Q4 $Ax = b$

Let $A \in R^{m\times n}$, $x \in R^n$ $b \in R^m$

Let $A = \begin{bmatrix} | & | & | & & | \\ a_1 & a_2 & a_3 & \cdots & a_n \\ | & | & | & & | \end{bmatrix}$

where $a_1, a_2 \ldots a_n \in R^m$.

$$Ax = \begin{bmatrix} | & | & | & & | \\ a_1 & a_2 & a_3 & \cdots & a_n \\ | & | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= x_1 \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} + x_2 \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} \cdots x_n \begin{bmatrix} | \\ a_n \\ | \end{bmatrix}$$

$\Rightarrow$ $Ax \in$ column space of (A) since it is a linear combination of A's column vectors.

for $b = Ax$ $b \in$ colum space of A

$\Rightarrow$ for $Ax = b$, to have a solution existing

$b \in$ column space (A)

for $Ax = b$ to have a unique solution if $Ax = b$ has a solution The following problem should have a unique sol^n

$$x_1 \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} + x_2 \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} \cdot x_n \begin{bmatrix} | \\ a_n \\ | \end{bmatrix} = b$$

we know that if $\begin{bmatrix} | \\ a_1 \\ | \end{bmatrix}, \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} \cdots \begin{bmatrix} | \\ a_n \\ | \end{bmatrix}$

$\Rightarrow$ the columns of A must be independent

$\Rightarrow$ columns of A form a basis of column space of A

$\Rightarrow$ If $b \in$ col. space (A) $\Rightarrow$ sol^n exists

& if column of A are a basis of col. space(A)

$\Rightarrow$ sol^n is unique.