Assignment -0

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Course Number : CS512

Semester – Fall 2018

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(A)

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}.$$

(1)
$$2A - B = 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$
$$= \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

(2)
$$\|A\| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$
$$= \sqrt{1 + 4 + 9}$$
$$= \sqrt{14}$$

Angle of A relative to the positive X axi is,

$$V_2 = (1,0,0)$$

$$\cancel{Angle} = \cancel{acos}(\cancel{A \cdot V_2})$$

$$= \cancel{cos}(\cancel{|A| \cdot |V_2|}).$$

$$Angle = \cos^{-1}\left(\frac{A \cdot V_2}{|A|\,|V_2|}\right)$$

$$= \cos^{-1}\left(\frac{(1,2,3)\cdot(1,0,0)}{\sqrt{14}\cdot 1}\right)$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{14}}\right)$$

$$= 1.3002^{\circ}$$

(3)

unit vector in the direction of A is,

We have $\|A\| = \sqrt{14}$ from the second question,

therefore, $u_A = \dfrac{\vec{A}}{|A|} = \left(\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}} \right)$

(4)

direction cosine of A.

$$\cos \alpha = \frac{A_x}{|A|} = \frac{1}{\sqrt{14}}$$

$$\cos \beta = \frac{A_y}{|A|} = \frac{2}{\sqrt{14}}$$

$$\cos \gamma = \frac{A_z}{|A|} = \frac{3}{\sqrt{14}}$$

$\cancel{(5)}$

$$\cancel{A \cdot B} = \begin{bmatrix} \cancel{1} \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} \cancel{4} \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} \cancel{4} \\ \cancel{10} \\ \cancel{18} \end{bmatrix}$$

(5)

$$\cancel{A \cdot B = |A| \times |B| \cos\theta}$$

$$A \cdot B = A_x \times B_x + A_y \times B_y + A_z \times B_z$$
$$= 1 \times 4 + 2 \times 5 + 3 \times 6$$
$$= 4 + 10 + 18$$
$$= 32$$

$$B \cdot A = B_x \times A_x + B_y \times A_y + B_z \times A_z$$
$$= 4 \times 1 + 5 \times 2 + 6 \times 3$$
$$= 32.$$

(6)

The angle between A and B is,

$$\text{Angle} = \cos^{-1}\left(\frac{A \cdot B}{|A|\,|B|}\right)$$
$$= \cos^{-1}\left(\frac{(1,2,3) \cdot (4,5,6)}{\sqrt{14}\ \sqrt{77}}\right)$$
$$= \cos^{-1}\left(\frac{32}{\sqrt{14}\sqrt{77}}\right)$$
$$= 0.2257$$

(7)

If A and X is perpendiculer where $X = (x, y, z)$ then $A \cdot X = 0$.

$$A \cdot X = x + 2y + 3z = 0$$
$$\therefore X = 2 + 2(2) + 3(-2) = 0$$

(8)

$$A \times B = (2\times6-3\times5 \quad 3\times4-1\times6 \quad 1\times5-2\times4)$$
$$= (-3 \quad 6 \quad -3)$$

$$B \times A = (5\times3-6\times2 \quad 6\times1-4\times3 \quad 4\times2-5\times1)$$
$$= (3 \quad -6 \quad 3)$$

also not possible.

(9)

→ vector which is perpendicular to both A and B is, X ~~where~~

$$\cancel{A \cdot X = x + 2y + 3z = 0}$$
$$\cancel{B \cdot X = 4x + 5y + 6z = 0}$$

given vector $\vec{A}$ and $\vec{B}$, we know that $\vec{A} \times \vec{B}$ is a vector that is perpendicular to both $\vec{A}$ and $\vec{B}$.

$$\vec{X} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$
$$= -3\hat{i} + 6\hat{j} - 3\hat{k}$$
$$= (-3, 6, -3)$$

Also, $A \cdot X$ and $B \cdot X$ is zero, so $X = (-3, 6, -3)$ is perpendicular to both the vectors.

(10)

→ To find a dependency we take constants x, y, z such that

$$xA + yB + zC = 0 :$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 3 & 0 \end{array}\right]$$

$$\frac{R_3}{3} \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right]$$

$$R_3 - R_1 \Rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$$\frac{R_3}{2} + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$C_3 + C_1 \to C_3$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 2 & 6 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_2 - R_3 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

~~$R_3 + R_1$~~

$R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$x + 3y = 0$$
$$2x + 6y = 0$$
$$z = 0.$$

$$\therefore y = \frac{-x}{3}$$

$$xA + yB + zC = 0$$
$$\therefore xA - \frac{xB}{3} = 0$$
$$\therefore 3A = B$$

This is not true, so A, B and C are linearly independent.

(11)

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= 4 + 10 + 18$$

$$= 32$$

$$B^T = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

$$A B^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

(B).

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

(1) $$2A - B = 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}.$$

(2)

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 10-3 & 5+2 & -20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}.$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6-1 \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-8+0 & 6+4+5 & 9-6-1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

(3) $$(AB)^T = \left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}^T$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

(4)

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix}$$

$$= +1(2-15) - 2(-4-0) + 3(20-0)$$
$$= -13 + 8 + 60$$
$$= 55.$$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix}$$

$$= 1(15-6) - 2(12+6) + 3(4+5)$$
$$= 9 - 36 + 27$$
$$= 0$$

(5)

To form an orthogonal set dot products of vector should be zero.

For, vector $A = (1,2,3) \cdot (4,-2,3)$
$= 4-4+9$
$= 9.$

Here, the dot product is not zero. So, A is not orthogonal set.

For, vector $B = (1,2,1) \cdot (2,1,-4)$
$= 2+2-4$
$= 0$

$(2,1,-4) \cdot (3,-2,1)$
$= 6-2-4$
$= 0$

$(3,-2,1) \cdot (1,2,1)$
$= 3-4+1$
$= 0.$

Here, the dot product is zero. So B is an orthogonal set.

Dt: / / Pg.no:

(6)

$$A^{-1} = \frac{1}{\det(A)} \operatorname{Adj}(A)$$

$$= \frac{1}{55} \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 20/55 & -5/55 & -2/55 \end{bmatrix}$$

$$= \begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 4/11 & -1/11 & -2/55 \end{bmatrix}$$

$$B^{-1} = \frac{1}{\det(B)} \operatorname{Adj}(B)$$

$$= \cancel{\frac{-1}{42} \begin{bmatrix} -7/42 & -4/42 & -9/42 \\ & & \\ & & \end{bmatrix}}$$

$$= \begin{bmatrix} 7/42 & 4/42 & 9/42 \\ 14/42 & 2/42 & -6/42 \\ 7/42 & -8/42 & 3/42 \end{bmatrix}$$

$$= \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 1/3 & 1/21 & -1/7 \\ 1/6 & -4/21 & 1/14 \end{bmatrix}.$$

(C) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$.

(1)

Eigenvalues of A are the roots of the characteristic equation $\det(A-\lambda I) = 0$.

$$\det\left(\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) = 0.$$

$$\begin{pmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{pmatrix} = 0$$

$$\therefore \lambda^2 - 3\lambda - 4 = 0$$

$$\therefore (\lambda+1)(\lambda-4) = 0$$

$$\therefore \lambda = -1 \text{ or } \lambda = 4$$

The eigenvalues are $= -1, 4$.

→ Solving $(A-\lambda I)$ with $\lambda = -1$.

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - (-1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$$

Reducing the matrix to row echelon form,

Swap matrix rows: $R_1 \leftrightarrow R_2$

$$\begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}$$

$$R_2 \leftarrow R_2 - \frac{2}{3} \cdot R_1.$$

$$= \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}$$

$$R_1 \leftarrow \frac{1}{3} \cdot R_1.$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

System associated with the eigenvalue λ

$$(A + 1I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = ($$

So, the equation is

$$x + y = 0$$

$$\therefore x = -y.$$

$$\therefore V = \begin{pmatrix} -y \\ y \end{pmatrix}, \text{ where } y \neq 0$$

$$\therefore \text{Let } y = 1 \text{ so, } \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

$\rightarrow$ eigenvector for $\lambda = 4$

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - 4\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}$$

Reduce to echelon form =

$$R_2 \leftarrow R_2 + R_1$$

$$\begin{pmatrix} -3 & 2 \\ 0 & 0 \end{pmatrix}$$

$$R_1 \leftarrow -\frac{1}{3} \cdot R_1.$$

$$\therefore \begin{pmatrix} 1 & -2/3 \\ 0 & 0 \end{pmatrix}.$$

$\rightarrow$ System with the eigenvalue $\lambda = 4$

$$\therefore (A - 4I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -2/3 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x - \frac{2}{3}y = 0$$

$$\therefore x = 2/3\, y$$

Let $y = 1.$, so, $\begin{pmatrix} 2/3 \\ 1 \end{pmatrix}$

$\rightarrow$ eigenvectors are $= \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2/3 \\ 1 \end{pmatrix}$

Dt: / / Pg.no:

(2)

$$V = \begin{bmatrix} -1 & 1 \\ 2/3 & 1 \end{bmatrix}$$

$$V^{-1} = \frac{1}{\det(V)} \operatorname{adj}(V)$$

$$= -\frac{3}{5} \begin{bmatrix} 1 & -1 \\ -2/3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3/5 & 3/5 \\ 2/5 & 3/5 \end{bmatrix}$$

$$\cancel{V^{-1}AV = \begin{bmatrix} -3/5 & 3/5 \\ 2/5 & 3/5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}}$$

$$\cancel{= \frac{1}{5} \begin{bmatrix} -1 & 3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2/3 \\ 1 & 1 \end{bmatrix}}$$

$$\cancel{= \frac{1}{5} \begin{bmatrix} -1+9 & -2+6 \\ 2+9 & 4+6 \end{bmatrix} \begin{bmatrix} -1 & 2/3 \\ 1 & 1 \end{bmatrix}}$$

$$\cancel{= \frac{1}{5} \begin{bmatrix} 8 & 4 \\ 11 & 10 \end{bmatrix} \begin{bmatrix} -1 & 2/3 \\ 1 & 1 \end{bmatrix}}$$

$$\cancel{= \frac{1}{5} \begin{bmatrix} -4 & 28/3 \\ -1 & 52/3 \end{bmatrix}}$$

$$=$$

$$V^{-1}AV = \begin{bmatrix} -3/5 & 3/5 \\ 2/5 & 3/5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2/3 & 1 \end{bmatrix}$$

$$= \cancel{\frac{1}{5}\begin{bmatrix} 8 & 4 \\ 11 & 10 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2/3 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 6/5 & 0 \\ 11/5 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2/3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6/5 & 6/5 \\ -19/15 & 21/5 \end{bmatrix}$$

(3)
→ Eigenvectors are $= [-1, 1]$ and $(2/3, 1)$

dot product ~~of~~ eigenvectors are $= \left(-1 \times \frac{2}{3}\right) + (1+1)$

$$= -\frac{2}{3} + 2$$

$$= 4/3$$

(4)
~~→ Value of the~~
→ Eigenvalue of the B ~~case~~ from the above formula $\det(B - \lambda I) = 0$ are $\lambda = 6$ or $\lambda = 1$.

→ Eigenvector for $\lambda = 1$ is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

→ Eigenvector for $\lambda = 6$ is $\begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$.

→ dot product for eigenvector of B is

$$= (2,1) \cdot (-1/2, 1)$$
$$= -1 + 1$$
$$= 0.$$

(5)

→ Here, the eigenvectors of B are linearly independent of each other. So, the matrix X will be invertible.

(1)

Here, $B = X \lambda X^{-1}$.
So, matrix B then called as diagonalizable.

(+1)

(D)

(1) $f(x) = x^2+3$

$$f'(x) = 2x$$

$$f''(x) = 2.$$

(2)

$$g(x,y) = x^2+y^2.$$

$$\frac{dg}{dx} = \frac{d}{dx}(x^2+y^2)$$

$$= 2x.$$

$$\therefore \frac{dg}{dy} = \frac{d}{dy}(x^2+y^2) = 2y.$$

(3) gradient vector $\nabla g(x,y) = (2x, 2y)$

$$\nabla g = \left(\frac{dg}{dx}, \frac{dg}{dy}\right)$$

$$\therefore \frac{dg}{dx} = 2x, \quad \frac{dg}{dy} = 2y.$$

$$\nabla g = (2x, 2y)$$

(4)

univariate gaussian distribution

$$p(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} e^{\left(\frac{-1}{2\sigma^2}(x-\mu)^2\right)}$$

probability density function of univariate normal distribution, is $= \frac{1}{\sqrt{2\pi}}$

where, $\mu=0$, $x=0$, $\sigma=1$