BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

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Question 1

Find all possible values of z satisfying

$$z^7 = -\frac{2187}{2} + \frac{2187\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z - 9i| - |z + 9i| = 13$ on the complex plane.

Solution:

Question 3

Describe the region $\left|\frac{z+8i}{z-8i}\right| \geq 3$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{8z} = 2\sqrt{3} - 2i$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\cot^{-1} z = \frac{1}{2i} \ln\left(\frac{z+i}{z-i}\right),$$

Solution:

Question 6

Solve for z where

$$\operatorname{cosech}^{-1} z = 7 + 8i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{2\sin(8z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 8z}{3z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 7z^2 + 6z - 5$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{3z-3}{3z+3i}$ at $z = i$.

Solution:

Question 12

Consider the function

$$f(z) = 2\sinh(4z) - 9\cos(3z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 2|z|^2 + 5z - 5\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 5e^{-3x}\cos(3y) - 4e^{2y}\sin(2x) + 12x^2y - 8x^2 - 4y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$v(x, y) = 4xe^{-8x} \cos(8y) + 4ye^{-8x} \sin(8y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution: