Lab Assignment – 5 (2022)
PH-566

1. Given a first order differential equation

$$\frac{dy}{dx} + 0.4y = 3e^{-x}$$

with the initial boundary condition, y(x=0)=5. Using Euler's method, calculate the value of y at x=3. In order to achieve an accuracy up to the 2^{nd} decimal place of y(x=3), what step size should be taken ?
Instead of using Euler's method, if you use the Runge-Kutta 2^{nd} and 4^{th} order method, what step size will be sufficient to achieve an accuracy up to the 2^{nd} decimal place?

2. Given a first order differential equation:

$$\frac{dy}{dx} = x + y^2, \;\; with \;\; boundary \;\; condition \;\; y(0) = 1$$

Compute y(0.1) using the Runge-Kutta 2^{nd} and 4^{th} order method taking a step size of h=0.1. How many iterations does the two methods take to achieve an accuracy up to 2^{nd} decimal place?

3. Solve the following differential equation using the Runge-Kutta 4^{th} order method for the range 0≤x≤2 (take a step size of h=0.1):

$$\frac{dy}{dx} = (x + y)\sin(xy), \;\; with \;\; boundary \;\; condition \;\; y(0) = 5$$

4. The concentration of salt x in a homemade soap maker tank is given as a function of time (t) by

$$\frac{dx}{dt} = 37.5 - 3.5x$$

At the initial time, t = 0, the salt concentration in the tank is 50 g/L. Using Euler and Runge-Kutta 2^{nd} order method and a step size of h = 1.5 min, numerically calculate the salt concentration after 3 minutes?
What is the % error of these numerically evaluated values with respect to the analytically exact result?
Find the same % error when the step size is taken to be h=0.1875 min