



**BRAC University**

Department of Mathematics and Natural Sciences

LECTURE ON

**Real Analysis (MAT221)**

# Monotone Sequences

## Monotone Convergence Theorem (MCT)

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CONDUCTED BY

**Partho Sutra Dhor**

Lecturer, BRAC University, Dhaka-1212

✉ [partho.dhor@bracu.ac.bd](mailto:partho.dhor@bracu.ac.bd) | ✉ [parthosutradhor@gmail.com](mailto:parthosutradhor@gmail.com)

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# Monotone Sequences

## Monotonically Increasing Sequence

A sequence  $\{a_n\}$  is said to be **monotonically increasing** if for all natural numbers  $n$ , the terms of the sequence satisfy the inequality

$$a_n \leq a_{n+1}.$$

## Monotonically Decreasing Sequence

A sequence  $\{a_n\}$  is said to be **monotonically decreasing** if for all natural numbers  $n$ , the terms of the sequence satisfy the inequality

$$a_n \geq a_{n+1}.$$



## Examples of Monotone Sequences

### ❓ Example

The sequence defined by

$$x_n = \frac{1}{n}$$

is a monotone sequence.



### ❓ Example

The sequence defined by  $x_1 = 3$  and

$$x_{n+1} = \frac{1}{4 - x_n}$$

is a monotone sequence.





### Example

The Sequence

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots$$

is a monotone sequence.



### Example

Let  $x_1 = 2$ , and define

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{2}{x_n} \right).$$


is a monotone sequence.





## Monotone Convergence Theorem (MCT)

### Monotone Convergence Theorem

If a sequence is monotone and bounded, then it converges. 



💡 **Aoc  $\implies$  MCT**

Let  $(a_n)$  be a monotone and bounded sequence. Using the Axiom of Completeness, show that the sequence  $(a_n)$  converges. 💡



### ❓ Problem

Prove that the sequence defined by  $x_1 = 3$  and

$$x_{n+1} = \frac{1}{4 - x_n}$$

converges and find the limit.





### 🔍 Problem

Prove that the sequence defined by  $x_1 = 1$  and

$$x_{n+1} = 4 - \frac{1}{x_n}$$

converges and find the limit.



### 🔍 Problem

Show that

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots$$

converges and find the limit.





### Problem

Let  $x_1 = 2$ , and define

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{2}{x_n} \right).$$

1. Show that  $x_n^2$  is always greater than 2, and then use this to prove that  $x_n - x_{n+1} \geq 0$ . Conclude that  $\lim x_n = \sqrt{2}$ .
2. Modify the sequence  $(x_n)$  so that it converges to  $\sqrt{c}$ .



# Thank You!

We'd love your questions and feedback.

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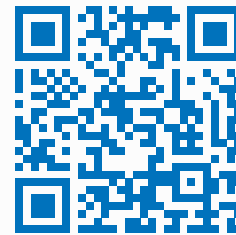
## Partho Sutra Dhor

Lecturer, BRAC University, Dhaka-1212

✉ [partho.dhor@bracu.ac.bd](mailto:partho.dhor@bracu.ac.bd) | ✉ [parthosutradhor@gmail.com](mailto:parthosutradhor@gmail.com)

 **@ParthoSutraDhor**

(Lectures, walkthroughs, and course updates)



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## References

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- [2] Terence Tao, *Analysis I*, 3rd Edition, Texts and Readings in Mathematics, Hindustan Book Agency, 2016.