BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: RUPAM MANDAL

Student ID: 24121270

Section: 12

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Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^7 = \frac{2187}{2} - \frac{2187\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z + 5i| + |z - 5i| = 15$ on the complex plane.

Solution:

Question 3

Describe the region $\left|\frac{z+4i}{z-4i}\right| \leq 6$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{4z} = -3\sqrt{3} + 3i$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\sec^{-1} z = \frac{1}{i} \ln \left(\frac{1 + \sqrt{1 - z^2}}{z} \right),$$

Solution:

Question 6

Solve for z where

$$\operatorname{cosech}^{-1} z = 3 - 5i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\tan z}{z}\right)^{\frac{9\sin(9z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 5z}{6z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 6z\bar{z} - 7z + 3\bar{z}$$

is not differentiable at $z = 0$.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{6}{5z+5}$ at $z = z_0$.

Solution:

Question 12

Consider the function

$$f(z) = 4\sin(7z) - 3\cosh(2z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 9ze^{-8z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 4\sin(2x)\cosh(2y) + 9x^2y - 3x^2 - 3y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 3xe^{-6x}\cos(6y) + 3ye^{-6x}\sin(6y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: