24th Sept

Vizing's thm.:

$(\Delta+1)$-colourable

proof:

Induction on $|E|$.

colour with which I am stuck is

(1) free at the source x

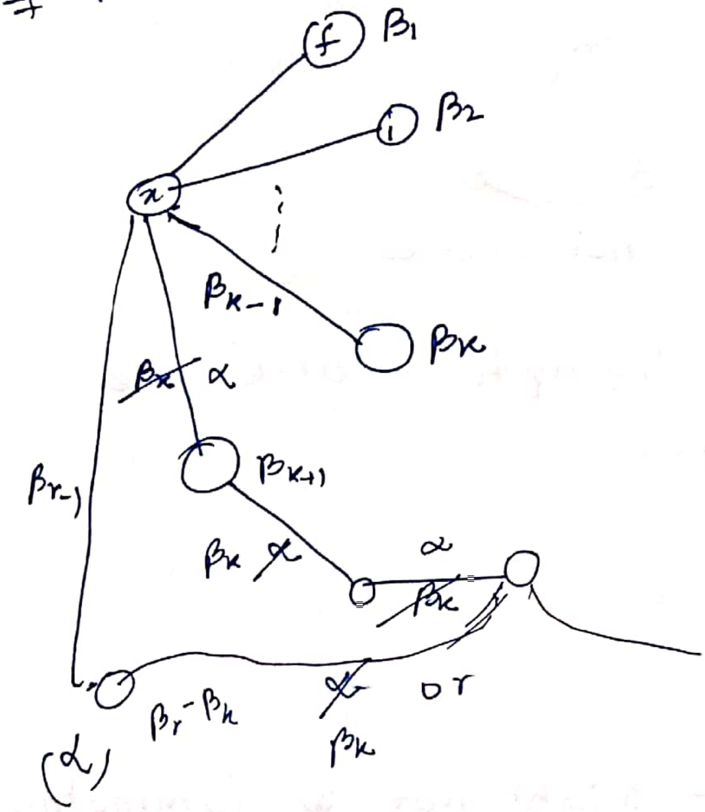
(2) already used.

~~ti~~ ~~1 = assum.~~

Case I

~~x has 2 free~~ Case II

~~f has 2 free~~



Directed Graphs (Digraphs)

A digraph D has a finite ~~set of~~ non-empty set of vertices V(D) and a set of ordered pairs of vertices A(D) : the set of arcs.

each arc has a source (~~head~~ tail) and a sink (~~tail~~ head)

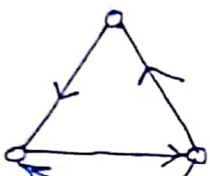
tail $\longrightarrow$ head

source. sink

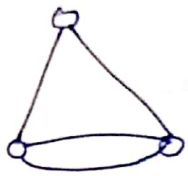
Take a digraph

Remove arrows from the arcs

$\rightarrow$ the underlying undirected graph



simple



not simple

D is a simple ~~gr~~ digraph if arcs are distinct & no loops.

isomorphic

adjacent

Incident to

adjacency matrix — Might not be symmetric in case of digraphs

di. walk — is a sequence of arcs

$v_0v_1, v_1v_2, v_2v_3, \dots, v_{m-1}v_m$

$v_0 \to v_1 \to v_2 \to \dots \to v_m$

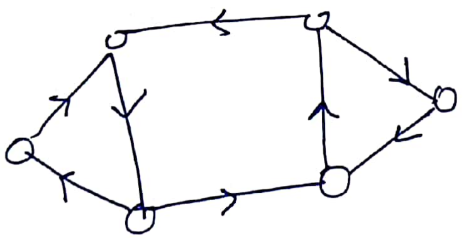
di trail : directed cycles.

D is connected if
D cannot be the ~~moon~~ union of two digraphs
≡ underlying graph is connected.

D is strongly connected if
~~∀ u, v~~ ∀ u,v ∈ V there is a directed path from u to v.

Road Map of a City :
one-way system.
if graph is not ~~a~~ strongly connected ie. no way to come back from v to u, you can only go u to v.

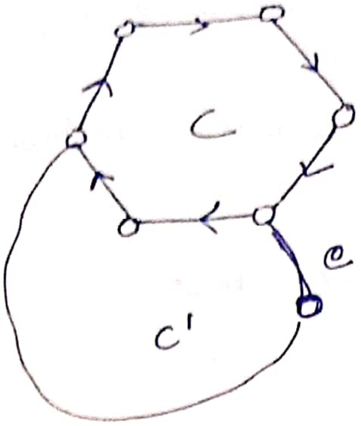
G is orientable
if each edge of G can be directed s.t. the resultant digraph is strongly connected.



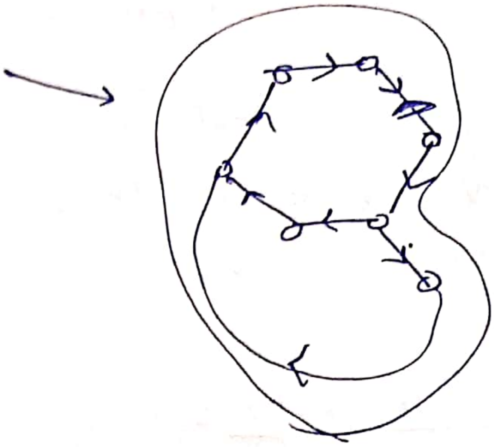
Theorem :
Let G be a connected graph. G is orientable iff each edge of G is contained in at least one cycle

Proof :
Given - G is ~~connected~~ contained in at least one cycle.

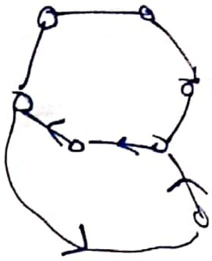
PTO



e will either belong to cycle containing upper part or lower part



or



Out degree — edges going out of the vertex ~~vertices of~~ whose deg. is a

In-degree — coming in.

Eulerian di. graph (iff proof) : $\forall$ vertex, in-degree = out-degree