



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 15**

 **Assignment-01**

**Course Code: MAT215**

Complex

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 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*


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### Question 1

Find all possible values of  $z$  such that

$$z^6 = 32\sqrt{3} - 32i$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

 Solution:

## Question 2

Consider the equation

$$|z + 7i| + |z - 7i| = 19$$

Describe the above locus in the complex plane.


 Solution:

### Question 3

Consider the inequality

$$|z + 6| + |z - 6| < 13$$

Describe the above locus in the complex plane.

 Solution:

### Question 4

Solve the following equation for  $z$ :

$$e^{4z} = \frac{7\sqrt{3}}{2} + \frac{7i}{2}$$

Express  $z$  as  $x + iy$  where  $x, y, \in \mathbb{R}$ .

 **Solution:**

### Question 5

Prove that

$$\sinh^{-1} z = \ln \left( z + \sqrt{z^2 + 1} \right),$$

 Solution:

### Question 6

Solve for  $z$ :

$$\operatorname{cosech}^{-1} z = 2 + 2i$$

 Solution:

### Question 7

Show that the limit,  $\lim_{z \rightarrow 0} \frac{\text{Im}\{z^2\}}{|z|^2}$  does not exist.


 Solution:



## Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\sin z}{z} \right)^{\frac{9 \sin(7z)}{z - \sin z}}$$

 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{9z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .

 **Solution:**

### Question 10

Using the definition show that

$$f(z) = 7z^2 + 5z - 8$$

is differentiable at all points. Also find the derivative.

 **Solution:**

### Question 11

Using the definition, find the derivative of

$$f(z) = \frac{7}{5z+3} \quad \text{at } z = z_0$$

.

 Solution:

## Question 12

Consider the function  $f(z)$  defined by

$$f(z) = 8 \sin(7z) - 6 \cosh(6z)$$

Using C-R equations determine whether the function is analytic or not.


 **Solution:**

### Question 13

Consider the function  $f(z)$  defined by

$$f(z) = 9|z|^2 + 6z - 7\bar{z}$$

Using C-R equations determine whether the function is analytic or not.


 **Solution:**

### Question 14

Show that the given function  $u$  defined by

$$u(x, y) = 8e^{-6x} \cos(6y) - 5e^{9y} \sin(9x) + 15x^2y - 5x^2 - 5y^3 + 5y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $\mathbf{u} + \mathbf{vi}$  becomes analytic.

 **Solution:**

### Question 15

Show that the given function  $v$  defined by

$$v(x, y) = 2xe^{-7x} \cos(7y) + 2ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $\mathbf{u}+\mathbf{vi}$  becomes analytic.

 **Solution:**