Image Processing and Visual Communications

Spatial Domain Linear Filtering

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Spatial Linear Filtering Systems

- **Linear Shift-Invariant System**

  - Linearity: “things (input/output pairs) can be added”
  - Shift-invariance: “behaviors (of system)do not change over space”

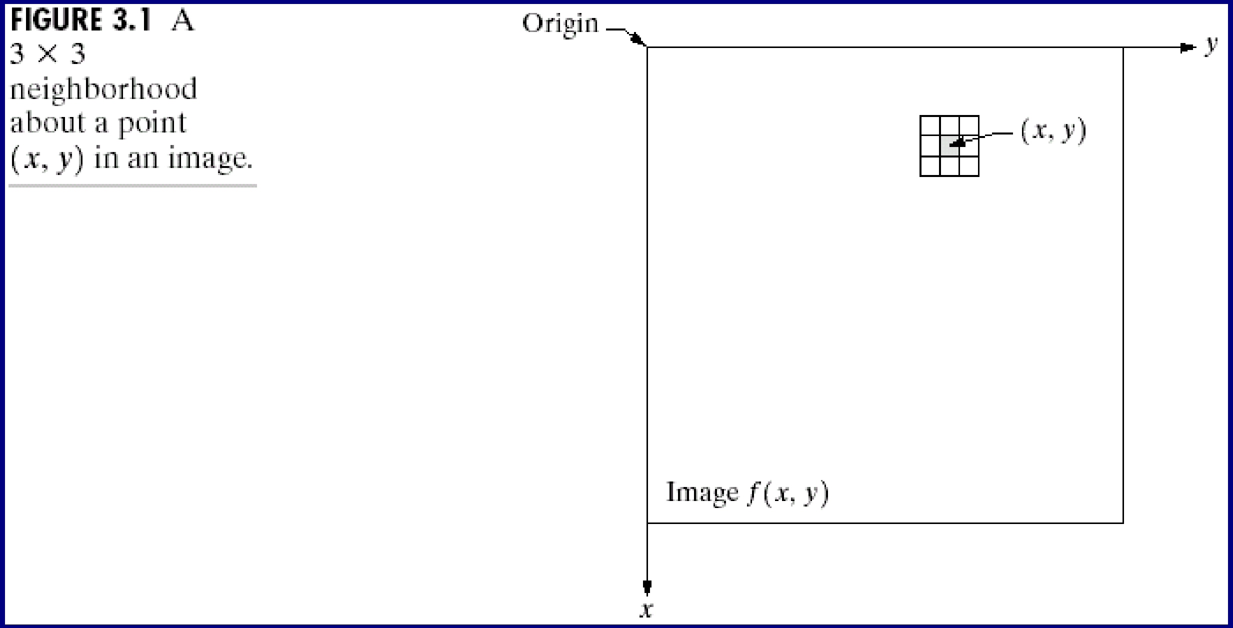
- **Filtering with LSI System**
  - Spatial domain → Convolution
  - Frequency domain → Multiplication (convolution theorem)

Impulse Response

- **The response of an LSI system to an impulse input**

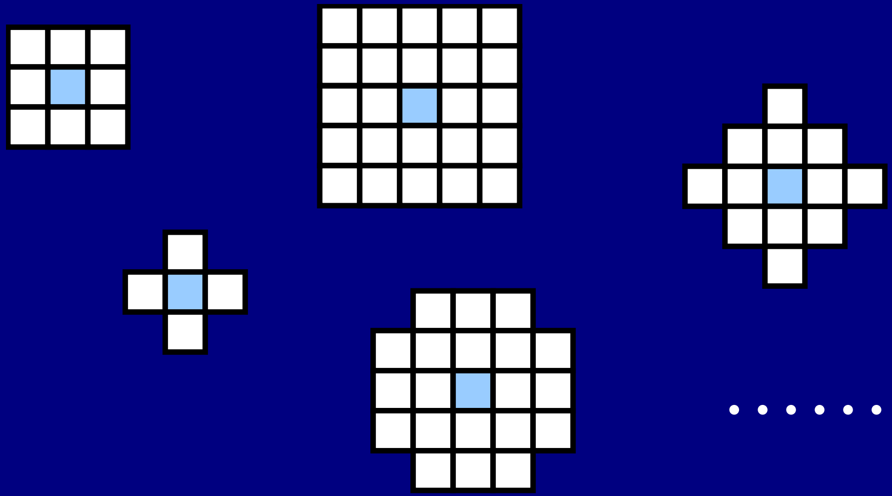
- **KEY: An LSI system can be completely characterized by its impulse response**
  - Given the impulse response of an LSI system, together with the input to the system, the output is uniquely determined

Spatial Neighborhood



From [Gonzalez & Woods]

choices of neighborhood:



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Masks, Windows, Filters and the Impulse Responses

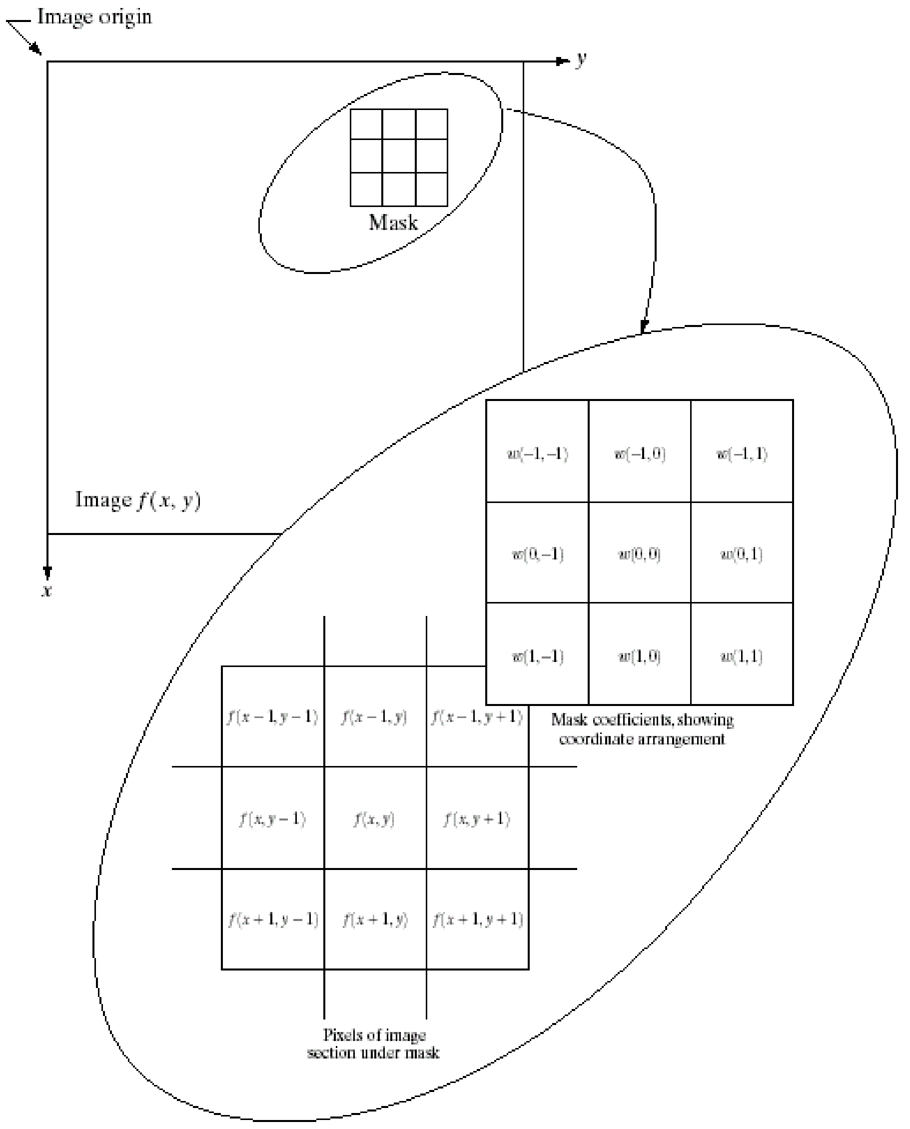


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

From [Gonzalez & Woods]

- **Spatial LSI Filter:**

  impulse response constrained within a local neighborhood

- **"Filter"**

  "Mask"

  "Window"

  "Impulse Response"

  often used interchangeably for LSI

2D Convolution

$$x(m,n) \longrightarrow \boxed{h(m,n)} \longrightarrow y(m,n)$$

$$y(m,n) = \sum_{k,l=-\infty}^{\infty} h(k,l)x(m-k,n-l) = h(m,n) \otimes x(m,n)$$

$$y(m,n) = \sum_{k,l=-\infty}^{\infty} h(m-k,n-l)x(k,l) = x(m,n) \otimes h(m,n)$$

$h(m, n)$ → impulse response (spatial linear filter)

$x(m, n)$ → input image

$y(m, n)$ → output image

Applications

- **Image Smoothing**
- **Image Enhancement**
- **Image Restoration**
  - Image denoising
  - Image deblurring
- **Edge Detection**
- **Filter Bank**
  - Image transformation
  - Frequency analysis
- ……

Image Smoothing: Average Filters

- **Average Filter**

$$h(m,n)=\frac{1}{N^2}\begin{bmatrix}1 & \dots & 1\\ \vdots & \ddots & \vdots\\ 1 & \dots & 1\end{bmatrix}$$

N: filter size

noisy

PSNR=20.2dB
noise std = 25

smoothed

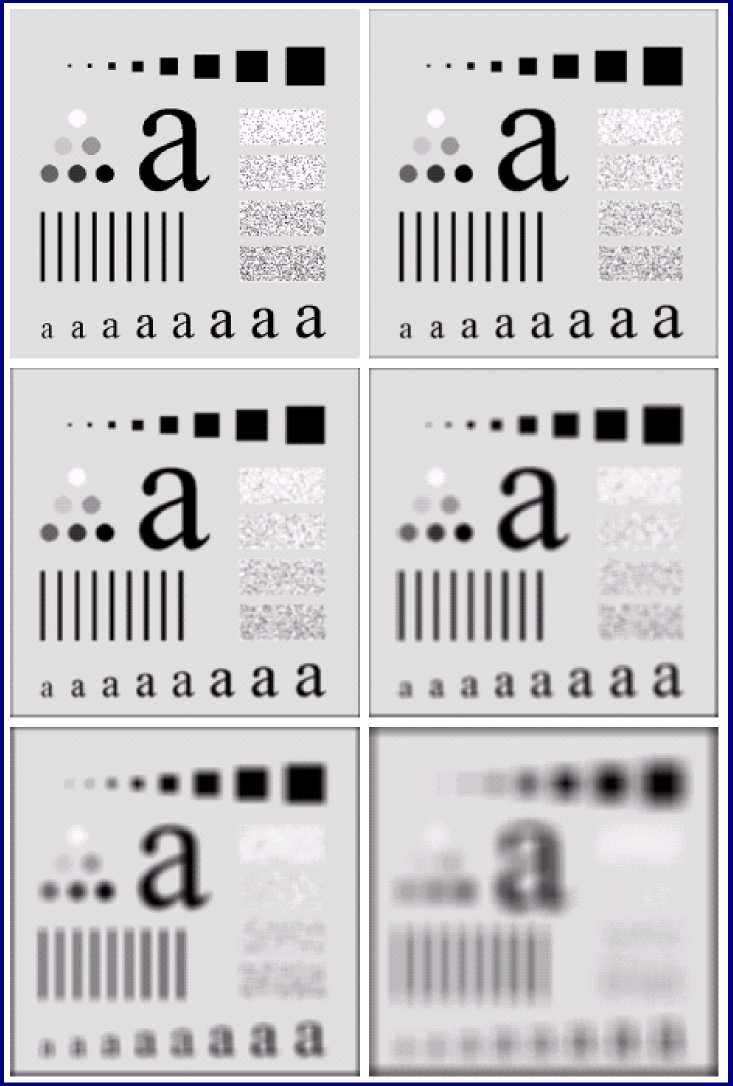
PSNR=23.8dB
3x3 window

smoothed

PSNR=22.0dB
5x5 window

From Prof. Xin Li

Image Smoothing: Average Filters



From [Gonzalez & Woods]

Original image size: 500x500
Average filtered images.
Filter sizes: 3, 5, 9, 15 and 35

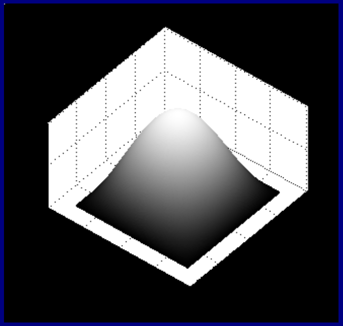
- **Effects**
  - Smoothing noise
  - Blurring edges

Image Smoothing: Gaussian Filters

- **Gaussian Filter**

$$h(m,n) = \frac{1}{Z}\exp\left[-\frac{m^2+n^2}{2\sigma^2}\right]$$

$$-N \le m,n \le N$$



noisy

PSNR=20.2dB
noise std = 25

smoothed

PSNR=24.4dB
σ=1

smoothed

PSNR=22.8dB
σ=1.5

From Prof. Xin Li

Image Smoothing Filter Example

- **Filter**

$$\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- **Input image: A 4x4, 4 bits/pixel**

1	8	6	6
6	3	11	8
8	8	9	10
9	10	10	7

- **Preprocessing: Zero-padding**

1	8	6	6
6	3	11	8
8	8	9	10
9	10	10	7

$\longrightarrow$

0	0	0	0	0	0
0	1	8	6	6	0
0	6	3	11	8	0
0	8	8	9	10	0
0	9	10	10	7	0
0	0	0	0	0	0

Image Smoothing Filter Example

- Move mask across the zero-padded image

$$\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

0	0	0	0	0	0
0	1	8	6	6	0
0	6	3	11	8	0
0	8	8	9	10	0
0	9	10	10	7	0
0	0	0	0	0	0

- Compute weighted sum

- Result:

2.6	4.3	6.2	4.3
4.0	6.5	8.0	7.2
6.5	7.7	9.5	7.3
6.0	7.8	7.7	5.7

$$\xrightarrow{\text{round}}$$

3	4	6	4
4	7	8	7
7	8	10	7
6	8	8	6

Sharpening Linear Filters

- **Discrete Approximation of Laplacian**

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

  - Zero at smooth regions
  - Sensitive to image details

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

Sharpening Linear Filters

a b
c d

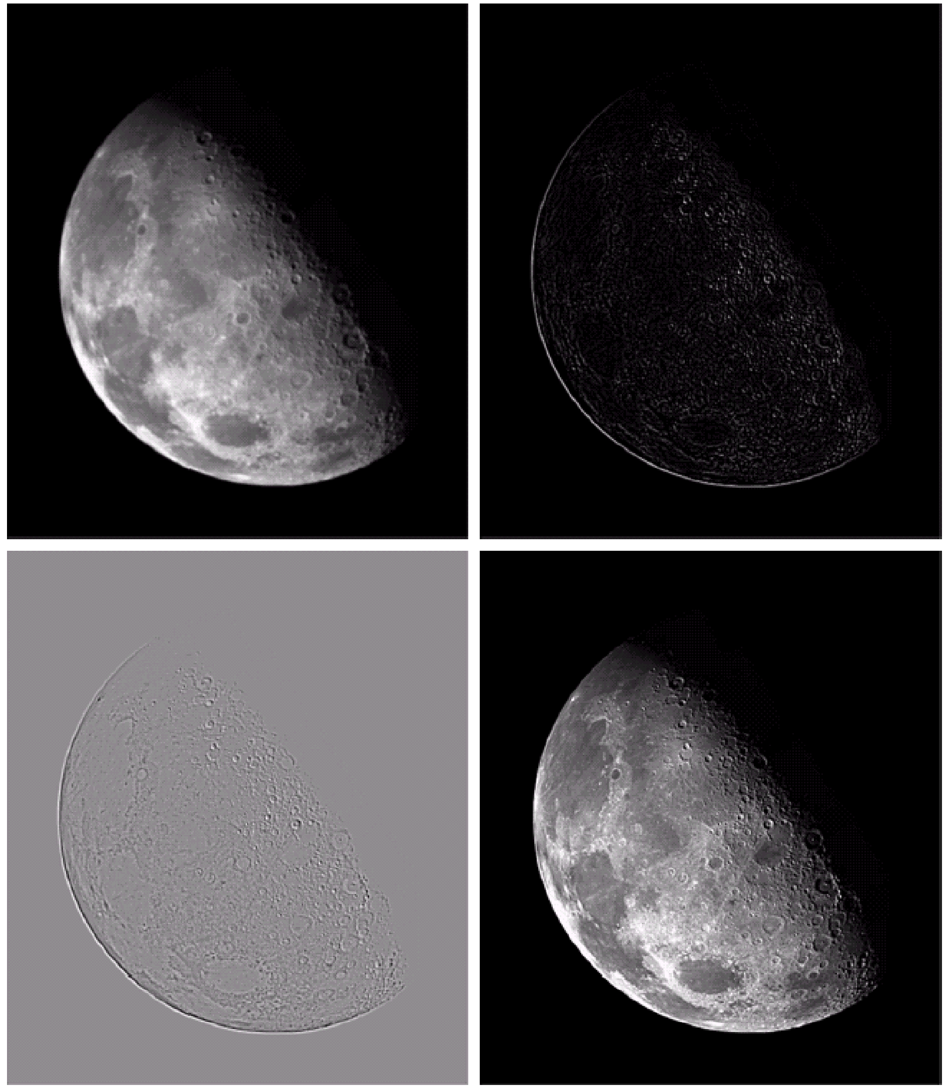


FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

From [Gonzalez & Woods]

Sharpening Linear Filters

- **Image Sharpening Idea:**
  combining Laplacian with the image itself

  – Case 1: Center coefficient of the Laplacian mask is positive

$$g(x,y) = f(x,y) + \nabla^2 f(x,y)$$

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

  – Case 2: Center coefficient of the Laplacian mask is negative

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

Sharpening Linear Filters

- **Combined sharpening filters**

$$g(x,y) = \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \otimes f(x,y) + \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array} \otimes f(x,y)$$

$$= \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 5 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array} \otimes f(x,y)$$

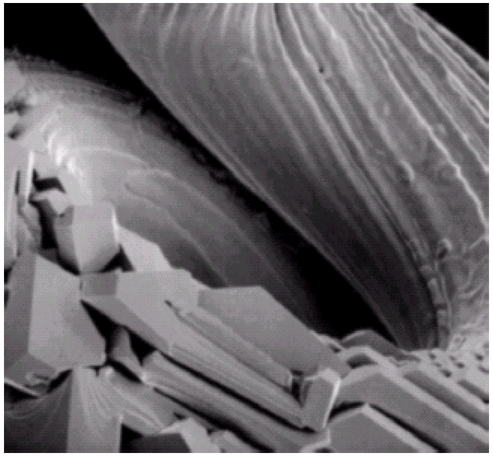
$$g(x,y) = \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \otimes f(x,y) + \begin{array}{|c|c|c|}\hline -1 & -1 & -1 \\ \hline -1 & 8 & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array} \otimes f(x,y)$$

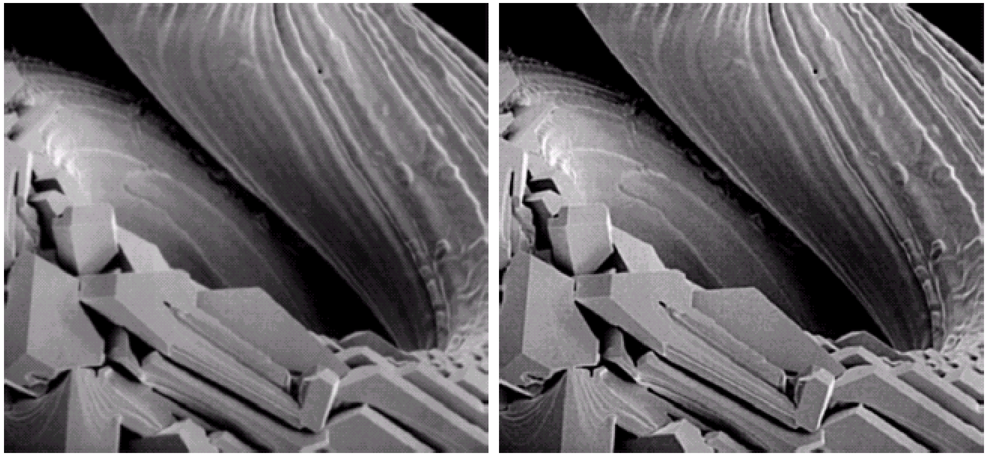
$$= \begin{array}{|c|c|c|}\hline -1 & -1 & -1 \\ \hline -1 & 9 & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array} \otimes f(x,y)$$

Sharpening Linear Filters

0	–1	0
–1	5	–1
0	–1	0

–1	–1	–1
–1	9	–1
–1	–1	–1





a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

From [Gonzalez & Woods]

Further Improvement

- **Advanced operators**

- **From spatial domain to transform domain filtering**
  - Frequency domain, wavelet domain
  - Better represented image features

- **From linear to nonlinear**
  - Median, order-statistics

- **From space-invariant to spatially adaptive**
  - Adapt to local image structures
    smooth regions, edges (orientations), textures …