BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT 215

Complex Variables and Laplace Transformations

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Section: 12

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^6 = -32\sqrt{3} - 32i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z + 7i| + |z - 7i| = 16$ on the complex plane.

Solution:

Question 3

Describe the region $\left|\frac{z+7i}{z-7i}\right| > 4$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{5z} = -\frac{3}{2} + \frac{3\sqrt{3}i}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\tan^{-1} z = \frac{1}{2i} \ln\left(\frac{1+iz}{1-iz}\right),$$

Solution:

Question 6

Solve for z where

$$\coth^{-1} z = 2 + 7i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}(z^2)}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{7\sin(2z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{6z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 6z^2 + 7z - 7$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{6}{z^2}$ at $z = 5 + 9i$.

Solution:

Question 12

Consider the function

$$f(z) = 4\sinh(8z) - 6\cos(2z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 7ze^{-2z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 6\sin(6x)\cosh(6y) \; + \; 6\,x^2y \; - \; 8x^2 \; - \; 2y^3 \; + \; 8y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 7\,xe^{-3x}\cos(3y) \;+\; 7\,ye^{-3x}\sin(3y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: