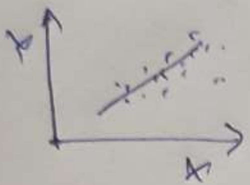
11T2023506. Assignment-2 Parth Agrawal

Deduce the LR model in the form for two parameter predictor and then proceed to solve the following assignment



$$h_w(x) = \underbrace{w_0 x_0}_{\text{fictitious feature}} + w_1 x_1$$

$$h_w(x) = w_0 + w_1 x_1$$

1) Let us consider m samples (x_i, y_i)
$i = 1, 2, 3 \ldots m$ rather we are. $(x^{(i)}, y^{(i)})$

2. Define an objective f^n. $J(w_0, w_1)$ could be an error prediction like. $J(w_0, w_1) = \sum_{i=1}^{m} \left(h_w(x^{(i)}) - y^{(i)}\right)^2$

For m samples.

$$J(w_0, w_1) = \left(w_0 + w_1 x^{(1)} - y^{(1)}\right)^2 + \left(w_0 + w_1 x^{(2)} - y^{(2)}\right)^2 + \ldots + \left(w_0 + w_1 x^{(m)} - y^{(m)}\right)^2$$

To minimize
$J(w_0, w_1)$ find.

$$\frac{\partial J(w)}{\partial w_0} = 2 \cdot \left\{ \left(w_0 + w_1 x^{(1)} - y^{(1)}\right) + \ldots \ldots \left(w_0 + w_1 x^{(m)} - y^{(m)}\right) \right\}.$$

$$\Rightarrow m w_0 + \sum_{i=1}^{m} x^{(i)} - \sum_{i=1}^{m} y^{(i)} = 0 \quad (1)$$

$$\frac{\partial J(w)}{\partial (w_1)} = 2 \left\{ \left(w_0 + w_1 x^{(1)} - y^{(1)}\right) x^{(1)} + \ldots \ldots \left(w_0 + w_1 x^{(m)} - y^{(m)}\right) x^{(m)} \right\}.$$

$$= w_0 \sum_{i=1}^{m} x^{(i)} + w_1 \sum_{i=1}^{m} \left(x^{(i)}\right)^2 - \sum_{i=1}^{m} x^{(i)} y^{(i)} = 0.$$

$$- (3)$$

Reformat (2) & (3)

$$A = \sum_{i=1}^{m} x^{(i)} \qquad B = \sum_{i=1}^{m} y^{(i)} \qquad C = \sum_{i=1}^{m} (x^{(i)})^2$$

$$D = \sum_{i=1}^{m} x^{(i)} y^{(i)}$$

$$m w_0 + A w_1 = B \quad -(4)$$
$$A w_0 + C w_1 = D \quad -(5)$$

$$w_0 + \frac{A}{m} w_1 = \frac{B}{m}$$
$$w_0 + \frac{C}{A} w_1 = \frac{D}{A}$$
$$- \qquad - \qquad -$$

$$w_1 \left(\frac{A}{m} - \frac{C}{A} \right) = \frac{B}{m} - \frac{D}{A}$$

$$\boxed{w_1 = \frac{AB - Dm}{A^2 - Cm}} \qquad \boxed{w_0 = \frac{AD - BC}{A^2 - Cm}}$$

$$w_1 = \frac{\sum_{i=1}^{m} x^{(i)} \sum_{i=1}^{m} y^{(i)} - m \sum_{i=1}^{m} x^{(i)} y^{(i)}}{\left(\sum_{i=1}^{m} x^{(i)} \right)^2 - m \sum_{i=1}^{m} (x^{(i)})^2}$$

$$w_0 = \frac{\sum_{i=1}^{m} x^{(i)} \sum_{i=1}^{m} x^{(i)} y^{(i)} - \sum_{i=1}^{m} y^{(i)} \sum_{i=1}^{m} (x^{(i)})^2}{\left(\sum_{i=1}^{m} (x^{(i)}) \right)^2 - m \sum_{i=1}^{m} (x^{(i)})^2}$$