BRAC University

Department of Mathematics and Natural Sciences

Total Points: 15

Assignment-01

Course Code: MAT215

Complex

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Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

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Question 1

Find all possible values of z such that

$$z^5 = 16\sqrt{2}\,(1+i)$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

Solution:

Question 2

Consider the equation

$$\left|\frac{z+9i}{z-9i}\right| = 2$$

Describe the above locus in the complex plane.

Solution:

Question 3

Consider the inequality

$$|z-6|-|z+6| \geq 8$$

Describe the above locus in the complex plane.

Solution:

Question 4

Solve the following equation for z:

$$e^{3z} = -6$$

Express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

Solution:

Question 6

Solve for z:

$$\sin^{-1} z = 4 + 9i$$

Solution:

Question 7

Show that the limit, $\lim_{z\to 0} \dfrac{\mathrm{Re}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\tan z}{z}\right)^{\frac{2\sin(9z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{4z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition show that

$$f(z) = 4z\bar{z} - 7z + 3\bar{z}$$

is not differentiable at $z = 0$.

Solution:

Question 11

Using the definition, find the derivative of

$$f(z) = \frac{4}{6z+4} \quad \text{at} \quad z = z_0$$

.

Solution:

Question 12

Consider the fuction $f(z)$ defined by

$$f(z) = 3\sinh(6z) - 6\cos(6z)$$

Using C-R equations determine whether the function is analytic or not.

Solution:

Question 13

Consider the fuction $f(z)$ defined by

$$f(z) = 8ze^{-4z}$$

Using C-R equations determine whether the function is analytic or not.

Solution:

Question 14

Show that the given function v defined by

$$v(x, y) = 5e^{-6x}\cos(6y) - 5e^{5y}\sin(5x) + 21x^2y - 9x^2 - 7y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate u of v such that **u+vi** becomes analytic.

Solution:

Question 15

Show that the given function v defined by

$$v(x, y) = 6xe^{-4x}\cos(4y) + 6ye^{-4x}\sin(4y)$$

is harmonic. Find the harmonic conjugate u of v such that **u+vi** becomes analytic.

Solution: