Homework - week 8

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1 Arcidiacono Study question:

$\frac{0.25*\exp(-1000)}{0.25*exp(-1000)+0.75*\exp(-1002)} = \frac{1}{1+\exp(-2)} = 0.7112346$

2 Aguirregibiria Part 1:

Question-1 As given in the graph S is the global minima. Suppose K is the fixed order cost. Since f is a K- convex it means that $f(S)+K$ will be achieved at one point. It is clear from the figure that point is s and also because of strict convexity it will be higher for each inventory level in (s, S).

3 Aguirregibiria Part 2:

Writting the bellman equation for this stochastic dyanmic programming:

Question-1

$\bar{V}(i,s) = E[\max_{q=0,1} U_{is}(q) + \epsilon_q + \beta *$ next period expected value at a given i].

$$\bar{V}(i,s) = E[\max_{q=0,1} U_{is}(q) + \beta * \tilde{V(i')}].$$

Substituting the value of i',

$$\bar{V}(i,s) = E[\max_{q=0,1} U_{is}(q) + \beta * \tilde{V}(i-s+q(Q-i+S))].$$

Using the formula from the notes:

$$\bar{V}(i,s) = \sum_{q=0,1} P_{i,s}(q) * (U_{i,s}(1) + \beta\tilde{V}_Q - \log(P_{i,s}(q)))$$

Taking the expectation w.r.t. s on both sides:

$$\tilde{V}_i = f_i + \beta\tilde{V}_Q + E[P_{i,s}(0)(U_{i,s}(0) + \beta\tilde{V}_{i-s} - \log(P_{i,s}(0))).$$

Question-2 From the definition of CCP

$$P_{i,s}(1) = \frac{\exp(\bar{v}_{i,s}(1))}{\exp(\bar{v}_{i,s}(0)) + \exp(\bar{v}_{i,s}(1))}.$$

This can be re-written as,

$$P_{i,s}(1) = \frac{1}{\exp(\bar{v}_{i,s}(0) - \bar{v}_{i,s}(1)) + 1}.$$

Calculations for choice specific value functions:

$$\bar{v}_{i,s}(0) = U_{i,s}(0) + \beta \tilde{V}_{i-s}.$$

$$\bar{v}_{i,s}(1) = U_{i,s}(1) + \beta \tilde{V}_{Q}.$$

We can get the CCPs in terms of U and f by replacing $\tilde{V}(\cdot)$ from the above equation.

Question-3

Since we are using moment conditions associated with optimality conditions for estimation. Using GMM method we can estimate the parameters associated with the costs. Since we are estimating 3 independent set of parameters which are associated with underage, overage and ship cost. Hence we need three moment conditions which involves these costs and hence these parameters.