Q5 To Prove $A(BC) = (AB)C$

Let $BC = P$

let $A \in R^{m \times p}$, $B \in R^{p \times q}$

$C \in R^{q \times n}$.

$\Rightarrow$ (i,j) entry in $A(BC) = AP$

is i^{th} row of A * j^{th} ~~row~~ column of P

$$\underbrace{i^{th} \text{ row of } A}_{a_i} \quad \underbrace{j^{th} \text{ column of } P}_{P_j}$$

$=$

$$a_i P_j = A_{i1} P_{1,j} + A_{i2} P_{2j} \cdots \cdots A_{ip} P_{pj}$$

$P_j = BC_j$ Thus

$$P_{xj} = B_{x1} C_{1j} + B_{x2} C_{2j} \cdots \cdots B_{sq} C_{qj}.$$

$$\Rightarrow a_i P_j = A_{i1}(B_{11} C_{1j} + B_{12} C_{1j} \cdots B_{1q} C_{qj})$$

$$+ A_{i2}(B_{21} C_{1j} + B_{22} C_{2j} \cdots B_{2q} C_{qj})$$

$$\vdots$$

$$A_{ip}(B_{p1} C_{1j} + B_{p2} C_{2j} \cdots B_{pq} C_{qj})$$

$$= ((A_{i1} B_{11}) + (A_{i2} B_{21}) \cdots (A_{ip} B_{p1})) \cdot C_{qj}$$

$$+ \vdots$$

$$= (A_{i1} B_{11} + A_{i2} B_{21} \cdots A_{ip} B_{p1}) C_{1j}$$

$$+$$

$$(A_{i1} B_{12} + A_{i2} B_{22} \cdots A_{pp} B_{p2}) C_{2j}$$

$$+$$

$$(A_{i1} B_{1q} + A_{i2} B_{2q} \cdots A_{ip} B_{pq}) C_{qj}$$

$$= (a_i B_1) C_{1j} + (a_i B_2) C_{2j} \cdots (a_i B_q) C_{qj}$$

let $D = AB$

$$\Rightarrow D_{ir} = (a_i B_r)$$

$$\Rightarrow (i,j) \text{ of } (AB) C = DC =$$

$$D_{i1} C_{1j} + D_{i2} C_{2j} \cdots D_{iq} C_{qj}$$

$$= (a_i B_1) C_{1j} + (a_i B_2) C_{2j} \cdots (a_i B_q) C_{qj}$$

$$= a_i P_j$$

$\Rightarrow (i,j)^{th}$ entry of $A(BC) = (i,j)^{th}$ entry of $(AB) C$

for $\forall i \in [1 \ldots m], j \in [1 \ldots n]$

$\Rightarrow$ Matrix multiplication is ~~commutative~~ associative.

(b) let $A = \begin{bmatrix} 5 & 7 \\ -3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix}$

$$AB = \begin{bmatrix} 68 & 3 \\ -3 & -6 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 7 \\ 48 & 63 \end{bmatrix}$$

$\Rightarrow AB \neq BA$

$\Rightarrow$ Matrix multiplication is not commutative

$A \in R^{p \times q}$ $B \in R^{q \times r}$, $C \in R^{r \times t}$

$(AB)C \rightarrow$ <u>no. of</u> computation for $A \times B$
$\qquad = p \times (2q-1) r$

There will be q multiplication + $(q-1)$ additions for $(AB)_{ij}$

$=$ Total computation $=$ ~~pqr~~ $p(2q-1)r$

for $(AB) \times C \rightarrow$ Total computations

$= p \times (2r-1)\, t$

$\Rightarrow$ $(AB)C$ has ~~tot~~ al

$p(2q-1)\, r$

$+ p(2r-1)\, t$

computations.

similarly

$A(BC)$ has $= q(2r-1)t + p(2q-1)t$

$\Rightarrow$ $p(2q-1)r + p(2r-1)t < q(2r-1)t$

$+ p(2q-1)t$

$\Rightarrow$ $\approx pqr + prt < qrt + pqt$

$\approx \dfrac{1}{t} + \dfrac{1}{q} < \dfrac{1}{p} + \dfrac{1}{r}.$