BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: OHI AHMED

Student ID: 24101194

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^5 = \frac{243}{2} - \frac{243\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z - 8i| - |z + 8i| = 14$ on the complex plane.

Solution:

Question 3

Describe the region $|z+8|+|z-8|<17$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{6z} = \frac{3\sqrt{3}}{2} + \frac{3i}{2}$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

Solution:

Question 6

Solve for z where

$$\operatorname{cosech}^{-1} z = 2 - 2i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Re}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{7\sin(3z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{2z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 7z^2 + 8z - 2$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{9z-8}{5z+2i}$ at $z = i$.

Solution:

Question 12

Consider the function

$$f(z) = 5\sinh(6z) - 2\cos(4z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 4ze^{-5z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$u(x, y) = 8 \sin(5x) \cosh(5y) + 27x^2y - 9x^2 - 9y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 2xe^{-9x}\cos(9y) + 2ye^{-9x}\sin(9y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: