



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: SUPORNO GHOSH

Student ID: 24301509

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^6 = 32\sqrt{3} - 32i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 7i| + |z - 7i| = 19$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z + 6| + |z - 6| < 13$ on the complex plane.

 Solution:

? Question 4

Solve the equation

$$e^{4z} = \frac{7\sqrt{3}}{2} + \frac{7i}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\sinh^{-1} z = \ln \left(z + \sqrt{z^2 + 1} \right),$$

 Solution:

? Question 6

Solve for z where

$$\operatorname{cosech}^{-1} z = 2 + 2i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Im}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{9 \sin(7z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{9z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 7z^2 + 5z - 8$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{7}{5z + 3}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 8 \sin(7z) - 6 \cosh(6z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 9|z|^2 + 6z - 7\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$u(x, y) = 8e^{-6x} \cos(6y) - 5e^{9y} \sin(9x) + 15x^2y - 5x^2 - 5y^3 + 5y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 Solution:

Question 15

Show that the function

$$v(x, y) = 2xe^{-7x} \cos(7y) + 2ye^{-7x} \sin(7y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**