



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^5 = 16\sqrt{2}(1 + i).$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 7| + |z - 7| = 15$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z - 4i| - |z + 4i| \leq 6$ on the complex plane.

 Solution:

? Question 4

Solve the equation

$$e^{5z} = -\frac{5}{2} + \frac{5\sqrt{3}i}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\operatorname{sech}^{-1} z = \ln \left(\frac{1 + \sqrt{1 - z^2}}{z} \right),$$

 Solution:

? Question 6

Solve for z where

$$\coth^{-1} z = 6 + 8i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{5 \sin(4z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 8z}{2z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 8z^2 + 9z - 3$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{4}{6z + 6}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 9 \sin(2z) - 5 \cosh(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 2|z|^2 + 7z - 6\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$u(x, y) = 4 \sin(9x) \cosh(9y) + 12x^2y - 4x^2 - 4y^3 + 4y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$u(x, y) = 3xe^{-2x} \cos(2y) + 3ye^{-2x} \sin(2y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution: