Homework 2: Operational Semantics for WHILE

CS 252: Advanced Programming Languages
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1 Introduction

For this assignment, you will implement the semantics for a small imperative language, named WHILE.

The language for WHILE is given in Figure 1. Unlike the Bool* language we discussed previously, WHILE supports *mutable references*. The state of these references is maintained in a *store*, a mapping of references to values. ("Store" can be thought of as a synonym for heap.) Once we have mutable references, other language constructs become more useful, such as sequencing operations ($e_1; e_2$).

2 Small-step semantics

The small-step semantics for WHILE are given in Figure 3. For the sake of brevity, these rules use *evaluation contexts* (C), which specify which *redex* will be evaluated next. The evaluation rules then apply to the "hole" ($\bullet$) in this context.

Most of these rules are fairly straightforward, but there are a couple of points to note with the [SS-WHILE] rule. First of all, this is the only rule that makes a more complex expression when it has finished.

Secondly, note the final value of this expression once the while loop completes. It will *always* be `false` when it completes. We could have created a special value, such as `null`, or we could have made the while loop a statement that returns no value. Both choices, however, would complicate our language needlessly.

3 YOUR ASSIGNMENT

Part 1: Rewrite the operational semantic rules for WHILE in LaTeX to remove the contexts (C[...]) and to use evaluation order rules instead. Submit both your LaTeX source and the generated PDF file.

Extend your semantics with features to handle boolean values. Specifically, add support for:

- `and`
- `or`
- `not`

The exact behavior of these new features is up to you, but should seem reasonable to most programmers.

Part 2: Once you have your semantics defined, download `WhileInterp.hs` and implement the `evaluate` function, as well as any additional functions you need. Your implementation must be consistent with your operational semantics, *including your extensions for* `and`, `or`, *and* `not`. Also, you may not change any type signatures provided in the file.

Finally, implement the interpreter to match your semantics.

Zip all files together into `hw2.zip` and submit to Canvas.

$e ::=$		*Expressions*
	x	variables/addresses
	v	values
	$x := e$	assignment
	$e; e$	sequential expressions
	$e\ op\ e$	binary operations
	`if` e `then` e `else` e	conditional expressions
	`while` (e) e	while expressions
	`and` e e	and operator
	`or` e e	or operator
	`not` e	not operator
$v ::=$		*Values*
	i	integer values
	b	boolean values
$op ::=$	$+ \mid - \mid * \mid / \mid > \mid >= \mid < \mid <=$	*Binary operators*

Figure 1: The WHILE language

Runtime Syntax:

$$\sigma \in Store = variable \to v$$

Evaluation Rules: $\boxed{e, \sigma \to e', \sigma'}$

[SS-VAR] $$\frac{x \in domain(\sigma)}{x, \sigma \to \sigma(x), \sigma}$$

[SS-ASSIGN-CONTEXT] $$\frac{e, \sigma \to e', \sigma'}{x := e, \sigma \to x := e', \sigma'}$$

[SS-ASSIGN-RED] $$\frac{}{x := v, \sigma \to v, \sigma[x := v]}$$

[SS-OP-BIN-CONTEXT1] $$\frac{e_1, \sigma \to e_1', \sigma'}{e_1\ op\ e_2, \sigma \to e_1'\ op\ e_2, \sigma'}$$

[SS-OP-BIN-CONTEXT2] $$\frac{e, \sigma \to e', \sigma'}{v\ op\ e, \sigma \to v\ op\ e', \sigma'}$$

[SS-OP-BIN-RED] $$\frac{v_3 = v_1\ op\ v_2}{v_1\ op\ v_2, \sigma \to v_3, \sigma}$$

[SS-SEQ-CONTEXT] $$\frac{e_1, \sigma \to e_1', \sigma'}{e_1; e_2, \sigma \to e_1'; e_2, \sigma'}$$

[SS-SEQ-RED] $$\frac{}{v; e, \sigma \to e, \sigma}$$

Figure 2: Small-step semantics for WHILE

$$[\text{SS-IFCONTEXT}] \quad \frac{e_1, \sigma \to e_1', \sigma'}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3, \sigma \to \texttt{if } e_1' \texttt{ then } e_2 \texttt{ else } e_3, \sigma'}$$

$$[\text{SS-IFTRUE}] \quad \frac{}{\texttt{if true then } e_1 \texttt{ else } e_2, \sigma \to e_1, \sigma}$$

$$[\text{SS-IFFALSE}] \quad \frac{}{\texttt{if false then } e_1 \texttt{ else } e_2, \sigma \to e_2, \sigma}$$

$$[\text{SS-WHILE}] \quad \frac{}{\texttt{while } (e_1)\ e_2, \sigma \to \texttt{if } e_1 \texttt{ then } e_2; \texttt{while } (e_1)\ e_2 \texttt{ else false}, \sigma}$$

$$[\text{SS-AND-CONTEXT}] \quad \frac{e_1, \sigma \to e_1', \sigma'}{\texttt{and } e_1\ e_2, \sigma \to \texttt{and } e_1'\ e_2, \sigma'}$$

$$[\text{SS-AND-FALSE}] \quad \frac{}{\texttt{and false } e, \sigma \to \texttt{false}, \sigma}$$

$$[\text{SS-AND-TRUE}] \quad \frac{}{\texttt{and true } e, \sigma \to e, \sigma}$$

$$[\text{SS-OR-CONTEXT}] \quad \frac{e_1, \sigma \to e_1', \sigma'}{\texttt{or } e_1\ e_2, \sigma \to \texttt{or } e_1'\ e_2, \sigma'}$$

$$[\text{SS-OR-FALSE}] \quad \frac{}{\texttt{or false } e, \sigma \to e, \sigma}$$

$$[\text{SS-OR-TRUE}] \quad \frac{}{\texttt{or true } e, \sigma \to \texttt{true}, \sigma}$$

$$[\text{SS-NOT-CONTEXT}] \quad \frac{e, \sigma \to e', \sigma'}{\texttt{not } e, \sigma \to \texttt{not } e', \sigma'}$$

$$[\text{SS-NOT-FALSE}] \quad \frac{}{\texttt{not false}, \sigma \to \texttt{true}, \sigma}$$

$$[\text{SS-NOT-TRUE}] \quad \frac{}{\texttt{not true}, \sigma \to \texttt{false}, \sigma}$$

Figure 3: Small-step semantics for WHILE (Continued)