



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

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 **Section: 12**

 **Semester: FALL 2025**

 **Submission Date: \_\_\_\_\_**

*Assigned by*


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### Question 1

Find all possible values of  $z$  satisfying


$$z^6 = \frac{729}{2} + \frac{729\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**


## Question 2

Describe the locus  $|z + 4| + |z - 4| = 11$  on the complex plane.

 Solution:

### Question 3

Describe the region  $|z - 6i| - |z + 6i| < 7$  on the complex plane.


 Solution:

### Question 4

Solve the equation

$$e^{2z} = 2\sqrt{2}(1 - i)$$


for  $z$  and express  $z$  as  $x + iy$  where  $x, y, \in \mathbb{R}$ .

 **Solution:**

### Question 5

Prove that


$$\operatorname{cosech}^{-1} z = \ln \left( \frac{1 + \sqrt{z^2 + 1}}{z} \right),$$

 Solution:

### Question 6


Solve for  $z$  where

$$\sinh^{-1} z = 4 + 8i$$

 Solution:

### Question 7


Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\text{Im}\{z^2\}}{|z|^2}$  does not exist.

 Solution:

## Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{2 \sin(6z)}{z - \sin z}}$$


 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 9z}{3z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .


 **Solution:**

### Question 10

Using the definition, show that


$$f(z) = 7z^2 + 9z - 7$$

is differentiable at all points. Also find the derivative.

 **Solution:**

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{4z - 9}{7z + 5i}$  at  $z = i$ .


 Solution:

## Question 12

Consider the function

$$f(z) = 4 \sinh(3z) - 2 \cos(5z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

### Question 13

Consider the function

$$f(z) = 6ze^{-9z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.


 **Solution:**

### Question 14

Show that the function

$$u(x, y) = 4 \sin(6x) \cosh(6y) + 24x^2y - 2x^2 - 8y^3 + 2y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.


 **Solution:**

### Question 15

Show that the function

$$u(x, y) = 6xe^{-9x} \cos(9y) + 6ye^{-9x} \sin(9y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 **Solution:**