ECE 105 MATH (TRIG, VECTORS AND CALCULUS) REVIEW

Work in groups. <u>AVOID</u> USING A CALCULATOR . Unless otherwise indicated, all angles are < 2π

A. <u>Trigonometry review (Page 1)</u>

Question 1. You walk up a straight hill, you have covered 3 m horizontally, and 4 vertically.

a) How long have you walked?

b) What is the tangent of the angle your hill slope makes with the vertical?

c) Find the sine of the angle.

d) Calculate the cosine from the sine.

Question 2. A line makes an angle of 30 degrees with the horizontal, the sine of the angle is negative, what is the standard designation of the angle? Pick all that apply. $-\pi/6, \pi/6, 7\pi/6, 5\pi/6, 13\pi/6, 11\pi/6$.

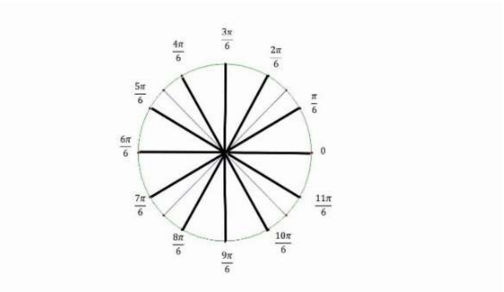
Question 3.

a) Two angles α and β are complimentary (α+β=π/2), show with a figure that the sine of one angle is equal to the cosine of the other, and vice-verse.

b) Show with a diagram that supplementary angles have the same sine but opposite cosines. Give a few examples of complimentary and supplementary angles.

Question 4. Important, do not use your calculator and do not convert to degrees, break up your circle according to the fractions indicated on the diagram, For example, for an angle $5\pi/6$, one breaks up the π into six equal parts, (the 2π into 12 equal parts) and picks the fifth partition from the positive x axis going counterclockwise as shown.

Draw and show sine and cosine, and indicate if positive or negative for



$$\boldsymbol{\frac{\pi}{6}, \frac{3\pi}{5}, \frac{13\pi}{6}, \frac{4\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{3}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{5\pi}{2}, \frac{6\pi}{3}, \frac{2\pi}{6} \frac{11\pi}{6}, \frac{7\pi}{3}, \frac{3\pi}{2}, 3\pi, 2\pi, \frac{5\pi}{12}, \frac{9\pi}{5}, \frac{11\pi}{7}, \frac{6\pi}{7}}$$

A. Trigonometry review (Page 2)

Question 5. Draw the unit circle for each of the values given, on your unit circle show the angle given. Show all possibilities, label the sine and cosine axes clearly.30°, 210°, $Sin^{-1}(-1/2)$, $Cos^{-1}(-1/2)$.

Question 6. Complete the following (list all possible values). The first line is an example, the value in red (sine) is given, and the rest of the table is filled based on that value. Use the unit circle to. Angles are measured from the positive x-axis counterclockwise. Again, do not use calculators unless indicated with a *. {* Use calculator when needed (only when there is a *)}.

Rad	deg	Sin	cos	tan	cot	sec	csc
$\frac{7\pi}{6}, \frac{11\pi}{6}$	210, 330	**-1/2**	$-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}$	$\sqrt{3}, -\sqrt{3}$	$-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$	-2
			+1/2				
			-1/2				
		0.3*					
				$-\sqrt{3}$			
					$-0.4*$		
						$2\frac{\sqrt{3}}{3}$	
							$-2\frac{\sqrt{3}}{3}$
		½					
				1			

B. Vector review (Page 1)

Question 1. Express the following vectors using Cartesian notation. The vectors here are given in magnitude angle notation, standard measure for angles is counterclockwise from positive *x*-axis. Length units are arbitrary. Decompose each vector into components on a diagram.
$(2, 30°)$, $(3, \pi/3)$, $(2, 4\pi/3)$, $(2, 5\pi/6)$, $(4, 3\pi/4)$, $(1, 3\pi/7)$, $(1, 2\pi/3)$, $(1.5, 5\pi/12)$

Question 2. Find the magnitude and angle for each of the following vectors:
$2\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}}$,$-3\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}$, $-3\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}}$,$2\hat{\boldsymbol{\imath}} - 4\hat{\boldsymbol{\jmath}}$.In general you may give the angle in any format you like, for example for one situation you many say 30 degrees below the positive *x*-axis, for another 20 degrees with the positive *y*-axis counterclockwise. Any format is ok as long as it is spelled out clearly. However, in this case, in addition to the form you prefer, please also include the standard measure of the angle (from the positive x-axis going counterclockwise).

Question 3. (do at home). a) Find the direction cosines (and angles) of the following vectors.
$\hat{\boldsymbol{\imath}} + \mathbf{2}\hat{\boldsymbol{\jmath}}$+$5\hat{\boldsymbol{k}}$, $4\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}} + 3\hat{\boldsymbol{k}}$, $2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} - 3\hat{\boldsymbol{k}}$, $-4\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} - 2\hat{\boldsymbol{k}}$.

Question 4. Add the following pairs of Forces **a)** add $(5\hat{\boldsymbol{\imath}} + 7\hat{\boldsymbol{\jmath}})$ N and $(2\hat{\boldsymbol{\imath}} - 4\hat{\boldsymbol{\jmath}})$ N, **b)** add (60 N, 40°) and (30 N, 85°), **c)** add (20 N, -30°) and **(**$12\hat{\boldsymbol{\imath}} - 28\hat{\boldsymbol{\jmath}}$) N. **Do so** by :

A) using Cartesian representation, whereby you find the components of each vector along each axis and then add the components on each axis.
B) By using the cosine law (or a variant). Express your results using magnitude angle notation, and Cartesian notation.

Question 6.

a) Write down the expression for the scalar (or dot) product between two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ using full formalism (e.g. the magnitude of a vector is expressed using magnitude bars and an arrow on top of symbol).

b) What is the value of the scalar product$\hat{\boldsymbol{\imath}}.\hat{\boldsymbol{\imath}}$? What about$\hat{\boldsymbol{\imath}}.\hat{\boldsymbol{\jmath}}$?

c) Find the scalar product of the following i) $4\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}}$, and $-2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}$, ii)$2\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{k}}$, and $-\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{k}}$, *iii*) $2\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} - 3\hat{\boldsymbol{k}}$, $4\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}} + 2\hat{\boldsymbol{k}}$.

d) Now find the angles between the vector pairs in part **c)**, but do so by making use of the scalar product.

B. Vector review (Page 2)

Question 7. Draw two arbitrary vectors $\vec{A}$ and $\vec{B}$ connecting at the origin or the tail with angle θ between them.

a) Use a diagram to illustrate to your classmate that only parallel components matter in the scalar product. Show that the scalar product between the two vectors can be expressed as $\vec{A}.\vec{B} = \overrightarrow{A_{\backslash\backslash}}.\vec{B}$. Break up the vector $\vec{A}$ into two components, one parallel $\left(\overrightarrow{A_{\backslash\backslash}}\right)$ and one perpendicular $\left(\overrightarrow{A_{\perp}}\right)$ to $\vec{B}$. Show that $\vec{A}.\vec{B} = \vec{A}.\vec{B}_{\backslash\backslash}$.

b) Write down an expression for the magnitude of the resultant of addition and illustrate by completing parallelogram.

Question 8. Draw two perpendicular vectors $\vec{A}$ and $\vec{B}$ connecting at the origin or the tail. Using the right hand rule to find direction of $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$. (use the version you know, but also use the three finger rule: line up thumb with first vector in product, index with second vector, and middle finger gives cross product. Keep the three fingers along mutually perpendicular directions. (XYZ).

Question 9.

a) Write down the expression for the vector (or cross) product between two vectors $\vec{A}$ and $\vec{B}$ using full formalism.

b) What is the result of the following cross products a) $\hat{\imath} \times \hat{\imath}$, b) $\hat{\imath}.\times(-\hat{\imath})$ c) $\hat{\imath} \times \hat{\jmath}$ d) $\hat{\jmath} \times \hat{\imath}$ e) $\hat{\jmath} \times \hat{k}$, f) $\hat{k} \times \hat{\jmath}$, g) $(-\hat{\jmath}) \times \hat{k}$, h)$\hat{\imath} \times (-\hat{k})$, i)$\hat{k} \times (-\hat{\imath})$.

c) Find the vector products of the following i) $2\hat{\imath} + 3\hat{\jmath}$, and $-2\hat{\imath} + 4\hat{\jmath}$, ii)$2\hat{\imath} - 3\hat{k}$, and $3\hat{\imath} - 2\hat{\jmath}$, $iii)$ $3\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, and $4\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$.

d) Now find he angles between the vector pairs in part **c)**, however, do so by making use of the vector product.

Question 10. Draw two arbitrary vectors (not perpendicular) connecting at origin. Illustrate pictorially to your classmate that only perpendicular components contribute to the vector product between two vectors. Show algebraically that $\vec{A} \times \vec{B} = \overrightarrow{A_{\perp}} \times \vec{B} = \vec{A} \times \vec{B}_{\perp}$. Apply right hand rule(s) to find direction.

3. Calculus Review. (25 min) (letters, a, b, c, A and ω are constants)

Question 11. find v(t), and a(t) for each for the following

1. $x(t) = at^3 + bt + c$

2. $y(t) = 5\sin(2t)$

3. $z(t) = tan(t)$

4. $y(t) = 5sec(2t)$

5. $x(t) = ln\left(\frac{2t}{7t^2+3}\right)$.

6. $z(t) = Ae^{ct}$

7. $x(t) = A\,sin(\omega t)$.

8. $x(t) = Acos(\,\omega t)$

Parts 7 and 8 above are very significant, can you comment on the relation between x(t) and a(t)? Compare to 6.

Question 12.

A. Integrate (indefinite) $1.\frac{t^3}{3}$ $2.cos(t)dt$ $3.sin(t)dt$ $4.sin(t)cos(t)dt$ $5.sin(t)cos^2(t)dt$ $6.\frac{sin(t)}{cos(t)}dt$

B. find v(t) and x(t)

7. $a(t) = at^3 + b$.

8. $a(t) = 6e^{2t}$.

9. $a(t) = -\omega^2 x(t)$

10. $a(t) = \omega^2 x(t)$