

DISCRETE STRUCTURES FOR COMPUTER SCIENCE  
(CS F222)

**ASSIGNMENT-1**

TEST TIME: 22-09-2019, 9:00AM – 1:00PM

CONTACT DETAILS:

- |                      |               |
|----------------------|---------------|
| • VAMSI NALLAPAREDDY | 2017A7PS0018H |
| • VARUN GUMMA        | 2017A7PS0165H |

## A. Dad's Gift

Your father gifted you an Amazon Gift Card worth  $X$  dollars. The card is redeemable only once, i.e. you can only use it on one cart. If the cart total is less than the card's worth, you lose the surplus amount.

There are multiple items on your wishlist. You want to buy some (possibly none or all) items from the wishlist using only the gift card by wasting as little money as possible. Determine the best possible subset of items.

### Input

The first line contains two space separated integers  $N$  ( $1 \leq N \leq 22$ ), the number of items in your wishlist and  $X$  ( $1 \leq X \leq 10^9$ ), the worth of the gift card.

The next line contains  $N$  space separated integers  $A_1, A_2, \dots, A_N$  ( $1 \leq A_i \leq 10^9$ ) where the  $A_i$  denotes the price of  $i^{\text{th}}$  item in your wishlist.

### Output

In the first line, print a single integer  $K$  ( $1 \leq K \leq N$ ) denoting the number of items you want to buy.

In the next line, print  $K$  unique space separated integers  $P_1, P_2, \dots, P_K$  ( $1 \leq P_i \leq N$ ) where  $P_i$  denotes the wishlist index of the item you want to buy, such that  $X - \sum A_{p_i}$  is minimum possible. If there are multiple possible answers, print any.

---

#### input

```
6 44
14 13 21 8 56 3
```

#### output

```
3
1 4 3
```

#### explanation

$A_1 + A_4 + A_3 = 43$ .  $44 - 43 = 1$  dollar wasted. You can see that no other subset has lesser wastage.

---

#### input

```
6 1000000000
1000000000 1000000000 1000000000 1000000000 1000000000 1000000000
```

#### output

```
1
2
```

---

## B. Course Registration

Rohan has just joined MIT and wants to take his favourite course(s) in the future. He notices that some courses have a few prerequisite courses which must be completed first, in order to be able to register to that course (See sample cases for clarity). Rohan has short-listed the courses he needs to do, but he is unable to figure out a correct order of completing those courses. Can you help him get a right order?

### Input

The first line contains a single integer  $N$  ( $1 \leq N \leq 10$ ), the number of courses shortlisted by Rohan. The next line contains  $N$  space separated integers  $A_1, A_2, \dots, A_N$  ( $1 \leq A_i \leq 10^9$ ) where the  $A_i$  denotes the course ID of  $i^{\text{th}}$  course he has shortlisted.

The next line contains a single integer  $M$  ( $1 \leq M \leq \frac{N(N-1)}{2}$ ), denoting the number of prerequisite-course pairs.

Each of the next  $M$  lines contains two space separated integers  $P_i, C_i$  ( $1 \leq P_i, C_i \leq 10^9$ ) which indicates course having ID  $P_i$  is the prerequisite of the course having ID  $C_i$ . Note that a course may have none or multiple prerequisites.

### Output

Output a single line of  $N$  space-separated integers  $B_1, B_2, \dots, B_N$  where  $B_i$  is the ID of the  $i^{\text{th}}$  course such that all the prerequisites of course  $B_i$  are completed before it. Formally, *for every*  $1 \leq i < j \leq N$ ,  $B_j$  must not be a prerequisite of  $B_i$ . If there are multiple answers, print any one answer.

---

### input

```
7
9 6 5 4 3 1 2
6
4 9
6 5
5 3
3 9
1 4
1 3
```

### output

```
1 2 6 5 3 4 9
```

### explanation

In the above case, courses bearing ID 1, 2 and 6 don't have any prereqs, and the above answer satisfies the conditions that a course is completed only after its prereq(s) is/are completed before.

## C. Bombed!

The Spinx brothers have planted a bomb in Tokyo's Metropolitan Police Headquarters. Quite conveniently, they left behind a string  $S$  (encrypted, of course) as the key to defuse the bomb, but there is more to it.

The police from their previous encounters with the Sphinx brothers have deduced a possible decryption. Some of the letters in the string need to be replaced with some other letter in accordance with the decryption pattern. It is possible that a decrypted letter can be decrypted/replaced again. You need to replace each letter in the string until it cannot further be replaced, to obtain the key. They challenge you to do this as quickly as possible, as the bomb in the headquarters is about to explode any time now.

### Input

The first line has the string  $S$  ( $1 \leq |S| \leq 10^5$ ) consisting of lowercase letters only. The next line has a single integer  $M$  ( $1 \leq M \leq 325$ ), denoting the number of replacements. The next  $M$  lines have two spaced characters,  $c_i$  and  $d_i$ , with  $d_i$  being the replacement of  $c_i$ .

### Output

Print the final string after all character replacements.

### Note

It is guaranteed that there are no cyclic replacements.

---

#### input

```
nineandeleven
5
n l
l u
u w
e a
a y
```

#### output

```
wiwywdywyvyw
```

---

## D. Class Assignment

Koro-sensei has recently taught relations to class 3E, and decided to conduct an assignment based on it for them. He prepared a binary relation  $\mathbf{R}$  on  $\{1, 2, 3 \dots N\}$  such that  $\mathbf{R} \subseteq \{1, 2, 3 \dots N\} \times \{1, 2, 3 \dots N\}$  and asked his students to judge if the given relation is reflexive, symmetric, transitive and hence if it is an equivalence.

With  $N$  being quite high, you decided to write a simple program to do the task for you and your classmates. So, can you come up with a basic algorithm and implementation to solve the problem?

### Input

The first line has two space separated integers  $N$  ( $1 \leq N \leq 500$ ), denoting the number of elements in the domain as well as the codomain, and  $M$  ( $1 \leq M \leq N^2$ ) denoting the number of elements in the relation. The next  $M$  lines have two space separated integers  $X$  and  $Y$  ( $1 \leq X, Y \leq N$ ), such that  $(X, Y) \in \mathbf{R}$ .

### Output

Output the type of relation i.e. *Reflexive*, *Symmetric*, *Transitive* or *Equivalence* (Print all satisfied conditions). If it doesn't satisfy any conditions, print -1.

---

#### input

```
4 8
1 1
3 4
3 1
3 3
4 4
1 3
4 3
2 2
```

#### output

```
Reflexive
Symmetric
```

---

#### input

```
3 2
1 3
2 2
```

#### output

```
Transitive
```

---

## E. Bakugo Vs Midoriya

All Might has planned training excercises for the first year students at UA High. He decided to divide the  $N$  students ( $N$  is even), into teams of two. Bakugo, however, refuses to be in the same team as Midoriya. Even after the relentless efforts of All Might, he doesn't seem to agree.

As their homeroom teacher, Aizawa, your task is to divide the students into two teams, with Bakugo and Midoriya as their respective captains. (See the sample case for better understanding)

### Input

The first line contains a single integer  $N$  ( $1 \leq N \leq 26$ ), denoting the number of students in UA High's first year. The next line has  $N$  unique space separated uppercase letters denoting the names of students (M for Midoriya, B for Bakugo and so on. *These two are certain to be in every input*).

### Output

Print all possibilities of forming the teams, *one for Midoriya and one for Bakugo*, i.e. **two teams**. *Make sure that both teams have equal number of members, and avoid duplicacy in the teams.*

---

#### input

6

Y M I B S H

#### output

M: Y I | B: S H

M: Y S | B: I H

M: Y H | B: S I

M: S H | B: Y I

M: S I | B: Y H

M: I H | B: Y S

---

## F. Chunin Exams

The Chunin Exams were approaching and the students were busy practicing their skills. At this point, Kakashi Sensei decides to test the wit of his three disciples. He gives them a set of numbers and asks them to report to him all maximal values in the Hasse Diagram of  $(S, \text{divides})$ . Help the trio outwit Kakashi Sensei. *S refers to the set of numbers given by Kakashi.*

### Input

The first line contains a single integer  $N$  ( $1 \leq N \leq 10^5$ ).

The next line contains  $N$  space-separated integers  $A_1, A_2, \dots, A_N$  ( $1 \leq A_i \leq 10^6$ ).

### Output

In the first line, print  $K$ , the number of maximal elements.

In the next line, print  $K$  integers, denoting all the maximal values obtained from the Hasse Diagram of  $(S, \text{divides})$ .

---

#### input

9

2 3 5 8 6 15 10 24 12

#### output

3

10 15 24

---

# G. Steins;Gate

After months of tedious research, Okabe Rintaro was finally able to figure out that all alternate realities (timelines) actually form an N-bit lattice (see the figure below for clarity). Each alternate reality (timeline) is associated with an N-bit binary number. He was also able to conclude that he can move from one timeline to another *if and only if* the corresponding N-bit binary numbers associated with the two realities *differed by one bit*, and the N-bit number associated with the destination timeline is strictly greater than that of the current timeline.

Okabe is now set on repairing the Phone Microwave, and meanwhile has asked you, SuperHacker Daru, to figure out the possible transitions he can make in the timeline lattice.

## Input

The first and only line has a single integer ( $1 \leq N \leq 24$ ), the size of the N-bit binary number associated with each timeline.

## Output

In the first line, print the number of edges in the lattice. In the following  $2^N$  lines, print the timeline he can reach from each timeline (See the example for better understanding).

---

### input

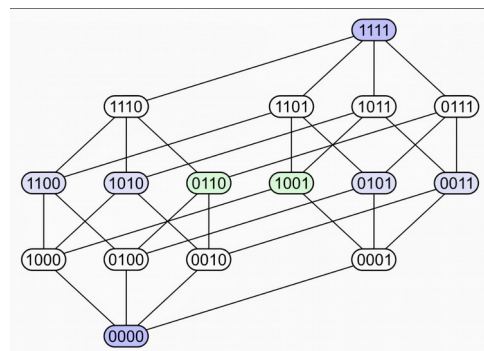
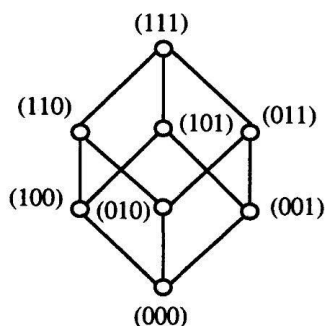
3

### output

12  
0: 1 2 4  
1: 3 5  
2: 3 6  
3: 7  
4: 5 6  
5: 7  
6: 7  
7:

### explanation

In the above case, Okabe can reach timelines 1, 2, or 4 from timeline 0, and can reach only timeline 7 from timeline 3, and cannot make any transitions from timeline 7 (as shown in the figure).





# H. Rotations

Write a basic program to check and print the words in a sentence which are rotations of a palindrome. For example, rotation of “qwerty” can be “ertyqw” or “yqwert” and so on.

## Input

The first line would contain the sentence string S ( $1 \leq |S| \leq 1000$ ).

## Output

Identify those words in the sentence which are rotations of a palindrome and print the corresponding palindromes. *If no such words exist in the sentence which satisfy the given condition, print an empty line (blank output).*

---

### input

redivider and ecarrac mean ammad

### output

redivider racecar madam

---

### input

just some random input

### output

---

# I. Key

As Alice nears the underground lab of Umbrella Corporation located in Raccoon City, she realizes that the door is locked. She notices a piece of paper with the Umbrella Corporation with a string comprising of alphabets and digits scribbled on it.

She quickly realizes that a permutation of these alphabets and digits could be a possible password. Help Alice out by coming up with all possible permutations of the string.

## Input

The first line would contain the sentence string  $S$  ( $1 \leq |S| \leq 10$ )

## Output

Print all possible permutations of the string.

## Note

There won't be any duplicate characters in the given string.

---

### input

abc

### output

abc

acb

bac

bca

cab

cba

---