DEPARTMENT OF MATHEMATICS

Indian Institute of Technology Guwahati

MA 201 (Mathematics III)

Time : 2 hours | September 17, 2018 | Total marks: $\boxed{30}$

Mid-Semester Examination

Answers without proper justification will fetch zero marks

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1. Find all complex numbers z such that $\text{Log}\,(e^z) = z$. $\boxed{2}$

2. Classify the singularity of the following functions at $z = 0$ with proper justification.
(i) $\sin\left(\dfrac{z-1}{z}\right)$ (ii) $\dfrac{1}{\sin z} - \dfrac{1}{z}$. $\boxed{2+2}$

3. Let f be an entire function. If $f(z) \neq 0$ for all $z \in \mathbb{C}$, then show that there exists an entire function g and a constant c such that $f(z) = ce^{g(z)}$ for all $z \in \mathbb{C}$. $\boxed{3}$

4. Let f be an entire function such that $f(\frac{1}{2}) = \frac{1}{2}$. If $|f(e^{i\theta})| > 1$ for all $\theta \in [0, 2\pi]$, then show that f has a zero in the unit disc. $\boxed{3}$

5. Let f be an entire function. If α is some positive irrational number and
$$\int_0^{2\pi} |f(re^{i\theta})| d\theta \leq r^\alpha \quad \forall\, r > 0,$$
then show that $f(z) = 0 \quad \forall z \in \mathbb{C}$. $\boxed{3}$

6. Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{3^n + 5^n} z^{2n}$. $\boxed{2}$

7. Does there exist an entire function such that $f\left(\dfrac{1}{n}\right) = \dfrac{i^{n!}}{n}$ for $n = 1, 2, \cdots$? Justify your answer. $\boxed{2}$

8. If u is a non-negative harmonic function on $\mathbb{C}$, then show that u is constant. $\boxed{3}$

9. Let f be an entire function. If $\lim\limits_{z\to\infty} |f(z)| = \infty$, then show that f is a polynomial. $\boxed{3}$

10. Prove that $\boxed{5}$
$$\int_0^\infty \frac{\cos ax}{(1+x^2)^2} dx = \frac{\pi(a+1)e^{-a}}{4}, \quad a > 0.$$

-:Paper Ends:-

Indian Institute of Technology, Guwahati
Department of Computer Science and Engineering

Mid Semester Examination
Semester I, 2018-19
CS 202: Discrete Mathematics

Total Marks: 65 Time: 2 Hours

Instructions: Please follow the instructions given below.

1. Write your answers neatly in clear handwriting.
2. Avoid writing unnecessarily long description.

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1. (a) Prove that $\sqrt{5}$ is an irrational number. $\boxed{3}$
   (b) Prove or disprove that there exists at least a pair of irrational numbers such that x^y is a rational number. $\boxed{2}$
   (c) Prove or disprove that the set of irrational numbers is uncountable. $\boxed{3}$
   (d) Prove that the sets $X = \{x \in [1, 2]\}$ and $Y = \{y \in [3, 7]\}$ are of the same size by showing a bijection between them. $\boxed{2}$

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. We say that f is *strictly increasing* if $f(x) < f(y)$, whenever $x < y$.
   (a) Show that a strictly increasing function f is always an injection, but may not be a surjection. $\boxed{3}$
   (b) Suppose a function f is a bijection and $f(0) = 0$ and $f(1) = 1$. Is it necessary that f is a strictly increasing function? Give justifiation. No marks will be given if justification is not given or is incorrect. $\boxed{2}$

3. R is an equivalence relation on a nonempty set A. Show that there exists a function f with A as its domain such that $(x, y) \in R$ iff $f(x) = f(y)$. $\boxed{3}$

4. Find an ordering of the tasks of a software project if fig 1 is the Hasse diagram for the tasks of the project. $\boxed{5}$

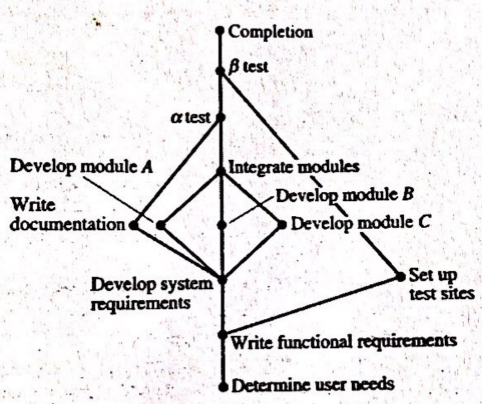

Figure 1: Hasse Diagram

5. Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is tautology without using the truth table.

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6. Use quantifiers and predicates with more than one variable to express the following statement: $\boxed{2}$

   `Every student in this class has been in at least one room of every building on campus.` Describe predicates and domains of variables clearly.

7. Show that the following statements are inconsistent: $\boxed{5}$

   ``If Amy does not take a course in discrete mathematics, then she will not graduate.''
   ``If Amy does not graduate, then she is not qualified for the job.''
   ``If Amy reads a discrete mathematics textbook, then she is qualified for the job.''
   ``Amy does not take a course in discrete mathematics but she reads a discrete mathematics textbook.''

8. Prove the validity of the following sequent using, among others, the rules =i and =e. Make sure that you indicate for each application of =e what the rule instances ϕ, t_1 and t_2 are. $\boxed{5}$

$$(x = 0) \vee ((x + x) > 0) \vdash (y = (x + x)) \rightarrow ((y > 0) \vee (y = (0 + x)))$$

9. Use quantifiers and predicates with more than one variable to express the following statement: $\boxed{3}$

   `Every student in this class has been in at least one room of every building on campus.` Describe predicates and domains of variables clearly.

10. A connected graph G has 8 vertices and 15 edges. Three vertices of the graph have degree 5 and three vertices are of degree 3. What are the possible degrees for the remaining two vertices. Give justification. No marks will be given if justification is not given or is incorrect. $\boxed{3}$

11. Prove that a tournament T has a *Hamiltonian cycle* iff it is strongly connected. $\boxed{12}$

12. Figure 2 shows a graph with a matching $M = \{ab, ci, kh, gf\}$. For each question below, a justification is required. No marks will be given if justification is not given or is incorrect.

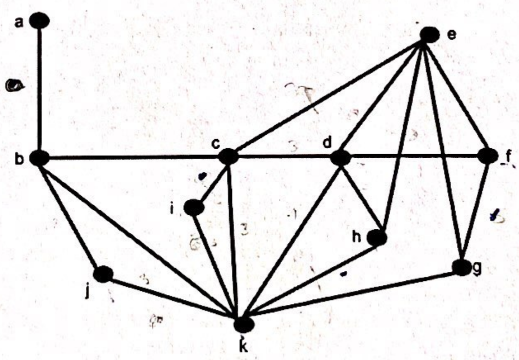


Figure 2: Garph G

   (a) Find an M-augmenting path in the graph or explain why none exists. $\boxed{2}$

   (b) Explain whether M is a maximal or a maximum or a perfect matching? $\boxed{3}$

   (c) If M is not maximum, find a matching that is maximum. $\boxed{5}$

INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Deptt. of Comp. Sc. & Engg. CS221 Digital Design

Mid Semester Exam. Date 19th Sep 2018 Time 09.00AM to 11.00AM

Full Marks=40

Part I : Basic Boolean Logic Implementation and Complex Logic Block

1) [4 Marks] Minimize the Boolean function $F(W, X, Y, Z) = \sum m(0, 2, 4, 9, 10, 15) + d(1, 3, 8)$ using Karnaugh-map Boolean function minimization.

2) [3+3 Marks] Minimize the Boolean function $F(W, X, Y, Z) = \sum m(1, 3, 8, 10, 15) + d(0, 2, 9)$ using Quine-McCluskey method. Derive all the required cube tables, generate all the require covers, and finally write the minimized Boolean function.

3) [3 Marks] Implement a two-to-four-line decoder with ENABLE signal using only NAND gates. Draw a neat diagram of your implemented circuit.

Part II : Arithmetic Circuit

4) [3 Marks] Given a Carry Select Adder of size N=1024 and group size of 4. How many multiplexers of what size you require to implement the carry select adder?

5) [5 Marks] Design a Booth multiplier (which uses 2 bit Booth recording) with a delay of complexity O(NlgN) complexity and area complexity of O(N), where size of multiplicand is N bits and size of multiplier is N bits. Draw a neat diagram of your designed circuit and explain your designed circuit.

Part III :Configurable Circuit

6) [3 Marks] Implement a 2x1 multiplexor using a three input LUT/CLB cells.

7) [3 Marks] Calculate the total number of configuration bit required to configure the interconnection part of an FPGA which have 100 number of switch matrix with each switch matrix having four vertical lines and four horizontal lines.

Part IV : Verilog HDL

8) [3+3+3 Marks] Write Verilog HDL module description of a two-to-four-line decoder with an extra Enable signal input.
   a) Using a data flow model.
   b) Using a structural model with system define primitive.
   c) Using a behavioural model using procedural statement.

9) [4 Marks] Write a proper Verilog HDL test-bench model to test the Verilog module of the previous question (Q8).

Indian Institute of technology, Guwahati,
Department of Computer Science and Engineering
Data Structure: (CS201): Mid-Semester Examination (Part B)

Date: 20th September 2018 **Duration: 2 Hours** **Full Marks: 40**

Answer All Questions

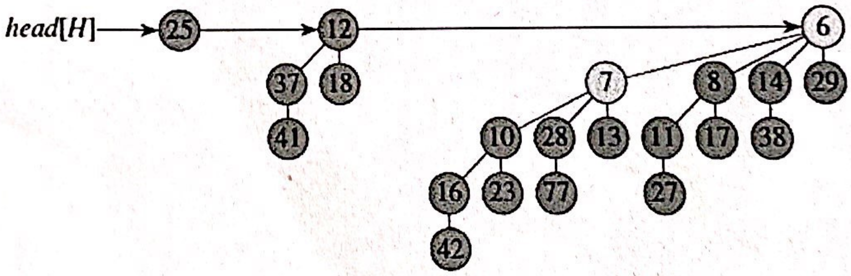
1. Use master method to give tight asymptotic bounds for the following recurrences. **(6)**
   (a) $T(n) = 2T(n/2) + n \log n$
   (b) $T(n) = 16T(n/4) + n^2$

2. Manan thinks he has discovered a remarkable property of binary search trees. Suppose that the search for key k in a binary search tree ends up in a leaf. Consider three sets: A, the keys to the left of the search path; B, the keys on the search path; and C, the keys to the right of the search path. Manan claims that any three keys $a \in A$, $b \in B$, and $c \in C$ must satisfy $a \leq b \leq c$. Are you agreeing with Manan's claim? Then justify his claim. Otherwise, give a counterexample to Manan's claim. **(4)**

3. Suppose that during the execution of BFS on a graph $G = (V, E)$, the queue Q contains the vertices $<v_1, v_2, \ldots, v_r>$, where v_1 is the head of Q and v_r is the tail. Then, prove that $d[v_r] \leq d[v_1] + 1$ and $d[v_i] \leq d[v_{i+1}]$ for $i = 1, 2, \ldots, r-1$. **(10)**

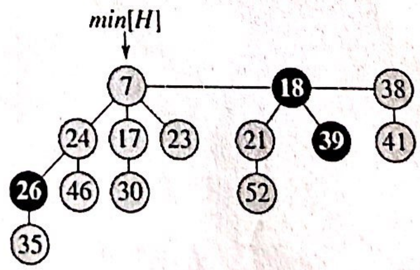
4. Write an efficient algorithm to find the smallest element in a max-heap assuming the heap is implemented using array. **(4)**

5. Show the binomial heap that results when the node with key 28 is deleted from the following binomial heap. **(6)**



6. Suppose there is no way to represent the key $-\infty$. Rewrite the BINOMIAL_HEAP_DELETE procedure to work correctly in this situation. It should still take O (log n) time. **(4)**

7. Show the resultant Fibonacci heap after extracting the minimum key from the following Fibonacci heap. **(6)**



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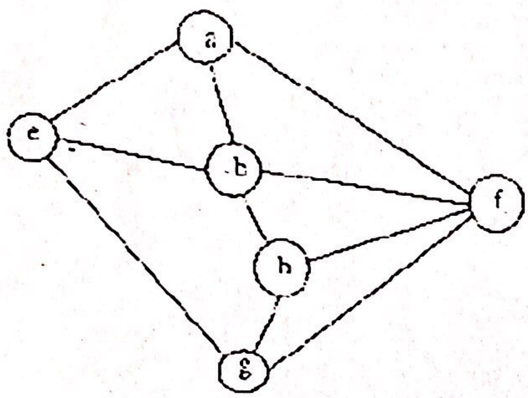
Indian Institute of technology, Guwahati,
Department of Computer Science and Engineering
Data Structure: (CS201): Class Test 2

Date: 8th September 2018 | Duration: 1 Hour | Full Marks: 20

Answer All Questions

1. Suppose that we have numbers between 1 and 1000 in a binary search tree and want to search for the number 363. Which of the following sequence could not be the sequence of nodes examined? Explain. **(3)**
(a) 924, 220, 911, 244, 898, 258, 362, 363.
(b) 925, 202, 911, 240, 912, 245, 363

2. Consider the following graph **(3)**



Among the following sequences
I) a b e g h f
II) a b f e h g ✗
III) a b f h g e
IV) a f g h b e

Which are depth first traversals of the above graph?

3. Write efficient non-recursive function(s) to construct Max-heap in bottom-up (Floyd's method) manner. Analyze the complexity of your function. **(6)**

4. Write an enhanced version of depth first search (DFS) that denotes each edge of a directed graph as any one of tree edge, back edge, forward edge and cross edge. **(8)**

Mayank Baranwal
170101084

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Data Structure (CS201) Class Test 1 Duration: 30 minutes Date: 21st Aug 2018 Total Marks: 15

1. Fill the blank lines for Morris Inorder Traversal Function given below. (7)

```
void MorrisInroderTraversal(struct tNode *root)
{
  struct tNode *current, *pre;

  if(root == NULL)
     return;

  current = root;
  while (current != NULL)
  {
    if (current->left == NULL)
    {
      printf("%d", current->data);
      current = current -> right;
    }
    else
    {
      /* Find the inorder predecessor of current */
      pre = current->left;

      while (pre->right !=NULL || pre-> right != current)
        pre = pre->right;

      /*Make current as right child of its inorder predecessor */
      if (pre->right == NULL)
      {
        pre -> right = current;
        current = current -> left;
      }

      //Revert the changes made in to restore the original
tree*/
      else
      {
        pre -> right = NULL;
        printf("%d ",current->data);
        current = current -> right;
      } /* End of if condition pre->right == NULL */
    } /* End of if condition current->left == NULL*/
  } /* End of while */
}
```

7

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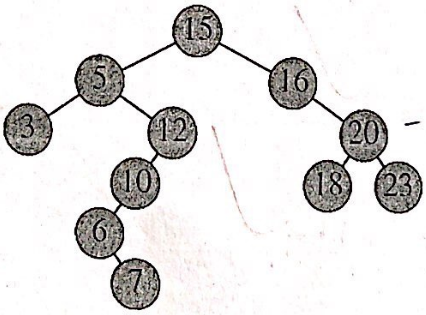
2. Write the BST_Predecessor(root, key) procedure. The function returns the pointer of the node that contains the predecessor key of the given key. You can assume that functions like BST_Minimum, BST_Maximum, and BST_Search are given to you. **(5)**

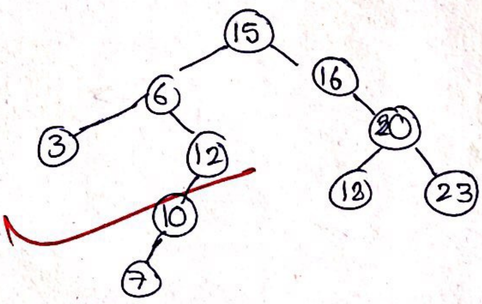
node* BST_Predecessor(node *root, int key){

```
if (!root)   return NULL;          // Binary tree does not exist
if (key == BST_Minimum (root))
        return NULL;              // No predecessor of minimum no.
node *temp = BST_Search (key);
if (!temp)  return NULL;           // key does not exist
if( temp -> left) {                // Max element of left subtree
    node* pred = BST-Maximum (temp -> left);   return pred; }
else {   node* prev= NULL;
    while (temp -> parent != NULL)
    {
        prev= temp;                        // Go up to predecessor
        temp = temp -> parent;
        if (temp -> right == prev)         // Right parent is predecessor
    }       ; return temp;
    ;
}}
}
```

3

3. Delete node 5 from the following BST. Draw the resultant BST tree after deletion. **(3)**





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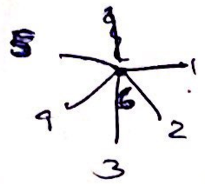
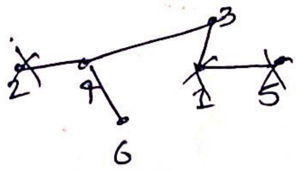


x 1 1 1

1 ✓
2 ✓
3 ✓
4
5 ✓
6

4 13 4



4 1 3 4

Open Time Examination
CS 202

Full marks – 60 — 18th November, 2018

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1. *Write your answers neatly in clear handwriting.*
2. *Avoid writing unnecessarily long description.*
3. *No clarifications will be provided.*
4. Don't Panic.

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1. If G is a connected planar simple graph, then G has a vertex of degree not exceeding five. [4 marks]

2. For any positive integer n, the number of all labelled trees with n vertices is n^{n-2}. [10 marks]

3. A number is called *triangular* if it is the sum of consecutive integers, beginning with 1. Prove the following statements:

   (a) The integer n is a triangular number if and only if $8n+1$ is a perfect square. [2 marks]

   (b) If t_n denotes the n^{th} triangular number, then without using Mathematical Induction, prove that for $n \geq 1$, [3 marks]

$$t_1 + t_2 + \cdots + t_n = \frac{n(n+1)(n+2)}{6}.$$

$n = \frac{i(i+1)}{2}$ 8+1

$4\,i(i+1) + 1$

$4i^2 + 4i + 1$

$4k+1$

$(2i+1)^2$

$8n+1 = \cancel{z}\,z^2$

$n = \frac{z^2-1}{8}$

$t_n = \frac{n(n+1)}{2}$

4. A *palindrome* is a number that reads the same backward as forward(for example, 121 are 13431 palindromes). Determine whether the following statement is correct or not with justification: Any palindrome with an even number of digits is divisible by 11. No marks will be given for incorrect, incomplete or no justification. [2 marks]

5. Determine the number of antisymmetric relations on a set with n elements. [4 marks]

6. Two tribes A and B leave peacefully in an isolated island. Persons beloing to the tribe A always tell the truth, whereas persons of tribe B community always lie. You encountered two inhabitants of the island I_1, I_2. I_1 says to you "At least one of us always lie", whereas I_2 does not say anything. Using the propositional logic formulation, determine which tribes I_1 and I_2 belong to (if feasible, otherwise shows it is not possible). [5 marks]

7. The set M consists of nine positive integers, none of which has a prime divisor larger than six. Prove that M has two elements whose product is the square of an integer. [5 marks]

8. There are four heaps of stones in our backyard. We rearrange them into five heaps. Prove that at least two stones are placed into a smaller heap. [5 marks]

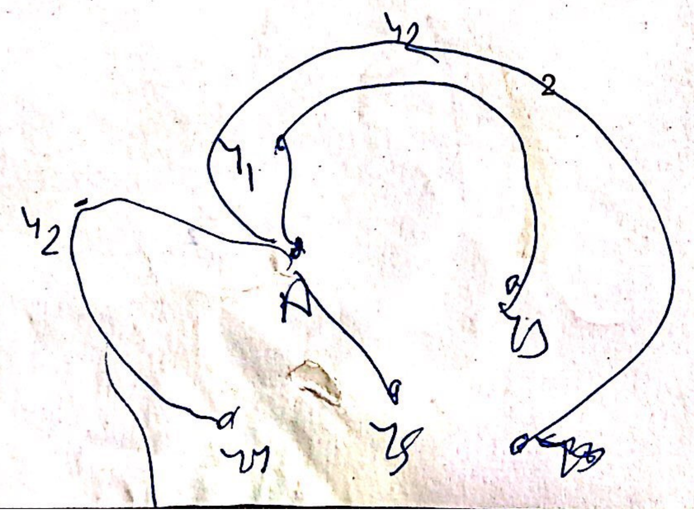
9. Prove that for all integers $n \geq 2$, we have [5 marks]

$$2^{n-2} \cdot n \cdot (n-1) = \sum_{k=2}^{n} k(k-1)\binom{n}{k}.$$

10. Show that a group is abelian if and only if for all pairs of elements a and b in the group, the following holds – $(ab)^{-1} = a^{-1}b^{-1}$. [5 marks]

11. Let x and y be members of the group G. Show that xy and yx have the same order. [5 marks]

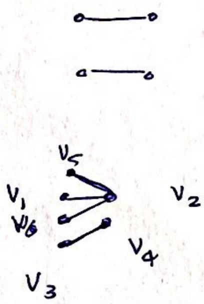
12. If every element x in a ring R satisfies $x^2 = x$, prove that R must be commutative. [5 marks]



End Semester Examination
CS 202

Full marks – 50 26^{th} November, 2018

1. *Start each answer on a fresh page.*
2. *Write your answers neatly in clear handwriting.*
3. *Avoid writing unnecessarily long description.*
4. *Each question carries five marks.*
5. *No clarifications will be provided.*
6. Don't Panic.



1. A game is played as follows. Two players alternately select distinct vertices v_0, v_1, v_2, ... of a graph G, where, for $i \geq 0$, v_{i+1} is required to be adjacent to v_i. The last player able to select a vertex wins the game. Show that the first player has a winning strategy iff G has no perfect matching.

p q

$\sim p \Rightarrow \sim q$

2. How many ways are there to place *some* rooks on an $n \times n$ chessboard so that no two of them attack each other?

3. Ramu goes on a four-week vacation. He takes with him forty chocolates in a box. He eats at least one chocolate each day, starting from day one. Prove that there exists a span of consecutive days during which he eats exactly fifteen chocolates.

k pairs of adj vertices v_i, v_{i+1}

Induction : For n vertices : No perfect match $\Rightarrow$ 1^{st} wins
G $\rightarrow$ perfect matching

No perfect $\exists T \subseteq V(G)$ s.t $|T| > |N_G(T)|$ Inductive hyp

4. In how many ways can one distribute n identical balls into k distinct bins?

5. Prove that in a sequence of n^2+1 distinct real numbers, there is either an increasing subsequence of length $n+1$ or a decreasing subsequence of length $n+1$.

6. If $p \neq 5$ is an odd prime, prove that either p^2-1 or p^2+1 is divisible by 10.

7. If H is a nonempty finite subset of a group G and H is closed under multiplication, then prove that H is a subgroup of G.

8. Prove that a finite integral domain is a field.

9. Using generating functions, find a closed-form for the following recurrence.

$$
\begin{aligned}
a_0 &= 0; \\
a_1 &= 1; \\
a_{n+2} &= 3a_{n+1} - 2a_n, \quad \text{for } n \geq 0.
\end{aligned}
$$

10. Let $n = 4k+2$, for some nonnegative integer k. Prove that exactly $1/4$ of all subsets of $[n]$ have a size that is divisible by four.

$p^2 \equiv -1 \pmod{10}$

$p^2 \equiv 1 \pmod{10}$

$\cancel{0}, \overset{\downarrow}{1}, \cancel{2}, 3, \cancel{4}, \cancel{5}, \cancel{6}, 7, \cancel{8}, \overset{\downarrow}{9}$

$10k+3$

End Semester Examination: MA 201 Mathematics III

Department of Mathematics, IIT Guwahati

Date: November 24, 2018 Time: 9:00–12:00 hours Maximum Marks: 50

Name: Mayank Balomural **Roll No:** 17C1010 84

Instructions:

(a) Write your name and roll number above as soon as you receive the question paper.

(b) Make sure that you are in the designated room as per the seating plan.

(c) Enter the Question number and the corresponding page number(s) in the front page of the booklet as instructed.

(d) Not adhering to Instructions (b) and (c) above will invite **penalty**.

(e) Questions are self-explanatory. No query will be entertained by the invigilators.

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1. Find the image of the points 0 and $-i$ in $\mathbb{C}$ on the Riemann sphere S^2 under the stereographic projection. **[2]**

2. Let $f(z) = z^2 \cos z$. If C_1 and C_2 are two smooth curves in the complex plane intersecting at origin with an angle $\frac{\pi}{6}$, then find the angle between the curves $f(C_1)$ and $f(C_2)$ at origin. **[2]**

3. Find the Möbius transform $w = T(z)$ that maps the points $z_1 = -i, z_2 = 0$ and $z_3 = i$ onto $w_1 = -1, w_2 = i$ and $w_3 = 1$ respectively. What is the image of the imaginary axis under T? **[3]**

4. Find the image of the lower half plane $\text{Im}(z) < 0$ under the map $w = \dfrac{i+z}{i-z}$. **[3]**

5. Find an integral surface for the initial value problem **[2+1+2]**
$$uu_x + xu_y = 1, \quad u\left(\frac{1}{2}x^2 + 1, \frac{1}{6}x^3 + x\right) = x.$$
Are there any other solutions? Explain lack of uniqueness and find at least two solutions.

6. Check the generalized transversality condition for the initial value problem **[3]**
$$u_x^2 + u_y^2 = 1, \quad u(\cos s, \sin s) = 0, \quad 0 \le s \le 2\pi$$
and comment on the existence and uniqueness of the solution.

7. Find general solution for the PDE **[3]**
$$x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} + xu_x + yu_y = 0, \quad x > 0.$$

8. Let $u(x,t)$ be the solution of the equation $u_{tt} - u_{xx} = 6t$ in the whole plane. Suppose that $u_x(x,t)$ is constant on the line $x = 1 + t$. Also assume that $u(x,0) = 1$ and $u(1,1) = 3$. Find a solution $u(x,t)$. Is this uniquely determined? Justify your answer. **[3+1]**

Please turn overleaf.....

9. Obtain the Fourier series expansion of $f(x)$ given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0, \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi. \end{cases}$$

Hence evaluate the infinite sum $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$. [3]

10. Given the Fourier series for the function $f(x) = x^2$, $-1 < x < 1$ as

$$f(x) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi x$$

find the Fourier series for $g(x) = x$, $-1 < x < 1$.
State under what conditions this type of derivation is possible. [2+1]

In questions 11-12 below, the solutions are required precisely for the given problem only. Any results derived prior to finding the solution to the given problem will not be worth any mark.

11. A string of length 2 metres, fastened to its ends, with $c = 30$ metre/sec, is initially at rest but is given an initial velocity of $300 \sin 4\pi x$ from its equilibrium position. Assuming that the string is homogenous, under uniform tension and not acted upon by any external force, find the vertical displacement $u(x,t)$ of the string at any point x at any time $t > 0$. [3]

12. Find the temperature $u(x,t)$ in a thin metal rod of length π which is perfectly insulated including the end $x = 0$ while the temperature at the end $x = \pi$ is maintained at zero degree Celsius such that with an initial temperature in the bar given as $u(x,0) = 100$. Take the thermal diffusivity of the rod to be one. [3]

13. Consider steady-state heat conduction outside a circular, planar disk of radius a whose temperature at the boundary is given to be a function of θ. Find the temperature distribution at any point outside the disk. [4]

14. (a) By considering an appropriate Fourier transform of the function $f(t) = e^{-at}$, $t > 0$, where $a > 0$ is a constant, evaluate the improper integral $\int_0^{\infty} \frac{x \sin kx}{a^2 + x^2} dx$. [3]
(b) Consider the heat conduction in a thin infinite rod of thermal diffusivity α with initial temperature distribution $\phi(x)$. By using Fourier transform, find the temperature distribution $u(x,t)$ at any point at any subsequent time $t > 0$. How would you approach this problem if the rod is of semi-infinite length? [2+1]

15. (a) Find the Laplace transform of $F(t) = \begin{cases} t+1, & 0 \le t \le 2, \\ 3, & t > 2. \end{cases}$ [2]
(b) Let $f(s) = \mathcal{L}\{F(t)\}$. By using the appropriate result(s), find [2]

$$\mathcal{L}^{-1}\left\{\frac{f(s)}{(s+a)^2 + b^2}\right\}.$$

(c) Using Laplace transform technique, solve the following initial value problem: [3]

$$\frac{d^2x}{dt^2} + x = 3\sin 2t, \quad x(0) = 3, \quad \frac{dx}{dt}(0) = 1.$$

$\boxed{\text{– The Paper Ends Here –}}$