



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: TOWSIF HASSAN

Student ID: 23341043

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^7 = 128i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z - 8i| - |z + 8i| = 12$ on the complex plane.

 **Solution:**

Question 3

Describe the region $|z + 9i| + |z - 9i| \leq 24$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{8z} = -9$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

Question 5

Prove that

$$\tanh^{-1} z = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right),$$

 Solution:

? Question 6

Solve for z where

$$\tanh^{-1} z = 9 - 4i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{2 \sin(6z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{7z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

? Question 10

Using the definition, show that

$$f(z) = 5z\bar{z} - 3z + 6\bar{z}$$

is not differentiable at $z = 0$.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{2}{z^2}$ at $z = 4 + 7i$.

 Solution:

Question 12

Consider the function

$$f(z) = 4 \sin(7z) - 7 \cosh(9z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 2|z|^2 + 3z - 6\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$u(x, y) = 6e^{-2x} \cos(2y) - 4e^{2y} \sin(2x) + 27x^2y - 8x^2 - 9y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution:

Question 15

Show that the function

$$v(x, y) = 2xe^{-7x} \cos(7y) + 2ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**