Homework 1 Solutions

1.

$$MAE = \frac{1}{16}(2+1+3+0+0+1+3+11+1+4+0+0+1+0+5+1)$$

$$= \frac{33}{16} = 2.0625$$

$$MSE = \frac{1}{16}(4+1+9+0+0+1+9+121+1+16+0+0+1+0+25+1)$$

$$= \frac{189}{16} = 11.8125$$

$$PSNR = 10 \log_{10} \frac{15^2}{MSE} = 12.8 \text{ dB}$$

2.

r		s	
3	$\longrightarrow$	5	①
15	$\longrightarrow$	11	②
?	$\longrightarrow$	8	
1	$\longrightarrow$	?	

(a) $① \Rightarrow \alpha \log_2(1+3)+\beta=5 \Rightarrow 2\alpha+\beta=5$

$② \Rightarrow \alpha \log_2(1+15)+\beta=11 \Rightarrow 4\alpha+\beta=11$

$$\Rightarrow \begin{cases} \alpha=3 \\ \beta=-1 \end{cases}$$

$$\Rightarrow s=T(r)=3\log_2(1+r)-1$$

(b) $r=1 \Rightarrow s=3\log_2(1+1)-1=2$

(c) $s=8 \Rightarrow 8=3\log_2(1+r)-1$

$\Rightarrow \log_2(1+r)=3$

$\Rightarrow 1+r=8$

$\Rightarrow r=7$

3.

(1) → (E) gamma transformation

(2) → (C) Inverting pixel intensities

(3) → (D) reducing contrast

(4) → (B) thresholding

(5) → (A) contrast stretch

4. (a) $r_{min} = 6, \quad r_{max} = 14$

$$s = \text{round}\left(15 \cdot \frac{r - r_{min}}{r_{max} - r_{min}}\right) = \text{round}\left(15 \cdot \frac{r-6}{14-6}\right) = \text{round}\left(\frac{15}{8}(r-6)\right)$$

r	s
6	round(0) = 0
7	round(1.875) = 2
11	round(9.375) = 9
12	round(11.25) = 11
13	round(13.125) = 13
14	round(15) = 15

0	13	11	13
11	0	2	11
13	2	2	11
15	9	9	15

(b)

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	0	0	0	0	2	3	0	0	0	2	4	3	2	0
Q(k)	0	0	0	0	0	0	2	5	5	5	5	7	11	14	16	16

Intermediate image

2	14	11	14
11	2	5	11
14	5	5	11
16	7	7	16

→ $r_{min} = 2$, $r_{max} = 16$

$$s = \text{round}\left[15 \frac{r - r_{min}}{r_{max} - r_{min}}\right] = \text{round}\left[\frac{15}{14}(r-2)\right]$$

r	s
2	round(0) = 0
5	round(3.2) = 3
7	round(5.4) = 5
11	round(9.6) = 10
14	round(12.9) = 13
16	round(15) = 15

→

0	13	10	13
10	0	3	10
13	3	3	10
15	5	5	15

(c) .....

5.

(a)

4	3.25	3.75	2.75
3.25	9.25	3.5	3
8.25	4	8.75	3.25
3.25	8.5	3.75	3.5

Filter1 →

Zero padding →

0	0	0	0	0	0
0	2	9	10	0	0
0	7	1	6	1	0
0	10	15	2	6	0
0	11	3	8	10	0
0	0	0	0	0	0

(b)

1.75	5.75	9.5	-1.5
1	-5	-1.5	-2
9	7	-1.75	2.5
7.25	0	2.75	9.5

Filter2 →

(c)

5.75	9	13.25	1.25
4.25	4.25	2	1
17.25	11	7	5.75
10.5	8.5	6.5	13

Filter3 →

Filter 3 = Filter 1 + Filter 2

⟹ filtered im3 = filtered im1 + filtered im1

because convolution is linear

6. (a)

Original

7	1	7	1
5	3	5	3
7	1	7	1
5	3	5	3

→ [2D-DFT] →

64	0	32	0
0	0	0	0
0	0	16	0
0	0	0	0

(b)

64	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0	0	32	0
0	0	0	0
0	0	0	0
0	0	0	0

0	0	0	0
0	0	0	0
0	0	16	0
0	0	0	0

→ [Inverse 2D-DFT] →

4	4	4	4
4	4	4	4
4	4	4	4
4	4	4	4

2	-2	2	-2
2	-2	2	-2
2	-2	2	-2
2	-2	2	-2

1	-1	1	-1
-1	1	-1	1
1	-1	1	-1
-1	1	-1	1

(c) Original image = ① + ② + ③

2D-DFT and Inverse 2D-DFT are both linear

7. (a)

$$G(u,v) = \frac{1}{H(u,v)} = \begin{bmatrix} 1 & -2.5+2.5j & Inf & -2.5-2.5j \\ -2.5+2.5j & -20j & Inf & 20 \\ Inf & Inf & Inf & Inf \\ -2.5-2.5j & 20 & Inf & 20j \end{bmatrix}$$

(b)

$$G(u,v) = \begin{cases} \frac{1}{H(u,v)}, & |H(u,v)| > 0.03 \\ 0, & else \end{cases} = \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & -20j & 0 & 20 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 20 & 0 & 20j \end{bmatrix}$$

(c)

$$G(u,v) = \begin{cases} \frac{1}{H(u,v)}, & |H(u,v)| > 0.1 \\ 0, & else \end{cases} = \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 0 & 0 & 0 \end{bmatrix}$$

(d)

$$G(u,v) = \begin{cases} \frac{1}{H(u,v)}, & |H(u,v)| > 0.1 \\ 0, & else \end{cases} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(e)

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K} = \begin{bmatrix} 0.8 & -0.606+0.606j & 0 & -0.606-0.606j \\ -0.606+0.606j & -0.198j & 0 & 0.198 \\ 0 & 0 & 0 & 0 \\ -0.606-0.606j & 0.198 & 0 & 0.198j \end{bmatrix}$$

$$K = \frac{\sigma_w^2}{\sigma_x^2} = \frac{25}{100} = 0.25$$

(f) Apply inverse 2D-DFT to $H(u,v)$, we have

$$h(m,n) = \frac{1}{160}\begin{bmatrix} 3 & 9 & 9 & 3 \\ 9 & 19 & 19 & 9 \\ 9 & 19 & 19 & 9 \\ 3 & 9 & 9 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0187 & 0.0563 & 0.0563 & 0.0187 \\ 0.0563 & 0.1187 & 0.1187 & 0.0563 \\ 0.0563 & 0.1187 & 0.1187 & 0.0563 \\ 0.0187 & 0.0563 & 0.0563 & 0.0187 \end{bmatrix}$$

(g) Apply inverse 2D-DFT to $G(u,v)$ designed in Question (c), we have

$$g(m,n) = \frac{1}{16}\begin{bmatrix} -9 & -9 & 1 & 1 \\ -9 & -9 & 1 & 1 \\ 1 & 1 & 11 & 11 \\ 1 & 1 & 11 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5625 & -0.5625 & 0.0625 & 0.0625 \\ -0.5625 & -0.5625 & 0.0625 & 0.0625 \\ 0.0625 & 0.0625 & 0.6875 & 0.6875 \\ 0.0625 & 0.0625 & 0.6875 & 0.6875 \end{bmatrix}$$

8. (a) $H(u,v) = 2D\text{-}DFT$ of $h(m,n)$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -2-2j & 0 & -2+2j \\ -2-2j & 2j & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2+2j & 2 & 0 & -2j \end{bmatrix}$$

(b)

$$G(u,v) = \begin{cases} \frac{1}{H(u,v)}, & |H(u,v)| > 1 \\ 0, & \text{else} \end{cases} = \begin{bmatrix} 0.05 & -0.25+0.25j & 0 & -0.25-0.25j \\ -0.25+0.25j & -0.5j & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ -0.25-0.25j & 0.5 & 0 & 0.5j \end{bmatrix}$$

(c)

$$G(u,v) = \begin{cases} \frac{1}{H(u,v)}, & |H(u,v)| > 3 \\ 0, & \text{else} \end{cases} = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(e) $K = \frac{\sigma_w^2}{\sigma_x^2} = \frac{100}{400} = 0.25$

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K} = \begin{bmatrix} 0.05 & -0.242+0.242j & 0 & -0.242-0.242j \\ -0.242+0.242j & -0.471j & 0 & 0.471 \\ 0 & 0 & 0 & 0 \\ -0.242-0.242j & 0.471 & 0 & 0.471j \end{bmatrix}$$

9. (a)

original image → zero-padding →

0	0	0	0	0	0
0	7	2	7	2	0
0	2	7	2	7	0
0	7	2	7	2	0
0	2	7	2	7	0
0	0	0	0	0	0

→ filtering →

3	4.17	3.33	3
4.17	3.67	5.33	3.33
3.33	5.33	3.67	4.17
3	3.33	4.17	3

→ rounding ↓

3	4	3	3
4	4	5	3
3	5	4	4
3	3	4	3

$$MAE = \frac{1}{16}(4+2+4+1+2+3+3+4+4+3+3+2+1+4+2+4) = 46/16 = 2.875$$

$$MSE = \frac{1}{16}(16+4+16+1+4+9+9+16+16+9+9+4+1+16+4+16) = 150/16 = 9.375$$

$$PSNR = 10\log_{10}\frac{15^2}{MSE} = 10\log_{10}\frac{225}{9.375} = 13.80\,dB$$

(b)

original image → replicate padding →

7	7	2	7	2	2
7	7	2	7	2	2
2	2	7	2	7	7
7	7	2	7	2	2
2	2	7	2	7	7
2	2	7	2	7	7

→ 3×3 median →

7	7	2	2
7	7	2	2
2	2	7	7
2	2	7	7

$$MAE = \frac{1}{16}(0+5+5+0+5+0+0+5+5+0+0+5+0+5+5+0) = 2.5$$

$$MSE = \frac{1}{16}(0+25+25+0+25+0+0+25+25+0+0+25+0+25+25+0) = 12.5$$

$$PSNR = 10\log_{10}\frac{15^2}{MSE} = 10\log_{10}\frac{225}{12.5} = 12.55\,dB$$

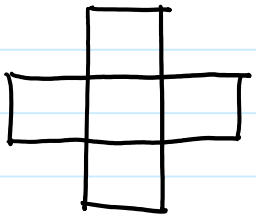
10.

9	8	7	6
8	7	13	5
7	6	5	4
6	1	4	3

replicate padding →

9	9	8	7	6	6
9	9	8	7	6	6
	8	7	13	5	5
7	7	6	5	4	4
6	6	1	4	3	3
6	6	1	4	3	3

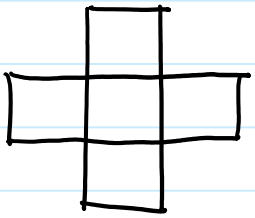
(a) Median filtering with



→

9	8	7	6
8	8	7	5
7	6	5	4
6	4	4	3

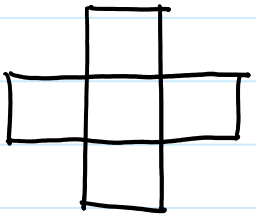
(b) Min filtering with



→

8	7	6	5
7	6	5	4
6	1	4	3
1	1	1	3

(c) Max filtering with



→

9	9	13	7
9	13	13	13
8	7	13	5
7	6	5	4

(d) OS filtering

$\{w_i\} = \{0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\}$ →

8.7	7.7	7.3	6
7.7	7.7	6.3	5.3
6.7	6	5	4.3
6	3.7	3.7	3.3

→

9	8	7	6
8	8	6	5
7	6	5	4
6	4	4	3

11.

$g_2(m,n) =$

2	1	1	-1	8
12	0	0	9	18
15	9	0	10	9
20	29	27	18	8
11	19	26	16	7

$g_1(m,n) =$

1	-1	2	1	10
-6	-1	0	10	18
-17	-10	1	19	27
-17	-11	1	18	17
-7	-11	-1	9	10

$|g_1(m,n)| + |g_2(m,n)| =$

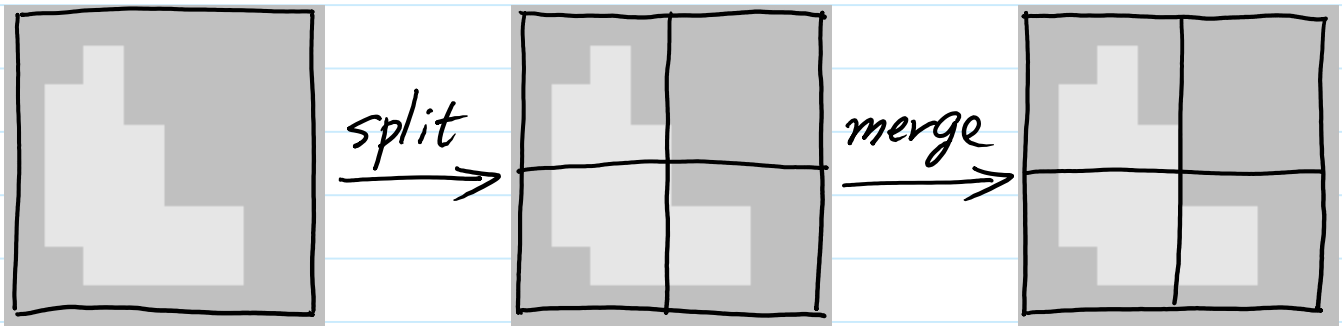
3	2	3	2	18
18	1	0	19	36
32	19	1	29	36
37	40	28	36	25
18	30	27	25	17

Edge map: $e(m,n) =$

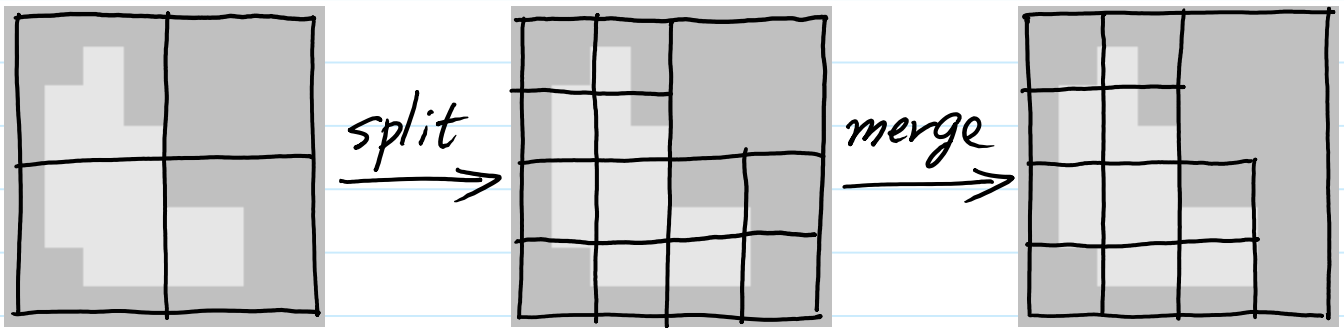
0	0	0	0	0
0	0	0	0	1
1	0	0	1	1
1	1	1	1	1
0	1	1	1	0

12.

Iteration 1:



Iteration 2:



Iteration 3: