



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z such that

$$z^7 = -64\sqrt{3} - 64i$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the above locus $|z - 7i| - |z + 7i| = 8$ in the complex plane.

 **Solution:**

Question 3

Describe the above locus $\left| \frac{z + 8i}{z - 8i} \right| \leq 6$ in the complex plane.

 Solution:

? Question 4

Solve the following equation for z :

$$e^{3z} = -\frac{7}{2} + \frac{7\sqrt{3}i}{2}$$

Express z as $x + iy$ where $x, y, \in \mathbb{R}$.

 **Solution:**

Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

 Solution:

? Question 6

Solve for z :

$$\tanh^{-1} z = 6 + 2i$$

 Solution:

? Question 7

Using the definition of limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{7 \sin(2z)}{z - \sin z}}$$

 **Solution:**

Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{6z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition show that

$$f(z) = 6z^2 + 8z - 2$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{9}{7z + 5}$ at $z = z_0$.

 Solution:

Question 12

Consider the function $f(z)$ defined by

$$f(z) = 3 \sin(3z) - 5 \cosh(4z)$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

Question 13

Consider the function $f(z)$ defined by

$$f(z) = 4ze^{-4z}$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

? Question 14

Show that the given function v defined by

$$v(x, y) = 2e^{-3x} \cos(3y) - 3e^{6y} \sin(6x) + 18x^2y - 3x^2 - 6y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate u of v such that $\mathbf{u} + \mathbf{v}\mathbf{i}$ becomes analytic.

 Solution:

? Question 15

Show that the given function u defined by

$$u(x, y) = 7xe^{-5x} \cos(5y) + 7ye^{-5x} \sin(5y)$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u} + \mathbf{v}\mathbf{i}$ becomes analytic.

 Solution: