CSC 503 Homework Assignment 6

Due October 1, 2014

September 24, 2014

The algorithm presented in lecture to calculate the most general unifier of a set S consists of the following steps.

- Step 0:
  - Set $S_0 = S$
  - Set $\sigma_0 = \epsilon$
- Step $k+1$:
  - If $|S_k| = 1$, return $\sigma_0 \cdots \sigma_k$
  - If the disagreement set $D(S_k)$ contains both a variable v and a term t in which v *does not occur*, then
    - Choose least such pair
    - Set $\sigma_{k+1} = \{t/v\}$
    - Set $S_{k+1} = S_k \sigma_{k+1}$
    - Proceed to step $k+2$
  - Otherwise, announce that S has no unifier

1. Apply the unification algorithm to each of the following sets. For each set, at each step i, show (a) the disagreement of S_i, (b) the substitution σ_i if there is one, or an explanation why there is no unifying substitution, (c) the result S_{i+1} of applying σ_i to S_i. If the set unifies, show also (d) the overall substitution $\sigma_0 \ldots \sigma_k$ expressed as a single substitution, not as a composition, and (e) the formula resulting from applying the most general unifier to the expressions in the set.

   In the following expressions, assume that a, b, c are constant symbols, f, g, h are function symbols, P, Q are predicate symbols, and u, v, w, x, y, z are variable symbols.

   (a) [25 points] $S = \{P(f(x), y), P(y, f(z))\}$
   Answer
   Initializing σ_0 to $\{\}$
   $S_0 = \{P(f(x), y), P(y, f(z))\}$
   $D(S_0) = \{f(x), y\}$

   $\sigma_1 = \{f(x)/y\}$
   $S_1 = \{P(f(x), f(x)), P(f(x), f(z))\}$
   $D(S_1) = \{x, z\}$

   $\sigma_2 = \{x/z\}$
   $S_2 = \{P(f(x), f(x))\}$
   $|S_2| = 1$

   $\sigma = \sigma_0 \cdot \sigma_1 \cdot \sigma_2$
   $\sigma = \{\} \cdot \{f(x)/y\} \cdot \{x/z\}$
   $\sigma = \{f(x)/y\} \cdot \{x/z\}$
   $\sigma = \{f(x)/y, x/z\}$
   Unification is feasible for above σ.

(b) [25 points] $S = \{P(f(x), f(f(y))), P(f(y), f(g(z)))\}$

Answer

Initializing σ_0 to $\{\}$

$S_0 = \{P(f(x), f(f(y))), P(f(y), f(g(z)))\}$

$D(S_0) = \{x, y\}$

$\sigma_1 = \{x/y\}$

$S_1 = \{P(f(x), f(f(x))), P(f(x), f(g(z)))\}$

$D(S_1) = \{f(x), g(z)\}$

There is no substitution for making 'f' and 'g' equal and hence we cannot unify these formulas.

Unification is not feasible.

(c) [25 points] $S = \{P(x, f(f(x))), P(y, y)\}$

Answer

Initializing σ_0 to $\{\}$

$S_0 = \{P(x, f(f(x))), P(y, y)\}$

$D(S_0) = \{x, y\}$

$\sigma_1 = \{x/y\}$

$S_1 = \{P(x, f(f(x))), P(x, x)\}$

$D(S_1) = \{x, f(f(x))\}$

No substitution is possible for this as we cannot substitute $f(f(x))$ for x as $f(f(x))$ contains x.

Unification is not feasible.

(d) [25 points] $S = \{Q(f(g(v), a), h(w, b)), Q(f(x, y), h(w, w)), Q(f(g(v), a), h(v, b))\}$

Answer

Initializing σ_0 to $\{\}$

$S_0 = \{Q(f(g(v), a), h(w, b)), Q(f(x, y), h(w, w)), Q(f(g(v), a), h(v, b))\}$

$D(S_0) = \{g(v), x\}$

$\sigma_1 = \{g(v)/x\}$

$S_1 = \{Q(f(g(v), a), h(w, b)), Q(f(g(v), y), h(w, w)), Q(f(g(v), a), h(v, b))\}$

$D(S_1) = \{a, y\}$

$\sigma_2 = \{a/y\}$

$S_2 = \{Q(f(g(v), a), h(w, b)), Q(f(g(v), a), h(w, w)), Q(f(g(v), a), h(v, b))\}$

$D(S_2) = \{w, v\}$

$\sigma_3 = \{v/w\}$

$S_3 = \{Q(f(g(v), a), h(v, b)), Q(f(g(v), a), h(v, v)))\}$

$D(S_3) = \{v, b\}$

$\sigma_5 = \{b/v\}$

$S_5 = \{Q(f(g(b), a), h(b, b)))\}$

$|S_5| = 1$

$\sigma = \sigma_0 \cdot \sigma_1 \cdot \sigma_2 \cdot \sigma_3 \cdot \sigma_4$

$\sigma = \{\} \cdot \{g(v)/x\} \cdot \{a/y\} \cdot \{v/w\} \cdot \{b/v\}$

$\sigma = \{g(v)/x\} \cdot \{a/y\} \cdot \{v/w\} \cdot \{b/v\}$
$\sigma = \{g(v)/x, a/y\} \cdot \{v/w\} \cdot \{b/v\}$
$\sigma = \{g(v)/x, a/y, v/w\} \cdot \{b/v\}$
$\sigma = \{g(b)/x, a/y, b/w, b/v\}$

Unification is feasible using the above σ.