



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: IBTIDA HOQUE CHOWDHURY

Student ID: 23201433

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^7 = 64\sqrt{2}(-1 - i).$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z - 9i| - |z + 9i| = 15$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z + 6i| + |z - 6i| \geq 15$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{9z} = -\frac{7}{2} + \frac{7\sqrt{3}i}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

? Question 5

Prove that

$$\cot^{-1} z = \frac{1}{2i} \ln \left(\frac{z+i}{z-i} \right),$$

 Solution:

? Question 6

Solve for z where

$$\tan^{-1} z = 2 + 3i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{7 \sin(9z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{8z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 8z^2 + 9z - 9$$

is differentiable at all points. Also find the derivative.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{8}{z^2}$ at $z = 4 + 8i$.

 Solution:

Question 12

Consider the function

$$f(z) = 8 \sin(6z) - 6 \cosh(2z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 6|z|^2 + 8z - 7\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$u(x, y) = 4 \sin(9x) \cosh(9y) + 15x^2y - 5x^2 - 5y^3 + 5y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$u(x, y) = 3xe^{-9x} \cos(9y) + 3ye^{-9x} \sin(9y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution: