🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

📋 Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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👥 **Section: 12**

📅 **Semester: FALL 2025**

🕐 Submission Date: _____

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❷ Question 1

Find all possible values of z satisfying

$$z^6 = \frac{729}{2} - \frac{729\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

✏️ **Solution:**

❷ Question 2

Describe the locus $|z + 5i| + |z - 5i| = 11$ on the complex plane.

✏️ **Solution:**

❓ **Question 3**

Describe the region $|z - 6| - |z + 6| \geq 7$ on the complex plane.

✏️ **Solution:**

❓ Question 4

Solve the equation

$$e^{5z} = \frac{3\sqrt{2}\,(1+i)}{2}$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❓ **Question 5**

Prove that

$$\cos^{-1} z = \frac{1}{i} \ln\left(z + \sqrt{z^2 - 1}\right),$$

✏️ **Solution:**

❓ **Question 6**

Solve for z where

$$\operatorname{cosec}^{-1} z = 4 + 6i$$

✏️ **Solution:**

❓ **Question 7**

Using the definition of a limit, show that $\lim\limits_{z\to 0}\dfrac{\text{Im}\{z^2\}}{|z|^2}$ does not exist.

✏️ **Solution:**

❷ Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\sin z}{z} \right)^{\frac{8 \sin(2z)}{z - \sin z}}$$

✎ **Solution:**

❷ Question 9

Consider the function
$$f(z) = \frac{\tan 9z}{5z}.$$
Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

✏️ **Solution:**

❷ Question 10

Using the definition, show that

$$f(z) = 8z^2 + 3z - 5$$

is differentiable at all points. Also find the derivative.

✏ **Solution:**

❓ Question 11

Using the definition, find the derivative of $f(z) = \dfrac{8z - 7}{3z + 9i}$ at $z = i$.

✏️ **Solution:**

❓ **Question 12**

Consider the function

$$f(z) = 6\sinh(8z) - 2\cos(9z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❓ Question 13

Consider the function

$$f(z) = 9|z|^2 + 5z - 5\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❷ Question 14

Show that the function

$$u(x, y) = 2e^{-3x} \cos{(3y)} - 5e^{9y} \sin{(9x)} + 6x^2 y - 4x^2 - 2y^3 + 4y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

✏️ **Solution:**

❓ **Question 15**

Show that the function

$$v(x, y) = 8xe^{-8x}\cos(8y) + 8ye^{-8x}\sin(8y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

✒️ **Solution:**