



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

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 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*


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### Question 1

Find all possible values of  $z$  satisfying


$$z^6 = 32 + 32\sqrt{3}i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:


## Question 2

Describe the locus  $|z + 8| + |z - 8| = 22$  on the complex plane.

 Solution:

### Question 3

Describe the region  $\left| \frac{z + 5i}{z - 5i} \right| > 7$  on the complex plane.


 Solution:

### Question 4

Solve the equation

$$e^{2z} = -3\sqrt{3} - 3i$$


for  $z$  and express  $z$  as  $x + iy$  where  $x, y, \in \mathbb{R}$ .

 Solution:

### Question 5

Prove that


$$\sin^{-1} z = \frac{1}{i} \ln \left( iz + \sqrt{1 - z^2} \right),$$

 Solution:

### Question 6


Solve for  $z$  where

$$\cot^{-1} z = 8 + 2i$$

 Solution:

### Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$  does not exist.


 Solution:



## Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{2 \sin(2z)}{z - \sin z}}$$


 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{4z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .


 **Solution:**

### Question 10

Using the definition, show that


$$f(z) = 8z^2 + 4z - 2$$

is differentiable at all points. Also find the derivative.

 **Solution:**

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{5}{4z+7}$  at  $z = z_0$ .


 Solution:

## Question 12

Consider the function

$$f(z) = 2 \sinh(9z) - 4 \cos(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

### Question 13

Consider the function

$$f(z) = 3ze^{-8z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

### Question 14

Show that the function

$$u(x, y) = 5 \sin(6x) \cosh(6y) + 24x^2y - 4x^2 - 8y^3 + 4y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.


 **Solution:**

### Question 15

Show that the function

$$v(x, y) = 4xe^{-2x} \cos(2y) + 4ye^{-2x} \sin(2y)$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $u + vi$  becomes analytic.

 **Solution:**