Date: 23rd January **Problem Set: 3** **Phy102**

Polar Coordinates and Newton's laws:

1. A bead of mass of mass m is free to slide on a thin rod rotating in a plane about one end with a constant angular velocity ω. Show that the position of the bead is given by $r = Ae^{-\gamma t} + Be^{\gamma t}$. What is γ? Neglect gravity. Find the constants A and B in terms initial conditions ($r(t=0) = r_0$ and $v(t=0) = v_0$). What is the situation when r decreases continually with time. (Exclude the case when the bead hits the origin). Refer to the problem 2.33 of Kleppner and Kolenkow for figure)

2. A mass m whirls around on a string, which passes through a ring as shown. Neglect gravity. Initially the mass is at a distance r_0 from the center and is revolving with an angular velocity ω_0. The string is pulled with a constant velocity V starting at $t = 0$ so that the radial distance to the mass decreases. Find i) $\omega(t)$ and ii) the force needed to pull the string. (Refer to Kleppner 2.34 and also example 2.7)

3. A particle of mass m is moving in a circular orbit r_0 in the XY plane with an angular speed ω_0. From time $t = 0$, the mass is subjected to a time dependent radial force $f(t)\,\hat{r}$.
a) What is the angular momentum vector $\vec{l}$ of the particle?
b) As a result of the applied force, the radius of the orbit decreases as $\dot{r} = -\lambda r$, where λ is a positive constant. Is the angular momentum still conserved?
c) Write down the equation of motion of the mass in polar co-ordinates and find out the form of $f(r)$, in terms of l, m and r.
d) Calculate the work done by the force in reducing the radius of the orbit from the initial value r_0 to a value r_1.

4. A river of width d flows with a constant velocity v_0 along its length. A man swims in the river with the same velocity v_0, with respect to the water, directed toward a fixed point O on the shore. He starts from the point A directly opposite to O on the other shore of the river.

(a) Find the equation of the trajectory in polar co-ordinates with O as origin and OA as the x-axis.

(b) At what distance from O will he land the shore?

5. A particle of mass m is constrained to move on the surface of an infinitely long cylinder of radius R. It is acted upon by a force $\vec{F} = -\hat{\imath}\,x - \hat{\jmath}\,y - \hat{k}z$

a) Write down the equation of motion of the particle using the appropriate cylindrical polar coordinates.
b) Which component of the angular momentum is conserved? What about the corresponding component of velocity
c) Analyze the motion in the z-direction.