🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

📋 Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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👥 Section: 12

📅 Semester: FALL 2025

🕐 Submission Date: _____

Assigned by

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❓ Question 1

Find all possible values of z such that

$$z^6 = 32\sqrt{2}\,(1 + i)$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

✏ **Solution:**

❷ Question 2

Describe the above locus $|z - 8i| - |z + 8i| = 13$ in the complex plane.

✏️ **Solution:**

❓ Question 3

Describe the above locus $|z - 8i| - |z + 8i| \geq 10$ in the complex plane.

✏️ **Solution:**

❓ Question 3

Describe the above locus $|z - 8i| - |z + 8i| \geq 10$ in the complex plane.

✏️ **Solution:**

❷ Question 4

Solve the following equation for z:

$$e^{5z} = \frac{3\sqrt{2}\,(-1+i)}{2}$$

Express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✎ **Solution:**

❓ **Question 5**

Prove that

$$\tan^{-1} z = \frac{1}{2i} \ln\left(\frac{1+iz}{1-iz}\right),$$

✏️ **Solution:**

❓ **Question 6**

Solve for z:

$$\sin^{-1} z = 4 + 6i$$

✒️ **Solution:**

❓ **Question 7**

Using the definition of limit, show that $\lim\limits_{z \to 0} \dfrac{\text{Re}\{z^2\}}{|z|^2}$ does not exist.

✏️ **Solution:**

❷ Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\tan z}{z} \right)^{\frac{7 \sin(4z)}{z - \sin z}}$$

✏️ **Solution:**

❓ Question 9

Consider the function

$$f(z) = \frac{\tan 3z}{8z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

✒ **Solution:**

❓ Question 10

Using the definition show that

$$f(z) = 3z\bar{z} - 9z + 5\bar{z}$$

is not differentiable at $z = 0$.

✏️ **Solution:**

❓ Question 11

Using the definition, find the derivative of $f(z) = \dfrac{7}{z^2}$ at $z = 4 + 7i$.

🖋 **Solution:**

❓ Question 12

Consider the fuction $f(z)$ defined by

$$f(z) = 3\sinh(5z) - 4\cos(9z)$$

Using C-R equations determine whether the function is analytic or not.

✏ **Solution:**

❓ Question 13

Consider the fuction $f(z)$ defined by

$$f(z) = 6|z|^2 + 2z - 4\bar{z}$$

Using C-R equations determine whether the function is analytic or not.

✏️ **Solution:**

❓ Question 14

Show that the given function v defined by

$$v(x, y) = 9e^{-8x}\cos(8y) - 7e^{6y}\sin(6x) + 12x^2y - 6x^2 - 4y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate u of v such that $\mathbf{u+vi}$ becomes analytic.

✏️ **Solution:**

❓ **Question 15**

Show that the given function v defined by

$$v(x, y) = 2xe^{-8x} \cos(8y) + 2ye^{-8x} \sin(8y)$$

is harmonic. Find the harmonic conjugate u of v such that $\mathbf{u+vi}$ becomes analytic.

✏️ **Solution:**