20).

There are $2n$ parts.

So, we can arrange them in $2n!$ ways.

Now, if we take ~~[illegible]~~ the parts of the respective stick together, we can arrange them in $n!$ ways.

and for each stick, the two parts can arrange themselves in $2!$ ways.

So, that the original stick can be formed.

So, the req. prob. $= \frac{2^n\, n!}{(2n)!}$

21). (a) $A = \{$1st roll results in 1$\}$

$\# A = 1 \times 4$

So, $P(A) = \cancel{\frac{1 \times 4}{4^2} = \frac{1}{4}}$ $\frac{4}{4 \times 4}$

$B = \{$2nd roll results in 2$\}$

$P(B) = \frac{1}{4}$

~~Now, $A \cap B = \{$~~

$\# A \cap B = 4 \times 4$

$\therefore P(A \cap B) = \frac{1}{16}$

$\therefore P(A \cap B) = P(A)\,P(B)$

so, A & B are ind.

(b). $B = \{$ sum of the two rolls is a 5 $\}$

$\{1, 2, 3, 4\}$

$(1,4), (4,1), (2,3), (3,2)$

~~Choose $\binom{4}{2}$ place to put say (1,4) in two place. Now, this (1,4) place may interchange their position i.e (4,1) and in any of these ways the left two other places may be put in 4×4 ways~~

~~So, For (1,4), the total choice is (so, that sum is 5) is $\binom{4}{2} \times 2! \times 4 \times 4$~~

~~Similarly, for (2,3).~~

so, $\cancel{P(B) = \frac{4!}{2!\times 2!\times 4} = \frac{4\times 3\times 2}{2\times 2\times 4}}$

total no. of events corr. to B $= 4$, $[(1,4),(4,1),(2,3),(3,2)]$

$\therefore P(B) = \frac{4}{4^2} = 1/4$

Now, # $A\cap B = 1$ $\therefore P(A\cap B) = \frac{1}{4^2}$

$\therefore P(A\cap B) = P(A)P(B)$

so, A & B ind.

(c). $A = \{$max. of the two rolls is 2$\}$

corr. events $(1,2), (2,1), (2,2)$

$P(A) = 3/16$

$B = \{$min. of the two rolls is 2$\}$

corr. events $(2,3), (3,2), (2,4), (4,2), (2,2)$

$\therefore P(B) = 5/16$

corr. events to $A\cap B$ $(2,2)$

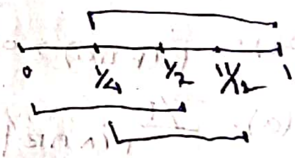
$\therefore P(A\cap B) = \frac{1}{16} \neq P(A)P(B)$

22). $P(A) = 1/2$, $P(B) = 3/4$, $P(C) = \frac{11}{12} - \frac{1}{4} = \frac{11-3}{12} = \frac{8}{12} = 2/3$

$P(A\cap B\cap C) = \frac{1}{2} - \frac{1}{4} = \frac{2-1}{4} = 1/4$ $\quad A\cap B\cap C = (1/4, 1/2)$

$= \frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}$

$= P(A)P(B)P(C)$



but $A\cap B = (1/4, 1/2)$

$\therefore P(A\cap B) = 1/4 \neq P(A)P(B)$ ind.

so, A & B are not ind.

i.e A, B, C are not ind.

23). $P(H_1) = 1/2$ $\quad [(H,H), (H,T), (T,H), (T,T)$

$P(H_2) = 1/2$, $P(H_1\cap H_2) = 1/4$

$P(H_1|D) = \frac{P(H_1\cap D)}{P(D)} = 1/2$

$P(H_2|D) = \frac{P(H_2\cap D)}{P(D)} = 1/2$

$P(H_1\cap H_2|D) = \frac{P(H_1\cap H_2\cap D)}{P(D)} = 0$

$P(H_1\cap H_2) = 0.25$

24). $P(H_b) = 0.99$

$P(H_r) = 0.01$

$D \equiv$ biere coin is selected.

$H_i =$ ith toss result in head, $i = 1, 2$

$P(H_1) = P(H_1 | 1\text{st coin choose})\, P(1\text{st coin choose})$

$+ P(H_2 | 2\text{nd} \ldots)\, P(2\text{nd coin} \ldots)$

$= 0.99 \times 0.5 + 0.01 \times 0.5$

$= 0.5$

$P(H_2) = 0.5$

$P(H_1 \cap H_2) = P(H_1 \cap H_2 | 1\text{st coin choose})\, P(1\text{st} \ldots)$

$+ P(H_1 \cap H_2 | 2\text{nd} \ldots)\, P(2\text{nd} \ldots)$

$= 0.5 \times 0.99^2 + 0.5 \times 0.01^2$ $\{(H,H), (H,T), (T, (T,T)\}$

$=$ Ans $= 0.4901$

$P(H_1 \cap D) = P(H_1 | D)\, P(D)$

$= 0.99 \times 0.5$

$P(H_2 | D) = 0.99.$

$P(H_1 \cap H_2 | D) = (0.99)^2$

25). $P(B \cap C) > 0$

(a). T/F $P(A \cap B | C) = P(A | B \cap C)\, P(B | C)$

$\rightarrow P(A \cap B | C) = \dfrac{P(A \cap B \cap C)}{P(C)} = \dfrac{P(A \cap B \cap C)}{P(B \cap C)} \times \dfrac{P(B \cap C)}{P(C)}$

$= P(A | B \cap C)\, P(B | C)$

(b). T/F $P(A \cap B | C) = P(A | C)\, P(B | C)$ if A & B are ind.

From question 23

$P(H_1 \cap H_2 | D) = 0$

$P(H_1 | D) = 0.5$

$P(H_2 | D) = 0.5$

H_1 & H_2 are ind.

but H_1 & H_2 are not conditional ind.

26).

$$P\left(\bigcap_{i=1}^{n} A_i^c\right) = P(A_1^c)\cdots P(A_n^c) \quad \because \left\{\begin{array}{l} A_1, \cdots, A_n \text{ ind.} \\ \text{so, } A_1^c, \cdots, A_n^c \text{ ind.} \end{array}\right.$$

$$= (1-P(A_1))\cdots(1-P(A_n^c))$$

$$\leq e^{-[P(A_1)+\cdots+P(A_n)]}$$

$$= e^{-\sum_{i=1}^{n} P(A_i)}$$

27).

(a) A & B are negatively associated
B & C are " "

[• A & B is negatively associated if $P(A\cap B) < P(A)P(B)$

• A & B is positively associated if $P(A\cap B) > P(A)P(B)$]

consider, two independent tosses of a fair coin
let $A = \{HT, HH\}$, $B = \{TT\}$, $C = \{TH, HH\}$

Then $P(A\cap B) = 0$, $P(A) = 1/2$, $P(B) = 1/4$

$\therefore P(A\cap B) < P(A)P(B)$

$P(B\cap C) = 0$, $P(C) = 1/2$

$\therefore P(B\cap C) < P(B)P(C)$

but $P(A\cap C) = 1/4$

$P(A)P(C) = \frac{1}{2}\cdot\frac{1}{2}$

$\therefore P(A\cap C) = P(A)P(C)$

so, A & C are not negatively associate

(b). let $A = \{HT, HH\}$, $B = \{HH, TH, HT\}$, $C = \{TH, HH\}$

$P(A) = \frac{1}{2} = P(C)$

$P(B) = 3/4$, $P(A\cap B) = 1/2$, $P(A)P(B) = \frac{1}{2}\times\frac{3}{4} = \frac{3}{8}$

$P(B\cap C) = 1/2$

$\therefore P(A\cap B) > P(A)P(B)$ $\quad [\frac{1}{2} > \frac{3}{8}$

$\cancel{P(A\cap C) > P(A)P(C)}$ so $\cancel{P(A)P(C) = 1/8}$

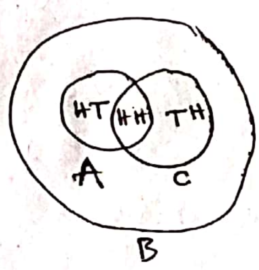
$P(B\cap C) > P(B)P(C)$

$P(B)P(C) = \frac{3}{4}\times\frac{1}{2} = 3/8$

Here, A & B positively asso.
also, B & C " "

but. $P(A\cap C) = 1/4 = P(A)\cdot P(C)$

so, A & C are not positively associate.



28). a). Let A & B be positively asso.

Then $P(A\cap B) > P(A)P(B)$ —— (1)

Now, $P(A\cap B^c) = P(A) - P(A\cap B)$

Now, $P(A) - P(A\cap B) < P(A)\,P(B^c)$

$\Leftrightarrow P(A)\,P(B) < P(A\cap B)$ by (1)

$\therefore P(A\cap B^c) = P(A) - P(A\cap B) < P(A)\,P(B^c)$

So, A & B^c are neg. asso.

(b). If A & B be neg. asso.

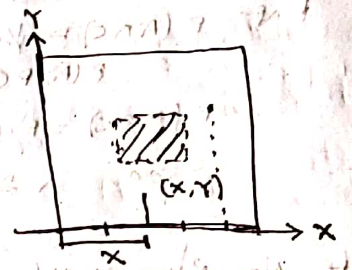
$P(A\cap B) < P(A)P(B)$ —— (2)

Now, $P(A\cap B^c) = P(A) - P(A\cap B) > P(A)\,P(B^c) \Leftrightarrow P(A)P(B) > P(A\cap B)$ by (2)

$\therefore P(A) - P(A\cap B) > P(A)\,P(B^c)$

So, A & B^c are +ve asso.

29). Let (X,Y) be the point



distance of (X,Y) from the nearest wall is

$Z = \min\{X, 1-X, Y, 1-Y\}$

find $P(Z \le 1/5)$

Now, $P\left(Z > \frac{1}{5}\right)$ $\Leftrightarrow$ $X > \frac{1}{5},\ 1-X > \frac{1}{5},\ Y > \frac{1}{5},\ 1-Y > \frac{1}{5}$

$1-X > \frac{1}{5}$

$\Rightarrow X < 4/5$

$\therefore \frac{1}{5} < X < 4/5,\ \frac{1}{5} < Y < 4/5$

$\therefore P\left(Z > \frac{1}{5}\right) = P\left(\frac{1}{5} < X < 4/5,\ \frac{1}{5} < Y < 4/5\right)$

$$= \frac{(4/5 - 1/5)\left(\frac{4}{5} - \frac{1}{5}\right)}{1}$$

$$= 3/5 \times 3/5 = 9/25$$

So, $P(Z \le 1/5) = 16/25$ (ans)

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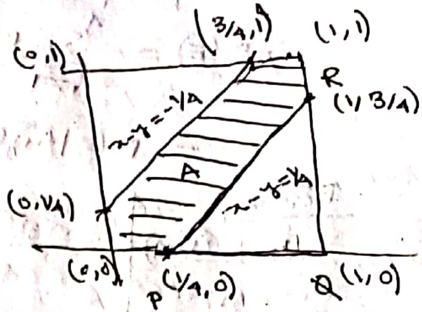
30). Let X & Y be r.v denoting the time of arrival from their schedule time.
Then according to the question
$X, Y \sim U(0,1)$

Now, They will meet iff $|X-Y| \le 1/4$

$-1/4 \le X-Y \le 1/4$



$\Delta PQR = \frac{1}{2} \times (1-1/4)(1-1/4)$

$= \frac{1}{2} \times \frac{9}{16}$

So,

$2\,\Delta PQR = 9/16$

So, $P(A) = \dfrac{1-9/16}{1\times 1} = 7/16$

$x-y=1/4$

31). He will go home as a winner either if he win the 1st time or he loss the 1st & win the 2nd.

$\therefore P(\ ) = \frac{1}{2} + \frac{1}{2}\times\frac{1}{2} = 3/4$

32). ~~1st person may toss~~

The total event in sample space is $2\times2\times2 = 8$

①	②	③
T	H	T

Now, let $A \equiv$ req. event.
1st person may toss in two ways, (H/T)
then 2nd & 3rd person have only one choice
Similarly, for start with 2nd & 3rd person.

So, $P(\text{Req.}\ )$

$= \dfrac{{}^3C_1 \times 2 \times 1 \times 1}{2\times2\times2}$

$= 3/4$

Counting process
(H, H, T), (H, T, H),
(T, T, H), (T, H, T)
(T, H, H), (H, T, T)

$6/8 = 3/4$

[let 1st person is diff. then 2nd & 3rd persons are same
(H, T, T), (T, H, H)

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33). Let E be the event that sum total is atleast 4
Let A_i be the event that the 1st roll results in $\{i\}$, $i = 1, 2, 3, 4$

$\therefore$ The req. prob.

$$\Rightarrow P(E) = \sum_{i=1}^{4} P(E|A_i) P(A_i)$$

$$= P(E|A_1)P(A_1) + P(E|A_2) \times P(A_2) + P(E|A_3)P(A_3) + P(E|A_4)P(A_4)$$

$$= \frac{2}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + 0 \times \frac{1}{4} + \frac{1}{4} \times 1$$

$$= 9/16$$

For A_1
(1, 3), (1, 4)

For A_2
(2, 2), (2, 3), (2, 4)

For A_3
no. 2nd toss

For A_4 $\rightarrow$ no 2nd toss

34). Let E_i denote the event that the student is upto date after the week i, $i = 1, 2, 3$

Let E_0 be the event that the student is upto date at the begining. So, $P(E_0) = 1$

$$P(E_1) = P(E_1|E_0)P(E_0) = 0.8$$

$$P(E_2) = P(E_2|E_1)P(E_1) + P(E_2|E_1^c)P(E_1^c)$$

$$= 0.8 \times 0.8 + 0.4 \times 0.2 = 0.72$$

$$P(E_3) = P(E_3|E_2)P(E_2) + P(E_3|E_2^c)P(E_2^c)$$

$$= 0.8 \times 0.72 + 0.4 \times 0.28 = 0.688$$

35). Let C_i, $i = 1, 2, 3$ denote the events that the car is behind door i.

Let X_i, $i = 1, 2, 3$ denote the event you choose the door i

D_i = event that host opens the door i, $i = 1, 2, 3$

Now,

$P(D_3|C_1, X_1) = 1/2$, $P(D_2|C_1, X_1) = 1/2$

$P(D_3|C_2, X_1) = 1$, $P(D_2|C_2, X_1) = 0$

$P(D_3|C_3, X_1) = 0$, $P(D_2|C_3, X_1) = 1$

$P(C_1) = 1/3$, $P(D_3 | x_1) = 1/2$, $P(D_2 | x_1) = 1/2$

Now,
$$P(C_1 | D_3, x_1) = \frac{P(C_1 \cap D_3 \cap x_1)}{P(D_3 \cap x_1)} = \frac{P(D_3 \cap C_1 \cap x_1)}{P(C_1 \cap x_1)} \times \frac{P(C_1 \cap x_1)}{P(D_3 \cap x_1)}$$

$$= \frac{P(D_3 | C_1, x_1) \cdot P(C_1, x_1)}{P(D_3, x_1)}$$

$$= \frac{1/2 \cdot P(C_1) \cdot \cancel{P(x_1)}}{P(D_3 | x_1) \cancel{P(x_1)}} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{1/2} = 1/3$$

$$P(C_2 | D_3, x_1) = \frac{P(C_2, D_3, x_1)}{P(D_3, x_1)} = \frac{P(D_3 | C_2, x_1) \, P(C_2, x_1)}{P(D_3 | x_1) \, P(x_1)}$$

$$= \frac{1 \times 1/3}{1/2} = 2/3$$

$$P(C_1 | D_2, x_1) = \frac{P(D_2 | C_1, x_1) \quad P(C_1, x_1)}{P(D_2 | x_1) \, P(x_1)} =$$

$$P(C_3 | D_2, x_1) =$$

$$P(C_3 | D_3, x_1) = \frac{P(D_3 | C_3, x_1) \quad P(C_3, x_1)}{P(D_3 | x_1) \, P(x_1)}$$

$$= 0$$

So, the wining probability increases with switching.

So, you tell to open another closed door.

3(c).

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?). The required event will happen if in the next 3 times, 2 black balls are placed & 1 white ball is placed.

($n=3$, $p = \frac{4}{6} = \frac{2}{3}$, success ≡ die shows {1,3,4,6})

$$\therefore P(\text{req.}) = {}^3C_2 \cdot \left(\tfrac{2}{3}\right)^2 \cdot \tfrac{1}{3} = 4/9.$$

37). Let A_{ij} denote the event that jth machine produces code i, $i = 0,1$, $j = 1,2,3,4$

req. prob.

using Bayes' th.

$$P(A_{01} | A_{14}) = \frac{P(A_{14} | A_{01})\ P(A_{01})}{P(A_{14} | A_{01})\ P(A_{01}) + P(A_{14} | A_{11})\ P(A_{11})}$$

Now, each code, gets changed maximum 3 times

$$M_1 \to M_2 \to M_3 \to M_4 \to$$

If M_1 produces code 0 & M_4 produces code 1, the code has changed odd no. of times.

Similarly, if both M_1 & M_4 produce code 1, then the code has changed even no. of times

$$\text{So, } P(A_{14} | A_{01}) = \binom{3}{1}\left(\tfrac{3}{4}\right)\left(\tfrac{1}{4}\right)^2 + \binom{3}{3}\left(\tfrac{3}{4}\right)^3$$

$$P(A_{14} | A_{11}) = \binom{3}{2}\left(\tfrac{3}{4}\right)^2\left(\tfrac{1}{4}\right) + \binom{3}{0}\left(\tfrac{1}{4}\right)^3$$

$$\therefore P(A_{14} | A_{11}) = 3/10$$

38) (a). $P(BCPM) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}$

b) $P(B^c C^c P^c M^c) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = 1/5$

c). $P(B \cap C^c \cap P^c \cap M^c) + P(B^c \cap C \cap P^c \cap M^c) + P(B^c \cap C^c \cap P \cap M^c) + P(B^c \cap C^c \cap P^c \cap M)$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} + \frac{1}{2} \times \frac{1}{3} \cdot \frac{3}{4} \times \frac{4}{5} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5}$ (∵ They are mutually exclusive & mutually ind.)

$= \frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \frac{1}{20}$

(d) ·

3) · $H_j \equiv$ the terrorist is hiding in house no. j, $j = 1, \ldots, n$

$P(H_j) = p_j$

$F_j \equiv$ search of the house no. j will fail to nab the terrorist there

$P(F_j \mid H_j) = r_j$

$\rightarrow$ $P(H_j \mid F_j) = \frac{P(H_j \cap F_j)}{P(F_j)} = \frac{P(F_j \mid H_j) P(H_j)}{P(F_j \mid H_j) P(H_j) + P(F_j \mid H_j^c) P(H_j^c)}$

$= \frac{r_j p_j}{r_j p_j + (1-p_j)}$ $\because P(F_j \mid H_j^c) = 1$

Now, $\frac{r_j p_j}{r_j p_j + (1-p_j)} < p_j$

$\Leftrightarrow$ $r_j < r_j p_j + 1 - p_j$ ~~$\Rightarrow r_j + p_j - r_j p_j < 1$~~

~~$P(A \cup B) = P(A) + P(B) - P(A \cap B)$~~

~~$\Leftrightarrow \frac{r_j}{r_j p_j + (1-p_j)} < 1$~~

$r_j (1-p_j) + p_j < 1 \Leftrightarrow r_j (1-p_j) < (1-p_j)$

So, $P(H_j \mid F_j) < P(H_j) = p_j$

$\Rightarrow P(H_j \cap F_j) < P(H_j) P(F_j)$

so, H_j & F_j -vely asso.

$$P(H_i|F_j) = \frac{P(H_i \cap F_j)}{P(F_j)} = \frac{P(F_j|H_i)\,P(H_i)}{P(F_j|H_i)\,P(H_i) + P(F_j|H_i^c)\,P(H_i^c)}$$

$i \neq j$

$P(F_j|H_i) = 1$

$$= \frac{p_i}{\cancel{p_i +} r_j p_j + 1 - p_j}$$

if $\cancel{\frac{p_i}{r_j p_j + 1 - p_j} > p_i}$

$\Leftrightarrow$ $r_j p_j + 1 - p_j < 1$

$P(H_i | F_j) > P(H_i)$

$\Leftrightarrow$ $P(H_i \cap F_j) > P(H_i)\,P(F_j)$

$\Leftrightarrow$ $\frac{p_i}{r_j p_j + 1 - p_j} > p_i$ $\Leftrightarrow$ $1 > r_j p_j + 1 - p_j$

$\Leftrightarrow$ $p_j - r_j p_j > 0$

$\Rightarrow$ $p_j(1 - r_j) > 0$ (which is true)

so, H_i & F_j +ve associated.

40). A be the event 6 turns up. eventually.

$A_n \equiv$ event 6 does not show in n ~~throw~~ thrown

Then $P(A_n) = (5/6)^n$

Now, $A^c \subseteq A_n$

$\Rightarrow$ $P(A^c) \leq P(A_n) = (5/6)^n \to 0$

so, $P(A) = 1$

9. $\frac{5}{10\times 1}$

41). $P \equiv$ Result shows positive

$D \equiv$ person has disease.

$$P(D|P) = \frac{P(D \cap P)}{P(P)}$$

$$= \frac{P(P|D)\,P(D)}{P(P|D)\,P(D) + P(P|D^c)\,P(D^c)}$$

$$= \frac{0.95 \times 0.005}{0.95 \times 0.005 + (1 - 0.005) \times 0.01}$$

42). P(aircraft present | alarm)

$$= \frac{P(\cancel{\text{aircraft}}\ \text{alarm} | \text{aircraft present})\ P(\text{air pre})}{P(\text{alarm}|\text{air pre})\ P(\text{air. pre}) + (\text{alarm}|\text{air. abs})\ P(\text{air. abs})}$$

$$= \frac{0.99 \times 0.05}{0.99 \times 0.05 + (1-0.05) \times 0.10}$$

$\therefore$ P(air pre & alarm) = P(alarm | air pres) P(air pre)

$$= 0.99 \times 0.05$$

P(air pres & no alarm)

$$= P(\text{no alarm} | \text{air pres})\ P(\text{air pre})$$

$$= (1-0.99) \times 0.05$$

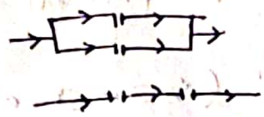
43). A = person has the disease
R = Report positive

$\cancel{P(R|A) =}$ $P(A|R) = \dfrac{P(R|A)P(A)}{P(R|A)P(A) + P(R|A^c)\ P(A^c)}$

$$= \frac{0.95 \times 0.001}{0.95 \times 0.001 + (1-0.95)(1-0.001)}$$

= Ans

44). (a). P(parallel system not work)



$= P(\text{all components not work})$

$= \prod_{i=1}^{n} (1 - p_i(t))$

$\therefore$ P (parallel system work)

$= P(\text{atleast one component work})$

$= 1 - \prod_{i=1}^{n} (1 - p_i(t))$

(b). P (series system work)

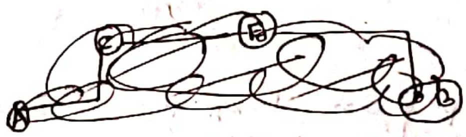
$= P(\text{all component has to be worked})$

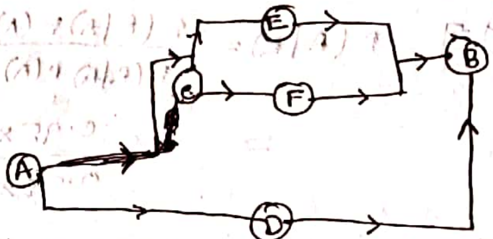
$= \prod_{i=1}^{n} p_i(t)$

(c). P (k-out system work)

$= \sum_{i=k}^{n} \binom{n}{i} (p(t))^i (1 - p(t))^{n-i}$

45).





$P\left[\overline{AD} \cap \overline{DB} \cup \left\{\overline{AC} \cap \left\{(\overline{CF} \cap \overline{FB}) \cup (\overline{CE} \cap \overline{EB})\right\}\right\}\right]$

$= P\left((\overline{AD} \cap \overline{DB}) \cup (\overline{AC} \cap \overline{CF} \cap \overline{FB}) \cup (\overline{AC} \cap \overline{CE} \cap \overline{EB})\right)$

$= P(\overline{AD} \cap \overline{DB}) + P(\overline{AC} \cap \overline{CF} \cap \overline{FB}) + P(\overline{AC} \cap \overline{CE} \cap \overline{EB})$
$- P(\overline{AD} \cap \overline{DB} \cap \overline{AC} \cap \overline{CF} \cap \overline{FB}) - P(\overline{AD} \cap \overline{DB} \cap \overline{AC} \cap \overline{CE} \cap \overline{EB})$
$- P(\overline{AC} \cap \overline{CF} \cap \overline{FB} \cap \cancel{\overline{AC}} \cap \overline{CE} \cap \overline{EB})$
$+ P(\overline{AD} \cap \overline{DB} \cap \overline{AC} \cap \overline{CF} \cap \overline{FB} \cap \cancel{\overline{AC}} \cap \overline{CE} \cap \overline{EB})$

$= 0.75 \times 0.95 + 0.90 \times 0.95 \times 0.85 + 0.90 \times 0.80 \times 0.90$
$- 0.75 \times 0.95 \times 0.90 \times 0.95 \times 0.85 - 0.75 \times 0.95 \times 0.90 \times 0.80 \times 0.90$
$- 0.90 \times 0.95 \times 0.85 \times 0.80 \times 0.90$
$+ 0.75 \times 0.95 \times 0.90 \times 0.95 \times 0.85 \times 0.80 \times 0.90$