OM 386: Demand Analytics and Pricing
Assignment #2

Please paste your answers within this file and save it as "HW2_eid1_eid2_eid3" (Where eids refer to your group members' EIDs) on Canvas at appropriate place. If you used MS Excel or any other software to arrive at your answers, please submit the relevant files/annotated code as well.

Write the names of your team members here:

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Q1: Use Logit Model to Estimate Auction Winning Probability for Fjord Motors

This problem is based on the "Fleet Sales Pricing at Fjord Motors" case and the associated dataset (Columbia Caseworks). https://www.study.net/content-bundle/view/30144289/materials

Mike Selvage and Dave Tobin are consultants working for a data analytics startup based in Austin TX. They are presently advising Fjord Motor's B2B sales team in revamping their bidding strategy for fleet sales auctions for their popular model *Coronet Elizabeth*.

Their first task is to come up with a method to predict the winning probability for any given price.

To assist their task, Fjord Motors experimented with about 4,000 bid prices in the past quarter. Messrs. Selvage and Tobin have been given access to this data that has the bid price, auction outcome and the order quantity. The MSRP for *Coronet Elizabeth* during the quarter was fixed $25,000.

Part A. Using the associated datasheet, Messrs. Selvage and Tobin decide to fit a two-parameter logit model that best estimates the probability of wining each bid as a function of ratio of the unit price to MSRP (i.e., PR=Unit price/MSRP):

$$\rho(PR) = \frac{\exp(a+b*PR)}{1+\exp(a+b*PR)},$$

where “a” and “b” are the parameters to be estimated and “p” is the winning probability as a function of PR. What are the values of “a” and “b” that maximize the sum of log likelihood based on the past data? (5 points)

Answer:

a = 7.756

b = -9.164

Note: All answers are rounded to 3 decimal digits

Sum (Log Likelihood)
-2261.87

Intercept	7.756228
PR coef	-9.16415

Part B. As Messrs. Selvage and Tobin continue their deliberations with Fjord sales team, they discover that bids 1 through 2,000 were to various police departments and that bids 2,001 through 4,000 were to corporate buyers. Using this new piece of information, they try to run new estimation by the following two approaches:

(1) They split the data into two parts: one dataset for police department bids and one dataset for corporate buyers. What are the corresponding values of “a” and “b” for each type of buyers if they follow the model in **Part A** and use MLE estimation? (5 points)

Answer:

For bids 1 – 2000 (Police departments):

a = 14.224

b = -20.01

Bids 1 - 2000	
Sum (Log Likelihood)	-905.858

Intercept	14.22415
PR coef	-20.0103

For bids 2001 – 4000 (Corporate buyers):

a = 27.875

b = -28.812

Bids 2001 - 4000			
Sum (Log Likelihood)	-124.34		
		Intercept	27.87499
		PR coef	-28.8116

Note: All answers are rounded to 3 decimal digits

(2) They also try another way: instead of splitting the data into two parts, they want to use the entire data. So, they create a dummy variable "police" (which equals 1 if for bids 1 through 2,000 and 0 for bids 2,001 through 4,000) and then they use the following model for the winning probability:

$$\rho(PR) = \frac{\exp(a+b*PR+c*police+d*PR*police)}{1+\exp(a+b*PR+c*police+d*PR*police)},$$

What will be the corresponding values for "a", "b", "c" and "d" from MLE estimation? Determine whether these results are essentially equivalent to those obtained in (1)? Please explain your conclusion. (5 points)

Answer:

a = 27.869

b = -28.805

c = -13.645

d = 8.795

Sum(Log Likeliho	-1030.2
Intercept	27.86879
PR coef	-28.8045
Police coef	-13.6452
PR*Police coef	8.794983

The results obtained are essentially equivalent to those obtained in (1)

For example:

Note: Assume PR = 1 in the examples below

Consider a corporate buyer bid (Value of binary variable Police = 0)

Thus, p(PR) = a + b*PR + c*0 + d*PR*0 = a+b*PR = 27.869 – 28.805(1) = **-0.936**

This is in essence almost identical to the value obtained in (1) which would be = 27.875 + (-28.812)(1) = **-0.937**

Now consider a police department bid (Value of binary variable Police = 1)

Thus, p(PR) = a + b*PR + c*1 + d*PR*1 = 27.869 + (-28.805)(1) + (-13.645)(1) + 8.795(1)(1) =

-5.786

This is in essence almost identical to the value obtained in (1) which would be = 14.224 + (-20.01)(1) = **-5.786**

By including the interaction term in the equation, we are accounting for the additive effect of different variables. This is equivalent to the approach tried earlier, in which we split up the two sets based on the kind of bid and calculated 2 sets of coefficients for each. In the end, for any particular kind of bid, the final regression estimate ends up being nearly identical.

Part C. Their further discussions with the sales team reveal that order size (i.e., units) is an important determinant for the winning probability. Therefore, Messrs. Selvage and Tobin add a new parameter "c" to capture this effect, and they come up with a new model as follows:

$$\rho(p) = \frac{\exp(a+b*PR+c*units)}{1+\exp(a+b*PR+c*units)}.$$

They continue to believe that the values of "a" and "b" should be different for the two types of sales, but the effect of the units should be the same. Try to run the MLE estimation for two sets of "a" and "b" values but one single "c" value based on the data. (8 points)

Answer:

Coefficients for Police sales -> a1, b1
Coefficients for Corporate sales -> a2, b2
Common units coefficient for both sales -> c

a1 = 14.969
b1 = -20.232
a2 = 28.886
b2 = -29.309
c = -0.029

Intercept (a1)	14.96926
PR coef (b1)	-20.2324
Units coef (c)	-0.0298
Intercept (a2)	28.8861
PR coef (b2)	-29.3086

Note: All answers are rounded to 3 decimal digits

Observations:

1. The intercept value for police sales (a1) is greater than the intercept value of corporate sales (a2) with both being positive. This indicates a higher level of baseline probability for Corporate buyers bids.
2. The coefficient for PR for police sales (b1) is smaller than coefficient for PR for corporate sales (b2). It can be interpreted that each unit increase in PR results in greater loss of probability for corporate bids compared to police bids keeping number of units constant.
3. The coefficient for units (c) is same for both models. It is quite small and negative indicating that an increase in units leads to a small loss in probability score keeping other factors constant.