



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Submission Date: _____

Assigned by

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Question 1

Find all possible values of z satisfying

$$z^6 = 32\sqrt{2}(1 + i).$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z - 8i| - |z + 8i| = 13$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z - 8i| - |z + 8i| \geq 10$ on the complex plane.

 Solution:

? Question 4

Solve the equation

$$e^{5z} = \frac{3\sqrt{2}(-1+i)}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\tan^{-1} z = \frac{1}{2i} \ln \left(\frac{1+iz}{1-iz} \right),$$

 Solution:

? Question 6

Solve for z where

$$\sin^{-1} z = 4 + 6i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{7 \sin(4z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 3z}{8z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

? Question 10

Using the definition, show that

$$f(z) = 3z\bar{z} - 9z + 5\bar{z}$$

is not differentiable at $z = 0$.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{7}{z^2}$ at $z = 4 + 7i$.

 Solution:

Question 12

Consider the function

$$f(z) = 3 \sinh(5z) - 4 \cos(9z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 6|z|^2 + 2z - 4\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$v(x, y) = 9e^{-8x} \cos(8y) - 7e^{6y} \sin(6x) + 12x^2y - 6x^2 - 4y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$v(x, y) = 2xe^{-8x} \cos(8y) + 2ye^{-8x} \sin(8y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution: