



# III BRAC University

Department of Mathematics and Natural Sciences

**Total Points: 150**

**Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

**Name: URNISHA CHAKMA**

**Student ID: 24321046**

**Section: 12**

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**Submission Date:** \_\_\_\_\_

*Assigned by*

**Partho Sutra Dhor**  
Lecturer, Department of MNS  
BRAC University

**?** Question 1

Find all possible values of  $z$  satisfying

$$z^7 = -64\sqrt{3} - 64i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

**Question 2**

Describe the locus  $|z - 7i| - |z + 7i| = 8$  on the complex plane.

 **Solution:**

**Question 3**

Describe the region  $\left| \frac{z+8i}{z-8i} \right| \leq 6$  on the complex plane.

 **Solution:**

**?** Question 4

Solve the equation

$$e^{3z} = -\frac{7}{2} + \frac{7\sqrt{3}i}{2}$$

for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 Solution:

**Question 5**

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left( \frac{z+1}{z-1} \right).$$

 Solution:

**?** Question 6

Solve for  $z$  where

$$\tanh^{-1} z = 6 + 2i$$

 Solution:

**?** Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$  does not exist.

 Solution:

**Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{7 \sin(2z)}{z - \sin z}}$$

 Solution:

## Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{6z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .

 **Solution:**

## Question 10

Using the definition, show that

$$f(z) = 6z^2 + 8z - 2$$

is differentiable at all points. Also find the derivative.

 Solution:

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{9}{7z + 5}$  at  $z = z_0$ .

 Solution:

## Question 12

Consider the function

$$f(z) = 3 \sin(3z) - 5 \cosh(4z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

### Question 13

Consider the function

$$f(z) = 4ze^{-4z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

**?** Question 14

Show that the function

$$v(x, y) = 2e^{-3x} \cos(3y) - 3e^{6y} \sin(6x) + 18x^2y - 3x^2 - 6y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $u + vi$  becomes analytic.

 Solution:

**?** Question 15

Show that the function

$$u(x, y) = 7xe^{-5x} \cos(5y) + 7ye^{-5x} \sin(5y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 Solution: