CSC 503 Homework Assignment 2

Due September 10, 2014

August 27, 2014

1. The formulas of propositional logic implicitly assume the binding priorities of the logical connectives put forward in Convention 1.3. Make sure that you fully understand those conventions by reinserting all omitted parentheses in the following abbreviated statements.

   (a) [10 points] $\neg\, p \rightarrow \neg\, p \vee q \rightarrow \neg\, r$

   Answer

   i. Highest Precedence to $\neg$.
   $(\neg p) \rightarrow (\neg p) \vee q \rightarrow (\neg r)$

   ii. Next precedence is given to $\vee$ and $\wedge$ equally.
   $(\neg p) \rightarrow ((\neg p) \vee q) \rightarrow (\neg r)$

   iii. Next precedence is given to $\rightarrow$ and it is right associative in nature.
   $(\neg p) \rightarrow (((\neg p) \vee q) \rightarrow (\neg r))$

   (b) [10 points] $r \rightarrow q \rightarrow p \vee (q \rightarrow \neg\, p \wedge r)$

   Answer

   i. Highest Precedence to $\neg$.
   $r \rightarrow q \rightarrow p \vee (q \rightarrow (\neg p) \wedge r)$

   ii. Next precedence is given to $\vee$ and $\wedge$ equally.
   $r \rightarrow q \rightarrow (p \vee (q \rightarrow ((\neg p) \wedge r)))$

   iii. Next precedence is given to $\rightarrow$ and it is right associative in nature.
   $r \rightarrow (q \rightarrow (p \vee (q \rightarrow ((\neg p) \wedge r))))$

2. [10 points] List all subformulas of the formula $(s \rightarrow r \vee l) \wedge (\neg q \vee r) \rightarrow (\neg(p \rightarrow s) \rightarrow r)$.

   Answer

   The subformulas are as follows.

   (a) l

   (b) p

   (c) q

   (d) r

   (e) s

   (f) $(r \vee l)$

   (g) $s \rightarrow (r \vee l)$

   (h) $\neg q$

   (i) $(\neg q \vee r)$

   (j) $(s \rightarrow (r \vee l) \wedge (\neg q \vee r))$

   (k) $p \rightarrow s$

   (l) $\neg(p \rightarrow s)$

(m) $(\neg(p \to s) \to r)$

(n) $((s \to (r \vee l)) \wedge (\neg q \vee r)) \to (\neg(p \to s) \to r)$.

3. [10 points] A formula is valid iff it computes T for all its valuations; it is satisfiable iff it computes T for at least one of its valuations. Is the formula $(p \vee \neg q) \wedge (q \vee \neg r)$ valid? Is it satisfiable?

Answer

The truth table for the formula $(p \vee \neg q) \wedge (q \vee \neg r)$ is as given below

p	q	r	$\neg q$	$\neg r$	$p \vee \neg q$	$q \vee \neg r$	$(p \vee \neg q) \wedge (q \vee \neg r)$
T	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	F	T	F	F
T	F	F	T	T	T	T	T
F	T	T	F	F	F	T	F
F	T	F	F	T	F	T	F
F	F	T	T	F	T	F	F
F	F	F	T	T	T	T	T

From the table we can see that the truth value of the formula does not compute T for all the evaluations, but it does for a few of them and hence the formula is **Satisfiable**.

4. [20 points] Does $\models (\neg(p \to q) \to (p \vee (\neg p \to q))) \to p$ hold? Justify your answer.

Answer

p	q	$p \to q$	$\neg(p \to q)$	$\neg p$	$\neg p \to q$	$p \vee (\neg p \to q)$	$\neg(p \to q) \to (p \vee (\neg p \to q))$
T	T	T	F	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T
F	F	T	F	T	F	F	T

Table (a)

p	$\neg(p \to q) \to (p \vee (\neg p \to q))$	$(\neg(p \to q) \to (p \vee (\neg p \to q))) \to p$
T	T	T
F	T	F

Table (b)

From the above truth tables we can see that the formula $\models (\neg(p \to q) \to (p \vee (\neg p \to q))) \to p$ depends on the p and hence the formula **Does Not Hold**.

5. Prove the validity of the following sequents. Use only the basic rules of natural deduction (no derived rules).

(a) [20 points] $r \to q, \neg r \to p, \neg q \to \neg p \vdash q$

Answer

1	$r \to q$	premise
2	$\neg r \to p$	premise
3	$\neg q \to \neg p$	premise
4	$\neg q$	assumption
5	$\neg p$	$\to$e, 3, 4
6	$\neg r$	assumption
7	p	$\to$e, 2, 6
8	$\perp$	$\neg$e, 5, 7
9	$\neg\neg r$	$\neg$i, 6–8
10	r	$\neg\neg$e, 9
11	q	$\to$e, 1, 10
12	$\perp$	$\neg$e, 4, 11
13	$\neg\neg q$	$\neg$i, 4–12
14	q	$\neg\neg$e, 13

(b) [20 points] $p \to (q \vee r), \neg q, \neg r \vdash \neg p$

Answer

1	$p \to (q \vee r)$	premise
2	$\neg q$	premise
3	$\neg r$	premise
4	p	assumption
5	$q \vee r$	$\to$e, 1, 4
6	q	assumption
7	$\perp$	$\neg$e, 2, 6
8	r	assumption
9	$\perp$	$\neg$e, 3, 8
10	$\perp$	$\vee$e, 5, 6–7, 8–9
11	$\neg p$	$\neg$i, 4–10