Q7 given a matrix $A \in R^{n \times n}$, A is invertible

Let $A = LU$ where U is a upper triangular matrix & L is a lower triangle Matrix

we multiply A with a matrix L_{ij} which makes The j^{th} row's j^{th} element zero.
doing this procedure for all $i \to [1 \to n]$
& $j\ [2 - i-1]$
will create a upper triangular matrix.

First we multiply with L_{21} which makes the a_{21} elements which means subtracting a multiple of row 1 from row 2 to make $a_{21} = 0$

similarly we can make $a_{31}, a_{41}, a_{n1} \ldots$ zero using $L_{21}, L_{31}, L_{41} \ldots L_{n1}$

~~The~~ We can similarly make $a_{32}, a_{42} \ldots a_{n-1\,2}$ zero.

This procedure is Repeated to form U.

$\cancel{L_{n-1\,n}\ L_{n-2}}$

$$L^{-1} A = U$$

$$L^{-1} = (L_{n(n-1)} \cdots L_{n2} \cdots L_{42} L_{32} L_{n1} \cdots L_{31} L_{21})$$

The matrix L_{ij} only changes the row i of a matrix and as a result a_{ij} becomes zero.

$$
\begin{array}{cc}
L_{ij} \longrightarrow & \begin{bmatrix} 1 & & & & & \\ 0 & 1 & & & & \\ 0 & 0 & 1 & & & \\ 0 & 0 & & 1 & & \\ 0 & 0 & & \times & 1 & \\ 0 & 0 & & & & 1 \end{bmatrix} \\
i \rightarrow \text{(row)} & \uparrow \\
 & j \text{ th column.}
\end{array}
$$

consider column j

$$
\begin{bmatrix} y_{1j} \\ y_{2j} \\ y_{3j} \\ \vdots \\ y_{nj} \end{bmatrix} \xrightarrow{L_{ij}} \begin{bmatrix} y_{1j} \\ y_{jj} \\ 0 \\ \vdots \\ y_{nj} \end{bmatrix}
$$

$$0 = y_{ij} + \frac{-(y_{ij})}{y_{jj}} y_{jj}$$

$$\Rightarrow L_{ij} \text{'s } (i,j)^{th} \text{ elements} = \frac{(-y_{ij})}{y_{jj}}$$

for eg $\rightarrow$ $A = \begin{bmatrix} 1 & 0 & 9 \\ 3 & 2 & 5 \\ 7 & 6 & 2 \end{bmatrix}$

$$L_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L_{21} A = \begin{bmatrix} 1 & 0 & 9 \\ 0 & 2 & -22 \\ 7 & 6 & 2 \end{bmatrix}$$

$$L_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow L_{31} A' = \begin{bmatrix} 1 & 0 & 9 \\ 0 & 2 & -22 \\ 0 & 6 & -61 \end{bmatrix}$$

$$L_{32} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$$L_{32} A'' \longrightarrow \begin{bmatrix} 1 & 0 & 9 \\ 0 & 2 & -22 \\ 0 & 0 & 5 \end{bmatrix}$$

hence $U = \begin{bmatrix} 1 & 0 & 9 \\ 0 & 2 & -22 \\ 0 & 0 & 5 \end{bmatrix}$

$$L = (L_{21}^{-1} L_{31}^{-1} L_{32}^{-1})$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 7 & 3 & 1 \end{bmatrix}$$