Lab Assignment 7 (2022)
PH-566

1. Consider the matrix equation AX=B, where

$$A = \begin{bmatrix} -4 & 1 & 0 & 0 \\ 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 1 & -4 \end{bmatrix} \quad and \quad B = \begin{bmatrix} -3 \\ -2 \\ -2 \\ -3 \end{bmatrix}$$

Here, the exact solution is given by X_e=[1, 1, 1, 1]
Use the Jacobi method to solve the above equation with a starting guess of $X^{(0)}=1.1X_e$. At each iteration number (i) record the error using the quantity $\log_{10}[ABS(X-X_e)]$. The error should diminish. Stop the iteration when the $ABS[x^{(k)} - x^{(k-1)}] < 10^{-6}$.

2. For the above problem, do the exact same thing using Gauss-Seidel method.
Compare this result with those obtained using Jacobi method.

3. Consider the following simple set of linear equations,

$$\begin{aligned} x_1 + 3x_2 - x_3 &= 5 \\ 3x_1 - x_2 \quad\quad &= 5 \\ x_2 + 2x_3 &= 1 \end{aligned}$$

Use the initial approximation $(x_1,x_2)=(0,0)$, show that both Jacobi and Gauss-Seidel method diverges.
In such cases, satisfying the condition involving strict diagonality helps to converge the solution.
Now, interchange the rows of the above system of linear equations to btain a system with a strictly digonally dominant coefficient matric. Then apply the Gauss-Seidel method to approximate the solution to two significant digit

4. Apply the Gauss-Seidel method to solve the following system of linear equations:

$$\begin{aligned} 4x_1 - x_2 - x_3 &= 18 \\ -x_1 + 4x_2 - x_3 - x_4 &= 18 \\ -x_2 + 4x_3 - x_4 - x_5 &= 4 \\ -x_3 + 4x_4 - x_5 - x_6 &= 4 \\ -x_4 + 4x_5 - x_6 - x_7 &= 26 \\ -x_5 + 4x_6 - x_7 - x_8 &= 16 \\ -x_6 + 4x_7 - x_8 &= 10 \\ -x_7 + 4x_8 &= 32 \end{aligned}$$