

# CSC 503 Homework Assignment 5

Due September 22, 2014

September 15, 2014

1. Use the predicates

$F(x, y) :$   $x$  is the father of  $y$   
 $M(x, y) :$   $x$  is the mother of  $y$   
 $H(x, y) :$   $x$  is the husband of  $y$   
 $S(x, y) :$   $x$  is the sister of  $y$   
 $B(x, y) :$   $x$  is the brother of  $y$

and the constant (nullary function) symbols

$j :$  John  
 $m :$  Mary

to translate the following English sentences into predicate logic. You are not allowed to use any predicate, function, or constant symbols other than the above.

(a) [4 points] Everyone has a mother.

**Answer**

$$\forall y \exists x (M(x, y))$$

(b) [4 points] Everyone has a father and a mother.

**Answer**

$$\forall z \exists x \exists y (F(x, z) \wedge M(y, z))$$

(c) [4 points] Whoever has a mother has a father.

**Answer**

$$\forall x (\exists y (M(y, x)) \rightarrow (\exists z (F(z, x))))$$

(d) [4 points] John is a grandfather

**Answer**

$$\exists x, y ((F(x, y) \vee M(x, y)) \wedge F(j, x))$$

(e) [4 points] All fathers are parents

**Answer**

$$\forall x, y (F(x, y) \rightarrow (F(x, y) \vee M(x, y)))$$

(f) [4 points] All husbands are spouses.

**Answer**

$$\forall x, y (H(x, y) \rightarrow (H(x, y) \vee H(y, x)))$$

(g) [4 points] No uncle is an aunt.

**Answer**

$$\neg \exists x \exists y \exists z (((F(y, x) \wedge B(z, y)) \vee (M(y, x) \wedge B(z, y))) \wedge ((F(y, x) \wedge S(z, y)) \vee (M(y, x) \wedge S(z, y))))$$

(h) [4 points] Nobody's grandmother is anybody's father.

**Answer**

$$\forall x, y, z, w (((M(z, x) \vee F(z, x)) \wedge M(y, z)) \rightarrow \neg F(y, w))$$

(i) [4 points] If Mary is her own mother, then she is her own grandmother.

**Answer**

$$M(m, m) \rightarrow \exists x ((M(x, m) \vee F(x, m)) \wedge M(m, x))$$

(j) [4 points] John's parents are husband and wife.

**Answer**

$$\exists x, y ((F(y, j) \wedge M(x, j)) \wedge H(y, x))$$

2. [20 points] Using only the basic natural deduction rules, find a proof for

$$\forall x \forall y (P(y) \rightarrow Q(x)) \vdash \exists y P(y) \rightarrow \forall x Q(x).$$

**Answer**

1	$\forall x \forall y (P(y) \rightarrow Q(x))$	premise
2	$\exists y P(y)$	assumption
3	$x_0$	assumption
4	$\forall y (P(y) \rightarrow Q(x_0))$	$\forall e, 1$
5	$y_0$	assumption
6	$P(y_0) \rightarrow Q(x_0)$	$\forall e, 4$
7	$Q(x_0)$	$\rightarrow e, 5, 6$
8	$Q(x_0)$	$\exists e, 2, 5-7$
9	$\forall x (Q(x))$	$\forall i, 3-8$
10	$\exists y P(y) \rightarrow \forall x Q(x)$	$\rightarrow i, 2-9$

Note:

In step 4 we are eliminating  $\forall x$

In step 6 we are eliminating  $\forall y$

In step 8 we are eliminating  $\exists y$

In step 9 we are introducing  $\forall x$

3. [20 points] Find a proof for

$$\exists x \forall y (P(x) \vee \neg Q(y)) \vdash \forall y \exists x (P(x) \vee \neg Q(y)).$$

### Answer

1	$\exists x \forall y (P(x) \vee \neg Q(y))$	premise
2	$y_0$	assumption
3	$x_0$   $\forall y (P(x_0) \vee \neg Q(y))$	assumption
4	$(P(x_0) \vee \neg Q(y_0))$	$\forall e, 3$
5	$\exists x (P(x) \vee \neg Q(y_0))$	$\exists i, 4$
6	$\exists x (P(x) \vee \neg Q(y_0))$	$\exists e, 1, 3-5$
7	$\forall y \exists x (P(x) \vee \neg Q(y))$	$\forall i, 2-6$

Note:

In step 4 we are eliminating  $\forall y$

In step 5 we are introducing  $\exists x$

In step 6 we are eliminating  $\exists x$

In step 7 we are introducing  $\forall y$

4. [20 points] Find a proof for

$$\forall x P(a, x, x), \forall z \forall y \forall x (P(x, y, z) \rightarrow P(f(f(x)), y, f(z))) \vdash P(f(f(a)), a, f(a)).$$

### Answer

1	$\forall x P(a, x, x)$	premise
2	$\forall z \forall y \forall x (P(x, y, z) \rightarrow P(f(f(x)), y, f(z)))$	premise
3	$P(a, a, a)$	$\forall e, 1$
4	$\forall y \forall x (P(x, y, a) \rightarrow P(f(f(x)), y, f(a)))$	$\forall e, 2$
5	$\forall x (P(x, a, a) \rightarrow P(f(f(x)), a, f(a)))$	$\forall e, 4$
6	$P(a, a, a) \rightarrow P(f(f(a)), a, f(a))$	$\forall e, 5$
7	$P(f(f(a)), a, f(a))$	$\rightarrow e, 3, 6$

Note:

In step 3 we are eliminating  $\forall x$

In step 4 we are eliminating  $\forall z$

In step 5 we are eliminating  $\forall y$

In step 6 we are eliminating  $\forall x$