Analyzing Systemic Risk with Network Centrality

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Marie Leaf ml2286, Parthit Patel pbp38, Sizhi Tan st728, Sidarth Wadhwa sw737

Introduction

Financial networks often have a few nodes that control the majority of the flow happening through the network. Node centrality is one of the most important metrics used to measure the importance of the nodes in a network to assess systemic risk and spillover effects during times of an economic crises (Jorge A. Chan-Lau, 2018). We explored the paradigm of a financial system, of a blockchain enabled community currency, Sarafu, and analysed the 'importance' of the nodes. We used the amount of financial transactions controlled by the nodes i.e the total number of incoming payment and total number of outgoing payments, to determine which of the centrality metrics that we used would be a good determinant of the importance of the node in the network. Gathering information about these metrics would empower decision makers to decide whether a certain node has too much influence over the system, whether and how its collapse would affect the overall network, and how would the overall network change if it were to happen.

Financial systems are characterized by a system of interactions between different entities. From a mathematical perspective, it can be formally captured by the concept of a network or a graph. Where the individual nodes are represented by financial entities, whether banks, corporations, companies or individuals and the edges are the financial payments or transactions that they have made to other nodes. Hence, it is possible to use algorithms from network theory to analyse the structure of these networks and find conclusions. Our group has analysed the topological properties of the network, analysing the static properties of the network. While this may not have captured the dynamic channels of interactions between the nodes in the systems, it gives us an overall picture of the importance of the nodes depending on the different centrality metrics and which one of the metrics that we used would be best suited to analysing the risk associated with the failure of a certain node in the network.

Literature Review

In researching systemic risk in financial networks we uncovered several papers that explored how centrality metrics could determine contagion effects and fragility of a liquidity squeeze during an economic downturn. In addition to exploring basic centrality metrics such as betweenness, closeness, and degree centrality, eigenvector centrality (specifically DebtRank which is inspired by PageRank) showed a promising metric in showing systemically risky nodes (Thurner, S., Poledna, S., 2013), and the velocity to which payment default risk can spread through a network of economic actors (Leinonen, Harry, 2005). The existence of networks of interpersonal relations has been empirically observed to constitute a fundamental factor in shaping the inter-institutional networks, or in accounting for the networks of risk-sharing agreements (Bartesaghi et al 2020).

In the analysis of financial and economic networks, the use of centrality measures is not so effective, as the classical ones provide a static view of the network, and even other measures based on dynamic processes, such as random walk-based centralities do not capture the changing conditions to which these networks could be subject in relatively short periods of time, (Bartesaghi et al 2020).

Methodology

Algorithms

One of the primary targets of this research is to assess whether the centrality of nodes explains their financial vulnerability or systemic importance. To document this issue, several centrality measures coming from network science have been applied to financial systems such as degree centrality, eigenvector centrality or Katz centrality to cite the most frequent (Gandica Y, Béreau S, Gnabo JY., 2020). In analyzing the structure of centrality metrics across nodes, capital allocations based on all other centrality measures help to improve the stability of the banking system in terms of expected total bankruptcy costs over at least some of the region (Alter, Adrian and Craig, Ben R. and Raupach, Peter, 2015).

In understanding the connectedness of a graph of economic transactions we chose to explore four different centrality metrics to rank the measures of influence the top nodes in the graph have:

1. **Betweenness Centrality :**

Betweenness Centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes (Wikipedia). In the The shortcoming of Betweenness Centrality in analyzing a large economic network however is that it is limited in assessing how many closed loops of obligation cycles it is involved in (Fleischman, Tomaž & Dini, Paolo. 2020).

The betweenness centrality of a node v is given by the expression:

$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

Where $\boldsymbol{\sigma}_{st}$ are all the paths from node s to t, and $\boldsymbol{\sigma}_{st}(v)$ is the number of paths from node s to node t that pass through node v, for paths where v is not the endpoint.

2. Closeness Centrality :

Closeness Centrality of a node is calculated as the reciprocal of the sum of the length of the shortest paths between the node and all other nodes in the graph. Thus, the more central a node is, the closer it is to all other nodes. This may indicate the time it takes for a financial collapse to spread.

$$C(x) = \frac{1}{\sum_y d(y,x)}.$$

Where d(y,x) is the distance between node x and node y, which is summed for all nodes y in the network.

3. Eigenvector Centrality :

We then applied the Eigenvector Centrality measure, which allows us to analyze directed as well as undirected graphs, as well as weighted graphs. This is more representative of how money flows through merchants within a financial system.

Eigenvector centrality (eigencentrality) is a measure of the influence of a node within a network. The relative scores that are assigned to nodes are based on the assertion that connections to high-scoring nodes contribute more to the score node being analyzed, rather than equal connections to low-scoring nodes. A node may have a general high degree score, because it has many connections, but a low eigen centrality because those connections have trivial or low scores of influence. A node may also have a high betweenness centrality, indicating that it connects disparate parts of a network, but a relatively low eigen centrality score if many of those connections are high distance from centers of power in the network.

Eigenvector centrality is also related to DebtRank or a Katz Centrality score.

$$x_i = \frac{1}{\lambda} \sum_{j \in M(i)} x_j$$

Where j ∈ M(i) means the sum over all nodes j, such that j and i are connected. And where **λ** is a constant.

4. **Degree Centrality :**

The Degree Centrality of a node is the number of links that a node has to other nodes. It could be interpreted to be the immediate risk that a node possesses on something (virus, information) to other nodes in the network.

$$C_D(v) = \deg(v)$$

Where the deg(v) is the number of links that a node has.

Dataset

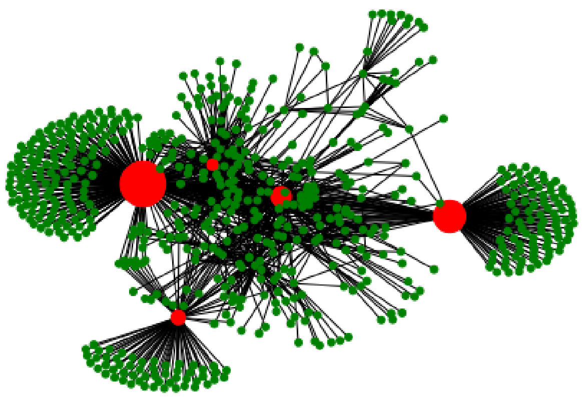
For our dataset, we used the publicly available Sarafu Transaction data from Grassroots Economics. Sarafu is a community inclusion currency distributed to roughly 55,000 users. For purposes of this project, we only ran the experiment and analyzed centrality metrics on the first 7500 transactions.

The dataset includes pseudonymized account information and transaction records for a digital community currency in Kenya. One unit of “Sarafu” is roughly equivalent in value to a Kenyan shilling. The Sarafu system has existed since 2010 and began operating digitally via USSD feature-code cellular technology in 2017.

Results

We plotted the graphs of the nodes in the network with the five highest values of centrality metrics. The nodes sizes show the order of the nodes (larger nodes size corresponds to higher metric value) The position of the nodes also give an insight into how the different nodes are ranked according to the different metrics.

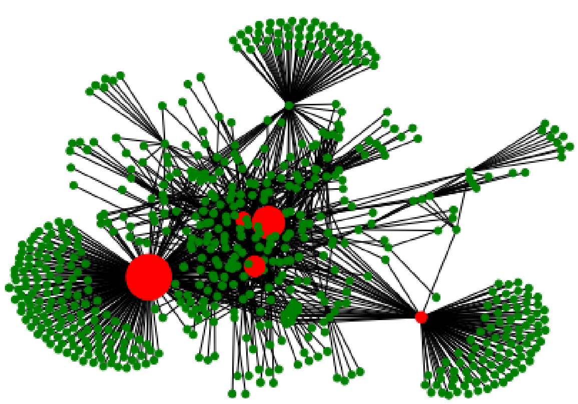
1. Betweenness Centrality



Mean Betweenness centrality 0.0032977489658986446
Top Nodes

1. 0x3700dd6c99f396c48e6658102ab2e18dc776d243 : 0.5585415927691739
2. 0xe6d39ad7c6be64673ec18af4a2eb02f2bea88e15 : 0.2865091694026129
3. 0xcb55fc893000a984a5ad73011f93d1540a5f0895 : 0.21815084129362786
4. 0x6e63c75540d889a365165811f974a8343e6d1f35 : 0.17909293201661206
5. 0x446090169a9198be7cea6df861a21999ebfeebc8 : 0.0504385174573413

2. Eigenvector Centrality

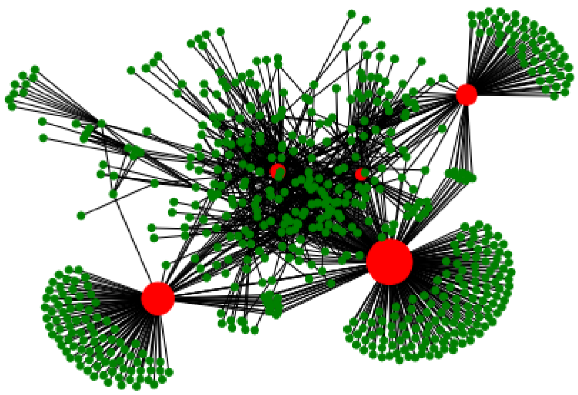


Mean Eigenvector centrality 0.023942244655297085

Top Nodes

1. 0x3700dd6c99f396c48e6658102ab2e18dc776d243 : 0.5453348291189044
2. 0xcb55fc893000a984a5ad73011f93d1540a5f0895 : 0.25739621094125725
3. 0x446090169a9198be7cea6df861a21999ebfeebc8 : 0.16455297277850275
4. 0x4d626e99d285852081f3529bd916e2852ff705fd : 0.15011632870281022
5. 0xe6d39ad7c6be64673ec18af4a2eb02f2bea88e15 : 0.1456155894036378

3. Degree Centrality

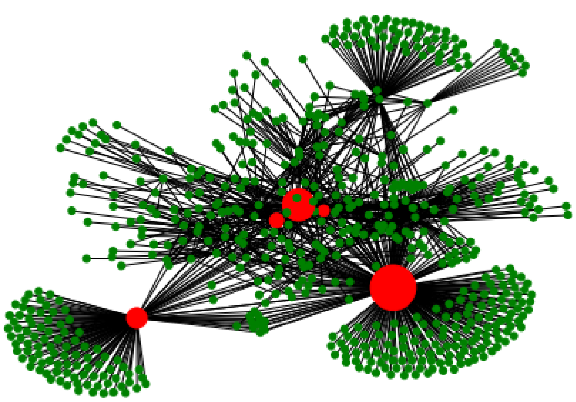


Mean Degree centrality 0.006403758842783233

Top Nodes

1. 0x3700dd6c99f396c48e6658102ab2e18dc776d243 : 0.424390243902439
2. 0xe6d39ad7c6be64673ec18af4a2eb02f2bea88e15 : 0.2016260162601626
3. 0x6e63c75540d889a365165811f974a8343e6d1f35 : 0.14308943089430895
4. 0xcb55fc893000a984a5ad73011f93d1540a5f0895 : 0.13658536585365855
5. 0x446090169a9198be7cea6df861a21999ebfeebc8 : 0.08292682926829269

4. Closeness Centrality



Mean Closeness centrality 0.3365937885030093

Top Nodes

1. 0x3700dd6c99f396c48e6658102ab2e18dc776d243 : 0.5710306406685237

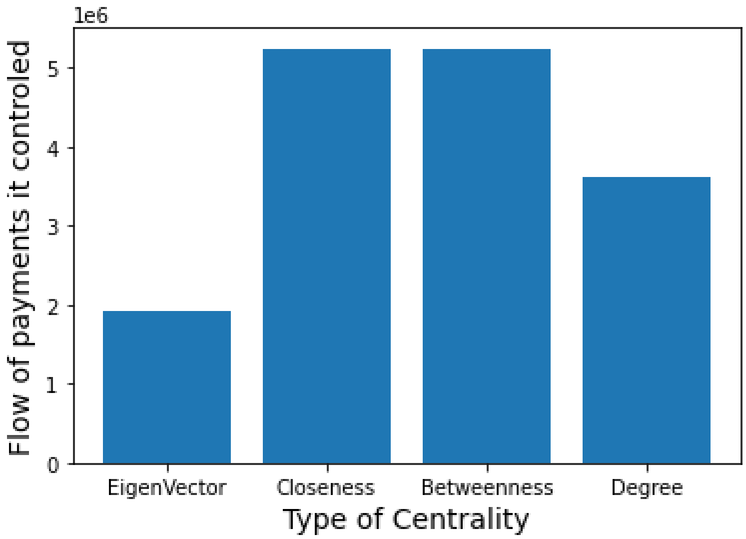
2. 0xcb55fc893000a984a5ad73011f93d1540a5f0895 : 0.5357142857142857
3. 0xe6d39ad7c6be64673ec18af4a2eb02f2bea88e15 : 0.4683929931454684
4. 0x4d626e99d285852081f3529bd916e2852ff705fd : 0.4552183567727609
5. 0xb7b1a85fed492df4a67f34a723408fc5f694f96b : 0.45387453874538747

The degree centrality of a node i counts the number of walks of length one starting at i (the degree of i). In contrast, the eigenvector centrality of node i gives the limit as k->∞ of the percentage of walks of length k which start at node i among all walks of length k. Thus, the degree centrality of nodes measures the local influence of i and the eigenvector centrality measures the global influence of i.

The graph below shows the number of transactions controlled by the top three nodes identified under the four types of centrality surveyed. Closeness and Betweenness centrality perform the best on this metric, which is to say, they find the nodes that control the highest flow of payments in the entire graph. However, it must be noted that this metric is not an end-all-be-all approach to 'rank' the types of centralities covered.

Two to three nodes seem to be identified as having great influence over the network by all the approaches, albeit with different ranks assigned to them. Node 0x3700dd6c99f396c48e6658102ab2e18dc776d243 had the highest value for all metrics, owing to the number of nodes that it was connected to. However, there were different node ranks for the highest metric values for all cases. The difference in the node ranking and their position in the graph gives us insight into the way rank is assigned by the different metrics used.

By summing the total incoming and outgoing payments of the top 3 nodes in each of the metrics, we see that Closeness and Betweenness centrality performed the best when considering the amount of payments controlled. Degree centrality performed the 3rd best, with Eigenvector being the least optimal indicator of the total number of payments controlled.



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