CSC 503 Homework Assignment 6

Due October 1, 2014

September 24, 2014

The algorithm presented in lecture to calculate the most general unifier of a set S consists of the following steps.

- Step 0:
 - Set $S_0 = S$
 - Set $\sigma_0 = \epsilon$
- Step k + 1:
 - If $|S_k| = 1$, return $\sigma_0 \cdots \sigma_k$
 - If the disagreement set $D(S_k)$ contains both a variable v and a term t in which v does not occur, then
 - * Choose least such pair
 - * Set $\sigma_{k+1} = \{t/v\}$
 - * Set $S_{k+1} = S_k \sigma_{k+1}$
 - * Proceed to step k+2
 - Otherwise, announce that S has no unifier
- 1. Apply the unification algorithm to each of the following sets. For each set, at each step i, show (a) the disagreement of S_i , (b) the substitution σ_i if there is one, or an explanation why there is no unifying substitution, (c) the result S_{i+1} of applying σ_i to S_i . If the set unifies, show also (d) the overall substitution $\sigma_0 \dots \sigma_k$ expressed as a single substitution, not as a composition, and (e) the formula resulting from applying the most general unifier to the expressions in the set.

In the following expressions, assume that a, b, c are constant symbols, f, g, h are function symbols, P, Q are predicate symbols, and u, v, w, x, y, z are variable symbols.

(a) [25 points]
$$S = \{P(f(x), y), P(y, f(z))\}$$

Answer

Initializing σ_0 to $\{\}$
 $S_0 = \{P(f(x), y), P(y, f(z))\}$
 $D(S_0) = \{f(x), y\}$
 $\sigma_1 = \{f(x)/y\}$
 $S_1 = \{P(f(x), f(x)), P(f(x), f(z))\}$
 $D(S_1) = \{x, z\}$
 $\sigma_2 = \{x/z\}$
 $S_2 = \{P(f(x), f(x))\}$
 $|S_2| = 1$
 $\sigma = \sigma_0 \cdot \sigma_1 \cdot \sigma_2$
 $\sigma = \{\} \cdot \{f(x)/y\} \cdot \{x/z\}$
 $\sigma = \{f(x)/y\} \cdot \{x/z\}$
 $\sigma = \{f(x)/y, x/z\}$

Unification is feasible for above σ .

(b) [25 points]
$$S = \{P(f(x), f(f(y))), P(f(y), f(g(z)))\}$$

Answer

Initializing
$$\sigma_0$$
 to $\{\}$

$$S_0 = \{P(f(x), f(f(y))), P(f(y), f(g(z)))\}$$

$$D(S_0) = \{x, y\}$$

$$\sigma_1 = \{x/y\}$$

$$S_1 = \{P(f(x), f(f(x))), P(f(x), f(g(z)))\}$$

$$D(S_1) = \{f(x), g(z)\}$$

There is no substitution for making 'f' and 'g' equal and hence we cannot unify these formulas.

Unification is not feasible.

(c) [25 points] $S = \{P(x, f(f(x))), P(y, y)\}$

Answer

Initializing
$$\sigma_0$$
 to $\{\}$
 $S_0 = \{P(x, f(f(x))), P(y, y)\}$
 $D(S_0) = \{x, y\}$
 $\sigma_1 = \{x/y\}$
 $S_1 = \{P(x, f(f(x))), P(x, x)\}$

No substitution is possible for this as we cannot substitute f(f(x)) for x as f(f(x)) contains x.

Unification is not feasible.

 $D(S_1) = \{x, f(f(x))\}\$

(d) [25 points] $S = \{Q(f(g(v), a), h(w, b)), Q(f(x, y), h(w, w)), Q(f(g(v), a), h(v, b))\}$

Answer

Initializing
$$\sigma_0$$
 to $\{\}$

$$S_0 = \{Q(f(g(v), a), h(w, b)), Q(f(x, y), h(w, w)), Q(f(g(v), a), h(v, b))\}$$

$$D(S_0) = \{g(v), x\}$$

$$\sigma_1 = \{g(v)/x\}$$

$$S_1 = \{Q(f(g(v), a), h(w, b)), Q(f(g(v), y), h(w, w)), Q(f(g(v), a), h(v, b))\}$$

$$D(S_1) = \{a, y\}$$

$$\sigma_2 = \{a/y\}$$

$$S_2 = \{Q(f(g(v), a), h(w, b)), Q(f(g(v), a), h(w, w)), Q(f(g(v), a), h(v, b))\}$$

$$D(S_2) = \{w, v\}$$

$$\sigma_3 = \{v/w\}$$

$$S_3 = \{Q(f(g(v), a), h(v, b)), Q(f(g(v), a), h(v, v)))\}$$

$$D(S_3) = \{v, b\}$$

$$\sigma_5 = \{b/v\}$$

$$S_5 = \{Q(f(g(b), a), h(b, b)))\}$$

$$|S_5| = 1$$

$$\sigma = \sigma_0 \cdot \sigma_1 \cdot \sigma_2 \cdot \sigma_3 \cdot \sigma_4$$

$$\sigma = \{\} \cdot \{g(v)/x\} \cdot \{a/y\} \cdot \{v/w\} \cdot \{b/v\}$$

$$\begin{split} \sigma &= \{g(v)/x\} \cdot \{a/y\} \cdot \{v/w\} \cdot \{b/v\} \\ \sigma &= \{g(v)/x, a/y\} \cdot \{v/w\} \cdot \{b/v\} \\ \sigma &= \{g(v)/x, a/y, v/w\} \cdot \{b/v\} \\ \sigma &= \{g(b)/x, a/y, b/w, b/v\} \end{split}$$

Unification is feasible using the above σ .