temperature of the Sun

We now have the information we need to calculate the brightness temperature of the Sun. We have the Sun's solid angle from Eqn. 6, the beam solid angle $\Omega_A$ from Eqn. 7, and the antenna temperature measured from the Sun $T_A$, using Eqn. 14, which go into Eqn. 15:

$$T_B = \frac{\Omega_A T_A}{\Omega_s} \quad \text{[K]}. \tag{15}$$

Estimate the uncertainty in your result in the usual way.

How does your result for the Sun's brightness temperature compare to the temperature usuually quoted for the Sun's photosphere (light emitting surface)? What do you think this implies?

## References

Several good books are available on radio astronomy. Miller is a free download from the internet, and is great as a starter. Kraus is the classic reference on radio astronomy. Burke & Graham-Smith is a good modern overview. Rohlfs & Wilson is the current standard reference for radio astronomers. Baars' paper is a useful summary on practicalities. William Lonc's book provides lots of practical examples of low-cost projects in radio astronomy using mass-produced hardware as far as possible.

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