



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT 215**

Complex Variables and Laplace Transformations

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 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*

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### Question 1

Find all possible values of  $z$  satisfying

$$z^7 = 64\sqrt{3} - 64i.$$


Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:



## Question 2


Describe the locus  $\left| \frac{z+7i}{z-7i} \right| = 3$  on the complex plane.

 Solution:



### Question 3

Describe the region  $|z + 8| + |z - 8| \geq 20$  on the complex plane.

 Solution:




### Question 4

Solve the equation

$$e^{7z} = 2\sqrt{2}(-1 - i)$$

for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .


 **Solution:**



### Question 5

Prove that

$$\sec^{-1} z = \frac{1}{i} \ln \left( \frac{1 + \sqrt{1 - z^2}}{z} \right),$$


 Solution:



### Question 6

Solve for  $z$  where


$$\operatorname{cosec}^{-1} z = 3 + 8i$$

 Solution:



### Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\text{Im}(z^2)}{|z|^2}$  does not exist.


 Solution:



## Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{7 \sin(2z)}{z - \sin z}}$$

 Solution:




### Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{9z}$$

. Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all points of discontinuity of  $f(z)$ .

 **Solution:**




### Question 10

Using the definition, show that

$$f(z) = 9z\bar{z} - 5z + 4\bar{z}$$


is not differentiable at  $z = 0$ .

 **Solution:**



### Question 11

Using the definition, find the derivative of  $f(z) = \frac{2z - 9}{7z + 8i}$  at  $z = i$ .

 Solution:




## Question 12

Consider the function

$$f(z) = 3 \sinh(6z) - 8 \cos(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**




### Question 13

Consider the function

$$f(z) = 5|z|^2 + 9z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**




### Question 14

Show that the function

$$v(x, y) = 6 \sin(8x) \cosh(8y) + 15x^2y - 3x^2 - 5y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $u + vi$  becomes analytic.

 **Solution:**




### Question 15

Show that the function

$$u(x, y) = 4xe^{-7x} \cos(7y) + 4ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 **Solution:**