



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT 215

Complex Variables and Laplace Transformations

Name: Sami Rahman

Student ID: 24142942

Section: 12

Semester: Fall 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^6 = 729i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 6| + |z - 6| = 18$ on the complex plane.

 **Solution:**

Question 3

Describe the region $|z + 6i| + |z - 6i| > 17$ on the complex plane.

 Solution:

? Question 4

Solve the equation

$$e^{6z} = \frac{7\sqrt{2}(-1+i)}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

? Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln\left(\frac{z+1}{z-1}\right).$$

 Solution:

? Question 6

Solve for z where

$$\coth^{-1} z = 3 + 7i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}(z^2)}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{4 \sin(4z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{7z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.

 **Solution:**

? Question 10

Using the definition, show that

$$f(z) = 3z\bar{z} - 8z + 5\bar{z}$$

is not differentiable at $z = 0$.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{9z - 2}{4z + 2i}$ at $z = i$.

 Solution:

Question 12

Consider the function

$$f(z) = 6 \sinh(6z) - 3 \cos(4z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 3|z|^2 + 4z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$v(x, y) = 8e^{-6x} \cos(6y) - 2e^{3y} \sin(3x) + 27x^2y - 8x^2 - 9y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$u(x, y) = 7xe^{-8x} \cos(8y) + 7ye^{-8x} \sin(8y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution: