BRAC University

Department of Mathematics and Natural Sciences

Lecture on

Real Analysis (MAT221)

Subsequences and Limit Points

Bolzano-Weierstrass Theorem (BWT)

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Conducted by

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Subsequence

Subsequence

Let (a_n) be a sequence of real numbers, and let

$$n_1 < n_2 < n_3 < n_4 < n_5 < \cdots$$

be an increasing sequence of natural numbers. Then the sequence

$$a_{n_1}, a_{n_2}, a_{n_3}, a_{n_4}, a_{n_5}, \cdots$$

is called a *subsequence* of (a_n) and is denoted by (a_{n_j}), where $j \in \mathbb{N}$ indexes the subsequence.

Examples of Subsequence

Theorem

Subsequences of a convergent sequence converge to the same limit as the original sequence.

Bolzano–Weierstrass Theorem

Bolzano–Weierstrass Theorem

Every bounded sequence contains a convergent subsequence.

NIP $\implies$ BWT

Let (a_n) be a bounded sequence. Using the Nested Interval property, show that there exists a convergent subsequence.

Aoc $\implies$ BWT

Let (a_n) be a bounded sequence. Using the Axiom of Completeness, show that there exists a convergent subsequence.

Theorem

Assume (a_n) is a bounded sequence with the property that every convergent subsequence of (a_n) converges to the same limit $a \in \mathbb{R}$. Show that (a_n) must converge to a.

Thank You!

We'd love your questions and feedback.

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References

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[2] Terence Tao, *Analysis I*, 3rd Edition, Texts and Readings in Mathematics, Hindustan Book Agency, 2016.