



BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

 **Assignment - 01**

Course Code: MAT215

Complex Variables & Laplace Transform

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 **Section: 12**

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 **Submission Date: _____**

Assigned by


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Question 1

Find all possible values of z satisfying

$$z^6 = 32\sqrt{2}(-1 - i).$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**


Question 2

Describe the locus $|z - 8| - |z + 8| = 12$ on the complex plane.

 Solution:

Question 3

Describe the region $|z + 7| + |z - 7| > 20$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{5z} = 3\sqrt{2}(1 - i)$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

 **Solution:**

Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

 Solution:

Question 6

Solve for z where

$$\tan^{-1} z = 9 - 8i$$

 Solution:

Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\text{Im}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{2 \sin(4z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 5z}{9z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.


 **Solution:**

Question 10

Using the definition, show that


$$f(z) = 5z\bar{z} - 4z + 8\bar{z}$$

is not differentiable at $z = 0$.

 **Solution:**

Question 11

Using the definition, find the derivative of $f(z) = \frac{4}{5z+2}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 8 \sin(8z) - 2 \cosh(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

Question 13

Consider the function

$$f(z) = 2ze^{-5z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

Question 14

Show that the function

$$u(x, y) = 4e^{-6x} \cos(6y) - 6e^{2y} \sin(2x) + 15x^2y - 2x^2 - 5y^3 + 2y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**

Question 15

Show that the function

$$u(x, y) = 7xe^{-4x} \cos(4y) + 7ye^{-4x} \sin(4y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**