



## **BRAC University**

Department of Mathematics and Natural Sciences

LECTURE ON

### **Real Analysis (MAT221)**

## **Monotone Sequences and Subsequences**

**Monotone Convergence, Bolzano-Weierstrass Theorem**

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CONDUCTED BY

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## Monotone Sequences

### ■ Monotonically Increasing Sequence

A sequence  $\{a_n\}$  is said to be **monotonically increasing** if for all natural numbers  $n$ , the terms of the sequence satisfy the inequality

$$a_n \leq a_{n+1}.$$

### ■ Monotonically Decreasing Sequence

A sequence  $\{a_n\}$  is said to be **monotonically decreasing** if for all natural numbers  $n$ , the terms of the sequence satisfy the inequality

$$a_n \geq a_{n+1}.$$

## Monotone Convergence Theorem (MCT)

### 💡 Monotone Convergence Theorem

If a sequence is monotone and bounded, then it converges.

## ② Problem

Prove that the sequence defined by  $x_1 = 3$  and

$$x_{n+1} = \frac{1}{4 - x_n}$$

converges and find the limit.

### ② Problem

Prove that the sequence defined by  $x_1 = 1$  and

$$x_{n+1} = 4 - \frac{1}{x_n}$$

converges and find the limit.



### ② Problem

Show that

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots$$

converges and find the limit.



## ② Problem

Let  $x_1 = 2$ , and define

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{2}{x_n} \right).$$

1. Show that  $x_n^2$  is always greater than 2, and then use this to prove that  $x_n - x_{n+1} \geq 0$ . Conclude that  $\lim x_n = \sqrt{2}$ .
2. Modify the sequence  $(x_n)$  so that it converges to  $\sqrt{c}$ .

# Thank You!

We'd love your questions and feedback.

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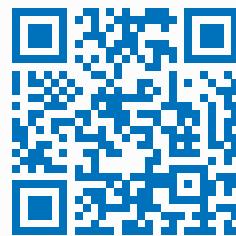
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(Lectures, walkthroughs, and course updates)



Scan for the channel

## References

- [1] Stephen Abbott, *Understanding Analysis*, 2nd Edition, Springer, 2015.
- [2] Terence Tao, *Analysis I*, 3rd Edition, Texts and Readings in Mathematics, Hindustan Book Agency, 2016.