3. a. 1, 4, 3, 2, 5, 6

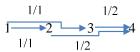
b.



c.



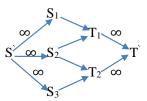
d.



4.

To create an equivalent max flow problem with only a single source and sink, you can add a single source that has edges connected to all of the sources from the original graph and give the edges from our new source to the original sources an infinite capacity. Then, we can do the same for the sinks, where all the original sinks now have edges of infinite capacity going to a single sink. Now, we have an equivalent max flow problem. For example, ...

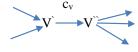




5.

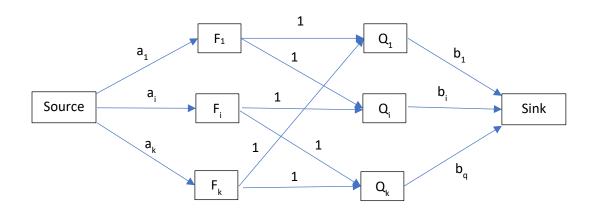
We can achieve this by splitting each intermediate vertex with capacity constraints into two vertices and connecting these vertices by one edge with capacity of the original intermediate vertex. Make all incoming edges go to the first vertex of the spilt and make all outgoing edges come out of the second vertex of the spilt. For example, if we can a vertex...





2. Dining Problem

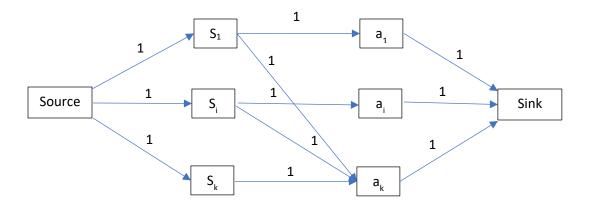
a. The capacity of the edges going toward the families is the amount of family members per family and the capacity of the tables is described in the outgoing edges from the tables. The edges from family to table is 1, since no two members of the same family can sit at the same table.



- b. No solution if flow conservation constraints are not met.
- c. We can see how many family members from each family are seated at a table and we can see which tables they sit at by looking at which q vertices are connected to a f vertex.

3. Committee Representation

a. Our flow network will have each subset have an incoming edge of capacity 1 since they must be represented by one object. Then, we connect the subsets to the objects that could possible represent then with an edge of capacity 1. Then, we have an outgoing edge from each object of capacity 1 to the sink. This will tell us if we can have an object represent a subset.



- b. We know that there is no solution if the max flow is less than the number of outgoing edges from the source (# of subsets, k).
- c. There is a solution if the max flow equals the number of subsets, k. We will know which object represents which subset by looking at which edges are at full capacity.