Ephemeris Calculation

By Parthvi Bhutani

Julian Date

- It is the number of elapsed days since the beginning of a cycle of 7980 years.
- The starting point for the first Julian Cycle began on January 1, 4713 BC and will end on January 22, 3268 AD.
- The objective of the system is to make it easy to compute the difference between two calendar dates.
- For example, the JD for 9:30 am on May 7, 2018 looks like this: 2458245.89583
    - ‘2458245’ days have passed since the beginning of the Julian Cycle.
    - ‘0.89583’ represents the time of the day.

Epoch

- It is a specific moment in time for which celestial coordinates of orbital elements are specified.
- From the value of Epoch, other orbital parameters can be calculated in order to predict the future positions of the orbital elements.
- For asteroid ‘Juno’, epoch value is 2458200.5 which corresponds to the date of 23rd March 2018.

Coordinate System for Celestial Objects

There are two planes that define the coordinate systems:

1. The ‘Ecliptic’ is the plane that contains Earth’s orbit
2. The Earth’s ‘Equatorial’ plane is the plane spanned by the Earth’s equator. It is perpendicular to Earth’s axis of rotation, and contains the Earth’s center.

Heliocentric Ecliptic Cartesian Coordinates

- There is a unique plane, parallel to Earth’s equatorial plane, passing through the Sun’s center.
- Earth passes through this plane twice a year, during the vernal (March) and the autumnal (September) equinox.
- An equinox is commonly regarded as the moment the plane of Earth's equator passes through the center of the Sun, which occurs twice each year, around 20 March and 22-23 September.
- In other words, it is the point in which the center of the visible sun is directly over the equator.
- On the day of an equinox, daytime and nighttime are of approximately equal duration all over the planet.

Heliocentric Ecliptic Cartesian Coordinates

- X- axis: It is defined as the direction of the vernal equinox as seen from the Sun.
- Y- axis: It is defined as the direction perpendicular to the X- axis, lying in the ecliptic in such a way that the Earth’s motion around the Sun is counter-clockwise wrt the X Y - Coordinates.
- Z- axis: It is defined as the axis perpendicular to the ecliptic such that the X Y Z - system is right-handed.

Heliocentric Ecliptic Cartesian Coordinates

- Let the position of celestial object in the solar system be $\mathbf{x}_\mathbf{a} = [x_a, y_a, z_a]$
- We can then find the heliocentric ecliptic coordinates as follows:

$$\mathbf{x} = r\mathbf{D}_3(\Omega)\mathbf{D}_1(i)\mathbf{D}_3(\omega)\begin{bmatrix}\cos\psi \\ \sin\psi \\ 0\end{bmatrix}$$

Heliocentric Ecliptic Cartesian Coordinates

- For any angle φ ε [0; 2π):

$$\mathbf{D}_1(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}, \qquad \mathbf{D}_3(\varphi) = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Heliocentric Ecliptic Cartesian Coordinates

The product of matrices $\mathbf{D}_3(\Omega)\mathbf{D}_1(i)\mathbf{D}_3(\omega)$ is explicitly given by:

$$\begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\cos i\sin\omega & -\cos\Omega\sin\omega - \sin\Omega\cos i\cos\omega & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\cos i\sin\omega & -\sin\Omega\sin\omega + \cos\Omega\cos i\cos\omega & -\cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & \cos i \end{bmatrix}$$

Geocentric Equatorial Cartesian Coordinates

Switch from heliocentric ecliptic cartesian coordinates to geocentric equatorial cartesian coordinates due to the following reasons:

- The equatorial coordinates are independent of the observer's location and the time of the observation, unlike the ecliptic coordinates.
- Thus only one set of coordinates is required for each object.

Geocentric Equatorial Spherical Coordinates

- First, we convert heliocentric ecliptic cartesian coordinates to geocentric ecliptic cartesian coordinates.
- If **x** and $\mathbf{x}_{\text{Earth}}$ are the object's and earth's heliocentric ecliptic cartesian coordinates respectively, then the object's geocentric ecliptic cartesian coordinates is given by

$$\tilde{\mathbf{x}} := \mathbf{x} - \mathbf{x}_{\text{Earth}}.$$

Geocentric Equatorial Spherical Coordinates

- We get the geocentric equatorial cartesian coordinates through

$$\bar{r}\prime = D_1(\epsilon)\,\bar{r}$$

Where ε = 23.4 degree is the obliquity of the ecliptic, which is the angle between the ecliptic and the equatorial.

Geocentric Equatorial Spherical Coordinates

- $\mathbf{x}'$ Is related to the geocentric equatorial spherical coordinates (α, δ) by

$$x_1^* = \cos\delta\cos\alpha$$
$$x_2^* = \cos\delta\sin\alpha$$
$$x_3^* = \sin\delta,$$

Where $\mathbf{x}^* = \mathbf{x}'/||\mathbf{x}'||.$

Geocentric Equatorial Spherical Coordinates

- The Equatorial coordinate system is basically the projection of latitude & longitude coordinate system that is used on Earth, onto the celestial sphere.
- Celestial equator is defined by projecting the earth’s equator onto the celestial sphere.
- The geometric spherical cartesian coordinates are ascension (α) and declination (δ).
- They are analogous to longitudes and latitudes.

Geocentric Equatorial Spherical Coordinates

- Lines of longitudes become lines of ascension, which indicates how far east is the object from where the celestial equator intersects the ecliptic (i.e, the vernal equinox). The units of ascension is Hours: Mins: Secs.
- Lines of latitudes become lines of declination, which indicates how far north or south of the celestial equator the object lies. The units of declination is degrees.

Geocentric Equatorial Spherical Coordinates

For asteroid ‘Juno’, the values obtained are as follows:

- Ascension = 358.2791314 degrees = 23 hrs 53 mins 6.99 secs,

  Where 1 degree = 1/15 hrs, 1 hr = 60 mins, & 1 min = 60 secs.

- Declination = 0.445411 degrees. ,