



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT 215

Complex Variables and Laplace Transformations

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👥 Section: 12

📅 Semester: Fall 2025

⌚ Submission Date: _____

Assigned by

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Question 1

Find all possible values of z satisfying

$$z^7 = 64\sqrt{3} - 64i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $\left| \frac{z+7i}{z-7i} \right| = 3$ on the complex plane.

 Solution:

? Question 3

Describe the region $|z + 8| + |z - 8| \geq 20$ on the complex plane.

 Solution:

? Question 4

Solve the equation

$$e^{7z} = 2\sqrt{2}(-1 - i)$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\sec^{-1} z = \frac{1}{i} \ln \left(\frac{1 + \sqrt{1 - z^2}}{z} \right),$$

 Solution:

? Question 6

Solve for z where

$$\operatorname{cosec}^{-1} z = 3 + 8i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}(z^2)}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{7 \sin(2z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{9z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.

 **Solution:**

? Question 10

Using the definition, show that

$$f(z) = 9z\bar{z} - 5z + 4\bar{z}$$

is not differentiable at $z = 0$.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{2z - 9}{7z + 8i}$ at $z = i$.

 Solution:

Question 12

Consider the function

$$f(z) = 3 \sinh(6z) - 8 \cos(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 5|z|^2 + 9z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$v(x, y) = 6 \sin(8x) \cosh(8y) + 15x^2y - 3x^2 - 5y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution:

Question 15

Show that the function

$$u(x, y) = 4xe^{-7x} \cos(7y) + 4ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**