Image Processing and Visual Communications

Intensity Transformation for Image Enhancement

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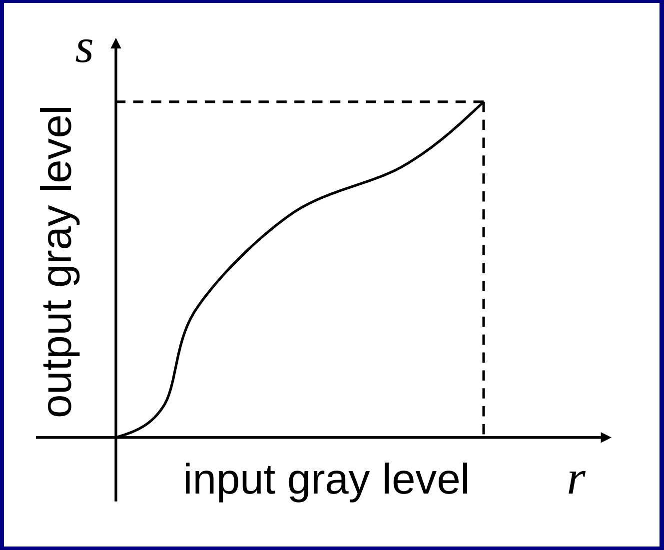
General Idea

- **Intensity transformation**
  - Also called point operation
  - Zero-memory operation

- **Map a given gray level to another level**

$$\boxed{s = T(r)}$$

typically monotonically increasing (but not always)



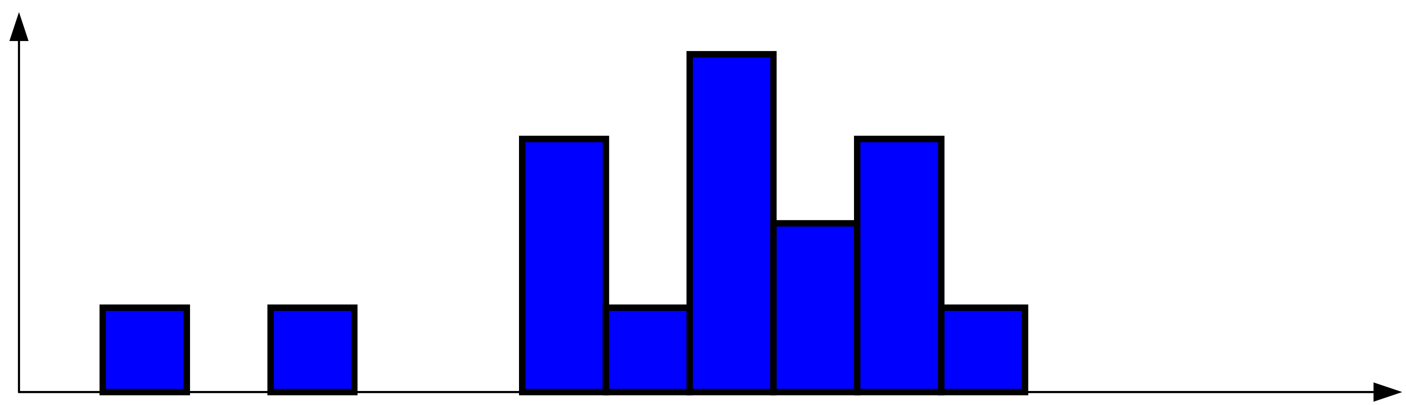
Intensity Histogram

- **Example**

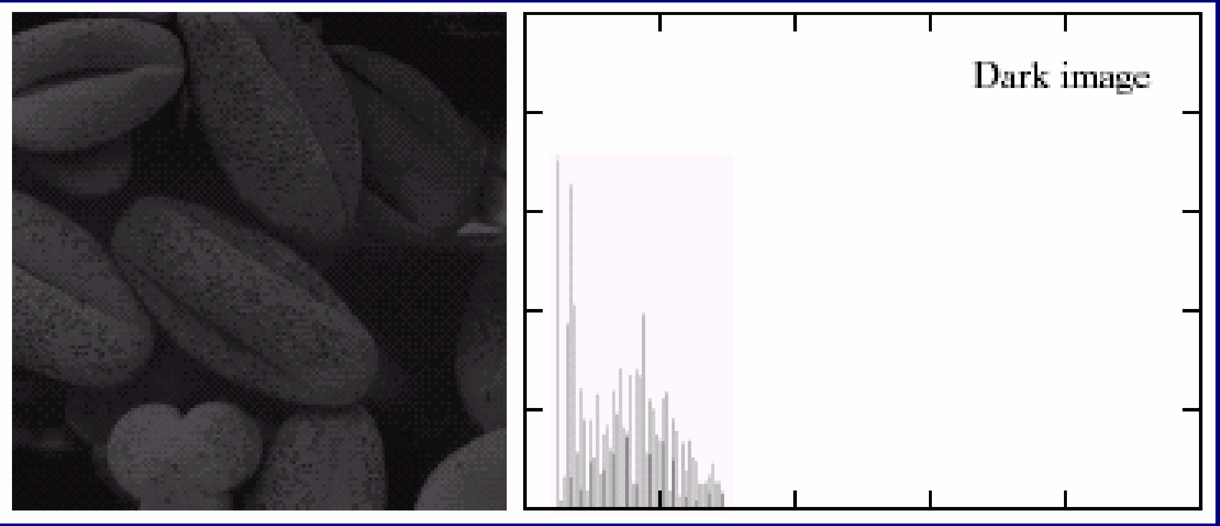
  a 4x4, 4bits/pixel image →

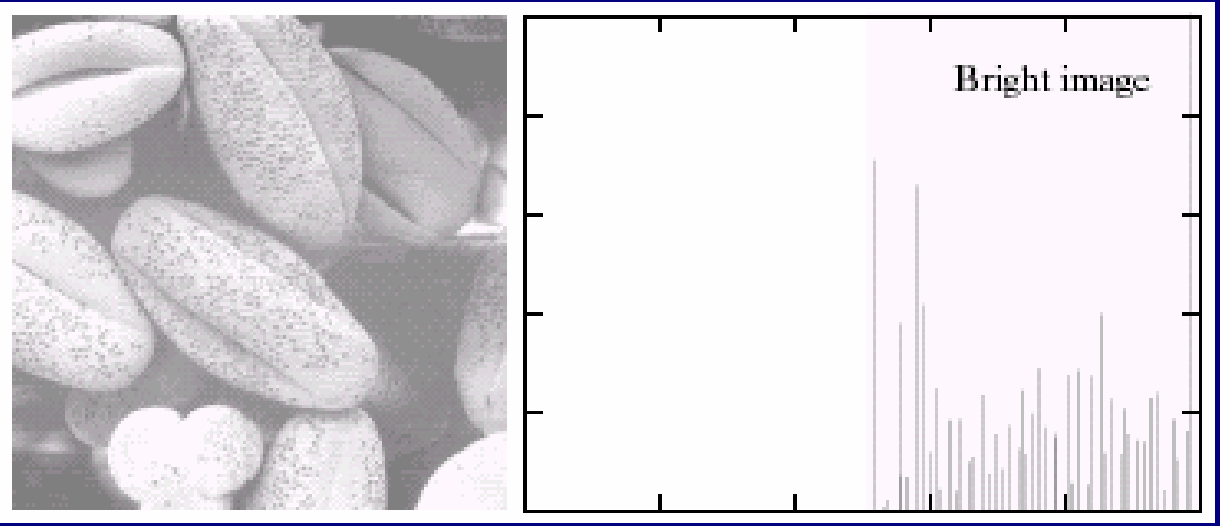
1	8	6	6
6	3	11	8
8	8	9	10
9	10	10	7

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	1	0	1	0	0	3	1	4	2	3	1	0	0	0	0



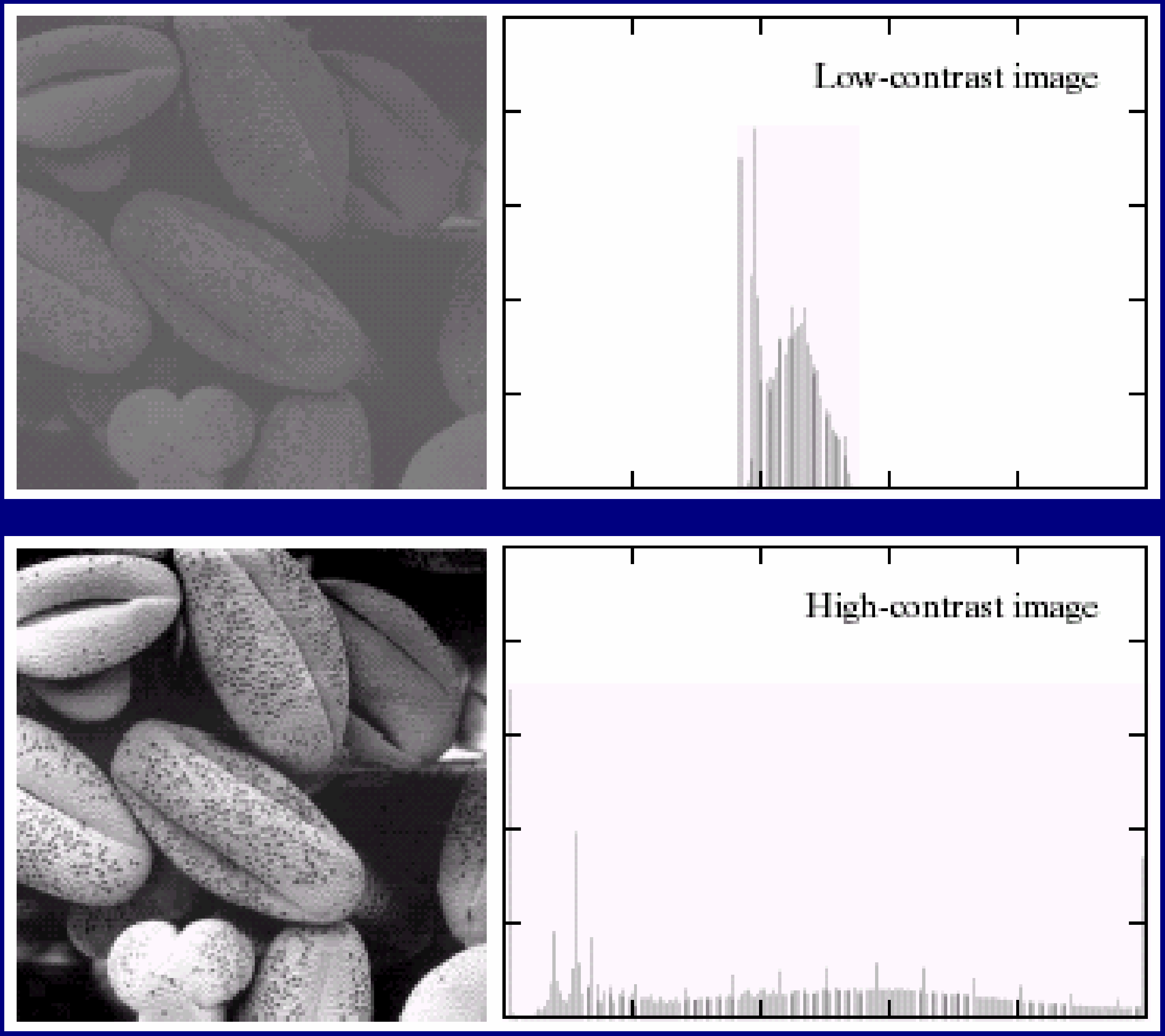
Intensity Histogram





From [Gonzalez & Woods]

Intensity Histogram



From [Gonzalez & Woods]

Basic Transformations

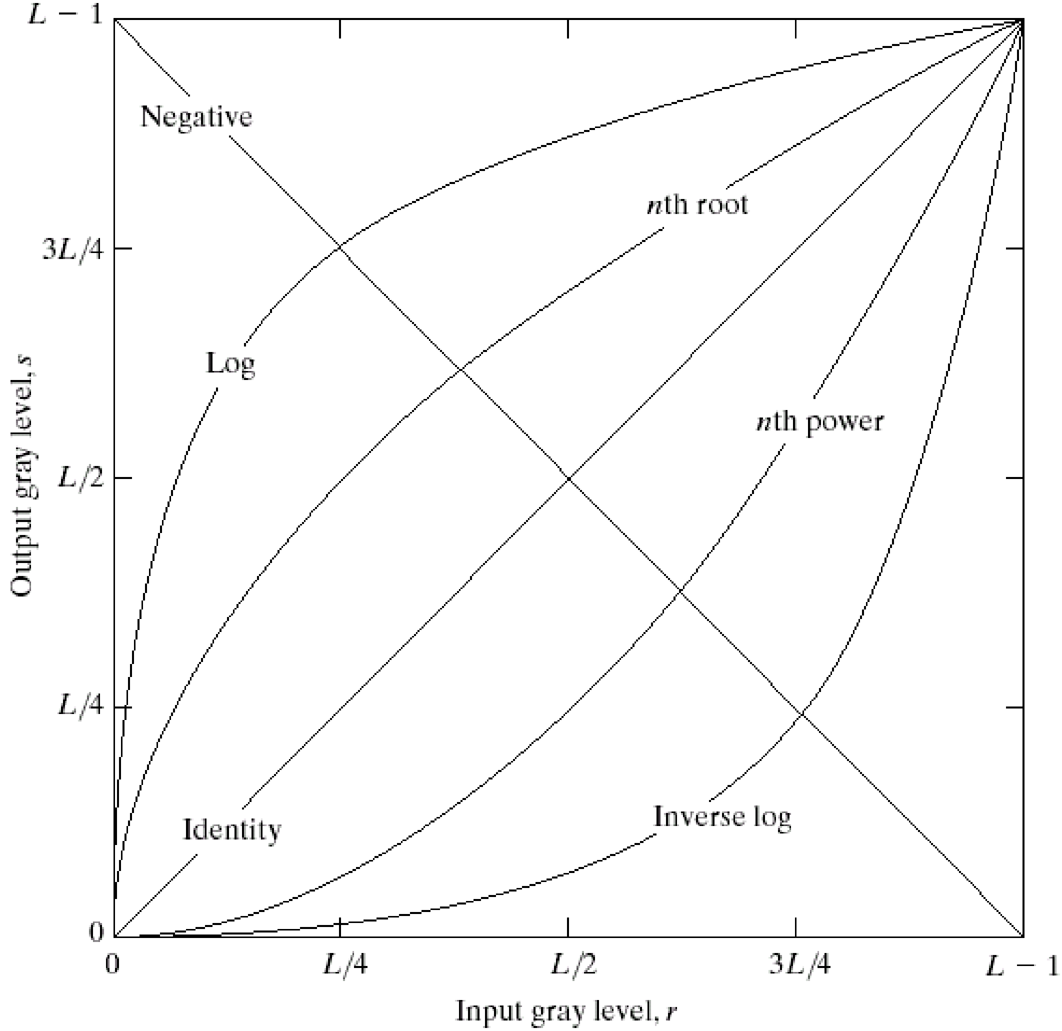


FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

Negative:

$$s = L - 1 - r$$

Log:

$$s = c\log(1 + r)$$

Inverse Log:

$$s = e^{cr} - 1$$

Power-law:

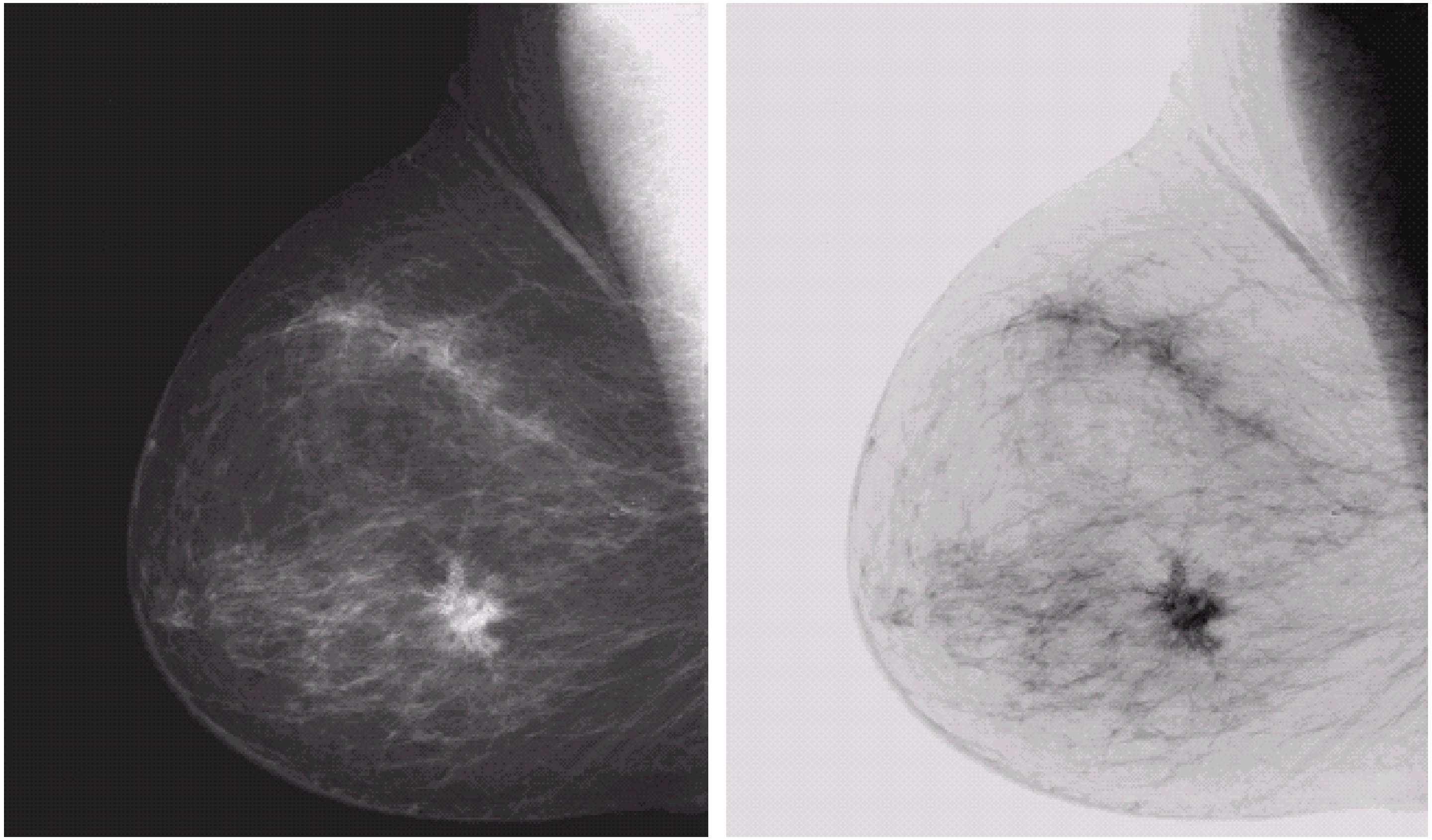
$$s = cr^{\gamma}$$

……

From [Gonzalez & Woods]

Negative Transformation

$$s = L - 1 - r$$



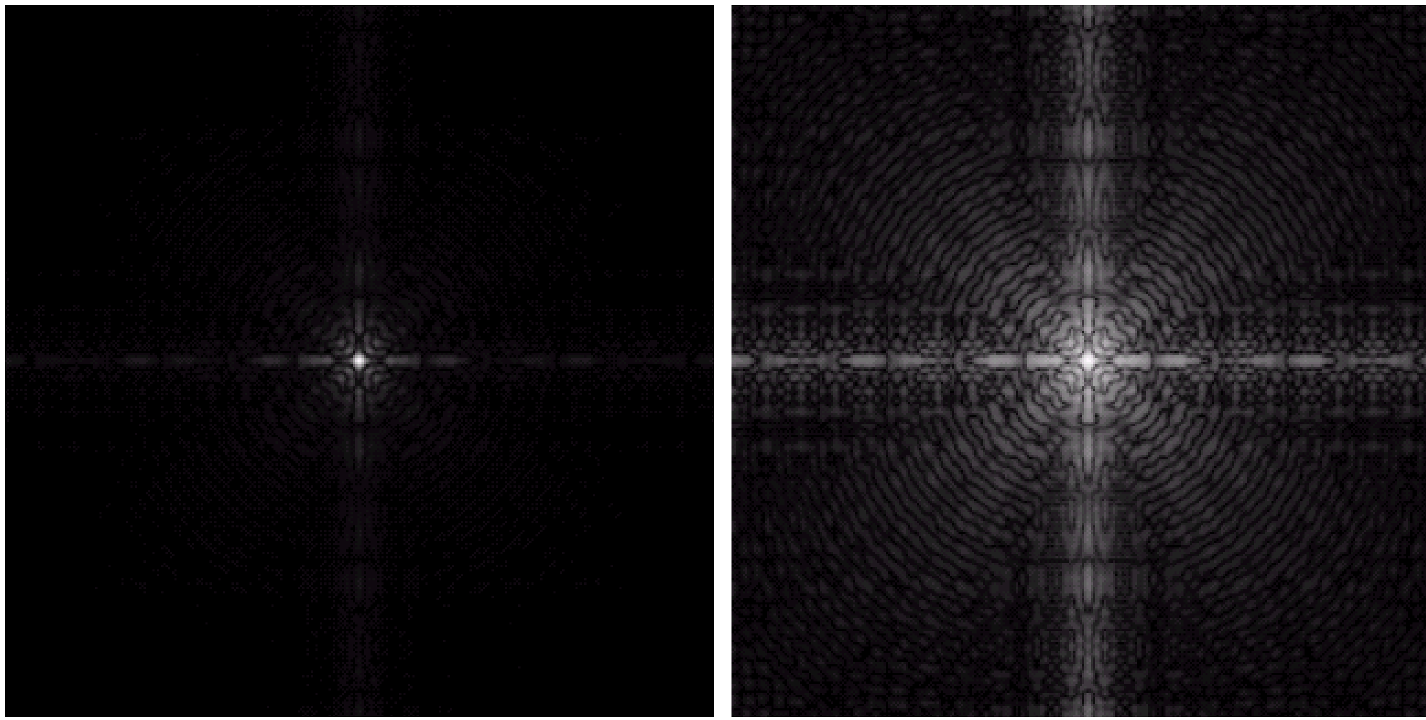
a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

From [Gonzalez & Woods]

Log Transformation

$$s = c \log(1 + r)$$



a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

From [Gonzalez & Woods]

Power-law (Gamma) Transformation

$$s = cr^{\gamma}$$

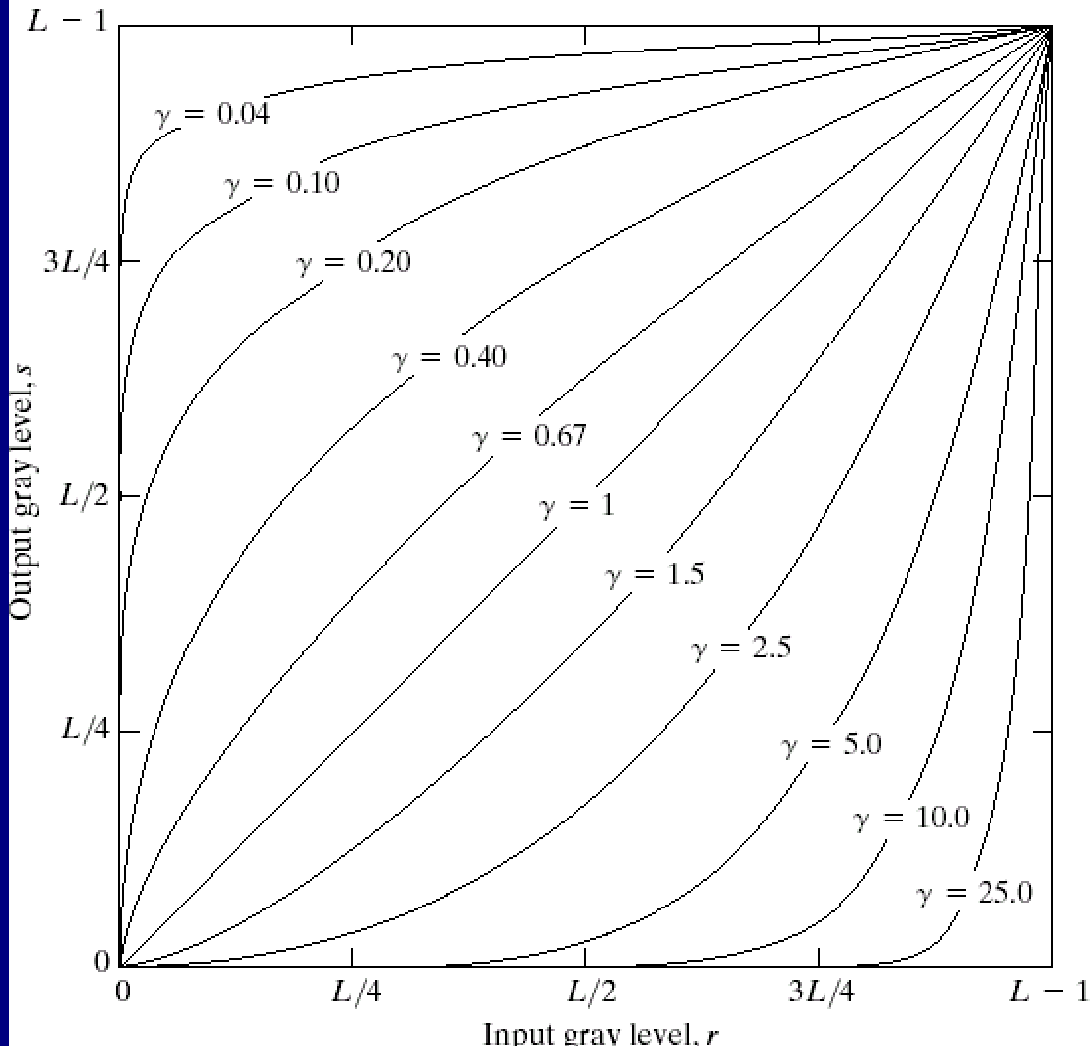
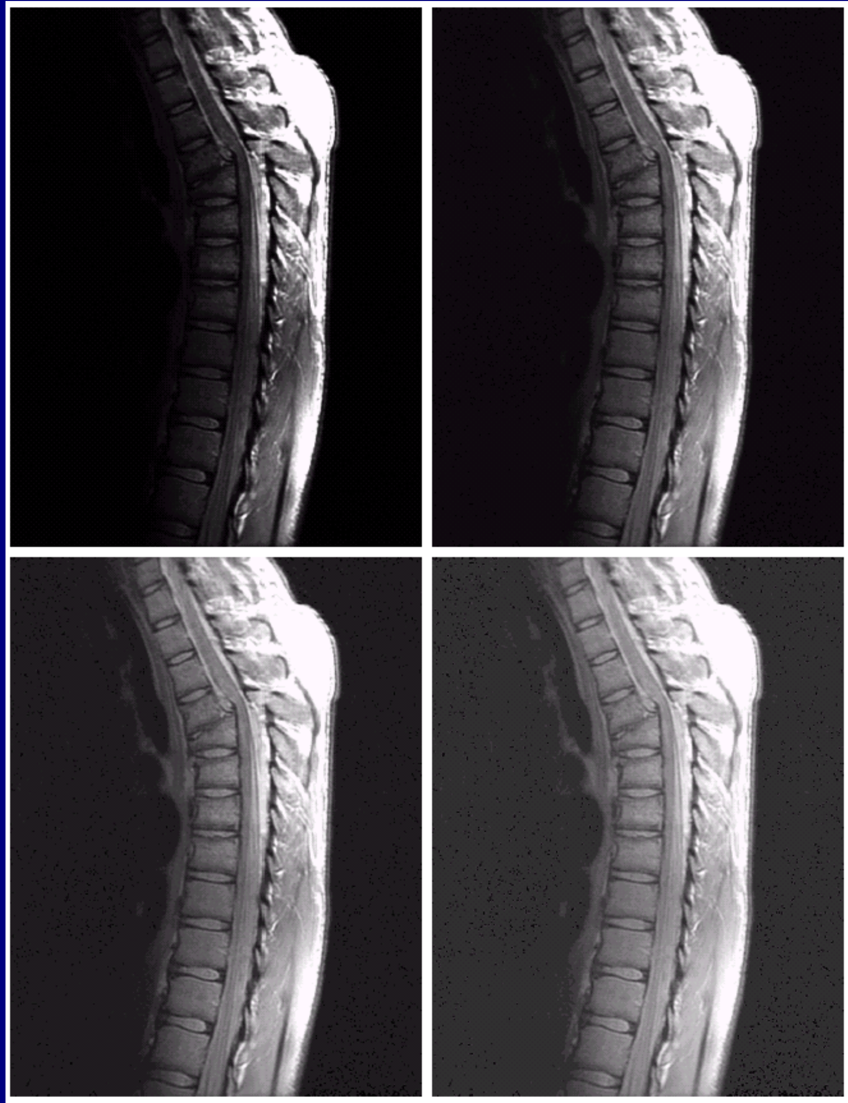


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

From [Gonzalez & Woods]

Power-law (Gamma) Transformation

$$s = cr^{\gamma}$$



a b
c d

FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

From [Gonzalez & Woods]

Power-law (Gamma) Transformation

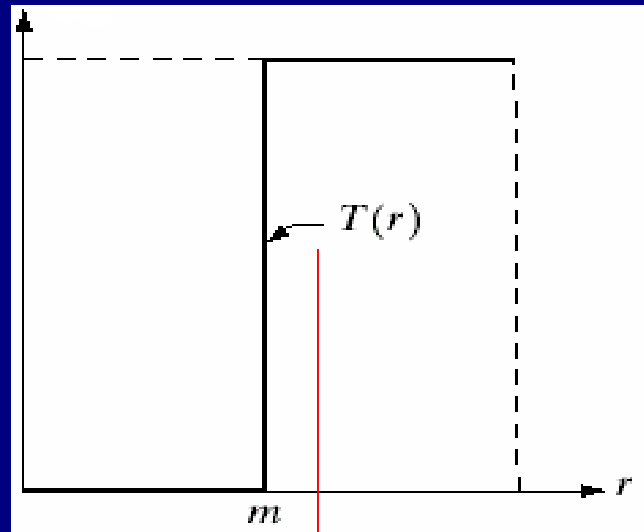
$$s = cr^{\gamma}$$

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

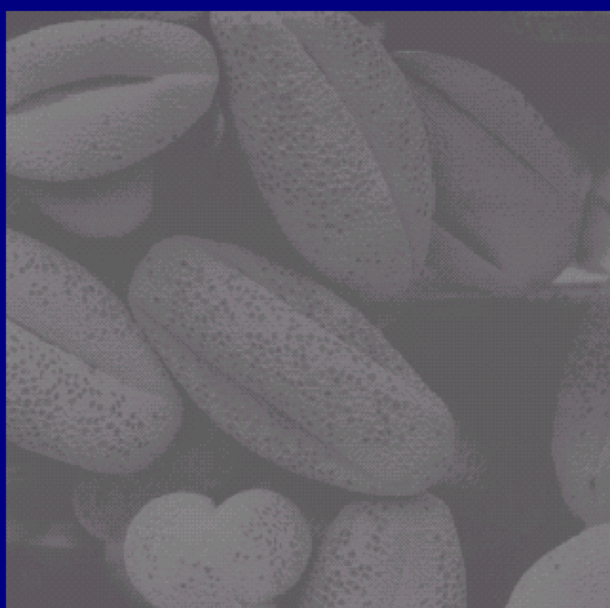
From [Gonzalez & Woods]

Thresholding



$$s = \begin{cases} 0 & \text{if } r \le m \\ c & \text{if } r > m \end{cases}$$

m : threshold



From [Gonzalez & Woods]

Example: Fixed Intensity Transformation

- A 4x4, 4bits/pixel image

1	8	6	6
6	3	11	8
8	8	9	10
9	10	10	7

passes through an intensity transformation

$$s = T(r) = round\left(\frac{1}{15} r^2\right)$$

1 → round(0.0667) = 0;

3 → round(0.6) = 1;

6 → round(2.4) = 2;

7 → round(3.2667) = 3;

8 → round(4.2667) = 4;

9 → round(5.4) =5;

10 → round(6.6667) = 7;

11 → round(8.0667) = 8;

The resulting image is:

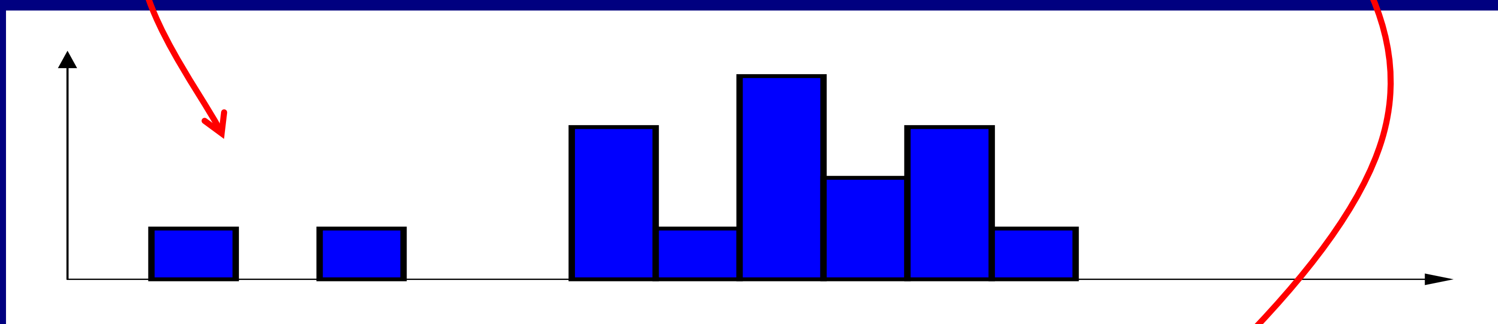
0	4	2	2
2	1	8	4
4	4	5	7
5	7	7	3

Example: Histogram Change

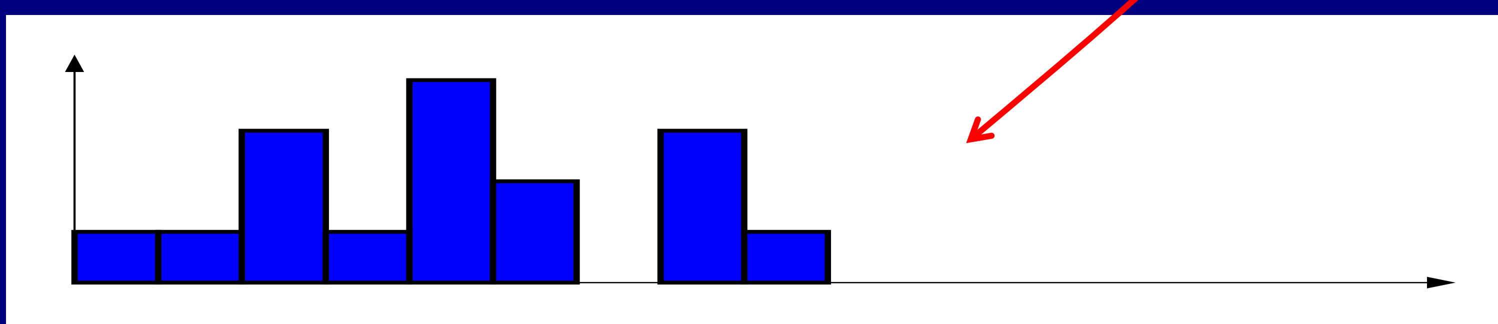
1	8	6	6
6	3	11	8
8	8	9	10
9	10	10	7

→

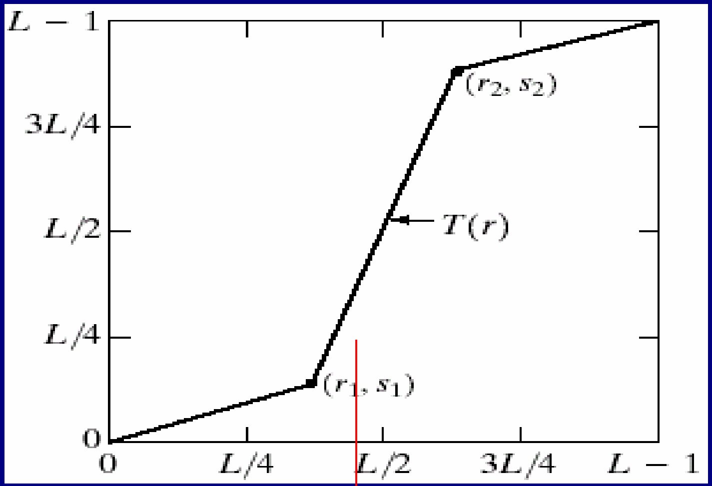
0	4	2	2
2	1	8	4
4	4	5	7
5	7	7	3

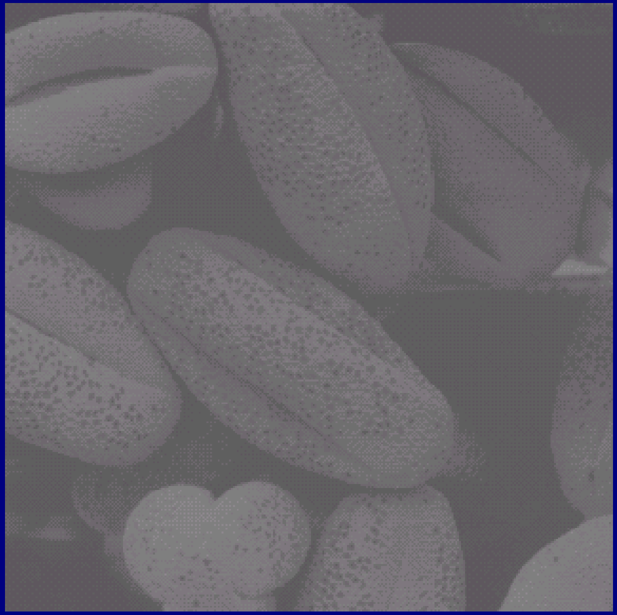


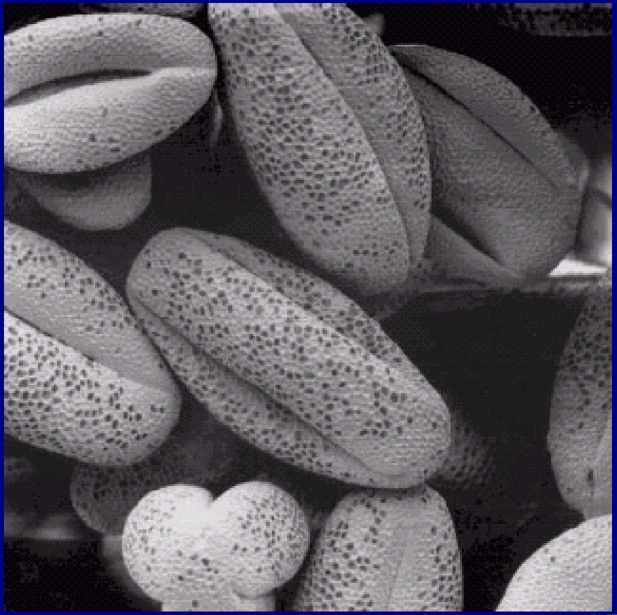
↓



Contrast Stretch: Make Best Use of the Dynamic Range







From [Gonzalez & Woods]

Contrast Stretch

General form:

$$s = \begin{cases} \dfrac{s_1}{r_1} \cdot r & 0 \le r < r_1 \\ \dfrac{s_2 - s_1}{r_2 - r_1} \cdot r + \dfrac{s_1 r_2 - s_2 r_1}{r_2 - r_1} & r_1 \le r \le r_2 \\ \dfrac{2^B - 1 - s_2}{2^B - 1 - r_2} \cdot r + (2^B - 1) \cdot \dfrac{s_2 - r_2}{2^B - 1 - r_2} & r_2 < r \le 2^B - 1 \end{cases}$$

Special case → Full-scale contrast stretch:

$$r_1 = r_{\min} \qquad s_1 = 0$$

$$r_2 = r_{\max} \qquad s_2 = 2^B - 1$$

$$\longrightarrow \quad s = (2^B - 1) \cdot \frac{r - r_{\min}}{r_{\max} - r_{\min}}$$

Typically used: $$s = round\left((2^B - 1) \cdot \frac{r - r_{\min}}{r_{\max} - r_{\min}} \right)$$

Example: Full-Scale Contrast Stretch

- Full-scale contrast stretch of a 4x4, 4bits/pixel image

4	8	6	6
6	4	11	8
8	8	9	10
8	11	10	7

- Find $r_{\min} = 4$ $r_{\max} = 11$ $2^B - 1 = 15$

$$s = round\left((2^B - 1) \cdot \frac{r - r_{\min}}{r_{\max} - r_{\min}} \right) = round\left(15 \cdot \frac{r-4}{11-4} \right) = round\left(\frac{15}{7}(r-4) \right)$$

4 → round(0) = 0;

6 → round(4.29) = 4;

7 → round(6.43) = 6;

8 → round(8.57) = 9;

9 → round(10.71) = 11;

10 → round(12.86) = 13;

11 → round(15) = 15;

The resulting image is:

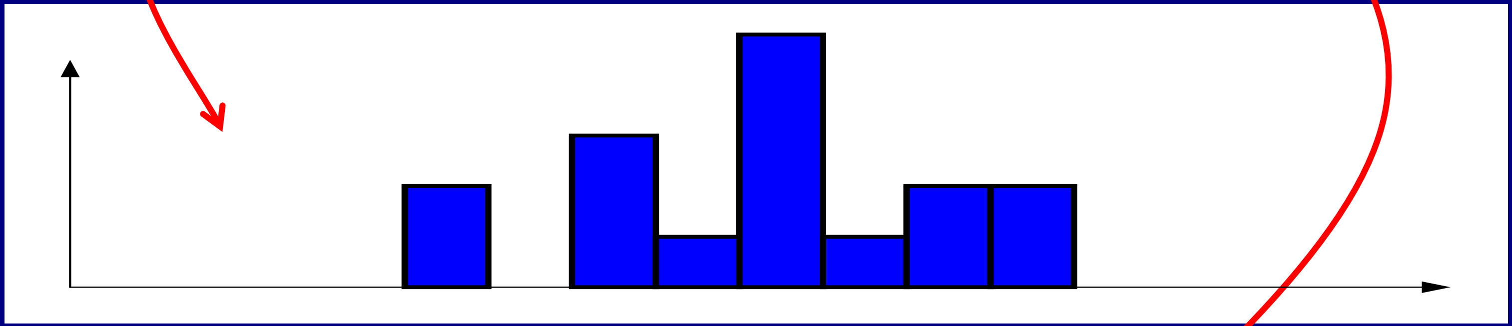
0	9	4	4
4	0	15	9
9	9	11	13
9	15	13	6

Example: Histogram Change

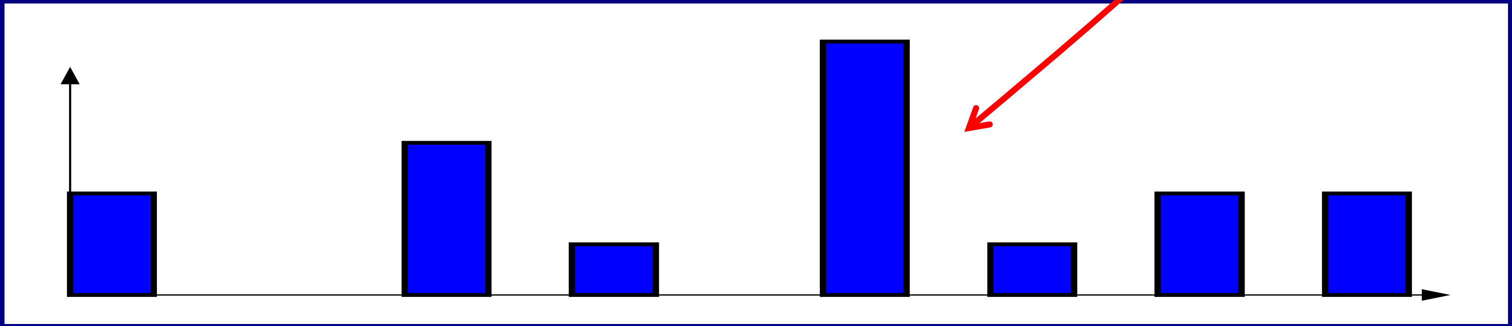
4	8	6	6
6	4	11	8
8	8	9	10
8	11	10	7

→

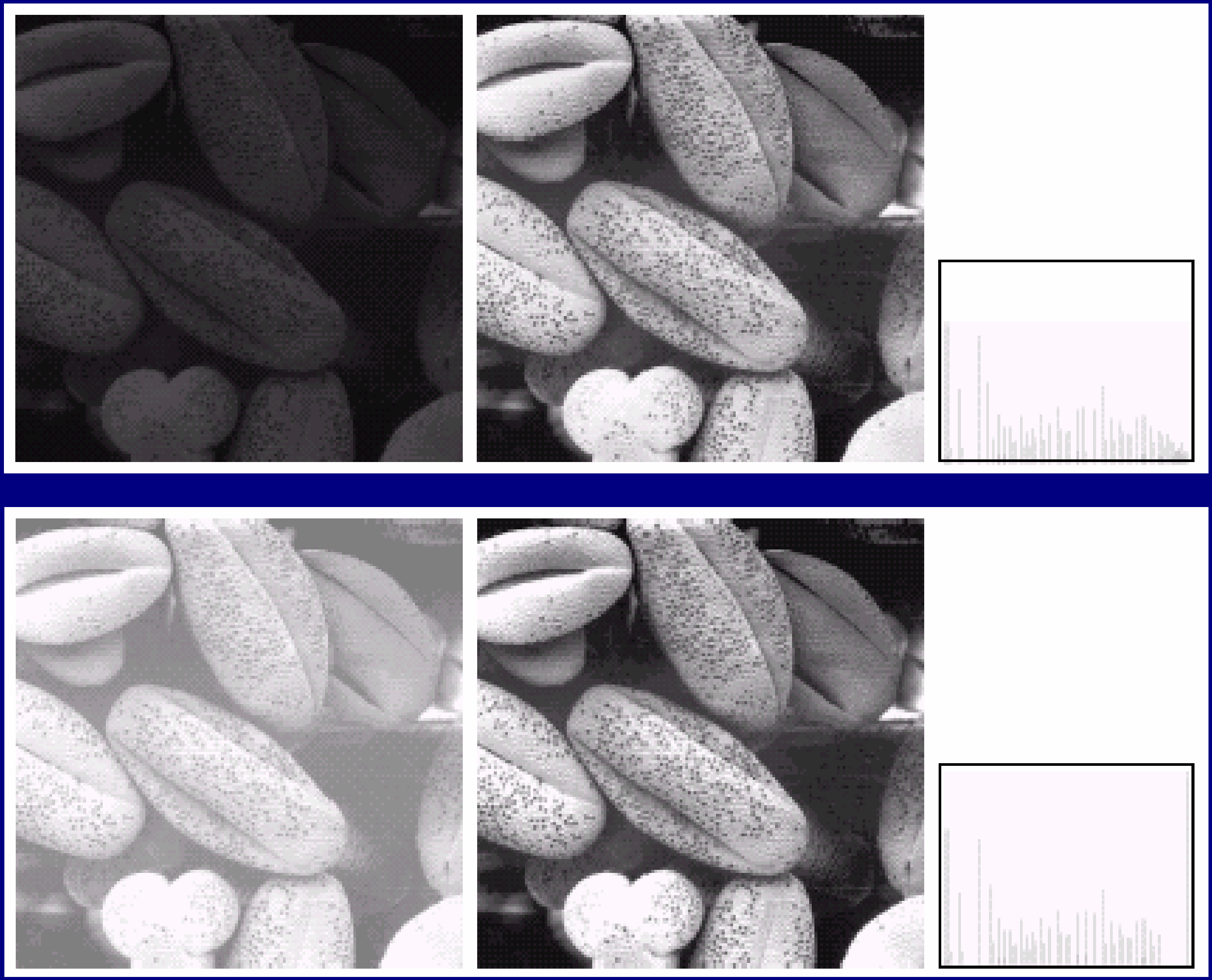
0	9	4	4
4	0	15	9
9	9	11	13
9	15	13	6



↓

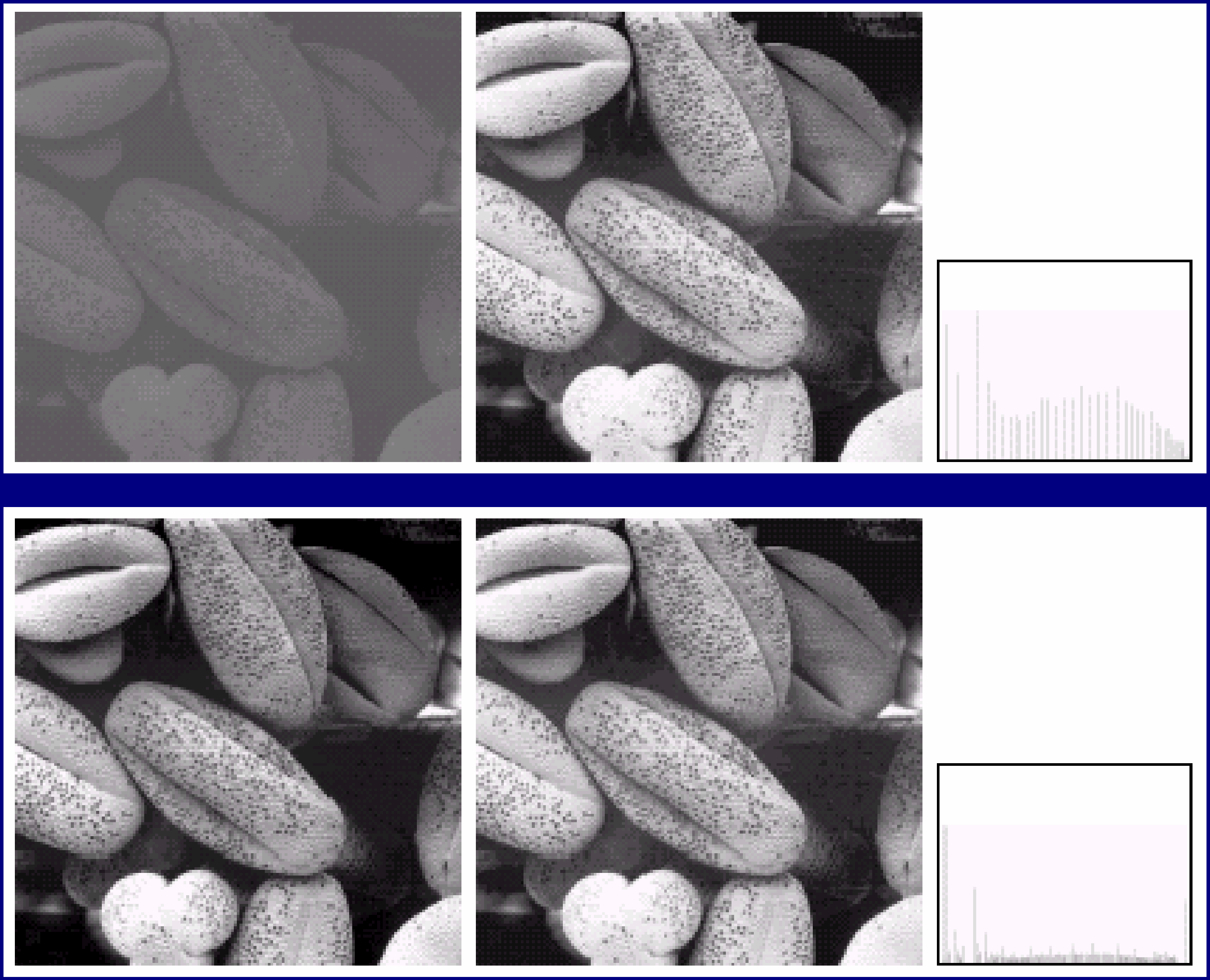


Histogram Equalization



From [Gonzalez & Woods]

Histogram Equalization



From [Gonzalez & Woods]

Example

- A 4x4, 4bits/pixel image

2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

- First try: full-scale contrast stretch $r_{\min} = 2$ $r_{\max} = 11$

$$s = round\left((2^B - 1) \cdot \frac{r - r_{\min}}{r_{\max} - r_{\min}} \right) = round\left(15 \cdot \frac{r-2}{11-2} \right) = round\left(\frac{5}{3}(r-2) \right)$$

2 → round(0) = 0;
3 → round(1.67) = 2;
8 → round(10.00) = 10;
9 → round(11.67) = 12;
10 → round(13.33) = 13;
11 → round(15) = 15;

The resulting image is:

0	10	12	12
0	2	13	12
10	2	2	15
10	2	13	15

Example: Histogram Change

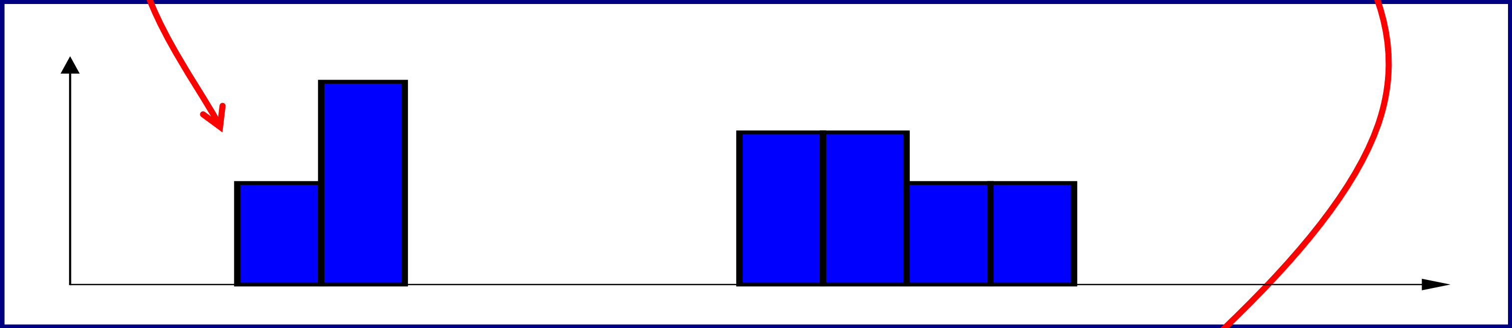
original

2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

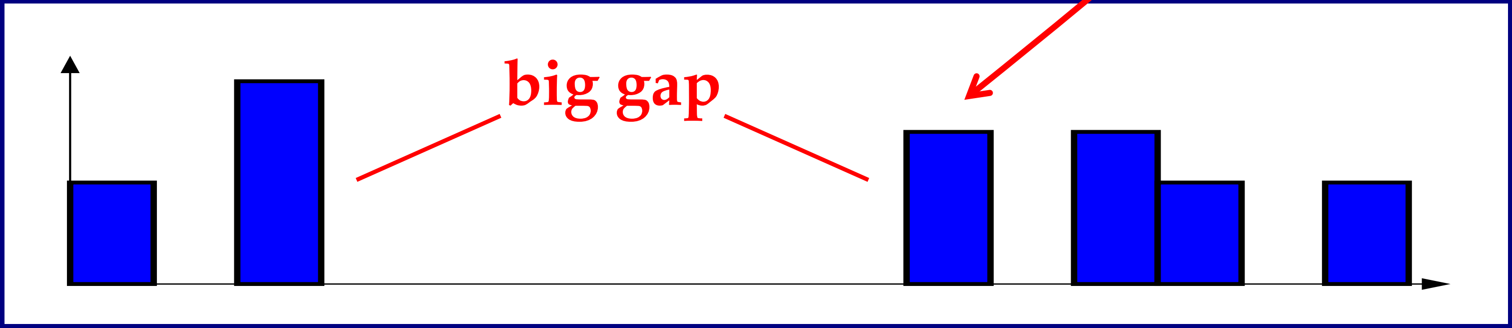
→

0	10	12	12
0	2	13	12
10	2	2	15
10	2	13	15

full-scale contrast stretch



↓

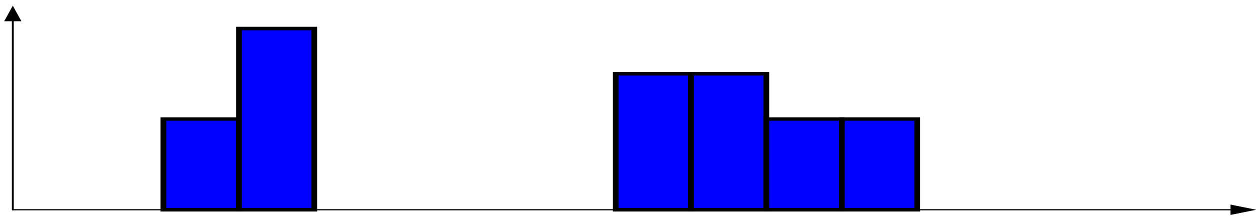


Cumulative Histogram

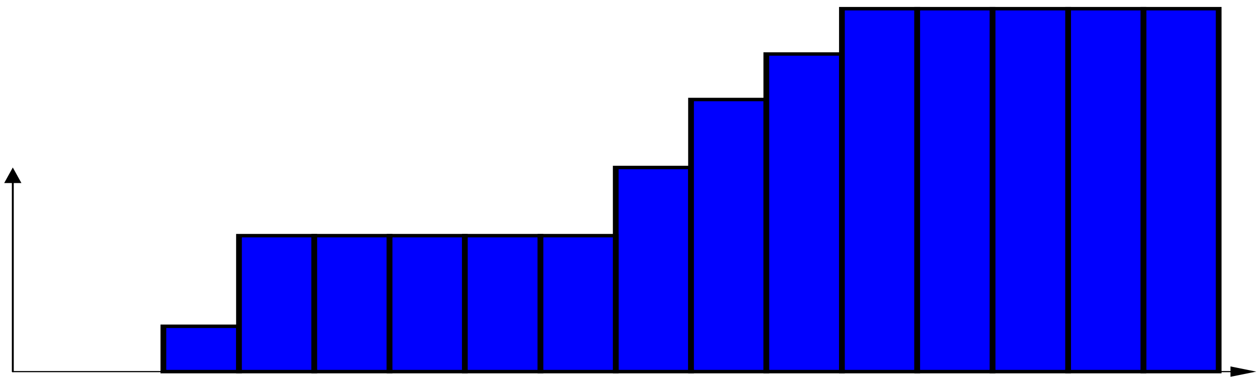
2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	2	4	0	0	0	0	3	3	2	2	0	0	0	0
Q(k)	0	0	2	6	6	6	6	6	9	12	14	16	16	16	16	16

H(k)



Q(k)



Intermediate Image

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	2	4	0	0	0	0	3	3	2	2	0	0	0	0
Q(k)	0	0	2	6	6	6	6	6	9	12	14	16	16	16	16	16

original

2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

→

2	9	12	12
2	6	14	12
9	6	6	16
9	6	14	16

intermediate image

Full-Scale Contrast Stretch of Intermediate Image

intermediate image

2	9	12	12
2	6	14	12
9	6	6	16
9	6	14	16

$r_{\min} = 2$ $r_{\max} = 16$

$$s = round\left((2^B - 1) \cdot \frac{r - r_{\min}}{r_{\max} - r_{\min}} \right) = round\left(15 \cdot \frac{r-2}{16-2} \right) = round\left(\frac{15}{14}(r-2) \right)$$

2 → round(0) = 0;

6 → round(4.29) = 4;

9 → round(7.50) = 8;

12 → round(10.71) = 11;

14 → round(12.86) = 13;

16 → round(15) = 15;

final result:

histogram equalized image

0	8	11	11
0	4	13	11
8	4	4	15
8	4	13	15

Histogram Comparison

2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

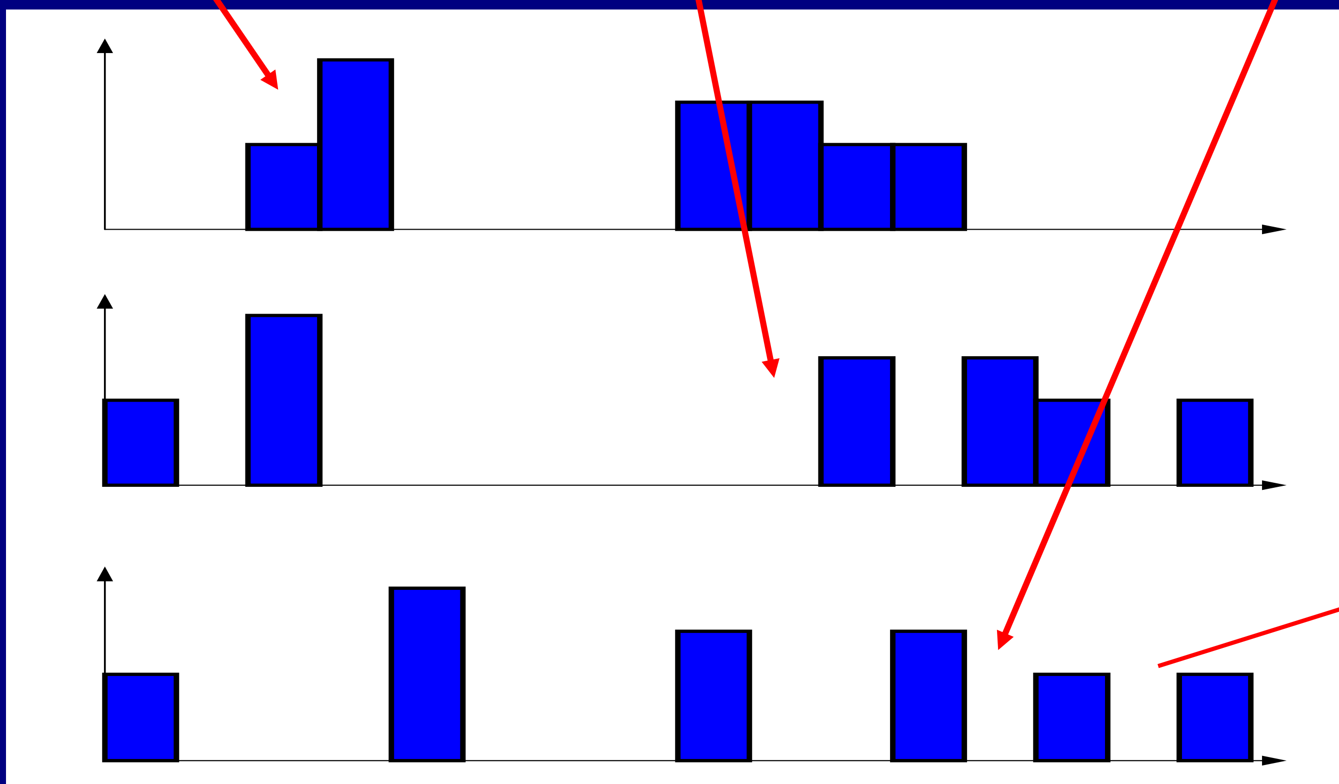
original

0	10	12	12
0	2	13	12
10	2	2	15
10	2	13	15

direct full-scale contrast stretch

0	8	11	11
0	4	13	11
8	4	4	15
8	4	13	15

histogram-equalized



more equalized

Summary of the Histogram Equalization Algorithm