

INTRODUCTORY CONCEPTS OF R PROGRAMMING

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What is R?

- R is a language and environment for statistical computing and graphics. It is a **GNU project** which is similar to the S language and environment which was developed at Bell Laboratories (formerly AT& T, now Lucent Technologies) by John Chambers and colleagues. R can be considered as a different implementation of S. There are some important differences, but much code written for S runs unaltered under R.
- R provides a wide variety of statistical (linear and nonlinear modelling, classical statistical tests, time-series analysis, classification, clustering, ...) and graphical techniques, and is highly extensible. The S language is often the vehicle of choice for research in statistical methodology, and R provides an Open Source route to participation in that activity

- One of R's strengths is the ease with which well-designed publication-quality plots can be produced, including mathematical symbols and formulae where needed. Great care has been taken over the defaults for the minor design choices in graphics, but the user retains full control.
- R is available as Free Software under the terms of the **Free Software Foundation's GNU General Public License** in source code form

The R environment

- R is an integrated suite of software facilities for data manipulation, calculation and graphical display
- It includes an effective data handling and storage facility
- A suite of operators for calculations on arrays, in particular matrices
- A large, coherent, integrated collection of intermediate tools for data analysis
- Graphical facilities for data analysis and display either on-screen or on hardcopy
- A well-developed, simple and effective programming language which includes conditionals, loops, user-defined recursive functions and input and output facilities.

Why should you adopt R Programming Language?

- R, SAS, and SPSS are three statistical languages. Of these three statistical languages, R is the only an open source. SAS is the most important private software business in the world. SPSS is now overseen by IBM. R Programming is extensible and hence, R groups are noted for its energetic contributions. Lots of Rs typical features can be written in R itself and hence, R has gotten faster over time and serves as a glue language.

• Installation

- R can be downloaded from one of the mirror sites in <http://cran.r-project.org/mirrors.html>. You should pick your nearest location.

• Using External Data

- R offers plenty of options for loading external data, including Excel, Minitab and SPSS files. We have included a tutorial titled [Data Import](#) on the subject for the purpose.

What is Command line

```
Console Terminal × Background Jobs ×
R 4.3.1 . /cloud/project/ ↗

R version 4.3.1 (2023-06-16) -- "Beagle Scouts"
Copyright (C) 2023 The R Foundation for Statistical Computing
Platform: x86_64-pc-linux-gnu (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

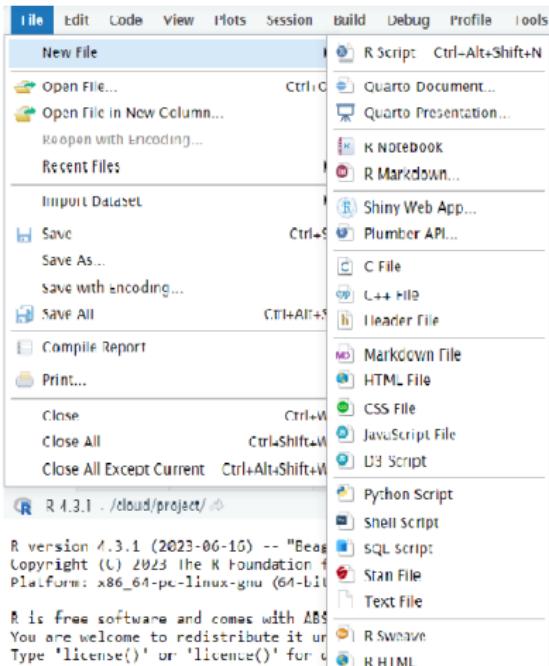
> Type the Commands here This is Command line
```

- Execution of commands in R is not menu driven. (Not like clicking over buttons to get outcome)
- We need to type the commands.
- Single line and multi line commands are possible to write.
- When writing multi-line programs, it is useful to use a text editor rather than execute everything directly at the command line.

Command line versus scripts

At this point R will
open a window entitled
Untitled-R Editor.

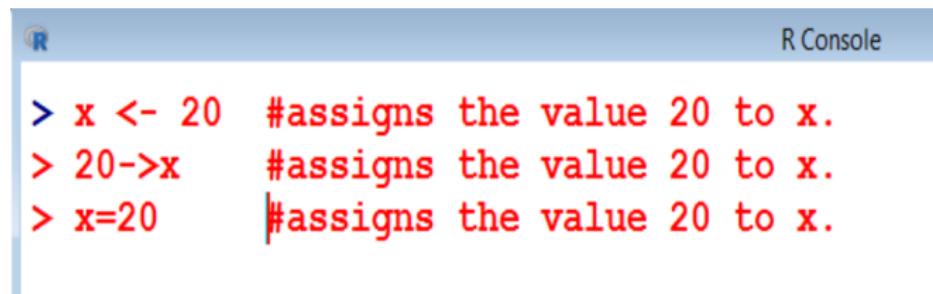
We may type and edit in this.



If we want to execute a line or a group of lines, just highlight them and press **Ctrl+R**.

Basics

- > is the prompt sign in R. you can enter numbers and perform calculations.
- The assignment operators are the left arrow with dash <- ,->and equal sign =.

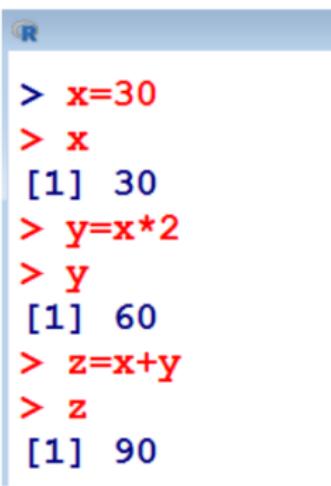


The image shows a screenshot of an R console window. The title bar says "R Console". The console area contains three lines of R code, each followed by a comment in red:

```
> x <- 20 #assigns the value 20 to x.  
> 20->x #assigns the value 20 to x.  
> x=20 #assigns the value 20 to x.
```

- `> x = 30` #assigns the value 30 to x.
- `> y = x * 2` #assigns the value 2*x to y.
- `> z = x + y` #assigns the value x + y to z.

Output:-



```
R
> x=30
> x
[1] 30
> y=x*2
> y
[1] 60
> z=x+y
> z
[1] 90
```

The image shows a screenshot of an R console window. The title bar has the letter 'R' in blue. The main area contains the R code and its corresponding output. The code consists of five lines starting with '>'. The first three lines define variables: 'x' is assigned the value 30, 'y' is assigned the value of 'x' multiplied by 2, and 'z' is assigned the value of 'x' plus 'y'. The last two lines are the results of printing 'x' and 'z', respectively, both showing the value 30 and 90 in blue brackets [1].

- # : The character # marks the beginning of a comment. All characters until the end of the line are ignored.

```
> # mu is the mean
```

```
> # x <- 20 is treated as comment only
```

- Capital and small letters are different.

```
> x=20
```

```
> X
```

```
Error: object 'X' not found
```

```
> x
```

```
[1] 20
```

```
> X=30
```

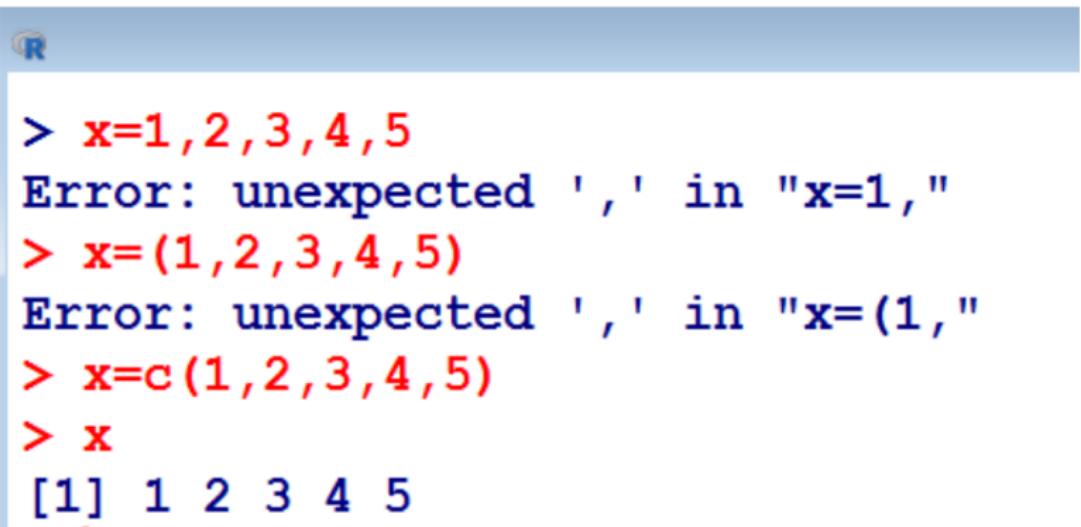
```
> X
```

```
[1] 30
```

```
> x+X
```

```
[1] 50
```

- The command `c(1,2,3,4,5)` combines the numbers 1,2,3,4 and 5 to a vector.



```
R> x=1,2,3,4,5
Error: unexpected ',', ' in "x=1,"
R> x=(1,2,3,4,5)
Error: unexpected ',', ' in "x=(1,"
R> x=c(1,2,3,4,5)
R> x
[1] 1 2 3 4 5
```

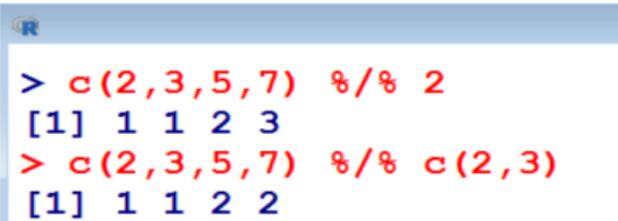
R as a Calculator

```
|> 2+3 # Command  
[1] 5  
> 2-3  
[1] -1  
> 2*3  
[1] 6  
> 2/3  
[1] 0.6666667  
> 2*3-4+5/6  
[1] 2.833333
```

```
R> 2^3          # Command  
[1] 8  
R> 2**3        # Command  
[1] 8  
R> 2^0.5        # Command  
[1] 1.414214  
R> 2**0.5        # Command  
[1] 1.414214  
R> 2^-0.5        # Command  
[1] 0.7071068  
R> 2^3^2        # Command  
[1] 512  
R> (2^3)^2        # Command  
[1] 64  
R> 2^(3^2)        # Command  
[1] 512
```

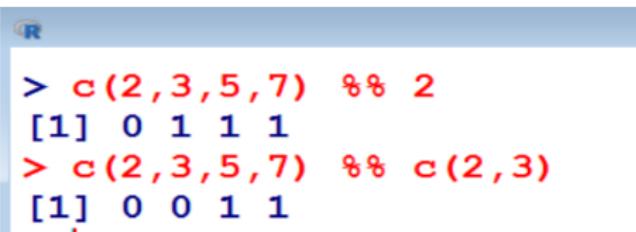
```
> c(2,3,5,7)^2          # command: application to a vector  
[1] 4 9 25 49  
> c(2,3,5,7)^c(2,3)      # command: application to a vector  
[1] 4 27 25 343  
> c(1,2,3,4,5,6)^c(2,3,4) # command: application to a vector  
[1] 1 8 81 16 125 1296  
> c(2,3,5,7)*3           # command: application to a vector  
[1] 6 9 15 21  
> c(2,3,5,7)*c(2,3)       # command: application to a vector  
[1] 4 9 10 21  
> c(1,2,3,4,5,6)*c(2,3,4) # command: application to a vector  
[1] 2 6 12 8 15 24  
> c(2,3,5,7)*c(-2,-3,-5,8) # command: application to a vector  
[1] -4 -9 -25 56  
> c(2,3,5,7) + 10         # command: application to a vector  
[1] 12 13 15 17
```

- Integer Division: Division in which the fractional part (remainder) is discarded



```
R> c(2,3,5,7) %/% 2
[1] 1 1 2 3
R> c(2,3,5,7) %/% c(2,3)
[1] 1 1 2 2
```

- $x \bmod y$: modulo operation finds the remainder after division of one number by another



```
R> c(2,3,5,7) %% 2
[1] 0 1 1 1
R> c(2,3,5,7) %% c(2,3)
[1] 0 0 1 1
```

Maximum: max & Minimum: min

```
> max(1.2, 3.4, -7.8)
[1] 3.4
> max( c(1.2, 3.4, -7.8) )
[1] 3.4
> min(1.2, 3.4, -7.8)
[1] -7.8
> min( c(1.2, 3.4, -7.8) )
[1] -7.8
```

Overview Over Further Functions

<code>abs()</code>	Absolute value
<code>sqrt()</code>	Square root
<code>round()</code> , <code>floor()</code> , <code>ceiling()</code>	Rounding, up and down
<code>sum()</code> , <code>prod()</code>	Sum and product
<code>log()</code> , <code>log10()</code> , <code>log2()</code>	Logarithms
<code>exp()</code>	Exponential function
<code>sin()</code> , <code>cos()</code> , <code>tan()</code> , <code>asin()</code> , <code>acos()</code> , <code>atan()</code>	Trigonometric functions
<code>sinh()</code> , <code>cosh()</code> , <code>tanh()</code> , <code>asinh()</code> , <code>acosh()</code> , <code>atanh()</code>	Hyperbolic functions

```
R> abs(-4)
[1] 4
> abs(c(-1,-2,-3,4,5))
[1] 1 2 3 4 5
> sqrt(4)
[1] 2
> sqrt(c(4,9,16,25))
[1] 2 3 4 5
> sum(c(2,3,5,7))
[1] 17
> prod(c(2,3,5,7))
[1] 210
> round(1.23)
[1] 1
> round(1.83)
[1] 2
```

- `log()` function computes natural logarithms (\ln) for a number or vector. `log10` computes common logarithms (\lg).`log2` computes binary logarithms (\log_2). `log(x,b)` computes logarithms with base b .



```
> log(10)
[1] 2.302585
> log10(10)
[1] 1
> log(9,base=3)
[1] 2
```

Complex functions

R

R Console

```
> var<-3+5i      # a variable with a complex value
> var
[1] 3+5i
> Re(var)        # real part
[1] 3
> Im(var)        # imaginary part
[1] 5
> Conj(var)      # complex conjugate
[1] 3-5i
> Mod(var)       # complex modulus
[1] 5.830952
> Arg(var)       # complex argument
[1] 1.030377
```

Matrix

- **Matrices are important objects in any calculation.** A matrix is a rectangular array with p rows and n columns.
- **Note:-**
 - The parameter `nrow` defines the row number of a matrix.
 - The parameter `ncol` defines the column number of a matrix.
 - The parameter `data` assigns specified values to the matrix elements.

- In R, a 4×2 -matrix X can be created with a following command:

```
> x <- matrix( nrow=4, ncol=2,data=c(1,2,3,4,5,6,7,8))  
> x
```

```
      [,1] [,2]  
[1,]    1    5  
[2,]    2    6  
[3,]    3    7  
[4,]    4    8
```

- One can access a single element of a matrix with $x[i,j]$:

```
> x[3,2]  
[1] 7
```

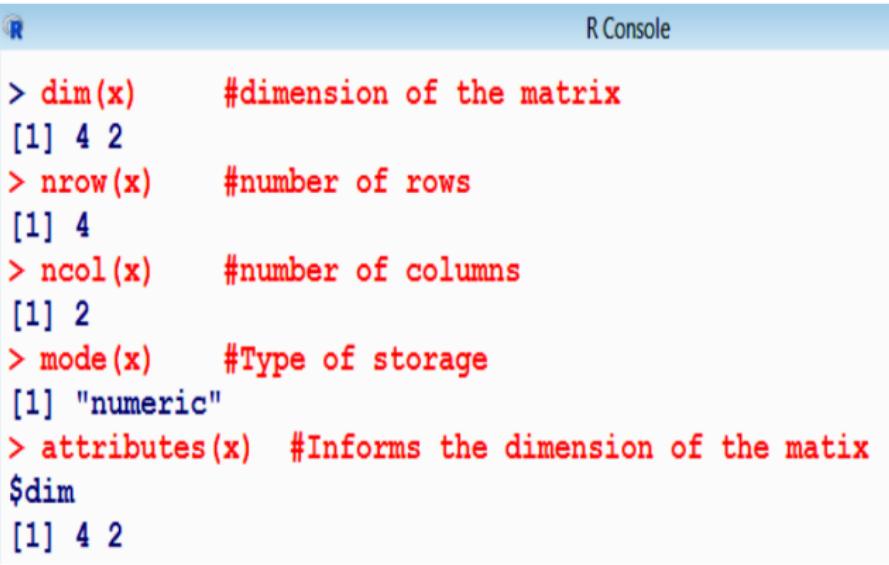
- In case, the data has to be entered row wise, then a 4×2 -matrix X can be created with

R

R Console

```
> x <- matrix( nrow=4, ncol=2,data=c(1,2,3,4,5,6,7,8) ,byrow=TRUE)
> x
     [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6
[4,]    7    8
```

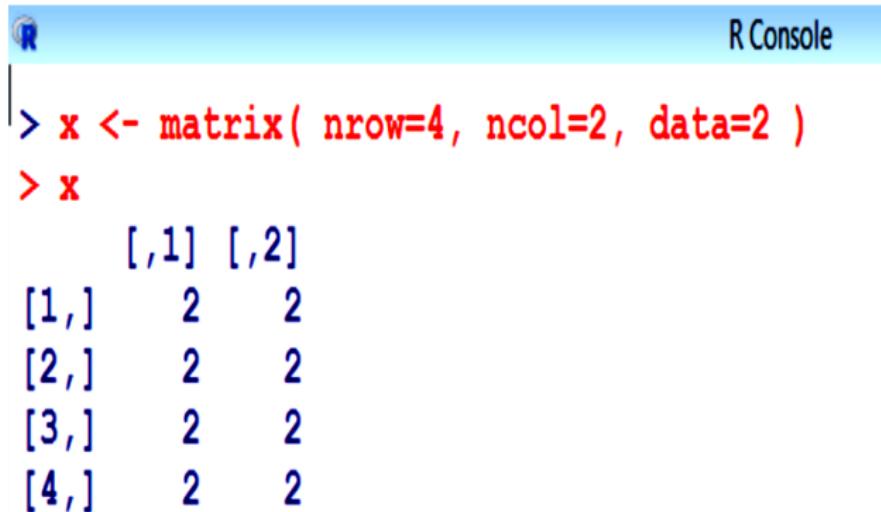
- We can get specific properties of a matrix:



The screenshot shows an R console window with a blue header bar containing the R logo and the text "R Console". The main area contains the following R code and its output:

```
> dim(x)      #dimension of the matrix
[1] 4 2
> nrow(x)     #number of rows
[1] 4
> ncol(x)     #number of columns
[1] 2
> mode(x)     #Type of storage
[1] "numeric"
> attributes(x) #Informs the dimension of the matix
$dim
[1] 4 2
```

- Assigning a specified number to all matrix elements:



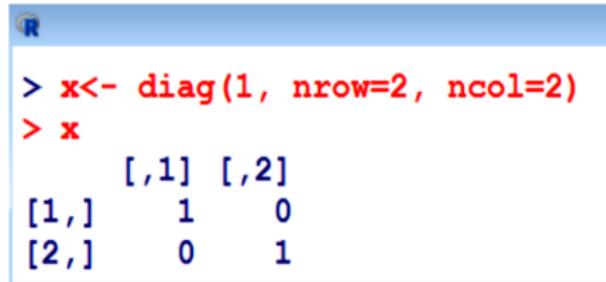
The screenshot shows the R console interface. The title bar says "R Console". The code input area contains the following R code:

```
> x <- matrix( nrow=4, ncol=2, data=2 )
> x
```

The output area displays the resulting matrix:

	[,1]	[,2]
[1,]	2	2
[2,]	2	2
[3,]	2	2
[4,]	2	2

- Construction of a diagonal matrix, here the identity matrix of a dimension 2:



The image shows a screenshot of an R console window. The title bar is blue with the letter 'R' in white. The main area contains R code and its output. The code is:
> x<- diag(1, nrow=2, ncol=2)
> x
[1,] [2,]
[1,] 1 0
[2,] 0 1

- Transpose of a matrix X : X'

```
R Console
> x <- matrix( nrow=4, ncol=2,data=c(1,2,3,4,5,6,7,8))
> x
     [,1] [,2]
[1,]    1    5
[2,]    2    6
[3,]    3    7
[4,]    4    8
> y<-t(x)
> y
     [,1] [,2] [,3] [,4]
[1,]    1    2    3    4
[2,]    5    6    7    8
```

- Multiplication of a matrix with a constant

```
R> 4*x
      [,1] [,2]
[1,]     4    8
[2,]    12   16
[3,]    20   24
[4,]    28   32
```

- Matrix multiplication: operator %*%

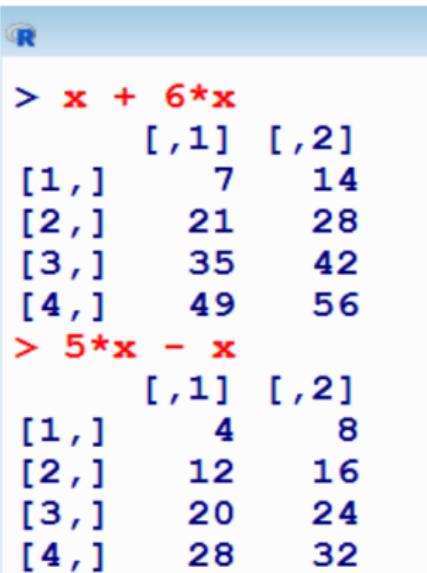
```
R> xt<-t(x) %*% x
> xt
      [,1] [,2]
[1,]    84   100
[2,]   100   120
```

- Cross product of a matrix X, $X'X$, with a function `crossprod`

```
R> xt <- crossprod(x)
> xt
     [,1] [,2]
[1,]   84  100
[2,]  100  120
>
```

- Note: Command `crossprod()` executes the multiplication faster than the conventional method with `t(x)%*%x`

- Addition and subtraction of matrices (of same dimensions!) can be executed with the usual operators + and -



The screenshot shows an R console window with a blue header bar containing the R logo. The console displays two R commands and their results:

```
> x + 6*x
      [,1] [,2]
[1,]    7   14
[2,]   21   28
[3,]   35   42
[4,]   49   56

> 5*x - x
      [,1] [,2]
[1,]    4    8
[2,]   12   16
[3,]   20   24
[4,]   28   32
```

The first command, `x + 6*x`, adds 6 times the matrix `x` to itself. The second command, `5*x - x`, subtracts the matrix `x` from 5 times itself.

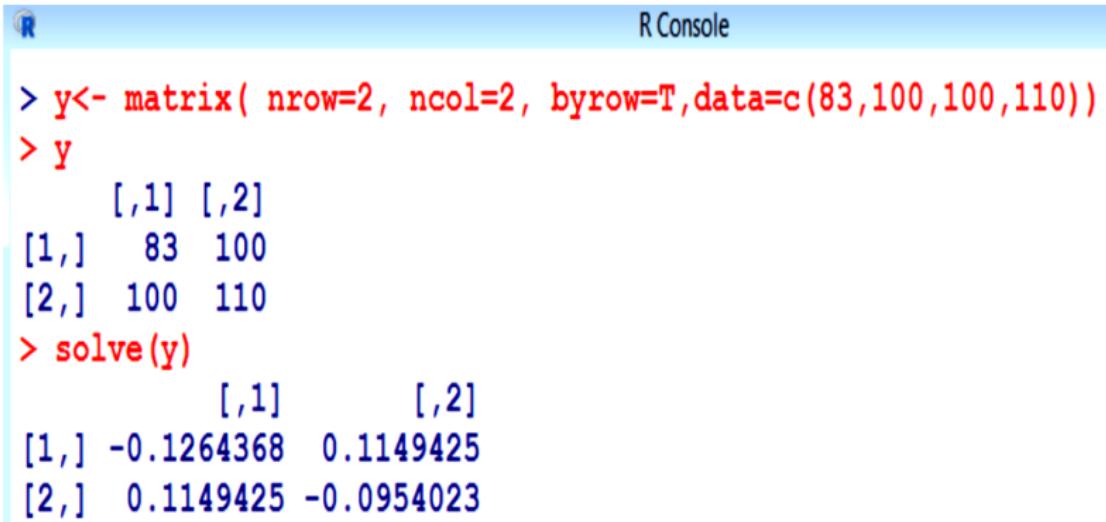
- Access to rows, columns or submatrices:

```
R           R Console

> x <- matrix( nrow=4, ncol=2,data=c(1,2,3,4,5,6,7,8),byrow=TRUE)
> x[2,]
[1] 3 4
> x[,2]
[1] 2 4 6 8
> x[,3]    #There is no third column
Error in x[, 3] : subscript out of bounds
> x[3:4, 1:2]
      [,1] [,2]
[1,]     5   6
[2,]     7   8
```

- Inverse of a matrix:

`solve()` finds the inverse of a positive definite matrix



R Console

```
> y<- matrix( nrow=2, ncol=2, byrow=T,data=c(83,100,100,110))
> y
     [,1] [,2]
[1,]   83   100
[2,]   100   110
> solve(y)
      [,1]      [,2]
[1,] -0.1264368  0.1149425
[2,]  0.1149425 -0.0954023
```

- $5x = 10$, what's x ?

```
R
> solve(5,10)
[1] 2
```

- Let's see two variables examples:
- $3x + 2y = 8$, $x + y = 2$; What's x and y ?

```
R Console
> a <- matrix(c(3,1,2,1),nrow=2,ncol=2)
> b <- matrix(c(8,2),nrow=2,ncol=1)
> solve(a,b)
     [,1]
[1,]    4
[2,]   -2
>
```

- **Eigen Values and Eigen Vectors:**

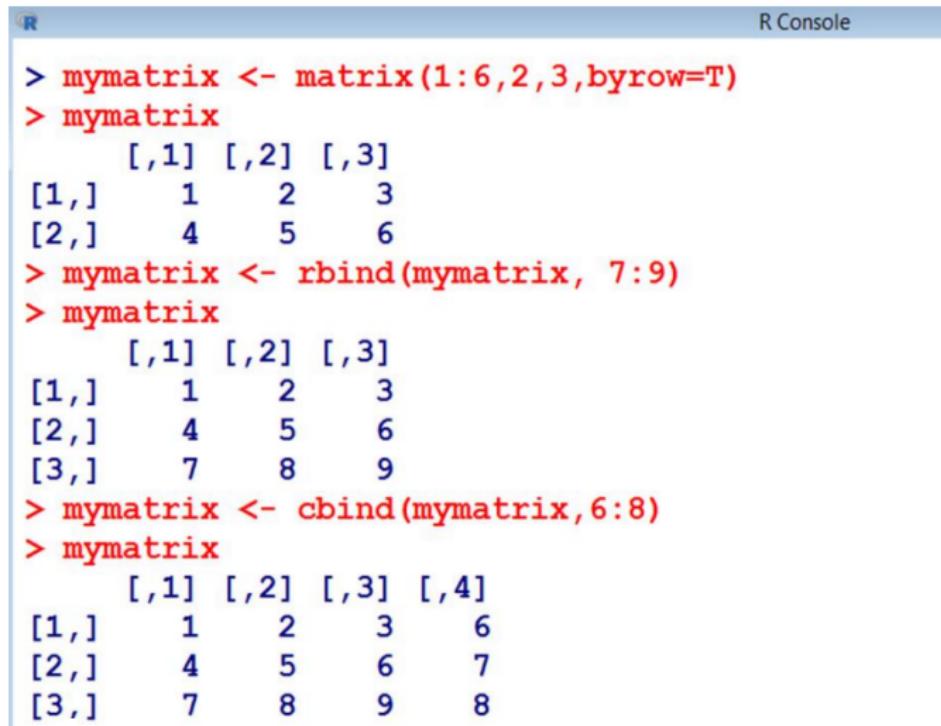
`eigen()` finds the eigen values and eigen vectors of a positive definite matrix

```
R Console

> y
      [,1] [,2]
[1,]    83   100
[2,]   100   110
> eigen(y)
$values
[1] 197.407136 -4.407136

$vectors
      [,1]          [,2]
[1,] 0.6581085 -0.7529231
[2,] 0.7529231  0.6581085
```

- Changing the matrix's elements



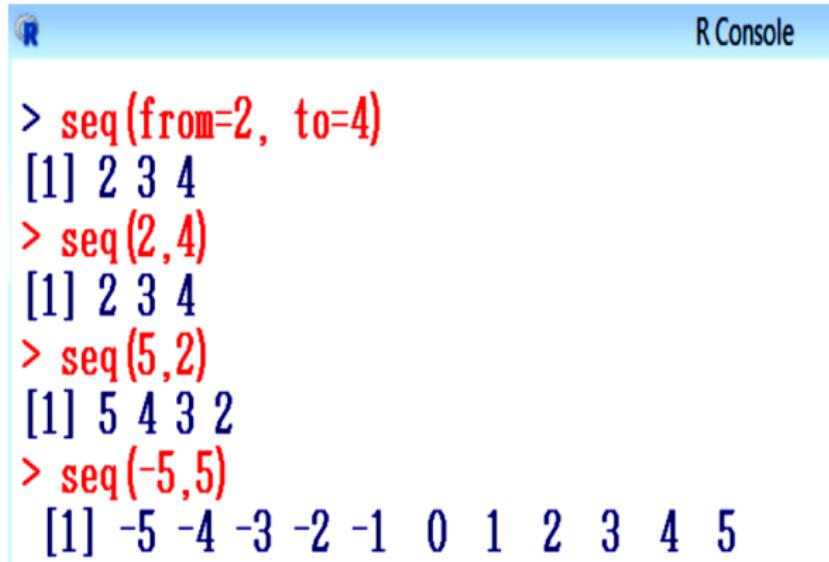
The screenshot shows an R console window with the title "R Console". The console displays the following R code and its output:

```
> mymatrix <- matrix(1:6, 2, 3, byrow=T)
> mymatrix
 [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
> mymatrix <- rbind(mymatrix, 7:9)
> mymatrix
 [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
> mymatrix <- cbind(mymatrix, 6:8)
> mymatrix
 [,1] [,2] [,3] [,4]
[1,]    1    2    3    6
[2,]    4    5    6    7
[3,]    7    8    9    8
```

```
> mymatrix[3,] <- 1:4          #changing an entire row
> mymatrix
     [,1] [,2] [,3] [,4]
[1,]    1    2    3    6
[2,]    4    5    6    7
[3,]    1    2    3    4
> mymatrix[,4] <- 7:9        #changing the entire column
> mymatrix
     [,1] [,2] [,3] [,4]
[1,]    1    2    3    7
[2,]    4    5    6    8
[3,]    1    2    3    9
> mymatrix <- mymatrix[-2,]  #deleting one row
> mymatrix
     [,1] [,2] [,3] [,4]
[1,]    1    2    3    7
[2,]    1    2    3    9
> mymatrix <- mymatrix[,-4]  #deleting one column
> mymatrix
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]
```

- **Sequences**

- A sequence is a set of related numbers, events, movements, or items that follow each other in a particular order. The regular sequences can be generated in R.



R Console

```
> seq(from=2, to=4)
[1] 2 3 4
> seq(2,4)
[1] 2 3 4
> seq(5,2)
[1] 5 4 3 2
> seq(-5,5)
[1] -5 -4 -3 -2 -1  0  1  2  3  4  5
```

- Sequence with constant increment and decrement:



R Console

```
> seq(from=10, to=20, by=2)
[1] 10 12 14 16 18 20
> seq(from=20, to=10, by=-2)
[1] 20 18 16 14 12 10
> seq(3, -2, by=-0.5)
[1] 3.0 2.5 2.0 1.5 1.0 0.5 0.0 -0.5 -1.0 -1.5 -2.0
> seq(to=10, length=10)
[1] 1 2 3 4 5 6 7 8 9 10
> seq(from=10, length=10)
[1] 10 11 12 13 14 15 16 17 18 19
```

```
> seq(from=10, length=10, by=-2)
[1] 10  8  6  4  2  0 -2 -4 -6 -8
> seq(from=10, length=5, by=-.2)
[1] 10.0 9.8 9.6 9.4 9.2
> x<-2
> seq(1, x, x/10)
[1] 1.0 1.2 1.4 1.6 1.8 2.0
> x<-50
> seq(0, x, x/10)
[1] 0  5 10 15 20 25 30 35 40 45 50
```

- Generating sequences of dates

```
R Console
```

```
> seq(as.Date("2010-01-01"), as.Date("2017-01-01"), by="years")
[1] "2010-01-01" "2011-01-01" "2012-01-01" "2013-01-01" "2014-01-01"
[6] "2015-01-01" "2016-01-01" "2017-01-01"
> seq(as.Date("2010-01-01"), as.Date("2017-11-30"), by="years")
[1] "2010-01-01" "2011-01-01" "2012-01-01" "2013-01-01" "2014-01-01"
[6] "2015-01-01" "2016-01-01" "2017-01-01"
> seq(as.Date("2017-11-30"), by = "days", length=6)
[1] "2017-11-30" "2017-12-01" "2017-12-02" "2017-12-03" "2017-12-04"
[6] "2017-12-05"
> seq(as.Date("2017-11-30"), by = "months", length=6)
[1] "2017-11-30" "2017-12-30" "2018-01-30" "2018-03-02" "2018-03-30"
[6] "2018-04-30"
> seq(as.Date("2017-11-30"), by = "years", length=6)
[1] "2017-11-30" "2018-11-30" "2019-11-30" "2020-11-30" "2021-11-30"
[6] "2022-11-30"
```

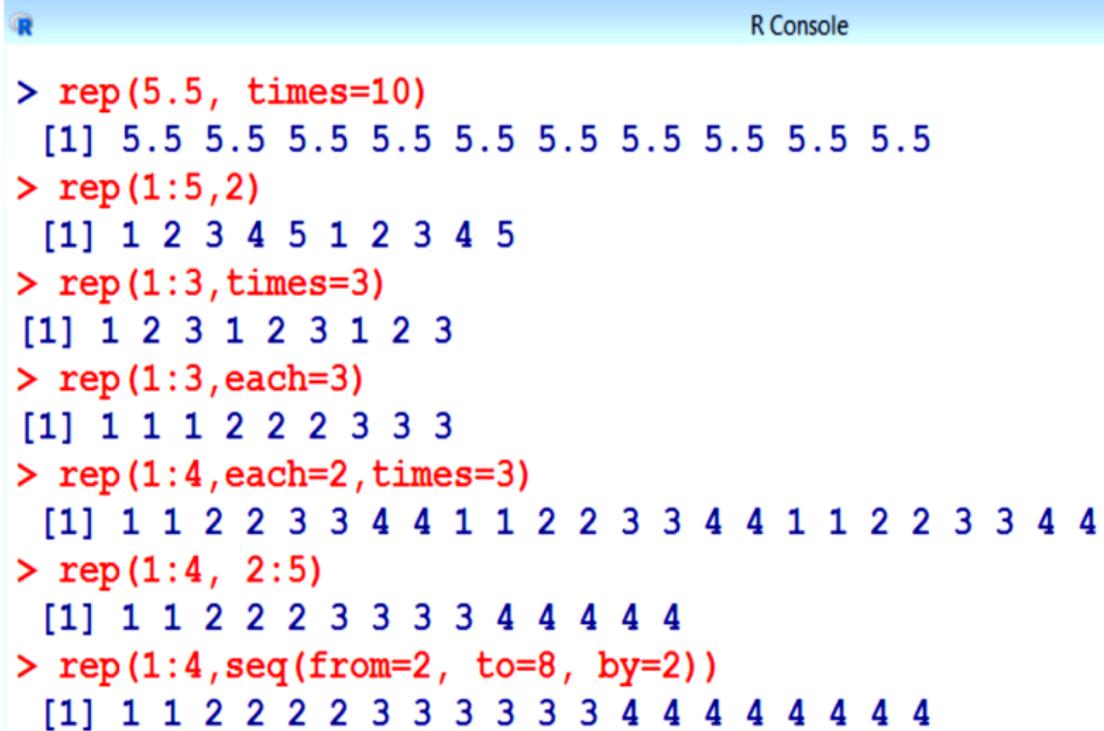
- Generating sequences of letters-lower case alphabets

```
R Console
> letters
[1] "a" "b" "c" "d" "e" "f" "g" "h" "i" "j" "k" "l" "m" "n" "o" "p" "q" "r" "s"
[20] "t" "u" "v" "w" "x" "y" "z"
> letters[1:3]
[1] "a" "b" "c"
> letters[3:1]
[1] "c" "b" "a"
> letters[21:23]
[1] "u" "v" "w"
> letters[5]
[1] "e"
> |
```

- sequence of uppercase alphabets

```
R Console
> LETTERS
[1] "A" "B" "C" "D" "E" "F" "G" "H" "I" "J" "K" "L" "M" "N" "O" "P" "Q" "R" "S"
[20] "T" "U" "V" "W" "X" "Y" "Z"
> LETTERS[1:3]
[1] "A" "B" "C"
> LETTERS[3:1]
[1] "C" "B" "A"
> LETTERS[21:24]
[1] "U" "V" "W" "X"
> LETTERS[5]
[1] "E"
```

- Repeats-the command `rep`



The screenshot shows an R console window with the title "R Console". It contains the following R code and its output:

```
> rep(5.5, times=10)
[1] 5.5 5.5 5.5 5.5 5.5 5.5 5.5 5.5 5.5 5.5 5.5 5.5
> rep(1:5,2)
[1] 1 2 3 4 5 1 2 3 4 5
> rep(1:3,times=3)
[1] 1 2 3 1 2 3 1 2 3
> rep(1:3,each=3)
[1] 1 1 1 2 2 2 3 3 3
> rep(1:4,each=2,times=3)
[1] 1 1 2 2 3 3 4 4 1 1 2 2 3 3 4 4 1 1 2 2 3 3 4 4
> rep(1:4, 2:5)
[1] 1 1 2 2 2 3 3 3 3 4 4 4 4 4 4
> rep(1:4,seq(from=2, to=8, by=2))
[1] 1 1 2 2 2 2 3 3 3 3 3 4 4 4 4 4 4 4
```

R

R Console

```
> x <- matrix(nrow=2, ncol=2, data=1:4, byrow=T)
> x
     [,1] [,2]
[1,]    1    2
[2,]    3    4
> rep(x,4)
[1] 1 3 2 4 1 3 2 4 1 3 2 4 1 3 2 4
```

- Repetition of characters

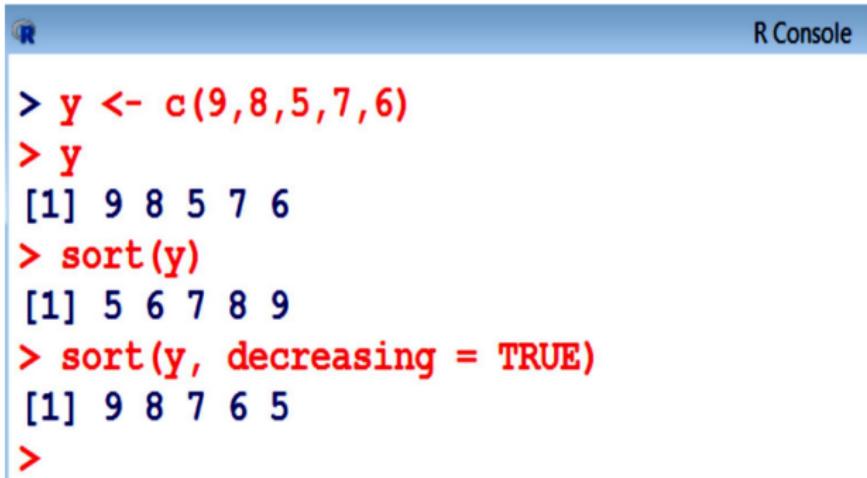
R

R Console

```
> rep(c("a", "b", "c"), 2)
[1] "a" "b" "c" "a" "b" "c"
> rep(c("sas", "scope", "site"), 2)
[1] "sas"   "scope" "site"  "sas"   "scope" "site"
>
```

- **Sorting**

sort function sorts the values of a vector in ascending order (by default) or descending order.

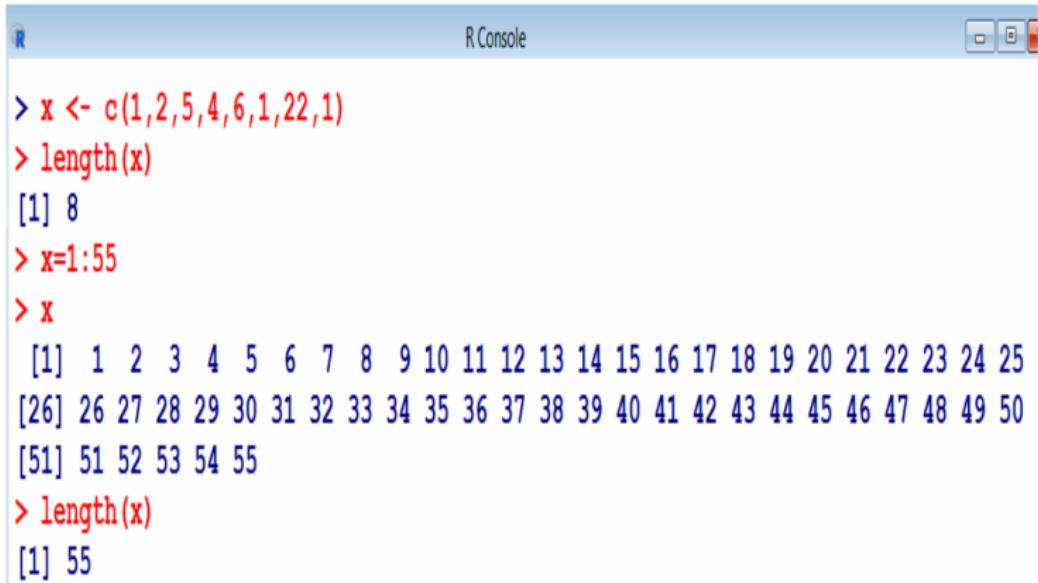


The screenshot shows an R console window with a blue header bar containing the R logo and the text "R Console". The main area contains the following R code and its output:

```
> y <- c(9,8,5,7,6)
> y
[1] 9 8 5 7 6
> sort(y)
[1] 5 6 7 8 9
> sort(y, decreasing = TRUE)
[1] 9 8 7 6 5
>
```

- **Length**

length() function gets or sets the length of a vector (list) or other objects

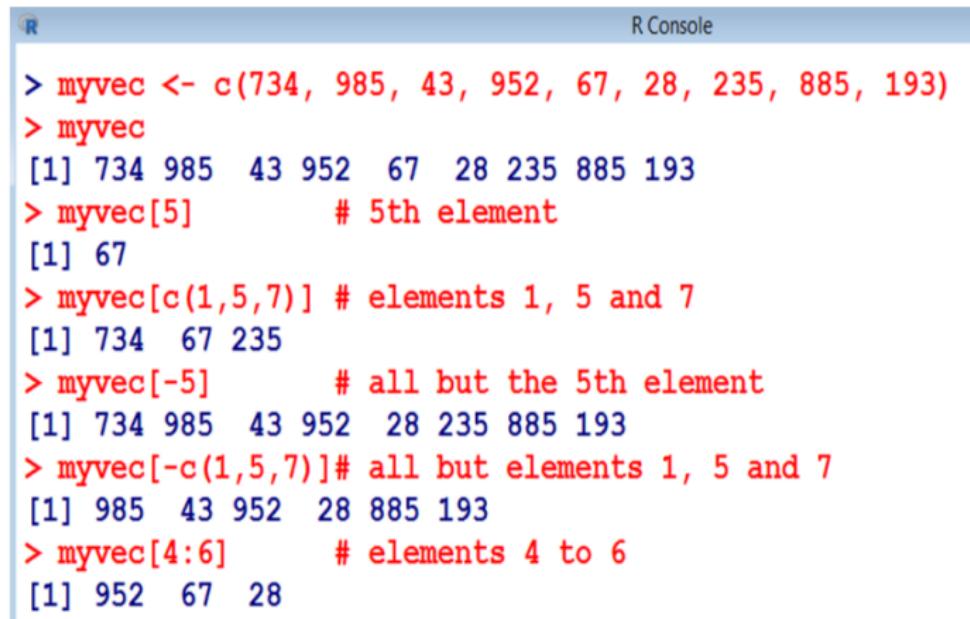


The screenshot shows the R console interface with the title "R Console". The console window displays the following R session:

```
> x <- c(1,2,5,4,6,1,22,1)
> length(x)
[1] 8
> x=1:55
> x
[1]  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
[26] 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
[51] 51 52 53 54 55
> length(x)
[1] 55
```

Data Structures in R

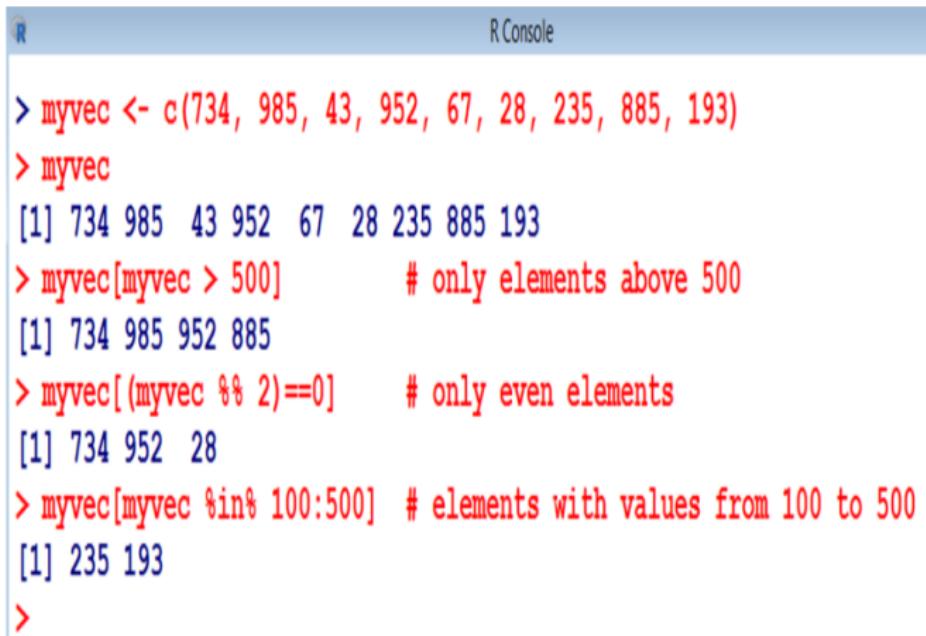
- Extracting vector elements by the element index(es)



The screenshot shows the R console interface with a blue header bar containing the R logo and the text "R Console". Below the header, several R commands are displayed in red, followed by their output in blue. The commands demonstrate various indexing techniques on a vector named "myvec".

```
> myvec <- c(734, 985, 43, 952, 67, 28, 235, 885, 193)
> myvec
[1] 734 985 43 952 67 28 235 885 193
> myvec[5]          # 5th element
[1] 67
> myvec[c(1,5,7)] # elements 1, 5 and 7
[1] 734 67 235
> myvec[-5]         # all but the 5th element
[1] 734 985 43 952 28 235 885 193
> myvec[-c(1,5,7)]# all but elements 1, 5 and 7
[1] 985 43 952 28 885 193
> myvec[4:6]        # elements 4 to 6
[1] 952 67 28
```

- Extracting vector elements by a logical expression



The screenshot shows an R console window with a blue header bar containing the letters 'R' and 'R Console'. The main area contains the following R code:

```
> myvec <- c(734, 985, 43, 952, 67, 28, 235, 885, 193)
> myvec
[1] 734 985 43 952 67 28 235 885 193
> myvec[myvec > 500]      # only elements above 500
[1] 734 985 952 885
> myvec[(myvec %% 2)==0]    # only even elements
[1] 734 952 28
> myvec[myvec %in% 100:500] # elements with values from 100 to 500
[1] 235 193
>
```

- set operations (union, intersection, asymmetric difference, equality and membership) on two vectors.
- Union() is not the same as concatenation c() because c() will duplicate values that are common to both vectors

```
R           R Console

> myvec1 <- c(3,6,7,8,12,23,94)
> myvec2 <- c(5,7,8,152,71,77)
> union(myvec1, myvec2)          # set union
[1] 3 6 7 8 12 23 94 5 152 71 77
> c(myvec1,myvec2)             # diff b/w union() and c()
[1] 3 6 7 8 12 23 94 5 7 8 152 71 77
> intersect(myvec1, myvec2)    # set intersection
[1] 7 8
> setdiff(myvec1, myvec2)      # set difference
[1] 3 6 12 23 94
>
```

• Missing data

R represents missing observations through the data value **NA**

We can detect missing values using **is.na**

R

R Console

```
> x <- NA          # assign NA to variable x
> is.na(x)         # is it missing?
[1] TRUE
```

- Now try a vector to know if any value is missing?

R

```
> x <- c(11, NA, 13)
> is.na(x)
[1] FALSE  TRUE FALSE
```

- Logical Operators and Comparisons

Operator	Executions
>	Greater than
\geq	Greater than or equal
<	Less than
\leq	Less than or equal
\equiv	Exactly equal to
\neq	Not equal to
!	Negation (not)

- **TRUE** and **FALSE** are reserved words denoting logical constants.

Operator	Executions
&, &&	and
,	or

Operator	Executions
xor()	either... or (exclusive)
isTRUE (x)	test if x is TRUE
TRUE	true
FALSE	false

R Console

```
> 8 > 7  
[1] TRUE  
> 7 < 5  
[1] FALSE  
> isTRUE(8<6)      #Is 8 less than 6?  
[1] FALSE  
> isTRUE(8>6)      #Is 8 greater than 6?  
[1] TRUE
```

R Console

```
> x <- 5  
> (x<10)&&(x > 2)      # && means AND  
[1] TRUE  
> (x < 10) || (x > 5) # || means OR  
[1] TRUE  
> (x > 10) || (x > 5)  
[1] FALSE
```

R Console

```
> x = 10
> y = 20
> (x == 10) & (y == 20) # == means exactly equal to
[1] TRUE
> (x == 10) & (y == 2)
[1] FALSE
> (x == 2) & (y == 3)
[1] FALSE
> (x == 1) & (y == 2)
[1] FALSE
> x = 1:6
> (x > 2) & (x < 5)
[1] FALSE FALSE TRUE TRUE FALSE FALSE
> x[(x > 2) & (x < 5)]
[1] 3 4
```

R Console

```
> x = 1:6
> (x > 2) | (x > 10)
[1] FALSE FALSE TRUE TRUE TRUE TRUE
>
> x[(x > 2) | (x > 10)]
[1] 3 4 5 6
```

R Console

```
> x = 1:6
> (x > 2) && (x < 5)
[1] FALSE
```

R Console

```
> (x[1] > 2) & (x[1] < 5)
[1] FALSE
```

- Conditional execution

`ifelse(test, yes, no)`



The screenshot shows an R console window with the title "R Console". The console output is as follows:

```
> x <- 1:10
> x
[1]  1  2  3  4  5  6  7  8  9 10
>
> ifelse( x<6, x^2, x+1 )
[1]  1  4  9 16 25  7  8  9 10 11
```

- Interpretation

If $x < 6$ (TRUE), then $x = x^2$ (YES) .

If $x \geq 6$ (FALSE), then $x = x + 1$ (NO).

- The **for** loop
- If the number of repetitions is known in advance a **for()** loop can be used.

```
R R Console
> for ( i in 1:5 ) { print( i^2 ) }
[1] 1
[1] 4
[1] 9
[1] 16
[1] 25
```

```
R R Console
> for ( i in c(2,4,6,7) ) { print( i^2 ) }
[1] 4
[1] 16
[1] 36
[1] 49
```

- The `while()` loop

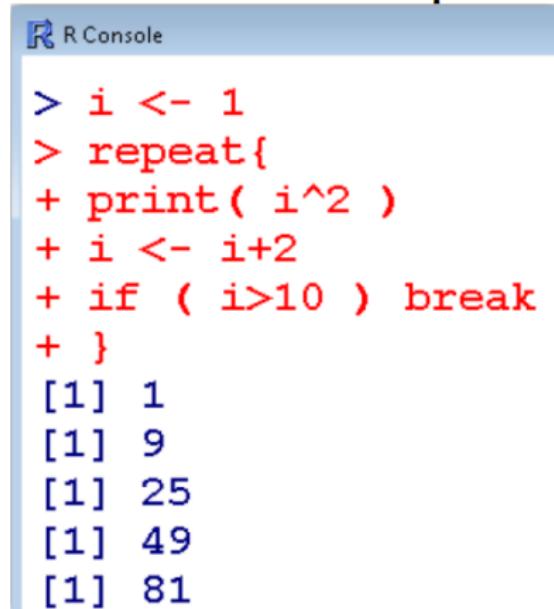
If the number of loops is not known in advance, e.g. when an iterative algorithm to maximize a likelihood function is used, one can use a `while()` loop.

R R Console

```
> i <- 1
> while (i<5) {
+ print(i^2)
+ i <- i+2
+
[1] 1
[1] 9
```

- The **repeat** loop

The repeat loop doesn't test any condition — in contrast to the while() loop — before entering the loop and also not during the execution of the loop.



R Console

```
> i <- 1
> repeat{
+ print( i^2 )
+ i <- i+2
+ if ( i>10 ) break
+ }
[1] 1
[1] 9
[1] 25
[1] 49
[1] 81
```

Lists

Mode:

Object	Example	Mode
Number	1.234	numeric
Vector of numbers	c(5, 6, 7, 8)	numeric
Character string	"India"	character
Vector of character strings	c("India", "USA")	character
Factor	factor(c("UP", "MP"))	numeric
List	list("India", "USA")	list
Data frame	data.frame(x=1:2, y=c("India", "USA"))	list
Function	print	function

```
R Console
> mode(1.234)
[1] "numeric"
> mode(c(5,6,7,8))
[1] "numeric"
> mode("India")
[1] "character"
> mode(c("India", "USA"))
[1] "character"
> mode(factor(c("UP", "MP")))
[1] "numeric"
> mode(list("India", "USA"))
[1] "list"
```

Data Frames

- In a data frame, we can combine variables of equal length, with each row in the data frame containing observations on the same unit.
- Data frames contain complete data sets that are mostly created with other programs (spreadsheet-files, software SPSS-files, Excel-files etc.).
- Variables in a data frame may be numeric (numbers) or categorical (characters or factors).
- Package “**MASS**” describes functions and datasets to support Venables and Ripley, “Modern Applied Statistics with S” (4th edition 2002)

R Console

> library(MASS)

> painters

	Composition	Drawing	Colour	Expression	School
Da Udine	10	8	16	3	A
Da Vinci	15	16	4	14	A
Del Piombo	8	13	16	7	A
Del Sarto	12	16	9	8	A
Fr. Penni	0	15	8	0	A
Guilio Romano	15	16	4	14	A

Rubens	18	13	17	17	G
Teniers	15	12	13	6	G
Van Dyck	15	10	17	13	G
Bourdon	10	8	8	4	H
Le Brun	16	16	8	16	H

```
> library(MASS)
```

```
> painters
```

	Composition	Drawing	Colour	Expression	School
Da Udine	10	8	16	3	A
Da Vinci	15	16	4	14	A
Del Piombo	8	13	16	7	A
Del Sarto	12	16	9	8	A
Fr. Penni	0	15	8	0	A
Guilio Romano	15	16	4	14	A

Rubens	18	13	17	17	G
Teniers	15	12	13	6	G
Van Dyck	15	10	17	13	G
Bourdon	10	8	8	4	H
Le Brun	16	16	8	16	H

R Console

```
> rownames(painters)
[1] "Da Udine"          "Da Vinci"           "Del Piombo"
[4] "Del Sarto"         "Fr. Penni"          "Guilio Romano"
[7] "Michelangelo"     "Perino del Vaga"   "Perugino"
[10] "Raphael"          "F. Zuccaro"        "Fr. Salviata"
[13] "Parmigiano"       "Primiticcio"      "T. Zuccaro"
[16] "Volterra"         "Barocci"           "Cortona"
[19] "Josepin"          "L. Jordaens"      "Testa"
[22] "Vanius"           "Bassano"           "Bellini"
[25] "Giorgione"        "Murillo"          "Palma Giovane"
[28] "Palma Vecchio"    "Pordenone"        "Tintoretto"
[31] "Titian"            "Veronese"          "Albani"
[34] "Caravaggio"        "Correggio"        "Domenichino"
[37] "Guercino"          "Lanfranco"        "The Carraci"
[40] "Durer"             "Holbein"           "Pourbus"
[43] "Van Leyden"        "Diepenbeck"       "J. Jordaens"
[46] "Otho Venius"       "Rembrandt"        "Rubens"
[49] "Teniers"           "Van Dyck"          "Bourdon"
```

R Console

```
> colnames(painters)
```

```
[1] "Composition" "Drawing"      "Colour"       "Expression"  "School"
```

```
R Console
> x <- 1:16
> y <- matrix(x, nrow=4, ncol=4)
> z <- letters[1:16]
> x
[1] 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
>
> y
     [,1] [,2] [,3] [,4]
[1,]    1    5    9   13
[2,]    2    6   10   14
[3,]    3    7   11   15
[4,]    4    8   12   16
>
> z
[1] "a" "b" "c" "d" "e" "f" "g" "h" "i" "j" "k" "l" "m"
[14] "n" "o" "p"
```

```
> datafr <- data.frame(x, y, z)
> datafr
   x X1 X2 X3 X4 z
1 1  1  5  9 13 a
2 2  2  6 10 14 b
3 3  3  7 11 15 c
4 4  4  8 12 16 d
5 5  1  5  9 13 e
6 6  2  6 10 14 f
7 7  3  7 11 15 g
8 8  4  8 12 16 h
9 9  1  5  9 13 i
10 10 2  6 10 14 j
11 11 3  7 11 15 k
12 12 4  8 12 16 l
13 13 1  5  9 13 m
14 14 2  6 10 14 n
15 15 3  7 11 15 o
16 16 4  8 12 16 p
```

Structure of the data

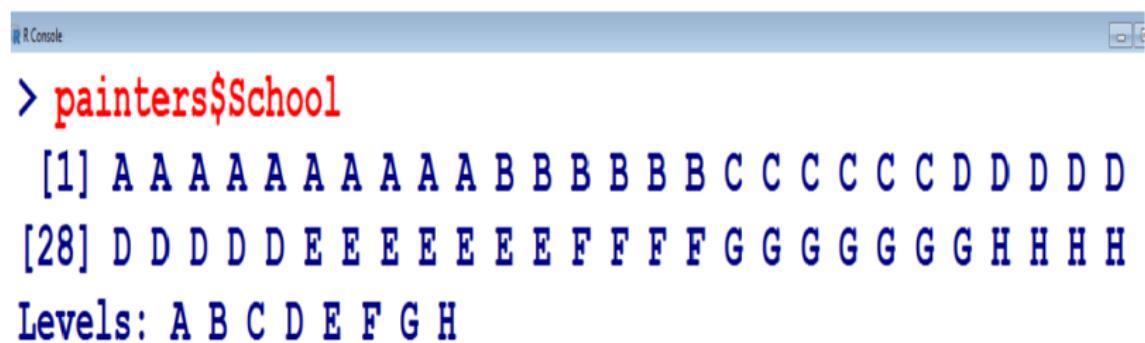
The result of `str` gives the dimension as well as the name and type of each variable.

```
R Console
```

```
> str(painters)
'data.frame': 54 obs. of 5 variables:
 $ Composition: int 10 15 8 12 0 15 8 15 4 17 ...
 $ Drawing    : int 8 16 13 16 15 16 17 16 12 18 ...
 $ Colour     : int 16 4 16 9 8 4 4 7 10 12 ...
 $ Expression : int 3 14 7 8 0 14 8 6 4 18 ...
 $ School     : Factor w/ 8 levels "A","B","C","D",...: 1 1 1 1 1 1 1 1 1 1 ...
```

Extract a variable from data frame using \$

Suppose we want to extract information on variable **School** from the data set **painters**.



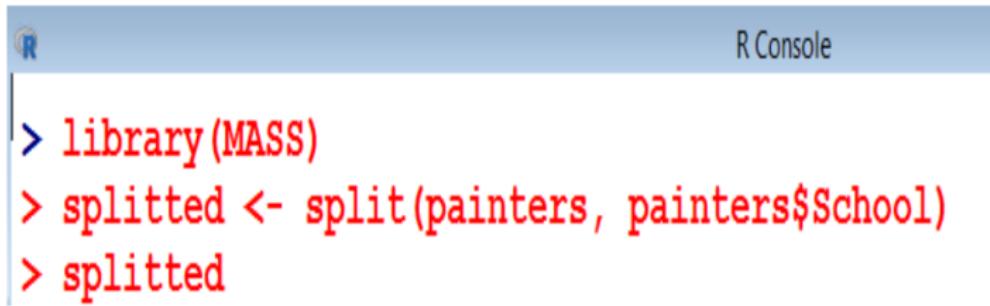
The screenshot shows an R console window. The title bar says "R Console". The code input is in red: > **painters\$School**. The output is in blue:
[1] A A A A A A A A A A A A B B B B B B C C C C C C D D D D D D
[28] D D D D D E E E E E E F F F F F G G G G G G G G H H H H H H
Levels: A B C D E F G H

- Subsets of a data frame can be obtained with `subset()` or with the second equivalent command:

```
> library(MASS)  
> subset(painters, School=='F')
```

	Composition	Drawing	Colour	Expression	School
Durer	8	10	10	8	F
Holbein	9	10	16	13	F
Pourbus	4	15	6	6	F
Van Leyden	8	6	6	4	F

- The command **split** partitions the data set by values of a specific variable. This should preferably be a factor variable.



The screenshot shows an R console window. The title bar says "R Console". The main area contains the following R code:

```
> library(MASS)
> splitted <- split(painters, painters$School)
> splitted
```

- **Importing CSV and Tabular Data Files**

We can change the current working directory as follows:

- `setwd("<location of the dataset>")`

- **Example**

```
>setwd("C:\\Users\\admin\\Desktop\\")  
>data=read.csv("stud.csv")
```

- Comma-separated values (CSV) files
- Data files have many formats and accordingly we have options for loading them.

```
>data=read.csv("C:\\Users\\admin\\Desktop\\Mokesh\\stud.csv")
```

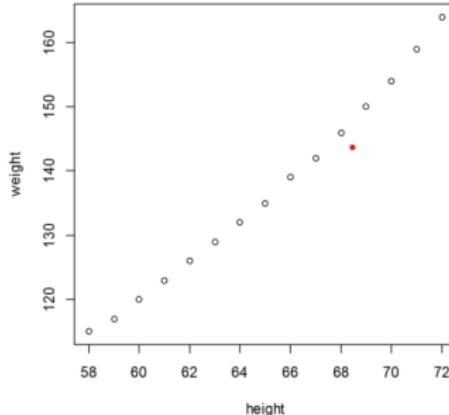
Or

```
>data=read.csv("C:/Users/admin/Desktop/Mokesh/stud.csv")
```

Graphics on R

- A simple plot `plot(X)` has each element of a discrete variable X plotted on the y-axis and the element's index on the x-axis

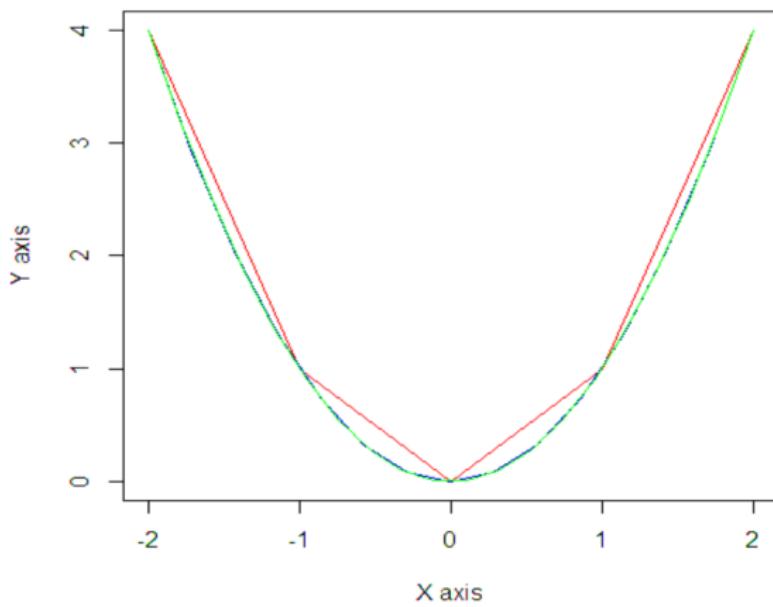
```
> # simple plot  
> women  #inbuilt data set  
  height weight  
1      58    115  
2      59    117  
3      60    120  
4      61    123  
5      62    126  
6      63    129  
7      64    132  
8      65    135  
9      66    139  
10     67    142  
11     68    146  
12     69    150  
13     70    154  
14     71    159  
15     72    164  
  
> plot(women)
```



Ac
Go

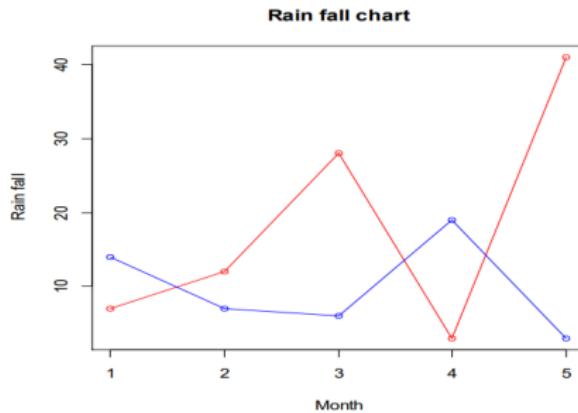
```
> # function plot  
> x = seq(-2,2)  
> y = x^2  
> # edgy graph!  
> plot(x,y,type="l",xlab="X axis",ylab="Y axis",main="Parabola", col = "red")  
> # better  
> sp <- spline(x, y) # spline interpolation of data points  
> lines(sp, col = "blue")  
> # much better  
> sp <- spline(x, y,n=20) # interpolation at n points spanning [xmin, xmax]  
> lines(sp, col = "green")
```

Parabola



A
G

```
>v <- c(7,12,28,3,41)  
>t <- c(14,7,6,19,3)  
> plot(v,type = "o", col = "red", xlab = "Month", ylab =  
"Rain fall",main = "Rain fall chart")  
>lines(t, type = "o", col = "blue")
```

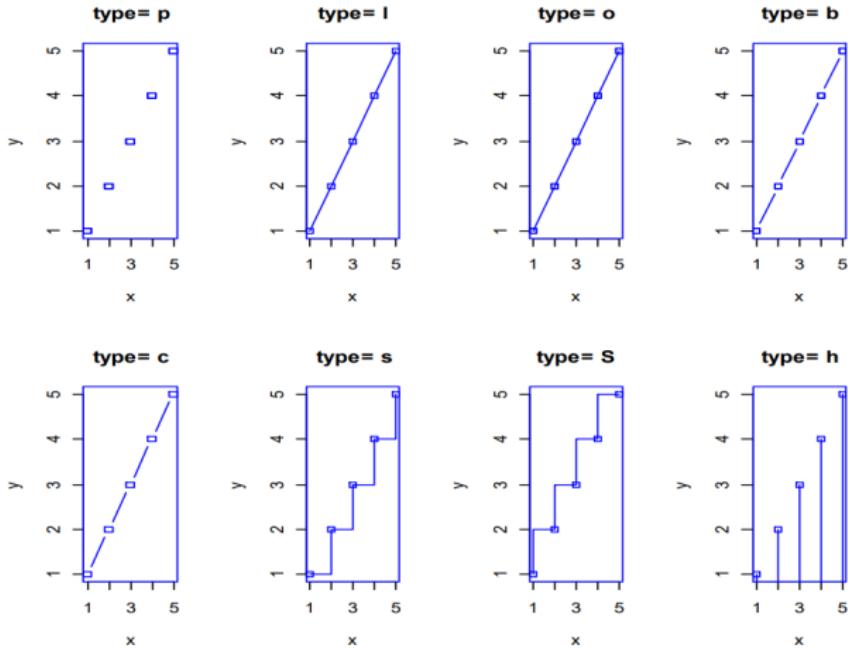


- **Line chart**

- A line chart is a simple plot with consecutive plots connected by lines

```
> x <- c(1:5)
> y <- x # create some data
> par(pch=22, col="blue") # plotting symbol and color
> par(mfrow=c(2,4)) # all plots on one page
> opts = c("p","l","o","b","c","s","S","h")
> for(i in 1:length(opts))
+ {
+ heading = paste("type=",opts[i])
+ plot(x, y, main=heading)
+ lines(x, y, type=opts[i])
+ }
```

Ac
Go

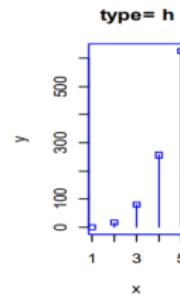
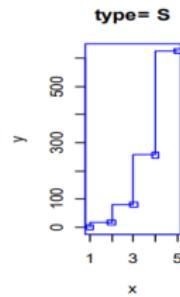
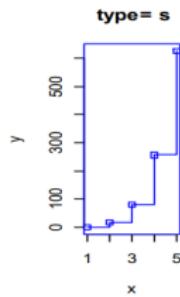
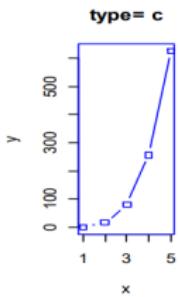
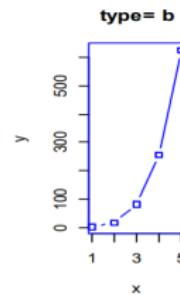
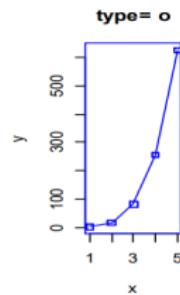
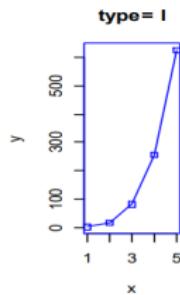
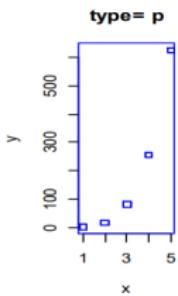


A
C

```
> x <- c(1:5)
> y <- x^4 # create some data
> par(pch=22, col="blue") # plotting symbol and color
> par(mfrow=c(2,4)) # all plots on one page
> opts = c("p","l","o","b","c","s","S","h")
> for(i in 1:length(opts))
+ {
+ heading = paste("type=",opts[i])
+ plot(x, y, main=heading)
+ lines(x, y, type=opts[i])
+ }
```

A

G



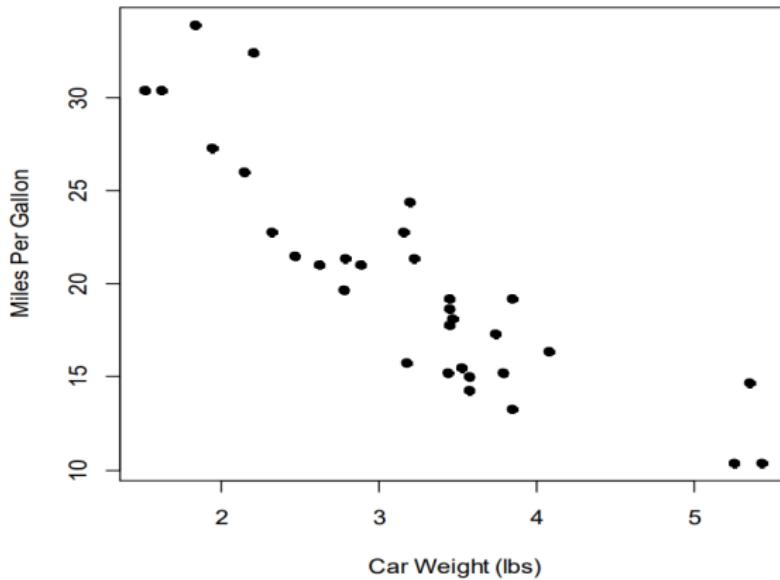
- Scatterplot

A scatterplot `plot(X, Y)` has each element of a variable Y plotted on the y-axis and the corresponding element for variable X on the x-axis

```
# scatterplot  
>attach(mtcars)  
>plot(wt, mpg, main="Weight / MPG graph",  
      xlab="Car Weight (lbs)", ylab="Miles Per Gallon",  
      pch=19)
```

Act

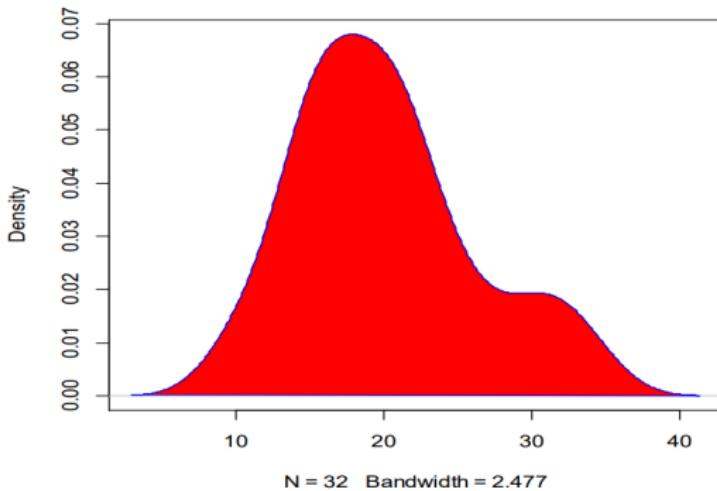
Weight / MPG graph



- **Kernel density plots**
- Kernel density plots nicely visualize the shape of a distribution. They can be better than histograms, even with normal curves because histograms are strongly affected by the number of bins used and by outliers.
- **# Kernel density plot**
- `>d <- density(mtcars$mpg) # kernel density estimates`
- `>plot(d)`
- **# Filled density plot**
- `>d <- density(mtcars$mpg)`
- `>plot(d, main="Kernel Density of Miles Per Gallon")`
- `>polygon(d, col="red", border="blue")`

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Kernel Density of Miles Per Gallon



- **boxplot(X)** is a plot that, if X is a vector, the vector elements are the heights of the bars in the plot, if X is a matrix, the matrix columns are the heights of the bars in the plot, stacked after the first bar (column)
- If the argument `beside=TRUE`, then the values in each column are juxtaposed, not stacked.
- The argument `horiz=TRUE` creates an horizontal barplot.

R

R Console

> **VADeaths**

	Rural Male	Rural Female	Urban Male	Urban Female
50-54	11.7	8.7	15.4	8.4
55-59	18.1	11.7	24.3	13.6
60-64	26.9	20.3	37.0	19.3
65-69	41.0	30.9	54.6	35.1
70-74	66.0	54.3	71.1	50.0

> **class(VADeaths)**

[1] "matrix"

> **dimnames(VADeaths)**

[[1]]

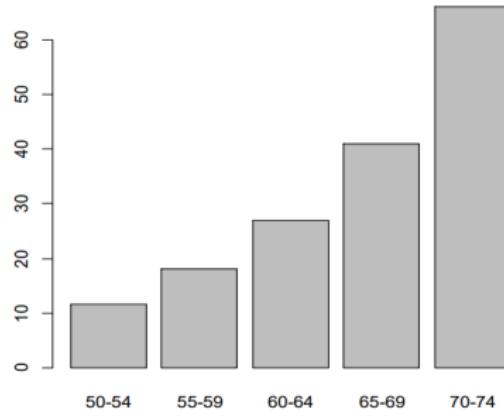
[1] "50-54" "55-59" "60-64" "65-69" "70-74"

[[2]]

[1] "Rural Male" "Rural Female" "Urban Male" "Urban Female"

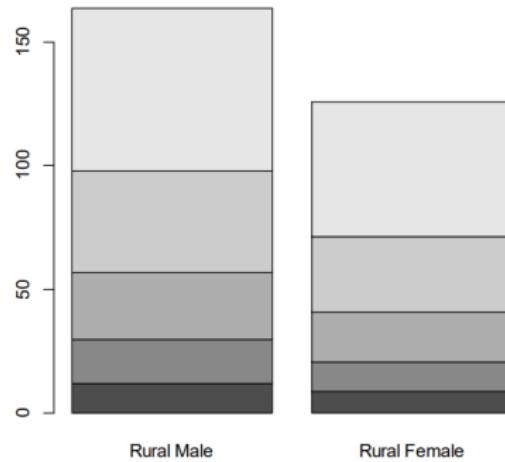
Act
Go to

- > simple barplot
- > barplot (VADeaths[, "Rural Male"])



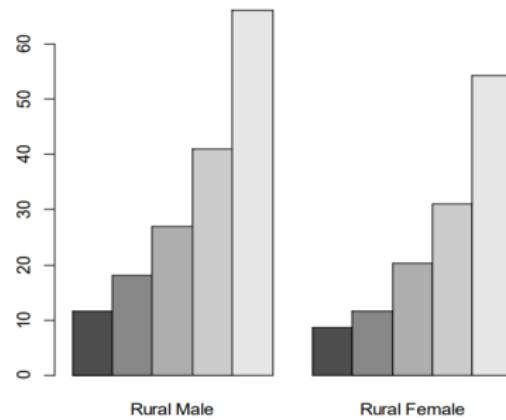
- # stacked barplots

```
> barplot(VADeaths[,c("Rural Male", "Rural Female")])
```

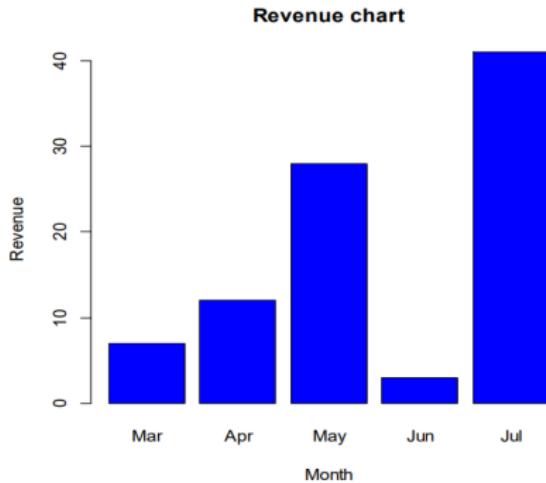


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- > # juxtaposed barplots
- > barplot(VADeaths [,c("Rural Male", "Rural Female")],beside=T)



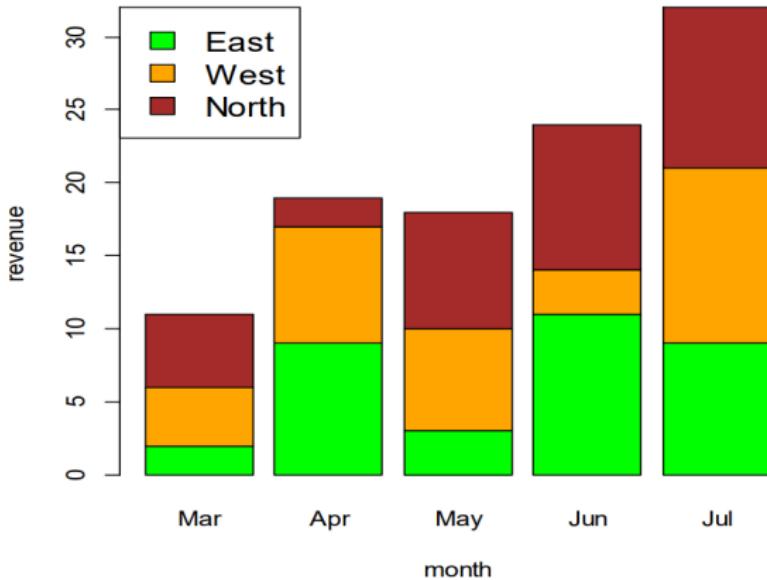
```
>H <- c(7,12,28,3,41)  
>M <- c("Mar","Apr","May","Jun","Jul")  
>barplot(H,names.arg = M,xlab = "Month",ylab =  
"Revenue",col="blue",main = "Revenue chart")
```



Example :-

```
>colors <- c("green","orange","brown")
>months <- c("Mar","Apr","May","Jun","Jul")
>regions <- c("East","West","North")
>Values <-
matrix(c(2,9,3,11,9,4,8,7,3,12,5,2,8,10,11),nrow = 3,ncol
= 5,byrow = TRUE)
>barplot(Values,main = "total revenue",names.arg =
months,xlab = "month",ylab = "revenue",col=colors)
>legend("topleft", regions, cex = 1.3, fill = colors)
```

total revenue



- # Simple Dotplot

```
>dotchart(mtcars$mpg,labels=row.names(mtcars),cex=.7,  
main="Gas Milage for Car Models",xlab="Miles Per  
Gallon")
```

- # Dotplot: Grouped Sorted and Colored

- # Sort by mpg, group and color by cylinder

- >x <- mtcars[order(mtcars\$mpg),] # sort by mpg

- >x\$cyl <- factor(x\$cyl) # it must be a factor

- >x\$color[x\$cyl==4] <- "red"

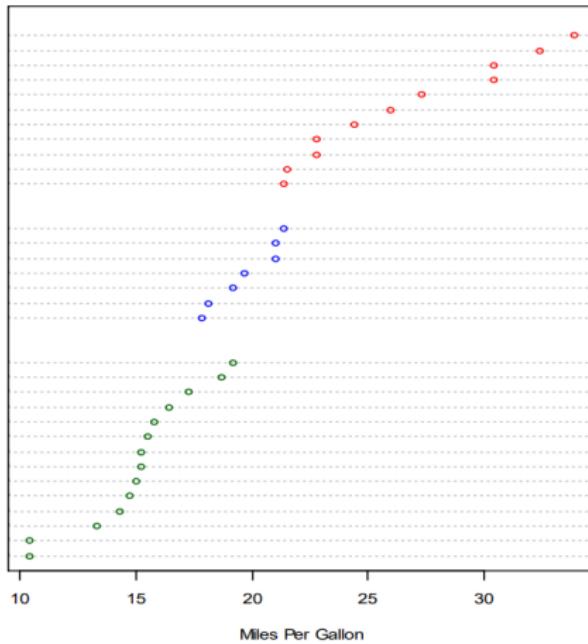
- >x\$color[x\$cyl==6] <- "blue"

- >x\$color[x\$cyl==8] <- "darkgreen"

- >dotchart(x\$mpg,labels=row.names(x),cex=.7,groups=
x\$cyl,main="Gas Milage for Car Models\ngrouped by
cylinder",xlab="Miles Per Gallon",gcolor="black",
color=x\$color)

Gas Milage for Car Models grouped by cylinder

- 4 Toyota Corolla
Fiat 128
Lotus Europa
Honda Civic
Fiat X1-9
Porsche 914-2
Merc 240D
Merc 230
Datsun 710
Toyota Corona
Volvo 142E
- 6 Hornet 4 Drive
Mazda RX4 Wag
Mazda RX4
Ferrari Dino
Merc 280
Valiant
Merc 280C
- 8 Pontiac Firebird
Hornet Sportabout
Merc 450SL
Merc 450SE
Ford Pantera L
Dodge Challenger
AMC Javelin
Merc 450SLC
Maserati Bora
Chrysler Imperial
Duster 360
Camaro Z28
Lincoln Continental
Cadillac Fleetwood



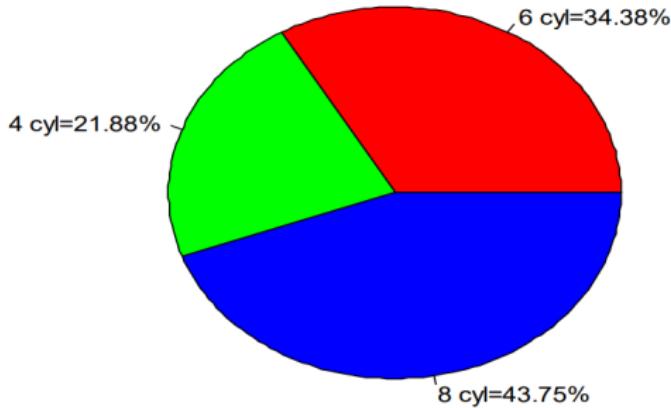
- **Pie**
- `pie(x)` draws a circle (pie) cut into segments (slices), each slice represents a unique value from the elements of `x` and the size of the slice and the relative frequency of each unique value is represented by the size of `t`

simple pie

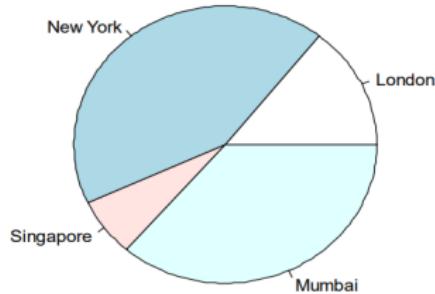
```
>pie(unique(mtcars$cyl), labels = unique(mtcars$cyl), main="Pie Chart of  
N. of cylinders") # pie with percentages and colors  
>with(mtcars, {  
>n.cyl <- unique(cyl)  
>percent.cyl <- round(table(cyl)/dim(mtcars)[1]*100,2)  
>lbls <- paste(n.cyl," cyl=",percent.cyl,"%",sep="")  
>pie(n.cyl, labels = lbls , main="Pie Chart of N. of cylinders",  
col=rainbow(length(lbls))))
```

Ac

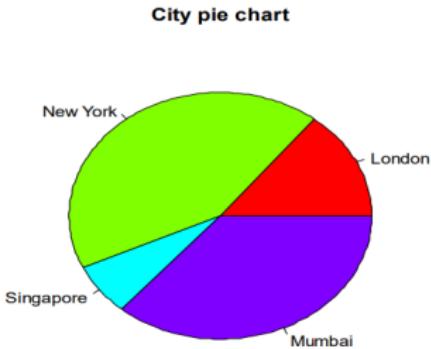
Pie Chart of N. of cylinders



```
>x <- c(21, 62, 10, 53)  
>labels <- c("London", "New York", "Singapore",  
"Mumbai")  
>pie(x,labels)
```

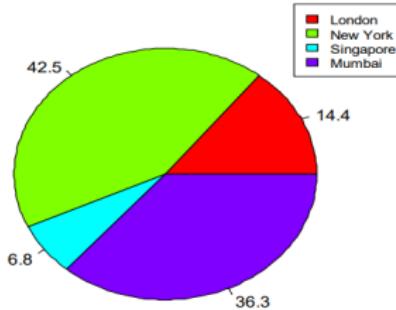


```
>x = c(21, 62, 10, 53)  
>labels = c("London", "New York", "Singapore",  
"Mumbai")  
>pie(x, labels, main = "City pie chart", col =  
rainbow(length(x)))
```



```
>x <- c(21, 62, 10, 53)  
>labels <- c("London", "New York", "Singapore", "Mumbai")  
>piepercent<- round(100*x/sum(x), 1)  
>pie(x, labels = piepercent, main = "City pie chart", col =  
rainbow(length(x)))  
>legend("topright", c("London", "New York", "Singapore",  
"Mumbai"), cex = 0.8, fill = rainbow(length(x)))
```

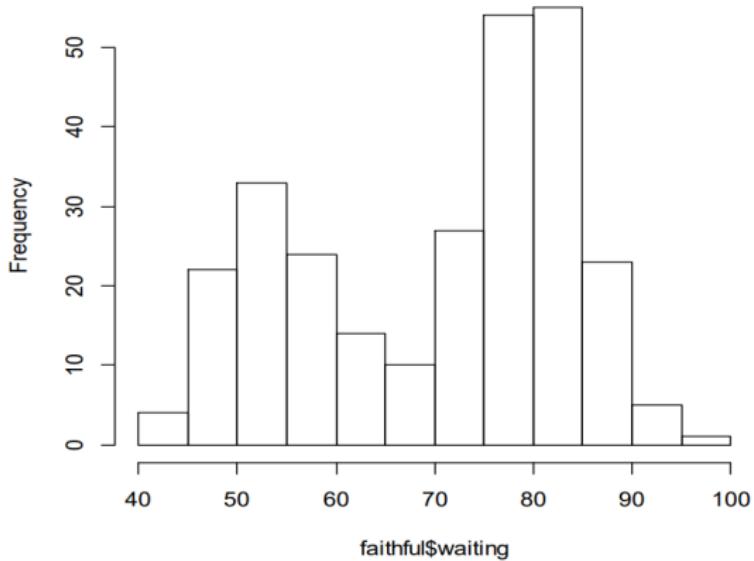
City pie chart



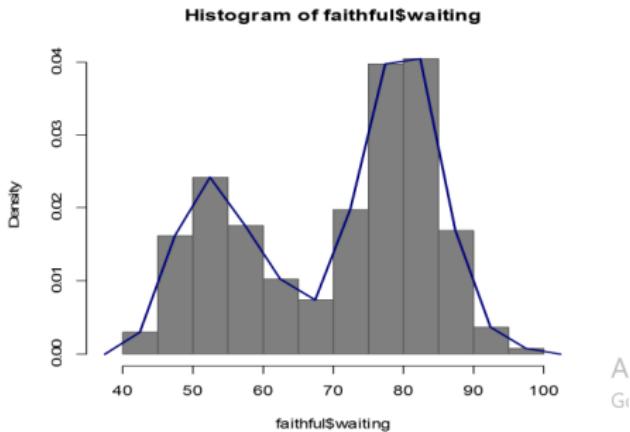
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- **histogram**
 - hist(X) is an histogram, a bar plot with the frequencies of the values in X on the y-axis and the ranges of values on the x-axis
 - A cumulative distribution curve is the proportion of X on the y-axis, up to the current position on the x-axis
-
- > # simple histogram
 - > hist(faithful\$waiting)

Histogram of faithful\$waiting



```
# draw the histogram  
>hist(faithful$waiting, prob = TRUE, xlim=range(xx) , border =  
"gray" , col="gray90")  
# adds the frequency polygon  
>lines(xx, yy, lwd=2, col = "royalblue")
```



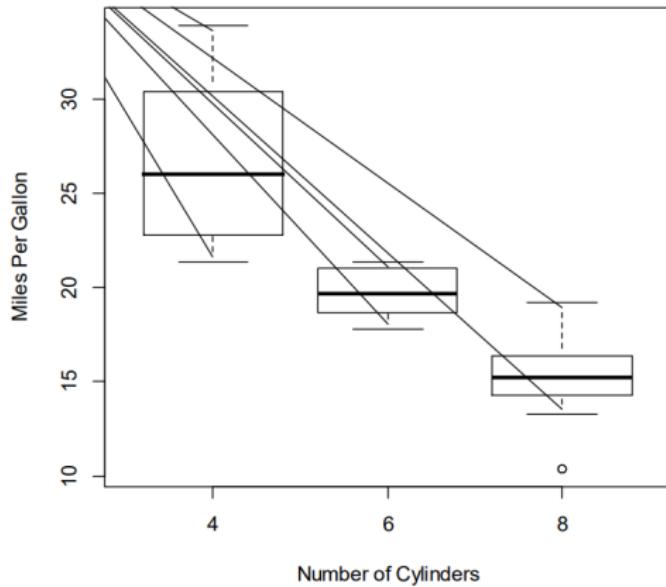
- **boxplot**

boxplot(X) is a box-and-whisker plot with the values of variable X, this is an effective way to summarize larger datasets.

```
# Boxplot of MPG by Car Cylinders
```

```
> boxplot(mpg~cyl,data=mtcars, main="Car Milage  
Data",xlab="Number of Cylinders", ylab="Miles Per  
Gallon")
```

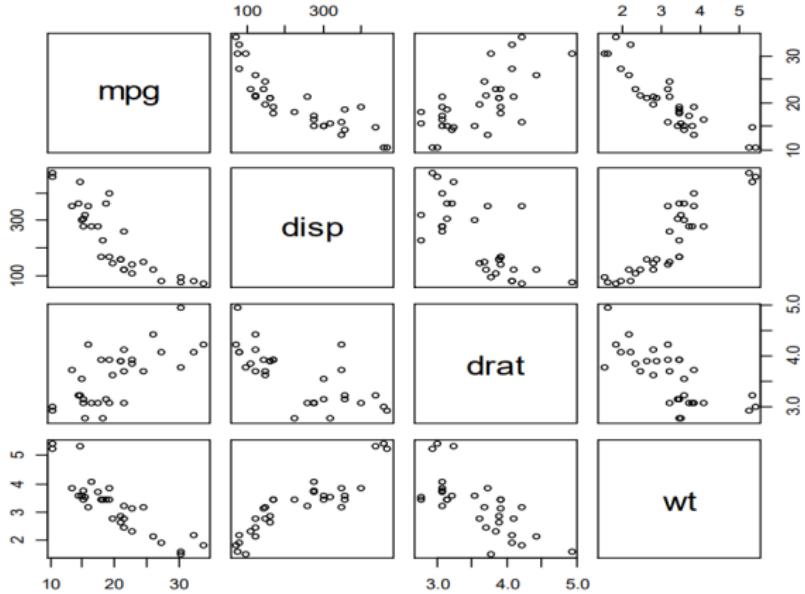
Car Milage Data



- **Pairs**

- pairs() shows a matrix with all the scatterplots for the columns of variable X
- `pairs(~mpg+disp+drat+wt,data=mtcars,
main="Scatterplot Matrix MPG, Displacement,Rear
axle ratio,Weight")`

Scatterplot Matrix MPG, Displacement,Rear axle ratio, Weight

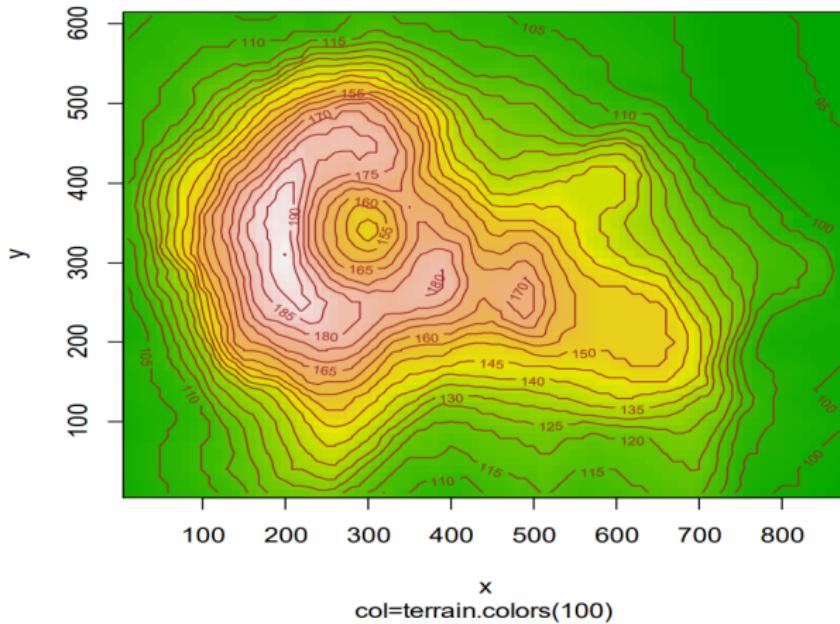


- **Contour**

- `contour(X,Y,Z)` draws a contour plot, with vector X for the rows, vector Y for the columns and matrix Z for the data

```
>x <- 10*(1:nrow(volcano)); x.at <- seq(100, 800, by=100)
>y <- 10*(1:ncol(volcano)); y.at <- seq(100, 600, by=100)
# Using Terrain Colors
>image(x, y, volcano, col=terrain.colors(100),axes=FALSE)
>contour(x, y, volcano, levels=seq(90, 200, by=5), add=TRUE,
  col="brown")
>axis(1, at=x.at)
>axis(2, at=y.at)
>box()
>title(main="Maunga      Whau      Volcano",      sub      =Ac
  "col=terrain.colors(100)", font.main=4)
  Go
```

Maunga Whau Volcano

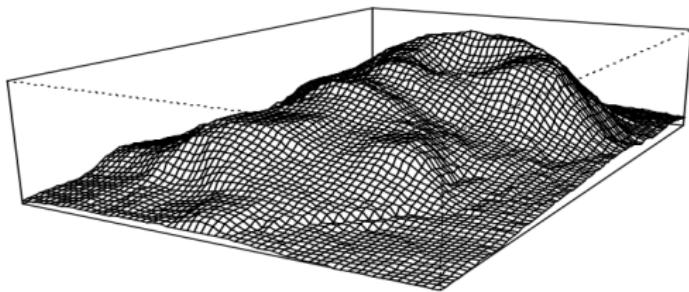


- **Persp**

`persp(X,Y,Z)` draws a 3d graph, with vector X for the rows, vector Y for the columns and matrix X for the data

(2) Visualizing a simple DEM model

```
>z <- 2 * volcano # Exaggerate the relief  
>x <- 10 * (1:nrow(z)) # 10 meter spacing (S to N)  
>y <- 10 * (1:ncol(z)) # 10 meter spacing (E to W)  
>persp(x, y, z, theta = 120, phi = 15, scale = FALSE, axes =  
FALSE)
```



• Tables

Example:

```
> library(MASS)  
>ships  
>table(ships$type)
```

A	B	C	D	E
8	8	8	8	8

```
>table(ships$type,ships$year)
```

	60	65	70	75
A	2	2	2	2
B	2	2	2	2
C	2	2	2	2
D	2	2	2	2
E	2	2	2	2

Example :-

```
>library(MASS)  
>USArrests  
>table(USArrests[,3])
```

32	39	44	45	48	50	51	52	53	54	56	57	58	59	60	62	63	65	66	67	68	70	72	73	74	75	
1	1	2	2	2	1	1	1	1	1	1	1	1	1	1	2	1	1	1	4	2	1	2	2	1	1	1
77	78	80	81	83	85	86	87	89	91																	
1	1	4	1	2	1	1	1	1	1																	

Ac

```
>table(cut(USArrests[,3],pretty(USArrests[,3])))
```

(30,40]	(40,50]	(50,60]	(60,70]	(70,80]	(80,90]	(90,100]
2	7	10	12	11	7	1
.....

Example :-

```
> airquality
```

```
> table(airquality[,4],airquality[,5])
```

```
>table(cut(airquality[,4],pretty(airquality[,4])),  
      airquality[,5])
```

	5	6	7	8	9
(50,60]	8	0	0	0	0
(60,70]	16	2	0	0	7
(70,80]	6	18	3	11	14
(80,90]	1	8	25	15	5
(90,100]	0	2	3	5	4

Example :-

```
> library(MASS)  
> cars
```

```
> head(cars)
  speed dist
1     4     2
2     4    10
3     7     4
4     7    22
5     8    16
6     9    10
```

```
> tail(cars)
  speed dist
45    23    54
46    24    70
47    24    92
48    24    93
49    24   120
50    25    85
```

```
> head(cars,3)
  speed dist
1     4     2
2     4    10
3     7     4
```

```
> tail(cars,3)
  speed dist
48    24    93
49    24   120
50    25    85
```

Thank You