



BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

 **Assignment - 01**

Course Code: MAT215

Complex Variables & Laplace Transform

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 **Section: 12**

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 **Submission Date: _____**

Assigned by


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Question 1

Find all possible values of z satisfying


$$z^7 = \frac{2187\sqrt{2}(-1+i)}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**


Question 2

Describe the locus $|z - 8i| - |z + 8i| = 11$ on the complex plane.

 Solution:

Question 3

Describe the region $|z + 7i| + |z - 7i| > 17$ on the complex plane.


 Solution:

Question 4

Solve the equation

$$e^{3z} = 1 + \sqrt{3}i$$


for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

Question 5

Prove that


$$\operatorname{cosech}^{-1} z = \ln \left(\frac{1 + \sqrt{z^2 + 1}}{z} \right),$$

 Solution:

Question 6


Solve for z where

$$\sec^{-1} z = 6 + 6i$$

 Solution:

Question 7


Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{7 \sin(7z)}{z - \sin z}}$$


 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 4z}{9z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.


 **Solution:**

Question 10

Using the definition, show that


$$f(z) = 3z\bar{z} - 2z + 8\bar{z}$$

is not differentiable at $z = 0$.

 **Solution:**

Question 11

Using the definition, find the derivative of $f(z) = \frac{5}{z^2}$ at $z = 6 + 9i$.


 Solution:

Question 12

Consider the function

$$f(z) = 3 \sinh(5z) - 8 \cos(9z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

Question 13

Consider the function

$$f(z) = 2ze^{-5z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

Question 14

Show that the function

$$u(x, y) = 9e^{-6x} \cos(6y) - 4e^{5y} \sin(5x) + 12x^2y - 6x^2 - 4y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.


 **Solution:**

Question 15

Show that the function

$$v(x, y) = 2xe^{-9x} \cos(9y) + 2ye^{-9x} \sin(9y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**