



# III BRAC University

Department of Mathematics and Natural Sciences

**Total Points: 150**

**Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

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**Section: 12**

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**Submission Date:** \_\_\_\_\_

*Assigned by*

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**Question 1**

Find all possible values of  $z$  satisfying

$$z^6 = 32\sqrt{2}(-1 - i).$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

**Question 2**

Describe the locus  $|z - 8| - |z + 8| = 12$  on the complex plane.

 **Solution:**

**?** Question 3

Describe the region  $|z + 7| + |z - 7| > 20$  on the complex plane.

 Solution:

**Question 4**

Solve the equation

$$e^{5z} = 3\sqrt{2}(1 - i)$$

for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 **Solution:**

**Question 5**

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left( \frac{z+1}{z-1} \right).$$

 Solution:

**?** Question 6

Solve for  $z$  where

$$\tan^{-1} z = 9 - 8i$$

 Solution:

**?** Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Im}\{z^2\}}{|z|^2}$  does not exist.

 Solution:

**Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\sin z}{z} \right)^{\frac{2 \sin(4z)}{z - \sin z}}$$

 Solution:

## Question 9

Consider the function

$$f(z) = \frac{\tan 5z}{9z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .

 **Solution:**

**?** Question 10

Using the definition, show that

$$f(z) = 5z\bar{z} - 4z + 8\bar{z}$$

is not differentiable at  $z = 0$ .

 Solution:

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{4}{5z + 2}$  at  $z = z_0$ .

 Solution:

## Question 12

Consider the function

$$f(z) = 8 \sin(8z) - 2 \cosh(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

### Question 13

Consider the function

$$f(z) = 2ze^{-5z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

**Question 14**

Show that the function

$$u(x, y) = 4e^{-6x} \cos(6y) - 6e^{2y} \sin(2x) + 15x^2y - 2x^2 - 5y^3 + 2y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 **Solution:**

**?** Question 15

Show that the function

$$u(x, y) = 7xe^{-4x} \cos(4y) + 7ye^{-4x} \sin(4y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 Solution: