🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

📋 Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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👥 **Section: 12**

📅 **Semester: FALL 2025**

🕐 **Submission Date:** _____

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❓ **Question 1**

Find all possible values of z satisfying

$$z^6 = \frac{729\sqrt{2}\,(-1-i)}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

✏️ **Solution:**

❷ Question 2

Describe the locus $|z + 8| + |z - 8| = 18$ on the complex plane.

✏️ **Solution:**

❓ Question 3

Describe the region $|z + 9| + |z - 9| \geq 22$ on the complex plane.

✏️ **Solution:**

❓ **Question 4**

Solve the equation

$$e^{7z} = -1 - \sqrt{3}i$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❓ **Question 5**

Prove that

$$\sinh^{-1} z = \ln\left(z + \sqrt{z^2 + 1}\right),$$

✐ **Solution:**

❓ **Question 6**

Solve for z where

$$\operatorname{cosech}^{-1} z = 4 - 2i$$

📝 **Solution:**

❓ **Question 7**

Using the definition of a limit, show that $\lim\limits_{z \to 0} \dfrac{\text{Im}\{z^2\}}{|z|^2}$ does not exist.

✏️ **Solution:**

❓ **Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\sin z}{z} \right)^{\frac{5 \sin(5z)}{z - \sin z}}$$

✏️ **Solution:**

❷ Question 9

Consider the function
$$f(z) = \frac{\tan 9z}{9z}.$$
Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

✏ **Solution:**

❷ Question 10

Using the definition, show that

$$f(z) = 7z^2 + 7z - 3$$

is differentiable at all points. Also find the derivative.

✏️ **Solution:**

❷ Question 11

Using the definition, find the derivative of $f(z) = \dfrac{7}{z^2}$ at $z = 2 + 8i$.

✏️ **Solution:**

❷ Question 12

Consider the function

$$f(z) = 5\sin(6z) - 4\cosh(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✎ **Solution:**

❓ Question 13

Consider the function

$$f(z) = 2ze^{-2z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

✏️ **Solution:**

❷ Question 14

Show that the function

$$v(x, y) = 2\sin(7x)\cosh(7y) + 15x^2y - 6x^2 - 5y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

✏️ **Solution:**

❓ **Question 15**

Show that the function

$$u(x, y) = 4xe^{-9x} \cos{(9y)} + 4ye^{-9x} \sin{(9y)}$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

✒️ **Solution:**