



# III BRAC University

Department of Mathematics and Natural Sciences

**Total Points: 150**

**Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

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**Section: 12**

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**Submission Date:** \_\_\_\_\_

*Assigned by*

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**Question 1**

Find all possible values of  $z$  such that

$$z^5 = 16\sqrt{2}(1 + i)$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

 **Solution:**

**Question 2**

Describe the above locus  $\left| \frac{z + 9i}{z - 9i} \right| = 2$  in the complex plane.

 Solution:

**?** Question 3

Describe the above locus  $|z - 6| - |z + 6| \geq 8$  in the complex plane.

 Solution:

**Question 4**

Solve the following equation for  $z$ :

$$e^{3z} = -6$$

Express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 **Solution:**

**Question 5**

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left( \frac{z+1}{z-1} \right).$$

 Solution:

**Question 6**

Solve for  $z$ :

$$\sin^{-1} z = 4 + 9i$$

 Solution:

**?** Question 7

Using the definition of limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$  does not exist.

 Solution:

**Question 8**

Using the L'Hospital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{2 \sin(9z)}{z - \sin z}}$$

 **Solution:**

## Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{4z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .

 **Solution:**

**?** Question 10

Using the definition show that

$$f(z) = 4z\bar{z} - 7z + 3\bar{z}$$

is not differentiable at  $z = 0$ .

 Solution:

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{4}{6z + 4}$  at  $z = z_0$ .

 Solution:

**Question 12**

Consider the function  $f(z)$  defined by

$$f(z) = 3 \sinh(6z) - 6 \cos(6z)$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

**Question 13**

Consider the function  $f(z)$  defined by

$$f(z) = 8ze^{-4z}$$

Using C-R equations determine whether the function is analytic or not.

 **Solution:**

**?** Question 14

Show that the given function  $v$  defined by

$$v(x, y) = 5e^{-6x} \cos(6y) - 5e^{5y} \sin(5x) + 21x^2y - 9x^2 - 7y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $\mathbf{u} + \mathbf{v}\mathbf{i}$  becomes analytic.

 Solution:

**?** Question 15

Show that the given function  $v$  defined by

$$v(x, y) = 6xe^{-4x} \cos(4y) + 6ye^{-4x} \sin(4y)$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $\mathbf{u} + \mathbf{v}\mathbf{i}$  becomes analytic.

 Solution: