Information and Instructions.

- This test is out of 30 marks, and there are 2 pages.
- Show all of your work. Submit your work to Crowdmark as you go. You have a total of 2 hours and 15 minutes - this includes the time you spend uploading your work.
- Do not take the amount of marks a question has as an indication for how long you answer 'should' be.
- You are not to use any software or the internet to help you answer the questions. A basic calculator is allowed. You may also use any of the materials/videos on the LEARN page.
- Piazza/Discord will be disabled. I will also probably not reply to any emails during the exam period.
- Do NOT discuss these problems with anyone until after Saturday, Oct. 23, 1:00 pm EST.
- Unless told otherwise, use any method you like to solve each question.

Question(s).

(1) (**4 marks**) A cat and a goose are each trying to determine $\int \frac{1}{3x}\,dx$. Their work is shown below.

Cat: I pulled out the 3 and used an integral table:

$$\begin{aligned}\int \frac{1}{3x}\,dx &= \frac{1}{3}\int \frac{1}{x}\,dx\\ &= \frac{1}{3}(\ln|x|+C)\\ &= \frac{1}{3}\ln|x|+D\end{aligned}$$

Goose: I used the substitution $u = 3x$. Then $du = 3dx$, so $dx = \frac{1}{3}du$, and with the help of an integral table we see that:

$$\begin{aligned}\int \frac{1}{3x}\,dx &= \frac{1}{3}\int \frac{1}{u}\,du\\ &= \frac{1}{3}(\ln|u|+C)\\ &= \frac{1}{3}(\ln|3x|+C)\\ &= \frac{1}{3}\ln|3x|+D\end{aligned}$$

The cat and the goose start fighting because their answers are different. Consider the following 4 statements:

(a) The cat has the correct answer and the goose has the incorrect answer.
(b) The cat has the incorrect answer and the goose has the correct answer.
(c) The cat and the goose both have the incorrect answer.
(d) The cat and the goose both have the correct answer.

Which of the above 4 statements is true? If your answer is (a), (b), or (c), explain the mistake(s) made by the cat and/or goose. If your answer is (d), explain how this is possible even though their answers are different.

(2) (**4 marks**) Determine the area bounded by the curves $y = x + 2$, $y = 14 - 2x$, and $x = 0$.

(3) (**4 marks**) **This question has 4 available marks. (a) is worth 4 marks and (b) is worth 3 marks. Do only one of (a) and (b).** [If you submit work for both (a) and (b), whatever comes first is what will be graded.]

(a) (**4 marks**) Find a function $g(x)$ so that $\displaystyle\int_3^7 g(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} \frac{8}{n^{3/2}}\sqrt{11n + 8i}$.

(b) (**3 marks**) Find numbers α and β so that $\displaystyle\int_\alpha^\beta \sqrt{x^2+3}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} \frac{5}{n}\sqrt{4 + \frac{10i}{n} + \frac{25i^2}{n^2}}$.

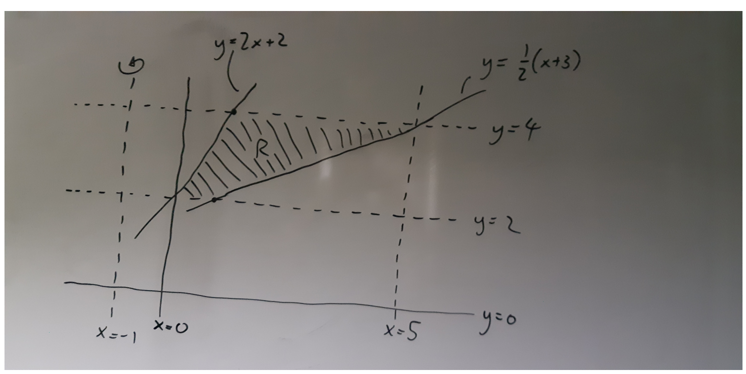
(4) (**5 marks**) Do both (a) and (b). They are separate questions.

(a) (**2 marks**) Given that $y = 1 + 2\tan\phi$, determine $\sin\phi$ in terms of y. [Your answer may not include any inverse trig functions.]

(b) (**3 marks**) For what integers A and B does the substitution $x = 3\sec\theta$ change the integral $\displaystyle\int \left(\sqrt{x^2-9}\right)^A x^B\,dx$ into $\displaystyle\frac{1}{9}\int \sin^2\theta\cos\theta\,d\theta$?

(5) (**5 marks**) Let $d > \sqrt{5}$ be a fixed real number. Given that $\displaystyle\int_d^\infty \frac{x}{(x^2-2)(x^2-5)}\,dx = \frac{1}{6}\ln(\frac{5}{2})$, determine the value of d. [**Hint**: I said several times that I would not force you to do partial fractions with 4 unknowns on the midterm. I was telling the truth.]

(6) (**5 marks**) Let R be the region in $\mathbb{R}^2$ bounded by $y = 2x+2$, $y = \frac{1}{2}(x+3)$, $y = 2$, and $y = 4$. Let V be the volume of the solid obtained by rotating R around the (vertical) line $x = -1$. Shown below is a high quality picture of the region R that is not drawn to scale. [Note that for (a) and (b) you are not being asked to calculate an integral(s) - just to write them down.]



(a) (**2 marks**) Express V as an integral (or sum of integrals) using the method of disks (some sources call this the method of washers, though our course notes do not).

(b) (**2 marks**) Express V as an integral (or sum of integrals) using the method of shells.

(c) (**1 marks**) Prove that $V \leq 70\pi$.

(7) (**3 marks**) Let $f(x)$ be a continuous function such that $\displaystyle\int f(x)\,dx = 3xf(x) + x^2 + C$. Express $\displaystyle\int_1^2 xf(x)\,dx$ in terms of $f(1)$, $f(2)$, and any other real numbers. [It is possible to determine $f(x)$ by solving a differential equation. Do NOT do this.]