



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^6 = 32 + 32\sqrt{3}i.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z + 8| + |z - 8| = 22$ on the complex plane.

 **Solution:**

Question 3

Describe the region $\left| \frac{z+5i}{z-5i} \right| > 7$ on the complex plane.

 **Solution:**

Question 4

Solve the equation

$$e^{2z} = -3\sqrt{3} - 3i$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

? Question 5

Prove that

$$\sin^{-1} z = \frac{1}{i} \ln \left(iz + \sqrt{1 - z^2} \right),$$

 Solution:

? Question 6

Solve for z where

$$\cot^{-1} z = 8 + 2i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\tan z}{z} \right)^{\frac{2 \sin(2z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{4z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 8z^2 + 4z - 2$$

is differentiable at all points. Also find the derivative.

 Solution:

Question 11

Using the definition, find the derivative of $f(z) = \frac{5}{4z + 7}$ at $z = z_0$.

 Solution:

Question 12

Consider the function

$$f(z) = 2 \sinh(9z) - 4 \cos(8z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 3ze^{-8z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 14

Show that the function

$$u(x, y) = 5 \sin(6x) \cosh(6y) + 24x^2y - 4x^2 - 8y^3 + 4y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

 **Solution:**

? Question 15

Show that the function

$$v(x, y) = 4xe^{-2x} \cos(2y) + 4ye^{-2x} \sin(2y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution: