CSC 503 Homework Assignment 5

Due September 22, 2014

September 15, 2014

1. Use the predicates

$$\begin{aligned} F(x,y): &\quad x \text{ is the father of } y \\ M(x,y): &\quad x \text{ is the mother of } y \\ H(x,y): &\quad x \text{ is the husband of } y \\ S(x,y): &\quad x \text{ is the sister of } y \\ B(x,y): &\quad x \text{ is the brother of } y \end{aligned}$$

and the constant (nullary function) symbols

$$\begin{aligned} j: &\quad \text{John} \\ m: &\quad \text{Mary} \end{aligned}$$

to translate the following English sentences into predicate logic. You are not allowed to use any predicate, function, or constant symbols other than the above.

(a) [4 points] Everyone has a mother.
Answer

$$\forall y \exists x (M(x,y))$$

(b) [4 points] Everyone has a father and a mother.
Answer

$$\forall z \exists x \exists y (F(x,z) \wedge M(y,z))$$

(c) [4 points] Whoever has a mother has a father.
Answer

$$\forall x (\exists y (M(y,x)) \rightarrow (\exists z (F(z,x))))$$

(d) [4 points] John is a grandfather
Answer

$$\exists x, y ((F(x,y) \vee M(x,y)) \wedge F(j,x))$$

(e) [4 points] All fathers are parents
Answer

$$\forall x, y (F(x,y) \rightarrow (F(x,y) \vee M(x,y)))$$

(f) [4 points] All husbands are spouses.
Answer

$$\forall x, y (H(x,y) \rightarrow (H(x,y) \vee H(y,x)))$$

(g) [4 points] No uncle is an aunt.
Answer

$$\neg\exists x\exists y\exists z(((F(y,x)\wedge B(z,y))\vee(M(y,x)\wedge B(z,y)))\wedge((F(y,x)\wedge S(z,y))\vee(M(y,x)\wedge S(z,y))))$$

(h) [4 points] Nobody's grandmother is anybody's father.
Answer

$$\forall x,y,z,w(((M(z,x)\vee F(z,x))\wedge M(y,z))\rightarrow\neg F(y,w))$$

(i) [4 points] If Mary is her own mother, then she is her own grandmother.
Answer

$$M(m,m)\rightarrow\exists x((M(x,m)\vee F(x,m))\wedge M(m,x))$$

(j) [4 points] John's parents are husband and wife.
Answer

$$\exists x,y((F(y,j)\wedge M(x,j))\wedge H(y,x))$$

2. [20 points] Using only the basic natural deduction rules, find a proof for

$$\forall x\forall y(P(y)\rightarrow Q(x))\vdash\exists yP(y)\rightarrow\forall xQ(x).$$

Answer

1	$\forall x\forall y(P(y)\rightarrow Q(x))$	premise
2	$\exists yP(y)$	assumption
3	x_0	assumption
4	$\forall y(P(y)\rightarrow Q(x_0))$	∀e, 1
5	y_0 $P(y_0)$	assumption
6	$P(y_0)\rightarrow Q(x_0)$	∀e, 4
7	$Q(x_0)$	→e, 5, 6
8	$Q(x_0)$	∃e, 2, 5–7
9	$\forall x(Q(x))$	∀i, 3–8
10	$\exists yP(y)\rightarrow\forall xQ(x)$	→i, 2–9

Note:
In step 4 we are eliminating $\forall x$
In step 6 we are eliminating $\forall y$
In step 8 we are eliminating $\exists y$
In step 9 we are introducing $\forall x$

3. [20 points] Find a proof for

$$\exists x\forall y(P(x)\vee\neg Q(y))\vdash\forall y\exists x(P(x)\vee\neg Q(y))).$$

Answer

1	$\exists x \forall y (P(x) \vee \neg Q(y))$	premise
2	y_0	assumption
3	x_0 $\forall y (P(x_0) \vee \neg Q(y))$	assumption
4	$(P(x_0) \vee \neg Q(y_0))$	$\forall$e, 3
5	$\exists x (P(x) \vee \neg Q(y_0))$	$\exists$i, 4
6	$\exists x (P(x) \vee \neg Q(y_0))$	$\exists$e, 1, 3–5
7	$\forall y \exists x (P(x) \vee \neg Q(y))$	$\forall$i, 2–6

Note:

In step 4 we are eliminating $\forall y$

In step 5 we are introducing $\exists x$

In step 6 we are eliminating $\exists x$

In step 7 we are introducing $\forall y$

4. [20 points] Find a proof for

$$\forall x P(a, x, x), \forall z \forall y \forall x (P(x, y, z) \rightarrow P(f(f(x)), y, f(z))) \vdash P(f(f(a)), a, f(a)).$$

Answer

1	$\forall x P(a, x, x)$	premise
2	$\forall z \forall y \forall x (P(x, y, z) \rightarrow P(f(f(x)), y, f(z)))$	premise
3	$P(a, a, a)$	$\forall$e, 1
4	$\forall y \forall x (P(x, y, a) \rightarrow P(f(f(x)), y, f(a)))$	$\forall$e, 2
5	$\forall x (P(x, a, a) \rightarrow P(f(f(x)), a, f(a)))$	$\forall$e, 4
6	$P(a, a, a) \rightarrow P(f(f(a)), a, f(a))$	$\forall$e, 5
7	$P(f(f(a)), a, f(a))$	$\rightarrow$e, 3, 6

Note:

In step 3 we are eliminating $\forall x$

In step 4 we are eliminating $\forall z$

In step 5 we are eliminating $\forall y$

In step 6 we are eliminating $\forall x$