



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z such that

$$z^6 = \frac{729\sqrt{2}(-1 - i)}{2}$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the above locus $\left| \frac{z+4i}{z-4i} \right| = 3$ in the complex plane.

 Solution:

Question 3

Describe the above locus $\left| \frac{z+9i}{z-9i} \right| \geq 2$ in the complex plane.

 Solution:

? Question 4

Solve the following equation for z :

$$e^{5z} = -\frac{5}{2} + \frac{5\sqrt{3}i}{2}$$

Express z as $x + iy$ where $x, y, \in \mathbb{R}$.

 Solution:

? Question 5

Prove that

$$\sin^{-1} z = \frac{1}{i} \ln \left(iz + \sqrt{1 - z^2} \right),$$

 Solution:

? Question 6

Solve for z :

$$\tanh^{-1} z = 5 - 9i$$

 Solution:

? Question 7

Using the definition of limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{8 \sin(8z)}{z - \sin z}}$$

 **Solution:**

Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{4z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition show that

$$f(z) = 4z^2 + 6z - 3$$

is differentiable at all points. Also find the derivative.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{3z - 2}{9z + 8i}$ at $z = i$.

 Solution:

Question 12

Consider the function $f(z)$ defined by

$$f(z) = 5 \sinh(7z) - 5 \cos(7z)$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

Question 13

Consider the function $f(z)$ defined by

$$f(z) = 8|z|^2 + 4z - 2\bar{z}$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

? Question 14

Show that the given function u defined by

$$u(x, y) = 6e^{-9x} \cos(9y) - 5e^{3y} \sin(3x) + 27x^2y - 9x^2 - 9y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u} + \mathbf{v}\mathbf{i}$ becomes analytic.

 Solution:

? Question 15

Show that the given function u defined by

$$u(x, y) = 2xe^{-8x} \cos(8y) + 2ye^{-8x} \sin(8y)$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u} + \mathbf{v}\mathbf{i}$ becomes analytic.

 Solution: