Experimental Generation of Polarization-Entangled Photons and Verification of Quantum Nonlocality

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This experiment aimed to generate polarization-entangled photon pairs using spontaneous parametric down-conversion in a two-crystal geometry with a violet laser diode source. Polarization-sensitive coincidence measurements were performed to analyze the polarization correlations of the entangled state. The measured data showed strong polarization correlations in agreement with quantum mechanical predictions. A CHSH Bell parameter of $S = 2.621 \pm 0.015$ was calculated, exceeding the local hidden variable bound by more than 40 standard deviations. These results confirm the successful generation of polarization-entangled photons and provide clear experimental evidence against local hidden variable theories.

I. INTRODUCTION

Entanglement is one of the most nonclassical features of quantum mechanics. Particles are said to be entangled if their state cannot be separated into single-particle states. Due to the advances in optical technologies, polarization-entangled photons have become a compelling platform for investigating this quantum mechanical phenomenon and testing fundamental concepts such as Bell inequalities [1]. These polarization-entangled photons can be generated through a nonlinear optical process known as spontaneous parametric down-conversion (SPDC).

In this experiment, two type-I phase-matched β-barium borate (BBO) crystals glued together with their optic axes oriented in perpendicular planes are used as an entanglement source. A 405 nm pump photon is down-converted into two entangled photons at 810 nm, which are emitted into a cone with a half-opening angle of approximately 3.70°- 4.20°, depending on the refractive index values used. As a result, the states of the downconverted photons are of the form [2]:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + e^{i\phi}|VV\rangle) \tag{1}$$

where $|V\rangle$ and $|H\rangle$ represent vertical and horizontal polarization, respectively, with the first and second positions in the ket representing the first and second photon, and ϕ is the phase difference of the two polarization components determined by the details of the phase matching and the crystal thickness.

By placing polarizers rotated to angles θ_1 and θ_2 in the two paths, the polarization of the downconverted photons is measured, and the probability of coincidence detection can be calculated to be

$$P(\theta_1,\theta_2) = \frac{1}{2}(\cos^2\frac{\phi}{2}\cos^2(\theta_1-\theta_2)+\sin^2\frac{\phi}{2}\cos^2(\theta_1+\theta_2)). \tag{2}$$

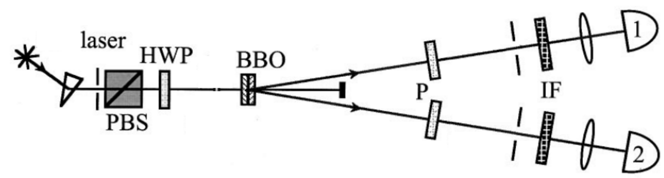

FIG. 1. Schematic of experimental setup. Symbols: PBS Polarizing Beam Splitter, HWP Half-Wave Plate, BBO BBO Crystals, P Polarizer, IF Longpass Filter. Adapted from [2].

In a special case where $\phi = 0$, this probability becomes

$$P(\theta_1,\theta_2) = \frac{1}{2}\cos^2(\theta_1-\theta_2), \tag{3}$$

which depends only on the relative angle of the polarizers $\theta_1 - \theta_2$.

For a fixed interval T of data acquisition, the recorded number of coincidences $N(\theta_1,\theta_2)$, assuming a constant flux of photon pairs, is given by

$$\begin{aligned} N(\theta_1,\theta_2) =& N_e(\cos^2\frac{\phi}{2}\cos^2(\theta_1-\theta_2) \\ &+ \sin^2\frac{\phi}{2}\cos^2(\theta_1+\theta_2)) + N_0 \end{aligned} \tag{4}$$

where N_e is the total number of entangled pair produced divided by 2 (from the expression of the probability), and N_0 is an offset to account for imperfections in the polarizers and alignment of the crystals, corresponding to the fact that some coincidences are observed even when the polarizers are set to $(\theta_1,\theta_2) = (0°, 90°)$. Assuming the generated photons are in a perfect $|HH\rangle+|VV\rangle$ state, the number of coincidence can be expressed as

$$N(\theta_1,\theta_2) = N_e\cos^2(\theta_1-\theta_2) + N_0 \tag{5}$$

II. EXPERIMENTAL SETUP

Figure 1 shows a schematic of the experimental setup. A THORLABS LDM405 Laser Diode Module produces a

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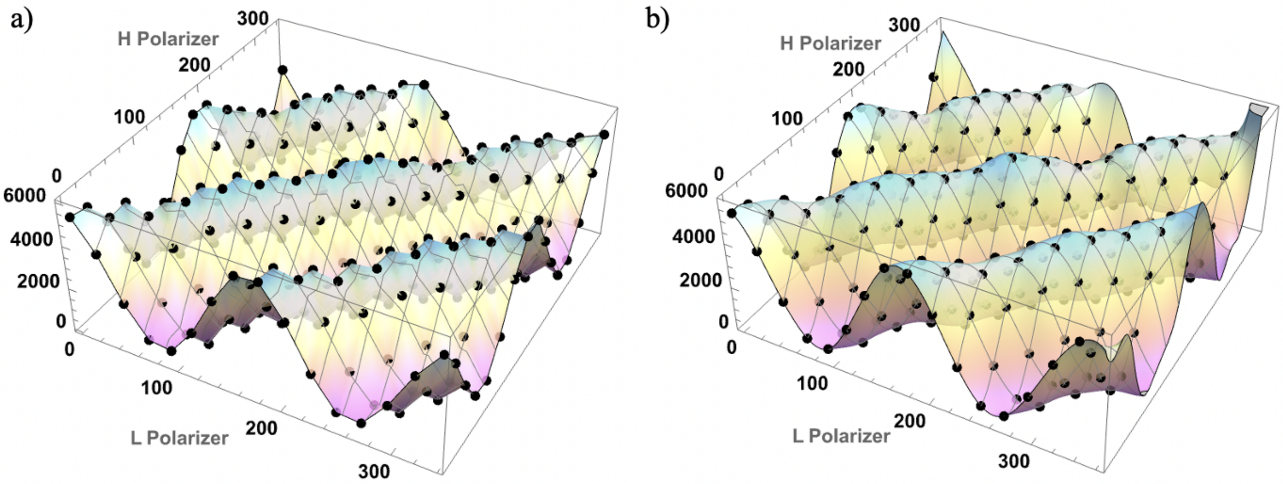

FIG. 2. Surface plots of (a) the raw coincidence counts and (b) the spline interpolated coincidence counts vs. polarization angles. The axis labeled L polarizer (H polarizer) corresponds to the angle of the polarizer on the lower (higher) rail.

beam of violet photons at 405 nm, which passes through a polarizing beam splitter to filter out the unwanted (horizontal) polarization. A dielectric-coated mirror (between PBS and HWP, not shown in the schematic) selectively reflects only photons at the desired wavelength into the setup. A half-wave plate then controls the polarization state of the pump photons before they reach the BBO crystals for SPDC. The position of the laser was adjusted such that the focus coincides with the plane of the BBO crystals for maximal conversion efficiency.

The rails are positioned at 4° from the middle line for capturing downconverted photons at 810 nm. This angle was selected to fall within the calculated emission range of the entangled photons and, combined with the angular acceptance of the lenses, ensures coverage of the entire range of possible emission angles. A red alignment laser was used to align the lenses, each with a focal length of 12.5 cm, so that the beams emerging from the crystals are focused onto the centers of the detectors—two single-photon counting modules (SPCMs). The polarizers were oriented such that their rotation does not distort the transmitted beams. After optimizing the alignment with the actual laser and crystals and setting both polarizers to vertical, coincidence count rates exceeding 1000 counts per second were observed. Finally, a laser power meter probe was placed between the rails to monitor the laser power throughout the experiment.

To generate the entangled state $|HH\rangle + |VV\rangle$, the half-wave plate preceding the BBO crystals was adjusted from an initial angle of 22.5°. The goal was to obtain equal coincidence count rates when the polarizers were set to (0°, 0°) and (90°, 90°). After fine-tuning, this balance was achieved at a half-wave plate angle of 27°, giving count rates of $520 \pm 15\ s^{-1}$ and $510 \pm 15\ s^{-1}$, respectively.

III. RESULTS

Using the data acquisition time of $T = 10$ seconds, the coincidence data versus polarization angles is plotted in Fig. 2. The dark count rate was 3.67 ± 0.88 per 10 seconds, and the laser power stayed within a 0.1 mW range throughout the measurement window (1.61-1.70 mW). The data was collected for polarization angles ranging from 0° to 337.5° with a step of 22.5°. There was, however, an angle offset of -9.186° caused by a mismatch between one of the polarizers' actual polarization and the scales on the rotation mount. This angle offset was corrected, and the mean coincidence count versus angle difference is plotted in Fig. 3a. The data points were fitted to Eq. 5, giving the best fit parameters $N_e = 4730.77 \pm 35.30$ and $N_0 = 455.33 \pm 18.51$ per 10 seconds. It is worth noting that if the 22.5° data points were excluded, the remaining data could be well fitted with a linear model (Fig. 3b), which would be consistent with the predictions of a local hidden variable theory. The entangled state's phase shift away from the desired $|HH\rangle + |VV\rangle$ state can then be estimated by choosing $\theta_1 - \theta_2 = 80.814°$ (with the angle offset corrected) and fitting the plot of coincidence count versus $\theta_1 + \theta_2$ to Eq. 4. From this fit, the phase shift is estimated to be $\phi = 37.18 \pm 2.28°$. The data points and their fit are shown in Fig. 4.

IV. ANALYSIS

To test the nonlocal nature of polarization entanglement, the measurement results can be used to evaluate the Clauser–Horne–Shimony–Holt (CHSH) Bell parameter S. According to local hidden variable theories

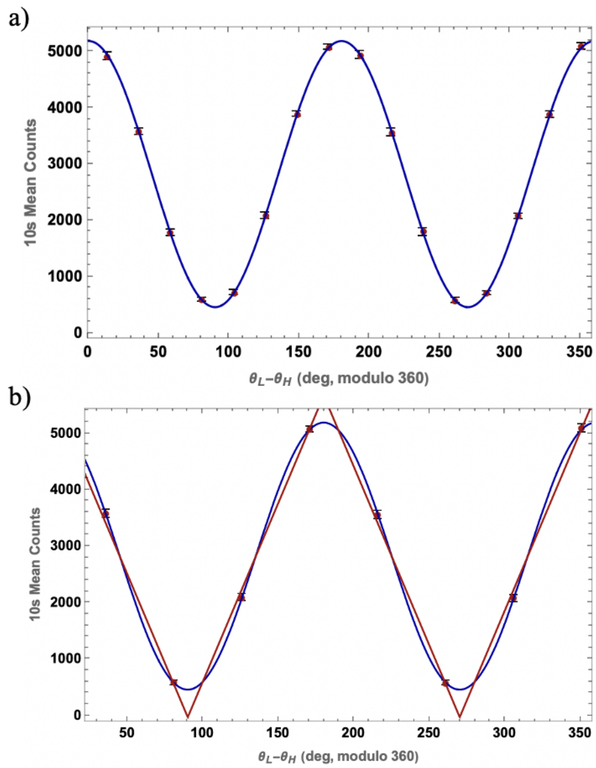

FIG. 3. Plots of the mean coincidence count versus angle difference after the angle offset correction (a) with and (b) without the 22.5° data points. The blue curves are a fit to Eq. 5, and the red curve is a fit to a linear model.

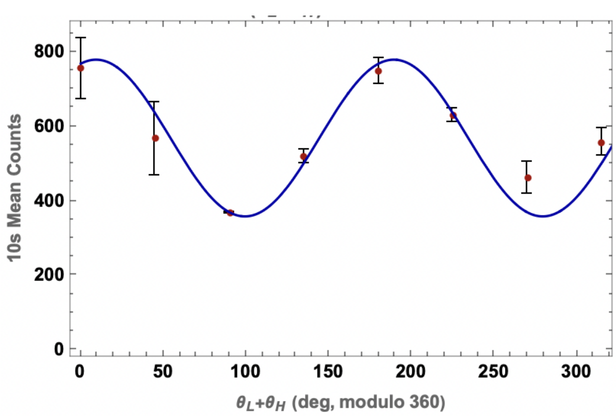

FIG. 4. Plot of the mean coincidence count versus angle sum for a fixed angle difference of 80.814° (corrected from 90°). The curve is a fit to Eq. 4 using this angle difference.

(HVTs), S cannot exceed 2. However, quantum mechanics predicts values up to $S^{(QM)} = 2\sqrt{2}$ for maximally entangled states. To calculate S, the correlation coefficients $E(\theta_1, \theta_2)$ must first be determined as follows:

$$E(\theta_1,\theta_2) \equiv P(\theta_1,\theta_2)+P(\theta_{1\perp},\theta_{2\perp})-P(\theta_1,\theta_{2_\perp})-P(\theta_{1\perp},\theta_2) \tag{6}$$

where $\theta_{i\perp} = \theta_i + 90°$. Then, the CHSH parameter can be calculated as

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b') \tag{7}$$

where a, a', b, b' are different polarizer angles. The probabilities P required for calculating E can be expressed in terms of the recorded number of coincidences N as

$$P(\theta_1,\theta_2) = N(\theta_1,\theta_2)/N_{tot}, \tag{8}$$

where $N_{tot} \equiv N(\theta_1,\theta_2) + N(\theta_{1\perp},\theta_{2\perp}) + N(\theta_1,\theta_{2_\perp}) + N(\theta_{1\perp},\theta_2)$ is the total number of pairs detected. The quantity $E(\theta_1,\theta_2)$ can then be expressed as

$$E(\theta_1,\theta_2) = \frac{N(\theta_1,\theta_2)+N(\theta_{1\perp},\theta_{2\perp})-N(\theta_1,\theta_{2_\perp})-N(\theta_{1\perp},\theta_2)}{N(\theta_1,\theta_2)+N(\theta_{1\perp},\theta_{2\perp})+N(\theta_1,\theta_{2_\perp})+N(\theta_{1\perp},\theta_2)}. \tag{9}$$

This means that a total of 16 measurements are needed. Table I shows the selected set of measurements for calculating S.

By choosing $a = 0°, a' = 45°, b = 22.5°, b' = 67.5°$ (see Eq. 7), the relevant correlation coefficients are determined to be

$$\begin{aligned} E(a,b) &= E(0°,22.5°) = 0.491, \\ E(a,b') &= E(0°,67.5°) = -0.969, \\ E(a',b) &= E(45°,22.5°) = 0.870, \\ E(a',b') &= E(45°,67.5°) = 0.291. \end{aligned} \tag{10}$$

From these, S is found to be

$$S = 2.621 > 2. \tag{11}$$

To verify the validity of this result, the statistical uncertainty can be computed. Since the uncertainty of N_i is $\sigma_{N_i} = \sqrt{N_i}$, the uncertainty of S is given by

$$\sigma_S = \sqrt{\sum_{i=1}^{16}\left(\sigma_{N_i}\frac{\partial S}{\partial N_i}\right)^2} = \sqrt{\sum_{i=1}^{16} N_i\left(\frac{\partial S}{\partial N_i}\right)^2}. \tag{12}$$

The expressions for $\left(\frac{\partial S}{\partial N_i}\right)^2$ can be evaluated as follows:

$$\left(\frac{\partial S}{\partial N_i}\right)^2 = \frac{4(N(\alpha,\beta_\perp)+N(\alpha_\perp,\beta))^2}{(N(\alpha,\beta)+N(\alpha_\perp,\beta_\perp)+N(\alpha_\perp,\beta)+N(\alpha,\beta_\perp))^4}$$
$$\text{for } N_i \in \{N(\alpha,\beta), N(\alpha_\perp,\beta_\perp)\}, \tag{13}$$

and

$$\left(\frac{\partial S}{\partial N_i}\right)^2 = \frac{4(N(\alpha,\beta)+N(\alpha_\perp,\beta_\perp))^2}{(N(\alpha,\beta)+N(\alpha_\perp,\beta_\perp)+N(\alpha_\perp,\beta)+N(\alpha,\beta_\perp))^4}$$
$$\text{for } N_i \in \{N(\alpha_\perp,\beta), N(\alpha,\beta_\perp)\} \tag{14}$$

θ_1 (°)	θ_2 (°)	N_1	N_2	N_{ac}	N
0	22.5	171589	208858	16.1	3440
0	67.5	169477	209299	16	125
0	112.5	168718	207993	15.8	1296
0	157.5	168669	208427	15.8	4731
45	22.5	173103	216115	16.8	4536
45	67.5	171529	216391	16.7	3107
45	112.5	166178	215199	16.1	312
45	157.5	167858	215494	16.3	1868
90	22.5	170010	209269	16	1119
90	67.5	169031	208992	15.9	4647
90	112.5	165121	208407	15.5	3631
90	157.5	167708	209140	15.8	24
135	22.5	168422	199837	15.1	326
135	67.5	166180	199720	14.9	1610
135	112.5	163254	198080	14.6	4622
135	157.5	165370	197989	14.7	3222

TABLE I. Single counts (N_1, N_2) and coincidence counts with accidental count and fit rate offset subtracted (N) for different polarization angles θ_1, θ_2 (without the angle offset correction). The acquisition time is again $T = 10$ seconds. The accidental coincidences ($N_{ac} = \tau N_1 N_2 / T$) are also shown, assuming $\tau = 4.5$ ns based on the comparator used.

This gives $\sigma_S = 0.015$. Thus, the final value of the CHSH Bell parameter is

$$S = 2.621 \pm 0.015, \tag{15}$$

a violation of the CHSH Bell inequality by more than 40 standard deviations. This result decisively rules out the HVTs and agrees with the predictions of quantum mechanics.

V. CONCLUSION

In summary, using spontaneous down-conversion in a simple two-crystal geometry with a violet laser diode as the source, we have successfully created polarization-entangled photon pairs described by the wavefunction $|\psi\rangle = (|HH\rangle + e^{i\phi}|VV\rangle)/\sqrt{2}$. Polarization-sensitive coincidence measurements reveal strong polarization correlations, in agreement with the predictions of quantum mechanics. Using this setup, we observed a CHSH Bell inequality violation exceeding 40 standard deviations, clearly ruling out local hidden variable theories.

ACKNOWLEDGMENTS

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[2] P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, Ultrabright source of polarization-entangled photons, Phys. Rev. A **60**, R773 (1999).