



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT 215

Complex Variables and Laplace Transformations

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^7 = -\frac{2187}{2} - \frac{2187\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 8i| + |z - 8i| = 19$ on the complex plane.

 **Solution:**

Question 3

Describe the region $\left| \frac{z+7i}{z-7i} \right| \geq 5$ on the complex plane.

 **Solution:**

Question 4

Solve the equation

$$e^{2z} = 4\sqrt{2}(1+i)$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

? Question 5

Prove that

$$\tanh^{-1} z = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right),$$

 Solution:

Question 6

Solve for z where

$$\operatorname{cosec}^{-1} z = 2 - 9i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}(z^2)}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{3 \sin(6z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{3z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.

 **Solution:**

? Question 10

Using the definition, show that

$$f(z) = 6z\bar{z} - 8z + 8\bar{z}$$

is not differentiable at $z = 0$.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{4}{5z + 5}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 5 \sinh(4z) - 9 \cos(5z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 5ze^{-9z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$v(x, y) = 2e^{-4x} \cos(4y) - 2e^{2y} \sin(2x) + 24x^2y - 9x^2 - 8y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution:

Question 15

Show that the function

$$u(x, y) = 6xe^{-9x} \cos(9y) + 6ye^{-9x} \sin(9y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**