Date: 23rd January Problem Set: 3 Phy102

Polar Coordinates and Newton's laws:

- 1. A bead of mass m is free to slide on a thin rod rotating in a plane about one end with a constant uuuuuuu velocit ω . Show that the position of the bead is given by $r = Ae^{-\gamma t} + Be^{\gamma t}$. What is γ ? Neglect gravity. Find the constants A and B in terms initial conditions $(r(t=0)=r_0)$ and $v(t=0)=v_0$. What is the situation when r decreases continually with time. (Exclude the case when the bead hits the origin). Refer to the problem 2.33 of Kleppner and Kolenkow for figure)
- 2. A mass m whirls around on a string, which passes through a ring as shown. Neglect gravity. Initially the mass is at a distance r_0 from the center and is revolving with an angular velocity ω_0 . The string is pulled with a constant velocity V starting at t=0 so that the radial distance to the mass decreases. Find i) $\omega(t)$ and ii) the force needed to pull the string. (Refer to Kleppner 2.34 and also example 2.7)
- 3. A particle of mass m is moving in a circular orbit r_0 in the XY plane with an angular speed ω_0 . From time t=0, the mass is subjected to a time dependent radial force f(t) \hat{r} .
- a) What is the angular momentum vector \vec{l} of the particle?
- b) As a result of the applied force, the radius of the orbit decreases as $\dot{r} = -\lambda r$, where λ is a positive constant. Is the angular momentum still conserved?
- c) Write down the equation of motion of the mass in polar co-ordinates and find out the form of f(r), in terms of l, m and r.
- d) Calculate the work done by the force in reducing the radius of the orbit from the initial value r_0 to a value r_1 .
- 4. A river of width d flows with a constant velocity v_0 along its length. A man swims in the river with the same velocity v_0 , with respect to the water, directed toward a fixed point O on the shore. He starts from the point A directly opposite to O on the other shore of the river.
- (a) Find the equation of the trajectory in polar co-ordinates with O as origin and OA as the x-axis.
- (b) At what distance from O will he land the shore?
- 5. A particle of mass m is constrained to move on the surface of an infinitely long cylinder of radius R. It is acted upon by a force $\vec{F} = -\hat{\imath} x \hat{\jmath} y \hat{k} z$
- a) Write down the equation of motion of the particle using the appropriate cylindrical polar coordinates.
- b) Which component of the angular momentum is conserved? What about the corresponding component of velocity
- c) Analyze the motion in the z-direction.