DISCRETE STRUCTURES FOR COMPUTER SCIENCE
(CS F222)

ASSIGNMENT-3

CONTACT DETAILS:

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A. No Game No Life

Sora and Shiro have finally collected all the race pieces and are ready to go all out against Tet. But Tet has an ace up his sleeve before they can challenge him. He teleports Sora and Shiro onto two vertices of a rooted tree. They want to challenge Tet (who is at the root, which is numbered 0) as soon as possible and can only do this together. Help Sora and Shiro find the *least common ancestor* of their current nodes such that they can challenge Tet quickly.

The least common ancestor of two nodes **u** and **v** is defined as the farthest node from the root which lies on the path from root to **u** as well as from root to **v**.

Input

The first line contains a single integer N ($1 \leq N \leq 1024$). The following N-1 lines contain two integers U_i and V_i ($0 \leq U_i, V_i \leq N-1$) each denoting an edge between U_i and V_i. The next line contains a single integer Q ($1 \leq Q \leq 1024$) denoting the number of times Tet teleports Sora and Shiro. Each of the following Q lines contain two integers X_i and Y_i ($0 \leq X_i, Y_i \leq N-1$) representing Sora's and Shiro's nodes respectively.

Output

Print Q lines where the $\mathbf{i^{th}}$ line contains a single integer representing the least common ancestor of X_i and Y_i.

---

input

```
7
0 1
0 2
0 3
2 4
5 2
6 2
4
1 3
4 3
4 5
2 6
```

output

```
0
0
2
2
```

B. Gym Battles

During his travels in the Orange Islands, Ash is perplexed as to what gym to choose next. So he formulates the entire archipelago in the form of a tree rooted at his present island (Island 0). He wishes to visit those islands that are the leaf nodes (the leaf nodes have gyms) of the tree that he has created. Misty jumps in and places a condition that he has to pick up *all the seashells present on each island he visits* and after reaching a leaf node, if the total sum of seashells present with him is equal to the magic number **X**, only then can he participate in a gym battle. Help Ash figure out his options of gyms that he can travel to.

Input

The first line contains a single integer N ($2 \le N \le 64$) denoting the number of islands in the archipelago. The following line contains N-1 space-separated integers P_1, P_2, ... P_{N-1} ($0 \le P_i \le N-1$) where the P_i denotes the parent of the $\mathbf{i}^{\mathbf{th}}$ vertex. The next line contains N space-separated integers A_0, A_1, A_2, ... A_{N-1} ($0 \le A_i \le 10^5$) where the A_i denotes the number of sea-shells that can be obtained on Island **i**. The last line of input contains the magic number X ($0 \le X \le 10^8$)

Output

Print all the possible paths Ash can traverse satisfying the given condition. The paths can be printed in any order, but make sure to print the elements in a path in the correct order.

---

input

```
10
0 1 2 2 0 5 5 7 7
5 4 11 7 2 8 13 4 5 1
22
```

output

```
0 1 2 4
0 5 7 8
```

C. Venetian Traveller

Shylock as a young merchant wanted to visit every single city in Italy. The only issue at hand was that he wanted to save money whilst doing so. He comes to you asking you for help. He adds a constraint that he wants to visit all the cities exactly once and *return back to his residence in city 0*. Help Shylock fulfill his wish of visiting every city in Italy. *It is guaranteed that some path covering all vertices exactly once always exists.*

Input

The first line contains a two space-separated integers N ($1 \le N \le 5$) denoting the number of cities in Italy, M ($1 \le M \le \frac{N(N-1)}{2}$) denoting the number bidirectional roads between those cities. The following M lines contain three space-separated integers U_i , V_i, and C_i ($0 \le U_i, V_i \le N-1$; $1 \le C_i \le 10^5$) each denoting an roadway between cities U_i and V_i with cost C_i.

Output

Print a single integer denoting the minimum cost of the tour you plan for Shylock.

---

input

```
4 6
0 1 20
1 3 25
1 2 30
0 3 10
0 2 15
3 2 35
```

output

```
80
```

explanation

```
The path followed in this case is 0-3-1-2-0, whose total cost
is 80. Note that, returning back to his residence at 0 at the
end is not considered revisiting it.
```

D. Taunts and Trials

Blair taunts Dan saying that he could never be good enough for Serena. Determined to prove him wrong, he asks him how he can prove his worth. Blair gives Dan a Binary Tree and asks him to find all the paths with a total cost of **X**. He mentions that you start out with an initial cost of 0 at the root node. Each edge will have a cost, the left pointing edges have a cost of -1 and those pointing right have a cost of +1. A completely befuddled Dan has pleaded you to help him out.

Input

The first line contains a single integer N ($1 \leq N \leq 1024$) denoting the number of vertices in the binary tree. The following N-1 lines contain two space-separated integers U_i , $V_{i,}$ and a character C_i ($0 \leq U_i, V_i \leq N-1$; $C_i \in \{L, R\}$) each denoting child node V_i is to the right of the parent node U_i if C_i is **R** or to the left if C_i is **L**. The next line contains a single integer Q ($1 \leq Q \leq 1024$) denoting the number queries Blair has for Dan. Each of the following Q lines contains a single integer X ($-10 \leq X_i \leq 10$) representing the cost. *It is given that the root is always numbered 0.*

Output

Print Q lines, where the **ith** line contains the indices of the nodes for which the cost of path from root to that node is $\mathbf{X_i}$ (**ith** query). *The indices can be printed in any order*. If a certain query cost does not return any nodes, print "No Nodes".

---

input

```
7
0 1 L
1 3 L
1 2 R
0 4 R
4 5 L
4 6 R
4
0
1
3
-1
```

output

```
0 2 5
4
No Nodes
1
```

E. Puzzle Simulation Finals

In his quest to become the best puzzle master in the world, Simone stumbles upon her biggest hurdle at the word finals of the Puzzle Simulation competition. The question given to her is of a string representing the inorder traversal of a certain Complete Binary Tree. She has to give the left and right children for any node that the examiner asks her. Help Simone win the World Finals.

Input

The first line contains a single integer N ($1 \le N \le 2^{20}$) denoting the number of vertices. The following line has N space-separated integers $A_0, A_1, A_2, \ldots A_{N-1}$ ($0 \le A_i \le N-1$) denoting the inorder traversal of the binary tree. The next line contains a single integer Q ($1 \le Q \le 10^5$) denoting the number of queries the examiner asks. Each of the following Q lines have a single integer X ($0 \le X \le N-1$), denoting the number of the node being queried. *It is given that the root is always numbered 0.*

Output

Print the left and right children of the queried node X (see sample case for better understanding). If any of them does not exist, print "NULL".

---

input

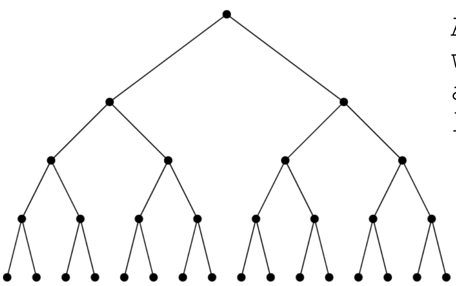
```
7
1 4 2 0 6 7 5
3
0
5
7
```

output

```
Left: 4
Right: 7

Left: NULL
Right: NULL

Left: 6
Right: 5
```



A **perfect binary tree** is a **binary tree** in which all interior nodes have two children and all leaves have the same depth or same level.

F. Ahmed’s Dilemma

Ahmed is a salesman who resides in the city **x** and he wants to travel to city **y**. To maximize his profits, he wants to stop by every single city in the path from **x** to **y** and set up a shop. The roads network in his country resembles an undirected graph. Ahmed has to come to you seeking help on creating a path for him from **x** to **y** *without visiting any city twice* but being able to maximize his profits i.e. *visit maximum cities in that path.*

Input

The first line contains two space-separated integers N ($1 \leq N \leq 10$) and M ($1 \leq M \leq \frac{N(N-1)}{2}$) the number of cities and number of roads between those cities in his Ahmed’s country. Each of the the following M lines contain two space-separated integers U and V ($0 \leq U_i, V_i \leq N-1$) denoting a bidirectional road between city U and city V. The last line has two space-separated integers X and Y ($0 \leq U_i, V_i \leq N-1$) denoting the Ahmed’s residence city and destination city (to which he wishes to travel to) respectively.

Output

Output a single sequence of numbers denoting the route map you’ll suggest to Ahmed. If there are multiple paths print any one. *Note that the path has to printed in correct order.*

---

input

```
9 9

0 1
0 2
2 3
2 4
1 5
3 4
3 6
4 7
5 8
0 7
```

output

```
0 2 3 4 7
```

explanation

```
Out of all the paths from 0 to 7, the given path 0-2-3-4-7 is
the longest of them all, i.e. he can earn maximum profit if
he follows this route as he visits maximum number of cities
in this path.
```

G. Hungry Bulbasaur

You see a tiny Bulbasaur longingly looking up at the leaves of a tree. You immediately think of helping the poor creature out. You analyze the tree and wish to help Bulbasaur (which is at the root of the tree) to reach the closest leaf node.

Input

The first line contains a single integer N ($1 \leq N \leq 1024$) denoting the number of vertices in the tree. The following line contains N-1 space separated integers, P_1, P_2, P_3, ... P_{N-1} ($0 \leq P_i \leq N-1$), where P_i is the parent of the **i**th vertex. *It is given that the root is always numbered 0.*

Output

Output a sequence of integers denoting the vertices that bulbasaur can reach fastest. *The sequence can be printed in any order.*

---

input

```
19
0 0 0 3 3 5 10 1 1 2 10 11 11 8 9 10 10 4
```

output

```
14 15 16 17 18 6 7
```

H. Birthday Gifts

You received two graphs as your birthday gifts from your best friends. As you have quite recently studied about graph isomorphism in class, you are curious if the the graphs you got as your gifts are isomorphic to each other or not, so you decide to write a program to check it.

Input

The first line contains two space-separated intergers N ($1 \le N \le 8$), M ($1 \le M \le \frac{N(N-1)}{2}$) denoting the number of vertices and edges in the first graph G_1. The following M lines contain two space-separated integers U_i, V_i ($0 \le U_i, V_i \le N-1$) each representing a bidirectional edge between vertex U_i and V_i .

The next line contains two space-separated intergers P ($1 \le P \le 8$), Q ($1 \le Q \le \frac{P(P-1)}{2}$) denoting the number of vertices and edges in the second graph G_2. The following Q lines contain two space-separated integers X_i, Y_i ($0 \le X_i, Y_i \le P-1$) each representing a bidirectional edge between vertex X_i and Y_i .

Output

If the given two graphs are isomorphic print “YES” followed by the bijective mapping between those two vertices of those two graphs (see sample case for better understanding), else print “NO”. The corresponding vertex pairs in the mapping can be printed in any order.

---

input

```
5 5
0 1
0 3
1 2
2 4
3 4
5 5
0 4
4 2
3 2
0 1
3 1
```

output

```
YES
0 – 0
1 – 4
2 – 2
4 – 3
3 – 1
```
