# Assignment 6

April 13, 2024

6. In this exercise, you will further analyze the Wage data set considered throughout this chapter.

[2]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import statsmodels.formula.api as smf
from sklearn.pipeline import Pipeline
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import PolynomialFeatures
from sklearn import metrics
from sklearn.model_selection import cross_val_score, train_test_split,␣
↪GridSearchCV
from sklearn.feature_selection import f_regression
from mlxtend.feature_selection import SequentialFeatureSelector as SFS
from mlxtend.plotting import plot_sequential_feature_selection as plot_sfs

%matplotlib inline

wage=pd.read_csv("https://raw.githubusercontent.com/JWarmenhoven/ISLR-python/
↪master/Notebooks/Data/Wage.csv")
wage.head()
```

[2]:
```
   Unnamed: 0  year  age      sex            maritl      race  \
0      231655  2006   18  1. Male  1. Never Married  1. White
1       86582  2004   24  1. Male  1. Never Married  1. White
2      161300  2003   45  1. Male        2. Married  1. White
3      155159  2003   43  1. Male        2. Married  3. Asian
4       11443  2005   50  1. Male       4. Divorced  1. White

         education              region        jobclass         health  \
0     1. < HS Grad  2. Middle Atlantic   1. Industrial      1. <=Good
1  4. College Grad  2. Middle Atlantic  2. Information  2. >=Very Good
2  3. Some College  2. Middle Atlantic   1. Industrial      1. <=Good
3  4. College Grad  2. Middle Atlantic  2. Information  2. >=Very Good
4       2. HS Grad  2. Middle Atlantic  2. Information      1. <=Good

  health_ins  logwage        wage
```

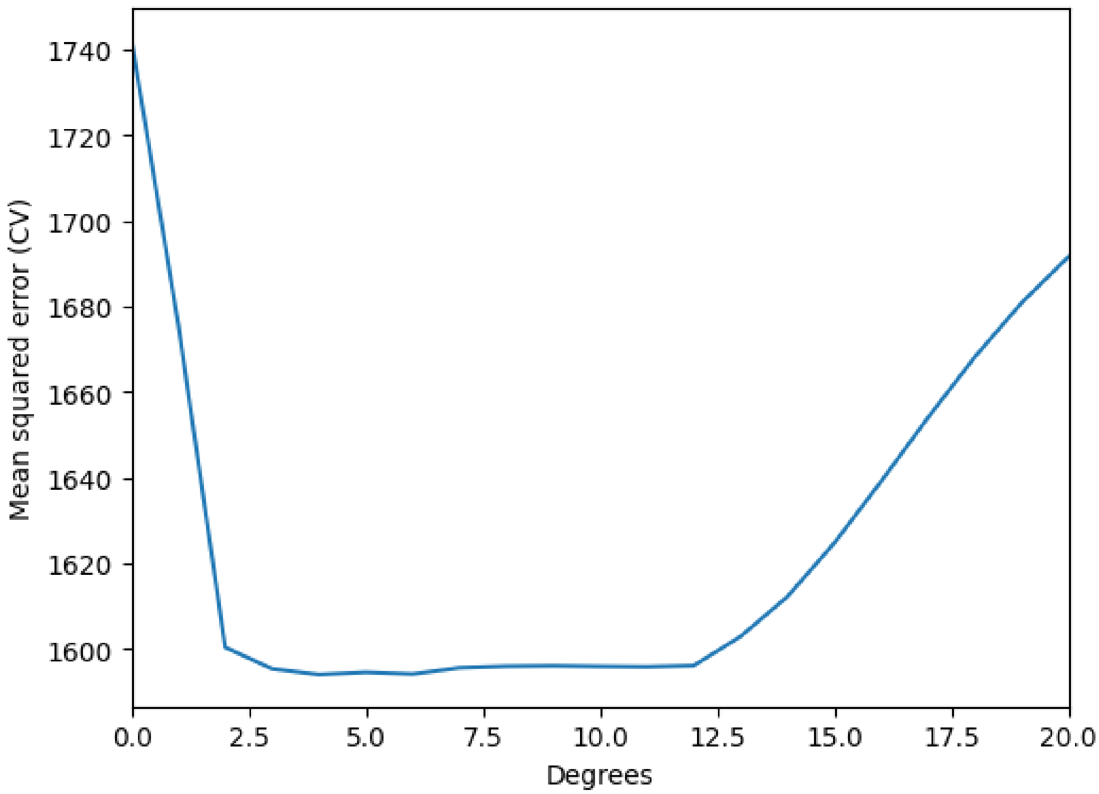
```
0      2. No  4.318063   75.043154
1      2. No  4.255273   70.476020
2     1. Yes  4.875061  130.982177
3     1. Yes  5.041393  154.685293
4     1. Yes  4.318063   75.043154
```

(a) Perform polynomial regression to predict wage using age. Use cross-validation to select the optimal degree d for the polynomial. What degree was chosen, and how does this compare to the results of hypothesis testing using ANOVA? Make a plot of the resulting polynomial ft to the data.

[2]:
```
np.random.seed(100)
```

[3]:
```
# Model variables
y = wage['wage']  # Assuming 'data' contains both 'age' and 'wage'
X = wage[['age']]  # Use double brackets to keep X as DataFrame

scores = []
for i in range(21):  # range(21) will iterate from 0 to 20
    model = Pipeline([('poly', PolynomialFeatures(degree=i)), ('linear',␣
 ↪LinearRegression())])
    model.fit(X, y)

    score = -cross_val_score(model, X, y, cv=10,␣
 ↪scoring='neg_mean_squared_error')  # Negative sign to make it positive
    scores.append(np.mean(score))

scores = np.array(scores)  # Convert scores to numpy array for better handling
```

[4]:
```
# Plot errors
x_plot = np.arange(0,21)

plt.plot(x_plot, scores)
plt.ylabel('Mean squared error (CV)')
plt.xlabel('Degrees')
plt.xlim(0,20)
plt.show()
```




select the optimal degree d for the polynomial

[5]:
```
print(np.where(scores == np.min(scores)))
```

```
(array([4]),)
```

Use cross-validation to select the optimal degree d for the polynomial which is 4.

hypothesis testing using ANOVA using statsmodels

[6]:
```
# Fitting polynomial models:
models=[]
for i in range(0,21):
    poly = PolynomialFeatures(degree=i)
    X_pol = poly.fit_transform(X)
    model = sm.GLS(y, X_pol).fit()
    models.append(model)
```

Performing Hypothesis Testing using ANOVA:

[7]:
```
sm.stats.anova_lm(models[0], models[1], models[2], models[3], models[4],␣
↪models[5], models[6],models[7],models[8],models[9],models[10], typ=1)
```

[7]:

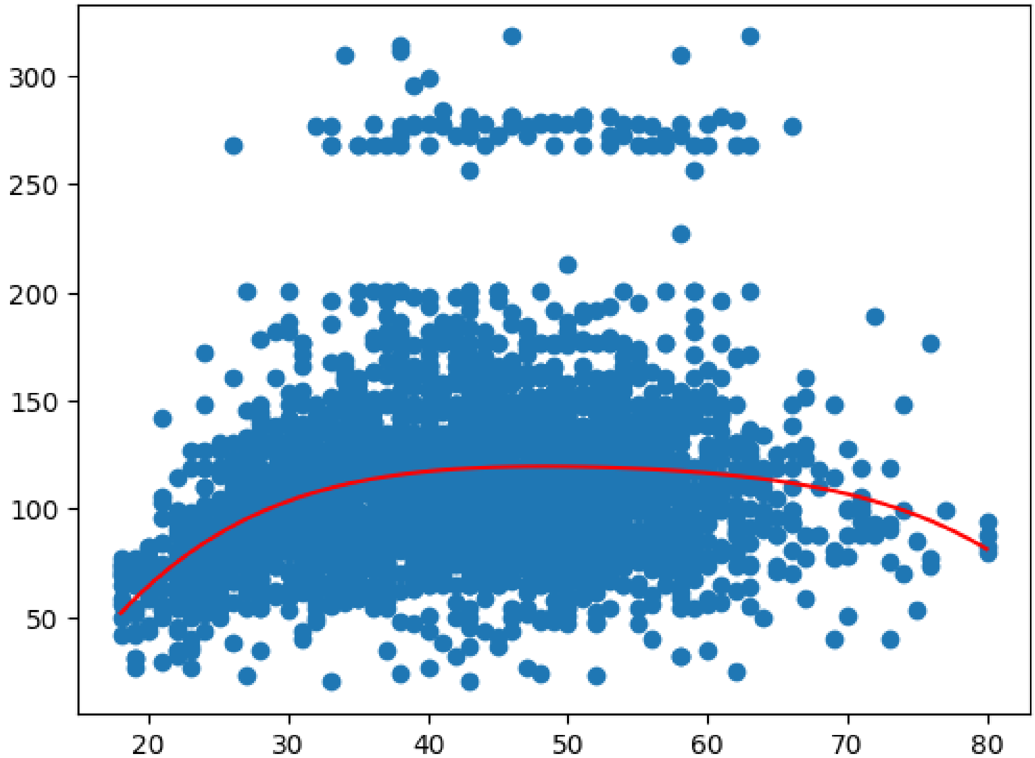
| | df_resid | ssr | df_diff | ss_diff | F | Pr(>F) |
|---|---|---|---|---|---|---|
| 0 | 2999.0 | 5.222086e+06 | 0.0 | NaN | NaN | NaN |
| 1 | 2998.0 | 5.022216e+06 | 1.0 | 199869.664970 | 125.379279 | 1.536176e-28 |
| 2 | 2997.0 | 4.793430e+06 | 1.0 | 228786.010128 | 143.518652 | 2.449935e-32 |
| 3 | 2996.0 | 4.777674e+06 | 1.0 | 15755.693664 | 9.883628 | 1.683872e-03 |
| 4 | 2995.0 | 4.771604e+06 | 1.0 | 6070.152124 | 3.807838 | 5.110636e-02 |
| 5 | 2994.0 | 4.770322e+06 | 1.0 | 1282.563017 | 0.804558 | 3.698061e-01 |
| 6 | 2993.0 | 4.766389e+06 | 1.0 | 3932.257630 | 2.466726 | 1.163856e-01 |
| 7 | 2993.0 | 4.764599e+06 | -0.0 | 1790.494445 | -inf | NaN |
| 8 | 2993.0 | 4.764136e+06 | -0.0 | 462.455755 | -inf | NaN |
| 9 | 2993.0 | 4.764981e+06 | -0.0 | -844.211529 | inf | NaN |
| 10 | 2993.0 | 4.771202e+06 | -0.0 | -6221.646214 | inf | NaN |

As F values drop, the coefficient's importance also does. Furthermore, the polynomial regression model is not significantly improved by degrees greater than 4. These results are consistent with the cross-validation results.

[8]:

```
opt_degree = 4 # storing the optimal degree for further use in␣
 ↪PolynomialFeatures and plot the polynomial regression:

model = Pipeline([('poly', PolynomialFeatures(degree = opt_degree)), ('linear',␣
 ↪LinearRegression())])
model.fit(X,y)

X_lin = np.linspace(18,80)[:,np.newaxis]
y_lin = model.predict(X_lin)

plt.scatter(X,y)
plt.plot(X_lin, y_lin,'-r');
```

```
/usr/local/lib/python3.10/dist-packages/sklearn/base.py:439: UserWarning: X does
not have valid feature names, but PolynomialFeatures was fitted with feature
names
  warnings.warn(
```




(b) Fit a step function to predict wage using age, and perform crossvalidation to choose the optimal number of cuts. Make a plot of the fit obtained.

[12]:
```
scores = []
for i in range(1,20):
    age_groups = pd.cut(wage['age'], i)
    df_dummies = pd.get_dummies(age_groups)

    X_cv = df_dummies
    y_cv = wage['wage']

    model.fit(X_cv, y_cv)
    score = cross_val_score(model, X_cv, y_cv, cv=10,␣
 ↪scoring='neg_mean_squared_error')
    scores.append(score)

scores = np.abs(scores) # converting negative MSE to postive using abs()␣
 ↪function.
```

[15]:
```
# Number of cuts that minimize the error
min_scores = []
for i in range(0,9):
```

```
    min_score = np.mean(scores[i,:])
    min_scores.append(min_score)

    print('Number of cuts: %i, error %.3f' % (i+1, min_score))

# Find the minimum score and corresponding number of cuts
min_index = min_scores.index(min(min_scores))
min_score = min(min_scores)
print()
print(f"The number of cuts that minimize the error is : {min_score} at the␣
 ↪number cut: {min_index + 1}")
```

```
Number of cuts: 1, error 1742.249
Number of cuts: 2, error 1734.212
Number of cuts: 3, error 1683.096
Number of cuts: 4, error 1635.559
Number of cuts: 5, error 1633.769
Number of cuts: 6, error 1627.065
Number of cuts: 7, error 1611.161
Number of cuts: 8, error 1602.094
Number of cuts: 9, error 1617.504

The number of cuts that minimize the error is : 1602.094431940049 at the number
cut: 8
```

[19]:
```
# Convert ages to groups of age ranges
n_groups = 8
age_groups = pd.cut(wage['age'], n_groups)
age_dummies = pd.get_dummies(age_groups)
# Add wage to the dummy dataset.
df_step = age_dummies.join(wage['wage'])
```

[20]:
```
df_step.head()
```

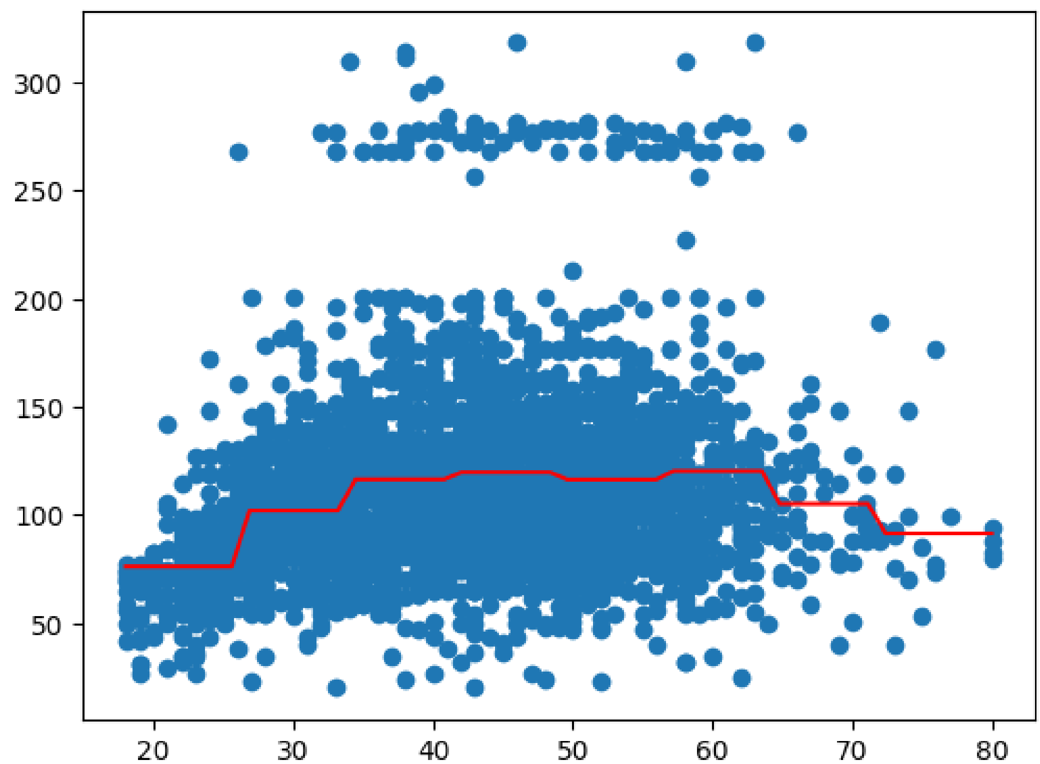
[20]:
```
   (17.938, 25.75]  (25.75, 33.5]  (33.5, 41.25]  (41.25, 49.0]  \
0             True          False          False          False
1             True          False          False          False
2            False          False          False           True
3            False          False          False           True
4            False          False          False          False

   (49.0, 56.75]  (56.75, 64.5]  (64.5, 72.25]  (72.25, 80.0]        wage
0          False          False          False          False   75.043154
1          False          False          False          False   70.476020
2          False          False          False          False  130.982177
3          False          False          False          False  154.685293
4           True          False          False          False   75.043154
```

[21]:
```
X_step = df_step.iloc[:,:-1]
y_step = df_step.iloc[:,-1]
```

[22]:
```
reg = sm.GLM(y_step, X_step).fit()
```

[23]:
```
X_aux = np.linspace(18,80)
groups_aux = pd.cut(X_aux, n_groups)
aux_dummies = pd.get_dummies(groups_aux)
```

[24]:
```
# Plot step function
X_step_lin = np.linspace(18,80)
y_lin = reg.predict(aux_dummies)

plt.scatter(X,y)
plt.plot(X_step_lin, y_lin,'-r');
```




10. This question relates to the College data set.

[4]:
```
df = pd.read_csv('https://www.statlearning.com/s/College.csv', index_col=0)

# Dummy variables
```

```
# The feature 'Private' is categorical. In order to use it in our models, we
↪need to use dummy variables.
df = pd.get_dummies(df)
df.head()
```

[4]:

```
                              Apps  Accept  Enroll  Top10perc  Top25perc  \
Abilene Christian University  1660    1232     721         23         52
Adelphi University            2186    1924     512         16         29
Adrian College                1428    1097     336         22         50
Agnes Scott College            417     349     137         60         89
Alaska Pacific University      193     146      55         16         44

                              F.Undergrad  P.Undergrad  Outstate  Room.Board  \
Abilene Christian University         2885          537      7440        3300
Adelphi University                   2683         1227     12280        6450
Adrian College                       1036           99     11250        3750
Agnes Scott College                   510           63     12960        5450
Alaska Pacific University             249          869      7560        4120

                              Books  Personal  PhD  Terminal  S.F.Ratio  \
Abilene Christian University    450      2200   70        78       18.1
Adelphi University              750      1500   29        30       12.2
Adrian College                  400      1165   53        66       12.9
Agnes Scott College             450       875   92        97        7.7
Alaska Pacific University       800      1500   76        72       11.9

                              perc.alumni  Expend  Grad.Rate  Private_No  \
Abilene Christian University           12    7041         60       False
Adelphi University                     16   10527         56       False
Adrian College                         30    8735         54       False
Agnes Scott College                    37   19016         59       False
Alaska Pacific University               2   10922         15       False

                              Private_Yes
Abilene Christian University         True
Adelphi University                   True
Adrian College                       True
Agnes Scott College                  True
Alaska Pacific University            True
```

(a) Split the data into a training set and a test set. Using out-of-state tuition as the response and the other variables as the predictors, perform forward stepwise selection on the training set in order to identify a satisfactory model that uses just a subset of the predictors.

[5]:

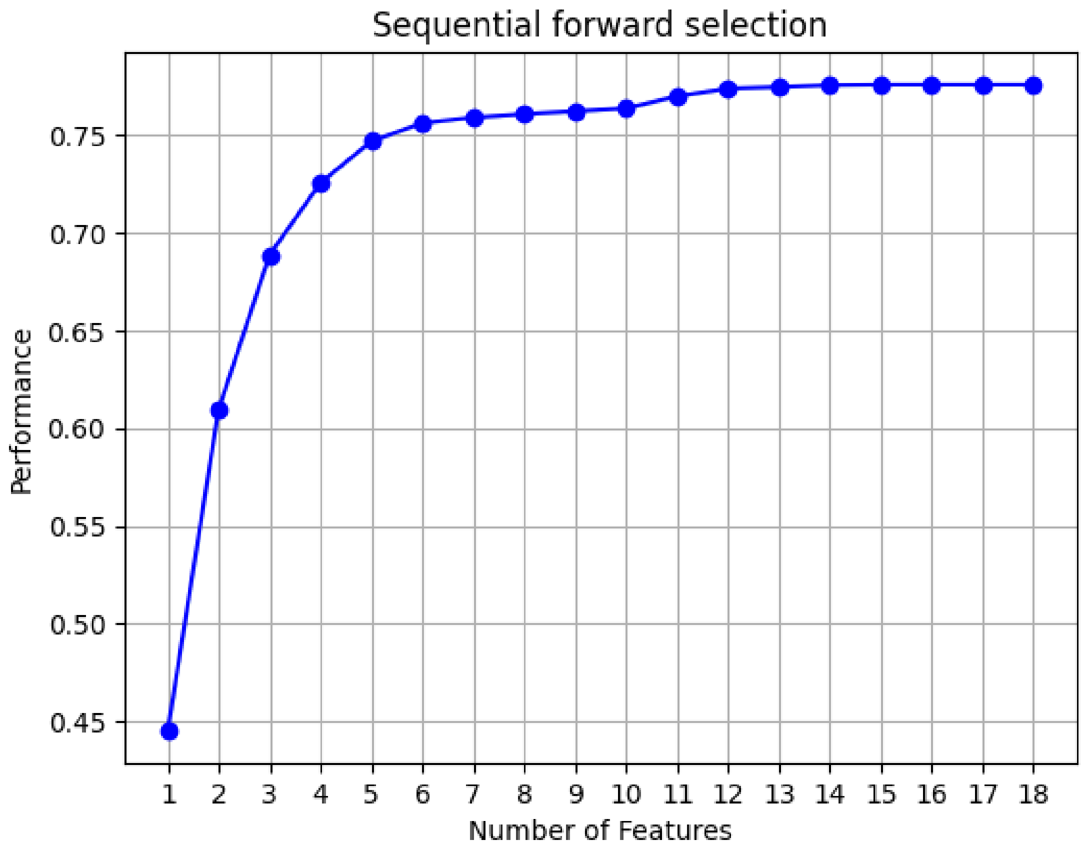
```
# Dataset
X = df.drop(['Outstate'], axis=1)
y = df['Outstate']
```

```
# Split into train and test subsets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=.2,␣
↪random_state=1)
```

[6]:

```
# Forward stepwise selection
lr = LinearRegression()

sfs = SFS(lr,
          k_features = 18,  # We have 18 features
          forward = True,
          floating = False,
          scoring = 'r2',
          cv = 0)

sfs = sfs.fit(X_train, y_train)  # as_matrix() to be readable by sfs

fig = plot_sfs(sfs.get_metric_dict())

#plt.title('Sequential forward selection (w. StdDev)')
plt.title('Sequential forward selection')
plt.grid()
plt.show()
```

```
/usr/local/lib/python3.10/dist-packages/numpy/core/_methods.py:206:
RuntimeWarning: Degrees of freedom <= 0 for slice
  ret = _var(a, axis=axis, dtype=dtype, out=out, ddof=ddof,
/usr/local/lib/python3.10/dist-packages/numpy/core/_methods.py:198:
RuntimeWarning: invalid value encountered in scalar divide
  ret = ret.dtype.type(ret / rcount)
```




We will choose six features. According to the graphic, adding more features after this point won't significantly improve performance.

[7]:
```
# Visualizing the results in dataframes
pd.DataFrame.from_dict(sfs.get_metric_dict()).T
```

[7]:

| | feature_idx | cv_scores \ |
|---|---|---|
| 1 | (14,) | [0.44548490593604373] |
| 2 | (14, 16) | [0.6098746812158646] |
| 3 | (7, 14, 16) | [0.6887746037816894] |
| 4 | (7, 13, 14, 16) | [0.7256619159122937] |
| 5 | (7, 10, 13, 14, 16) | [0.7468820442179642] |
| 6 | (7, 10, 13, 14, 15, 16) | [0.7562065682189014] |
| 7 | (7, 9, 10, 13, 14, 15, 16) | [0.7590373434320926] |
| 8 | (3, 7, 9, 10, 13, 14, 15, 16) | [0.7607756220059703] |
| 9 | (3, 7, 9, 10, 12, 13, 14, 15, 16) | [0.7622330250671435] |
| 10 | (1, 3, 7, 9, 10, 12, 13, 14, 15, 16) | [0.7637006442613157] |
| 11 | (0, 1, 3, 7, 9, 10, 12, 13, 14, 15, 16) | [0.7699026270388485] |
| 12 | (0, 1, 2, 3, 7, 9, 10, 12, 13, 14, 15, 16) | [0.7738280911029343] |
| 13 | (0, 1, 2, 3, 7, 9, 10, 11, 12, 13, 14, 15, 16) | [0.7746038967301329] |

```
14  (0, 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16)   [0.7756133330814865]
15  (0, 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 1…   [0.7758770748808218]
16  (0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14…   [0.7758802240596209]
17  (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,…   [0.7758802349925187]
18  (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,…   [0.7758802349925187]

   avg_score                                     feature_names ci_bound  \
1   0.445485                                         (Expend,)      NaN
2   0.609875                              (Expend, Private_No)      NaN
3   0.688775                  (Room.Board, Expend, Private_No)      NaN
4   0.725662      (Room.Board, perc.alumni, Expend, Private_No)      NaN
5   0.746882  (Room.Board, PhD, perc.alumni, Expend, Private…      NaN
6   0.756207  (Room.Board, PhD, perc.alumni, Expend, Grad.Ra…      NaN
7   0.759037  (Room.Board, Personal, PhD, perc.alumni, Expen…      NaN
8   0.760776  (Top10perc, Room.Board, Personal, PhD, perc.al…      NaN
9   0.762233  (Top10perc, Room.Board, Personal, PhD, S.F.Rat…      NaN
10  0.763701  (Accept, Top10perc, Room.Board, Personal, PhD,…      NaN
11  0.769903  (Apps, Accept, Top10perc, Room.Board, Personal…      NaN
12  0.773828  (Apps, Accept, Enroll, Top10perc, Room.Board, …      NaN
13  0.774604  (Apps, Accept, Enroll, Top10perc, Room.Board, …      NaN
14  0.775613  (Apps, Accept, Enroll, Top10perc, Room.Board, …      NaN
15  0.775877  (Apps, Accept, Enroll, Top10perc, Top25perc, R…      NaN
16   0.77588  (Apps, Accept, Enroll, Top10perc, Top25perc, P…      NaN
17   0.77588  (Apps, Accept, Enroll, Top10perc, Top25perc, F…      NaN
18   0.77588  (Apps, Accept, Enroll, Top10perc, Top25perc, F…      NaN

   std_dev std_err
1      0.0     NaN
2      0.0     NaN
3      0.0     NaN
4      0.0     NaN
5      0.0     NaN
6      0.0     NaN
7      0.0     NaN
8      0.0     NaN
9      0.0     NaN
10     0.0     NaN
11     0.0     NaN
12     0.0     NaN
13     0.0     NaN
14     0.0     NaN
15     0.0     NaN
16     0.0     NaN
17     0.0     NaN
18     0.0     NaN
```

[8]:
```
# Variables that we will choose
print('Variables: %s, %s, %s, %s, %s, %s' % (X.columns[16], X.columns[7], X.
↪columns[10], X.columns[13], X.columns[14], X.columns[15]))
```

```
Variables: Private_No, Room.Board, PhD, perc.alumni, Expend, Grad.Rate
```

[9]:
```
selected_features = ["Private_No", "Room.Board", "PhD", "perc.alumni",␣
↪"Expend", "Grad.Rate"]
selected_features
```

[9]:
```
['Private_No', 'Room.Board', 'PhD', 'perc.alumni', 'Expend', 'Grad.Rate']
```

(b) Fit a GAM on the training data, using out-of-state tuition as the response and the features selected in the previous step as the predictors. Plot the results, and explain your findings.

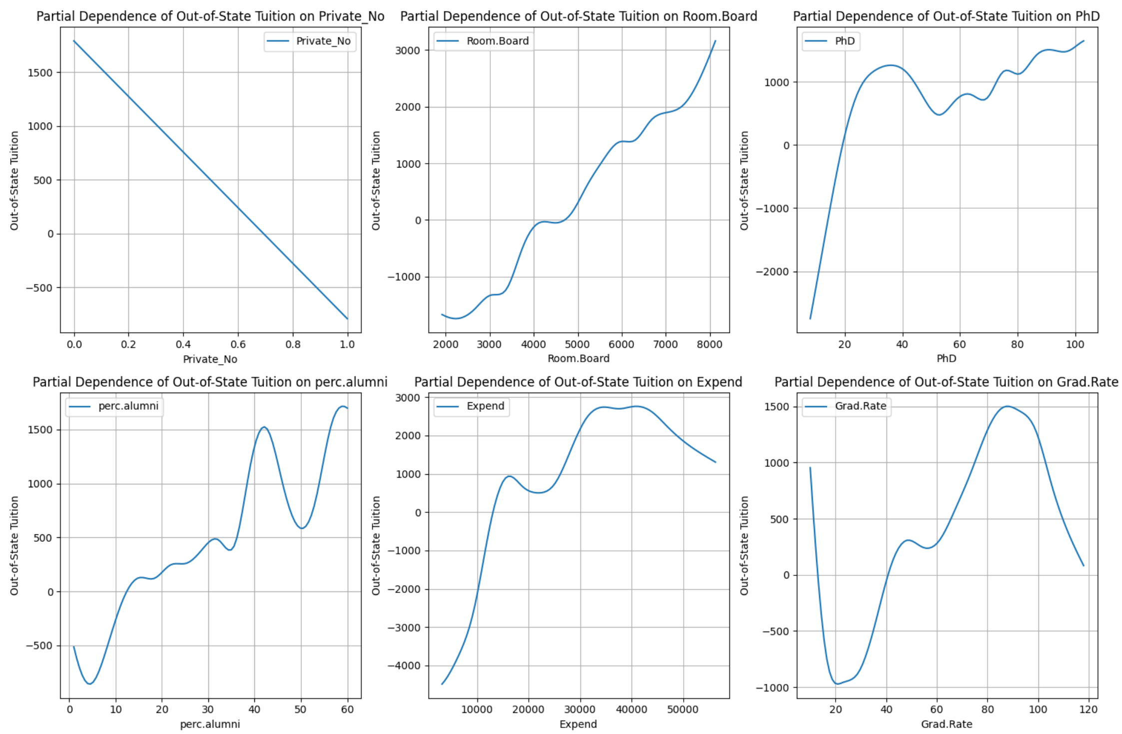
[10]:
```
X_train_selected = X_train[selected_features]
X_test_selected = X_test[selected_features]
```

[12]:
```
!pip install pygam
```

```
Collecting pygam
  Downloading pygam-0.9.1-py3-none-any.whl (522 kB)
                              522.0/522.0

kB 3.2 MB/s eta 0:00:00
Requirement already satisfied: numpy>=1.25 in
/usr/local/lib/python3.10/dist-packages (from pygam) (1.25.2)
Requirement already satisfied: progressbar2<5.0.0,>=4.2.0 in
/usr/local/lib/python3.10/dist-packages (from pygam) (4.2.0)
Requirement already satisfied: scipy<1.12,>=1.11.1 in
/usr/local/lib/python3.10/dist-packages (from pygam) (1.11.4)
Requirement already satisfied: python-utils>=3.0.0 in
/usr/local/lib/python3.10/dist-packages (from progressbar2<5.0.0,>=4.2.0->pygam)
(3.8.2)
Requirement already satisfied: typing-extensions>3.10.0.2 in
/usr/local/lib/python3.10/dist-packages (from python-
utils>=3.0.0->progressbar2<5.0.0,>=4.2.0->pygam) (4.11.0)
Installing collected packages: pygam
Successfully installed pygam-0.9.1
```

[13]:
```
from pygam import LinearGAM, s, f, l

# Fit GAM model
gam = LinearGAM().fit(X_train_selected, y_train)

plt.figure(figsize=(15, 10))
for i, feature in enumerate(selected_features):
    plt.subplot(2, 3, i + 1)
```

```
    XX = gam.generate_X_grid(term=i)
    partial_dep = gam.partial_dependence(term=i, X=XX)
    plt.plot(XX[:, i], partial_dep, label=feature)
    plt.title("Partial Dependence of Out-of-State Tuition on {}".
↪format(feature))
    plt.xlabel(feature)
    plt.ylabel("Out-of-State Tuition")
    plt.grid(True)
    plt.legend()

plt.tight_layout()
plt.show()
```




[14]:
```
gam.summary()
```

```
LinearGAM
=============================================== 
==========================================================
Distribution:                        NormalDist Effective DoF:
54.5714
Link Function:                     IdentityLink Log Likelihood:
-9907.585
Number of Samples:                          621 AIC:
19926.3128
```

```
                                                    AICc:
19937.4524
                                                    GCV:
4016346.628
                                                    Scale:
3386490.0647
                                                    Pseudo R-Squared:
0.8075
================================================================================
==========================
Feature Function                  Lambda               Rank         EDoF
P > x        Sig. Code
================================= ==================== ============ ============
============ ============
s(0)                              [0.6]                20           3.3
1.11e-16     ***
s(1)                              [0.6]                20           11.6
1.11e-10     ***
s(2)                              [0.6]                20           11.7
1.49e-01
s(3)                              [0.6]                20           11.3
1.25e-02     *
s(4)                              [0.6]                20           8.0
1.11e-16     ***
s(5)                              [0.6]                20           8.6
2.75e-04     ***
intercept                                              1            0.0
1.11e-16     ***
================================================================================
==========================
Significance codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

WARNING: Fitting splines and a linear function to a feature introduces a model
identifiability problem
         which can cause p-values to appear significant when they are not.

WARNING: p-values calculated in this manner behave correctly for un-penalized
models or models with
         known smoothing parameters, but when smoothing parameters have been
estimated, the p-values
         are typically lower than they should be, meaning that the tests reject
the null too readily.

<ipython-input-14-dec6a6acdaaa>:1: UserWarning: KNOWN BUG: p-values computed in
this summary are likely much smaller than they should be.

Please do not make inferences based on these values!
```

```
Collaborate on a solution, and stay up to date at:
github.com/dswah/pyGAM/issues/163
```

```
gam.summary()
```

Interpretation: Private institutions usually charge greater out-of-state tuition, which is influenced by a number of variables like graduation rates, alumni percentages, Ph.D. program availability, and availability of boarding rooms. On the other hand, tuition costs frequently drop due to individual reasons. There is a sharp rise in spending at first, which is followed by a slowdown and then a small decline.

(c) Evaluate the model obtained on the test set, and explain the results obtained.

[15]:
```python
# Evaluating the model on the test dataset
test_score = gam.score(X_test_selected, y_test)
print("Test Set R^2 Score:", test_score)
```

```
Test Set R^2 Score: 0.7687301916198483
```

Six factors were shown to be highly important in our research using out-of-state tuition as the response variable, which helps to explain the variation in tuition costs. These predictors include the following: the percentage of alumni who donate to the university ('perc.alumni'), the cost of boarding rooms ('Room.Board'), the graduation rate ('Grad.Rate'), the number of Ph.D. holders among faculty members ('PhD'), and whether or not the university is private ('Private_No').

With these six predictors, our Generalized Additive Model (GAM) model produced a Test Set R^2 Score of roughly 0.769. This suggests that our model can account for roughly 76.87% of the variation in out-of-state tuition costs. This high R^2 value indicates that the chosen variables together offer insightful information about the factors influencing out-of-state tuition costs.

(d) For which variables, if any, is there evidence of a non-linear relationship with the response?

The partial dependence charts from Part (b) are available for inspection. There may be a non-linear relationship between the predictor and the responder if the curves are non-linear. Also, we have the evidence from the summary of GAM model, which suggest that:

Interpretation: The graphs illustrating the partial dependence of 'out-of-state tuition' on 'perc.alumni' and 'graduation rate' are non-linear. This indicates that the relationship between the variables is not constant and that the graphs do not follow a straight line. Also, we have evidence from the summary of GAM model which suggest that demonstrates a strong nonlinear link between 'spending' and 'out-of-state' tuition.