



# III BRAC University

Department of Mathematics and Natural Sciences

**Total Points: 150**

**Assignment - 01**

**Course Code: MAT215**

Complex Variables & Laplace Transform

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**Section: 12**

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**Submission Date:** \_\_\_\_\_

*Assigned by*

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## Question 1

Find all possible values of  $z$  satisfying

$$z^5 = 32.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

**Question 2**

Describe the locus  $|z - 8i| - |z + 8i| = 13$  on the complex plane.

 **Solution:**

**Question 3**

Describe the region  $\left| \frac{z+7i}{z-7i} \right| < 4$  on the complex plane.

 **Solution:**

**?** Question 4

Solve the equation

$$e^{4z} = \frac{9\sqrt{3}}{2} + \frac{9i}{2}$$

for  $z$  and express  $z$  as  $x + iy$  where  $x, y \in \mathbb{R}$ .

 Solution:

**?** Question 5

Prove that

$$\sinh^{-1} z = \ln \left( z + \sqrt{z^2 + 1} \right),$$

 Solution:

**?** Question 6

Solve for  $z$  where

$$\tan^{-1} z = 9 - 9i$$

 Solution:

**?** Question 7

Using the definition of a limit, show that  $\lim_{z \rightarrow 0} \frac{\operatorname{Im}\{z^2\}}{|z|^2}$  does not exist.

 Solution:

**Question 8**

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left( \frac{\sin z}{z} \right)^{\frac{8 \sin(3z)}{z - \sin z}}$$

 Solution:

## Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{9z}.$$

Is  $f(z)$  continuous at  $z = 0$ ? If not, redefine  $f$  at  $z = 0$  so that  $f(z)$  becomes continuous. Also, find all the points of discontinuity of  $f(z)$ .

 **Solution:**

**?** Question 10

Using the definition, show that

$$f(z) = 7z\bar{z} - 2z + 3\bar{z}$$

is not differentiable at  $z = 0$ .

 Solution:

**?** Question 11

Using the definition, find the derivative of  $f(z) = \frac{4z - 3}{3z + 6i}$  at  $z = i$ .

 Solution:

## Question 12

Consider the function

$$f(z) = 6 \sinh(2z) - 8 \cos(3z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

### Question 13

Consider the function

$$f(z) = 9|z|^2 + 5z - 8\bar{z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

**?** Question 14

Show that the function

$$u(x, y) = 3 \sin(8x) \cosh(8y) + 9x^2y - 7x^2 - 3y^3 + 7y^2$$

is harmonic. Find the harmonic conjugate  $v$  of  $u$  such that  $u + vi$  becomes analytic.

 Solution:

**?** Question 15

Show that the function

$$v(x, y) = 7xe^{-9x} \cos(9y) + 7ye^{-9x} \sin(9y)$$

is harmonic. Find the harmonic conjugate  $u$  of  $v$  such that  $u + vi$  becomes analytic.

 Solution: