



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: MOHAMMAD ZAWAAD MOSTOFA

Student ID: 24121143

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

? Question 1

Find all possible values of z satisfying

$$z^5 = -16 - 16\sqrt{3}i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $\left| \frac{z+4i}{z-4i} \right| = 6$ on the complex plane.

 Solution:

? Question 3

Describe the region $|z - 7i| - |z + 7i| \leq 13$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{3z} = 3\sqrt{2}(1+i)$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

Question 5

Prove that

$$\tanh^{-1} z = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right),$$

 Solution:

? Question 6

Solve for z where

$$\sinh^{-1} z = 4 - 3i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{8 \sin(2z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 4z}{3z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 8z^2 + 9z - 4$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{6}{9z + 7}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 2 \sinh(9z) - 8 \cos(9z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 6ze^{-7z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$v(x, y) = 8 \sin(2x) \cosh(2y) + 9x^2y - 9x^2 - 3y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$u(x, y) = 9xe^{-8x} \cos(8y) + 9ye^{-8x} \sin(8y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution: