



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT 215

Complex Variables and Laplace Transformations

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^6 = -\frac{729\sqrt{3}}{2} + \frac{729i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z - 6i| - |z + 6i| = 10$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z - 9i| - |z + 9i| \leq 11$ on the complex plane.

 Solution:

? Question 4

Solve the equation

$$e^{7z} = \frac{7\sqrt{2}(1-i)}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\operatorname{cosech}^{-1} z = \ln \left(\frac{1 + \sqrt{z^2 + 1}}{z} \right),$$

 Solution:

Question 6

Solve for z where

$$\cos^{-1} z = 3 - 7i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}(z^2)}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{3 \sin(5z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{9z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.

 **Solution:**

? Question 10

Using the definition, show that

$$f(z) = 7z\bar{z} - 4z + 6\bar{z}$$

is not differentiable at $z = 0$.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{2}{4z + 9}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 2 \sinh(9z) - 9 \cos(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 8|z|^2 + 6z - 6\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$v(x, y) = 4e^{-6x} \cos(6y) - 4e^{6y} \sin(6x) + 18x^2y - 5x^2 - 6y^3 + 5y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution:

Question 15

Show that the function

$$u(x, y) = 4xe^{-7x} \cos(7y) + 4ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**