



BRAC University

Department of Mathematics and Natural Sciences

LECTURE ON

Real Analysis (MAT221)

Constructing the Rationals

Equivalence Relation and Constructing the Rational Numbers


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CONDUCTED BY

Partho Sutra Dhor

Lecturer, BRAC University, Dhaka-1212

✉ partho.dhor@bracu.ac.bd | ✉ parthosutradhor@gmail.com

For updates subscribe on  [@ParthoSutraDhor](#)

Cartesian Product and Relation

Relation Between Two Sets


A *relation* R from a set A to a set B is any subset of the Cartesian product $A \times B$, i.e.

$$R \subseteq A \times B.$$

If $(a, b) \in R$, we say that “ a is related to b by R ,” written as $a R b$ or sometimes $a \sim b$.


Equivalence Relation and Examples

Equivalence Relation

A relation \sim on a set A is called an **equivalence relation** if 

1. **Reflexive:** $a \sim a$, for all $a \in A$.
2. **Symmetric:** If $a \sim b$, then $b \sim a$, for all $a, b \in A$.
3. **Transitive:** If $a \sim b$ and $b \sim c$, then $a \sim c$, for all $a, b, c \in A$.

Important examples of equivalence relations

1. **Congruence modulo n on \mathbb{Z} :** For a fixed integer $n > 1$, define 

$$a \sim b \iff a \equiv b \pmod{n}.$$

2. **Rational Difference on \mathbb{R} :** On the set of real numbers, define

$$a \sim b \iff a - b \in \mathbb{Q}.$$

3. **Congruent Triangles:** For triangles $\triangle ABC$ and $\triangle DEF$, define

$$\triangle ABC \sim \triangle DEF \iff \triangle ABC \cong \triangle DEF$$

4. **Parallel Lines in the Plane:** For lines l_1 and l_2 in \mathbb{R}^2 , define

$$l_1 \sim l_2 \iff l_1 \parallel l_2.$$

5. **Same Slope of Nonzero Vectors:** In $\mathbb{R}^2 \setminus \{(0, 0)\}$, define

$$(x_1, y_1) \sim (x_2, y_2) \iff \exists k \neq 0 \text{ such that } (x_1, y_1) = k(x_2, y_2).$$

❓ Problem

Congruence modulo n on \mathbb{Z} : For a fixed integer $n > 1$, define 

$$a \sim b \iff a \equiv b \pmod{n}.$$

Prove that \sim is an equivalence relation.

❓ Problem

Rational Difference on \mathbb{R} : On the set of real numbers, define

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🔍 Problem

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Prove that \sim is an equivalence relation.

Equivalence Classes

Equivalence Class

If \sim is an equivalence relation on a set A and $a \in A$, the **equivalence class** of a is

$$[a] = \{ x \in A \mid x \sim a \}.$$

Problem

Define the equivalence relation \sim on \mathbb{Z} by

$$a \sim b \iff a \equiv b \pmod{5} \iff a - b \text{ is divisible by } 5.$$

Find the equivalence class of 3.

Problem

In $\mathbb{R}^2 \setminus \{(0, 0)\}$, define the equivalence relation \sim by



$$(x_1, y_1) \sim (x_2, y_2) \iff \exists k \neq 0 \text{ such that } (x_1, y_1) = k(x_2, y_2).$$

Find the equivalence class of $(1, 2)$.

Partitions from Equivalence Relations

Integers to Rational Numbers

“God made the integers; all else is the work of man.”

— Leopold Kronecker (1823–1891)

Rational Numbers to Reals (complete)

Reals to Complex (algebraically closed)

Basic Topology of \mathbb{R}

Sequence and Limit

Functional Limits and Continuity

The Derivative

Sequences and Series of Functions

The Riemann Integral

Thank You!

We'd love your questions and feedback.

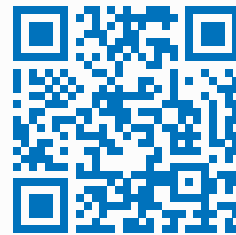
Partho Sutra Dhor

Lecturer, BRAC University, Dhaka-1212

✉ partho.dhor@bracu.ac.bd | ✉ parthosutradhor@gmail.com

 **@ParthoSutraDhor**

(Lectures, walkthroughs, and course updates)



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References

- [1] Stephen Abbott, *Understanding Analysis*, 2nd Edition, Springer, 2015.
- [2] Terence Tao, *Analysis I*, 3rd Edition, Texts and Readings in Mathematics, Hindustan Book Agency, 2016.