



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT 215**

Complex Variables and Laplace Transformations

 **Name: Nusrat Miah**


 **Student ID: 24312256**

 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*


 **Partho Sutra Dhor**  
Lecturer, Department of MNS  
BRAC University

### Question 1

Find all possible values of  $z$  satisfying

$$z^7 = -\frac{2187}{2} - \frac{2187\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**


## Question 2

Describe the locus  $|z + 8i| + |z - 8i| = 19$  on the complex plane.

 Solution:

### Question 3

Describe the region  $\left| \frac{z+7i}{z-7i} \right| \geq 5$  on the complex plane.

 Solution:

### Question 4

Solve the equation

$$e^{2z} = 4\sqrt{2}(1 + i)$$

for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 **Solution:**

### Question 5

Prove that

$$\tanh^{-1} z = \frac{1}{2} \ln \left( \frac{1+z}{1-z} \right),$$

 Solution:

### Question 6

Solve for  $z$  where

$$\operatorname{cosec}^{-1} z = 2 - 9i$$

 Solution:

### Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}(z^2)}{|z|^2}$  does not exist.

 Solution:



## Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{3 \sin(6z)}{z - \sin z}}$$

 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{3z}$$

. Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all points of discontinuity of  $f(z)$ .


 **Solution:**

### Question 10

Using the definition, show that


$$f(z) = 6z\bar{z} - 8z + 8\bar{z}$$

is not differentiable at  $z = 0$ .

 **Solution:**

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{4}{5z+5}$  at  $z = z_0$ .


 Solution:

## Question 12

Consider the function

$$f(z) = 5 \sinh(4z) - 9 \cos(5z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

### Question 13

Consider the function

$$f(z) = 5ze^{-9z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

### Question 14

Show that the function

$$v(x, y) = 2e^{-4x} \cos(4y) - 2e^{2y} \sin(2x) + 24x^2y - 9x^2 - 8y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $u + vi$  becomes analytic.

 **Solution:**

### Question 15

Show that the function

$$u(x, y) = 6xe^{-9x}\cos(9y) + 6ye^{-9x}\sin(9y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 **Solution:**