



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Submission Date: _____

Assigned by

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? Question 1

Find all possible values of z satisfying

$$z^5 = -243.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 5| + |z - 5| = 13$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z - 9| - |z + 9| > 14$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{7z} = 3\sqrt{3} + 3i$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

? Question 5

Prove that

$$\sin^{-1} z = \frac{1}{i} \ln \left(iz + \sqrt{1 - z^2} \right),$$

 Solution:

Question 6

Solve for z where

$$\cosh^{-1} z = 5 - 9i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{3 \sin(7z)}{z - \sin z}}$$

 Solution:

? Question 9

Consider the function

$$f(z) = \frac{\tan 4z}{4z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 7z^2 + 2z - 6$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{7}{8z+7}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 4 \sin(5z) - 9 \cosh(6z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 5|z|^2 + 3z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 14

Show that the function

$$v(x, y) = 8 \sin(9x) \cosh(9y) + 27x^2y - 2x^2 - 9y^3 + 2y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**

? Question 15

Show that the function

$$u(x, y) = 6xe^{-7x} \cos(7y) + 6ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution: