CSC-591: Foundations of Data Science
T/Th. 12:50-2:05pm. EBI-1005.

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W6: 9/22-24/15

Administrative

- Updated Weekly Schedule (on Moodle)
- HW-2: To be posted (9/22-23/15)
  - DUE: 10/4/15
- 1st Midterm: 10/6/15
- Feedback (HWs)
  - What worked best for enhancing your understanding and what didn't?
  - Anything (change) that you would like to see in HW2?

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So far

- 1st Module is completed
  - Exploratory Data Analysis
  - Summary Statistics, Histograms, etc.
  - Basic Probability (including set operations)
  - Basic Linear/Matrix Algebra, Calculus
  - Probability distributions
  - Parameter estimation
  - Sampling distribution, CLT, C.I.
  - Hypothesis testing
- 1st Midterm will be based on 1st module
  - All these topics will be reviewed on 10/5

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Today

- Regression Analysis (covered in 3 lectures)

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Learning

- Observe a phenomena
- Construct a model of that phenomena
- Make predictions using the model

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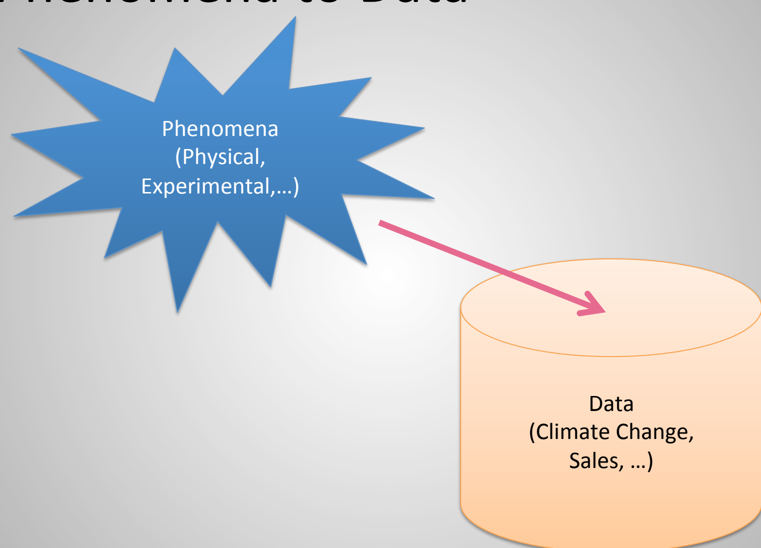
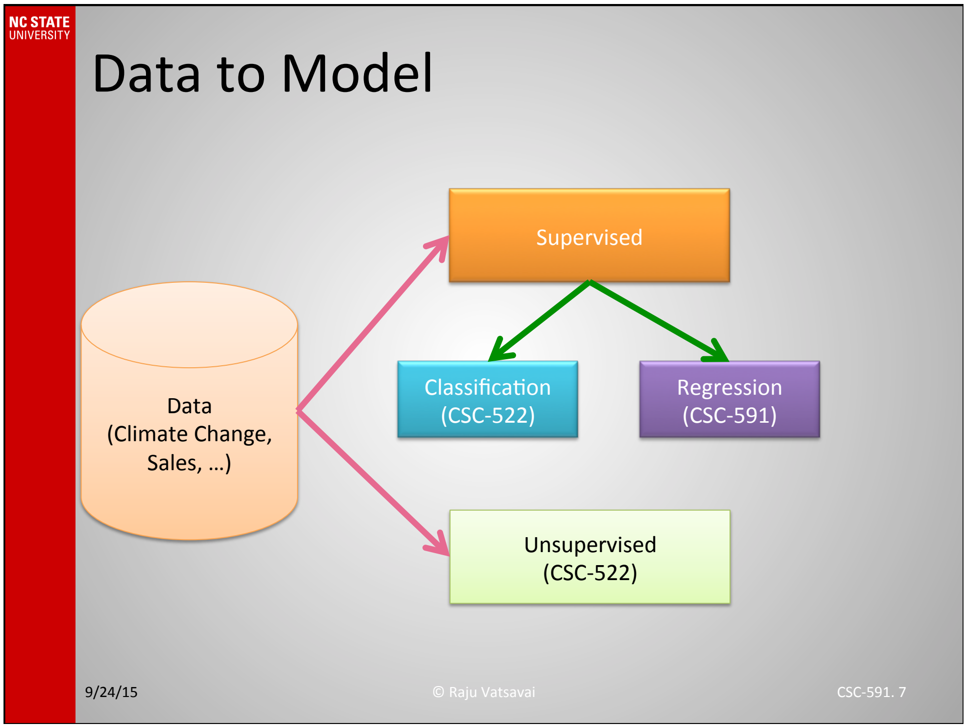
Phenomena to Data

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Data to Model
Supervised
Data
(Climate Change,
Sales, …)
Classification
(CSC-522)
Regression
(CSC-591)
Unsupervised
(CSC-522)
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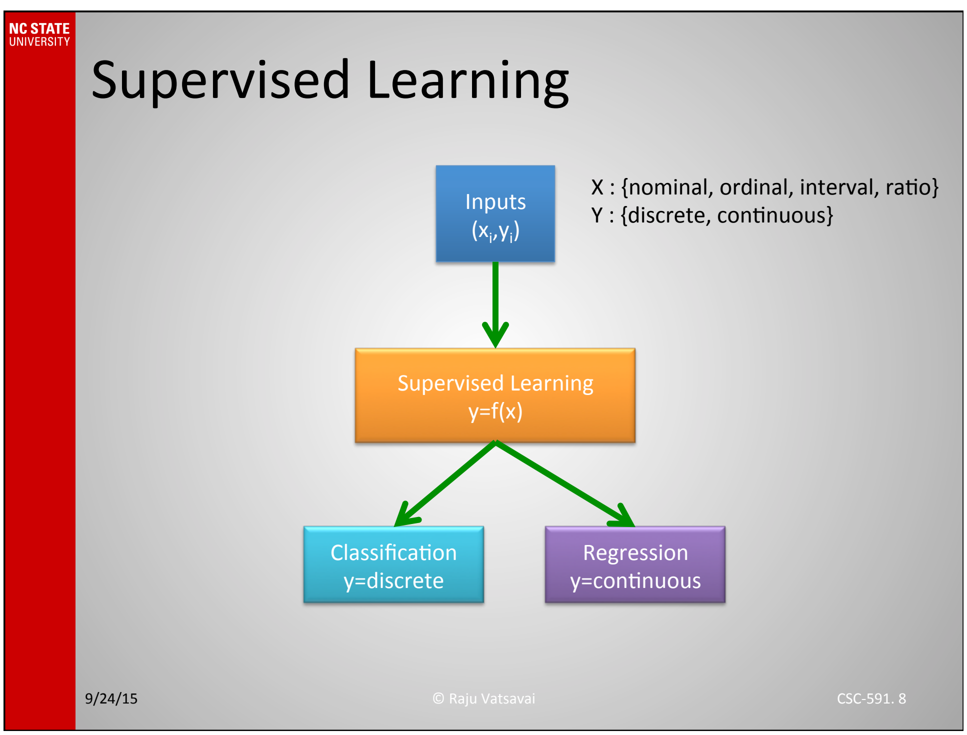
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Supervised Learning
Inputs
(x_i, y_i)
X : {nominal, ordinal, interval, ratio}
Y : {discrete, continuous}
Supervised Learning
y=f(x)
Classification
y=discrete
Regression
y=continuous
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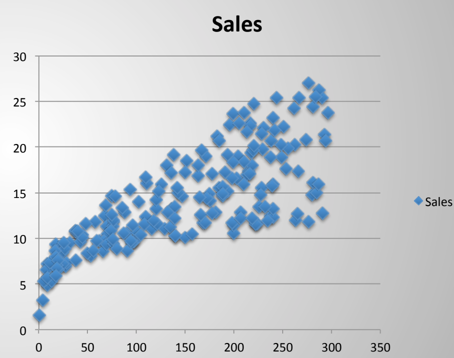
Ex: Advertising Data

TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	58.5	18.5
180.8	10.8	58.4	12.9
8.7	48.9	75	7.2
57.5	32.8	23.5	11.8
120.2	19.6	11.6	13.2
8.6	2.1	1	4.8
199.8	2.6	21.2	10.6
66.1	5.8	24.2	8.6
214.7	24	4	17.4
23.8	35.1	65.9	9.2



Is there a relationship between advertising budget (TV) and sales?

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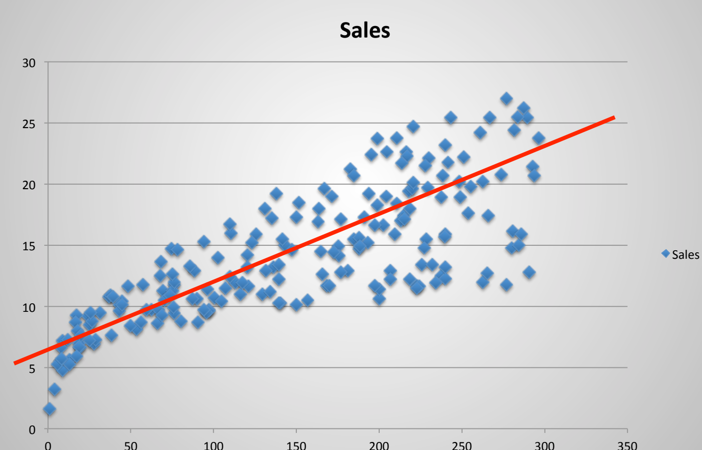
Can We Establish y = f(x)



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Regression

- Regression is a method for studying the relationship between response variable Y and covariates X.
  - The covariate is also called predictor variable or explanatory variable or a feature
- The term "regression" is due to Sir Francis Galton (1822-1911) who noticed that tall and short men tend to have their sons with heights closer to the mean. He called this "regression towards the mean"

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Simple Linear Regression

- SLR studies the relationship between a response variable *y* and a single explanatory variable *x*.
- SLR assumes that for each value of x the observed values of the response variable y are Normally distributed with a mean (μ_y) that depends on *x*.
  - The mean of response variable, μ_y changes as *x* changes. The means all lie on a straight line. That is, $\mu_y = \beta_0 + \beta_1 x$.
  - Individual responses of *y* with the same *x* vary according to a Normal distribution. These Normal distributions all have the same standard deviation.

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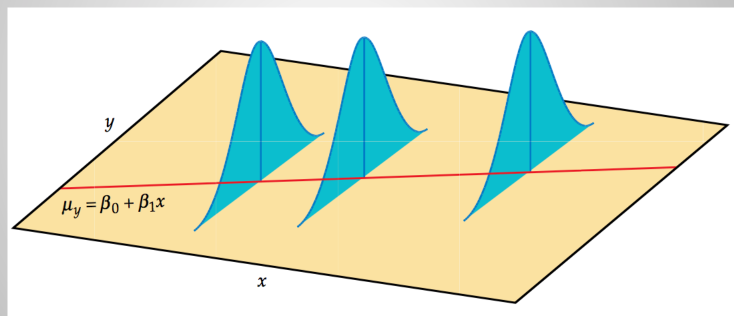
Simple Linear Regression

- The SLR model ($\mu_y = \beta_0 + \beta_1 x$), with intercept β_0 and slope β_1, assumes that all means (μ_y) lie on a line when plotted against *x*. This is the population regression line.



The statistical model for linear regression; the mean response is a straight-line function of the explanatory variable.

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Marginal Distributions

- Sales Data

TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	58.5	18.5
180.8	10.8	58.4	12.9
8.7	48.9	75	7.2
57.5	32.8	23.5	11.8
120.2	19.6	11.6	13.2
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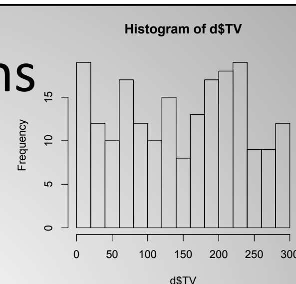

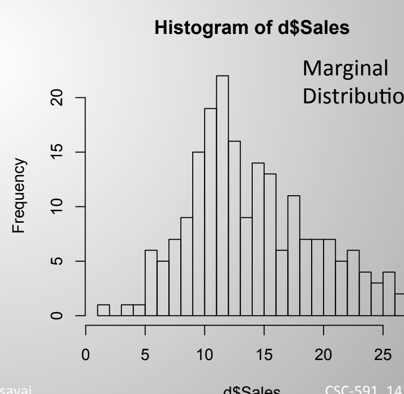

Marginal Distributions

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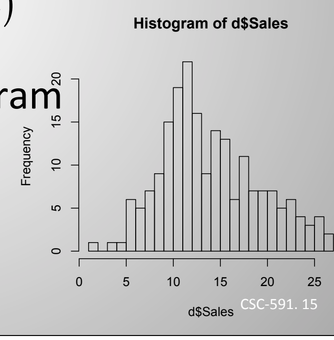
Least Squares

- How to find "middle" via least squares?
- Let Y_i be sales, i = 1..200, then the middle μ is the one that minimizes

$$\sum_i^n \left(Y_i - \mu\right)^2$$



- This is the center of histogram
- Then, $\mu = \bar{Y}$

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Show that $\mu = \bar{Y}$

Proof is demonstrated in the class.
You should try it as practice question.

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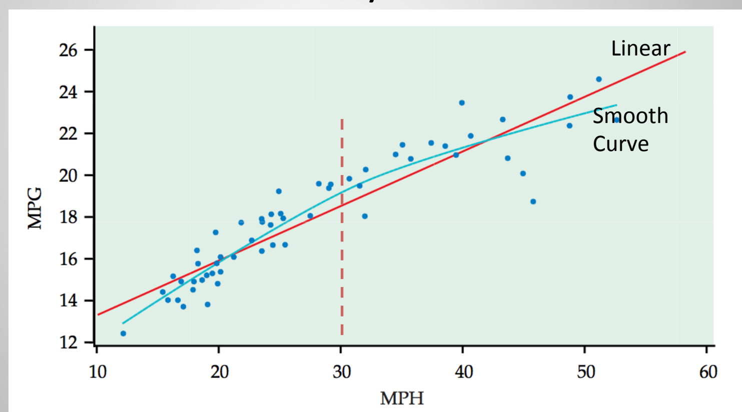
Relationship is Approximate Linear

- Consider the relationship between car driven speed and fuel efficiency



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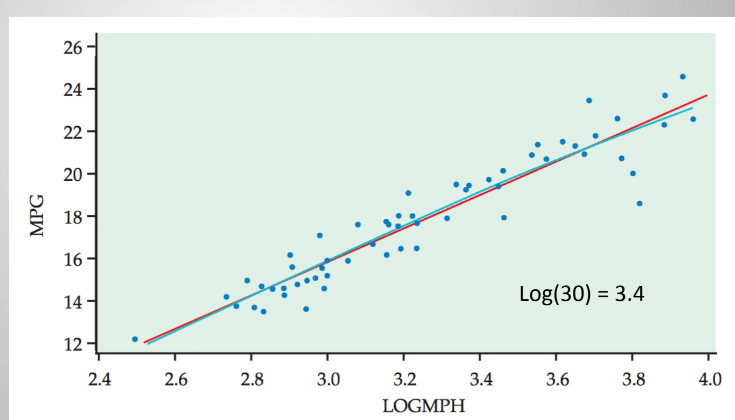
Options

- Confine SLR to a region (e.g., up to 30MPH)
- Transformation (e.g., log)



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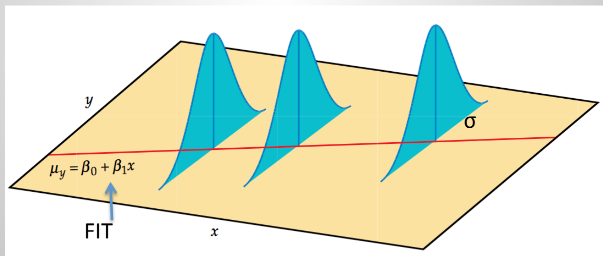
SLR Model

- Population regression gives mean value. However, we can't observe this line on sample data. The statistical model for linear regression consists of population regression line and a description of the variation of *y* about the line. That is, data = fit + residual. The residual part represents deviations of the data from the line of population means. We assume that these deviations are Normally distributed with standard deviation σ.



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SLR Model

SIMPLE LINEAR REGRESSION MODEL

Given n observations of the explanatory variable x and the response variable y,

$$(x_1, y_1),\ (x_2, y_2), \ldots,\ (x_n, y_n)$$

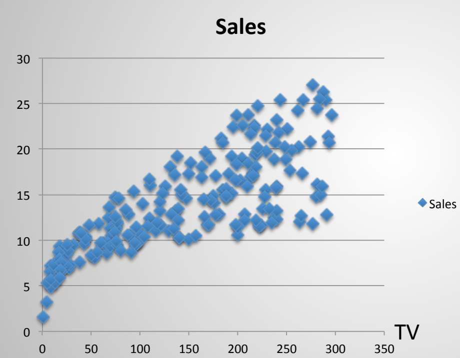
the **statistical model for simple linear regression** states that the observed response y_i when the explanatory variable takes the value x_i is

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

Here $\beta_0 + \beta_1 x_i$ is the mean response when $x = x_i$. The deviations ϵ_i are assumed to be independent and Normally distributed with mean 0 and standard deviation σ.

The parameters of the model are β_0, β_1, and σ.

Correlation



The correlation measures the direction and strength of the relationship between two quantitative variables, written as *r.*

$$r = \frac{1}{n-1}\sum\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

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Regression Parameters

- Least-squares line:

$$\hat{y} = b_0 + b_1 x$$

- Intercept

$$b_0 = \bar{y} - b_1\bar{x}$$

- Slope

$$b_1 = r\frac{s_y}{s_x}$$

- Residual, e_i = observed response – predicted response

$$e_i = y_i - b_0 - b_1 x_i$$

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Regression Parameters

- For SLR, the estimate of σ^2 is the average of squared residual

$$s^2 = \frac{\sum e_i^2}{n-2} = \frac{\sum (y_i - \hat{y}_i)^2}{n-2}$$

- The quantity (n – 2) is called the degrees of freedom for s^2
- The estimate of σ is given by $s = \sqrt{s^2}$

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Properties

- Sum of residuals is 0
- Sum of squared residuals is minimum (this is the constraint to be satisfied in deriving least squares estimators of the regression parameters)
- Sum of the observed values of Y_i equals sum of the fitted values $\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} \hat{Y}_i$
- Sum of the weighted residuals is zero when residual in the i^{th} trail is weighted by x_i

$$\sum_{i=1}^{n} X_i e_i = 0 \qquad \text{Likewise,} \qquad \sum_{i=1}^{n} \hat{Y}_i e_i = 0$$

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Example

- Consider linear regression model with μ_y = 40.5 – 2.5*x* and σ = 2.0
  - What is the slope of the population regression line
  - What is y when x = 10?
  - Between what two values would approximately 95% of the observed responses, *y*, fall when x = 10?

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Acknowledgements

- G. James, et. al., Moore, et. al. Caffo, et. al.