BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

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Question 1

Find all possible values of z satisfying

$$z^7 = -2187i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z+9|+|z-9|=25$ on the complex plane.

Solution:

Question 3

Describe the region $\left|\frac{z+4i}{z-4i}\right| > 5$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{9z} = 2$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\sin^{-1} z = \frac{1}{i} \ln\left(iz + \sqrt{1 - z^2}\right),$$

Solution:

Question 6

Solve for z where

$$\sinh^{-1} z = 9 - 8i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{3\sin(8z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{4z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 4z^2 + 8z - 6$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{6z-7}{9z+8i}$ at $z = i$.

Solution:

Question 12

Consider the function

$$f(z) = 9\sin(7z) - 3\cosh(9z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 4ze^{-9z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 5e^{-2x}\cos(2y) - 6e^{4y}\sin(4x) + 12x^2y - 5x^2 - 4y^3 + 5y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 7xe^{-4x}\cos(4y) + 7ye^{-4x}\sin(4y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: