BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT 215

Complex Variables and Laplace Transformations

Name: Sami Islam

Student ID: 24165329

Section: 12

Semester: Fall 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^5 = 16\sqrt{2}\,(1+i)\,.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z-7|-|z+7|=9$ on the complex plane.

Solution:

Question 3

Describe the region $\left|\frac{z+4i}{z-4i}\right| > 1$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{4z} = 5i$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\operatorname{sech}^{-1} z = \ln\left(\frac{1+\sqrt{1-z^2}}{z}\right),$$

Solution:

Question 6

Solve for z where

$$\sec^{-1} z = 5 + 9i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}(z^2)}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{8\sin(4z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 9z}{3z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 8z^2 + 8z - 4$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{8}{7z+2}$ at $z = z_0$.

Solution:

Question 12

Consider the function

$$f(z) = 3\sin(6z) - 5\cosh(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 6ze^{-4z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 2e^{-7x}\cos(7y) \; - \; 9e^{8y}\sin(8x) \; + \; 18\,x^2y \; - \; 2x^2 \; - \; 6y^3 \; + \; 2y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$v(x, y) = 8\,xe^{-7x}\cos(7y) \;+\; 8\,ye^{-7x}\sin(7y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution: