



BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

 **Assignment - 01**

Course Code: MAT 215

Complex Variables and Laplace Transformations

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 **Section: 12**

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 **Submission Date: _____**

Assigned by


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Question 1

Find all possible values of z satisfying


$$z^7 = 64\sqrt{2}(1 + i).$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**


Question 2

Describe the locus $|z - 5i| - |z + 5i| = 5$ on the complex plane.

 Solution:

Question 3

Describe the region $|z + 7| + |z - 7| < 21$ on the complex plane.


 Solution:

Question 4

Solve the equation

$$e^{6z} = 3 + 3\sqrt{3}i$$


for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

Question 5

Prove that

$$\cosh^{-1} z = \ln(z + \sqrt{z^2 - 1}),$$

 Solution:

Question 6

Solve for z where

$$\cos^{-1} z = 3 - 4i$$

 Solution:

Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}(z^2)}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{2 \sin(3z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 4z}{3z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.


 **Solution:**

Question 10

Using the definition, show that


$$f(z) = 7z^2 + 5z - 8$$

is differentiable at all points. Also find the derivative.

 **Solution:**

Question 11

Using the definition, find the derivative of $f(z) = \frac{2}{8z+9}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 2 \sin(7z) - 9 \cosh(6z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

Question 13

Consider the function

$$f(z) = 3|z|^2 + 9z - 9\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

Question 14

Show that the function

$$u(x, y) = 6e^{-3x} \cos(3y) - 2e^{2y} \sin(2x) + 18x^2y - 5x^2 - 6y^3 + 5y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**

Question 15

Show that the function

$$u(x, y) = 6xe^{-6x}\cos(6y) + 6ye^{-6x}\sin(6y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**