Q2 $x \in R^n$ $\quad 1_n \equiv \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

$\text{avg}(x) : R^n \longrightarrow R$

such that let $x \in R^n = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$$\text{avg}(x) = \frac{(x_1 + x_2 \cdots x_n)}{n}$$

$$= \frac{1}{n} [1 \; 1 \; 1 \; 1 \; 1 \; 1 \cdots 1] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \frac{1}{n} 1_n^T x$$

$$\text{std}(x) = \frac{\|x - \text{avg}(x) 1_n\|_2}{\sqrt{n}}$$

$$= \frac{\|x - \text{avg}(x) \cdot 1_n\|_2}{\sqrt{n}}$$

To Prove

(a) $\text{avg}(\alpha x + \beta 1_n) = \alpha \, \text{avg}(x) + \beta$

$$\text{avg}(\alpha x + \beta 1_n) = \frac{1}{n} (1_n^T (\alpha x + \beta 1_n)$$

$$= \frac{\alpha}{n} 1_n^T x + \frac{\beta}{n} 1_n^T 1_n$$

$1_n^T 1_n = n$

$$\Rightarrow \cancel{\alpha}\ \operatorname{avg}(\alpha x + \beta 1_n) = \alpha\left(\frac{1}{n} 1_n^T x\right) + \beta$$

$$= \alpha \operatorname{avg}(x) + \beta$$

(b) $\operatorname{std}(\alpha x + \beta 1_n)$

$$= \frac{\|\alpha x + \beta 1_n - \operatorname{avg}(\alpha x + \beta 1_n) 1_n\|_2}{\sqrt{n}}$$

$$= \frac{\|\alpha x + \cancel{\beta 1_n} - (\alpha \operatorname{avg}(x) 1_n + \cancel{\beta 1_n})\|_2}{\sqrt{n}}$$

$$= \frac{\|\alpha x - \alpha \operatorname{avg}(x) 1_n\|_2}{\sqrt{n}}$$

$$= \frac{|\alpha| \|x - \operatorname{avg}(x) 1_n\|_2}{\sqrt{n}}$$

$$= |\alpha| \operatorname{std}(x)$$

($\|\alpha x\|_2$ operator $= \alpha \|x\|_2$)