



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

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 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*


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### Question 1

Find all possible values of  $z$  satisfying

$$z^7 = -2187i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:


## Question 2

Describe the locus  $|z + 5i| + |z - 5i| = 14$  on the complex plane.

 Solution:

### Question 3

Describe the region  $|z + 6i| + |z - 6i| \geq 16$  on the complex plane.

 Solution:

### Question 4

Solve the equation

$$e^{4z} = \frac{3\sqrt{2}(1-i)}{2}$$


for  $z$  and express  $z$  as  $x + iy$  where  $x, y, \in \mathbb{R}$ .

 **Solution:**

### Question 5

Prove that

$$\cos^{-1} z = \frac{1}{i} \ln \left( z + \sqrt{z^2 - 1} \right),$$

 Solution:

### Question 6

Solve for  $z$  where

$$\tanh^{-1} z = 9 - 3i$$

 Solution:

### Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\text{Im}\{z^2\}}{|z|^2}$  does not exist.

 Solution:



## Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{5 \sin(8z)}{z - \sin z}}$$

 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 9z}{6z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .


 **Solution:**

### Question 10

Using the definition, show that


$$f(z) = 6z^2 + 9z - 4$$

is differentiable at all points. Also find the derivative.

 **Solution:**

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{2z - 5}{3z + 4i}$  at  $z = i$ .


 Solution:

## Question 12

Consider the function

$$f(z) = 9 \sinh(5z) - 2 \cos(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

### Question 13

Consider the function

$$f(z) = 9|z|^2 + 8z - 2\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

### Question 14

Show that the function

$$u(x, y) = 5 \sin(7x) \cosh(7y) + 21x^2y - 6x^2 - 7y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 **Solution:**

### Question 15

Show that the function

$$v(x, y) = 7xe^{-4x} \cos(4y) + 7ye^{-4x} \sin(4y)$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $u + vi$  becomes analytic.

 **Solution:**