DISCRETE STRUCTURES FOR COMPUTER SCIENCE
(CS F222)

ASSIGNMENT- 4

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A. Sub-Tree Weights

You are given a tree consisting of N vertices. Your task is to write a simple program to find the node that has minimum sub-tree weight. *Minimum sub-tree weight* of a node **X** is defined as the sum of weights of all the nodes in the subtree of X (rooted at X). *Leaf nodes are also considered subtrees by themselves.*

Input

The first line contains a single integer N ($1 \le N \le 150$) denoting the number of vertices in the given tree. The following N-1 lines contain two integers U_i and V_i ($0 \le U_i, V_i \le N-1$) each denoting a directed edge between vertices U_i and V_i. The last line contains N space separated integers $W_0, W_1, \ldots W_{N-1}$ ($-10^5 \le W_i \le 10^5$) with the $\mathbf{i^{th}}$ element denoting the weight of the $\mathbf{i^{th}}$ node. *The tree is rooted at 0.*

Output

Print a pair integers denoting the number of the vertex which is the root of the subtree which has the minimum sub-tree weight and also the weight of the corresponding subtree. *In case of multiple answers, print any.*

---

input

```
6
0 1
0 5
1 2
2 3
1 4
10 1 -2 -7 -1 2
```

output

```
1 -9
```

---

input

```
8
0 1
0 2
0 5
1 6
2 3
2 4
5 7
100 -2 -2 5 6 -9 -1 -4
```

output

```
5 -13
```

B. Experiment

Leonard Hofstader has recently designed an experiment to prove Sheldon's theories. The experiment involves a few nodes and an interaction field between a few of them. The interaction field between the nodes consumes a lot of power. Hence, Leonard asks you, Howard, to remove a few interaction fields between those nodes, such that the power consumed of the system is *minimum* and *all the nodes are connected by the interaction fields.*

Input

The first line contains a single integer N ($2 \le N \le 10^4$), M ($1 \le M \le \frac{N(N-1)}{2}$) denoting the number of nodes and the number of interaction fields between those nodes. The following M lines contain three space-separated integers U_i, V_i and E_i ($0 \le U_i, V_i \le N-1$; $1 \le E_i \le 10^5$) each denoting an interaction field (bidirectional) between U_i and V_i and the power consumed by the corresponding interaction field respectively. *It is guaranteed that the graph is connected and does not contain loops or multiple edges.*

Output

Print a single integer denoting the minimum power consumed by the system after all modifications. Also, print the interaction fields present in the system after modifications. *The edges can be printed in any order*. If there are multiple answers, print any.

input

```
6 11
0 1 10
1 2 11
2 5 12
5 4 1
4 3 2
3 0 5
0 4 1
3 1 10
1 5 2
2 4 5
1 4 1
```

output

```
10
0 4
1 4
3 4
2 4
5 4
```

C. Tough Question

Koro-sensei has planned yet another tough assignment for the students of class 3E. He gives them a binary tree and asks them to find the subtree that has *maximum weight and is a Binary Search Tree as well*. Nagisa is quite stuck with this assignment and has approached you, Karuma, for help. Help Nagisa find the solution. *The weight of a (binary) sub-tree is defined as the sum of weights of the corresponding root, left and right subtrees. Leaf nodes are not considered as subtrees by themselves in this case, i.e. the subtree must consist of atleast two nodes.*

`BST property`: *The data in the left child (if it exists) should be strictly less than that of the parent, and the data of the right child (if it exists) should be strictly greater than that of the parent.*

Input

The first line contains a single integer N ($2 \leq N \leq 1024$) denoting the number of vertices in the tree. The following N-1 lines contain two space-separated integers U_i , V_i, and a character C_i ($0 \leq U_i$, $V_i \leq N$-1; $C\ i \in \{L, R\}$) each denoting child node V_i is to the right of the parent node U_i if C_i is **R** or to the left if C_i is **L**. The last line of input has N space-separated integers W_0, W_1, … W_{N-1} ($-10^5 \leq W_i \leq 10^5$) where the $\mathbf{W_i}$ denotes the weight of the $\mathbf{i^{th}}$ vertex. *The tree is rooted at 0.*

Output

Print a single integer denoting the vertex number of the node, which is the root of the subtree that has maximum weight and is a BST as well. If multiple answers exist, print any. In case of no answer, print "No such node".

---

input

```
6
0 5 R
0 2 L
5 4 L
5 1 R
4 3 R
10 5 7 -2 -1 2
```

output

```
No such node
```

---

input

```
7
0 5 R
0 2 L
5 1 L
5 4 R
2 6 L
2 3 R
12 3 10 14 5 6 -1
```

output

```
2
```

D. Swapped Nodes

Leonard bought Sheldon a Binary Search Tree for his birthday. However, Penny decided to prank Sheldon and swapped the data of two nodes on the BST. On receiving the tree, Sheldon immediately saw through the prank and asked Penny to set right the tree (swap the nodes back). Penny having long forgotten the two nodes she had swapped, asks you, Raj, to set it right as soon as possible.

<u>`BST property`</u>`:` *The data in the left child (if it exists) should be strictly less than that of the parent, and the data of the right child (if it exists) should be strictly greater than that of the parent.*

Input

The first line contains a single integer N ($2 \le N \le 1024$) denoting the number of vertices in the tree. The following N-1 lines contain two space-separated integers U_i , V_i, and a character C_i ($0 \le U_i , V_i \le$ N-1; C i ∈ {L, R}) each denoting child node V_i is to the right of the parent node U_i if C_i is **R** or to the left if C_i is **L**. The following line has N separated integers A_0, A_1, … A_{N-1} ($-10^5 \le A_i \le 10^5$) with A_i denoting the data in the $\mathbf{i^{th}}$ vertex of the BST. *The tree is rooted at 0 and it is guaranteed that the data of all the nodes will be unique.*

Output

Print two integers X, Y denoting the vertex numbers of the nodes whose data were swapped.

---

input

```
7
0 5 L
0 1 R
5 2 L
1 3 L
1 4 R
5 6 R
5 100 -2 91 3 201 4
```

output

```
4 5
```

explanation

```
We can see that on constructing the tree, nodes 4 and 5 do not follow the BST principles. On swapping them again, the BST will be correct.
```

E. Traversals

You are given a the inorder and pre-order traversals of a binary tree. Can you come up with a simple solution that will allow you to find the children nodes of any node queried?

Input

The first line contains a single integer N ($2 \le N \le 1024$) denoting the number of vertices in the tree. The next two lines constains N space-separated integers A_0, A_1, … A_{N-1} ($0 \le A_i \le N-1$) and B_0, B_1, … B_{N-1} ($0 \le B_i \le N-1$) denoting the inorder and pre-order traversal respectively. The next line contains a single integer Q ($2 \le N \le 1024$) denoting the number of queries. The following Q lines contain a single integer $\mathbf{X_i}$ denoting the node being queried. *The tree is rooted at 0.*

Output

For each query print two integers U, V denoting the left and right children of the queried node. If any child node does not exist, print **X** (see the sample case for better understanding). *Note that U and V have to printed in the correct order; U denotes the left child and V denotes the right child.*

---

input

```
11
6 5 1 7 2 3 0 4 9 8 10
0 1 5 6 2 7 3 4 8 9 10
4
5
1
7
4
```

output

```
6 X
5 2
X X
X 8
```

F. Project Submission

Arman lives in Armenia which has N cities and M bidirectional roads and for his Geography projects he was asked to study the road system in the country. Arman decides to model the country's road system as an undirected weighted graph connecting the cities with the roads. To complete the analysis, Arman needs to determine the sum of the minimum distances between each pair of cities. Help Arman complete his Geography assignment.

Input

The first line contains a single integer N ($2 \le N \le 150$), M ($1 \le M \le \frac{N(N-1)}{2}$) denoting the number of cities and the number of bidirectional roads between those cities. The following M lines contain three space-separated integers U_i, V_i and S_i ($0 \le U_i, V_i \le N-1$; $1 \le S_i \le 10^5$) each denoting a road between U_i and V_i of length S_i.

Output

Print a single integer denoting the sum of minimum distances between each pair cities.

input

```
5 6
0 2 5
3 4 1
1 0 3
2 1 1
3 2 4
3 1 2
```

output

```
32
```

G. Cabbages

Shin Chan comes by a cabbage patch which was modelled after a binary tree. Standing at the root node (which is always node 0) he decides to cross the patch by doing a *zigzag traversal* i.e. *from left to right, then right to left for the next level and so on* (see the sample case for better understanding). Help Shin Chan figure out the route.

Input

The first line contains a single integer N ($2 \le N \le 1024$) denoting the number of vertices in the tree. The following N-1 lines contain two space-separated integers U_i , V_i, and a character C_i ($0 \le U_i , V_i \le N-1$; $C\ i \in \{L, R\}$) each denoting child node V_i is to the right of the parent node U_i if C_i is **R** or to the left if C_i is **L**.

Output

Print a sequence of integers denoting the path Shin Chan must follow (see the sample case for better understanding). *Note that, in this question order matters, and each level must be printed correctly in the correct order as followed by Shin Chan.*

---

input

```
5
0 1 L
0 2 R
2 3 L
2 4 R
```

output

```
0
1 2
4 3
```

H. Homework

As a part of his English assignment, Shizuka was given a list of words. Her task is to find the shortest unique prefix to represent each of the words. Help Shizuka solve the assignment.

Input

The first line contains a single integer N ($2 \le N \le 1024$) denoting the number of strings Shizuka has for her homework. Each of the following N lines have a string $\mathbf{S_i}$ ($1 \le |S_i| \le 10^4$) denoting the $\mathbf{i^{th}}$ word of her homework.

Output

Print N lines, where the $\mathbf{i^{th}}$ line as the smallest unique prefix used to represent the $\mathbf{i^{th}}$ string of her homework.

---

input

```
4
zebra
dog
duck
dove
```

output

```
z
dog
du
dov
```

explanation

For the first string, z is enough as it can uniquely represent the word ‘zebra’. To represent the second and fourth string, we need 3 characters in the prefix as we cannot represent them uniquely in under two characters (*do* and *do* is common in both).
