🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 15

📋 Assignment-01

Course Code: MAT215

Complex

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👥 **Section: 12**

📅 **Semester: FALL 2025**

🕐 **Submission Date:** _____

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❓ Question 1

Find all possible values of z such that

$$z^7 = -64\sqrt{3} - 64i$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

✏️ **Solution:**

❷ Question 2

Consider the equation

$$|z - 7i| - |z + 7i| = 8$$

Describe the above locus in the complex plane.

✏️ **Solution:**

❓ **Question 3**

Consider the inequality

$$\left|\frac{z + 8i}{z - 8i}\right| \leq 6$$

Describe the above locus in the complex plane.

✏️ **Solution:**

❷ Question 4

Solve the following equation for z:

$$e^{3z} = -\frac{7}{2} + \frac{7\sqrt{3}i}{2}$$

Express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❓ Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

✏️ **Solution:**

❷ **Question 6**

Solve for z:

$$\tanh^{-1} z = 6 + 2i$$

✏ **Solution:**

❓ **Question 7**

Show that the limit, $\lim\limits_{z \to 0} \dfrac{\text{Re}\{z^2\}}{|z|^2}$ does not exist.

✏️ **Solution:**

❓ Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\tan z}{z} \right)^{\frac{7 \sin(2z)}{z - \sin z}}$$

✏️ **Solution:**

❷ Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{6z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

🖊 **Solution:**

❓ Question 10

Using the definition show that

$$f(z) = 6z^2 + 8z - 2$$

is differentiable at all points. Also find the derivative.

✏️ **Solution:**

❷ Question 11

Using the definition, find the derivative of

$$f(z) = \frac{9}{7z + 5} \quad \text{at} \quad z = z_0$$

.

✎ **Solution:**

❓ **Question 12**

Consider the fuction $f(z)$ defined by

$$f(z) = 3\sin(3z) - 5\cosh(4z)$$

Using C-R equations determine whether the function is analytic or not.

✏ **Solution:**

❓ Question 13

Consider the fuction $f(z)$ defined by

$$f(z) = 4ze^{-4z}$$

Using C-R equations determine whether the function is analytic or not.

✏️ **Solution:**

❓ **Question 14**

Show that the given function v defined by

$$v(x, y) = 2e^{-3x}\cos(3y) - 3e^{6y}\sin(6x) + 18x^2y - 3x^2 - 6y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate u of v such that $\mathbf{u+vi}$ becomes analytic.

✏ **Solution:**

❓ Question 15

Show that the given function u defined by

$$u(x, y) = 7xe^{-5x}\cos(5y) + 7ye^{-5x}\sin(5y)$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u+vi}$ becomes analytic.

✏️ **Solution:**