



III BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: RAISA ZAHIN

Student ID: 24101491

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^5 = -\frac{243}{2} + \frac{243\sqrt{3}i}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

 **Solution:**

Question 2

Describe the locus $|z - 4i| - |z + 4i| = 1$ on the complex plane.

 **Solution:**

? Question 3

Describe the region $|z - 7i| - |z + 7i| \geq 11$ on the complex plane.

 Solution:

Question 4

Solve the equation

$$e^{4z} = -\frac{5\sqrt{3}}{2} - \frac{5i}{2}$$

for z and express z as $x + iy$ where $x, y \in \mathbb{R}$.

 **Solution:**

? Question 5

Prove that

$$\cot^{-1} z = \frac{1}{2i} \ln \left(\frac{z+i}{z-i} \right),$$

 Solution:

Question 6

Solve for z where

$$\operatorname{cosech}^{-1} z = 5 - 7i$$

 Solution:

? Question 7

Using the definition of a limit, show that $\lim_{z \rightarrow 0} \frac{\operatorname{Re}\{z^2\}}{|z|^2}$ does not exist.

 Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{\frac{9 \sin(6z)}{z - \sin z}}$$

 Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 5z}{8z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

 **Solution:**

Question 10

Using the definition, show that

$$f(z) = 3z^2 + 3z - 3$$

is differentiable at all points. Also find the derivative.

 Solution:

? Question 11

Using the definition, find the derivative of $f(z) = \frac{3z - 5}{5z + 4i}$ at $z = i$.

 Solution:

Question 12

Consider the function

$$f(z) = 8 \sin(5z) - 8 \cosh(5z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

Question 13

Consider the function

$$f(z) = 6ze^{-4z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

 Solution:

? Question 14

Show that the function

$$v(x, y) = 4e^{-3x} \cos(3y) - 9e^{8y} \sin(8x) + 24x^2y - 8x^2 - 8y^3 + 8y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution:

? Question 15

Show that the function

$$v(x, y) = 5xe^{-6x} \cos(6y) + 5ye^{-6x} \sin(6y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

 Solution: