🏛 BRAC University

Department of Mathematics and Natural Sciences

Total Points: 15

📋 Assignment-01

Course Code: MAT215

Complex

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👥 **Section: 12**

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🕐 **Submission Date:** _____

Assigned by

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❓ Question 1

Find all possible values of z such that

$$z^6 = \frac{729\sqrt{2}\,(-1-i)}{2}$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

✏️ **Solution:**

❓ Question 2

Consider the equation

$$\left| \frac{z + 4i}{z - 4i} \right| = 3$$

Describe the above locus in the complex plane.

🖊 **Solution:**

❓ Question 3

Consider the inequality

$$\left|\frac{z + 9i}{z - 9i}\right| \geq 2$$

Describe the above locus in the complex plane.

🖊 **Solution:**

❓ **Question 4**

Solve the following equation for z:

$$e^{5z} = -\frac{5}{2} + \frac{5\sqrt{3}i}{2}$$

Express z as $x + iy$ where $x, y, \in \mathbb{R}$.

✏️ **Solution:**

❷ **Question 5**

Prove that

$$\sin^{-1} z = \frac{1}{i} \ln\left(iz + \sqrt{1-z^2}\right),$$

✏️ **Solution:**

❓ **Question 6**

Solve for z:

$$\tanh^{-1} z = 5 - 9i$$

📝 **Solution:**

❓ **Question 7**

Show that the limit, $\lim\limits_{z \to 0} \dfrac{\text{Re}\{z^2\}}{|z|^2}$ does not exist.

✏️ **Solution:**

❓ Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \to 0} \left(\frac{\tan z}{z} \right)^{\frac{8 \sin(8z)}{z - \sin z}}$$

✏️ **Solution:**

❓ **Question 9**

Consider the function
$$f(z) = \frac{\tan 2z}{4z}.$$
Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

🖊 **Solution:**

❓ Question 10

Using the definition show that
$$f(z) = 4z^2 + 6z - 3$$
is differentiable at all points. Also find the derivative.

✏️ **Solution:**

❷ Question 11

Using the definition, find the derivative of

$$f(z) = \frac{3z - 2}{9z + 8i} \quad \text{at} \quad z = i$$

.

✏ **Solution:**

❓ Question 12

Consider the fuction $f(z)$ defined by

$$f(z) = 5\sinh(7z) - 5\cos(7z)$$

Using C-R equations determine whether the function is analytic or not.

✏️ **Solution:**

❓ Question 13

Consider the fuction $f(z)$ defined by

$$f(z) = 8|z|^2 + 4z - 2\bar{z}$$

Using C-R equations determine whether the function is analytic or not.

✏️ **Solution:**

❓ **Question 14**

Show that the given function u defined by

$$u(x,y) = 6e^{-9x}\cos(9y) - 5e^{3y}\sin(3x) + 27x^2 y - 9x^2 - 9y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u+vi}$ becomes analytic.

✒ **Solution:**

❓ **Question 15**

Show that the given function u defined by

$$u(x, y) = 2xe^{-8x}\cos(8y) + 2ye^{-8x}\sin(8y)$$

is harmonic. Find the harmonic conjugate v of u such that $\mathbf{u+vi}$ becomes analytic.

✒ **Solution:**