OPNS 523 - Assignment 1

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1 Question 1

Definition of $E[N_r(x,y)]$:

Since $N_r(x,y) = \min_{f\in F_p(r)}\{C_{rf}(x_f,y_f)\}$, where C_{rf} represents firm $f's$ expected per unit cost to supply a given contract r. It follows that $E[N_r(x,y)]$ represents the expected minimum cost with all the firms participating in the bid which can produce the part given in the contract.

Calculations for closed form expression for $E[N_r(x,y)]$:

From the definition of $N_r(x,y)$,

$$E[N_r(x,y)] = E[\min_{f\in F_p(r)}\{C_{rf}(x_f,y_f)\}]. \tag{1}$$

From the definition of C_{rf},

$$C_{rf} = \bar{C}_{rf} + \bar{C}_r + \eta_{rf},$$

where η_{rf} is the supplier firm cost and $\bar{C}_r$ represents the cost that is independent of the plant.

Substituting the value of C_{rf} in (1), we get

$$E[N_r(x,y)] = E[\min_{f\in F_p(r)}\{\bar{C}_{rf} + \bar{C}_r + \eta_{rf}\}].$$

which can be re-written as

$$-E[N_r(x,y)] = E[\max_{f\in F_p(r)}\{-\bar{C}_{rf} - \bar{C}_r - \eta_{rf}\}],$$

where expectation is over η_{rf}.

Since $-\eta_{rf}$ is assumed to be distributed as Gumbel(0), hence it follows that

$$-(\bar{C}_{rf} + \bar{C}_r + \eta_{rf}) \sim \text{ Gumbel } (-\bar{C}_{rf} - \bar{C}_r).$$

Since we know that following equation holds true if each α_n is distributed as Gumbel distribution.

$$\max_n \alpha_n \sim \text{ Gumbel (LSE}(\alpha)\text{, where LSE}(\alpha) = \log(\sum_{n=1}^{n} \exp(\alpha_n))$$

It follows that,

$$-E[N_r(x,y)] = E[\text{ Gumbel(LSE }(r))],$$

where $\text{LSE}(r) = \log(\sum_{f=1}^{F_p(r)} \exp(-\bar{C}_{rf} - \bar{C}_r))$.

Using the expectation of Gumbel distribution we get

$$E[N_r(x,y)] = -(LSE(r) + \gamma), \tag{2}$$

where γ is Euler's constant.

Reasons behind using nested logit structure for error terms :

η_{rf} is the firm specific shock which captures the cost of obtaining specialized skill at a firm(and its plants) specific level, fixed costs and the cost due to interpersonal relationships developed. This is realized only after deciding the location of the plants based on the local factors.

ϵ_{ri} on the other hand is a plant specific shock such as a productivity shock. This is only realized after the auction takes place.

It doesn't make sense at all to capture both the errors at just the firm level because then we wouldn't be able to capture the variation due to the auction results. On the other hand, we could have distributed the η of a firm among the ϵ of its plants. This would have captured all the effects that is desired. However, it would not be possible to differentiate between the the two effects. Also, there is a time lag between the two shocks which is difficult to capture in a single variable.

2 Question 2: coding assignment

Objective of this problem is to optimize the foreign suppliers locations of the plants given the location of assembler and competing domestic supplier.

In the data set we have one foreign supplier with 6 plants and one domestic supplier with 4 plants. There are 12 different assembly contracts.

A firm wants to maximize its profit over locations of plants conditional on the location of assemblies and domestic competitor and as given in the paper it is equivalent to solving the following,

$$\psi(f,r) = \min_{x_f, y_f} \sum_r E[N_r(x_f, y_f, x_{-f}, y_{-f})] + \sum_f \phi_f(x_f, y_f). \tag{3}$$

In the optimization problem above, (x_f, y_f) denotes the location of plants of foreign supplier, (x_{-f}, y_{-f}) denotes the location of plants of domestic supplier, r denotes the contract and ϕ_f denotes the fixed cost of firm f.

Since we assume fixed cost to be 0 for each firm, hence (3) can be written as,

$$\psi(f,r) = \min_{x_f, y_f} \sum_r E[N_r(x_f, y_f, x_{-f}, y_{-f})] \tag{4}$$

First we estimate $\bar{C}_{rf}$, then using result from question 1, we can compute the expression for $E[N_r(x_f, y_f, x_{-f}, y_{-f})]$.

We denote the estimation of $\bar{C}_{rf}$ by $\bar{C}_{trf}$.
From the definition of $\bar{C}_{trf}$,

$$\bar{C}_{trf} = E[\min_{i \in I_f} \{\bar{C}_{tri} + \epsilon_{ri}\}], \text{where expectation is over } \epsilon_{ri}.$$

We assume that $\epsilon_{ri} \sim$ Gumbel(0). Hence using similar argument we get,

$$\bar{C}_{trf} = - \text{ LSE(r}'),$$

where $\text{LSE}(r') = \log \sum_{i=1}^{I_f} \exp(-\bar{C}_{tri})$.

Using this estimate $\bar{C}_{trf}$ and (2), (3) can be simplified to the expression below.

$$\psi(f,r) = \min_{x_f, y_f} - \sum_r \log(\sum_{i=1}^{I_f} \exp(-\bar{C}_{tri}) + \sum_{i=1}^{I_{-f}} \exp(-\bar{C}_{tri})),$$

where I_f represents the number of plants of foreign supplier which is given as 6 and I_{-f} represents number of plants of domestic competitor supplier which is given as 4.

Result of the code

We attach the result of optimal location of foreign supplier plants and total cost, i.e., $\psi(f,r)$.

```
[1] "Optimal location: "
         V1         V2
1 0.7564461 0.76857731
2 0.3555549 0.20475116
3 0.6847111 0.44032821
4 0.2229684 0.41993008
5 0.2107923 0.41995108
6 0.4172147 0.05074672
[1] "Minimum cost: "
[1] 7.904424
```

Figure 1: R output of the optimal location of supplier plants using initial location from "Solution.rds" and iterating 10000 times