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👥 **Section: 12**

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🕐 Submission Date: _____

Assigned by

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❷ Question 1

Find all possible values of z satisfying

$$z^5 = \frac{243}{2} - \frac{243\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

✒ **Solution:**

❷ Question 2

Describe the locus $|z - 6i| - |z + 6i| = 5$ on the complex plane.

✏ **Solution:**

❓ Question 3

Describe the region $|z + 6| + |z - 6| \leq 19$ on the complex plane.

✏️ **Solution:**

❓ Question 4

Solve the equation

$$e^{7z} = -\frac{3\sqrt{3}}{2} + \frac{3i}{2}$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❓ **Question 5**

Prove that

$$\cosh^{-1} z = \ln\left(z + \sqrt{z^2 - 1}\right),$$

✏️ **Solution:**

❓ **Question 6**

Solve for z where

$$\cot^{-1} z = 3 + 4i$$

✏️ **Solution:**

❓ **Question 7**

Using the definition of a limit, show that $\lim\limits_{z \to 0} \dfrac{\text{Im}\{z^2\}}{|z|^2}$ does not exist.

✏️ **Solution:**

❓ **Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\sin z}{z} \right)^{\frac{6 \sin(4z)}{z - \sin z}}$$

✏️ **Solution:**

❓ Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{3z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

✏️ **Solution:**

❓ Question 10

Using the definition, show that

$$f(z) = 6z\bar{z} - 7z + 9\bar{z}$$

is not differentiable at $z = 0$.

✏️ **Solution:**

❓ Question 11

Using the definition, find the derivative of $f(z) = \dfrac{9}{4z + 5}$ at $z = z_0$.

✏️ **Solution:**

❷ Question 12

Consider the function
$$f(z) = 3\sin(7z) - 5\cosh(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❓ Question 13

Consider the function

$$f(z) = 4ze^{-3z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❷ Question 14

Show that the function

$$u(x, y) = 8e^{-7x} \cos{(7y)} - 2e^{8y} \sin{(8x)} + 24x^2 y - 6x^2 - 8y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

✏️ **Solution:**

❓ **Question 15**

Show that the function

$$v(x, y) = 8xe^{-7x}\cos(7y) + 8ye^{-7x}\sin(7y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

✒️ **Solution:**