



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^6 = \frac{729}{2} + \frac{729\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 4| + |z - 4| = 11$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z - 6i| - |z + 6i| < 7$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{2z} = 2\sqrt{2}(1 - i)$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

Question 5

Prove that

$$\operatorname{cosech}^{-1} z = \ln \left(\frac{1 + \sqrt{z^2 + 1}}{z} \right),$$

 Solution:

? Question 6

Solve for z where

$$\sinh^{-1} z = 4 + 8i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{2 \sin(6z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 9z}{3z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 7z^2 + 9z - 7$$

is differentiable at all points. Also find the derivative.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{4z - 9}{7z + 5i}$ at $z = i$.

 Solution:

Question 12

Consider the function

$$f(z) = 4 \sinh(3z) - 2 \cos(5z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 6ze^{-9z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$u(x, y) = 4 \sin(6x) \cosh(6y) + 24x^2y - 2x^2 - 8y^3 + 2y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$u(x, y) = 6xe^{-9x} \cos(9y) + 6ye^{-9x} \sin(9y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution: