Q1.

		Predicted Class		
		C_1	C_2	Reject
True Class	C_1	0	λ	c
	C_2	S	0	c

(a) Expected loss when Predicted Class is C_1 $= P(C_1|x)\cdot 0 + S(1-P(C_1|x))$

$$\Rightarrow EL(C_1) = S(1-h(x))$$

$$\boxed{\Rightarrow \frac{\partial EL(C_1)}{\partial h(x)} = -S}$$

$\therefore$ It is a decreasing function of $h(x)$ as $S > 0$

Similarly,

$$EL(C_2) = r(h(x)) + 0\cdot(1 - h(x))$$

$$\Rightarrow EL(C_2) = r\,h(x)$$

$$\boxed{\Rightarrow \frac{\partial EL(C_2)}{\partial h(x)} = r}$$

As $r > 0$, it is an increasing function of $h(x)$

(b) Given Loss Matrix is –

		Predicted Class		
		C_1	C_2	Reject
True Class	C_1	0	r	c
	C_2	s	0	c

P.T.O

Expected loss while choosing $C_1 = 0\, h(x) + s(1-h(x))$

$= s(1-h(x)) \geq 0$

$(\text{as } s > 0 \;\&\; 1-h(x) \geq 0)$

Similarly, Expected loss while choosing $C_2 = r\, h(x) + 0(1-h(x))$

$= r\, h(x) \geq 0$

$(\text{as } r > 0 \;\&\; h(x) \geq 0)$

$\boxed{\text{Clearly, both of these are } \geq 0}$

$\boxed{\text{expected loss to Reject} = 0}$

So, Reject will <u>always</u> be the <u>best option</u> due to <u>0 expected loss</u>

$\boxed{\therefore \text{ Best option is to Reject all instances of } x}$

(c) Given Loss Matrix -

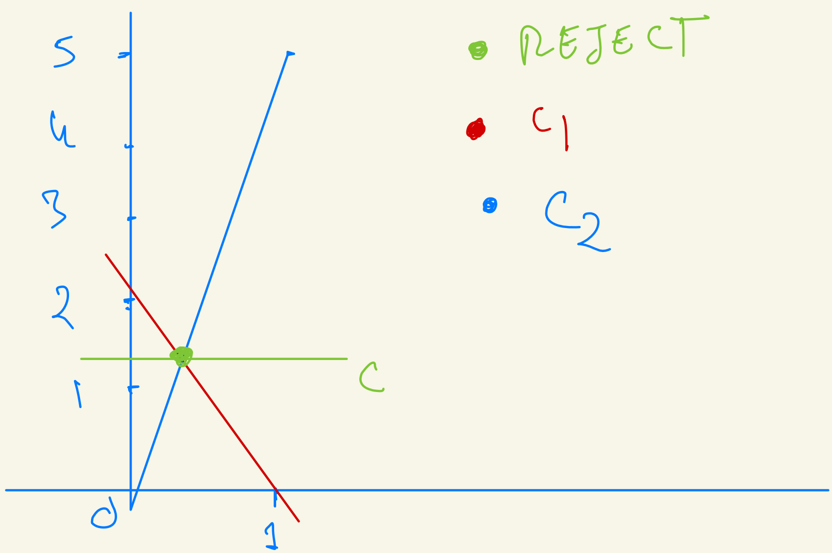
		Predicted		
		C_1	C_2	Reject
Truth	C_1	0	5	C
	C_2	2	0	C

Expected loss while choosing $C_1 = 2(1-h(x))$

" " " " $C_2 = 5h(x)$

" " " " for Reject Option = C.

Plotting these, we get -



Clearly, we can see that the point of intersection pt. of lines L_1 & L_2 will give us our reqd. solution.

$$\therefore \quad 2(1-h(x)) = 5(h(x))$$

$$\boxed{\Rightarrow h(x) = 2/7}$$

For $h(x) = 2/7$, expected loss for both C_1 & $C_2 = 2\left(1-\frac{2}{7}\right)$
$= 10/7$

$\therefore$ For $c = \frac{10}{7}$, we will never use the Reject Option.

$\therefore$ $c = \frac{10}{7}$ is the required minimum value

(d)

Predicted Class

True Class

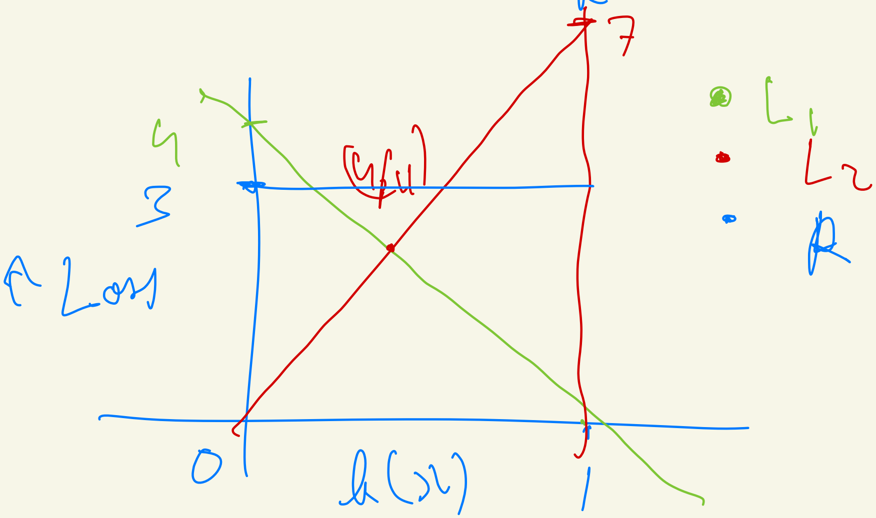
	C_1	C_2	Reject
C_1	0	7	3
C_2	4	0	3

exp. loss for choosing C_1, $L_1 = 4 - 4h(x)$

exp. loss for choosing C_2, $L_2 = 7h(x)$

exp. loss for Reject option $= 3 + 3(1 - h(x))$

$\Rightarrow L_R = 3$



From the plot, it seems like there is no feasible solution for Reject option

Let us check algebraically.

$$L_1 \geq L_R$$

$$\Rightarrow 4 - 4h(x) \geq 3$$

$$\boxed{\Rightarrow h(x) \leq 1/4} \rightarrow (i)$$

Also, $L_2 \geq L_R$

$$\Rightarrow 7h(x) \geq 3$$

$$\boxed{\Rightarrow h(x) \geq 3/7} \rightarrow (ii)$$

From (i) & (ii),

$h(x) \leq \frac{1}{4}$ and $h(x) \geq 3/7$.

$\boxed{\Rightarrow \text{No solution exists}}$

$\boxed{\text{2. There is no region where we choose Reject option}}$

Similarly for c_1:

$$L_1 \le L_2$$

$$\Rightarrow 4 - 4h(x) \le 7h(x)$$

$$\Rightarrow h(x) \ge 4/11 \to \text{(iii)}.$$

$$L_1 \le R$$

$$\Rightarrow 4 - 4h(x) \le 3$$

$$\Rightarrow h(x) \ge 1/4 \to \text{(iv)}$$

From (iii) & (iv),

$$\boxed{\text{we choose } c_1, \; h(x) \ge 4/11}$$

Similarly, for c_2:

$$L_2 \le L_1$$

$$\Rightarrow 7h(x) \le 4 - 4h(x)$$

$$\Rightarrow h(x) \le 4/11 \to \text{(v)}$$

$$L_2 \le R$$

$$\Rightarrow h(x) \le 3/7 \to \text{(vi)}$$

From (v) & (vi)

$\boxed{\text{To choose } c_2 ,\ h(x) \leq 4/11}$

Summarizing, we get —

To choose c_1: $h(x) \geq 4/11$

To choose c_2: $h(x) \leq 4/11$

To reject: No such $h(x)$ exists

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