BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z such that

$$z^6 = 32\sqrt{3} - 32i$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the above locus $|z + 7i| + |z - 7i| = 19$ in the complex plane.

Solution:

Question 3

Describe the above locus $|z+6|+|z-6|<13$ in the complex plane.

Solution:

Question 4

Solve the following equation for z:

$$e^{4z} = \frac{7\sqrt{3}}{2} + \frac{7i}{2}$$

Express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\sinh^{-1} z = \ln\left(z + \sqrt{z^2 + 1}\right),$$

Solution:

Question 6

Solve for z:

$$\operatorname{cosech}^{-1} z = 2 + 2i$$

Solution:

Question 7

Using the definition of limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{9\sin(7z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{9z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition show that

$$f(z) = 7z^2 + 5z - 8$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{7}{5z+3}$ at $z = z_0$.

Solution:

Question 12

Consider the fuction $f(z)$ defined by

$$f(z) = 8\sin(7z) - 6\cosh(6z)$$

Using C-R equations determine whether the function is analytic or not.

Solution:

Question 13

Consider the fuction $f(z)$ defined by

$$f(z) = 9|z|^2 + 6z - 7\bar{z}$$

Using C-R equations determine whether the function is analytic or not.

Solution:

Question 14

Show that the given function u defined by

$$u(x, y) = 8e^{-6x}\cos(6y) - 5e^{9y}\sin(9x) + 15x^2y - 5x^2 - 5y^3 + 5y^2$$

is harmonic. Find the harmonic conjugate v of u such that **u+vi** becomes analytic.

Solution:

Question 15

Show that the given function v defined by

$$v(x, y) = 2xe^{-7x}\cos(7y) + 2ye^{-7x}\sin(7y)$$

is harmonic. Find the harmonic conjugate u of v such that **u+vi** becomes analytic.

Solution: