



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

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Question 1

Find all possible values of z satisfying

$$z^7 = -2187i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 5i| + |z - 5i| = 14$ on the complex plane.

 **Solution:**

Question 3

Describe the region $|z + 6i| + |z - 6i| \geq 16$ on the complex plane.

 Solution:

? Question 4

Solve the equation

$$e^{4z} = \frac{3\sqrt{2}(1-i)}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

? Question 5

Prove that

$$\cos^{-1} z = \frac{1}{i} \ln \left(z + \sqrt{z^2 - 1} \right),$$

 Solution:

? Question 6

Solve for z where

$$\tanh^{-1} z = 9 - 3i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{5 \sin(8z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 9z}{6z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 6z^2 + 9z - 4$$

is differentiable at all points. Also find the derivative.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{2z - 5}{3z + 4i}$ at $z = i$.

 Solution:

Question 12

Consider the function

$$f(z) = 9 \sinh(5z) - 2 \cos(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 9|z|^2 + 8z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$u(x, y) = 5 \sin(7x) \cosh(7y) + 21x^2y - 6x^2 - 7y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$v(x, y) = 7xe^{-4x} \cos(4y) + 7ye^{-4x} \sin(4y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution: