



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

 **Name: TAWHID HASAN**


 **Student ID: 23201593**

 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*


 **Partho Sutra Dhor**  
Lecturer, Department of MNS  
BRAC University

### Question 1

Find all possible values of  $z$  satisfying


$$z^6 = 32 - 32\sqrt{3}i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:


## Question 2

Describe the locus  $\left| \frac{z+9i}{z-9i} \right| = 7$  on the complex plane.

 Solution:

### Question 3

Describe the region  $\left| \frac{z + 5i}{z - 5i} \right| > 3$  on the complex plane.


 Solution:

### Question 4

Solve the equation

$$e^{3z} = 4i$$


for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 Solution:

### Question 5

Prove that


$$\operatorname{sech}^{-1} z = \ln \left( \frac{1 + \sqrt{1 - z^2}}{z} \right),$$

 Solution:

### Question 6


Solve for  $z$  where

$$\tanh^{-1} z = 6 + 4i$$

 Solution:

### Question 7


Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$  does not exist.

 Solution:

## Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{6 \sin(5z)}{z - \sin z}}$$


 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 9z}{3z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .


 **Solution:**

### Question 10

Using the definition, show that


$$f(z) = 2z^2 + 3z - 6$$

is differentiable at all points. Also find the derivative.

 **Solution:**

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{9}{z^2}$  at  $z = 4 + 7i$ .


 Solution:

## Question 12

Consider the function

$$f(z) = 3 \sin(6z) - 5 \cosh(2z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

### Question 13

Consider the function

$$f(z) = 6|z|^2 + 6z - 9\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

### Question 14

Show that the function

$$u(x, y) = 2e^{-7x} \cos(7y) - 2e^{5y} \sin(5x) + 24x^2y - 2x^2 - 8y^3 + 2y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.


 **Solution:**

### Question 15

Show that the function

$$u(x, y) = 9xe^{-7x} \cos(7y) + 9ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 **Solution:**