# Lab Assignment – 6 (2022)
# PH-566

**1.** Suppose you have developed a differential equation solution technique that has a local truncation error of order $O(h^n)$, and suppose E(h) and E(h/2) denote local truncation error estimates using step sizes of length h and h/2, respectively. What is the relation between E(h) and E(h/2), when h<<1 ?
(Note: This property of errors on successively refined grids (i.e. grids of size h, h/2, h/4 … etc.) is extremely useful when testing numerical codes on problems where exact solution is not known. It gives rise to an easy way to test that the order of accuracy of your method is what you expect, and hence that you have not made a coding mistake.)

**2.** Solve the following differential equation using the Euler's and Runge-Kutta 2nd order method for the range $0 \le x \le 2$ (take a step size of h=0.1):

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 1, \quad with \quad boundary \quad condition \quad y(0) = y'(0) = 0$$

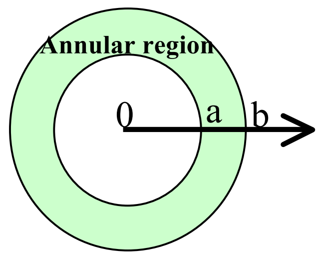
What is the absolute difference in the value of $y(x)$ at $x$=1, as obtained from the two methods.

**3.** Consider the following 2nd order differential equation

$$y'' = x(y')^2 - y^2,$$

with the boundary conditions, $y(0)=1, y'(0)=0$.
Solve the above differential equation to compute $y(0.2)$ correct to 4-decimal places using Euler and Runge-Kutta 2nd order methods (choose 50 subintervals).

**4.** Consider the circular membrane depicted in the adjoining figure in the right. When the membrane is under constant pressure, $p$, the displacement $y$(out of the page) as a function of radial position $r$ can be modeled using the following differential equation,




$$\frac{d^2y}{dr^2} + \frac{1}{r}\frac{dy}{dr} + \frac{p}{\tau} = 0.$$

Here, the parameter $\tau > 0$ is the membrane tension. The boundary condisitons at the inner and outer radius of the annular membrane are $y(a)=y(b)=0$.
Use Euler's and Runge-Kutta 2nd order method, compute the displacement $y$ at the middle of the annular region, considering n=200 subintervals between $[0,b]$. Use $p$=200, $\tau = 80$, a=1.0 and b=1.5