



BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

 **Assignment - 01**

Course Code: MAT215

Complex Variables & Laplace Transform

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 **Section: 12**

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 **Submission Date: _____**

Assigned by


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Question 1

Find all possible values of z satisfying


$$z^6 = 64.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:


Question 2

Describe the locus $|z + 6| + |z - 6| = 13$ on the complex plane.

 Solution:

Question 3

Describe the region $|z + 8i| + |z - 8i| < 18$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{7z} = -3i$$


for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\sin^{-1} z = \frac{1}{i} \ln \left(iz + \sqrt{1 - z^2} \right),$$

 Solution:

Question 6

Solve for z where

$$\tan^{-1} z = 3 - 3i$$

 Solution:

Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{2 \sin(7z)}{z - \sin z}}$$


 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 9z}{6z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.


 **Solution:**

Question 10

Using the definition, show that


$$f(z) = 5z\bar{z} - 5z + 3\bar{z}$$

is not differentiable at $z = 0$.

 **Solution:**

Question 11

Using the definition, find the derivative of $f(z) = \frac{9z - 4}{7z + 4i}$ at $z = i$.

 Solution:

Question 12

Consider the function

$$f(z) = 3 \sinh(9z) - 4 \cos(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

Question 13

Consider the function

$$f(z) = 5ze^{-3z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

Question 14

Show that the function

$$v(x, y) = 4e^{-5x} \cos(5y) - 3e^{8y} \sin(8x) + 18x^2y - 3x^2 - 6y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**

Question 15

Show that the function

$$v(x, y) = 7xe^{-2x} \cos(2y) + 7ye^{-2x} \sin(2y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**