Q.2) Using

$$P(y_i | w, x_i) = \frac{1}{1+e^{-y_i w^T x_i}}$$

obj func : $\min_{w} \frac{\lambda}{2}\|w\|^2 + \sum_{i=1}^{N} \ln(1+e^{-y_i w^T x_i})$

$$E(w) = \frac{\delta LL}{\delta w}$$

$$= \frac{\delta}{\delta w}\left(\frac{\lambda}{2}\|w\|^2 + \sum_{i=1}^{N} \ln(1+e^{-y_i w^T x_i})\right)$$

Let $\alpha_i = y_i w^T x_i$

$O_i = \sigma(\alpha_i)$

$$\therefore E(w) = \lambda w + \frac{\delta}{\delta w}\left(\sum_{i=1}^{N} \ln(O_i)^{-1}\right)$$

Applying chain rule:

$$\frac{\delta \alpha_i}{\delta w} = \frac{\delta}{\delta w}(y_i w^T x_i) = y_i x_i$$

$$\frac{\delta O_i}{\delta \alpha_i} = \frac{\delta}{\delta \alpha_i}(\sigma(\alpha_i)) = \frac{\delta}{\delta \alpha_i}\left(\frac{1}{1+e^{-\alpha_i}}\right)$$

$$= \frac{-e^{-\alpha_i}}{1+e^{-\alpha_i}}$$

$$= (1 - O_i)\, O_i$$

$$\frac{\delta LL}{\delta O_i} = \frac{\delta}{\delta O_i}(\ln (O_i)^{-1}) = -(O_i)^{-1}$$

Using chain rule:

$$\frac{\delta LL}{\delta w} = \frac{\delta LL}{\delta O_i} \times \frac{\delta O_i}{\delta \alpha_i} \times \frac{\delta \alpha_i}{\delta w}$$

Substituting above for $E(w)$

$$E(w) = \lambda w + \sum_{i=1}^{N} \left[y_i x_i (1-O_i) O_i \times \frac{-1}{O_i} \right]$$

$$= \lambda w + \sum_{i=1}^{N} y_i x_i (O_i - 1)$$

$$= \lambda w + \sum_{i=1}^{N} O_i (x_i^T y_i)^T - \sum_{i=1}^{N} x_i^T y_i$$

Taking negative LL:

$$E(w) = \sum_{i=1}^{N} x_i^T y_i - \lambda w - \sum_{i=1}^{N} O_i (x^T y_i)^T$$

$$\boxed{w^+ = w - \eta (E(w))}$$

2.5) $f(\omega) = \frac{\lambda}{2}||\omega||^2 + \sum_{i=1}^{N} \ln\left(1 + e^{-y_i \omega^T x_i}\right)$

s.t $y_i(\omega^T x_i + b) \geq 1$

$\Rightarrow 1 - y_i \omega^T x_i \leq 0$

$L(\omega, b) = f(\omega) + \beta g(\omega)$

$= f(\omega) + \beta\left(\sum_{i=1}^{N} (1 - y_i \omega^T x)\right)$

$\frac{\delta}{\delta x} L(\omega, b) = \sum_{i=1}^{N} \frac{e^{-y_i \omega^T x_i}}{1 + e^{-y_i \omega^T x_i}} \times -y_i \omega^T$

$- \beta \sum_{i=1}^{N} y_i \omega^T = 0$

$\therefore \beta = -\sum_{i=1}^{N} \frac{e^{-y_i \omega^T x_i}}{1 + e^{-y_i \omega^T x_i}} = -\sum_{i=1}^{N} \frac{1}{1 + e^{-y_i \omega^T x_i}}$