BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

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Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

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Question 1

Find all possible values of z satisfying

$$z^7 = 64\sqrt{2}\,(1+i)\,.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $\left|\frac{z+5i}{z-5i}\right| = 5$ on the complex plane.

Solution:

Question 3

Describe the region $|z + 9i| + |z - 9i| < 25$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{5z} = 6i$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\coth^{-1} z = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right).$$

Solution:

Question 6

Solve for z where

$$\sinh^{-1} z = 3 - 3i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Im}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{2\sin(2z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 2z}{2z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 7z^2 + 8z - 4$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{3z-9}{6z+8i}$ at $z = i$.

Solution:

Question 12

Consider the function

$$f(z) = 7\sinh(9z) - 5\cos(9z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 2ze^{-3z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$u(x, y) = 7 \sin(9x) \cosh(9y) + 15x^2y - 9x^2 - 5y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$u(x, y) = 9xe^{-2x}\cos(2y) + 9ye^{-2x}\sin(2y)$$

is harmonic. Find the harmonic conjugate v of u such that $u + vi$ becomes analytic.

Solution: