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🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

📋 Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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👥 **Section: 12**

📅 **Semester: FALL 2025**

🕐 Submission Date: _____

Assigned by

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❓ Question 1

Find all possible values of z satisfying

$$z^5 = \frac{243\sqrt{3}}{2} - \frac{243i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

✏️ **Solution:**

❷ Question 2

Describe the locus $|z + 5| + |z - 5| = 16$ on the complex plane.

🖊 **Solution:**

❓ Question 3

Describe the region $|z + 5i| + |z - 5i| \geq 17$ on the complex plane.

✏️ **Solution:**

❓ **Question 4**

Solve the equation

$$e^{3z} = -3\sqrt{3} - 3i$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❓ **Question 5**

Prove that

$$\cos^{-1} z = \frac{1}{i} \ln\left(z + \sqrt{z^2 - 1}\right),$$

✏️ **Solution:**

❓ **Question 6**

Solve for z where

$$\cos^{-1} z = 8 - 8i$$

✏ **Solution:**

❓ **Question 7**

Using the definition of a limit, show that $\lim\limits_{z \to 0} \dfrac{\text{Im}\{z^2\}}{|z|^2}$ does not exist.

🖋 **Solution:**

❓ Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\tan z}{z} \right)^{\frac{5 \sin(4z)}{z - \sin z}}$$

✏️ **Solution:**

❓ Question 9

Consider the function
$$f(z) = \frac{\tan 2z}{4z}.$$
Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

✏️ **Solution:**

❓ Question 10

Using the definition, show that

$$f(z) = 8z^2 + 9z - 9$$

is differentiable at all points. Also find the derivative.

✏️ **Solution:**

❷ Question 11

Using the definition, find the derivative of $f(z) = \dfrac{5}{z^2}$ at $z = 3 + 7i$.

✏️ **Solution:**

❓ Question 12

Consider the function

$$f(z) = 2\sin(9z) - 9\cosh(5z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❓ **Question 13**

Consider the function

$$f(z) = 8|z|^2 + 5z - 8\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✒ **Solution:**

❓ **Question 14**

Show that the function

$$u(x, y) = 3\sin(6x)\cosh(6y) + 18x^2y - 9x^2 - 6y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

✏️ **Solution:**

❓ Question 15

Show that the function

$$u(x, y) = 3xe^{-2x}\cos(2y) + 3ye^{-2x}\sin(2y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

✏️ **Solution:**