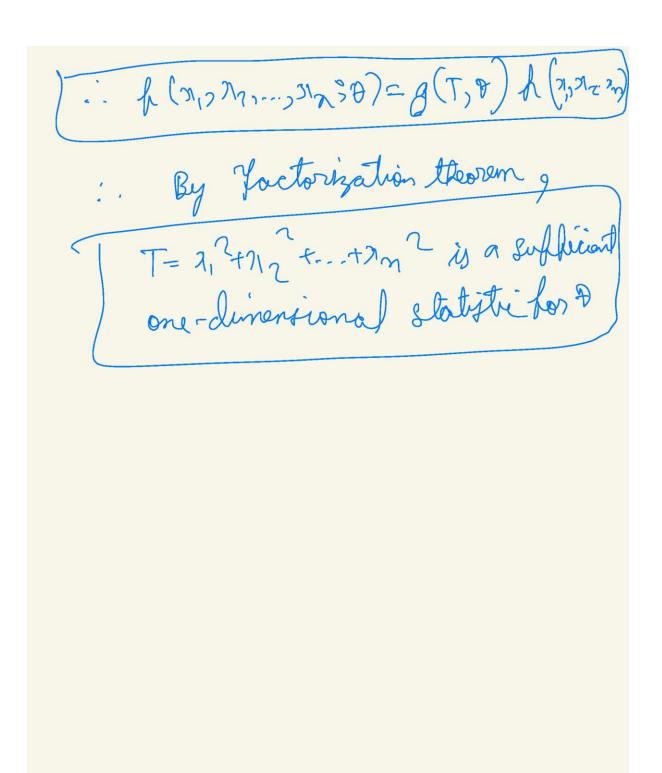
## Unsupervised Learning HW1 Submitted By: Parthiv Borgohain (pb25347)

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$$k(x_1, x_2, \dots, x_n; \theta) = \frac{1}{\theta \sqrt{\pi}} e^{-\frac{2x_1^2}{\theta x_2}}$$

$$= \left(\frac{1}{\theta \sqrt{\pi}} e^{-\frac{2x_1^2}{\theta x_2}}\right) \left(\frac{1}{\theta \sqrt{\pi}} e^{-\frac{$$

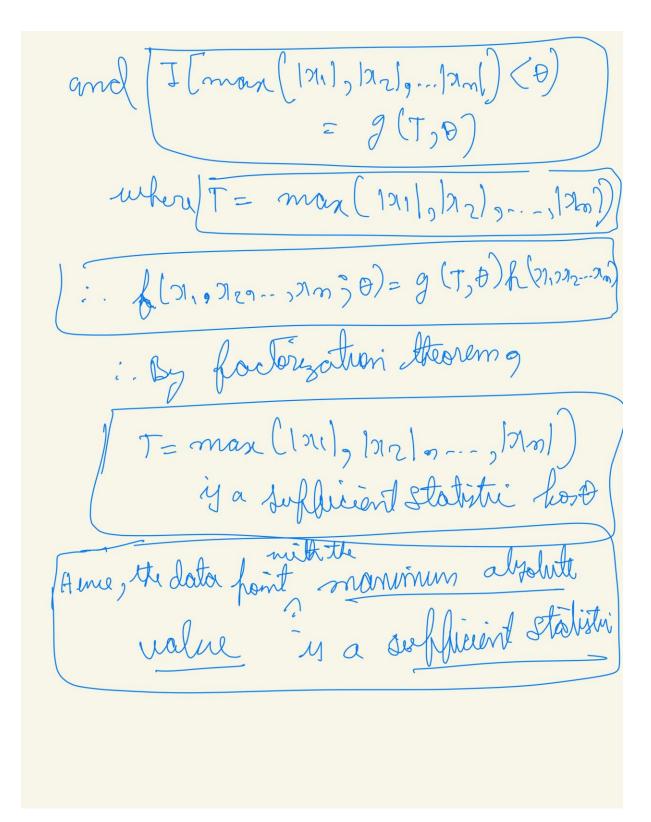


So, clearly via the working shown in the attached images above, we have shown via the factorization theorem that -

 $T = x_1^2 + x_2^2 - \dots + x_n^2$  is a sufficient one-dimensional statistic for  $\theta$ .

**Q2.** 

$$\frac{1}{10} = \frac{1}{10} = \frac{1}{10}$$

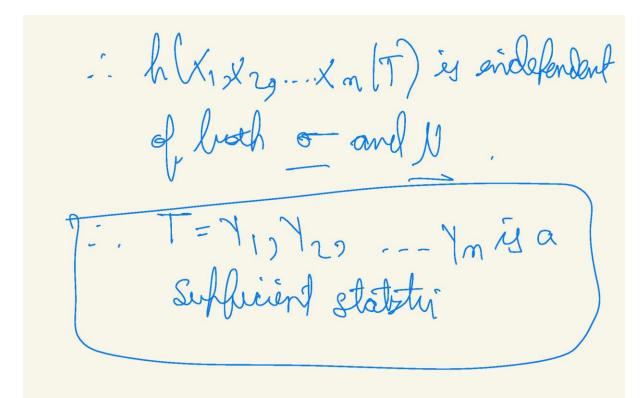


So, clearly via the working shown in the attached images above, we have shown via the factorization theorem that -

 $max(|x_1|,|x_2|,...,|x_n|)$  is a one-dimensional sufficient statistic for  $\theta$ .

guien 9 f (Nill, 0) = 1 e 52 hor - 2/ 11/20 j=42, ..., n 2 where -2<N200, 5>0 are unknown variables. Nous of (X12X29 .... 27m) T= 7/19 /29-- 7/m) L(T= 1/19/29...(1) /m) = lete know that 1, 1/2... I'm ard 21, 22. ... Xn in sortel horm

.. The joint frobability of X, 1x, 2, ... , Xm, Y19729--- In is the same of the frombility of Xighten. [ [ Xexx3--62x dx] ] . . . [ [ [ Xexx3-62x dx] ] = 人(メノノショーラメか) From Dywe get -((X1,1/29,-,,Xm)T)= ((X1,9/2,...Xm)



In the above attached images, we stated that-

$$f(X_1, X_2, ..., X_n | T) = \frac{f(X_1, X_2, ..., X_n)}{f(T = Y_1, Y_2, ..., Y_n)} = \frac{1}{n!}$$

As given as a hint in the question prompt, this is the case because we are trying to compute the probability of  $X_1, X_2, ..., X_n$  occurring in a particular order given they occurred in the sequence

 $Y_1, Y_2, ..., Y_n$  such that  $Y_1 \le Y_2 \le ..., \le Y_n$ . This probability is  $\frac{1}{n!}$ . This is because there are **n! possible permutations** and each of these permutations is equally likely. This gives us the probability of  $\frac{1}{n!}$ 

This is independent of both  $\mu$  and  $\sigma^2$  (the parameters). So, the statistic  $T=Y_1,Y_2,...,Y_n$  is sufficient for given  $\mu$  and  $\sigma^2$ .