Analysis of the COVID-19 pandemic by SIR model and machine learning technics for forecasting

Babacar Mbaye Ndiaye, Lena Tendeng, Diaraf Seck

Abstract. This work is a trial in which we propose SIR model and machine learning tools to analyze the coronavirus pandemic in the real world. Based on the public data from [3], we estimate main key pandemic parameters and make predictions on the inflection point and possible ending time for the real world and specifically for Senegal. The coronavirus disease 2019, by World Health Organization, rapidly spread out in the whole China and then in the whole world. Under optimistic estimation, the pandemic in some countries will end soon, while for most part of countries in the world (US, Italy, etc.), the hit of anti-pandemic will be no later than the end of April.

Keywords. coronavirus, COVID-19, pandemic, SIR model, stochastic, machine learning, forecasting.

1. Introduction

The pandemic of COVID-19 was firstly reported in Wuhan in December 2019 and then quickly spread out international wide, see Figure 1.

Since some decades ago mathematical community and specialists in computer sciences investigate biological questions to propose models describing subjects such as diseases, physiological phenomena. One of these models are formulated with the help of Differential Equations (SIR models, reaction diffusions equations, etc.) while others use computer sciences (see [20, 13, 8, 7, 5]).

In this work, for the mathematical modeling part, we are going to use the SIR model without demography because the lifespan of the disease which is an influenza disease will be very short compared to the lifespan of humans (even if the flu lasted 2 years which would be exceptional, 2 years do not represent much compared to the average longevity of a man). Similarly for

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this type of disease it is also assumed that there will be no significant renewal of the population while it is raging. The last influenzas that seriously affected humanity did not last long (H1N1 for example) even if this new virus hitherto seriously defies science.

Our aims are first, to collect carefully the pandemic data from the [3], e.g. https://www.tableau.com/covid-19-coronavirus-data-resources, from January 21, 2020 to April 02, 2020. In the second step, we propose two technics, a SIR model and machine learning tools, to analyze the coronavirus pandemic in the worldwide.

Worldwide infected countries with COVID-19

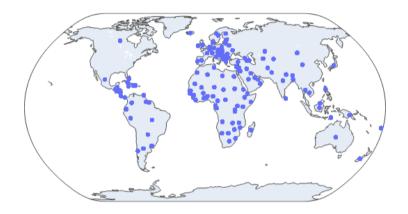


FIGURE 1. All infected countries with COVID-19 pandemic

The article is organized as follows. In the section 2, we present some auxiliary results of the SIR model (model, estimation of parameters) and the machine learning technics for forecasting, followed by the simulations and results in section 3. Finally, in the section 4, we present conclusions and perspectives.

2. Methodology

2.1. SIR model

In this work, we use the classical Kermack-Mckendrick SIR model to describe the transmission of the COVID-19 virus pandemic[9, 10]. The SIR model is a compartmental model for modeling how a disease spreads through a population. Its an acronym for Susceptible, Infected, Recovered. The total population is assumed to be constant and divided into three classes.

In this model, naturel death and birth rate are not considered because the outstanding period of the disease is much shorter than the lifetime of the

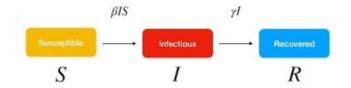


FIGURE 2. SIR model diagram

human.

$$\begin{cases} \frac{dS}{dt} = -\beta IS \\ \frac{dI}{dt} = \beta IS - \gamma I \\ \frac{dR}{dt} = \gamma I \end{cases}$$
 (2.1)

- S is the number of individuals susceptible to be infected at time t.
- *I* is the number of both asymtotimatic and symptomatic infected individuals at time *t*.
- R is the number of recovered persons at time t.
- The parameters β and γ are the transmission rate through exposure of the disease and the rate of recovering.

Fore more details about SIR, see [22].

2.2. Estimation of parameters

Parameters identification methods fall into two broad categories: parametric and non-parametric. The aim of non-parametric methods is to determine models by direct technics, without establishing a class of models a priori. They are called nonparametric because they do not involve a vector of parameters to be sought to represent the model.

Parametric methods are based on model structures chosen a priori and parameterized. In this case, the goal is to find a vector of parameters θ , which will make it possible to obtain a model representing the behavior as close as possible to the real system. The search for the best model becomes a problem of estimating θ .

Parameter identification belongs to the topic of inverse problems[11, 19, 21]. It is widely studied and there is an abundant literature. But the subject is still interesting and there are still open problems. There are numerous approaches for the estimation of parameters. In this paper, we shall give a brief overview and quote the least square method. For details and the discovery of other methods see for instance [21].

If we look at the problem in general first, we have data y_i , i = 1, ..., n where each y_i was measured at a specific t_i point; on the other hand, we have a model, i.e. a function f depending on time on the one hand and on the other hand a set of parameters that will be noted $P = (p_1, p_2, ..., p_k)$. It is assumed that the amount (scalar or vector) that is tracked is described by $q(t) = f(t, p_1, p_2, ..., p_k)$ that is noted by f(t, P). We do not know the value

of $p_i, j = 1, ..., k$ but with measurement errors, we must have the relationship

$$y_i = f(t_i, P), \quad i = 1, ..., n$$

and then that y_i is the measure of $q(t_i)$.

The idea is therefore to look for all the values $p_1, p_2, ..., p_k$, which, in a sense, makes the differences $y_i - f(t_i, P)$ as close as possible to 0 for all i = 1, ..., n simultaneously.

One natural method is the least square one, by setting

$$\mathcal{E}(P) = \sum_{i=1}^{n} [y_i - f(t_i, P)]^2$$

to be minimized on a subset S of \mathbb{R}^k . This set may be open of closed. Even it is quite possible to study cases where Euclidean metric is replace by a Riemannian one usually noted g.

Remark 2.1. To estimate parameters, it is possible to use the observer theory by first ensuring that the model is well observable and identifiable.

2.3. Existence

First let us remind that the minimization problems in the previous section are very classical in the cases we shall consider. That is why, we do not emphasize on the theoretical aspects.

For the SIR model, it is also classical to have existence and uniqueness whenever convenient Cauchy data are considered. The Cauchy Lipschitz theorem ensure existence and uniqueness of local solution. And even the recovery theorem is equivalent to Cauchy Lipschitz theorem.

2.4. Forecasting using Prophet

Machine learning technics for forecasting is a branch of computer science where algorithms learn from data. Algorithms can include artificial neural networks, deep learning, association rules, decision trees, reinforcement learning and bayesian networks [1, 2, 12].

Forecasting involves taking models fit on historical data and using them to predict future observations. We use this technic for the COVID-19 prediction in the real world.

We use Prophet [15, 18], a procedure for forecasting time series data based on an additive model where non-linear trends are fit with yearly, weekly, and daily seasonality, plus holiday effects. It works best with time series that have strong seasonal effects and several seasons of historical data. Prophet is robust to missing data and shifts in the trend, and typically handles outliers well.

For the average method, the forecasts of all future values are equal to the average (or mean) of the historical data. If we let the historical data be denoted by $y_1, ..., y_T$, then we can write the forecasts as

$$\hat{y}_{T+h|T} = \bar{y} = (y_1 + y_2 + \dots + y_T)/T$$

The notation $\hat{y}_{T+h|T}$ is a short-hand for the estimate of y_{T+h} based on the data $y_1, ..., y_T$.

A prediction interval gives an interval within which we expect y_t to lie with a specified probability. For example, assuming that the forecast errors are normally distributed, a 95% prediction interval for the h-step forecast is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma_h}$$

where σ_h is an estimate of the standard deviation of the h-step forecast distribution.

We perform one week ahead forecast with Prophet, with 97% prediction intervals (multiplier=2.17). Here, no tweaking of seasonality-related parameters and additional regressors are performed.

For the data preparation, when we are forecasting at country level, for small values (Senegal for example), it is possible for forecasts to become negative. To counter this, we round negative values to zero.

3. Numerical simulations and results

In this section, we present the simulations of SIR model and the forecating technic for the prediction of the pandemic.

The numerical tests were performed by using the Python with panda [16]. The numerical experiments were executed on a computer intel(R) Core-i7 CPU 2.60GHz, 24.0Gb of RAM, under UNIX system.

3.1. Data analysis

As cited in the introduction, the simulations are carried out from data in [3] (https://www.tableau.com/covid-19-coronavirus-data-resources), from January 21, 2020 to April 02, 2020. According to this daily reports, we first analyse and make some data preprocessing before simulations. The cumulative numbers of confirmed cases and deaths cases are illustrated in Figure 3 and Figure 4 (curve and histogram).



FIGURE 3. Worldwide cases - confirmed and deaths

Worldwide COVID-19 cases - Confirmed & Deaths

Worldwide COVID-19 cases - Confirmed & Deaths

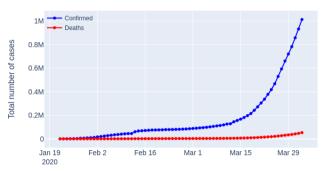


FIGURE 4. Worldwide cases - confirmed and deaths

3.2. SIR model

We use the SIR model simulations for the Senegal case study. For the initial values, well use normalized population values for our S_0 , R_0 , etc. So if we assume we have 10k people in our population, and we begin with four exposed person and the remaining susceptible, we have: N=10k (a sample), $I_0=4$, $R_0=0$, $S_0=N-I_0-R_0$.

For the contact rate (β) and the mean recovery rate (γ), we have: $\beta = 0.25$ and $\gamma = 1/30$ (in 1/days). Susceptible people in blue curve remain constant

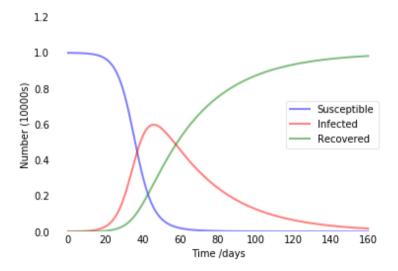


Figure 5. Prediction with SIR for the Senegal

for less than twenty days which could correspond to the incubation periode of the disease.

After the day 20, there is a rapid decrease in the blue curve which show the fast transmission of COVID-19.

Infected people in red line appear right after day 20. Where the red curve and blue curve cross, at about day 40, is the first day where more people are in the infected compartement than the susceptible one.

The peak of infected people is reached at day 40 approximately with 6000 infected persons (60% of infected persons). The model predicts a significant number of recoverers so few deaths due to the disease.

3.3. Forecast for the pandemic of COVID-19

Here, we perform predictions for the worldwide (180 countries, from January 21, 2020 to April 02, 2020) and four selected countries China, Italy, Iran and Senegal. Most importantly, with the model and parameters in hand, we can

ds	\hat{y}	\hat{y}_{lower}	\hat{y}_{upper}
2020-04-03	9.740387e + 05	9.259386e + 05	1.017860e + 06
2020-04-04	1.024429e+06	9.780116e + 05	1.072039e+06
2020-04-05	1.074312e+06	1.029720e + 06	1.123804e+06
2020-04-06	1.125356e+06	1.074945e+06	1.172525e+06
2020-04-07	1.177793e+06	1.129130e + 06	1.229721e+06

TABLE 1. Predicted cumulative confirmed cases in the near future ~April 07, 2020.

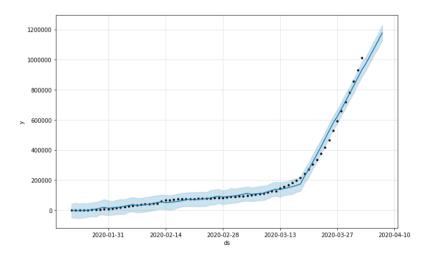


FIGURE 6. Forcasting - confirmed cases in the worldwide

carry out simulations for a longer time and forecast the potential tendency of the COVID-19 pandemic. For all figures, the predicted cumulative number of confirmed cases and the number of deaths cases are plotted for a shorter

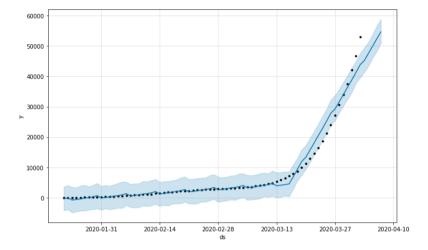


FIGURE 7. Forcasting - deaths cases in the worldwide

period of next 5 days.

Under optimistic estimation, the pandemic in some countries (like China) will end soon within few weeks, while for most part of countries in the world (Italy, Iran and Senegal), the hit of anti-pandemic will be no later than the end of April.

We can summarize our basic predictions as follows, for the worldwide and by country:

- For the worldwide, (see Figure 6 and Figure 7), overall, except in a period, the validation data show a well agreement with our forecast and all fall into the 97% confidence interval (shaded area). We see the number of cases is almost equal to our predictions, showing each country of the whole world must take strict measures to stop the pandemic. At ~April 07, 2020 we may obtain > 1 million 200,000 confirmed cases (see Table 1).
- For China (see Figure 8), based on optimistic estimation, the pandemic of COVID-19 would soon be ended within few weeks in China.
- For Italy (see Figure 9) and Iran (see Figure 10), the success of antipandemic will be no later than the middle of Avril. The situation in Italy and Iran are still very severe. We expect it will end up at the beginning of May.
- For Senegal (see Figure 11), we don't have enough data, because the first case appear in the beginning of March. And we are delighted to see cases are lower than our predictions, showing the nation wide anti-pandemic measures in Senegal come into play.

Remark 3.1. In Figure 6 to 11, the dots (color black) are the real data, the blue line is \hat{y} , the shaded area represents the confidence interval (97%).

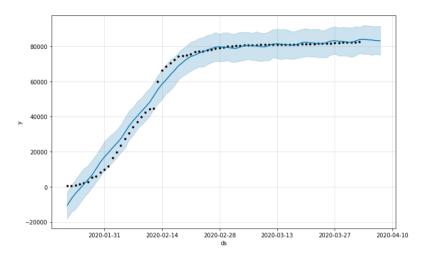


Figure 8. Forcasting - confirmed in China

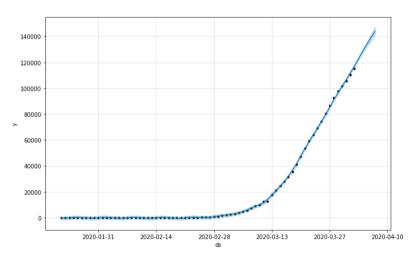


FIGURE 9. Forcasting - confirmed in Italy

Finally, due to the inclusion of suspected cases with clinical diagnosis into confirmed cases (quarantined cases), we can see severe situation in some cities, which requires much closer attention.

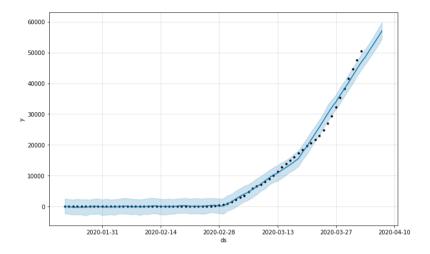


Figure 10. Forcasting - confirmed in Iran

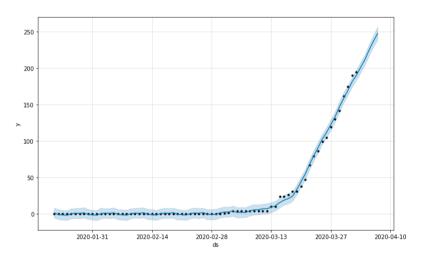


Figure 11. Forcasting - confirmed in Senegal

4. Conclusion and perspectives

In this article, we propose the generalized SIR model and forecasting (machine learning) to analysis the coronavirus pandemic (COVID-19). The authors think that the part of identification could be deepen. Indeed, one way should be to to consider in general the identification of systems. It is the operation of determining the dynamic model of a process (system) from input / output measurements. Knowledge of the dynamic model is necessary for the design and implementation of an efficient regulatory system.

In other words, the identification of a real dynamical system (called object) is

to characterize another system (called model), starting from the experimental knowledge of the inputs and outputs so as to obtain an identity of behavior. In practice, the purpose of identification is generally to determine the conduct model, which can be used to simulate, control or regulate a process. This model can be physical (in the sense of analog or digital simulator and reduced model), or an abstract model (mathematical model, i.e. system of algebraic or differential equations (ODE or PDE)). Finally, note that some addressed questions in parameter identification are the recognizing of sets (domains, curves, etc.).

Finally, note that some addressed questions in parameter identification are the recognition of sets (domains, curves, etc.). And a final remark is the identification of geodesics: In almost every epidemic, there is a phenomenon of super-propagation (super spreader)- when a person transmits an infection to a large number of people. And super spreader phenomena may appear. The role of super-spreaders it is the fact, noticed by Erds [4], that in a random graph there is always a small number of very connected people, through which almost all geodesics pass. How could it be possible to identify these facts in this pandemic?

The stochastic aspect would be very interesting because we are not even sure to have for the case of Senegal all the data. In addition there are infected people who do not develop the disease and spread it (the case of some children as it seems) not to mention the case of asymptomatics that are not monitored and are the cause of community transmission. There is also the probability of touching an infected object: too much hazard in the transmission of the disease. Let us propose among many possibilities the following stochastic SIR model:

$$\begin{cases} \frac{dS}{dt} = -\beta IS + \sigma_1 dW_1 \\ \frac{dI}{dt} = \beta IS - \gamma I + \sigma_2 dW_2 \\ \frac{dR}{dt} = \gamma I \end{cases}$$

$$(4.1)$$

- S is the number of individuals susceptible to be infected at time t.
- I is the number of both asymtotimatic and symptomatic infected individuals at time t.
- R is the number of recovered persons at time t.
- The parameters β and γ are the transmission rate through exposure of the disease and the rate of recovering.
- σ_1 and σ_2 are volatility rates that may depend on the time t, the suspects and the infected. W_1 and W_2 are classical Brownian motions.

For this model, the complete theoretical study could be an interesting part such as existence and uniqueness results and qualitative properties. Let us quote for the readers that there is a Cauchy Lipschitz Theorem for (4.1), see for example [6].

Remark 4.1. We would like to underline that the case of Senegal must be deepen in the sense that there are not yet enough data. And it should be

better to look for getting more data and even consider a stochastic model to take into account the difficulties that can be met in the information system to collect enough data.

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