



# III BRAC University

Department of Mathematics and Natural Sciences

**Total Points: 150**

**Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

**Name: SABAH ISLAM SAFIA**

**Student ID: 24310009**

**Section: 12**

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**Submission Date:** \_\_\_\_\_

*Assigned by*

**Partho Sutra Dhor**  
Lecturer, Department of MNS  
BRAC University

**Question 1**

Find all possible values of  $z$  satisfying

$$z^6 = \frac{729\sqrt{2}(-1 - i)}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

**Question 2**

Describe the locus  $\left| \frac{z+4i}{z-4i} \right| = 3$  on the complex plane.

 Solution:

**Question 3**

Describe the region  $\left| \frac{z + 9i}{z - 9i} \right| \geq 2$  on the complex plane.

 **Solution:**

**?** Question 4

Solve the equation

$$e^{5z} = -\frac{5}{2} + \frac{5\sqrt{3}i}{2}$$

for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 Solution:

**?** Question 5

Prove that

$$\sin^{-1} z = \frac{1}{i} \ln \left( iz + \sqrt{1 - z^2} \right),$$

 Solution:

**?** Question 6

Solve for  $z$  where

$$\tanh^{-1} z = 5 - 9i$$

 Solution:

**?** Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$  does not exist.

 Solution:

**Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{8 \sin(8z)}{z - \sin z}}$$

 Solution:

## Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{4z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .

 **Solution:**

## Question 10

Using the definition, show that

$$f(z) = 4z^2 + 6z - 3$$

is differentiable at all points. Also find the derivative.

 Solution:

**?** Question 11

Using the definition, find the derivative of  $f(z) = \frac{3z - 2}{9z + 8i}$  at  $z = i$ .

 Solution:

## Question 12

Consider the function

$$f(z) = 5 \sinh(7z) - 5 \cos(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

### Question 13

Consider the function

$$f(z) = 8|z|^2 + 4z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

**?** Question 14

Show that the function

$$u(x, y) = 6e^{-9x} \cos(9y) - 5e^{3y} \sin(3x) + 27x^2y - 9x^2 - 9y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 Solution:

**?** Question 15

Show that the function

$$u(x, y) = 2xe^{-8x} \cos(8y) + 2ye^{-8x} \sin(8y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 Solution: