BRAC University

Department of Mathematics and Natural Sciences

Total Points: 150

Assignment - 01

Course Code: MAT215

Complex Variables & Laplace Transform

Name: SADIA RAHMAN SUPTY

Student ID: 24101042

Section: 12

Semester: FALL 2025

Submission Date: _____

Assigned by

Partho Sutra Dhor
Lecturer, Department of MNS
BRAC University

Question 1

Find all possible values of z satisfying

$$z^7 = -2187.$$

Locate them on the complex plane. Show that they lie on a circle, and determine its radius. Also, find the angular distance between two adjacent roots.

Solution:

Question 2

Describe the locus $|z+4|+|z-4|=15$ on the complex plane.

Solution:

Question 3

Describe the region $|z-7|-|z+7|<11$ on the complex plane.

Solution:

Question 4

Solve the equation

$$e^{4z} = 2\sqrt{2}\,(-1 - i)$$

for z and express z as $x + iy$ where $x, y, \in \mathbb{R}$.

Solution:

Question 5

Prove that

$$\operatorname{sech}^{-1} z = \ln\left(\frac{1+\sqrt{1-z^2}}{z}\right),$$

Solution:

Question 6

Solve for z where

$$\sec^{-1} z = 5 + 9i$$

Solution:

Question 7

Using the definition of a limit, show that $\lim_{z\to 0} \dfrac{\mathrm{Re}\{z^2\}}{|z|^2}$ does not exist.

Solution:

Question 8

Using L'Hôpital's rule, evaluate

$$\lim_{z\to 0}\left(\frac{\tan z}{z}\right)^{\frac{2\sin(5z)}{z-\sin z}}$$

Solution:

Question 9

Consider the function

$$f(z) = \frac{\tan 6z}{8z}.$$

Is $f(z)$ continuous at $z = 0$? If not, redefine f at $z = 0$ so that $f(z)$ becomes continuous. Also, find all the points of discontinuity of $f(z)$.

Solution:

Question 10

Using the definition, show that

$$f(z) = 8z^2 + 4z - 8$$

is differentiable at all points. Also find the derivative.

Solution:

Question 11

Using the definition, find the derivative of $f(z) = \dfrac{2}{6z+2}$ at $z = z_0$.

Solution:

Question 12

Consider the function

$$f(z) = 3\sinh(6z) - 4\cos(2z).$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 13

Consider the function

$$f(z) = 8ze^{-8z}.$$

Using the Cauchy–Riemann equations, determine whether the function is analytic.

Solution:

Question 14

Show that the function

$$v(x, y) = 5e^{-2x}\cos(2y) - 5e^{5y}\sin(5x) + 27x^2y - 9x^2 - 9y^3 + 9y^2$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution:

Question 15

Show that the function

$$v(x, y) = 7xe^{-2x}\cos(2y) + 7ye^{-2x}\sin(2y)$$

is harmonic. Find the harmonic conjugate u of v such that $u + vi$ becomes analytic.

Solution: