



BRAC University

Department of Mathematics and Natural Sciences

LECTURE ON

Real Analysis (MAT221)

Constructing the Rationals

Equivalence Relation and Constructing the Rational Numbers

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CONDUCTED BY

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Cartesian Product and Relation

Relation Between Two Sets

A *relation* R from a set A to a set B is any subset of the Cartesian product $A \times B$, i.e.

$$R \subseteq A \times B.$$

If $(a, b) \in R$, we say that “ a is related to b by R ,” written as $a R b$ or sometimes $a \sim b$.

Equivalence Relation and Examples

Equivalence Relation

A relation \sim on a set A is called an **equivalence relation** if 

1. **Reflexive:** $a \sim a$, for all $a \in A$.
2. **Symmetric:** If $a \sim b$, then $b \sim a$, for all $a, b \in A$.
3. **Transitive:** If $a \sim b$ and $b \sim c$, then $a \sim c$, for all $a, b, c \in A$.

Important examples of equivalence relations

1. **Congruence modulo n on \mathbb{Z} :** For a fixed integer $n > 1$, define 

$$a \sim b \iff a \equiv b \pmod{n}.$$

2. **Rational Difference on \mathbb{R} :** On the set of real numbers, define

$$a \sim b \iff a - b \in \mathbb{Q}.$$

3. **Congruent Triangles:** For triangles $\triangle ABC$ and $\triangle DEF$, define

$$\triangle ABC \sim \triangle DEF \iff \triangle ABC \cong \triangle DEF$$

4. **Parallel Lines in the Plane:** For lines l_1 and l_2 in \mathbb{R}^2 , define

$$l_1 \sim l_2 \iff l_1 \parallel l_2.$$

5. **Same Slope of Nonzero Vectors:** In $\mathbb{R}^2 \setminus \{(0, 0)\}$, define

$$(x_1, y_1) \sim (x_2, y_2) \iff \exists k \neq 0 \text{ such that } (x_1, y_1) = k(x_2, y_2).$$

② Problem

Congruence modulo n on \mathbb{Z} : For a fixed integer $n > 1$, define

$$a \sim b \iff a \equiv b \pmod{n}.$$

Prove that \sim is an equivalence relation.

② Problem

Rational Difference on \mathbb{R} : On the set of real numbers, define

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Prove that \sim is an equivalence relation.

Equivalence Classes

Equivalence Class

If \sim is an equivalence relation on a set A and $a \in A$, the **equivalence class** of a is

$$[a] = \{x \in A \mid x \sim a\}.$$



Problem

Define the equivalence relation \sim on \mathbb{Z} by



$$a \sim b \iff a \equiv b \pmod{5} \iff a - b \text{ is divisible by } 5.$$

Find the equivalence class of 3.

② Problem

In $\mathbb{R}^2 \setminus \{(0, 0)\}$, define the equivalence relation \sim by

$$(x_1, y_1) \sim (x_2, y_2) \iff \exists k \neq 0 \text{ such that } (x_1, y_1) = k(x_2, y_2).$$

Find the equivalence class of $(1, 2)$.

Partitions from Equivalence Relations

Integers to Rational Numbers

“God made the integers; all else is the work of man.”

— Leopold Kronecker (1823–1891)

Rational Numbers to Reals (complete)

Reals to Complex (algebraically closed)

Basic Topology of \mathbb{R}

Sequence and Limit

Functional Limits and Continuity

The Derivative

Sequences and Series of Functions

The Riemann Integral

Thank You!

We'd love your questions and feedback.

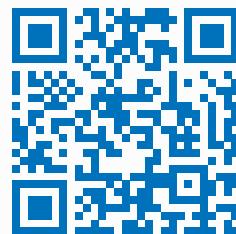
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(Lectures, walkthroughs, and course updates)



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References

- [1] Stephen Abbott, *Understanding Analysis*, 2nd Edition, Springer, 2015.
- [2] Terence Tao, *Analysis I*, 3rd Edition, Texts and Readings in Mathematics, Hindustan Book Agency, 2016.