



# **BRAC University**

Department of Mathematics and Natural Sciences

**Total Points: 150**

 **Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

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 **Section: 12**

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 **Submission Date: \_\_\_\_\_**

*Assigned by*


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### Question 1

Find all possible values of  $z$  such that

$$z^7 = -64\sqrt{3} - 64i$$

Locate them in the complex plane. Show that they are contained in a circle and find the radius of that circle. Also find the angular distance between two adjacent roots.

 Solution:


## Question 2

Describe the above locus  $|z - 7i| - |z + 7i| = 8$  in the complex plane.

 Solution:

### Question 3

Describe the above locus  $\left| \frac{z+8i}{z-8i} \right| \leq 6$  in the complex plane.

 Solution:

### Question 4

Solve the following equation for  $z$ :

$$e^{3z} = -\frac{7}{2} + \frac{7\sqrt{3}i}{2}$$

Express  $z$  as  $x + iy$  where  $x, y, \in \mathbb{R}$ .

 **Solution:**

### Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left( \frac{z+1}{z-1} \right).$$

 Solution:

## Question 6

Solve for  $z$ :

$$\tanh^{-1} z = 6 + 2i$$

 Solution:

### Question 7

Using the definition of limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$  does not exist.

 Solution:



## Question 8

Using the L'Hospital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\tan z}{z} \right)^{\frac{7 \sin(2z)}{z - \sin z}}$$

 Solution:

### Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{6z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .

 **Solution:**

### Question 10

Using the definition show that


$$f(z) = 6z^2 + 8z - 2$$

is differentiable at all points. Also find the derivative.

 **Solution:**

### Question 11

Using the definition, find the derivative of  $f(z) = \frac{9}{7z+5}$  at  $z = z_0$ .

 Solution:

## Question 12

Consider the function  $f(z)$  defined by

$$f(z) = 3 \sin(3z) - 5 \cosh(4z)$$

Using C-R equations determine whether the function is analytic or not.


 **Solution:**

### Question 13

Consider the function  $f(z)$  defined by

$$f(z) = 4ze^{-4z}$$

Using C-R equations determine whether the function is analytic or not.


 **Solution:**

### Question 14

Show that the given function  $v$  defined by

$$v(x, y) = 2e^{-3x} \cos(3y) - 3e^{6y} \sin(6x) + 18x^2y - 3x^2 - 6y^3 + 3y^2$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $\mathbf{u} + \mathbf{vi}$  becomes analytic.

 **Solution:**

### Question 15

Show that the given function  $u$  defined by

$$u(x, y) = 7xe^{-5x} \cos(5y) + 7ye^{-5x} \sin(5y)$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $\mathbf{u} + \mathbf{vi}$  becomes analytic.

 **Solution:**