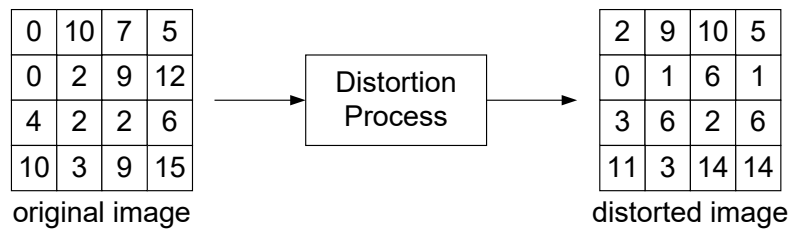


Instructor: Prof. Zhou Wang

Homework 1

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- The use of calculators is allowed.
 - Work on the Homework independently. Group work not allowed.
 - Submit your Homework individually in one single file that includes your answers to all questions.
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1. A 4×4, 4bits/pixel original image passes through a distortion process, resulting in a distorted image, as shown below

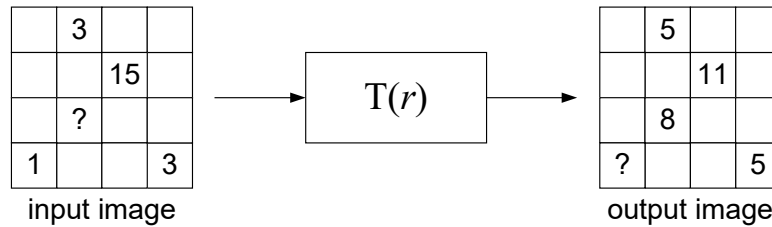


Compute the mean absolute error (MAE), the mean squared error (MSE) and the peak signal-to-noise ratio (PSNR) between the original and the distorted images.

2. A 4×4, 4bits/pixel image passes through a point-wise intensity transformation given by

$$s = T(r) = \alpha \log_2(1+r) + \beta$$

where α and β are unknown parameters. Only a few pixels are available in the input and the output images, as shown below.



- (a) Find α and β .
- (b) What's the value of the pixel with the “?” mark in the output image.
- (c) What's the value of the pixel with the “?” mark in the input image.

3. An original image (the intensity values of the pixels are normalized to be between 0 and 1) passes through several point-wise intensity transformations listed below

(1) $s = r^2$

(2) $s = 1 - r$

(3) $s = 0.5r + 0.25$

(4) $s = \begin{cases} 0 & r \leq 0.7 \\ 1 & \text{else} \end{cases}$

(5) $s = \begin{cases} 0 & r < 0.25 \\ 2r - 0.5 & 0.25 \leq r \leq 0.75 \\ 1 & 0.75 < r \end{cases}$

The original and transformed images are shown below:



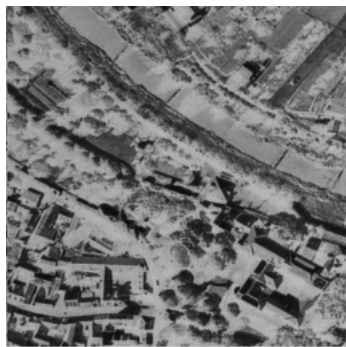
Original



(A)



(B)



(C)



(D)



(E)

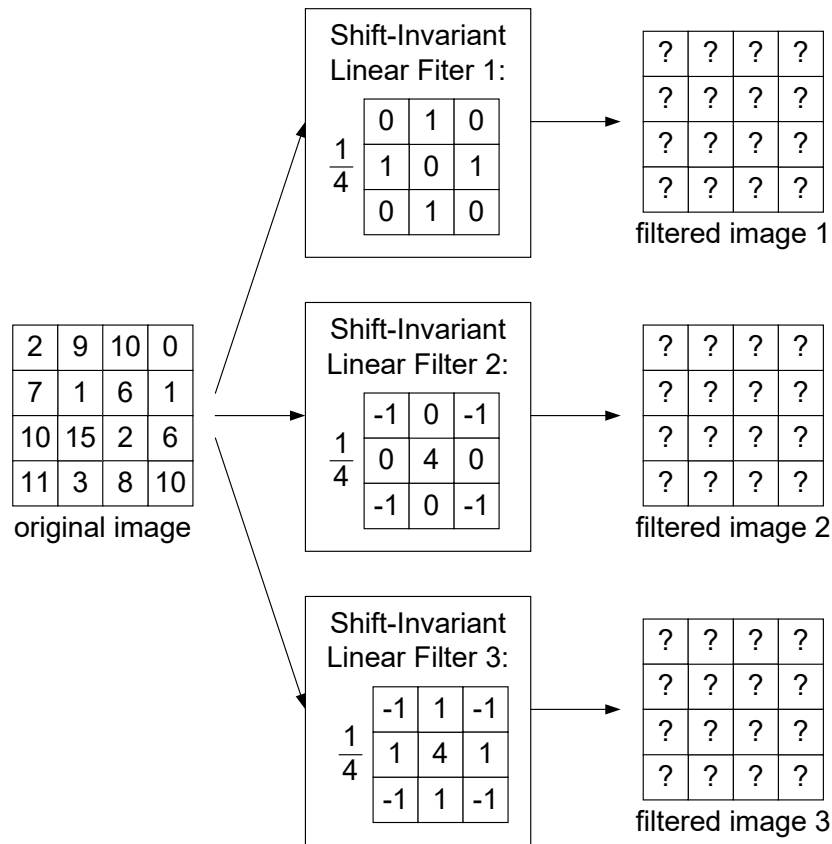
For each resulting image (A)-(E), determine the corresponding intensity transformation that creates it. Briefly explain your answers.

4. A 4×4, 4bits/pixel original image is given by

6	13	12	13
12	6	7	12
13	7	7	12
14	11	11	14

- (a) Apply full-scale contrast stretch to the image. Show your work and sketch the resulting image.
- (b) Apply histogram equalization to the image. Show your work and sketch the resulting image.
- (c) Sketch the histograms of the original image, the full-scale contrast stretched image, and the histogram-equalized image.

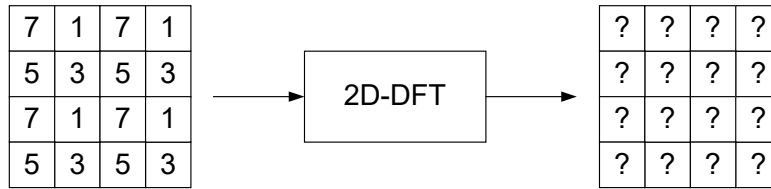
5. A 4×4 gray-scale original image passes through three spatial linear shift-invariant filters, resulting in three filtered images.



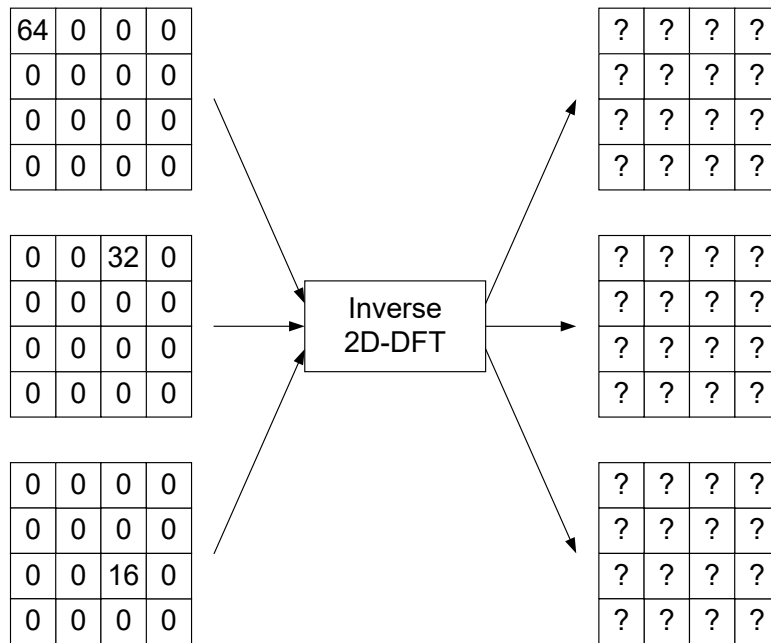
- Find the first filtered image (Use zero-padding of the original image and do not round the resulting image pixels into integers).
- Find the second filtered image (Use zero-padding of the original image and do not round the resulting image pixels into integers).
- Find the third filtered image (Use zero-padding of the original image and do not round the resulting image pixels into integers). Hint: Can you obtain the image from the results of (a) and (b)?

6. Two dimensional discrete Fourier transform (2D-DFT).

(a) Compute the 2D-DFT of the 4×4 gray-scale original image given below

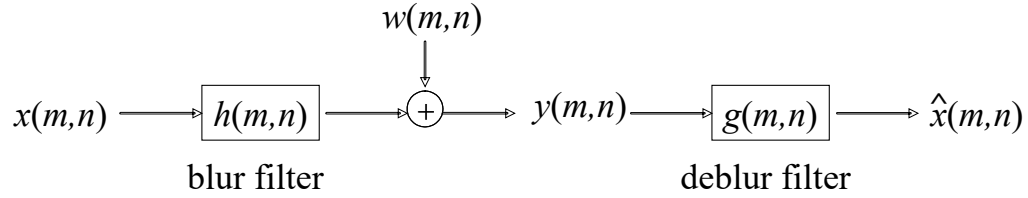


(b) Compute the inverse 2D-DFT of the following three 4×4 patterns, respectively



(c) Add the three resulting images in (b) and compare it with the original image. Comment on it.

7. An image distortion (blurring followed by additive white Gaussian noise contamination) and restoration model is shown as below. The signal variance and the noise variance are given by $\sigma_x^2 = 100$ and $\sigma_w^2 = 25$, respectively.

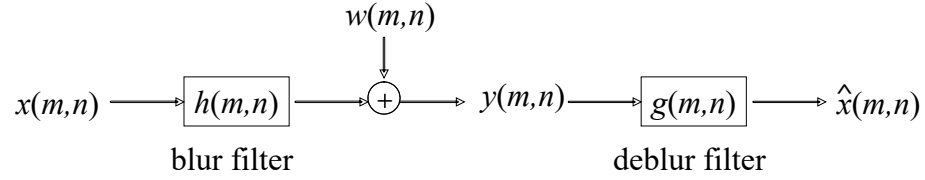


The 2D-DFT of the blur filter $h(m,n)$ is given by

$$H(u,v) = \begin{bmatrix} 1 & -0.2 - 0.2j & 0 & -0.2 + 0.2j \\ -0.2 - 0.2j & 0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2 + 0.2j & 0.05 & 0 & -0.05j \end{bmatrix}$$

- Design a deblur filter $G(u,v)$, i.e., the 2D-DFT of $g(m,n)$, using the inverse filtering approach;
- Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 0.03$;
- Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 0.1$;
- Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 0.3$;
- Design a deblur filter $G(u,v)$ using the Wiener filtering approach;
- Find $h(m,n)$;
- Find $g(m,n)$ for pseudo-inverse filtering with $\delta = 0.1$, i.e., the corresponding spatial domain deblur filter designed for Question (c).

8. An image distortion (blurring followed by additive white Gaussian noise contamination) and restoration model is shown in the figure below. The signal variance and the noise variance are given by $\sigma_x^2 = 400$ and $\sigma_w^2 = 100$, respectively.



The blur filter $h(m,n)$ is known as

$$h(m,n) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

- Find $H(u,v)$, i.e., the blur filter in the frequency (2D-DFT) domain.
- Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 1$.
- Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 3$.
- Design a deblur filter $G(u,v)$ using the Wiener filtering approach.

9. A 4×4, 4bits/pixel original image is shown as below

7	2	7	2
2	7	2	7
7	2	7	2
2	7	2	7

(a) The original image passes through a linear shift-invariant filter given by

$\frac{1}{6}$	0	1	0
	1	2	1
	0	1	0

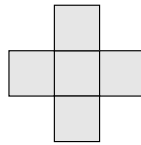
Find the filtered image (use zero-padding of the original image), and round the resulting image pixels to integers. Compute the mean absolute error (MAE), the mean squared error (MSE), and the peak signal to noise ratio (PSNR) between the original image and the filtered image (after rounding).

(b) The original image passes through a 3×3 median filter. Find the filtered image (use replicate-padding of the original image). Compute the MAE, the MSE, and the PSNR between the original and the filtered images.

10. A 4×4 image is given by

9	8	7	6
8	7	13	5
7	6	5	4
6	1	4	3

- (a) Filter the image using a Median filter (after replicate-padding), where the filter mask is given by



- (b) Filter the image using a Min filter (after replicate-padding) using the same filter mask as in (a).
- (c) Filter the image using a Max filter (after replicate-padding) using the same filter mask as in (a).
- (d) Filter the image using an Order Statistics filter (after replicate padding) using the same filter mask as in (a). The weighting vector of the order statistics filter is defined as

$$\{w_i\} = \left\{0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right\}.$$

Round the resulting pixels to integers.

11. A 7×7 image is given by

3	3	1	3	3	3	4
0	3	3	3	3	3	3
3	3	3	2	3	3	12
12	3	3	3	3	12	12
10	12	2	3	3	12	12
12	14	12	12	12	12	11
11	12	12	12	10	12	12

Use Prewitt gradient operator to find its edges. Use

$$|g(m,n)| = |g_1(m,n)| + |g_2(m,n)|$$

to estimate the gradient magnitude, and use $T = 22$ as the threshold for edge detection. Note: no padding of the boundaries, and the result will be a 5×5 binary edge image.

12. An image is given below



Use iterative quadtree split-and-merge algorithm to segment the image. Give the result after each step (split or merge) in each iteration.