



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z such that

$$z^7 = 64\sqrt{2}(1 + i)$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the above locus $\left| \frac{z+5i}{z-5i} \right| = 5$ in the complex plane.

 Solution:

? Question 3

Describe the above locus $|z + 9i| + |z - 9i| < 25$ in the complex plane.

 Solution:

Question 4

Solve the following equation for z :

$$e^{5z} = 6i$$

Express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

? Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

 Solution:

Question 6

Solve for z :

$$\sinh^{-1} z = 3 - 3i$$

 Solution:

? Question 7

Using the definition of limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{2 \sin(2z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{2z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition show that

$$f(z) = 7z^2 + 8z - 4$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{3z - 9}{6z + 8i}$ at $z = i$.

 Solution:

Question 12

Consider the function $f(z)$ defined by

$$f(z) = 7 \sinh(9z) - 5 \cos(9z)$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

Question 13

Consider the function $f(z)$ defined by

$$f(z) = 2ze^{-3z}$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

? Question 14

Show that the given function u defined by

$$u(x, y) = 7 \sin(9x) \cosh(9y) + 15x^2y - 9x^2 - 5y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u} + \mathbf{v}\mathbf{i}$ becomes analytic.

 Solution:

? Question 15

Show that the given function u defined by

$$u(x, y) = 9xe^{-2x} \cos(2y) + 9ye^{-2x} \sin(2y)$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u} + \mathbf{v}\mathbf{i}$ becomes analytic.

 Solution: