BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

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Assigned by

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Question 1

Find all possible values of z satisfying

$$z^7 = \frac{2187\sqrt{2}\,(1+i)}{2}.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z-4|-|z+4|=3$ on the complex plane.

Solution:

Question 3

Describe the region $|z - 7i| - |z + 7i| < 7$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{2z} = -\frac{3}{2} - \frac{3\sqrt{3}i}{2}$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\sec^{-1} z = \frac{1}{i} \ln \left(\frac{1 + \sqrt{1 - z^2}}{z} \right),$$

Solution:

Question 6

Solve for z where

$$\cos^{-1} z = 2 - 5i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{2\sin(6z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 7z}{4z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 5z^2 + 9z - 5$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{4}{z^2}$ at $z = 9 + 7i$.

Solution:

Question 12

Consider the function

$$f(z) = 6\sinh(4z) - 4\cos(7z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 7ze^{-8z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$u(x, y) = 8e^{-5x}\cos(5y) - 3e^{5y}\sin(5x) + 27x^2y - 9x^2 - 9y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 7xe^{-4x}\cos(4y) + 7ye^{-4x}\sin(4y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: