uuuu:23rd January **Problem Set: 3** **Phy102**

Polar Coordinates and uuuuuu's laws

1. A bead of mass of mass m is free to slide on a thin rod rotating in a plane about one end with a constant uuuuuuu velocity ω. Show that the position of the bead is given by $r = Ae^{-\gamma t} + Be^{\gamma t}$. What is γ? Neglect gravity. Find the constants A and B in terms initial conditions ($r(t=0) = r_0$ and $v(t=0) = v_0$). What is the situation when r decreases continually with time. (Exclude the case when the bead hits the origin). Refer to the problem 2.33 of Kleppner and Kolenkow for figure)

2. A mass m whirls around on a string, which passes through a ring as shown. Neglect gravity. Initially the mass is at a distance r_0 from the center and is revolving with an uuuuuuu velocity ω_0. The string is pulled with a constant velocity V starting at $t = 0$ so that the radial distance to the mass decreases. Find i) $\omega(t)$ and ii) the force needed to pull the string. (Refer to Kleppner 2.34 and also example 2.7)

3. A particle of mass m is moving in a circular orbit r_0 in the XY plane with an uuuuuuu speed ω_0. From time $t = 0$, the mass is subjected to a time dependent radial force $f(t)\,\hat{r}$.
a) What is the uuuuuuu momentum vector $\vec{l}$ of the particle?
b) As a result of the applied force, the radius of the orbit decreases as $\dot{r} = -\lambda r$, where λ is a positive constant. Is the uuuuuuu momentum still conserved?
c) Write down the equation of motion of the mass in polar co-ordinates and find out the form of $f(r)$, in terms of l, m and r.
d) Calculate the work done by the force in reducing the radius of the orbit from the initial value r_0 to a value r_1.

4. A river of width d flows with a constant velocity v_0 along its length. A man swims in the river with the same velocity v_0, with respect to the water, directed toward a fixed point O on the shore. He starts from the point A directly opposite to O on the other shore of the river.

(a) Find the equation of the trajectory in polar co-ordinates with O as origin and OA as the x-axis.

(b) At what distance from O will he land the shore?

5. A particle of mass m is constrained to move on the surface of an infinitely long cylinder of radius R. It is acted upon by a force $\vec{F} = -\hat{\imath}\,x - \hat{\jmath}y - \hat{k}z$

a) Write down the equation of motion of the particle using the appropriate cylindrical polar coordinates.
b) Which component of the uuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuu
component of velocity
c) Analyze the motion in the z-direction.