BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

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Question 1

Find all possible values of z satisfying

$$z^5 = -16 - 16\sqrt{3}i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $\left|\frac{z+4i}{z-4i}\right| = 2$ on the complex plane.

Solution:

Question 3

Describe the region $|z-6|-|z+6|<8$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{9z} = \frac{3}{2} + \frac{3\sqrt{3}i}{2}$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

Solution:

Question 6

Solve for z where

$$\operatorname{cosec}^{-1} z = 6 + 3i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\tan z}{z}\right)^{\frac{5\sin(2z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{6z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 7z^2 + 8z - 5$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{5}{z^2}$ at $z = 9 + 6i$.

Solution:

Question 12

Consider the function

$$f(z) = 8\sin(5z) - 6\cosh(6z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 6|z|^2 + 6z - 4\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$u(x, y) = 9 \sin(2x) \cosh(2y) + 24x^2y - 2x^2 - 8y^3 + 2y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 8xe^{-7x}\cos(7y) + 8ye^{-7x}\sin(7y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: