🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

📋 Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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👥 **Section: 12**

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🕐 **Submission Date: _____**

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❷ Question 1

Find all possible values of z satisfying

$$z^5 = 16\sqrt{2}\,(1+i)\,.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

✒️ **Solution:**

❷ Question 2

Describe the locus $\left| \dfrac{z + 9i}{z - 9i} \right| = 2$ on the complex plane.

✏️ **Solution:**

❓ Question 3

Describe the region $|z - 6| - |z + 6| \geq 8$ on the complex plane.

✏️ **Solution:**

❓ **Question 4**

Solve the equation

$$e^{3z} = -6$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❓ Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

✏️ **Solution:**

❓ **Question 6**

Solve for z where

$$\sin^{-1} z = 4 + 9i$$

✏ **Solution:**

❓ **Question 7**

Using the definition of a limit, show that $\lim\limits_{z \to 0} \dfrac{\text{Re}\{z^2\}}{|z|^2}$ does not exist.

✏️ **Solution:**

❓ **Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\tan z}{z} \right)^{\frac{2\sin(9z)}{z - \sin z}}$$

✏ **Solution:**

❓ **Question 9**

Consider the function

$$f(z) = \frac{\tan 6z}{4z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

🖊 **Solution:**

❷ Question 10

Using the definition, show that

$$f(z) = 4z\bar{z} - 7z + 3\bar{z}$$

is not differentiable at $z = 0$.

✏ **Solution:**

❓ **Question 11**

Using the definition, find the derivative of $f(z) = \dfrac{4}{6z + 4}$ at $z = z_0$.

✏️ **Solution:**

❷ Question 12

Consider the function

$$f(z) = 3\sinh(6z) - 6\cos(6z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏ **Solution:**

❓ **Question 13**

Consider the function

$$f(z) = 8ze^{-4z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❓ Question 14

Show that the function

$$v(x, y) = 5e^{-6x}\cos(6y) - 5e^{5y}\sin(5x) + 21x^2y - 9x^2 - 7y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

✏️ **Solution:**

❷ Question 15

Show that the function

$$v(x, y) = 6xe^{-4x} \cos(4y) + 6ye^{-4x} \sin(4y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

✏ **Solution:**