



BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

 **Assignment - 01**


Course Code: MAT215

Complex Variables & Laplace Transform

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 **Section: 12**

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 **Submission Date: _____**

Assigned by


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Question 1

Find all possible values of z satisfying


$$z^7 = 64\sqrt{3} - 64i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:


Question 2

Describe the locus $|z - 6i| - |z + 6i| = 6$ on the complex plane.

 Solution:

Question 3

Describe the region $|z + 4i| + |z - 4i| > 10$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{4z} = 6$$


for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\cos^{-1} z = \frac{1}{i} \ln \left(z + \sqrt{z^2 - 1} \right),$$

 Solution:

Question 6

Solve for z where

$$\cot^{-1} z = 8 + 7i$$

 Solution:

Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\text{Im}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{4 \sin(8z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{3z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.


 **Solution:**

Question 10

Using the definition, show that


$$f(z) = 7z\bar{z} - 8z + 3\bar{z}$$

is not differentiable at $z = 0$.

 **Solution:**

Question 11

Using the definition, find the derivative of $f(z) = \frac{4}{4z+8}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 7 \sin(9z) - 3 \cosh(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

Question 13

Consider the function

$$f(z) = 8|z|^2 + 7z - 5\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

Question 14

Show that the function

$$u(x, y) = 2e^{-6x} \cos(6y) - 4e^{7y} \sin(7x) + 27x^2y - 8x^2 - 9y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**

Question 15

Show that the function

$$v(x, y) = 9xe^{-7x} \cos(7y) + 9ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**