Boulder

Reinforcement Learning; Simulated Environments

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Continuous Control with Deep Reinforcement Learning

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human arm $\rightarrow$ 7 degrees of freedom

$a_i \in \{-k, 0, k\}, i = 1, \ldots, 7$

$3^7 = 2187 \rightarrow$ dimensionality of action space

Background

$\mathcal{E} \rightarrow$ environment (maybe stochastic)

$t \rightarrow$ time step

$x_t \rightarrow$ observation

$a_t \rightarrow$ action $\quad a_t \in \mathbb{R}^N$

$r_t \rightarrow$ reward

$s_t = x_1, a_1, x_2, a_2, \ldots, a_{t-1}, x_t \rightarrow$ state

partially observed environment

$s_t = x_t \rightarrow$ fully observed environment

$\pi : \mathcal{S} \rightarrow \mathcal{P}(\mathcal{A})$ — $\mathcal{S}$: states; $\mathcal{P}(\mathcal{A})$: prob. distributions; set of legal actions; π: policy

$p(s_1) \rightarrow$ initial state distribution

$p(s_{t+1}|s_t, a_t) \rightarrow$ transition dynamics

$r(s_t, a_t) \rightarrow$ reward function

MDP: Markov Decision Process

$$R_t = \sum_{i=t}^{T} \gamma^{i-t} r(s_i, a_i) \rightarrow \text{future discounted return}$$

$$J = \mathbb{E}_{r_i, s_i \sim \mathcal{E}, a_i \sim \pi}[R_1]$$

$\rho^\pi \rightarrow$ discounted state visitation distribution for a policy π

$$Q^\pi(s_t, a_t) = \mathbb{E}_{r_{i \geq t}, s_{i>t} \sim \mathcal{E}, a_{i>t} \sim \pi}[R_t | s_t, a_t]$$

action-value function: expected return after taking action a_t in state s_t and thereafter following policy π

$$Q^\pi(s_t, a_t) = \mathbb{E}_{r_t, s_{t+1} \sim \mathcal{E}}\left[r(s_t, a_t) + \gamma \mathbb{E}_{a_{t+1} \sim \pi}[Q^\pi(s_{t+1}, a_{t+1})]\right]$$

Bellman Equation

$\mu : \mathcal{S} \rightarrow \mathcal{A}$

if deterministic target policy

$$Q^\pi(s_t, a_t) = \mathbb{E}_{r_t, s_{t+1} \sim \mathcal{E}}\left[r(s_t, a_t) + \gamma Q^\pi(s_{t+1}, a_{t+1})\right]$$

It is therefore possible to learn Q^π off-policy

$\beta \rightarrow$ stochastic behavior policy

$$\mu(s) = \arg\max_a Q(s, a) \rightarrow \text{greedy policy}$$

$$L(\theta^Q) = \mathbb{E}_{s_t \sim \rho^\beta, a_t \sim \beta, r_t \sim \mathcal{E}}[(Q(s_t, a_t|\theta^Q) - y_t)^2]$$

$$y_t = r(s_t, a_t) + \gamma Q(s_{t+1}, \mu(S_{t+1})|\theta^Q)$$

⋆ replay buffer

⋆ separate target network

$\mu(s|\theta^\mu) \rightarrow$ actor function

$Q(s, a) \rightarrow$ critic (Q-Learning & Bellman Equation)

$$\begin{aligned}\nabla_{\theta^\mu} J &\approx \mathbb{E}_{s_t \sim \rho^\beta}\left[\nabla_{\theta^\mu} Q(s, a|\theta^Q)|_{s=s_t, a=\mu(s_t|\theta^\mu)}\right] \\ &= \mathbb{E}_{s_t \sim \rho^\beta}\left[\nabla_a Q(s, a|\theta^Q)|_{s=s_t, a=\mu(s_t)} \nabla_{\theta_\mu} \mu(s|\theta^\mu)|_{s=s_t}\right]\end{aligned}$$

$$L(\theta^\mu) = -\mathbb{E}_{s_t \sim \rho^\beta}[Q(s_t, \underbrace{\mu(s_t|\theta^\mu)}_{a_t} |\theta^Q)]$$

Deterministic Policy Gradient (DPG) algorithm

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

https://www.youtube.com/watch?v=tJBIqkC1wWM&feature=youtu.be

Lillicrap, Timothy P., et al. "Continuous control with deep reinforcement learning." *arXiv preprint arXiv:1509.02971* (2015).

Trust Region Policy Optimization

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$(\mathcal{S}, \mathcal{A}, P, c, \rho_0, \gamma) \rightarrow$ infinite-horizon discounted Markov Decision Process (MDP)

$\mathcal{S} \rightarrow$ finite set of states

$\mathcal{A} \rightarrow$ finite set of actions

$P \rightarrow$ transition probability distribution

$P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$

$c \rightarrow$ cost function

$c : \mathcal{S} \rightarrow \mathbb{R}$

$\rho_0 \rightarrow$ distribution of initial state s_0

$\rho_0 : \mathcal{S} \rightarrow \mathbb{R}$

$\gamma \in (0, 1) \rightarrow$ discount factor

$\pi \rightarrow$ stochastic policy

$\pi : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$

$\eta(\pi) \rightarrow$ expected discounted cost

$$\eta(\pi) = \mathbb{E}_{s_0, a_0, \ldots}[\sum_{t=0}^{\infty} \gamma^t c(s_t)]$$

$$s_0 \sim \rho_0(s_0), a_t \sim \pi(a_t|s_t), s_{t+1} \sim P(s_{t+1}|s_t, a_t)$$

$Q_\pi(s_t, a_t) \rightarrow$ state-action value function

$$Q_\pi(s_t, a_t) = \mathbb{E}_{s_{t+1}, a_{t+1}, \ldots}[\sum_{l=0}^{\infty} \gamma^l c(s_{t+l})]$$

$V_\pi(s_t) \rightarrow$ value function

$$V_\pi(s_t) = \mathbb{E}_{a_t, s_{t+1}, \ldots}[\sum_{l=0}^{\infty} \gamma^l c(s_{t+l})]$$

$A_\pi(s_t, a_t) \rightarrow$ advantage function

$$A_\pi(s_t, a_t) = Q_\pi(s_t, a_t) - V_\pi(s_t)$$

$\rho_\pi(s) \rightarrow$ (unnormalized) discounted visitation frequencies

$$\rho_\pi(s) = (P(s_0 = s) + \gamma P(s_1 = s) + \gamma^2 P(s_2 = s) + \ldots)$$

where $s_0 \sim \rho_0$ and the actions are chosen according to π

Algorithm 1 Approximate policy iteration algorithm guaranteeing non-increasing expected cost η

Initialize π_0. See the paper for proof!

for $i = 0, 1, 2, \ldots$ until convergence **do**

Compute all advantage values $A_{\pi_i}(s, a)$.

Solve the constrained optimization problem

$$\pi_{i+1} = \arg\min_{\pi} \left[L_{\pi_i}(\pi) + \left(\overbrace{\frac{2\epsilon\gamma}{(1-\gamma)^2}}^{C} \right) D_{\mathrm{KL}}^{\max}(\pi_i, \pi) \right]$$

where $\epsilon = \max_s \max_a |A_\pi(s, a)|$

and $L_{\pi_i}(\pi) = \eta(\pi_i) + \sum_s \rho_{\pi_i}(s) \sum_a \pi(a|s) A_{\pi_i}(s, a)$

end for

$D_{\mathrm{KL}}^{\max}(\pi, \tilde{\pi}) = \max_s D_{\mathrm{KL}}(\pi(\cdot|s) \parallel \tilde{\pi}(\cdot|s))$

In practice, if we used the penalty coefficient C above recommended by the theory, the step sizes would be very small.

$\eta(\theta) := \eta(\pi_\theta)$

$L_\theta(\tilde{\theta}) := L_{\pi_\theta}(\pi_{\tilde{\theta}})$, and $D_{\mathrm{KL}}(\theta \parallel \tilde{\theta}) := D_{\mathrm{KL}}(\pi_\theta \parallel \pi_{\tilde{\theta}})$

$\overline{D}_{\mathrm{KL}}^{\rho}(\theta_1, \theta_2) := \mathbb{E}_{s \sim \rho}[D_{\mathrm{KL}}(\pi_{\theta_1}(\cdot|s) \parallel \pi_{\theta_2}(\cdot|s))]$

$$\underset{\theta}{\text{minimize}}\, L_{\theta_{\text{old}}}(\theta)$$ trust region constraint

$$\text{subject to } \overline{D}_{\mathrm{KL}}^{\rho_{\theta_{\text{old}}}}(\theta_{\text{old}}, \theta) \leq \delta.$$

$$\underset{\theta}{\text{minimize}} \sum_s \rho_{\theta_{\text{old}}}(s) \sum_a \pi_\theta(a|s) A_{\theta_{\text{old}}}(s, a)$$

$$\text{subject to } \overline{D}_{\mathrm{KL}}^{\rho_{\theta_{\text{old}}}}(\theta_{\text{old}}, \theta) \leq \delta.$$

$$\sum_s \rho_{\theta_{\text{old}}}(s)[\ldots] = \frac{1}{1-\gamma} \mathbb{E}_{s \sim \rho_{\theta_{\text{old}}}}[\ldots]$$

$$\sum_a \pi_\theta(a|s_n) A_{\theta_{\text{old}}}(s_n, a) =$$

importance sampling

$$\mathbb{E}_{a \sim q}\left[\frac{\pi_\theta(a|s_n)}{q(a|s_n)} A_{\theta_{\text{old}}}(s_n, a) \right]$$

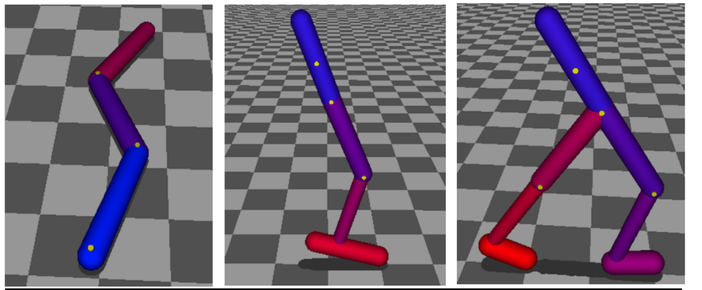
Replace advantage values by Q-values which only changes the objective by a constant.

$$\underset{\theta}{\text{minimize}}\, \mathbb{E}_{s \sim \rho_{\theta_{\text{old}}}, a \sim q}\left[\frac{\pi_\theta(a|s)}{q(a|s)} Q_{\theta_{\text{old}}}(s, a) \right]$$

$$\text{subject to } \mathbb{E}_{s \sim \rho_{\theta_{\text{old}}}}[D_{\mathrm{KL}}(\pi_{\theta_{\text{old}}}(\cdot|s) \parallel \pi_\theta(\cdot|s))] \leq \delta.$$

$q(a|s) = \pi_{\theta_{\text{old}}}(a|s)$

TRPO



http://sites.google.com/site/trpopaper/

Schulman, John, et al. "Trust region policy optimization." *International conference on machine learning*. 2015.

Conjugate Gradient Method

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TRPO objective function

$$\pi_{k+1} = \arg\max_{\pi'} \mathcal{L}_{\pi_k}(\pi') \text{ s.t. } \bar{D}_{KL}(\pi' || \pi_k) \leq \delta$$

Linear Approximation

$$\mathcal{L}_{\theta_k}(\theta) \approx \mathcal{L}_{\theta_k}(\theta_k) + g^T(\theta - \theta_k)$$

Quadratic Approximation

$$\bar{D}_{KL}(\theta||\theta_k) \approx \bar{D}_{KL}(\theta_k||\theta_k) + \nabla_\theta \bar{D}_{KL}(\theta||\theta_k)|_{\theta_k}(\theta - \theta_k) + \frac{1}{2}(\theta - \theta_k)^T H(\theta - \theta_k)$$

$$\mathcal{L}_{\theta_k}(\theta) \approx g^T(\theta - \theta_k) \qquad g \doteq \nabla_\theta \mathcal{L}_{\theta_k}(\theta)|_{\theta_k}$$

$$\bar{D}_{KL}(\theta||\theta_k) \approx \frac{1}{2}(\theta - \theta_k)^T H (\theta - \theta_k) \qquad H \doteq \nabla^2_\theta \bar{D}_{KL}(\theta||\theta_k)|_{\theta_k}$$

$$\theta_{k+1} = \arg\max_\theta g^T(\theta - \theta_k) \text{ s.t. } \frac{1}{2}(\theta - \theta_k)^T H(\theta - \theta_k) \leq \delta$$

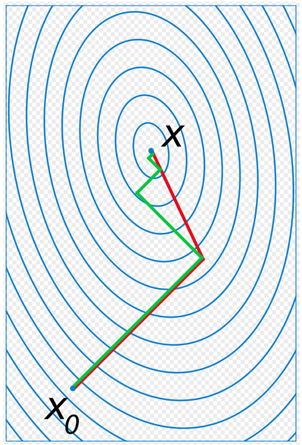
$$\theta_{k+1} = \theta_k + \sqrt{\frac{2\delta}{g^T H^{-1} g}} H^{-1} g$$

natural policy gradient

$$H^{-1}g = x \implies Hx = g$$

$$Hx = \nabla_\theta \left(\left(\nabla_\theta \bar{D}_{KL}(\theta||\theta_k) \right)^T x \right)$$

$$\arg\min_x f(x) = \frac{1}{2} x^T H x - x^T g \qquad \nabla f(x) = Hx - g$$



Conjugate gradient method

$r_0 = g - Hx_0 = -\nabla f(x_0)$

$p_0 = r_0$

$x_1 = x_0 + \alpha_0 p_0$

$f(x_1) = f(x_0 + \alpha_0 p_0) =: g(\alpha_0)$

$$g'(\alpha_0) = 0 \implies \alpha_0 = \frac{p_0^T(g - Hx_0)}{p_0^T H p_0}$$

$r_1 = g - Hx_1 = -\nabla f(x_1)$

$$p_1 = r_1 - \frac{p_0^T H r_1}{p_0^T H p_0} p_0 \rightarrow \text{Gram-Schmidt orthonormalization}$$

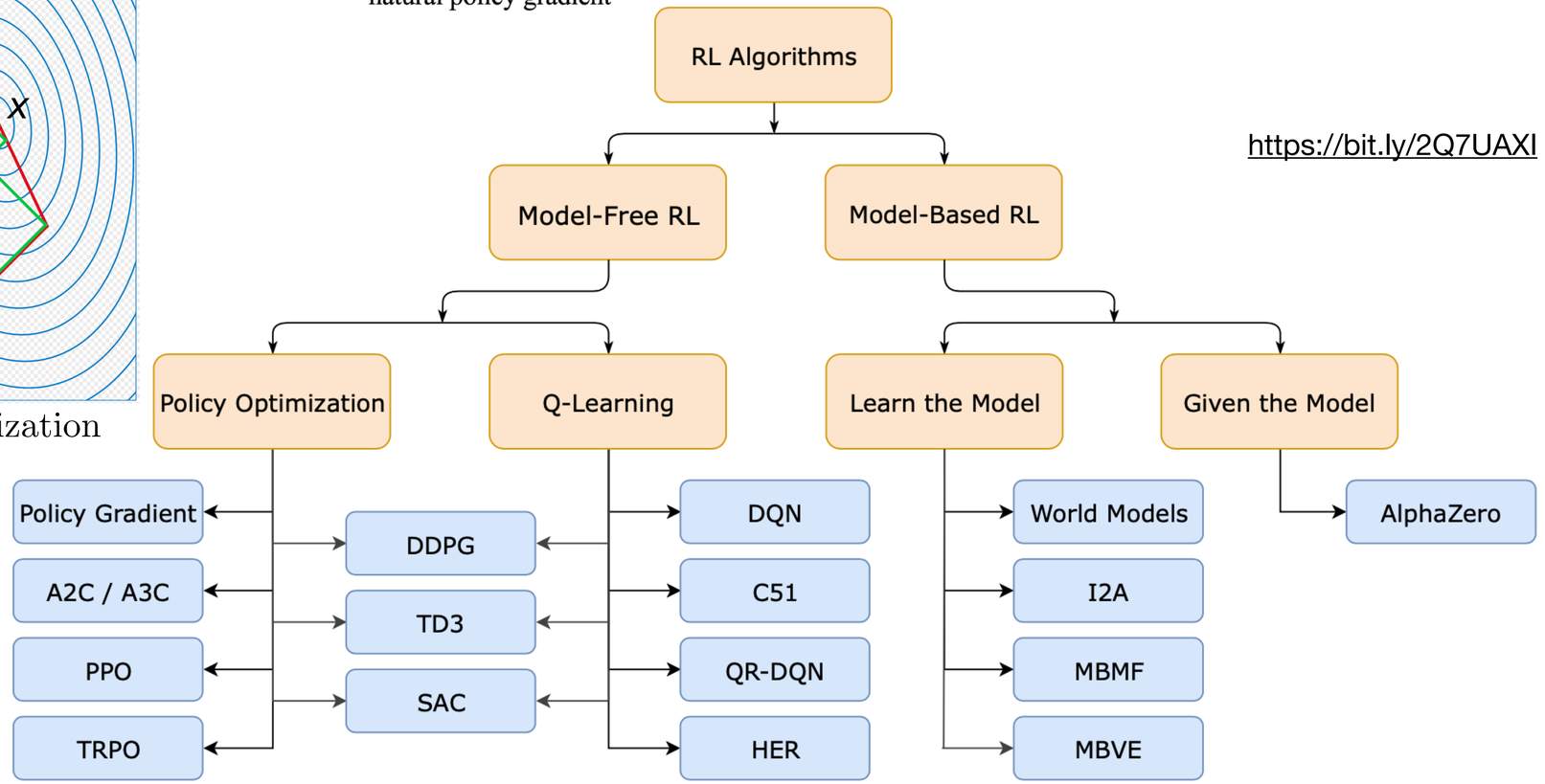
$$p_0^T H p_1 = p_0^T H r_1 - \frac{p_0^T H r_1}{p_0^T H p_0} p_0^T H p_0 = 0$$

$\implies p_0$ and p_1 are conjugate w.r.t. H.

$x_2 = x_1 + \alpha_1 p_1$

$$\alpha_1 = \frac{p_1^T(g - Hx_1)}{p_1^T H p_1}$$

https://bit.ly/2Q7UAXI



Asynchronous Methods for Deep Reinforcement Learning

YouTube Video

value-based model-free methods

$Q(s, a; \theta) \rightarrow$ approximate action-value function

$Q^*(s, a) \approx Q(s, a; \theta) \rightarrow$ Q-Learning

$$L_i(\theta_i) = \mathbb{E}[\underbrace{(r + \gamma \max_{a'} Q(s', a'; \theta_{i-1})}_{\text{one-step return}} - Q(s, a; \theta_i))^2]$$

$$\underbrace{r_t + \gamma r_{t+1} + \ldots + \gamma^{n-1} r_{t+n-1} + \max_a \gamma^n Q(s_{t+n}, a)}_{n\text{-step return}}$$

A single reward directly affects the values of n preceding state action pairs

policy-based model-free methods

$\pi(a|s; \theta)$

gradient ascent on $\mathbb{E}[R_t]$

$$\underbrace{\nabla_\theta \log \pi(a_t|s_t; \theta) R_t}_{\text{as an unbiased estimate of } \nabla_\theta \mathbb{E}[R_t]} \rightarrow \text{REINFORCE}$$

$$\nabla_\theta \log \pi(a_t|s_t; \theta)(R_t - \underbrace{b_t(s_t)}_{\text{baseine}})$$

$b_t(s_t) \approx V^\pi(s_t)$

$R_t - b_t \approx$ advantage of action a_t in state s_t

$$A(a_t, s_t) = \underbrace{Q(a_t, s_t)}_{\approx R_t} - \underbrace{V(s_t)}_{\approx b_t} \qquad \underbrace{\text{actor}}_{\pi} - \underbrace{\text{critic}}_{b}$$

Asynchronous Advantage Actor-Critic (A3C)

Algorithm Asynchronous advantage actor-critic - pseudocode for each actor-learner thread.

// Assume global shared parameter vectors θ and θ_v and global shared counter $T = 0$
// Assume thread-specific parameter vectors θ' and θ'_v
Initialize thread step counter $t \leftarrow 1$
repeat
https://www.youtube.com/watch?v=0xo1Ldx3L5Q&feature=youtu.be
  Reset gradients: $d\theta \leftarrow 0$ and $d\theta_v \leftarrow 0$.
  Synchronize thread-specific parameters $\theta' = \theta$ and $\theta'_v = \theta_v$
  $t_{start} = t$
  Get state s_t
https://www.youtube.com/watch?v=Ajjc08-iPx8&feature=youtu.be
  repeat
    Perform a_t according to policy $\pi(a_t|s_t; \theta')$
    Receive reward r_t and new state s_{t+1}
    $t \leftarrow t + 1$
    $T \leftarrow T + 1$
https://www.youtube.com/watch?v=nMR5mjCFZCw&feature=youtu.be
  until terminal s_t **or** $t - t_{start} == t_{max}$
  $R = \begin{cases} 0 & \text{for terminal } s_t \\ V(s_t, \theta'_v) & \text{for non-terminal } s_t \text{// Bootstrap from last state} \end{cases}$
  for $i \in \{t-1, \ldots, t_{start}\}$ **do**
    $R \leftarrow r_i + \gamma R$
    Accumulate gradients wrt θ': $d\theta \leftarrow d\theta + \nabla_{\theta'} \log \pi(a_i|s_i; \theta')(R - V(s_i; \theta'_v))$
    Accumulate gradients wrt θ'_v: $d\theta_v \leftarrow d\theta_v + \partial (R - V(s_i; \theta'_v))^2 / \partial \theta'_v$
  end for
  Perform asynchronous update of θ using $d\theta$ and of θ_v using $d\theta_v$.
until $T > T_{max}$

$$\nabla_\theta \log \pi(a_t|s_t; \theta) \underbrace{A(s_t, a_t; \theta, \theta_v)}_{\sum_{i=0}^{k-1} \gamma^i r_{t+i} + \gamma^k V(s_{t+k}; \theta_v) - V(s_t; \theta_v)} + \beta \nabla_\theta \underbrace{H(\pi(s_t; \theta))}_{\text{entropy}}$$

Mnih, Volodymyr, et al. "Asynchronous methods for deep reinforcement learning." *International conference on machine learning*. 2016.

High-Dimensional Continuous Control Using Generalized Advantage Estimation

Credit assignment problem: The long time delay between actions and their positive or negative effect on rewards.

$$\hat{g} = \frac{1}{N}\sum_{n=1}^{N}\sum_{t=0}^{\infty}\hat{A}_t^n \nabla_\theta \log \pi_\theta(a_t^n \mid s_t^n) \rightarrow \text{policy gradient estimator}$$

$\hat{A}_t \rightarrow$ estimate of the discounted advantage function $A^{\pi,\gamma}(s_t, a_t)$

$\gamma \rightarrow$ discount factor

$n \rightarrow$ indexes over a batch of episodes

Increase the probability of better-than-average actions and decrease the probability of worse-than-average actions.

$V \rightarrow$ an approximate value function

$$\begin{aligned}
\hat{A}_t^{(1)} &:= \delta_t^V && = -V(s_t) + r_t + \gamma V(s_{t+1}) \\
\hat{A}_t^{(2)} &:= \delta_t^V + \gamma\delta_{t+1}^V && = -V(s_t) + r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) \\
\hat{A}_t^{(3)} &:= \delta_t^V + \gamma\delta_{t+1}^V + \gamma^2\delta_{t+2}^V && = -V(s_t) + r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 V(s_{t+3}) \\
\hat{A}_t^{(k)} &:= \sum_{l=0}^{k-1}\gamma^l\delta_{t+l}^V && = -V(s_t) + r_t + \gamma r_{t+1} + \cdots + \gamma^{k-1} r_{t+k-1} + \gamma^k V(s_{t+k})
\end{aligned}$$

$\hat{A}_t^{(k)} \rightarrow$ an estimator of the advantage function, which is only un-biased when $V = V^{\pi,\gamma}$

the bias generally becomes smaller as $k \rightarrow \infty$

$$\hat{A}_t^{(\infty)} = \sum_{l=0}^{\infty}\gamma^l\delta_{t+l}^V = -V(s_t) + \sum_{l=0}^{\infty}\gamma^l r_{t+l} \rightarrow \text{un-biased}$$

However, as $k \rightarrow \infty$, the variance becomes larger.

Higher variance necessitates using more samples!

Generalized Advantage Estimator GAE(γ, λ)

$$\begin{aligned}
\hat{A}_t^{\mathrm{GAE}(\gamma,\lambda)} &:= (1-\lambda)\left(\hat{A}_t^{(1)} + \lambda\hat{A}_t^{(2)} + \lambda^2\hat{A}_t^{(3)} + \ldots\right) \rightarrow \text{exponentially-weighted average} \\
&= (1-\lambda)(\delta_t^V + \lambda(\delta_t^V + \gamma\delta_{t+1}^V) + \lambda^2(\delta_t^V + \gamma\delta_{t+1}^V + \gamma^2\delta_{t+2}^V) + \ldots) \\
&= (1-\lambda)(\delta_t^V(1 + \lambda + \lambda^2 + \ldots) + \gamma\delta_{t+1}^V(\lambda + \lambda^2 + \lambda^3 + \ldots) \\
&\qquad + \gamma^2\delta_{t+2}^V(\lambda^2 + \lambda^3 + \lambda^4 + \ldots) + \ldots) \\
&= (1-\lambda)\left(\delta_t^V\left(\frac{1}{1-\lambda}\right) + \gamma\delta_{t+1}^V\left(\frac{\lambda}{1-\lambda}\right) + \gamma^2\delta_{t+2}^V\left(\frac{\lambda^2}{1-\lambda}\right) + \ldots\right) \\
&= \sum_{l=0}^{\infty}(\gamma\lambda)^l\delta_{t+l}^V
\end{aligned}$$

Special Cases

$$\begin{aligned}
\mathrm{GAE}(\gamma, 0): &\quad \hat{A}_t := \delta_t && = r_t + \gamma V(s_{t+1}) - V(s_t) \\
\mathrm{GAE}(\gamma, 1): &\quad \hat{A}_t := \sum_{l=0}^{\infty}\gamma^l\delta_{t+l} && = \sum_{l=0}^{\infty}\gamma^l r_{t+l} - V(s_t)
\end{aligned}$$

γ-just $\rightarrow$ un-biased

$\mathrm{GAE}(\gamma, 1)$ is γ-just regardless of the accuracy of V, but it has high variance due to the sum of terms. $\mathrm{GAE}(\gamma, 0)$ is γ-just for $V = V^{\pi,\gamma}$ and otherwise induces bias, but it typically has much lower variance. The generalized advantage estimator for $0 < \lambda < 1$ makes a compromise between bias and variance, controlled by parameter λ.

Value Function Estimation (trust region & conjugate gradient algorithm)

$$\underset{\phi}{\text{minimize}} \quad \sum_{n=1}^{N}\|V_\phi(s_n) - \hat{V}_n\|^2 \quad \text{subject to} \quad \frac{1}{N}\sum_{n=1}^{N}\frac{\|V_\phi(s_n) - V_{\phi_{\text{old}}}(s_n)\|^2}{2\sigma^2} \le \epsilon$$

$\hat{V}_t = \sum_{l=0}^{\infty}\gamma^l r_{t+l}$

$\frac{1}{N}\sum_{n=1}^{N}\|V_{\phi_{\text{old}}}(s_n) - \hat{V}_n\|^2$

TRPO + GAE

https://sites.google.com/site/gaepapersupp/

Deal with the non-stationarity of the incoming data

Schulman, John, et al. "High-dimensional continuous control using generalized advantage estimation." *arXiv preprint arXiv:1506.02438* (2015).

Proximal Policy Optimization Algorithms

YouTube Video

Policy Gradient Methods

$$\hat{g} = \hat{\mathbb{E}}_t[\nabla_\theta \log \pi_\theta(a_t|s_t)\hat{A}_t] \rightarrow \text{policy gradient estimator}$$

- $\hat{\mathbb{E}}_t$: empirical average over a finite batch of samples
- π_θ: policy
- $\hat{A}_t$: estimator of the advantage function at timestep t

$$L^{PG}(\theta) = \hat{\mathbb{E}}_t[\log \pi_\theta(a_t|s_t)\hat{A}_t]$$

Trust Region Methods

$$\max_\theta \quad \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}\hat{A}_t\right] \rightarrow \text{linear approximation}$$

conjugate gradient algorithm

$$\text{s.t.} \quad \hat{\mathbb{E}}_t[\text{KL}(\pi_{\theta_{\text{old}}}(.|s_t)\|\pi_\theta(.|s_t))] \leq \delta \rightarrow \text{quadratic approximation}$$

$$\max_\theta \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}\hat{A}_t - \beta\text{KL}(\pi_{\theta_{\text{old}}}(.|s_t)\|\pi_\theta(.|s_t))\right]$$

Clipped Surrogate Objective

$$r_t(\theta) := \frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)} \implies r_t(\theta_{\text{old}}) = 1$$

$r_t(\theta)$: prob. ratio

$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}\hat{A}_t\right] = \hat{\mathbb{E}}_t[r_t(\theta)\hat{A}_t]$$

CPI: Conservative Policy Iteration

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\Big[\min(\underbrace{r_t(\theta)\hat{A}_t}_{L^{CPI}}, \underbrace{\text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)}_{\text{removes the incentive for moving } r_t \text{ outside of the interval } [1-\epsilon, 1+\epsilon]}\hat{A}_t)\Big]$$

min: pessimistic lower bound

$$L_t^{CLIP+VF+S}(\theta) = \hat{\mathbb{E}}_t\big[L_t^{CLIP}(\theta) - c_1 L_t^{VF}(\theta) + c_2 S[\pi_\theta](s_t)\big]$$

$L_t^{VF}(\theta)$: $(V_\theta(s_t) - V_t^{\text{targ}})^2$; $S[\pi_\theta](s_t)$: entropy bonus

$$\hat{A}_t = -V(s_t) + r_t + \gamma r_{t+1} + \cdots + \gamma^{T-t+1} r_{T-1} + \gamma^{T-t} V(s_T)$$

$$\hat{A}_t = \delta_t + (\gamma\lambda)\delta_{t+1} + \cdots + \cdots + (\gamma\lambda)^{T-t+1}\delta_{T-1},$$

where $\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$

same as above if $\lambda = 1$

Algorithm 1 PPO, Actor-Critic Style

```
for iteration=1,2,... do
    for actor=1,2,...,N do
        Run policy π_θold in environment for T timesteps
        Compute advantage estimates Â_1,...,Â_T
    end for
    Optimize surrogate L wrt θ, with K epochs and minibatch size M ≤ NT
    θ_old ← θ
end for
```

Schulman, John, et al. "Proximal policy optimization algorithms." *arXiv preprint arXiv:1707.06347* (2017).

Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor

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YouTube Video

Model-free Deep Reinforcement Learning
– very high sample complexity
– brittle convergence properties (meticulous hyper-parameter tuning)

Continuous state and action spaces

$(\mathcal{S}, \mathcal{A}, p, r) \rightarrow$ infinite-horizon Markov decision process (MDP)

$\mathcal{S} \rightarrow$ state space (continuous)

$\mathcal{A} \rightarrow$ action space (continuous)

$p : \mathcal{S} \times \mathcal{S} \times \mathcal{A} \rightarrow [0, \infty)$

└ unknown state transition probability
probability density of the next state $s_{t+1} \in \mathcal{S}$
given the current state $s_t \in \mathcal{S}$ and action $a_t \in \mathcal{A}$

$r : \mathcal{S} \times \mathcal{A} \rightarrow [r_{\min}, r_{\max}]$

└ reward emitted on each transition

$\rho_\pi(s_t) \rightarrow$ state marginal of the trajectory distribution induced by a policy $\pi(a_t|s_t)$

$\rho_\pi(s_t, a_t) \rightarrow$ state-action marginal

Maximum Entropy Reinforcement Learning

$$J(\pi) = \sum_{t=0}^{T} \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \rho_\pi} \left[r(\mathbf{s}_t, \mathbf{a}_t) + \alpha \mathcal{H}(\pi(\,\cdot\,|\mathbf{s}_t)) \right]$$

$Q : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$

└ soft Q-value

$\mathcal{T}^\pi \rightarrow$ Bellman backup operator

$$\mathcal{T}^\pi Q(\mathbf{s}_t, \mathbf{a}_t) \triangleq r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \mathbb{E}_{\mathbf{s}_{t+1} \sim p} \left[V(\mathbf{s}_{t+1}) \right]$$

$$V(\mathbf{s}_t) = \mathbb{E}_{\mathbf{a}_t \sim \pi} \left[Q(\mathbf{s}_t, \mathbf{a}_t) - \log \pi(\mathbf{a}_t|\mathbf{s}_t) \right]$$

└ soft state value function

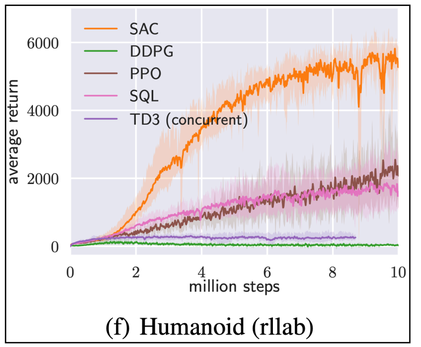
Lemma (Soft Policy Evaluation)

$$Q^{k+1} = \mathcal{T}^\pi Q^k \implies Q^k \rightarrow \text{soft } Q\text{-value as } k \rightarrow \infty$$

Lemma (Soft Policy Improvement)

$$\pi_{\text{new}} = \arg\min_{\pi' \in \Pi} \mathrm{D}_{\mathrm{KL}} \left(\pi'(\,\cdot\,|\mathbf{s}_t) \,\middle\|\, \frac{\exp\left(Q^{\pi_{\text{old}}}(\mathbf{s}_t, \,\cdot\,)\right)}{Z^{\pi_{\text{old}}}(\mathbf{s}_t)} \right)$$

$$\implies Q^{\pi_{\text{new}}}(s_t, a_t) \geq Q^{\pi_{\text{old}}}(s_t, a_t), \forall (s_t, a_t) \in \mathcal{S} \times \mathcal{A}$$



(f) Humanoid (rllab)

Theorem (Soft Policy Iteration)

Repeated application of soft policy evaluation and soft policy improvement converges to a policy π^* such that $Q^{\pi^*}(s_t, a_t) \geq Q^\pi(s_t, a_t), \forall \pi \in \Pi$.

Soft Actor-Critic

$V_\psi(s_t) \rightarrow$ parametrized state value function

$Q_\theta(s_t, a_t) \rightarrow$ parametrized soft Q-function

$\pi_\phi(a_t|s_t) \rightarrow$ tractable policy

$$J_V(\psi) = \mathbb{E}_{\mathbf{s}_t \sim \mathcal{D}} \left[\frac{1}{2} \left(V_\psi(\mathbf{s}_t) - \mathbb{E}_{\mathbf{a}_t \sim \pi_\phi} \left[Q_\theta(\mathbf{s}_t, \mathbf{a}_t) - \log \pi_\phi(\mathbf{a}_t|\mathbf{s}_t) \right] \right)^2 \right]$$

$$J_Q(\theta) = \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \mathcal{D}} \left[\frac{1}{2} \left(Q_\theta(\mathbf{s}_t, \mathbf{a}_t) - \hat{Q}(\mathbf{s}_t, \mathbf{a}_t) \right)^2 \right]$$

$$\hat{Q}(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \mathbb{E}_{\mathbf{s}_{t+1} \sim p} \left[V_{\bar{\psi}}(\mathbf{s}_{t+1}) \right]$$

$\bar{\psi}$ can be an exponentially moving average of the value network weights

$$J_\pi(\phi) = \mathbb{E}_{\mathbf{s}_t \sim \mathcal{D}} \left[\mathrm{D}_{\mathrm{KL}} \left(\pi_\phi(\,\cdot\,|\mathbf{s}_t) \,\middle\|\, \frac{\exp\left(Q_\theta(\mathbf{s}_t, \,\cdot\,)\right)}{Z_\theta(\mathbf{s}_t)} \right) \right]$$

$\mathbf{a}_t = f_\phi(\epsilon_t; \mathbf{s}_t) \rightarrow$ reparameterization trick

$$J_\pi(\phi) = \mathbb{E}_{\mathbf{s}_t \sim \mathcal{D}, \epsilon_t \sim \mathcal{N}} \left[\log \pi_\phi(f_\phi(\epsilon_t; \mathbf{s}_t)|\mathbf{s}_t) - Q_\theta(\mathbf{s}_t, f_\phi(\epsilon_t; \mathbf{s}_t)) \right]$$

defined implicitly in terms of f_ϕ

can be differentiated through!

Algorithm 1 Soft Actor-Critic

Initialize parameter vectors $\psi, \bar{\psi}, \theta, \phi$.
for each iteration **do**
  for each environment step **do**
    $\mathbf{a}_t \sim \pi_\phi(\mathbf{a}_t|\mathbf{s}_t)$
    $\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$
    $\mathcal{D} \leftarrow \mathcal{D} \cup \{(\mathbf{s}_t, \mathbf{a}_t, r(\mathbf{s}_t, \mathbf{a}_t), \mathbf{s}_{t+1})\}$
  end for
  for each gradient step **do**
    $\psi \leftarrow \psi - \lambda_V \hat{\nabla}_\psi J_V(\psi)$
    $\theta_i \leftarrow \theta_i - \lambda_Q \hat{\nabla}_{\theta_i} J_Q(\theta_i)$ for $i \in \{1, 2\}$
    $\phi \leftarrow \phi - \lambda_\pi \hat{\nabla}_\phi J_\pi(\phi)$
    $\bar{\psi} \leftarrow \tau\psi + (1 - \tau)\bar{\psi}$
  end for
end for

Haarnoja, Tuomas, et al. "Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor." *International Conference on Machine Learning*. PMLR, 2018.

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Questions?

YouTube Playlist