ECEN 5623

Rate Monotonic Derivation

Be Boulder.

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Rate Monotonic Theory

The Complete Derivation

RM Assumptions and Constraints

- A1: All Services Requested on Periodic Basis, the Period is Constant
- A2: Completion-Time < Period
- A3: Service Requests are Independent (No Known Phasing)
- A4: Run-time is Known and Deterministic (WCET may be Used)
- C1: Deadline = Period by Definition
- C2: Fixed Priority, Preemptive, Run-to-Completion Scheduling
- <u>Critical Instant</u>: longest response time for a service occurs when all system services are requested simultaneously (maximum interference case for lowest priority service)
- No Other Shared Resources – Not in Paper, but key assumption – e.g. shared memory

Derivation of RM LUB for 2 Tasks

<u>Claim</u> - General RM Least Upper Bound: $U = \sum_{i=1}^{m} (Ci / Ti) \le m(2^{\frac{1}{m}} - 1)$

(Guarantees that all Service Releases Can meet Deadlines)

<u>Goal</u> - Derive RM LUB For 2 Tasks: $U = C_1 / T_1 + C_2 / T_2 \le 2(2^{\frac{1}{2}} - 1) \le 0.83$

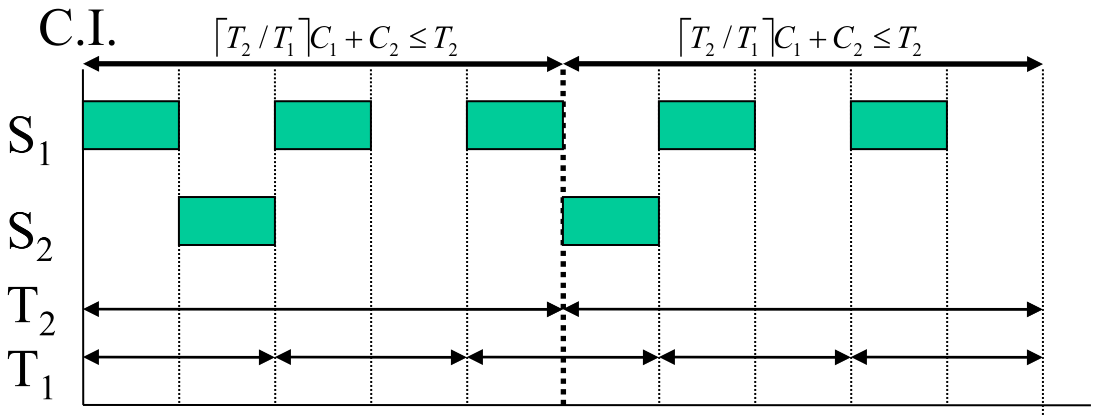
For a System, Can All C's fit in largest T over LCM time?

Given: Services S_1, S_2 with periods T_1 and T_2 and C_1 and C_2, Assume $T_2 > T_1$

E.g. T_1=2, T_2=5, C_1=1, C_2= 1, then if prio(S_1) > prio(S_2), we can see that ...

$U = 1/2 + 1/5 = 0.7$

$U = 0.7 \le 2(2^{\frac{1}{2}} - 1) \le 0.83$



Can You Safely Exceed LUB?

YES!

In some cases, but RM LUB will never pass an infeasible system

RM LUB is a SUFFICIENT Feasibility Test

RM LUB is NOT a NECESSARY and SUFFICIENT Feasibility Test

(Systems with Harmonic Periods can often have U=1.0 and be feasible!)

<u>Example Where RM LUB is Safely Exceeded</u>:

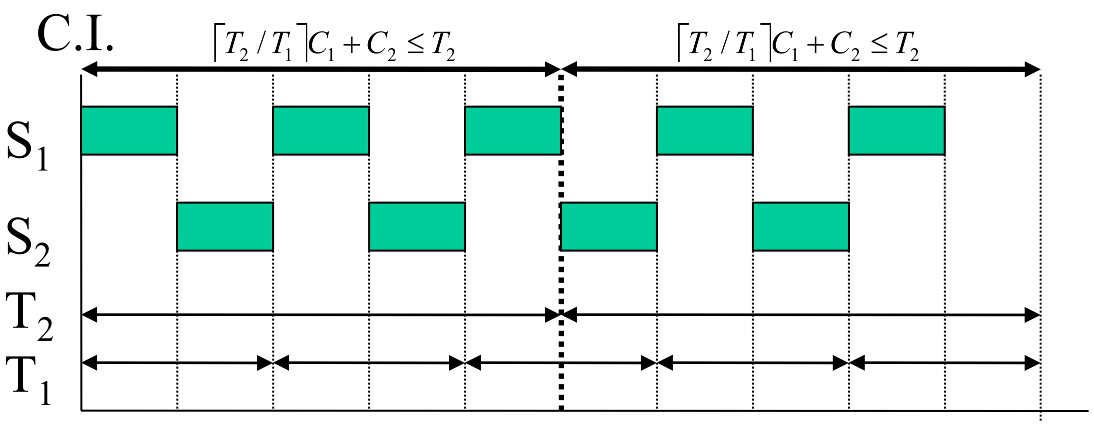
Given: Services S_1, S_2 with periods T_1 and T_2 and C_1 and C_2, Assume $T_2 > T_1$

E.g. T_1=2, T_2=5, C_1=1, **C_2= 2**, then if prio(S_1) > prio(S_2), note that:

$$U = 1/2 + 2/5 = 0.9$$

$$U = 0.9 > 2(2^{\frac{1}{2}} - 1)$$

$$U = 0.9 > 0.83$$



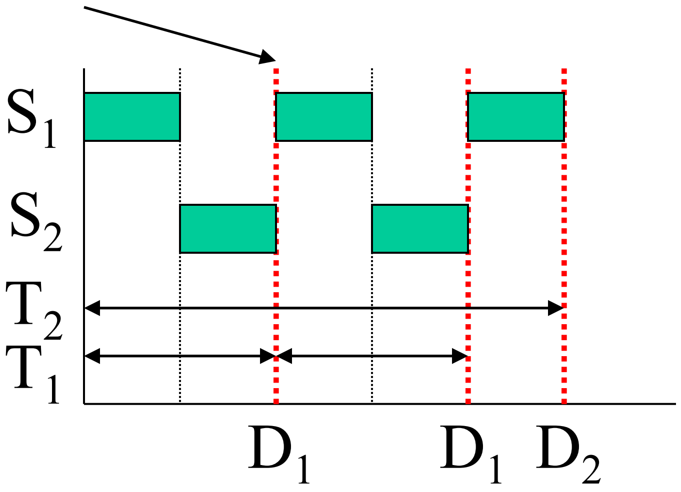
RM Priority Assignment Policy

<u>RM Policy</u>:

Given: services S_1, S_2 with periods T_1 and T_2 and C_1 and C_2 with $T_2 > T_1$

E.g. T_1=2, T_2=5, C_1=1, C_2=2, then if **prio(S_1) > prio(S_2)**, note that:

S_1 Makes Deadline if prio(S_1) > prio(S_2)

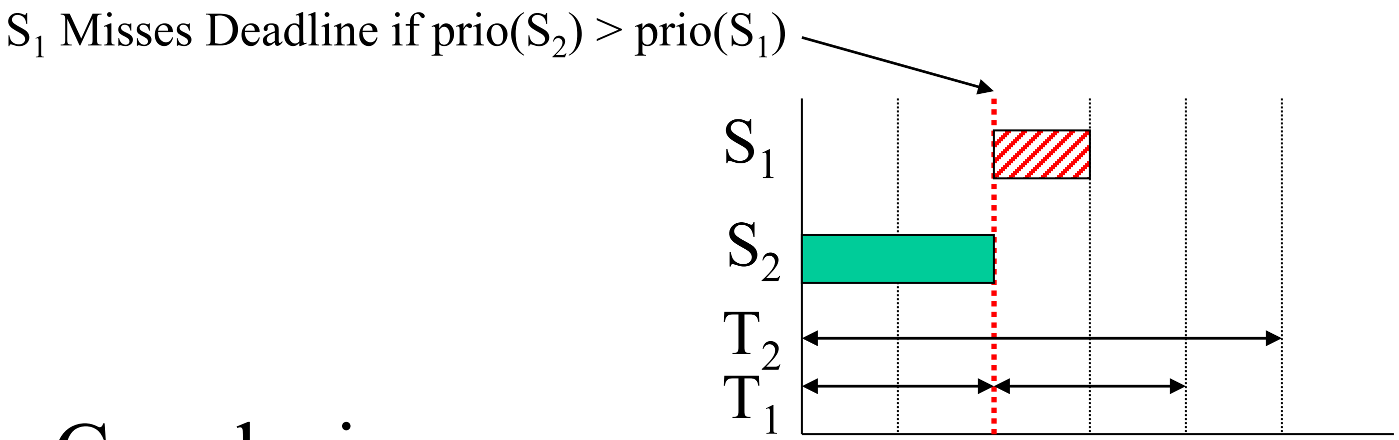


RM Priority Assignment Policy

Consider Alternative Policy:

Given: services S_1, S_2 with periods T_1 and T_2 and C_1 and C_2 with $T_2 > T_1$

E.g. T_1=2, T_2=5, C_1=1, C_2=2, then if **prio(S_2) > prio(S_1)**, note that:



Conclusion:

If $\{S_n\}$ feasible with prio(S_2) > prio(S_1), then $\{S_n\}$ is also always feasible given prio(S_1) > prio(S_2), but converse is not necessarily TRUE!!

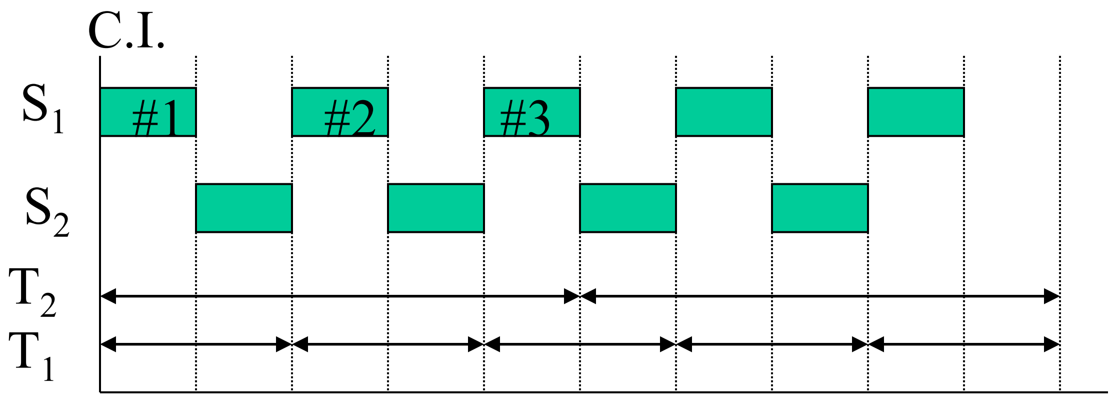
Therefore, prio(S_1) > prio(S_2) is OPTIMAL

RM LUB Derivation

Finding RM Safe Upper Bound (LUB):

If system feasible over LCM with RM policy, then by Worst-Case Analysis, it is safe!

Note that there can be up to $\lceil T_2 / T_1 \rceil$ releases of S_1 during T_2!



LUB Derivation Strategy:

Case 1: C_1 short enough to fit all 3 releases in T_2 (fits S_2 critical time zone)

Case 2: C_1 too large to fit last release in T_2 (doesn't fit S_2 critical time zone)

Examine U in both cases to find common U bound.

RM LUB Derivation

Case 1 (All 3 C_1 releases fit in T_2):

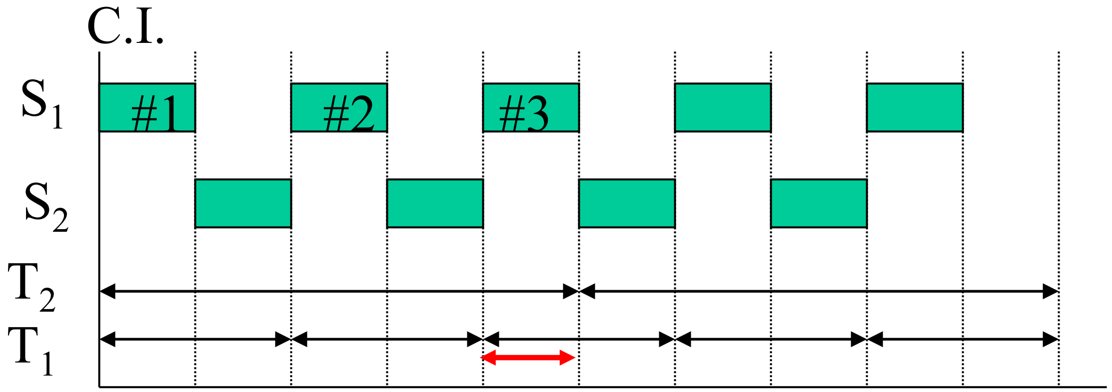
$$C_1 \le T_2 - T_1 \lfloor T_2 / T_1 \rfloor$$

(I.e, C_1 is small enough to fit into fractional 3rd T_1 shown below as ⟷)

$$C_2 = T_2 - C_1 \lceil T_2 / T_1 \rceil$$

(I.e, $C_2 = T_2$ - Interference from C_1 releases)

$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$



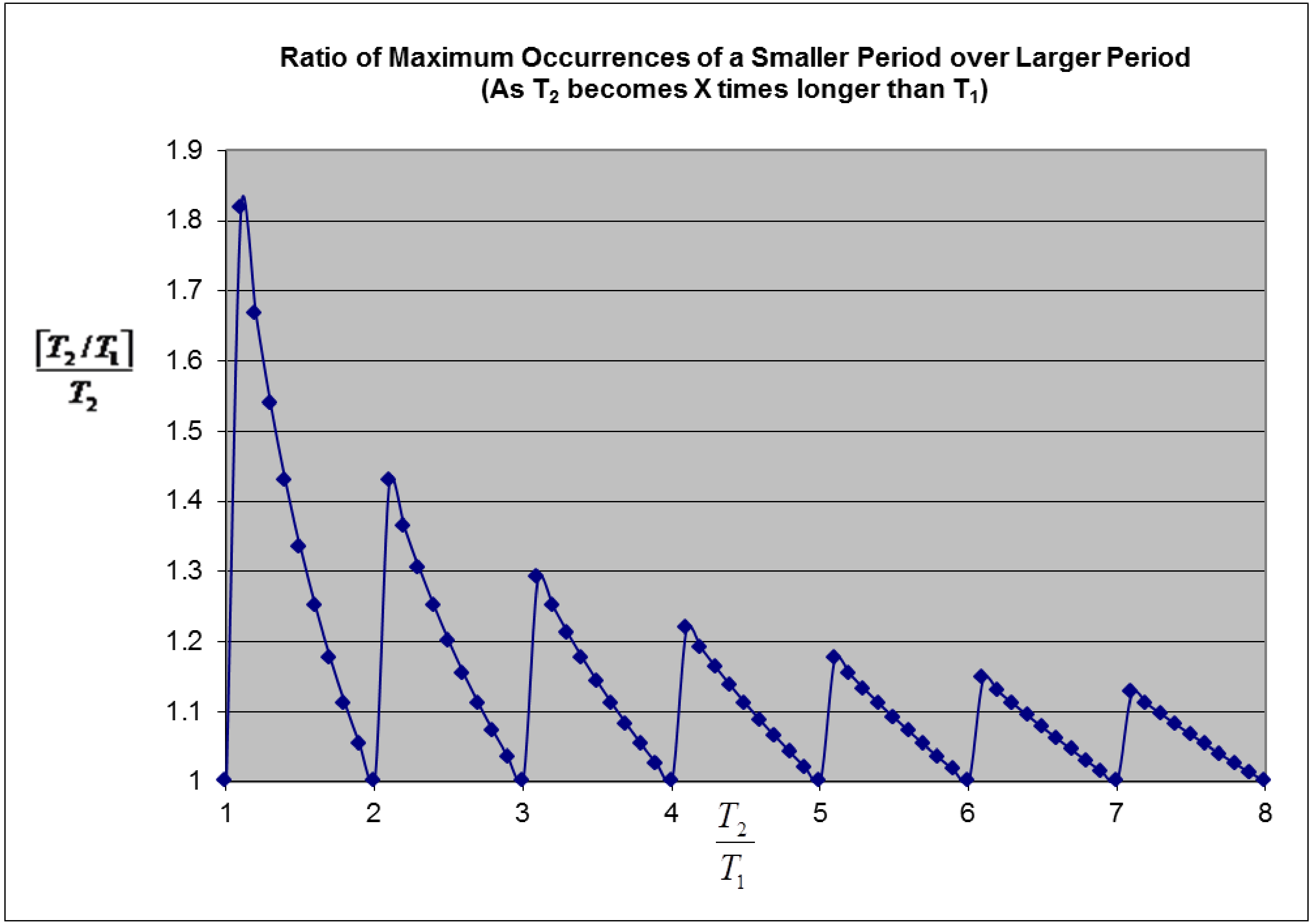
Plugging C_2 Expression into U:

$$U = \frac{C_1}{T_1} + \frac{[T_2 - C_1 \lceil T_2 / T_1 \rceil]}{T_2}$$

U monotonically decreases with increasing C_1 when ($T_2 > T_1$)

$$U = 1 + C_1 \left[(1/T_1) - \frac{\lceil T_2 / T_1 \rceil}{T_2} \right]$$

Proof that U Monotonically Decreases in Case-1



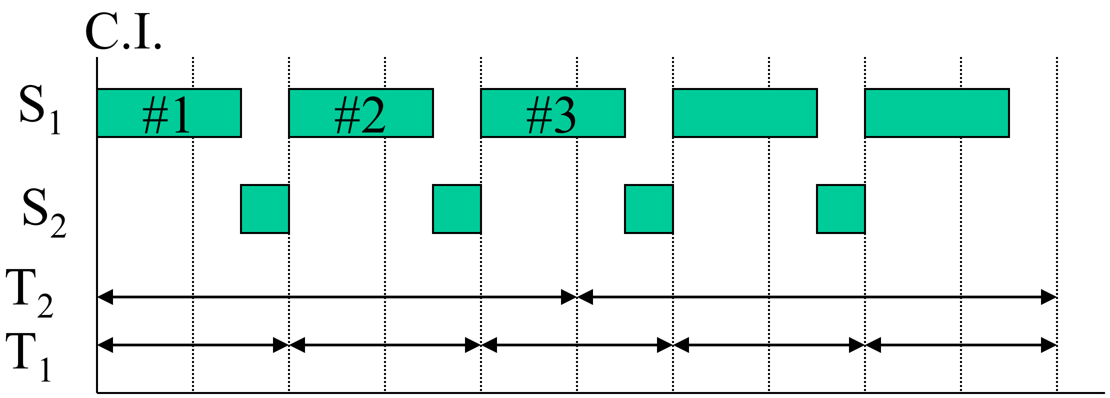
RM LUB Derivation

Case 2 (Last C_1 release does not fit in T_2):

$$C_1 \geq T_2 - T_1 \lfloor T_2 / T_1 \rfloor$$

$$C_2 = T_1 \lfloor T_2 / T_1 \rfloor - C_1 \lfloor T_2 / T_1 \rfloor$$

$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$



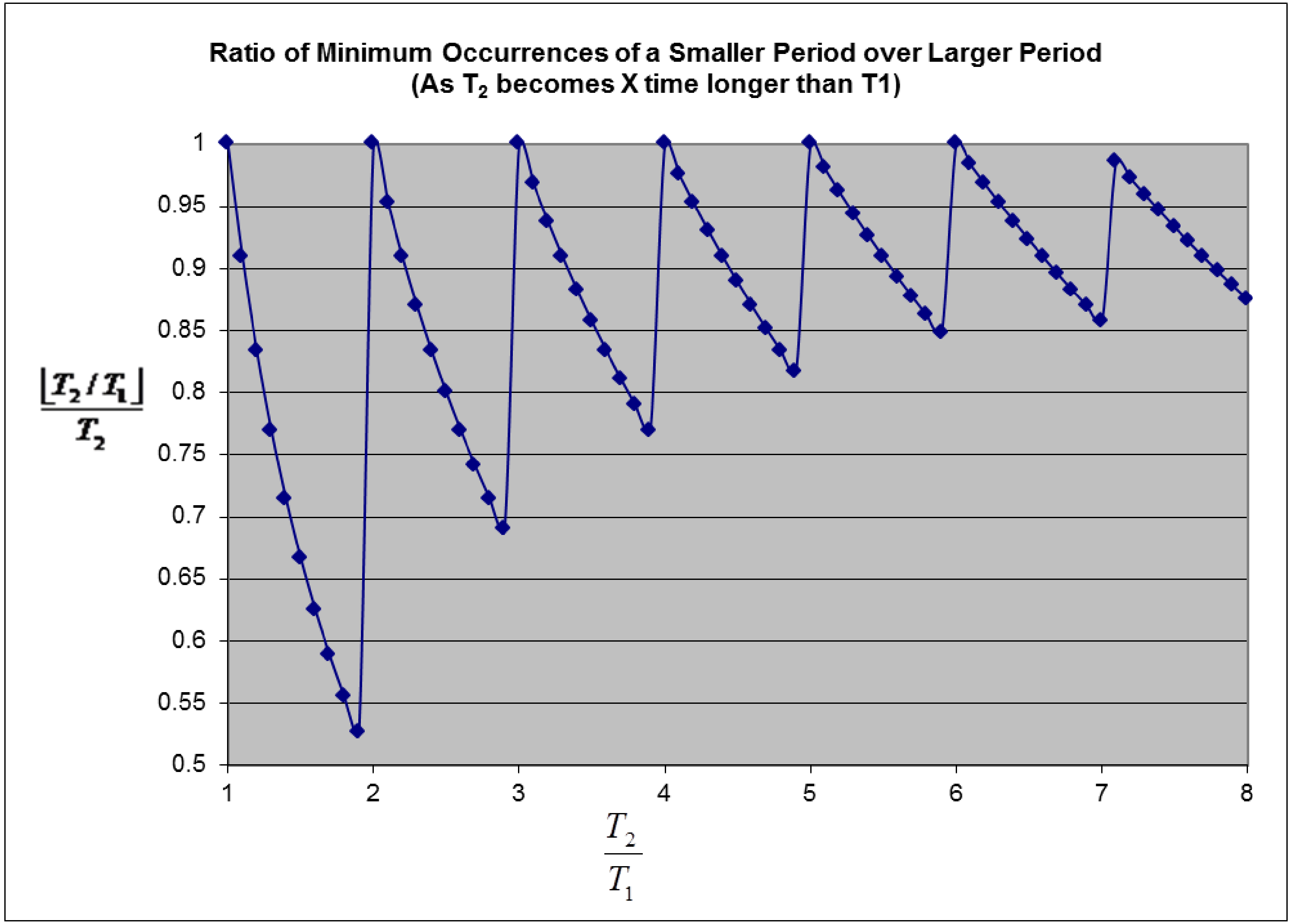
Plugging C_2 Expression into U:

$$U = \frac{C_1}{T_1} + \frac{\left[T_1 \lfloor T_2 / T_1 \rfloor - C_1 \lfloor T_2 / T_1 \rfloor\right]}{T_2}$$

U monotonically increases with increasing C_1 when (T_2>T_1)

$$U = (T_1 / T_2) \lfloor T_2 / T_1 \rfloor + C_1 \left[(1/T_1) - (1/T_2) \lfloor T_2 / T_1 \rfloor\right]$$

Proof that U Monotonically Increases in Case-2



RM LUB Derivation

<u>Given Cases 1 and 2</u> (T_1=2, T_2=5)

Case 1

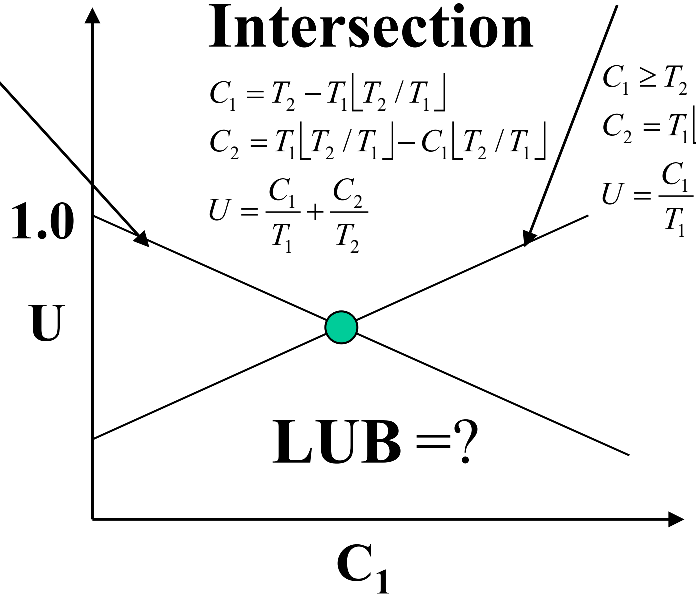
$$C_1 \le T_2 - T_1\lfloor T_2/T_1 \rfloor$$
$$C_2 = T_2 - C_1\lceil T_2/T_1 \rceil$$
$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$

Intersection

$$C_1 = T_2 - T_1\lfloor T_2/T_1 \rfloor$$
$$C_2 = T_1\lfloor T_2/T_1 \rfloor - C_1\lfloor T_2/T_1 \rfloor$$
$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$

Case 2

$$C_1 \ge T_2 - T_1\lfloor T_2/T_1 \rfloor$$
$$C_2 = T_1\lfloor T_2/T_1 \rfloor - C_1\lfloor T_2/T_1 \rfloor$$
$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$



T_1=	2	T_2=	5
Case 1		**Case 2**	
C_1	U	C_1	U
0	1	0	0.8
0.1	0.99	0.1	0.81
0.2	0.98	0.2	0.82
0.3	0.97	0.3	0.83
0.4	0.96	0.4	0.84
0.5	0.95	0.5	0.85
0.6	0.94	0.6	0.86
0.7	0.93	0.7	0.87
0.8	0.92	0.8	0.88
0.9	0.91	0.9	0.89
1	0.9	1	0.9

Case 1: $U = 1 + C_1\left[(1/T_1) - \frac{\lceil T_2/T_1 \rceil}{T_2}\right]$

Case 2: $U = (T_1/T_2)\lfloor T_2/T_1 \rfloor + C_1\left[(1/T_1) - (1/T_2)\lfloor T_2/T_1 \rfloor\right]$

Note: we want the LUB for any given T_2 and T_1, not the ones assumed here in particular, so the general LUB could be something other than 0.9, and must be found in terms of T_1 and T_2 only for general LUB.

RM LUB Derivation

Given Cases 1 and 2 (T_1=2, T_2=5)

Case 1

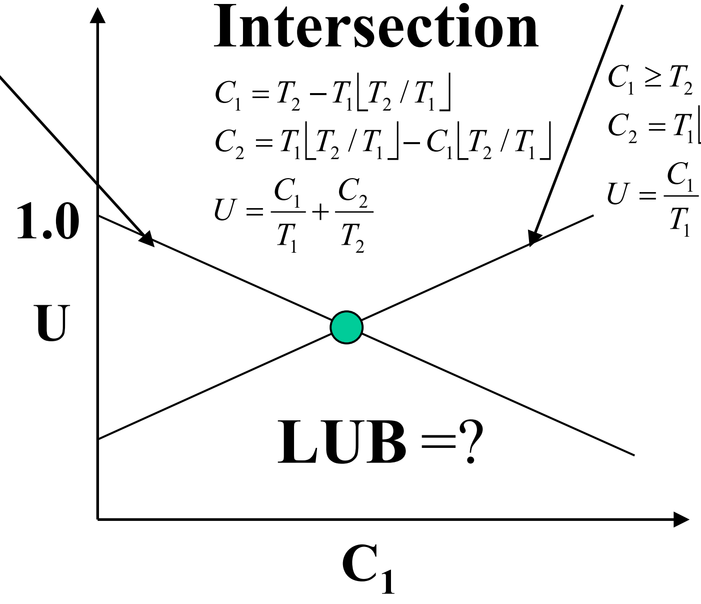
$$C_1 \le T_2 - T_1\lfloor T_2/T_1 \rfloor$$
$$C_2 = T_2 - C_1\lceil T_2/T_1 \rceil$$
$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$

Intersection

$$C_1 = T_2 - T_1\lfloor T_2/T_1 \rfloor$$
$$C_2 = T_1\lfloor T_2/T_1 \rfloor - C_1\lfloor T_2/T_1 \rfloor$$
$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$

Case 2

$$C_1 \ge T_2 - T_1\lfloor T_2/T_1 \rfloor$$
$$C_2 = T_1\lfloor T_2/T_1 \rfloor - C_1\lfloor T_2/T_1 \rfloor$$
$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$



T_1=	2	T_2=	5
Case 1		**Case 2**	
C_1	U	C_1	U
0	1	0	0.8
0.1	0.99	0.1	0.81
0.2	0.98	0.2	0.82
0.3	0.97	0.3	0.83
0.4	0.96	0.4	0.84
0.5	0.95	0.5	0.85
0.6	0.94	0.6	0.86
0.7	0.93	0.7	0.87
0.8	0.92	0.8	0.88
0.9	0.91	0.9	0.89
1	0.9	1	0.9

Case 1: $U = 1 + C_1\left[(1/T_1) - \frac{\lceil T_2/T_1 \rceil}{T_2}\right]$

Case 2: $U = (T_1/T_2)\lfloor T_2/T_1 \rfloor + C_1\left[(1/T_1) - (1/T_2)\lfloor T_2/T_1 \rfloor\right]$

Note: What if C1 = 0.5 in Case 1, can we make a schedule assuming that? What if C1 = 0.5 in Case2, can we make a workable schedule using RM?
What about C1 = 1.5 in both cases?

Can we make a workable schedule using RM policy if $C_1 = 0.5$ for Case 2 with $T_1 = 2$ and $T_2 = 5$?

A Yes, Always

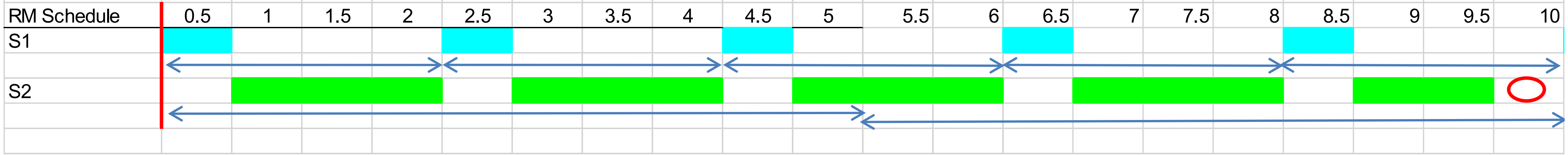
B No, Never

C Sometimes

RM LUB Derivation

Case 1 (T_1=2, T_2=5) at Upper Bound (but not the LUB)

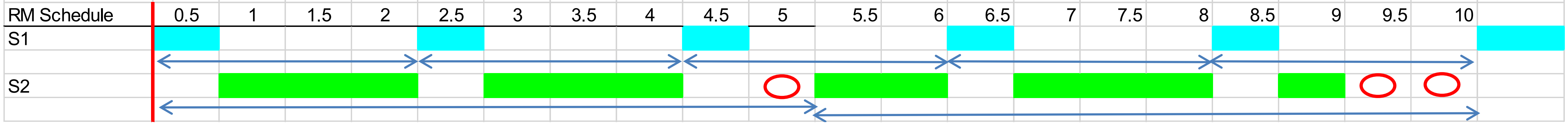
Case 1, C1 = 0.5	Service	Period		Freq f	f0 mult	WCET		Utility				
	S1	T1	2	0.5	2.5	C1	0.5	U1	25.0%	**LCM =**	**10**	C2 = T2 - C1 *Ceil(T2/T1)
	S2	T2	5	0.2	1	C2	3.5	U2	70.0%	**LUB =**	**82.8%**	
										Utot =	**95.0%**	



RM LUB Derivation

Case 2 (T_1=2, T_2=5) at Upper Bound? (but not the LUB)

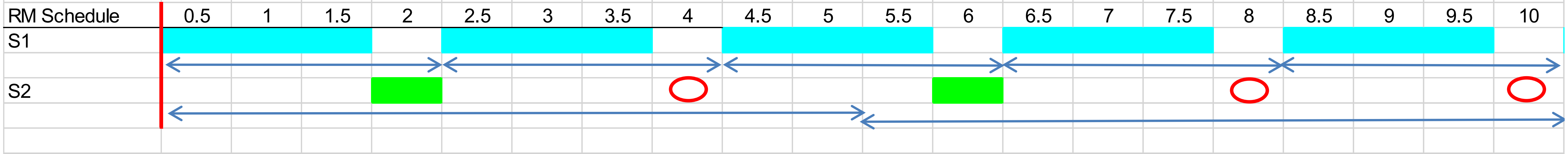
Case 2, C1 = 0.5	Service	Period		Freq f	f0 mult	WCET		Utility					
	S1	T1	2	0.5	2.5	C1	0.5	U1	25.0%	**LCM =**	**10**		C2 = T1*Floor(T2/T1) - C1 *Floor(T2/T1)
	S2	T2	5	0.2	1	C2	3	U2	60.0%	**LUB =**	**82.8%**		
										Utot =	**85.0%**		



RM LUB Derivation

Case 1 (T_1=2, T_2=5) at Upper Bound? (but not the LUB)

Case 1, C1 = 0.5	Service	Period		Freq f	f0 mult	WCET		Utility					
	S1	T1	2	0.5	2.5	C1	1.5	U1	75.0%	**LCM =**	**10**		C2 = T2 - C1 *Ceil(T2/T1)
	S2	T2	5	0.2	1	C2	0.5	U2	10.0%	**LUB =**	**82.8%**		
										Utot =	**85.0%**		



RM LUB Derivation

Case 2 (T_1=2, T_2=5) at Upper Bound (but not the LUB)

Case 2, C1 = 1.5	Service	Period		Freq f	f0 mult	WCET		Utility												
	S1	T1	2	0.5	2.5	C1	1.5	U1	75.0%	**LCM =**	**10**		C2 = T1*Floor(T2/T1) - C1 *Floor(T2/T1)							
	S2	T2	5	0.2	1	C2	1	U2	20.0%	**LUB =**	**82.8%**									
										Utot =	**95.0%**									
RM Schedule	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
S1																				
S2																				

RM LUB Derivation

Intersection of Case 1 & 2 is Least Upper Bound

Plug In Intersection for C_1 and C_2 into U to get expression in terms of T_1 and T_2 only: $U = 1 - (T_1/T_2)[\lceil T_2/T_1 \rceil - (T_2/T_1)][(T_2/T_1) - \lfloor T_2/T_1 \rfloor]$

(Substitute for both C_1 and C_2 in U expression and simplify)

Let $I = \lfloor T_2/T_1 \rfloor$ and $f = (T_2/T_1) - \lfloor T_2/T_1 \rfloor$ so, $U = 1 - \left(\frac{f(1-f)}{(T_2/T_1)} \right)$

I is the integer number of times that T_1 occurs during T_2

f is the fractional time of the last release for T_1 during T2, noting that if ***f*** =0, then T_1 and T_2 are harmonic, and therefore U=1, an uninteresting ideal case.

Substituting ***I*** and ***f*** into the U expression above and simplifying, we get:

$U = 1 - (T_1/T_2)[\lceil T_2/T_1 \rceil - (T_2/T_1)][(T_2/T_1) - \lfloor T_2/T_1 \rfloor]$ (noting that 1+floor(N+/-0.d) = ceiling(N+/-0.d) when f non-zero)

$U = 1 - (T_1/T_2)[1 + \lfloor T_2/T_1 \rfloor - (T_2/T_1)][(T_2/T_1) - \lfloor T_2/T_1 \rfloor]$

$U = 1 - (T_1/T_2)[1 - ((T_2/T_1) - \lfloor T_2/T_1 \rfloor)][(T_2/T_1) - \lfloor T_2/T_1 \rfloor]$

So, $U = 1 - (T_1/T_2)(1-f)(f)$, Re-arranged to obtain: $U = 1 - \left(\frac{f(1-f)}{(T_2/T_1)} \right)$

RM LUB Derivation

$$U = 1 - \left(\frac{f(1-f)}{(T_2 / T_1)} \right)$$ Can also be expressed as:

$$U = 1 - \left(\frac{f(1-f)}{\lfloor T_2 / T_1 \rfloor + (T_2 / T_1) - \lfloor T_2 / T_1 \rfloor} \right)$$ By adding and subtracting the same denominator term to get:

$$U = 1 - \left(\frac{f(1-f)}{(I+f)} \right)$$ smallest I is 1, and LUB for U occurs when I is minimized, so: $$U = 1 - \left(\frac{(f - f^2)}{(1+f)} \right)$$

Now taking the derivative of U w.r.t. f, and solving for extreme, we get:

$$dU / df = \frac{(1+f)(1-2f) - (f - f^2)(1)}{(1+f)^2} = 0$$

Solving for f, we get: $f = (2^{1/2} - 1)$

And, plugging f back into U, we get: $U = 2(2^{1/2} - 1)$ The RM LUB!

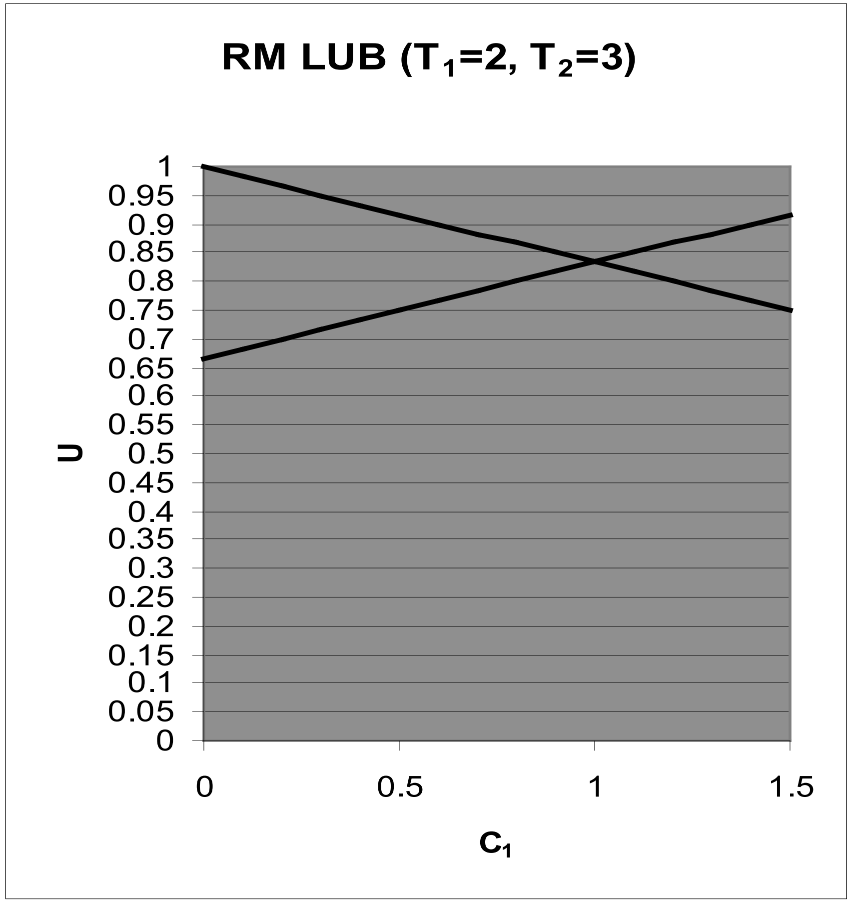
RM LUB Scenario for 2 Services

(With T_1=2, T_2=3, We Show U=0.83 Worst-Case Graphically)

<u>Given Case 1:</u> $U = (T_1 / T_2)\lfloor T_2 / T_1 \rfloor + C_1\left[(1/T_1) - (1/T_2)\lfloor T_2 / T_1 \rfloor\right]$

<u>And Case 2:</u> $U = 1 + C_1\left[(1/T_1) - \frac{\lceil T_2 / T_1 \rceil}{T_2}\right]$

T_1=	2		T_2=	3	
Case 1			**Case 2**		
C_1	U	C_2	C_1	U	C_2
0	1	3	0	0.666667	2
0.1	0.983333	2.8	0.1	0.683333	1.9
0.2	0.966667	2.6	0.2	0.7	1.8
0.3	0.95	2.4	0.3	0.716667	1.7
0.4	0.933333	2.2	0.4	0.733333	1.6
0.5	0.916667	2	0.5	0.75	1.5
0.6	0.9	1.8	0.6	0.766667	1.4
0.7	0.883333	1.6	0.7	0.783333	1.3
0.8	0.866667	1.4	0.8	0.8	1.2
0.9	0.85	1.2	0.9	0.816667	1.1
1	0.833333	1	1	0.833333	1
1.1	0.816667	0.8	1.1	0.85	0.9
1.2	0.8	0.6	1.2	0.866667	0.8
1.3	0.783333	0.4	1.3	0.883333	0.7
1.4	0.766667	0.2	1.4	0.9	0.6
1.5	0.75	0	1.5	0.916667	0.5



RM LUB Scenario for 2 Services

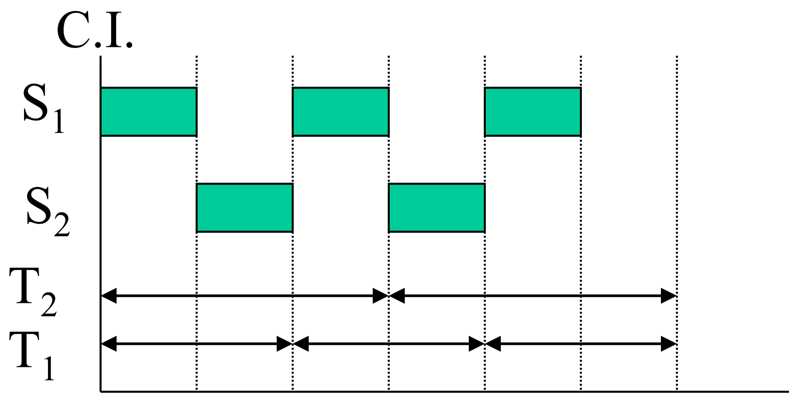
(Showing U=0.83 Worst-Case Timing Diagram)

<u>Given Cases 1 and 2 again:</u>

T_1=2, T_2=3, C_1=1, C_2=1 at U=0.83

Note: 5 out of 6 Time Units Used Over LCM

Such That U=0.83, the RM LUB!



Mistakes

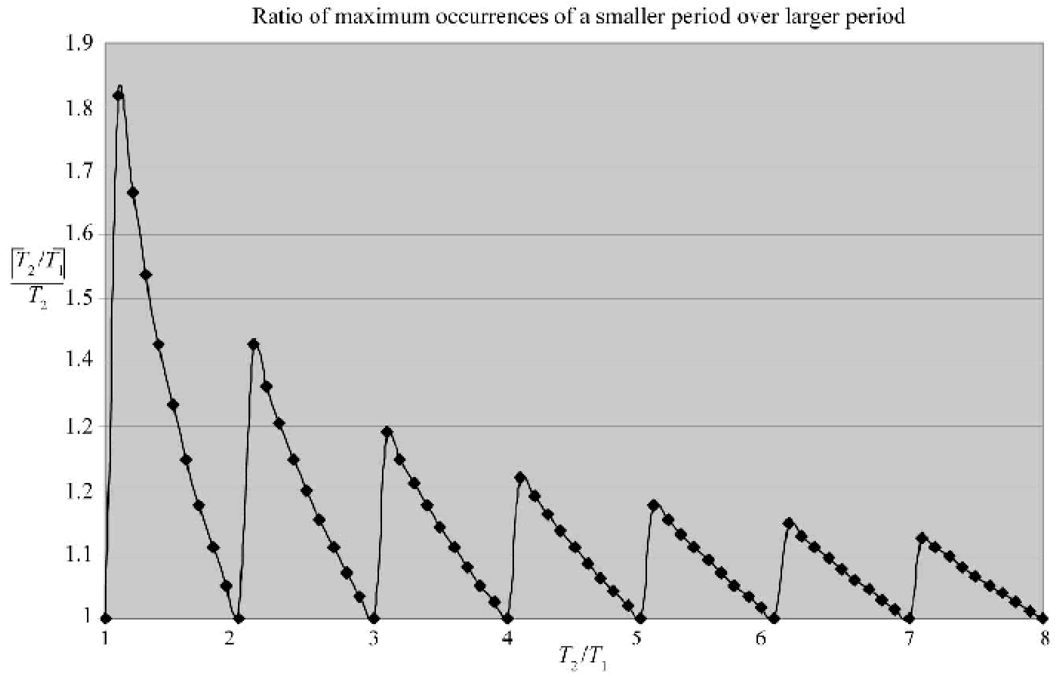


Figure 3.6 Case 1 Relationship of T_2 and T_1

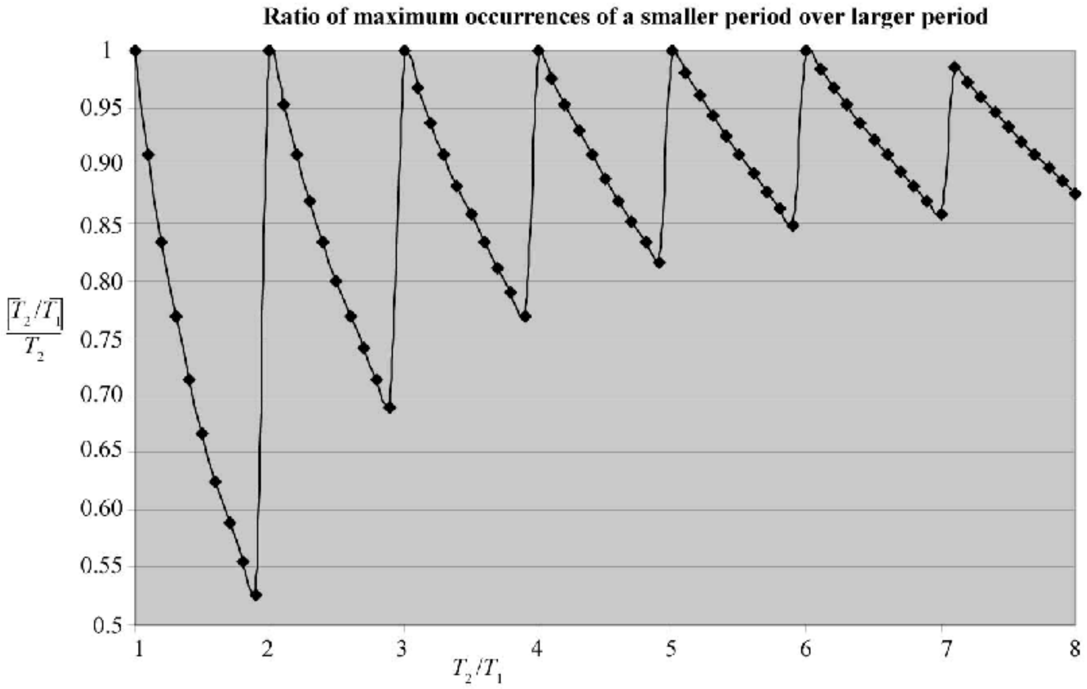


Figure 3.8 Case 2 Relationship of T_2 and T_1

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Mistakes

$$C_1 \geq T_2 - T_1 \left\lfloor T_2 / T_1 \right\rfloor$$

$$C_2 = C_1 \left\lfloor T_2 / T_1 \right\rfloor - C_1 \left\lfloor T_2 / T_1 \right\rfloor$$

$$U = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$

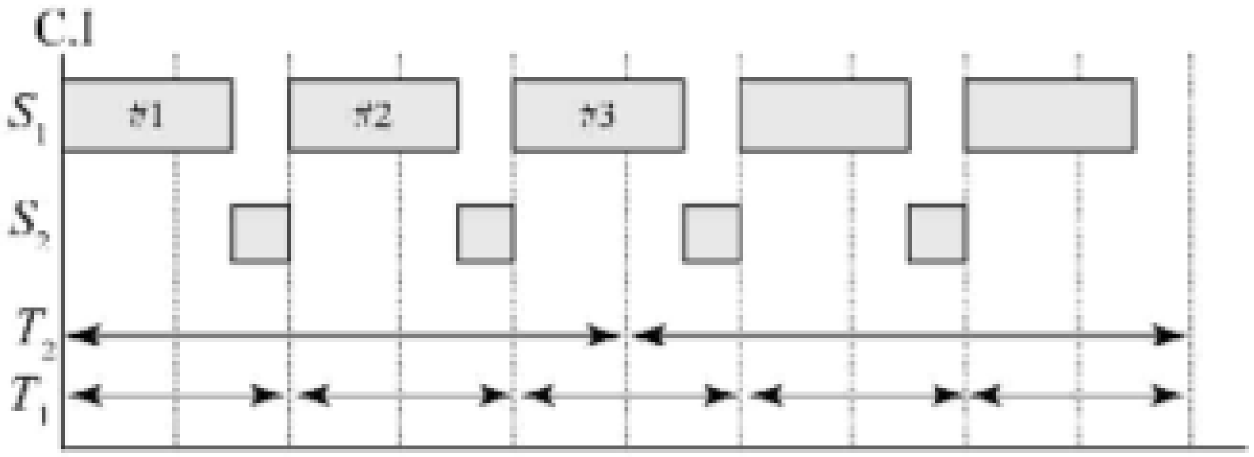

Figure 3.7 Case 2 Overrun of Critical Time Zone by S_1

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RM FAQ

- Where in the paper does it say that T must equal D?
  - T=D is in section 4 above Theorem 1 in Liu and Layland paper
- Why does Floor(x.d)+1 = Ceiling(x.d)?, because if x.d=1 or any integer, this is not true.
  - Floor(x.d) + 1 = Ceiling(x.d) iff x.d is not an integer value, which is always true if T_2 is not a multiple of T_1
- If $T_2 > T_1$, but T_2 is a multiple of T_1 (if T_1 and T_2 are harmonic), then U=1, and doesn't this violate the RM LUB?
  - Yes, it does, but remember the RM LUB is only sufficient, not N&S, and therefore it will pessimistically fail service sets that can actually be scheduled
  - The RM LUB will correctly fail all service sets than in fact can't be scheduled, so it is sufficient
- Why Not Just Use EDF Scheduling Policy if it Can Achieve 100% Utility and Be Shown to Meet Deadlines?
  - What Happens When EDF is Overloaded?

Rate Monotonic Understanding

- Task/thread blocking is [choose best]:
  A. When a thread/task requests a resource other than the CPU that is not available
  B. Preemption by a higher priority thread/task
  C. Interrupt preemption of thread/task
  D. Suspension of a thread/task due to an exception [e.g divide by zero]
  E. Due to a call to sleep() or yield()

- Asymmetric Multi-Processing [choose best]:
  A. Implements load balancing on multi-core CPUs
  B. Runs one RTOS instance on a multi-core CPU
  C. Runs an RTOS instance on each CPU core
  D. Can't make use of message passing between CPU cores
  E. None of the above

Rate Monotonic Understanding

- The RM LUB and policy will fail some feasible service sets
  A. TRUE
  B. FALSE

- The RM policy assigns highest priority to the highest frequency service
  A. TRUE
  B. FALSE

- The only way to implement an RM service design is with an RTOS
  A. TRUE
  B. FALSE