BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: MAHBUBA KHANOM MOHUA

Student ID: 24101101

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^5 = 16\sqrt{2}\,(1+i)\,.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $\left|\frac{z+9i}{z-9i}\right| = 2$ on the complex plane.

Solution:

Question 3

Describe the region $|z + 5i| + |z - 5i| > 14$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{7z} = \frac{3\sqrt{2}\,(-1+i)}{2}$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\sec^{-1} z = \frac{1}{i} \ln \left(\frac{1 + \sqrt{1 - z^2}}{z} \right),$$

Solution:

Question 6

Solve for z where

$$\cos^{-1} z = 5 + 2i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Re}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{3\sin(5z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{6z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 9z^2 + 4z - 6$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{8}{6z+6}$ at $z = z_0$.

Solution:

Question 12

Consider the function

$$f(z) = 7\sin(4z) - 7\cosh(4z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 3|z|^2 + 8z - 4\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 2\sin(8x)\cosh(8y) + 27x^2y - 6x^2 - 9y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 4xe^{-9x}\cos(9y) + 4ye^{-9x}\sin(9y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: