🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

📋 Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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👥 **Section: 12**

📅 **Semester: FALL 2025**

🕐 **Submission Date: _____**

Assigned by

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❓ Question 1

Find all possible values of z satisfying

$$z^6 = 729.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

✎ **Solution:**

❷ Question 2

Describe the locus $|z - 5i| - |z + 5i| = 6$ on the complex plane.

✏️ **Solution:**

❷ **Question 2**

Describe the locus $|z - 5i| - |z + 5i| = 6$ on the complex plane.

✏️ **Solution:**

❓ Question 3

Describe the region $|z + 7i| + |z - 7i| < 18$ on the complex plane.

✏️ **Solution:**

❓ Question 4

Solve the equation

$$e^{3z} = -\frac{7\sqrt{3}}{2} + \frac{7i}{2}$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❓ **Question 5**

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

✏ **Solution:**

❓ **Question 6**

Solve for z where

$$\tanh^{-1} z = 6 - 2i$$

🖊 **Solution:**

❓ Question 7

Using the definition of a limit, show that $\lim\limits_{z \to 0} \dfrac{\text{Re}\{z^2\}}{|z|^2}$ does not exist.

✏ **Solution:**

❓ **Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\tan z}{z} \right)^{\frac{2\sin(5z)}{z - \sin z}}$$

✏️ **Solution:**

❷ Question 9

Consider the function
$$f(z) = \frac{\tan 4z}{6z}.$$
Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

✏ **Solution:**

❷ Question 10

Using the definition, show that

$$f(z) = 7z\bar{z} - 8z + 7\bar{z}$$

is not differentiable at $z = 0$.

✎ **Solution:**

❓ Question 11

Using the definition, find the derivative of $f(z) = \dfrac{3z - 7}{2z + 8i}$ at $z = i$.

✏️ **Solution:**

❓ Question 12

Consider the function

$$f(z) = 7\sin(7z) - 4\cosh(2z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❓ Question 13

Consider the function

$$f(z) = 6|z|^2 + 2z - 8\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❷ Question 14

Show that the function

$$v(x, y) = 2\sin(3x)\cosh(3y) + 21x^2y - 8x^2 - 7y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

✏️ **Solution:**

❓ Question 15

Show that the function

$$v(x, y) = 8xe^{-4x}\cos{(4y)} + 8ye^{-4x}\sin{(4y)}$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

✒️ **Solution:**