



BRAC University

Department of Mathematics and Natural Sciences

LECTURE ON

Real Analysis (MAT221)

Monotone Sequences

Monotone Convergence Theorem (MCT)

NOVEMBER 02, 2025

CONDUCTED BY

Partho Sutra Dhor

Lecturer, BRAC University, Dhaka-1212

partho.dhor@bracu.ac.bd | parthosutradhor@gmail.com

For updates subscribe on [@ParthoSutraDhor](#)

Monotone Sequences

■ Monotonically Increasing Sequence

A sequence $\{a_n\}$ is said to be **monotonically increasing** if for all natural numbers n , the terms of the sequence satisfy the inequality

$$a_n \leq a_{n+1}.$$

■ Monotonically Decreasing Sequence

A sequence $\{a_n\}$ is said to be **monotonically decreasing** if for all natural numbers n , the terms of the sequence satisfy the inequality

$$a_n \geq a_{n+1}.$$

Examples of Monotone Sequences

① Example

The sequence defined by

$$x_n = \frac{1}{n}$$

is a monotone sequence.



② Example

The sequence defined by $x_1 = 3$ and

$$x_{n+1} = \frac{1}{4 - x_n}$$

is a monotone sequence.

② Example

The Sequence

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots$$

is a monotone sequence.

② Example

Let $x_1 = 2$, and define

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{2}{x_n} \right).$$

is a monotone sequence.

Monotone Convergence Theorem (MCT)

💡 Monotone Convergence Theorem

If a sequence is monotone and bounded, then it converges.

Aoc \implies MCT

Let (a_n) be a monotone and bounded sequence. Using the Axiom of Completeness, show that the sequence (a_n) converges.

② Problem

Prove that the sequence defined by $x_1 = 3$ and

$$x_{n+1} = \frac{1}{4 - x_n}$$

converges and find the limit.

② Problem

Prove that the sequence defined by $x_1 = 1$ and

$$x_{n+1} = 4 - \frac{1}{x_n}$$

converges and find the limit.

② Problem

Show that

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots$$

converges and find the limit.



② Problem

Let $x_1 = 2$, and define

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{2}{x_n} \right).$$

1. Show that x_n^2 is always greater than 2, and then use this to prove that $x_n - x_{n+1} \geq 0$. Conclude that $\lim x_n = \sqrt{2}$.
2. Modify the sequence (x_n) so that it converges to \sqrt{c} .

Limit Superior

Definition

The **limit superior** of a bounded sequence (a_n) is defined as

$$\limsup_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(\sup_{k \geq n} a_k \right).$$



Explain

Why limit superior exists?



Limit Inferior

Definition

The **limit inferior** of a bounded sequence (a_n) is defined as

$$\liminf_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(\inf_{k \geq n} a_k \right).$$



Explain

Why limit inferior exists?



② Problem

Find the Limit Superior and Limit Inferior of the sequence

$$x_n = (-1)^n + \frac{1}{n}$$



Theorem

For every bounded sequence a_n

$$\liminf_{n \rightarrow \infty} a_n \leq \limsup_{n \rightarrow \infty} a_n$$



Theorem

Any sequence a_n converges if and only if

$$\liminf_{n \rightarrow \infty} a_n = \limsup_{n \rightarrow \infty} a_n$$



Thank You!

We'd love your questions and feedback.

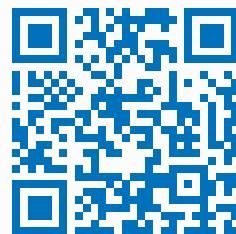
Partho Sutra Dhor

Lecturer, BRAC University, Dhaka-1212

 partho.dhor@bracu.ac.bd |  parthosutradhor@gmail.com



(Lectures, walkthroughs, and course updates)



Scan for the channel

References

- [1] Stephen Abbott, *Understanding Analysis*, 2nd Edition, Springer, 2015.
- [2] Terence Tao, *Analysis I*, 3rd Edition, Texts and Readings in Mathematics, Hindustan Book Agency, 2016.