Assignment - 3

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* Semester = Fall 2018.

(1)

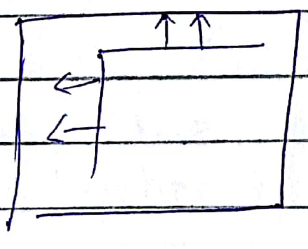
(a)

* Principle of corner detection =

→ For a image, we define a window having neighbourhood points, where corners represent interests points, so goal is to identify these points.

→ The steps to detect a corner in a local window are =

1) Find the correlation matrix in the window.
2) compute eigenvalues of the matrix
3) check if $\lambda_1 \lambda_2 > Z$.



→ The gradient in that window is seen, if there are more than one directions it is said to have a corner.

(b)

→ To find principal direction of gradient orientation in the local path,

We use, $\sum_{i=0}^{n} P_i P^T$ → as correlation matrix comprising of P_i = points in neighbourhood.

Here, we find direction of minimum projection a direction subject to be perpendicular to all previous directions.

~~Here,~~
Here, direction is eigen vectors of correlation metrix. and projections are propositional to eigen values.

(c)

→ Given gradient vectors are
$= \{ (0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (1,1), (1,2), (1,3) \}$

→ Correlation matrix $= \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i \\ \Sigma y_i x_i & \Sigma y_i^2 \end{bmatrix}$

$$= \begin{bmatrix} 0+0+0+0+0+1+1+1+1 & 0+0+0+0+0+0+1+2+3 \\ 0+0+0+0+0+0+1+2+3 & 0+1+4+9+16+0+1+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 6 & 44 \end{bmatrix}.$$

(d)

→ The eigenvalues of the gradient corelation matrix λ_1 and λ_2 should be greater than the threshold z.

$$\lambda_1 \cdot \lambda_2 > 'z'$$

Thus, we detect a corner in that neighbourhood where both λ_1 and λ_2 are large enough.

(e)

→ non maximum suppression helps in finding a unique corner for a location when we are using a multiple windows.

→ As for point in image multiple windows will find multiple corners.

→ Steps for calculating non-maximum suppressions are :

(1) compute $\lambda_1 \cdot \lambda_2$ for all windows.

(2) select windows with $\lambda_1 \cdot \lambda_2 > \tau$ and sort in decreasing order.

(3) Select the top of the list as corner and delete all other corners in its neighbourhood from the list.

(4) Stop when detecting x% of the points as corners.

(f)

→ In harris corner detection,

$$C(G) = \det(G) - k\,tr^2(G)$$

where, G = gradient correlation matrix
$tr(G)$ = trace of G.

Thus, we do not consider eigen values of the gradient corelation matrix directly instead we find determinent and trace.

(g)

→ To determine p we project gradient onto edge and choose p minimum projector.

→ Localisation of point p

$$p = C^{-1} V$$
$$= C^{-1} \sum_i \nabla I(P_i) \cdot \nabla I(P_i)^T \cdot P_i$$

where $C = \sum \nabla I(P_i) \cdot \nabla I(P_i)^T$ is a corelation matrix.

* Condition for solution to exist =

→ $\lambda_1 \cdot \lambda_2 > Z$ so that C is a non-singular and we can get inverse of that.

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(h)

→ Feature points characterization using HOG.

(1) Take a window.
(2) split each patch into cells which can be possibly overlapping.
(3) create orientation histogram in each cell using edge or gradient directions, possibly weighted by distance from center or gradient magnitude.
(4) concatenate histograms and we get feature vector.

→ good characterization of feature points are = (1) Translation invarient.
(2) rotation invarient.
(3) scale invarient.
(4) illumination invarient.

(i)
→ SIFT =
(a) take a large window.
(b) split into blocks.
(c) compute gradient vector in each block.
(d) combine all the gradient vector, into orientation histogram over smaller sub region.

(2)

(a)

→ If you use the slope and y-intercept to calculate a and b in $y = ax + b$ you obtain values $a \in [-\infty, \infty]$ and $b \in [-\infty, \infty]$.

→ That's why the implicit line equation is used instead.

(b)

$\theta = 45$,

$d = 10$.

→ Equation is, $x\cos\theta + y\sin\theta - d = 0$

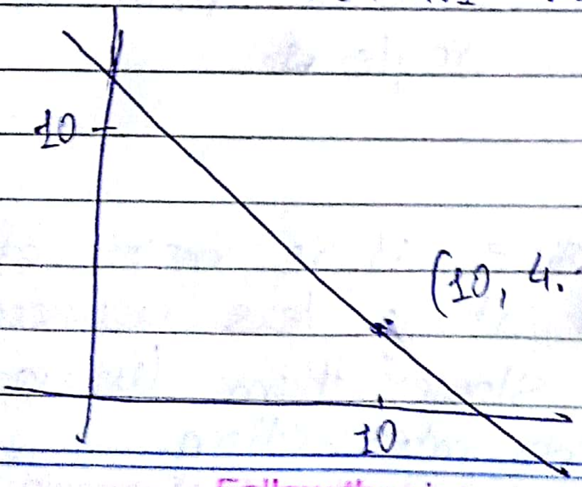
$$x(\cos 45^\circ) + y(\sin 45^\circ) - 10 = 0$$

$$\therefore x \times \frac{1}{\sqrt{2}} + y \times \frac{1}{\sqrt{2}} - 10 = 0$$

$$\therefore x + y - 10\sqrt{2} = 0$$

→ detection point is (10, 4.1) which satisfy this equation

$$\therefore 10 + 4.1 - 10\sqrt{2} \approx 0$$



So this satisfy the explicit line equation.

(c)
→ When using the polar representation of lines the vote of each point in the image looks like sinusoidal curve in the parameter plane.

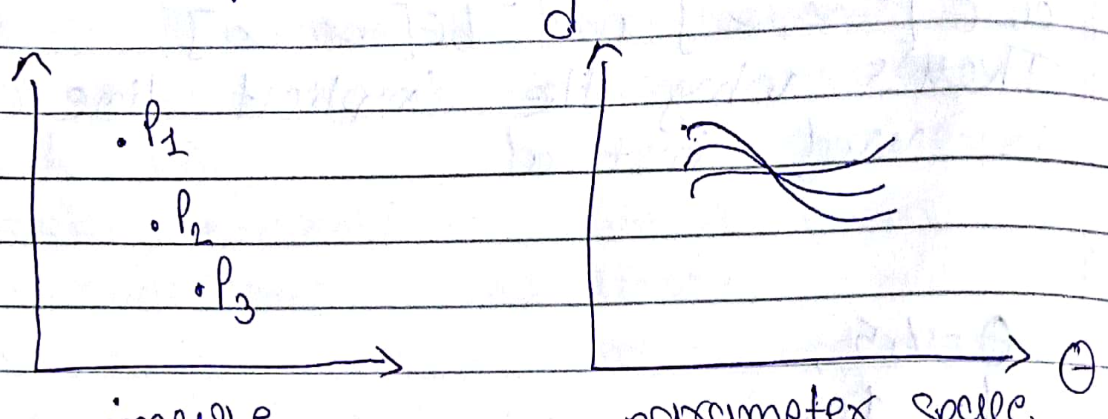


image. parameter space.

(d)
→ When different lines which represent points in image, has a intersection point, the distance and angle of that point represent the line.

Equation is,

$x\cos\theta + y\sin\theta - d = 0$

where d = distance from origin.

θ = angle

(e)
→ Larger the bin size it is more efficient and faster but it is less accurate.
→ Smaller bin is slower than larger bin and it gives no intersection.

(f)

→ If the normal at each point is known we can find ∇I at voting point and compute θ, so the range will become $(\theta - \Delta, \ldots, \theta + \Delta)$.

(g)

→ When using Hough transform for circles, the number of ~~the~~ dimensions of the parameter space is 3

(3).

(a) the disadvantage of the equation $y = ax + b$ is that the geometric distance and real point are not minimized. thus it will result in non-accurate fitting.
↳ Lines with higher slopes cannot be fitted accurately.

(b)
→ normal (1, 2)
distance = 2 from origin.

equation is $l^T x = 0$, where l^T has 3 coefficient → a, b and c.

$$\text{so, } \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \begin{bmatrix} x & y & 1 \end{bmatrix} \Rightarrow x + 2y - 2 = 0$$

so, $l = [1, 2, -2]$

(c)
→ We use implicit line equation to minimize geometric distance.
$l^T x = 0$, where all points 'x' should be on the line l.

→ objective function to be minimized.

$$E(l) = \sum_{i=1}^{n} (l^T x_i)^2$$

$$= l^T \underbrace{\left(\sum_{i=1}^{n} (x_i x_i^T) \right)}_{C} l.$$

$$E(l) = l^T C l. \quad \left(\because C = \sum_{i=1}^{n} (x_i x_i^T) \right]$$

$$l^* = \underset{l}{\operatorname{argmin}} \; E(l) \Rightarrow \Delta E(l) = 0$$

$$Cl = 0$$

where, $C = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i \\ \Sigma x_i & \Sigma y_i & n \end{bmatrix}$

Scanned by CamScanner

(d)

→ points are = {(0,1), (1,3), (2,6)}.

$$C = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i \\ \Sigma x_i & \Sigma y_i & n \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+4 & 0+3+12 & 0+\cancel{3}+2 \\ 0+3+12 & 1+9+36 & 1+3+6 \\ 0+1+2 & 1+3+6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 15 & 3 \\ 15 & 46 & 10 \\ 3 & 10 & 3 \end{bmatrix}$$

(e)

→ Explicit equation

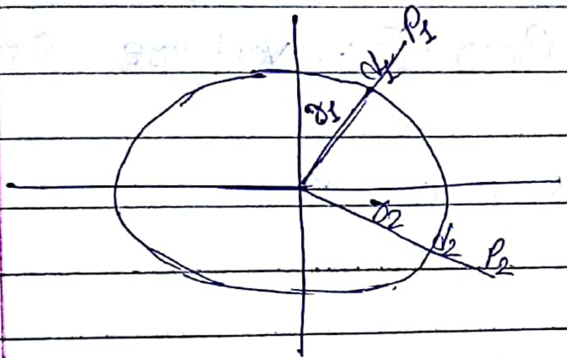
$$\left(\frac{x-x_0}{a}\right)^2 + \left(\frac{y-y_0}{b}\right)^2 = 1.$$

→ constraints on parameters a, b, c, d, e, f that guarantees the model will be an ellipse is $b^2 - 4ac < 0$.

(f)

→ equation for fitting ellipse is :-
$\sum_{i=1}^{n} (l^T P_i)^2$ → where $l^T P_i$ is algebric distance.

$q_i = l^T P_i$ and q_i (proportional) $\frac{d_i}{d_i + r_i}$



So, for $q_1 > q_2$

$$\frac{d_i}{d + r_1} > \frac{d}{d + r_2}$$

So points close to short axis will affect more the fitting.

(g)

→ $d(p,f) = |p-x| = \dfrac{|f(p)|}{|\nabla f(x)|}$ ← algebric distance

geometric distance

~~x~~ x is closet point.

→ The problem is what is the closet point X is.

(h)

→ $$E[\phi(s)] = \int_{\phi_s} \left(\alpha(s) E_{continuity} + \beta(s) E_{curvature} + \gamma(s) E_{image} \right) ds$$

→ Econtinuity, Ecurvature, Eimage are energy terms.

→ $\alpha(s), \beta(s), \gamma(s)$ are coefficients.

→ $\alpha(s) \cdot E_{continuity} + \beta(s) \cdot E_{curvature}$ are internal parameters.

→ $\gamma(s) \cdot E_{image}$ is external parameter.

(i)

→ Continuity of discrete curve:

$$E_{continuity} = \left| \frac{d\phi}{ds} \right|^2 \Rightarrow \underbrace{\Sigma \left| P_i - P_{i-1} \right|^2}_{\text{distance between neighbouring points}}$$

Curvature of discrete curve :-

$$E_{curvature} = \left| \frac{d^2\phi}{\partial s^2} \right|^2 \Rightarrow \Sigma \left| (P_{i+1} - P_i) - (P_i - P_{i-1}) \right|^2$$

(j)

→ The continuity of active contours will be $|P_i - P_{i-1}| - d$ to allow for sharp corners.