CSC 503 Homework Assignment 1

Due September 8, 2014

August 25, 2014

1. Use $\neg$, $\rightarrow$, $\wedge$, and $\vee$ to express the following declarative sentences in propositional logic over atoms p, q, r, etc. First state what your propositional atoms mean in self-contained English sentences, then give the translation of the sentence.

   (a) [10 points] If Alice flew to San Francisco, then Alice was not in Raleigh yesterday.
   Answer

   p : Alice flew to San Francisco.
   q : Alice was in Raleigh yesterday.
   $p \rightarrow \neg q$

   (b) [10 points] Either Bob called his father after he became sunburned or Bob's mother called him.
   Answer

   p : Bob Called his father after he became sunburned
   q : Bob's mother called him (Bob's father).
   $p \vee q$

   (c) [10 points] Carol's pet zebra is black and white.
   Answer

   p : Carol's pet zebra is black
   q : Carol's pet zebra is white
   $p \wedge q$

2. [10 points] Why is the expression $p \vee q \wedge r$ problematic? Justify your answer.

   Answer

   The given expression is problematic since it can be interpreted in multiple ways and both the interpretations have different truth values as shown below.

   1. $(p \vee q) \wedge r$

p	q	r	$(p \vee q)$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

2. $p \vee (q \wedge r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

3. [10 points] Compute the complete truth table of the formula $((p \to p) \to q) \to q$.

   Answer

   Truth table of the formula $((p \to p) \to q) \to q$ is as follows

p	q	$p \to p$	$(p \to p) \to q$	$((p \to p) \to q) \to q$
T	T	T	T	T
T	F	T	F	T
F	T	T	T	T
F	F	T	F	T

4. [10 points] Show that the sequent $(p \vee q) \to r, \neg r \wedge \neg q \vdash \neg p \to q$ is not valid by finding a valuation in which the truth values of the formulas to the left of $\vdash$ are T and the truth value of the formula to the right of $\vdash$ is F.

   Answer

   This sequent can be proved wrong using the natural deduction as shown below

1	$(p \vee q) \to r$	premise
2	$\underline{\neg r \wedge \neg q}$	premise
3	$\neg r$	$\wedge e_1$, 2
4	$\neg q$	$\wedge e_2$, 2
5	$\quad \underline{p \vee q}$	assumption
6	$\quad r$	$\to$e, 1, 5
7	$\quad \bot$	$\neg$e, 3, 6
8	$\neg(p \vee q)$	$\neg$i, 5–7
9	$\quad \underline{p}$	assumption
10	$\quad p \vee q$	$\vee i_1$, 9
11	$\quad \bot$	$\neg$e, 8, 10
12	$\neg p$	$\neg$i, 9–11
13	$\quad \underline{\neg p \to q}$	assumption
14	$\quad q$	$\to$e, 12, 13
15	$\quad \bot$	$\neg$e, 4, 14
16	$\neg(\neg p \to q)$	$\neg$i, 13–15

This shows that when premises $(p \vee q) \to r, \neg r \wedge \neg q$ are true then $\neg(\neg p \to q)$ is also true. Hence the expected conclusion $\neg p \to q$ has to be false. Thus for an instance, for a set of values of (p, q, r) as (T, T, T), the L.H.S. is of the sequent is true and the R.H.S. is false. Also notice that this is only one of the sample solution and the solution is not affected for any value of r.

5. Prove the validity of the following sequents. Use only the basic rules of natural deduction (no derived rules).

   (a) [20 points] $\vdash (p \vee q) \to (s \to ((p \vee q) \wedge s))$
   Answer

1	$\quad \underline{p \vee q}$	assumption
2	$\quad\quad \underline{s}$	assumption
3	$\quad\quad (p \vee q) \wedge s$	$\wedge$i, 1, 2
4	$\quad s \to ((p \vee q) \wedge s)$	$\to$i, 2–3
5	$(p \vee q) \to (s \to ((p \vee q) \wedge s))$	$\to$i, 1–4

   (b) [20 points] $q \to \neg p \vdash ((\neg p \wedge q) \to q) \wedge (q \to (\neg p \wedge q))$
   Answer

1	$q \to \neg p$	premise
2	$\quad \neg p \wedge q$	assumption
3	$\quad q$	$\wedge\text{e}_2$, 2
4	$(\neg p \wedge q) \to q$	$\to$i, 2–3
5	$\quad q$	assumption
6	$\quad \neg p$	$\to$e, 1, 5
7	$\quad \neg p \wedge q$	$\wedge$i, 5, 6
8	$q \to (\neg p \wedge q)$	$\to$i, 5–7
9	$((\neg p \wedge q) \to q) \wedge (q \to (\neg p \wedge q))$	$\wedge$i, 4, 8