



BRAC University

Department of Mathematics and Natural Sciences

LECTURE ON

Real Analysis (MAT221)

Subsequences and Limit Points

Bolzano-Weierstrass Theorem (BWT)

NOVEMBER 02, 2025

CONDUCTED BY

Partho Sutra Dhor

Lecturer, BRAC University, Dhaka-1212

partho.dhor@bracu.ac.bd | parthosutradhor@gmail.com
For updates subscribe on [@ParthoSutraDhor](#)

Subsequence

Subsequence

Let (a_n) be a sequence of real numbers, and let



$$n_1 < n_2 < n_3 < n_4 < n_5 < \dots$$

be an increasing sequence of natural numbers. Then the sequence

$$a_{n_1}, a_{n_2}, a_{n_3}, a_{n_4}, a_{n_5}, \dots$$

is called a *subsequence* of (a_n) and is denoted by (a_{n_k}) , where $k \in \mathbb{N}$ indexes the subsequence.

Examples of Subsequence

Theorem

Any subsequence of a convergent sequence converge to the same limit as the original sequence.

Bolzano–Weierstrass Theorem

Bolzano–Weierstrass Theorem

Every bounded sequence contains a convergent subsequence.

NIP \implies BWT

Let (a_n) be a bounded sequence. Using the Nested Interval property, show that there exists a convergent subsequence.

Aoc \implies BWT

Let (a_n) be a bounded sequence. Using the Axiom of Completeness, show that there exists a convergent subsequence.

Theorem

Assume (a_n) is a bounded sequence with the property that every convergent subsequence of (a_n) converges to the same limit $a \in \mathbb{R}$.

Show that (a_n) must converge to a .

Theorem

Every sequence contains a monotonic subsequence.



Limit Point or Cluster Point

Limit Point

A real number l is said to be a limit point or cluster point of a sequence (a_n) if for every $\varepsilon > 0$, there exists a term a_m such that

$$|a_m - l| < \varepsilon.$$

Examples of Limit Points

Theorem

If a sequence (a_n) converges to l , then l is the only limit point of the sequence.

Bolzano–Weierstrass Theorem (Limit Point Version)

💡 Bolzano–Weierstrass Theorem

Every bounded sequence has at least one limit point.



Limit Superior and Inferior

Limit Superior

The Limit Superior of a bounded sequence is the largest limit point.



Limit Inferior

The Limit Inferior of a bounded sequence is the smallest limit point.



Thank You!

We'd love your questions and feedback.

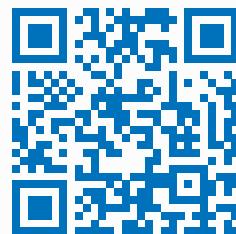
Partho Sutra Dhor

Lecturer, BRAC University, Dhaka-1212

 partho.dhor@bracu.ac.bd |  parthosutradhor@gmail.com



(Lectures, walkthroughs, and course updates)



Scan for the channel

References

- [1] Stephen Abbott, *Understanding Analysis*, 2nd Edition, Springer, 2015.
- [2] Terence Tao, *Analysis I*, 3rd Edition, Texts and Readings in Mathematics, Hindustan Book Agency, 2016.