



BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

 **Assignment - 01**

Course Code: MAT 215

Complex Variables and Laplace Transformations

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 **Section: 12**

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 **Submission Date: _____**

Assigned by


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Question 1

Find all possible values of z satisfying

$$z^5 = -16\sqrt{3} + 16i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 Solution:


Question 2

Describe the locus $|z + 7| + |z - 7| = 15$ on the complex plane.

 Solution:

Question 3

Describe the region $|z + 4i| + |z - 4i| \leq 14$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{6z} = 5$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 Solution:

Question 5

Prove that

$$\operatorname{sech}^{-1} z = \ln \left(\frac{1 + \sqrt{1 - z^2}}{z} \right),$$

 Solution:

Question 6

Solve for z where

$$\cos^{-1} z = 8 + 4i$$

 Solution:

Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\text{Im}(z^2)}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{8 \sin(7z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 4z}{8z}$$

. Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all points of discontinuity of $f(z)$.


 **Solution:**

Question 10

Using the definition, show that


$$f(z) = 2z^2 + 8z - 2$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{3}{7z+3}$ at $z = z_0$.


 Solution:

Question 12

Consider the function

$$f(z) = 8 \sinh(5z) - 4 \cos(3z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

Question 13

Consider the function

$$f(z) = 4ze^{-7z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 **Solution:**

Question 14

Show that the function

$$v(x, y) = 5 \sin(3x) \cosh(3y) + 18x^2y - 6x^2 - 6y^3 + 6y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**

Question 15

Show that the function

$$v(x, y) = 5xe^{-6x}\cos(6y) + 5ye^{-6x}\sin(6y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 **Solution:**