Q 6 let $A \in R^{m \times n}$

Then B is defined as a left inverse of A
$\in R^{n \times m}$

if $BA = I^{n \times n}$

for a left inverse of A to exist
A's column should be linearly independent

(i) $A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \in R^{5 \times 1}$

since A's colums are independent (only 1 colum)
and The column $\neq 0$
$\Rightarrow$ ~~A is in~~ A has a left inverse.

eg $[1/2 \; 0 \; 0 \; 1/2 \; 0]^T$ is a left inverse
of A

Let x be a ~~fe~~ left inverse of A
let Y be the set of All matrices such that
$yA = 0 \;\forall\; y \in Y$

Let $x = [a, b, c, d, e]$

where $a, b, c, d, e \in R$

for $xA = I$

$\Rightarrow a + d = \cancel{0}\ 1$

$=$ Any left inverse of A is of the form

$[a, b, c, 1-a, e]$

for $yA = 0$ $\quad [a', b', c', d', e'] \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 0$

$\Rightarrow a' = -d'$

$\Rightarrow$ y is of the form $[a', b', c', -a', e']$

Then any ~~vector~~ matrix $B \in R^{1 \times 5}$ such that $BA = I$

$B = [b_1\ b_2\ b_3\ 1-b_1\ b_5]$

can be represented as $x + y$

where $y = [b_1 - x_1, b_2 - x_2\ b_3 - x_3\ x_1 - b_1, b_5 - x_5]$

& $yA = 0$

(b) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \\ 3 & 3 \end{bmatrix}$

let $a_1 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$, $a_2 = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$

for a_1, a_2 to be dependent

$$a_1 x_1 + a_2 x_2 = 0$$

for atleast one of $x_1 ; x_2 \neq 0$

$$\Rightarrow \begin{bmatrix} 2x_1 \\ 0 \\ 3x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -2x_2 \\ 3x_3 \end{bmatrix} = 0$$

$\Rightarrow x_1 = 0, \; x_2 = 0$

$\Rightarrow a_1, a_2$ are linearly independent.

$\Rightarrow$ Inverse exists

eg $\rightarrow$ $\begin{bmatrix} 1 & -1/2 & -1/3 \\ 0 & -1/2 & 0 \end{bmatrix}$ is a left inverse.

let $X = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$

if $XA = I \Rightarrow 2a+3e = 1, \quad 2b+3f=0$
$-2c+3e = 0 \quad -2d+3f = 1$

$\Rightarrow X$ is of the form

$$\begin{bmatrix} a & \frac{(1-2a)}{2} & (1-2a)/3 \\ b & -\frac{(1+2b)}{2} & -\frac{2b}{3} \end{bmatrix}$$

if $YA = 0 \Rightarrow Y$ is of the form

$$\begin{bmatrix} a & -a & -2a/3 \\ b & -b & -2b/3 \end{bmatrix}$$

$\Rightarrow$ Any $z \in$ Set of All inverse.

can be formed as $(x+y)$

where $xA = I, \quad yA = 0$