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ASSIGNMENT - 1.

1) $\begin{bmatrix} 0 & 10 & 7 & 5 \\ 0 & 2 & 9 & 12 \\ 4 & 2 & 2 & 6 \\ 10 & 3 & 9 & 15 \end{bmatrix} \longrightarrow \boxed{\text{distortion Process}} \longrightarrow \begin{bmatrix} 2 & 9 & 10 & 5 \\ 0 & 1 & 6 & 1 \\ 3 & 6 & 2 & 6 \\ 11 & 3 & 14 & 14 \end{bmatrix}$

Original Image — distorted Image.

Mean Absolute error :- $\frac{1}{n}$(sum(|original image val − distorted|))

Mean Square Error :- $\frac{1}{M*N} \sum_{i=1}^{M} \sum_{j=1}^{N} (x_{ij} - y_{ij})^2$

$M*N$ = image dimension

x_{ij} = pixel value of original image at rows & col", i, j

peak signal-to-noise ratio :- $10 \log_{10} \frac{L^2}{MSE}$

→ Mean absolute error (MAE) :-

n = 16 pixels in the image.

sum($\underbrace{\text{abs}(\text{orig" img} - \text{distorted img"})}_{\text{Abs}}$) (for each pixel)

$= \text{abs}(0-2) + \text{abs}(10-9) + \text{abs}(7-10) + \text{abs}(5-5) +$
$\text{abs}(0-0) + \text{abs}(2-1) + \text{abs}(9-6) + \text{abs}(12-1) +$
$\text{abs}(4-3) + \text{abs}(2-6) + \text{abs}(2-2) + \text{abs}(6-6) +$
$\text{abs}(10-11) + \text{abs}(3-3) + \text{abs}(9-14) + \text{abs}(15-14).$

$= 2+1+3+0+0+1+3+11+1+4+0+0+1+0+5+1$

$MAE = \frac{1}{n}(\text{sum}(\text{Abs})) = \frac{1}{16}(33) = 2.06$

$\boxed{MAE = 2.06}$

Mean squared Error :- $\frac{1}{M*N} \sum_{i=1}^{M}\sum_{j=1}^{N} (x_{ij} - y_{ij})^2$

$M = 4$; $N = 4$.

$$= \frac{1}{16}\Big((2-2)^2 + (10-9)^2 + (7-10)^2 + (5-5)^2 + (0-0)^2 + (2-1)^2 +$$
$$(9-6)^2 + (12-1)^2 + (4-3)^2 + (2-6)^2 + (2-2)^2 + (6-6)^2 +$$
$$(10-11)^2 + (3-3)^2 + (9-14)^2 + (15-14)^2\Big)$$

$$= \frac{1}{16}\big(4+1+9+0+0+1+9+121+16+0+0+1+0+$$
$$1+25+1\big)$$

$$= \frac{1}{16}(189)$$

$$\boxed{MSE = 11.81}$$

Peak signal to Noise ratio :- $10\log_{10}\left(\frac{L^2}{MSE}\right)$

L is the dynamic range of pixel (0 to 15)
(or) maximum possible pixel value. $\Rightarrow 15-0=15$

$$10\log_{10}\left(\frac{15^2}{11.81}\right) = 12.79 \text{ dB}$$

$$\boxed{PSNR = 12.79 \text{ dB}}$$

2) point wise intensity transformation given by :

$$S = T(r) = \alpha \log_2(1+r) + \beta.$$

	3		
		15	
	?		
1			3

$$\longrightarrow \boxed{T(r)} \longrightarrow$$

	5		
		11	
	8		
?			5

a) find α & β.
b) find missing pixel value in original (input) image
c) find missing pixel value in output image.

a). $3 \longrightarrow \boxed{T(r)} \longrightarrow 5$ (output is 5 after applying T(r) to 3).

$$5 = \alpha \log_2(1+3) + \beta \quad \text{— (1)}$$

$$15 \longrightarrow \boxed{T(r)} \longrightarrow 11$$

$$11 = \alpha \log_2(1+15) + \beta. \quad \text{— (2)}$$

$$3 \longrightarrow \boxed{T(r)} \longrightarrow 5$$

$$5 = \alpha \log_2(1+3) + \beta. \quad \text{— (3)}$$

Solving equations (1) & (2) $\Rightarrow$ $\alpha = 3$
$\beta = -1$.

$$\therefore \boxed{\alpha = 3 \ \& \ \beta = -1}$$

b) Missing value in input image

$$S = T(r) = 3\log_2(1+r) - 1.$$

$8 = T(x)$ where, suppose x as missing val

$$8 = 3\log_2(1+x) - 1$$

$$9 = 3\log_2(1+x)$$

$$3 = \log_2(1+x)$$ [$\log_a n = x$ then, $n = a^x$]

$$2^3 = 1+x$$

$$\Rightarrow \boxed{x = 7}$$

c) Missing value in output image.

$S = T(r) = 3\log_2(1+r) - 1$, s is the missing value in o/p

$$S = T(1)$$

$$S = 3\log_2(1+1) - 1$$

$$S = 3(1) - 1$$

$$\boxed{S = 2}$$

3) Here, the assumption is the black pixel value corresponds to a '0' and a white, corresponds to 1.

a) $s = r^2$

Considering some pixel values in between 0 & 1

$r = 0.25 \Rightarrow s = 0.06$

$r = 0.5 \Rightarrow s = 0.25$

$r = 0.6 \Rightarrow s = 0.36$

$r = 0 \Rightarrow s = 0$

$r = 1 \Rightarrow s = 1$

Except for black & white, all other values result in a pixel value lower than the original pixel., so the image would be darker than the original

So $s = r^2$ corresponds to image E.

b) $s = 1 - r$

considering pixel values between 0 & 1

$r = 0.25 \Rightarrow s = 1 - 0.25 = 0.75$

$r = 0.6 \Rightarrow s = 1 - 0.6 = 0.4$

$r = 1 \Rightarrow s = 1 - 1 = 0$

$r = 0 \Rightarrow s = 1 - 0 = 1$

The values are inverted, so the closest representation would be 'C'.

c) $s = 0.5r + 0.25$

consider some example pixel values between 0 & 1.

$r = 0.25 \Rightarrow s = 0.375$ $\quad$ $r = 1 \Rightarrow s = 0.75$

$r = 0.6 \Rightarrow s = 0.55$ $\quad$ $r = 0 \Rightarrow s = 0.25$

We can see from the examples that, if we take input pixel value > 0.5, we get result as less than the original input while an input value < 0.5 gives us a value greater than original pixel.
Using this, we can safely say, the lighter regions of the image become darker and darker regions become lighter.

This can be seen in image (D).

d) $S = \begin{cases} 0, & r \leq 0.7 \\ 1, & \text{else} \end{cases}$

the o/p would be a binary image which has only black and white colors. This can be seen in image (B).

e) $S = \begin{cases} 0, & r \leq 0.25 \\ 2r - 0.5, & 0.25 \geq r \leq 0.75 \\ 1, & r \geq 0.75 \end{cases}$

Ex's :-
$r = 0.1 \Rightarrow S = 0$ $\quad r = 0.6 \Rightarrow S = 0.7$
$r = 0.25 \Rightarrow S = 0$ $\quad r = 0.75 \Rightarrow S = 1$
$r = 0.4 \Rightarrow S = 0.3$ $\quad r = 1 \Rightarrow S = 1$

The o/p pixel value is lower than original pixel value if the original value is < 0.5 & the o/p pixel value is greater than original pixel value, if the original pixel value is > 0.5.
This makes the image brighter (increases the contrast), thus the closest representation would be (A).

Hence contrast means darks would become darker & lights would become lighter

4). Image =

6	13	12	13
12	6	7	12
13	7	7	12
14	11	11	14

a) Apply full-scale stretch.
b) Histogram equalisation.
c) Histograms of original image, full scale contrast stretched image and histogram equalised image

a) full scale stretch :- every pixel changed using below equn.

$$S = round\left((2^B - 1) \cdot \frac{r - r_{min}}{r_{max} - r_{min}}\right).$$

B = bits / pixel = $2^4 - 1 = 15$.

r_{min} = min. pixel value in the image = 6.
r_{max} = max. pixel value in the image = 14.
r = current pixel value.

$6 \rightarrow round\left(15 \cdot \left(\frac{6-6}{14-6}\right)\right) = 0$

$(14-6=8)$

$7 \rightarrow round\left(15 \cdot \left(\frac{7-6}{14-6}\right)\right) = 2.$

$11 \rightarrow round\left(15 \cdot \left(\frac{11-6}{8}\right)\right) = 9.$

$12 \rightarrow round\left(15 \cdot \left(\frac{12-6}{8}\right)\right) = 11.$

$13 \rightarrow round\left(15 \cdot \left(\frac{13-6}{8}\right)\right) = 13$

$14 \rightarrow round\left(15 \cdot \left(\frac{14-6}{8}\right)\right) = 15.$

∴ Image with full contrast stretch is:

0	13	11	13
11	0	2	11
13	2	2	11
15	9	9	15

b) Histogram Equalisation :-

6	13	12	13
12	6	7	12
13	7	7	12
14	11	11	14

Histogram equalisation Algorithm :-

orig. img → Histogram H(K) → Cummulative histogram Q(K) → Intermediate img. ↓ Full contrast stretch ↓ Histogram equalised img.

K	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(K)	0	0	0	0	0	0	2	3	0	0	0	2	4	3	2	0
Q(K)	0	0	0	0	0	0	2	5	5	5	5	7	11	14	16	16

Intermediate img :-

2	14	11	14
11	2	5	11
14	5	5	11
16	7	7	16

full contrast stretch of intermediate img. ⇒ $\text{round}\left(2^B - 1\left(\frac{r - r_{min}}{r_{max} - r_{min}}\right)\right)$

$$= \text{round}\left(15 \times \left(\frac{r-2}{16-2}\right)\right) = \text{round}\left(\frac{15}{14}(r-2)\right)$$

$$\text{round}\left(1.07(r-2)\right).$$

$2 \rightarrow \text{round}\left(\frac{15}{14}(2-2)\right) = 0$

$5 \rightarrow \text{round}\left(\frac{15}{14}(5-2)\right) = 3$

$7 \rightarrow \text{round}\left(\frac{15}{14}(7-2)\right) = 5$

$11 \rightarrow \text{round}\left(\frac{15}{14}(11-2)\right) = 10$

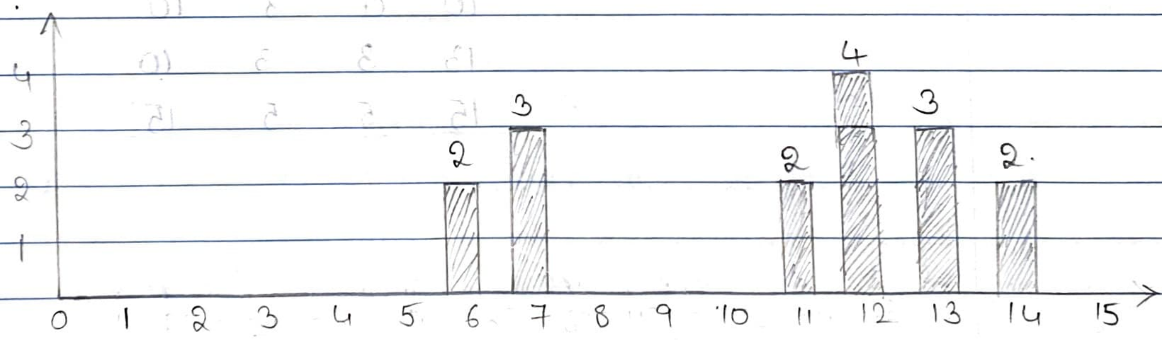
$14 \rightarrow \text{round}\left(\frac{15}{14}(14-2)\right) = 13$

$16 \rightarrow \text{round}\left(\frac{15}{14}(16-2)\right) = 15$

Histogram Equalised img =

0	13	10	13
10	0	3	10
13	3	3	10
15	5	5	15

c).



→ The above sketch is the Histogram of the original image

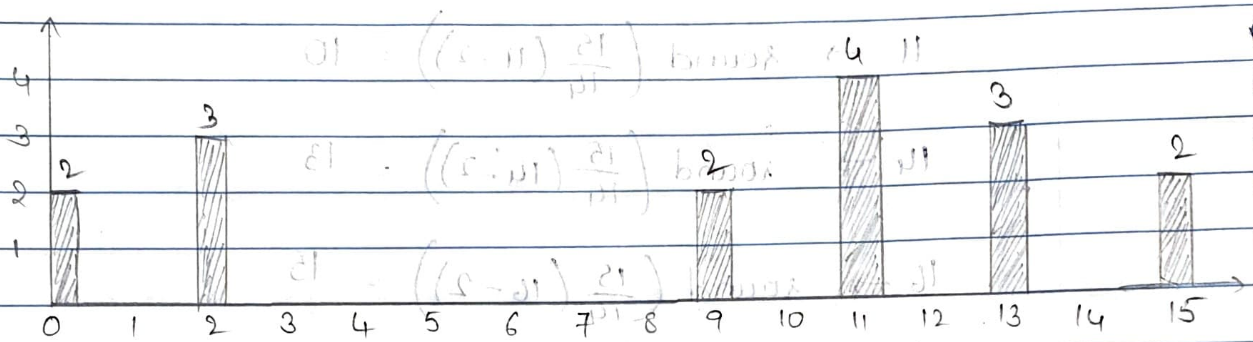
original image =

6	13	12	13
12	6	7	12
13	7	7	12
14	11	11	14

Histogram sketch of full contrast stretch image.

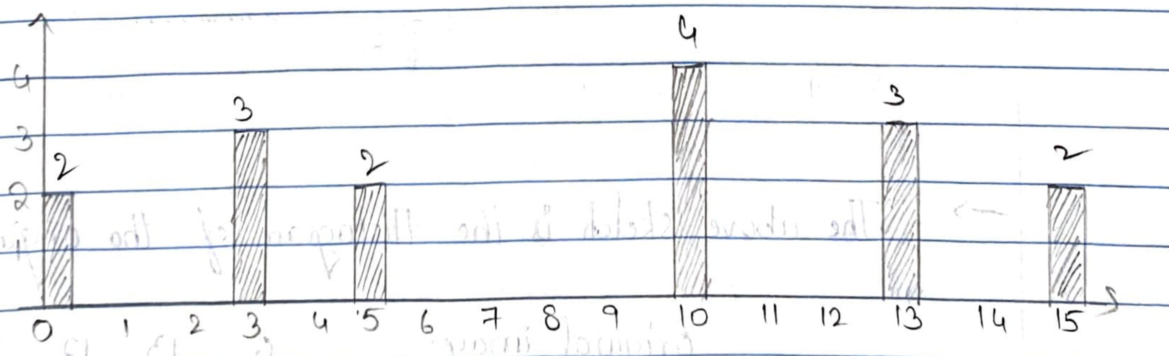
=

0	13	11	13
11	0	2	11
13	2	2	11
15	9	9	15



Histogram sketch of Histogram Equalised image.

0	13	10	13
10	0	3	10
13	3	3	10
15	5	5	15.



5).

2	9	10	0
7	1	6	1
10	15	2	6
11	3	8	10

$\frac{1}{4}$ $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ → a) filtered image 1 4x4 gray-scale.

$\frac{1}{4}$ $\begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ → b) filtered gray-scale image 2 (4x4)

$\frac{1}{4}$ $\begin{bmatrix} -1 & 1 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ → c) filtered 4x4 gray-scale image 3.

a)

i) Adding zero padding to original image.

0	0	0	0	0	0
0	2	9	10	0	0
0	7	1	6	1	0
0	10	15	2	6	0
0	11	3	8	10	0
0	0	0	0	0	0

ii) Using the shift invariant linear filter - 1 :- $\frac{1}{4} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

iii) O/p image 1. =

4	3.25	3.75	2.75
3.25	9.25	3.5	3
8.25	14	8.75	3.25
3.25	8.5	3.75	3.5

b) Using the shift invariant linear filter - 2 :- $\frac{1}{4} * \begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

i) O/p image 2 =

1.75	5.75	9.5	-1.5
1	-5	-1.5	-2.
9	7	-1.75	2.5
7.25	0	2.75	9.5

c) Using the shift invariant linear filter-3 :- $\frac{1}{4} * \begin{bmatrix} -1 & 1 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

i) original Img :-

2	9	10	0
7	1	6	1
10	15	2	6
11	3	8	10

ii) with zero padding :-

0	0	0	0	0	0
0	2	9	10	0	0
0	7	1	6	1	0
0	10	15	2	6	0
0	11	3	8	10	0
0	0	0	0	0	0

iii) Applying the filter :-

iv) o/p image 3 =

5.75	9	13.25	1.25
4.25	4.25	2.[illegible]	[illegible]1
17.25	11	7	5.75
10.5	8.5	6.5	13

Consider the first filter is added to the second filter, it looks ~~something~~ like the third filter, thus adding the o/p's of first two gives the third output image.

$$g(x,y) = \frac{1}{4} * \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{\text{filter - 1}} \otimes f(x,y) + \frac{1}{4} * \underbrace{\begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}}_{\text{filter-2}} \otimes f(x,y)$$

$$g(x,y) = \frac{1}{4} * \underbrace{\begin{bmatrix} -1 & 1 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}}_{= \text{ filter 3}} \otimes f(x,y)$$

where $f(x,y)$ = i/p
$g(x,y)$ = o/p.

6) 2D DFT of the 4x4 gray-scale original image.

$$\begin{matrix} 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \\ 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \end{matrix} \longrightarrow \text{2D-DFT} \longrightarrow ?_{4\times4}$$

i) To compute 2D-DFT of an image X, as a matrix, we use the formula

$$\tilde{X} = f_N X f_N$$

where N is the dimension and here it is 4.

$$\Rightarrow f_N = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & \cdots & W_N^{N-1} \\ 1 & W_N^2 & \vdots & & & \vdots \\ \vdots & \vdots & \vdots & & & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & \cdots & W_N^{(N-1)^2} \end{bmatrix} \quad \text{where } W_N = e^{-j2\pi/4}$$

ii) $$\Rightarrow F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

iii) $$\tilde{X} = F_4 X F_4 = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}}_{4\times4} \underbrace{\begin{bmatrix} 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \\ 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \end{bmatrix}}_{4\times4} \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}}_{4\times4}$$

$$\tilde{X} = \begin{bmatrix} 24 & 8 & 24 & 8 \\ 0 & 0 & 0 & 0 \\ 4 & -4 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\tilde{X} = \begin{bmatrix} 64 & 0 & 32 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b) Inverse 2D-DFT :- $x = \frac{1}{N^2} f_N^* \tilde{X} f_N^*$

$$\tilde{X} = \begin{bmatrix} 64 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$f_N^* = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^{-1} & \cdots & & W_N^{1-N} \\ \vdots & \vdots & & & \vdots \\ & & & & -(N-1)^2 \\ 1 & W_N^{1-N} & \cdots & & W_N \end{bmatrix} \Rightarrow f_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & +1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$x = \frac{1}{4^2} f_4^* \tilde{X} f_4^* = \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 64 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 64 & 0 & 0 & 0 \\ 64 & 0 & 0 & 0 \\ 64 & 0 & 0 & 0 \\ 64 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 64 & 64 & 64 & 64 \\ 64 & 64 & 64 & 64 \\ 64 & 64 & 64 & 64 \\ 64 & 64 & 64 & 64 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

$$\therefore \; x = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

b iii) Inverse 2D-DFT of $\tilde{x} = \begin{bmatrix} 0 & 0 & 32 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$x = \frac{1}{4^2} F_4^* \tilde{x} F_4^* = \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 0 & 0 & 32 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 0 & 0 & 32 & 0 \\ 0 & 0 & 32 & 0 \\ 0 & 0 & 32 & 0 \\ 0 & 0 & 32 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$x = \frac{1}{16} \begin{bmatrix} 32 & -32 & 32 & -32 \\ 32 & -32 & 32 & -32 \\ 32 & -32 & 32 & -32 \\ 32 & -32 & 32 & -32 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \end{bmatrix}$$

$$\tilde{x} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

biii) $X = \frac{1}{4^2} F_4^* \tilde{X} F_4^* = \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix}$

$$= \frac{1}{16} \begin{bmatrix} 0 & 0 & 16 & 0 \\ 0 & 0 & -16 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & -16 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix}$$

$$X = \frac{1}{16} \begin{bmatrix} 16 & -16 & 16 & -16 \\ -16 & 16 & -16 & 16 \\ 16 & -16 & 16 & -16 \\ -16 & 16 & -16 & 16 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

c) Adding result images in section (b).

$\Rightarrow X_{res} = X_1 + X_2 + X_3$

$$= \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

$$X_{res} = \begin{bmatrix} 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \\ 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \end{bmatrix}$$ $\Rightarrow$ this is equal to the original image given in (a).

7) Given $H(u,v) = \begin{bmatrix} 1 & -0.2-0.2j & 0 & -0.2+0.2j \\ -0.2-0.2j & 0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2+0.2j & 0.05 & 0 & -0.05j \end{bmatrix}$

a) Deblur filter $G(u,v)$ using inverse filtering approach :-

$$G(u,v) = 1/H(u,v).$$

$$G(u,v) = \begin{bmatrix} 1/1 & 1/(-0.2-0.2j) & 1/0 & 1/(-0.2+0.2j) \\ 1/(-0.2-0.2j) & 1/0.05j & 1/0 & 1/0.05 \\ 1/0 & 1/0 & 1/0 & 1/0 \\ 1/(-0.2+0.2j) & 1/0.05 & 1/0 & 1/(-0.05j) \end{bmatrix}$$

$$\Rightarrow G(u,v) = \begin{bmatrix} 1 & -2.5+2.5j & \infty & -2.5-2.5j \\ -2.5+2.5j & -20j & \infty & 20 \\ \infty & \infty & \infty & \infty \\ -2.5-2.5j & 20 & \infty & 20j \end{bmatrix}$$

b) deblur filter $G(u,v)$, using pseudo-inverse filtering approach, $\delta = 0.03$.

$$G(u,v) = \begin{cases} 1/H(u,v), & \delta < H(u,v) \\ 0, & \delta \geq H(u,v) \end{cases}$$

The magnitude of $H(U,V)$ is compared to δ. ($=0.03$)

(Ex: $\sqrt{(-0.2)^2+(-0.2)^2} > 0.03 \Rightarrow 1/H(U,V)$)

$$\therefore G(U,V) = \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & -20j & 0 & 20 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 20 & 0 & 20j \end{bmatrix}$$

c) deblur filter $G(U,V)$ using pseudo inverse filtering, $\delta = 0.1$

$$G(U,V) = \begin{cases} 1/H(U,V), & \delta < H(U,V) \\ 0, & \delta \geq H(U,V) \end{cases}$$

$$G(U,V) = \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.5+2.5j & 0 & 0 & 0 \end{bmatrix}$$

($\because$ The magnitude of each element of $H(U,V)$ is compared to the given ($\delta = 0.1$), to determine $G(U,V)$)

Ex =

$\sqrt{(-0.2)^2+(-0.2)^2} = 0.28 > 0.1 \Rightarrow 1/H(U,V)$

$\sqrt{(0.05)^2} = 0.05 < 0.1 \Rightarrow G(U,V) = 0$

d) Deblur filter $G(u,v)$, using pseudo inverse filtering method, $\delta = 0.3$.

$$G(u,v) = \begin{cases} 1/H(u,v), & \delta < H(u,v) \\ 0, & \delta \geq H(u,v) \end{cases}$$

Comparing magnitude of $H(u,v)$ to δ, to determine $G(u,v)$

Ex :- $\sqrt{(-0.2)^2 + (-0.2)^2} = 0.28 < 0.3 \Rightarrow G(u,v) = 0$

$\sqrt{(1)^2} = 1 > 0.3 \Rightarrow G(u,v) = 1/H(u,v)$.

$$\therefore G(u,v) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

e) filter design using Weiner filter & $\sigma_x^2 = 100$; $\sigma_\omega^2 = 25$

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}, \text{ where } K = \frac{\sigma_\omega^2}{\sigma_x^2}$$

$$K = \frac{25}{100} = 0.25$$

$$H^*(u,v) = \begin{bmatrix} 1 & -0.2+0.2j & 0 & -0.2-0.2j \\ -0.2+0.2j & -0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2-0.2j & 0.05 & 0 & 0.05j \end{bmatrix}$$

$$|H(u,v)| = \begin{bmatrix} 1 & 0.283 & 0 & 0.283 \\ 0.283 & 0.05 & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ 0.283 & 0.05 & 0 & 0.05 \end{bmatrix}$$

$$|H(u,v)|^2 = \begin{bmatrix} 1 & 0.08 & 0 & 0.08 \\ 0.08 & 0.0025 & 0 & 0.0025 \\ 0 & 0 & 0 & 0 \\ 0.08 & 0.0025 & 0 & 0.0025 \end{bmatrix}$$

$$G(u,v) = \frac{\begin{bmatrix} 1 & -0.2+0.2j & 0 & -0.2-0.2j \\ -0.2+0.2j & -0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2-0.2j & 0.05 & 0 & 0.05j \end{bmatrix}}{\begin{bmatrix} 1 & 0.08 & 0 & 0.08 \\ 0.08 & 0.0025 & 0 & 0.0025 \\ 0 & 0 & 0 & 0 \\ 0.08 & 0.0025 & 0 & 0.0025 \end{bmatrix} + 0.25}$$

$$G(u,v) = \begin{bmatrix} 0.8 & -0.6+0.6j & 0 & -0.6-0.6j \\ -0.6+0.6j & -0.2j & 0 & 0.2 \\ 0 & 0 & 0 & 0 \\ -0.6-0.6j & 0.2 & 0 & 0.2j \end{bmatrix}$$

f) $h(m,n) = \frac{1}{N^2} f_N^* \cdot H(U,V) \cdot f_N^*$

$N = 4$; $f_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$

$$= \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 & -0.2-0.2j & 0 & -0.2+0.2j \\ -0.2-0.2j & 0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2+0.2j & 0.05 & 0 & -0.05j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 0.6 & -0.15-0.15j & 0 & -0.15+0.15j \\ 1.4 & -0.25-0.25j & 0 & -0.25+0.25j \\ 1.4 & -0.25-0.25j & 0 & -0.25+0.25j \\ 0.6 & -0.15-0.15j & 0 & -0.15+0.15j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$h(m,n) = \frac{1}{16} \begin{bmatrix} 0.3 & 0.9 & 0.9 & 0.3 \\ 0.9 & 1.9 & 1.9 & 0.9 \\ 0.9 & 1.9 & 1.9 & 0.9 \\ 0.3 & 0.9 & 0.9 & 0.3 \end{bmatrix}$$

$$\therefore h(m,n) = \begin{bmatrix} 0.019 & 0.056 & 0.056 & 0.019 \\ 0.056 & 0.119 & 0.119 & 0.056 \\ 0.056 & 0.119 & 0.119 & 0.056 \\ 0.019 & 0.056 & 0.056 & 0.019 \end{bmatrix}$$

g) $g(m,n)$ for $G(u,v)$ obtained in (c) with $\delta = 0.1$

$$G(u,v) \text{ with } \delta = 0.1 = \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 0 & 0 & 0 \end{bmatrix}$$

$$g(m,n) = \frac{1}{N^2} f_N^* \, G(u,v) \, f_N^* \quad \text{where}$$

$$N = 4 \;\; \& \;\; f_N^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} -4 & -2.5+2.5j & 0 & -2.5-2.5j \\ -4 & -2.5+2.5j & 0 & -2.5-2.5j \\ 6 & -2.5+2.5j & 0 & -2.5-2.5j \\ 6 & -2.5+2.5j & 0 & -2.5-2.5j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} -9 & -9 & 1 & 1 \\ -9 & -9 & 1 & 1 \\ 1 & 1 & 11 & 11 \\ 1 & 1 & 11 & 11 \end{bmatrix} = \begin{bmatrix} -0.56 & -0.56 & 0.062 & 0.062 \\ -0.56 & -0.56 & 0.062 & 0.062 \\ 0.062 & 0.062 & 0.69 & 0.69 \\ 0.062 & 0.062 & 0.69 & 0.69 \end{bmatrix}$$

8) Given $h(m,n) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

a) $H(u,v) = ?$ $\Rightarrow$ $H(u,v) = F_N \cdot h(m,n) F_N$ ($\because N = 4$)

& $f_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

$$H(u,v) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 6 & 4 \\ 0 & -1-j & -1-j & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1+j & -1+j & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$H(u,v) = \begin{bmatrix} 20 & -2-2j & 0 & -2+2j \\ -2-2j & 2j & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2+2j & 2 & 0 & -2j \end{bmatrix}$$

b) $G(u,v)$ using pseudo inverse filtering approach :-

$\delta = 1$

$$|H(u,v)| = \begin{bmatrix} 20 & 2.8284 & 0 & 2.8284 \\ 2.8284 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 2.8284 & 0 & 0 & 0 \end{bmatrix}$$

$$G(u,v) = \begin{cases} 1/H(u,v) & , \ |H(u,v)| > 1 \\ 0 & , \ |H(u,v)| \leq 1 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1/20 & 1/-2-2j & 0 & 1/-2+2j \\ 1/-2-2j & 1/2j & 0 & 1/2 \\ 0 & 0 & 0 & 0 \\ 1/-2+2j & 1/2 & 0 & 1/-2j \end{bmatrix}$$

$$G(u,v) = \begin{bmatrix} 0.05 & -0.25+0.25j & 0 & -0.25-0.25j \\ -0.25+0.25j & -0.5j & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ -0.25-0.25j & 0.5 & 0 & 0.5j \end{bmatrix}$$

c) $G(u,v)$, using pseudo inverse filter, where $\delta = 3$.

$$G(u,v) = \begin{cases} 1/H(u,v), & |H(u,v)| > 3 \\ 0, & |H(u,v)| \leq 3. \end{cases}$$

$$G(u,v) = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

d) $G(u,v)$ using Wiener filtering approach :-

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}$$

$\sigma_x^2 = 400$

$\sigma_w^2 = 100$

$K = \frac{\sigma_w^2}{\sigma_x^2} = \frac{100}{400} = 0.25$

$$H^*(u,v) = \begin{bmatrix} 20 & -2+2j & 0 & -2-2j \\ -2+2j & -2j & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2-2j & 2 & 0 & 2j \end{bmatrix}$$

$$|H(u,v)|^2 + K = \begin{bmatrix} 400.25 & 8.25 & 0.25 & 8.25 \\ 8.25 & 4.25 & 0.25 & 4.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 8.25 & 4.25 & 0.25 & 4.25 \end{bmatrix}$$

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}$$

$$= \frac{\begin{bmatrix} 20 & -2+2j & 0 & -2-2j \\ -2+2j & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2-2j & 2 & 0 & 2j \end{bmatrix}}{\begin{bmatrix} 400.25 & 8.25 & 0.25 & 8.25 \\ 8.25 & 4.25 & 0.25 & 4.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 8.25 & 4.25 & 0.25 & 4.25 \end{bmatrix}}$$

$$\therefore G(u,v) = \begin{bmatrix} 0.05 & -0.24+0.24j & 0 & -0.24-0.24j \\ -0.24+0.24j & -0.47j & 0 & 0.47 \\ 0 & 0 & 0 & 0 \\ -0.24-0.24j & 0.47 & 0 & 0.47j \end{bmatrix}.$$

9) Given image =

7	2	7	2
2	7	2	7
7	2	7	2
2	7	2	7

Given, Linear shift invariant filter = $\frac{1}{6}$

0	1	0
1	2	1
0	1	0

Zero padding for the image given :-

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 2 & 7 & 2 & 0 \\ 0 & 2 & 7 & 2 & 7 & 0 \\ 0 & 7 & 2 & 7 & 2 & 0 \\ 0 & 2 & 7 & 2 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Applying the shift invariant filter to the zero-padded image.

~~3 2.67 3 2.67~~

~~3.8 2.5 5 2.5~~

~~2.6~~

$$\frac{1}{6}\left(1\times 0 + 1\times 0 + 7\times 2 + 2\times 1 + 2\times 1\right) = 3$$

Similarly, =

3	4.167	3.33	3
4.167	3.67	5.33	3.33
3.33	5.33	3.67	4.167
3	3.67	4.167	3

final image obtained after rounding image pixels.

$$= \begin{matrix} 3 & 4 & 3 & 3 \\ 4 & 4 & 5 & 3 \\ 3 & 5 & 4 & 4 \\ 3 & 4 & 4 & 3 \end{matrix}$$

Mean Absolute error :- $\frac{1}{n}$ (sum |orig. image val − distorted image val|)

where n is no. of pixels. ∴ $|A| = abs(A)$.

$$= \frac{1}{16}\Big(|7-3| + |2-4| + |7-3| + |2-3| + |2-4| + |7-4| + |2-5| + |7-3| + |7-3| + |2-5| + |7-4| + |2-4| + |2-3| + |7-4| + |2-4| + |7-3|\Big)$$

$$= \frac{1}{16}\Big(4+2+4+1+2+3+3+4+4+3+3+2+1+3+2+4\Big)$$

$$= \frac{1}{16}(45) = 2.81.$$

Mean squared error :- $\frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (x_{ij} - y_{ij})^2$ $M=4$, $N=4$

$$= \frac{1}{16}\Big(4^2 + 2^2 + 4^2 + 1^2 + 2^2 + 3^2 + 3^2 + 4^2 + 4^2 + 3^2 + 3^2 + 2^2 + 1^2 + 3^2 + 2^2 + 4^2\Big)$$

$$MSE = \frac{1}{16}(143) = 8.94$$

$$PSNR = 10\log_{10}\left(\frac{L^2}{MSE}\right) = 10\log_{10}\left(\frac{15^2}{8.94}\right)$$

$$PSNR = 14 \text{ dB}$$

b) Using a median filter, compute MAE, MSE & PSNR.

Median filter is a 3x3 filter.

Using replicate-padding of the original image:

orig. img. =

$$\begin{matrix} 7 & 2 & 7 & 2 \\ 2 & 7 & 2 & 7 \\ 7 & 2 & 7 & 2 \\ 2 & 7 & 2 & 7 \end{matrix}$$

replicate-padding:

$$\begin{matrix} 7 & 7 & 2 & 7 & 2 & 2 \\ 7 & 7 & 2 & 7 & 2 & 2 \\ 2 & 2 & 7 & 2 & 7 & 7 \\ 7 & 7 & 2 & 7 & 2 & 2 \\ 2 & 2 & 7 & 2 & 7 & 7 \\ 2 & 2 & 7 & 2 & 7 & 7 \end{matrix}$$

A median filter takes the median filter, when the mask is applied to the image.

→ Sliding the 3x3 median filter mask through the 6x6 padded image, we get a 4x4 image.

i) Median (7, 7, 2, 7, 7, 2, 2, 2, 7) = 7. (after rearranging them in order)

Likewise we get,

7	7	2	2
7	7	2	2
2	2	7	7
2	2	7	7

Mean absolute error :- $\frac{1}{n}$ (sum (abs (orig img - distorted img))

$$= \frac{1}{16}\Big(|7-7|+|2-7|+|7-2|+|2-2|+|7-2|+|7-7|+ |2-2|+|7-2|+|2-7|+|2-2|+|7-7|+|2-7|+ |2-2|+|2-7|+|2-7|+|7-7|\Big)$$

$$= 1/16\,(0+5+5+0+5+0+5+0+5+0+5+0+0+5+5+0)$$

$$= 1/16\,(40) = 0.4.$$

Mean square error :- $\frac{1}{M \times N}\sum_{i=1}^{M}\sum_{j=1}^{N}(x_{ij}-y_{ij})^2$ $\begin{cases} M=4 \\ N=4 \end{cases}$

$$= 1/16\,(0^2+5^2+5^2+0^2+5^2+0^2+5^2+0^2+0^2+5^2+5^2+0^2+0^2+5^2+5^2+0^2)$$

$$= 1/16\,(200) \Rightarrow \text{MSE} = 12.5$$

$$PSNR = 10\log_{10}\left(\frac{L^2}{MSE}\right)$$

$$= 10\log_{10}\left(\frac{15^2}{12.5}\right)$$

$$= 10\log_{10}(18)$$

$$\therefore \boxed{PSNR = 12.55\ dB}$$

10).

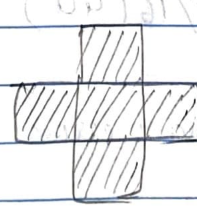
Given 4x4 image,

$$\begin{matrix} 9 & 8 & 7 & 6 \\ 8 & 7 & 13 & 5 \\ 7 & 6 & 5 & 4 \\ 6 & 1 & 4 & 3 \end{matrix}$$

a) Median filter after replicate padding :-

$$\begin{matrix} 9 & 9 & 8 & 7 & 6 & 6 \\ 9 & 9 & 8 & 7 & 6 & 6 \\ 8 & 8 & 7 & 13 & 5 & 5 \\ 7 & 7 & 6 & 5 & 4 & 4 \\ 6 & 6 & 1 & 4 & 3 & 3 \\ 6 & 6 & 1 & 4 & 3 & 3 \end{matrix}$$

Median filter mask :-



Applying the median filter on the first 3 rows & columns.

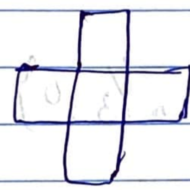
Median (9,9,9,8,8) = 9.

Likewise, doing it on other columns, we get.

Resulting image :-

$$\begin{matrix} 9 & 8 & 7 & 6 \\ 8 & 8 & 7 & 5 \\ 7 & 6 & 5 & 4 \\ 6 & 4 & 4 & 3 \end{matrix}$$

b) Image using a min. filter after replicate padding

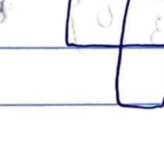
min() on

$$\begin{matrix} 9 & 9 & 8 & 7 & 6 & 6 \\ 9 & 9 & 8 & 7 & 6 & 6 \\ 8 & 8 & 7 & 13 & 5 & 5 \\ 7 & 7 & 6 & 5 & 4 & 4 \\ 6 & 6 & 1 & 4 & 3 & 3 \\ 6 & 6 & 1 & 4 & 3 & 3 \end{matrix}$$

Resulting image :

$$\begin{matrix} 8 & 7 & 6 & 5 \\ 7 & 6 & 5 & 4 \\ 6 & 1 & 4 & 3 \\ 1 & 1 & 1 & 3 \end{matrix}$$

c) Image using a max. filter after replicate padding.

max() on

$$\begin{matrix} 9 & 9 & 8 & 7 & 6 & 6 \\ 9 & 9 & 8 & 7 & 6 & 6 \\ 8 & 8 & 7 & 13 & 5 & 5 \\ 7 & 7 & 6 & 5 & 4 & 4 \\ 6 & 6 & 1 & 4 & 3 & 3 \\ 6 & 6 & 1 & 4 & 3 & 3 \end{matrix}$$

Resulting image :-

9	9	13	7
9	13	13	13
8	7	13	5
7	6	5	4

d)

Filter the replicate padded image with order-statistics filter $w_i = \{0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\}$

$$w_i = \{0, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, 0\} \text{ on } \begin{bmatrix} 9 & 9 & 8 & 7 & 6 & 6 \\ 9 & 9 & 8 & 7 & 6 & 6 \\ 8 & 8 & 7 & 13 & 5 & 5 \\ 7 & 7 & 6 & 5 & 4 & 4 \\ 6 & 6 & 1 & 4 & 3 & 3 \\ 6 & 6 & 1 & 4 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} & 9 & \\ 9 & 9 & 8 \\ & 8 & \end{bmatrix} * \{0, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, 0\}$$

$$\Rightarrow \text{round}\left(\text{sum}(8, 8, 9, 9, 9) * \{0, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, 0\}\right)$$

$$= 9.$$

Similarly, applying the filter on the image, we get,

Resulting image:

9	8	7	6
8	8	6	5
7	6	5	4
6	4	4	3

11) Given 7X7 image,

$$
\begin{matrix}
3 & 3 & 1 & 3 & 3 & 3 & 4 \\
0 & 3 & 3 & 3 & 3 & 3 & 3 \\
3 & 3 & 3 & 2 & 3 & 3 & 12 \\
12 & 3 & 3 & 3 & 3 & 12 & 12 \\
10 & 12 & 2 & 3 & 3 & 12 & 12 \\
12 & 14 & 12 & 12 & 12 & 11 & 11 \\
11 & 12 & 12 & 12 & 10 & 12 & 12
\end{matrix}
$$

Prewitt gradient operator, using

$$g(m,n) = |g_1(m,n)| + |g_2(m,n)|.$$

Given threshold is $T = 22$

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \& \quad G_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Sliding the G_x mask on the image, one stride at a time.

$$\text{sum}\left(\begin{bmatrix} 3 & 3 & 1 \\ 0 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \right) = (-3+1+3-3+3) = (+1)$$

Similarly,

$$g_1(m,n) = \begin{matrix} 1 & -1 & 2 & 1 & 10 \\ -6 & -1 & 0 & 10 & 18 \\ -17 & -10 & 1 & 19 & 27 \\ -17 & -11 & 1 & 18 & 17 \\ -7 & -11 & -1 & 9 & 10 \end{matrix}$$

Now sliding G_y filter mask on the image we get

$$\text{sum}\left(\begin{bmatrix} 3 & 3 & 1 \\ 0 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} * \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}\right) = (-3-3-1+3+3+3) = 2$$

Similarly,

$$g_2(m,n) = \begin{matrix} 2 & 1 & 1 & -1 & 8 \\ 12 & 0 & 0 & 9 & 18 \\ 15 & 9 & 0 & 10 & 9 \\ 20 & 29 & 27 & 18 & 8 \\ 11 & 19 & 26 & 16 & 7 \end{matrix}$$

$$|g(m,n)| = |g_1(m,n)| + |g_2(m,n)|$$

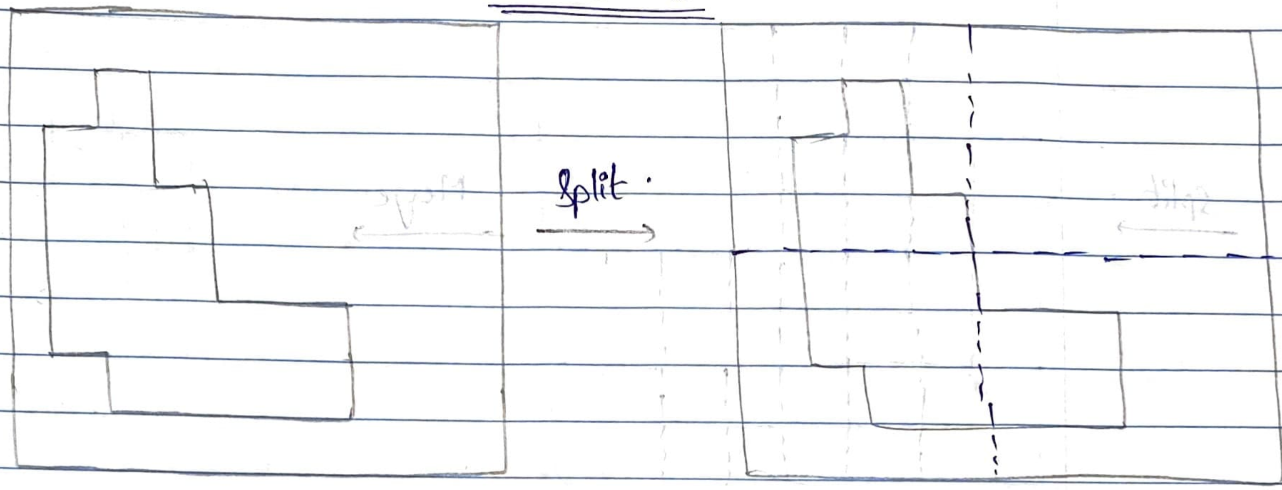
$$\therefore\ g(m,n) = \begin{matrix} 3 & 2 & 3 & 2 & 18 \\ 18 & 1 & 0 & 19 & 36 \\ 32 & 19 & 1 & 29 & 36 \\ 37 & 40 & 28 & 36 & 25 \\ 18 & 30 & 27 & 25 & 17 \end{matrix}$$

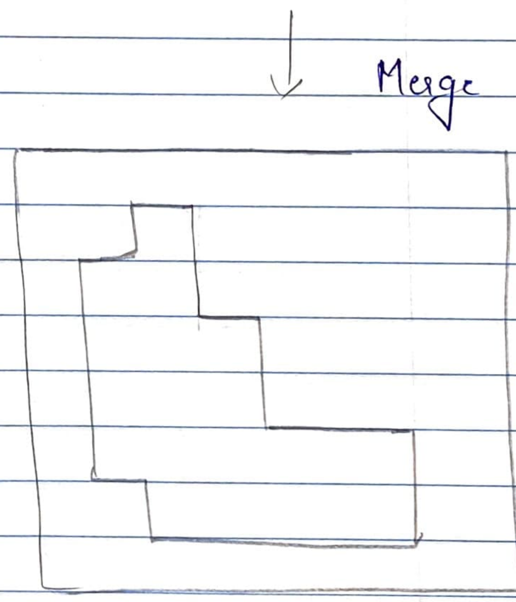
Given threshold is $T = 22$ so, removing the elements < 22 (i.e., adding a '0' in that place) and adding a 1 at each place where the element > 22

$$\therefore \text{ edge map } e(m,n) = \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{matrix}$$

12). Iterative quad tree Split & merge.

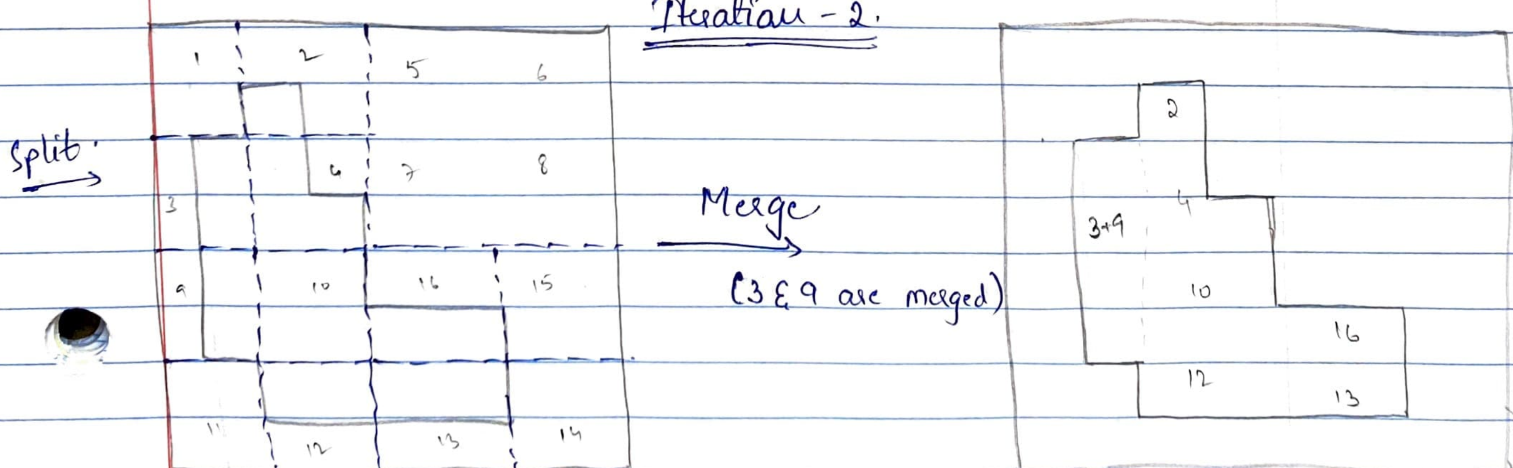
Iteration - 1





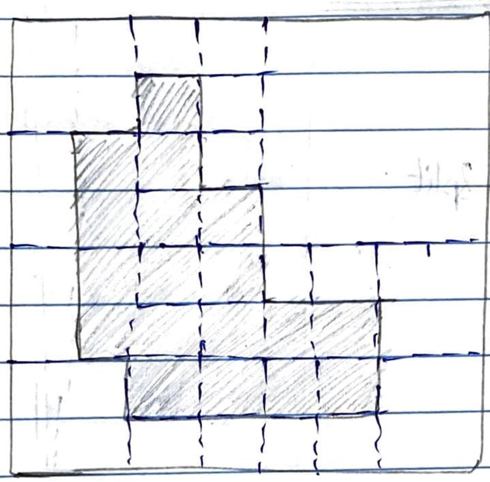
Splitting every non-uniform region to 4 and merging all adjacent uniform regions.

Iteration - 2.

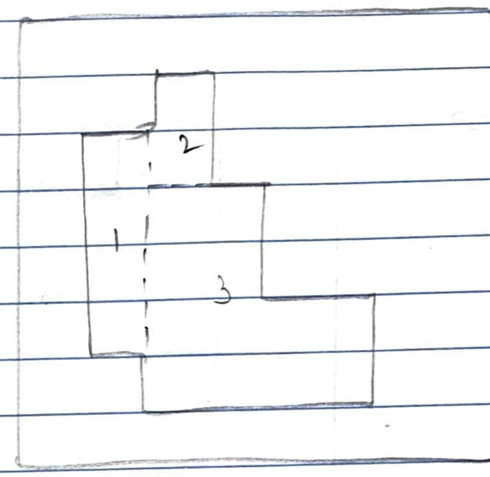


Iteration - 3

Split →



Merge →



3 regions