



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT 215**

Complex Variables and Laplace Transformations

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 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*


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### Question 1

Find all possible values of  $z$  satisfying

$$z^6 = 729i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:


## Question 2

Describe the locus  $|z + 6| + |z - 6| = 18$  on the complex plane.

 Solution:

### Question 3

Describe the region  $|z + 6i| + |z - 6i| > 17$  on the complex plane.


 Solution:

### Question 4

Solve the equation

$$e^{6z} = \frac{7\sqrt{2}(-1+i)}{2}$$

for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 **Solution:**

### Question 5

Prove that


$$\coth^{-1} z = \frac{1}{2} \ln \left( \frac{z+1}{z-1} \right).$$

 Solution:

### Question 6

Solve for  $z$  where

$$\coth^{-1} z = 3 + 7i$$

 Solution:

### Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\text{Im}(z^2)}{|z|^2}$  does not exist.

 Solution:



## Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\sin z}{z} \right)^{\frac{4 \sin(4z)}{z - \sin z}}$$

 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{7z}$$

. Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all points of discontinuity of  $f(z)$ .


 **Solution:**

### Question 10

Using the definition, show that


$$f(z) = 3z\bar{z} - 8z + 5\bar{z}$$

is not differentiable at  $z = 0$ .

 **Solution:**

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{9z - 2}{4z + 2i}$  at  $z = i$ .

 Solution:

## Question 12

Consider the function

$$f(z) = 6 \sinh(6z) - 3 \cos(4z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

### Question 13

Consider the function

$$f(z) = 3|z|^2 + 4z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

### Question 14

Show that the function

$$v(x, y) = 8e^{-6x} \cos(6y) - 2e^{3y} \sin(3x) + 27x^2y - 8x^2 - 9y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $u + vi$  becomes analytic.

 **Solution:**

### Question 15

Show that the function

$$u(x, y) = 7xe^{-8x}\cos(8y) + 7ye^{-8x}\sin(8y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 **Solution:**