Convex Methods for Spacecraft Rendezvous and Docking

Parth Shrotri

***Abstract*— Autonomous spacecraft rendezvous and docking is a problem of increasing relevance as commercial spacecraft operators seek to extend the operational life-span of existing satellites through repair and on-orbit refueling. In the human spaceflight sector, there is increased interest in reducing reliance on human operators during critical phases of the mission. Rendezvous and docking are inherently risky operational phases and, as such, there are several requirements on trajectories for conducting such operations, some of which are non-convex. In this paper, I present a framework for autonomous rendezvous and docking using linearized dynamics and convex optimization techniques to generate open-loop control inputs. I then test the performance of my approach in a non-linear orbital dynamics environment. I demonstrate it's validity as a guidance and control scheme by verifying compliance with all constraints.**

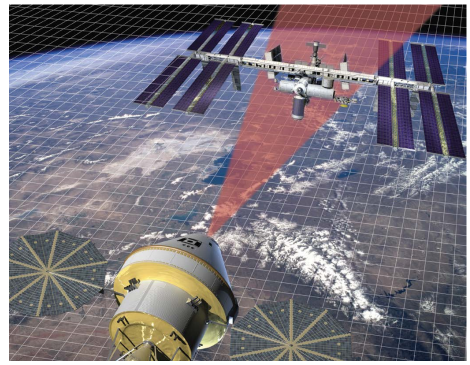

Fig. 1. Illustration of a spacecraft approaching the ISS

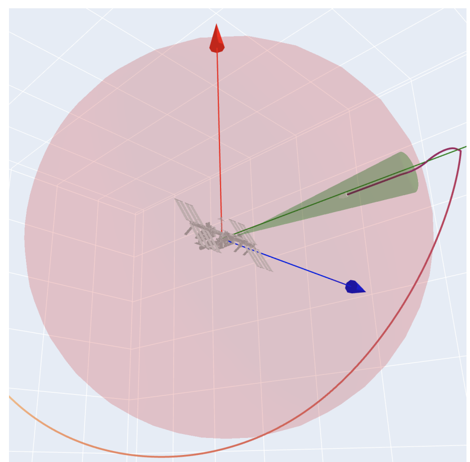

Fig. 2. Illustration of a constrained rendezvous

I. Introduction

Rendezvous and docking are operational phases in spaceflight during which a spacecraft, known as the chaser, approaches and connects with another, known as the leader. This enables crew and cargo resupply to crewed spacecraft, as well as refueling and repair for robotic spacecraft. Orbital rendezvous and docking have been used for several decades, starting during NASA's Project Gemini in the mid-1960s. The first case of autonomous docking using onboard guidance, navigation, and control systems (GN&C) can be found in the Soviet Union's space program in the late 1960s but did not see regular use until the development of the Kurs docking system for the Soyuz spacecraft. In contrast, legacy American spacecraft such as the Gemini and Apollo capsules as well as the Space Shuttle, rendezvous and docking were flown manually by an astronaut. American spacecraft did not incorporate autonomy into their crewed vehicle's design until the late 2010s with SpaceX's Crew/Cargo Dragon and Boeing's Starliner. Figure 1 illustrates the scenario we are discussing: a spacecraft approaching the International Space Station (ISS) using its onboard GN&C system.

Rendezvous and docking are inherently hazardous operations, as bringing two spacecraft in close proximity without careful trajectory design, analysis, and execution can result in a collision resulting in the loss of mission or, in the worst case, the vehicle and crew. These are clearly unacceptable consequences; therefore, modern spacecraft that use autonomous guidance and control must plan these trajectories with safety in mind. Specifically, this means designing trajectories for a system that is non-linear with several strict constraints, some of which are non-convex. An illustration of constrained rendezvous is shown in Figure 2. As the number of missions grows, particularly in the on-orbit refueling and repair sector, it becomes necessary for these spacecraft to perform trajectory planning autonomously with minimal operator input and reliance on Earth-based infrastructure, such as trajectory generation by a centralized command and control system.

This work aims to demonstrate one such approach to solving this problem. I begin by describing the dynamics of the system and the constraints involved. I demonstrate how we can use a linear model to describe the non-linear system for control design. I then demonstrate how convex optimization techniques can be applied to the system to enforce safety constraints. Specifically, I formulate non-convez problems as solutions to a sequential convex program (SCP) [1]. Then I describe my implementation of my proposed solution method. Finally, I use a non-linear orbital dynamics simulation to test and validate that my solution method meets all constraints and produces acceptable results.

A. Related work

The paper entitled "Autonomous Rendezvous and Docking Implementation for Small Satellites Using Model Predictive Control" [2] describes a framework in which constraints are determined based on the phase of the operation: rendezvous, approach, and docking. The rendezvous phase is when the chasing spacecraft repositions itself to align with the docking target. This paper uses a linearized non-convex keep-out constraint to enforce a collision avoidance constraint during the rendezvous phase and line-of-sight (LOS) constraint for the approach and docking phase using a quadratic program. The paper uses a LOS constraint so that a vehicle's image-based relative navigation system always has the docking target in its field of view.

The paper entitled "Autonomous Trajectory Planning for Rendezvous and Proximity Operations by Conic Optimization" [3] only discusses the docking phase. They use a different state constraint, one which uses a defined docking corridor, modeled as a second-order cone. This results in the optimization problem being a second-order cone program. This method is preferred for spacecraft where a docking corridor is explicitly defined in the trajectory requirements for visiting vehicles.

Both papers use the Clohessy-Wiltshire (CW) equations [4] that linearize the relative dynamics of the system, as this produces a linear time-invariant approximation of the relative orbital dynamics.

B. Statement of contributions

This project contributes to solving the problem of autonomous rendezvous and docking by combining the approaches used in [2] and [3]. This work also examines the application of these guidance and control methods in environments with orbital perturbations.

II. Problem Formulation

A. Dynamics

Orbital dynamics are described using the following equation:

$$\ddot{r} = -\frac{\mu r}{\|r\|^3} + a_{\text{perturb}}(r) + u \tag{1}$$

where r is the position vector $[x, y, z]^T$ of the chaser in a planet-centered inertial frame (such as ECI [5]), $a_{\text{perturb}}(r)$ is the acceleration due to perturbing acceleration such as J_2 [6] or other planetary bodies, u is the acceleration control input. μ is the standard gravitational parameter for the planet being orbited. This can be rewritten as a set of first-order differential equations:

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \ddot{r}[0] \\ \ddot{r}[1] \\ \ddot{r}[2] \end{bmatrix} \tag{2}$$

Where $\mathbf{X} = [x, y, z, v_x, v_y, v_z]^T$ represent the position and velocity of a spacecraft in a Cartesian inertial space. The chaser state vector $\mathbf{X}_c$ and the leader state vector $\mathbf{X}_l$ both take the form of $\mathbf{X}$ and propagated by integrating Equation (2) over a given time interval Δt. It is clear that this is a non-linear dynamical system. Since we want to define the state of the chaser relative to the state of the leader, we can define the relative state in the inertial frame: $\mathbf{X}_{\text{rel},c} = \mathbf{X}_c - \mathbf{X}_l$. Thus, the relative orbital dynamics are also given by a set of non-linear equations.

B. Constraints

The problem involves generating a set of control inputs that a chaser spacecraft executes to rendezvous and dock with the leader spacecraft. The resulting chaser state trajectory must satisfy the following constraints, derived from the rendezvous and docking requirements for the ISS [7]. During the rendezvous phase, the chaser shall remain outside $r_{\text{keep-out}}$ meters. Written mathematically, the keep-out constraint can be written as:

$$\|p_{\text{rel, }c}\|_2 > r_{\text{keep-out}} \tag{3}$$

where $p_{\text{rel, }c}$ refers to the position vector of the chaser relative to the leader. It is clear that this is a non-convex constraint. The red shaded region in Figure 3 illustrates this keep-out region. During the docking phase, the chaser shall remain within a docking corridor: θ degrees from the docking axis. This constraint can be written as:

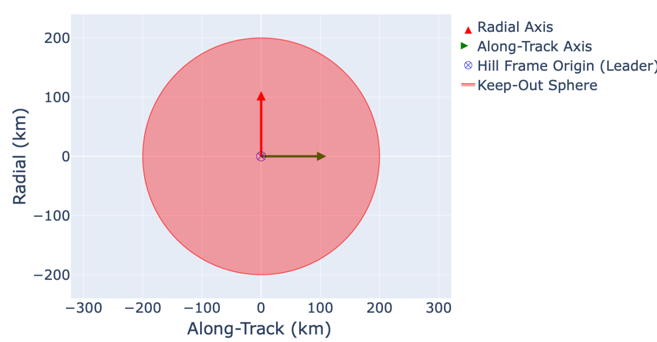

Fig. 3. Keep-Out Region

$$\|p_{\text{rel, }c}\|_2 \cos\left(\frac{\theta}{2}\right) \leq p_{\text{rel, }c} \cdot d \tag{4}$$

where d is the unit vector that describes the docking axis of the leader spacecraft. This is a second-order cone constraint. Note that this is a two-sided cone. We want to ensure that the chaser stays on the front side of the docking target, so we add the additional constraint:

$$p_{\text{rel, }c} \cdot d \geq 0 \tag{5}$$

The green shaded region in Figure 4 illustrates the docking corridor.

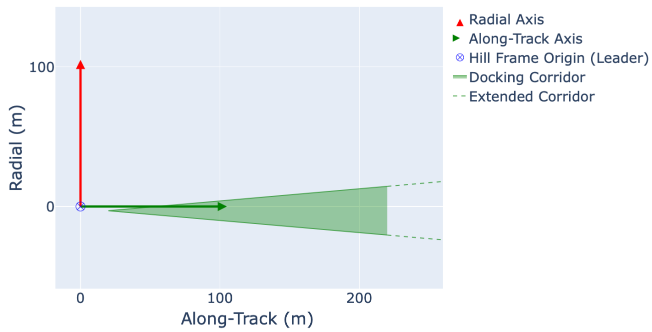

Fig. 4. Docking Corridor

III. Proposed Solution

A. Dynamics

In order to solve this optimization problem, we require linear dynamics. This can be accomplished by using the CW equations. This system makes the following assumptions: the leader's orbit is circular and the chaser orbit is circular or elliptical. This dynamical system is defined in a new frame, centered on the leader spacecraft, called the Hill frame, and is illustrated in Figure 5.

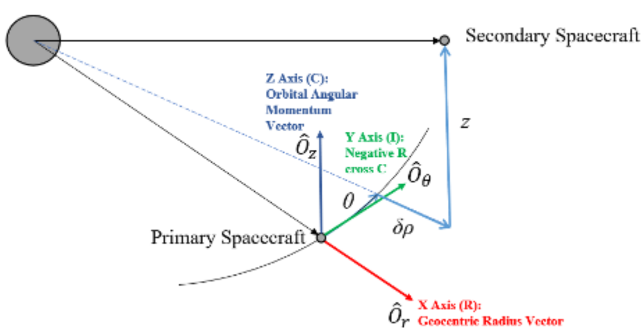

Fig. 5. Hill Frame

The CW equations are defined as a linear time-invariant system:

$$\dot{x} = Ax + Bu \tag{6}$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{7}$$

x is the state of the chaser in the Hill frame, and u is the commanded acceleration in the Hill frame, and $n = \sqrt{\mu/a^3}$, where a is the leader's orbit radius.

B. Constrained Optimization

As the rendezvous phase optimization problem is has a non-convex constraint, we must describe this in a convex form. We can frame this as a sequential convex problem which linearizes the non-convex constraint. By using this approach, we can leverage efficient convex optimization techniques. By iteratively solving this problem, we can enforce the non-convex constraint. The specific optimization problem we aim to solve in the rendezvous phase is given by Equation (8).

$$\begin{aligned} \min_{u} \quad & \gamma s^2 + (x(t_f) - x_{\text{des}})^T Q_f (x(t_f) - x_{\text{des}}) + u^T R u \\ \text{s.t.} \quad & x(0) = x_{\text{init}} \\ & x(t+1) = x(t) + (Ax(t) + Bu(t))\Delta t \quad \forall t \\ & \|u(t)\|_\infty \leq u_{\max} \quad \forall t \\ & G_t x(t) + h_t \geq -s \quad \forall t \\ & s \geq 0 \end{aligned} \tag{8}$$

In this problem Δt is the chosen time step size and x_{des} is the desired state at the end of the rendezvous phase. The first constraint ensures that the initial state of the solution matches the true initial state of the system. The second constraint ensures that the solution satisfies the dynamics. The third constraint ensures that the maximum control input u in any axis in the Hill frame does not exceed some value $u_{\max}$. The fourth constraint ensures that the trajectory avoids the linearized non-convex keep-out region described in Equation (3). The last constraint ensures that the problem is feasible, allowing some violation in the keep-out region. By successively solving this problem, we can drive s to zero ensuring that the constraint is not violated. Figure 6 demonstrates how the optimal trajectory without the keep-out constraint violates the constraint and how successive iterations result in a trajectory that does not violate the constraint. Note that there are optimization variables that must be manually tuned based on the engineer's judgment: γ (slack penalty), Q_f (penalty on terminal state error), and R (penalty on control input).

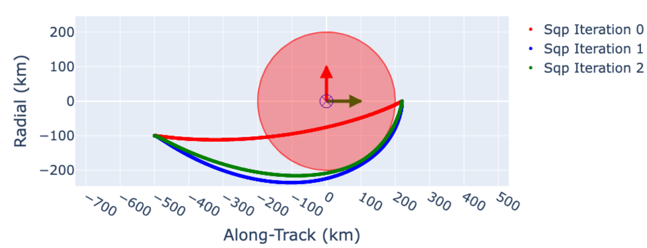

Fig. 6. Sequential Convex Programming Iterations

The docking phase trajectory constraint is already framed as a second-order cone. Therefore, this can be solved as a second-order cone program without any further manipulation. The docking phase optimization problem is described

TABLE I

RENDEZVOUS CONSTRAINTS AND OPTIMIZATION PARAMETERS

Parameter	Value
x_{des}	$[-3 \quad 240 \quad 0 \quad 0 \quad 0 \quad 0]$
Δt	1 sec
t_f	500 sec
$r_{\text{keep-out}}$	200m
$u_{\max}$	$0.032\text{m}/\text{sec}^2$
Q_f	$\text{diag}([1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1])$
R	$\text{diag}([1 \quad 1 \quad 1])$
γ	10^5

by Equation (9).

$$
\begin{aligned}
\min_{u} \quad & (x - x_{\text{des}})^T Q (x - x_{\text{des}}) + u^T R u \\
\text{s.t.} \quad & x(0) = x_{\text{init}} \\
& x(t+1) = x(t) + (Ax(t) + Bu(t))\Delta t \quad \forall t \\
& \|u(t)\|_\infty \leq u_{\max} \quad \forall t \\
& \|p_{\text{error}}(t)\|_2 \cos\left(\frac{\theta}{2}\right) \leq p_{\text{error}}(t) \cdot d \quad \forall t \\
& p_{\text{error}}(t) \cdot d \geq 0 \quad \forall t \\
& \|v_{\text{error}}(t) \cdot d\|_2 \leq v_{\max} \quad \forall t
\end{aligned} \tag{9}
$$

In this problem Δt is the chosen time step size and x_{des} is the desired state at the end of the docking phase. p_{error} and v_{error} are the differences between the desired position and the desired velocity, respectively. These are not optimization parameters; they are components of the state vector x. The first constraint ensures that the initial state of the solution matches the true initial state of the system. The second constraint ensures that the solution satisfies the dynamics. The third constraint ensures that the maximum control input u in any axis in the Hill frame does not exceed some value $u_{\max}$. The fourth and fifth constraints ensure the chaser remains within the defined corridor as defined by Equations (4 and 5). The sixth constraint limits the magnitude of the closure rate along the docking axis to some value $v_{\max}$. Note that there are optimization variables that must be tuned based on the engineer's judgment: Q the penalty on the state error and R the penalty on control input.

IV. EXPERIMENTAL RESULTS

A. Experiment Setup

In order to implement, simulate, and validate the proposed solution, we must first define the values of the various constraints and optimization parameters for both phases. The parameters for the rendezvous phase are tabulated in Table I. The parameters for the docking phase are tabulated in Table II.

The initial state of the leader spacecraft using Keplerian elements are defined in Table III. The initial state of the chaser spacecraft is defined relative to that of the leader. The relative initial state of the chaser in the Hill frame is given in Table IV. This chaser state was chosen because this passively satisfies the keep-out constraint while also producing a trajectory that closely matches the first-order dynamics described by the CW equations. An illustration of this relative state is shown in Figure 7 where the image plane is aligned with the orbital plane of the leader.

I then simulate the trajectory of both spacecraft in the inertial frame using the non-linear dynamics described in Equation (1) for 890sec. The states of both spacecraft are allowed to propagate without any control inputs applied for 60sec. Once this time has elapsed, the rendezvous phase starts and the chaser will solve the optimization problem described in Equations (8 and 9) and execute the resulting open-loop control inputs over the prescribed time period. The

TABLE II

DOCKING CONSTRAINTS AND OPTIMIZATION PARAMETERS

Parameter	Value
x_{des}	$[-3 \quad 20 \quad 0 \quad 0 \quad 0 \quad 0]$
Δt	1 sec
t_f	60 sec
θ	10°
d	$[0 \quad 1 \quad 0]$
$u_{\max}$	$0.032\text{m}/\text{sec}^2$
$v_{\max}$	$1\text{m}/\text{sec}$
Q	$\text{diag}([1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1])$
R	$\text{diag}([1 \quad 1 \quad 1])$

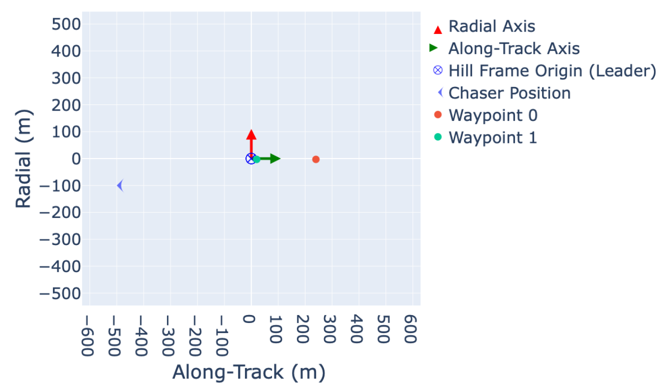

Fig. 7. Initial Condition

TABLE III

INITIAL STATE OF THE LEADER SPACECRAFT

Keplerian Element	Value
Semi-Major Axis	6795.138km
Eccentricity	0.007
Inclination	51.6°
Right Ascension of Ascending Node	0°
Argument of Periapse	0°
True Anomaly	0°

TABLE IV

INITIAL RELATIVE STATE OF THE CHASER SPACECRAFT

Relative State Component	Value
Radial Distance	−100m
Along-Track Distance	−500m
Cross-Track Distance	0m
Radial Rate	0m/sec
Along-Track Rate	0.225m/sec
Cross-Track Rate	0m/sec

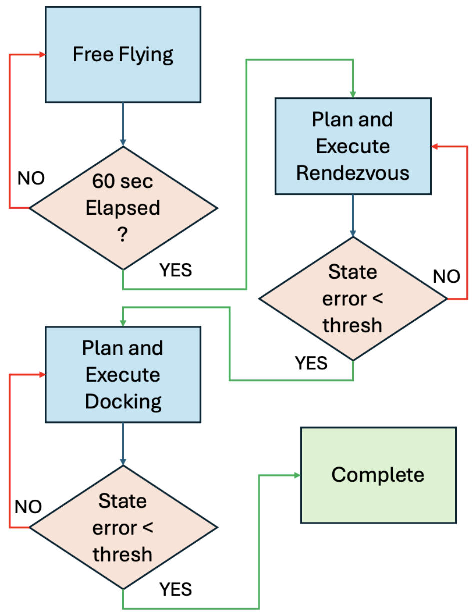

Fig. 8. Phase Switching Logic Flow Diagram

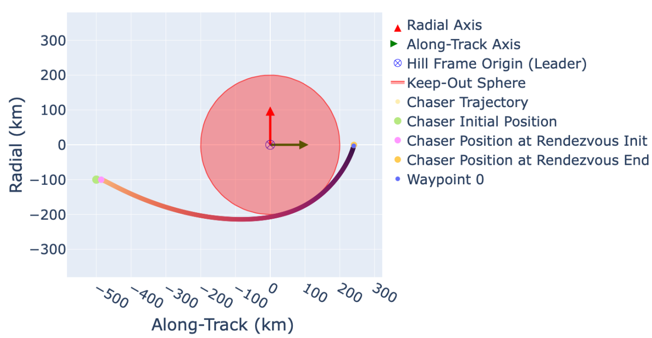

Fig. 9. Simulated Rendezvous Trajectory

chaser uses the logic flow illustrated in Figure 8 to determine when to switch between the free-flying, rendezvous, docking, and docked modes.

B. Simulation Results

We can now analyze the results of the simulation and determine if the chaser was able to successfully rendezvous and dock without violating the constraints. First, looking at the rendezvous phase, we can see the resulting trajectory of the chaser after the control inputs are applied in Figure 9. Figure 10 is a plot of the distance from the chaser to the leader as a function of time. We can clearly see that the range never decreases below the constraint value of 200m. Although satisfaction of the control input constraint is guaranteed by the convex optimization approach, we must demonstrate that this constraint is not violated. The commanded acceleration as a function of time during the rendezvous phase is shown in Figure 11. Thus, for the rendezvous phase, we can state that for this test case, all the constraints are met using the proposed approach method.

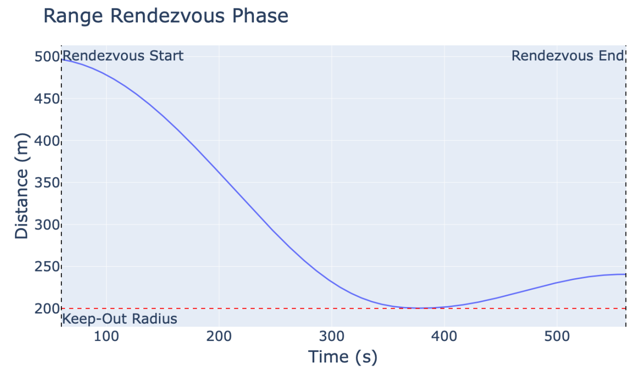

Fig. 10. Range as a Function of Time During Rendezvous Phase

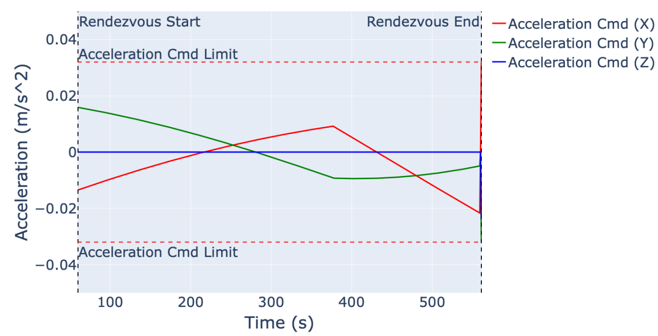

Fig. 11. Control Inputs as a Function of Time During Rendezvous Phase

We can analyze the docking phase by plotting the trajectory with the docking corridor constraint as shown in Figure 12. Here we can clearly see that the terminal state matches the desired state and that the trajectory does not violate the docking corridor.

Similar to the rendezvous phase, we have a guarantee that the control input constraint will not be violated. However, we will still plot the commanded acceleration as a function of time during the rendezvous phase in Figure 13.

In the docking phase, we also have additional constraints on the closure rate along the docking axis. Since the true system is non-linear, we do not have guarantees of constraint

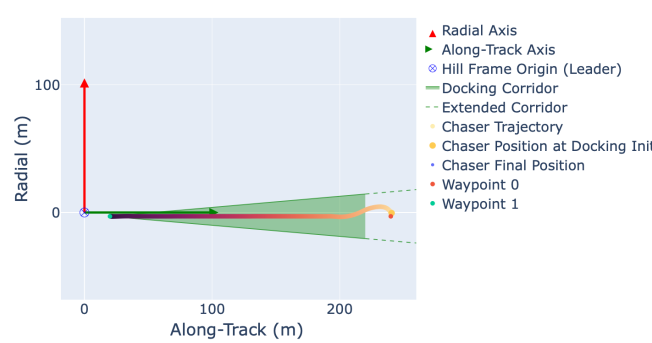

Fig. 12. Simulated Docking Trajectory

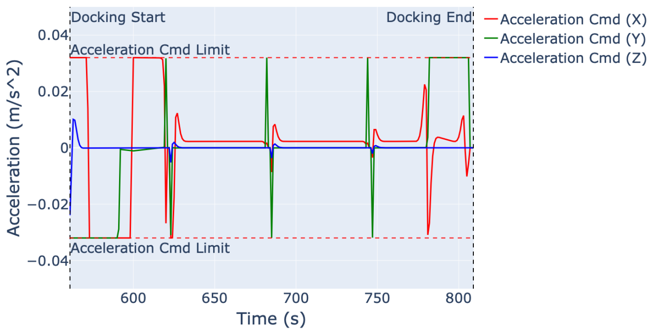

Fig. 13. Control Inputs as a Function of Time During Docking Phase

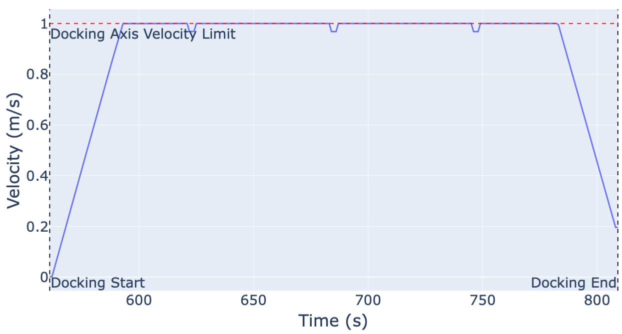

Fig. 14. Velocity along Docking Axis as a Function of Time

compliance, so we must demonstrate this. We can see a plot of the closure rate along the docking axis as a function of time in Figure 14. Here we see that the velocity constraint is met and that at the end of the docking phase, the chaser comes into contact with the leader with a velocity of approximately 0.2m/sec, which is sufficiently low for safe docking. Thus, it is clear that all the constraints are met in the docking phase.

This shows that the proposed approach is valid for the test case presented. The simulation and plotting code used to generate these results can be found at this Github repository [8]. It is important to note that re-planning due to deviation from the planned trajectory is not performed in my set up. For trajectories over sufficiently long periods of time, it is possible that constraint violation may occur using this framework without re-planning; however, I found that it was not necessary for the test cases I used. However, should such a need arise, the only necessary change is to change the mode switching logic in Figure 8 to allow re-planning if significant deviation occurs.

V. Conclusions and Future Work

In this paper, I have described the non-linear dynamics and non-convex constraints involved in spacecraft rendezvous, and docking. I discussed how using the Clohessy-Wiltshire equations results in a linear time-invariant system, as well as how sequential convex programming can be used to solve non-convex problems using convex optimization techniques. I then implement my proposed solution and demonstrate its validity under non-linear dynamics.

One major area of improvement is developing a formulation of the convex optimization problem that does not prescribe a final time. Using this approach, we can instead solve for a time-optimal trajectory while also minimizing control input. Further work can also be done to determine how to generate trajectories which will not violate constraints in the event that control input capability is lost. In other words, develop a means of generating fail-safe trajectories. We can also further improve this work by adding more accurate modeling of the attitude control and orbital maneuvering thrusters to the convex optimization framework. This can include incorporating their geometry to solve the classical ”thruster allocation problem” and/or include minimum thrust constraints. Finally, we can also apply this work to dual-quaternion-based constrained rendezvous and docking as a means of combined position and attitude control. This will allow us to solve the pose optimization problem with attitude-pointing constraints and trajectory constraints using a single optimization problem.

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