BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^5 = -16\sqrt{3} + 16i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $\left|\frac{z+7i}{z-7i}\right| = 5$ on the complex plane.

Solution:

Question 3

Describe the region $\left|\frac{z+4i}{z-4i}\right| \geq 3$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{7z} = 5i$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\tan^{-1} z = \frac{1}{2i} \ln\left(\frac{1+iz}{1-iz}\right),$$

Solution:

Question 6

Solve for z where

$$\operatorname{cosec}^{-1} z = 3 - 5i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Re}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\tan z}{z}\right)^{\frac{9\sin(3z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 3z}{8z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 9z\bar{z} - 6z + 2\bar{z}$$

is not differentiable at $z = 0$.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{7}{z^2}$ at $z = 4 + 4i$.

Solution:

Question 12

Consider the function

$$f(z) = 3\sinh(2z) - 7\cos(3z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 6ze^{-5z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 9\sin(8x)\cosh(8y) + 15x^2y - 8x^2 - 5y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 7xe^{-6x}\cos(6y) + 7ye^{-6x}\sin(6y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: