report_TP8

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1 Series 8, Introduction to Computational Finance, Dimitris Proios,

1.0.1 Exercise 1) Implement a Minority Game.

- Problem description

Agent An agent will not make a decision based on other agents but only based on hist own history and on the random choice of initial S strategies. At every round he will be choosing the action $a_{i,S}^{\mu(t)}$ where: - S = strategy chosen - i = index of agent - $\mu(t)$ = the past record of victories for the specific agent for the given iteration

Simulation: In every turn all agent will be called to give a choice based on those the simulatino engine will give: - to the minority a win (1) - the majority a loss (-1).

Strategy selection:

- strategy selection followes the folloing patter initially we select a number of S strategies from the possible 2^{2^M}
- in every iteration will throw a roulette calculation and select from the S strategies
- if he won based on some strategy a sccore is assigned to bias the selecction towards that strategy.
- Important note: in the experimentation I also tried strategies totally at totally random, and filter between a set of random strategies.

[1]:
```
import random
import numpy as np
import itertools
import copy
import time
import matplotlib.pyplot as plt
import statistics

def getStrategy(M):
    return "".join([str (random.choice([0,1])) for i in range(0,2**M)])


def roullette(choices, weights):
```

```
    choices_dict = {}
    for index, k in enumerate(choices):
          choices_dict[k]=weights[index]
    max = sum(choices_dict.values())
    pick = random.uniform(0, max)
    current = 0
    for key, value in choices_dict.items():
        current += value
        if current > pick:
            return key


class Agent():
    def __init__(self, strategies, possible_history, M):
        # for M = 3, 8 answers for 8 combination
        assert len(strategies[0]) == len(possible_history)
        self.history = [str (random.choice([0,1])) for i in range(0,M)]
        self.history = "".join(self.history)
        self.strategy_scores= [0 for _ in range(S)]
        self.strategies = strategies
        self.current_choice = random.choice([0,1])
        self.chosen_strategy_index = random.choice(range(S))

    def choose(self, iteration_number, possible_historyDict):
        # chosen_strategy_index = random.choice(range(S))
        chosen_strategy_index = roullette(range(S), self.strategy_scores)
        chosen_strategy = self.strategies[self.chosen_strategy_index]
        index = possible_historyDict[self.history]
        a = int(chosen_strategy[index])
        if a == 0:
          a = -1
        self.current_choice = a
        return a

    def set_agent_history(self, A):
        self.history=self.history[1:]
        g =- A  * self.current_choice
        # agent lost belongs to majority
        if g > 0:
            self.history += "1"
            self.strategy_scores[self.chosen_strategy_index]+=1
            # agent won belongs to minority
        if g < 0:
            self.history += "0"


class MinorityGame():
```

```
    def __init__(
        self,
        N,
        M,
        S,
        possible_history,
        iterations_per_simulation,
        possible_historyDict
    ):
        self.A = []
        self.agents = []
        for i in range(N):
            # store available strategies per agent
            self.strategies = []
            self.strategies = [getStrategy(M) for _ in range(S)]
            self.agents.append(Agent(self.strategies, possible_history, M))

        for iteration_number in range(0, iterations_per_simulation):
            moves = []
            for agent_index in range(0,N):
                moves.append(self.agents[agent_index].choose(iteration_number,␣
↪possible_historyDict))
            self.A.append(sum(moves))
            # A in the last position contains who lost an d who won
            for agent_index in range(0,N):
                self.agents[agent_index].set_agent_history(
                    self.A[-1]
                )

    def get_Variance_A(self):
        # !fun fact np.variance is not behaving same as statistics.variance 2␣
↪hours lost there :D
        # statistics.variance([1,0,-1]) =1
        # np.var([1,0,-1])=0,66
        return np.var(self.A)

    def get_A(self):
        return self.A
```

1.1 Experiments

1.1.1 Exercicse 2)

Draw the curve describing how the variance σ^2/N of the participation A depends on the parameter $a = 2^M/N$. Here N denotes the number of agents and the length of the historical window taken into consideration by an agent.

Solution We run an experiment for S = 2, M = 1..15, iterations = 300 and N=101

[2]:
```
def experiment(
    N,
    M,
    S,
    iterations_per_simulation
):
        possible_history = ["".join(seq) for seq in itertools.product("01",␣
↪repeat=M)]
        possible_historyDict=dict(enumerate(possible_history))
        possible_historyDict=dict(map(reversed, possible_historyDict.items()))
        return MinorityGame(
                    N,
                    M,
                    S,
                    possible_history,
                    iterations_per_simulation,
                    possible_historyDict
        )
```

[3]:
```
N=101
M=10
S=2
iterations_per_simulation = 500

res = experiment(
    N,
    M,
    S,
    iterations_per_simulation
)
res.get_Variance_A()
```

[3]: 94.323136

2 When S = 2 (Sbeing the number of strategies), what is the critical value $_c$ for which σ^2/N reaches a minimum ?

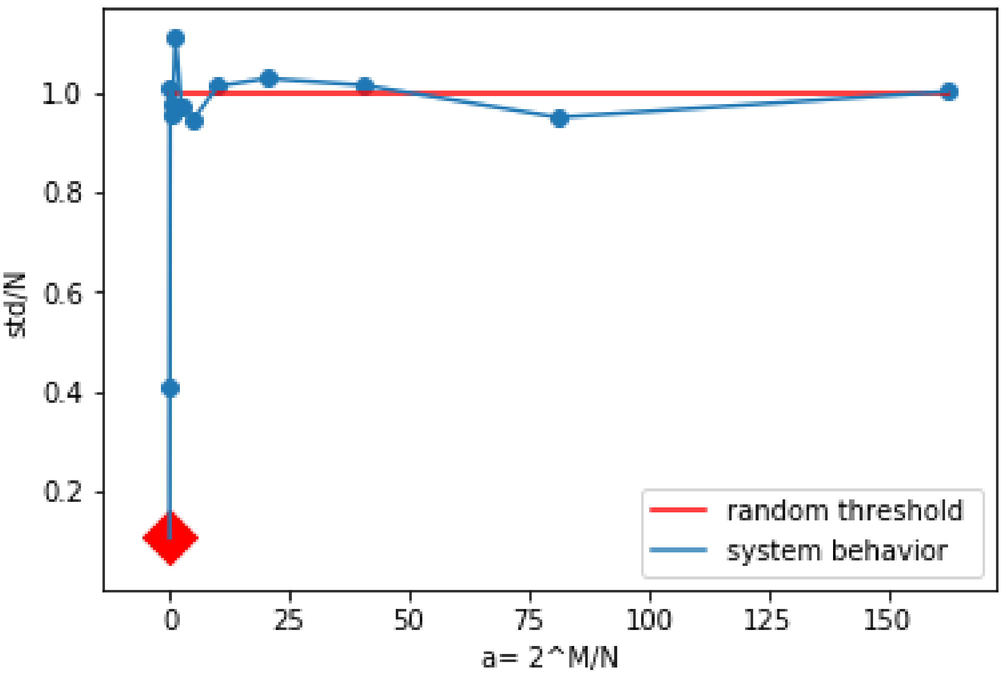
2.0.1 Experiment 1

I create two experiments to answer the question above. In the first we keep N steady and modify M. Due to exponential calcultion time which we otherwise would have to compensate with exponential storage, I couldn't manage to complete experiments for bigger than M = 20. That said if we ignore the first iterations we can see that variance converge towards 1 which is expected. I am not sure if the critical a parameter is correct since it is always found in the M = 1 which is weird and not complying to the theory. I suspect some issue with my code and this is verified by the second experiment where I iterate through N (number of agents) keeping M steady.

2.0.2 Some notes that may faciliate deducting issues with my code.

- initial conditions are random : (The setting of history and the choice of the strategies)
- choice of strategy is based on history, I reward strategies with +1 score and I through weighted probability using roullete algorithm to perform its calculation .
- random choice (ignoring history) and strategy application demonstrate similar behavior.

[4]:
```
S=2
simulations_varying_M = []
MS= range(2,15)
N=101
iterations_per_simulation = 300
for M in MS:
        simulations_varying_M.append(
            experiment(
                N,
                M,
                S,
                iterations_per_simulation
            )
        )



sigmasN = np.asarray([s.get_Variance_A() for s in simulations_varying_M]) /␣
 ↪float(N)
alpha_list = np.asarray([2**M for M in MS]) /float(N)

plt.plot(alpha_list, [1 for _ in range(0, len(alpha_list))], 'r-')
plt.scatter(alpha_list, sigmasN)
plt.plot(alpha_list, sigmasN)
plt.scatter(x=np.argmin(min(sigmasN)), y=min(sigmasN), c='red', marker='D',␣
 ↪s=200)
plt.xlabel("a= 2^M/N")
plt.ylabel("std/N")
plt.legend(["random threshold ", "system behavior"])
plt.show()
a_c= min(alpha_list)
print("a_c = ", a_c)
alpha_list, [s.get_Variance_A() for s in simulations_varying_M]
```




```
a_c =  0.039603960396039604
```

[4]:
```
(array([3.96039604e-02, 7.92079208e-02, 1.58415842e-01, 3.16831683e-01,
        6.33663366e-01, 1.26732673e+00, 2.53465347e+00, 5.06930693e+00,
        1.01386139e+01, 2.02772277e+01, 4.05544554e+01, 8.11089109e+01,
        1.62217822e+02]),
 [10.868622222222221,
  41.54222222222222,
  101.64493333333331,
  98.78555555555555,
  96.27,
  112.35128888888889,
  98.24782222222224,
  95.5404,
  102.18039999999999,
  103.74648888888888,
  102.38306666666666,
  95.8712888888889,
  101.09382222222223])
```

I also tried to keep M steady and loop through N

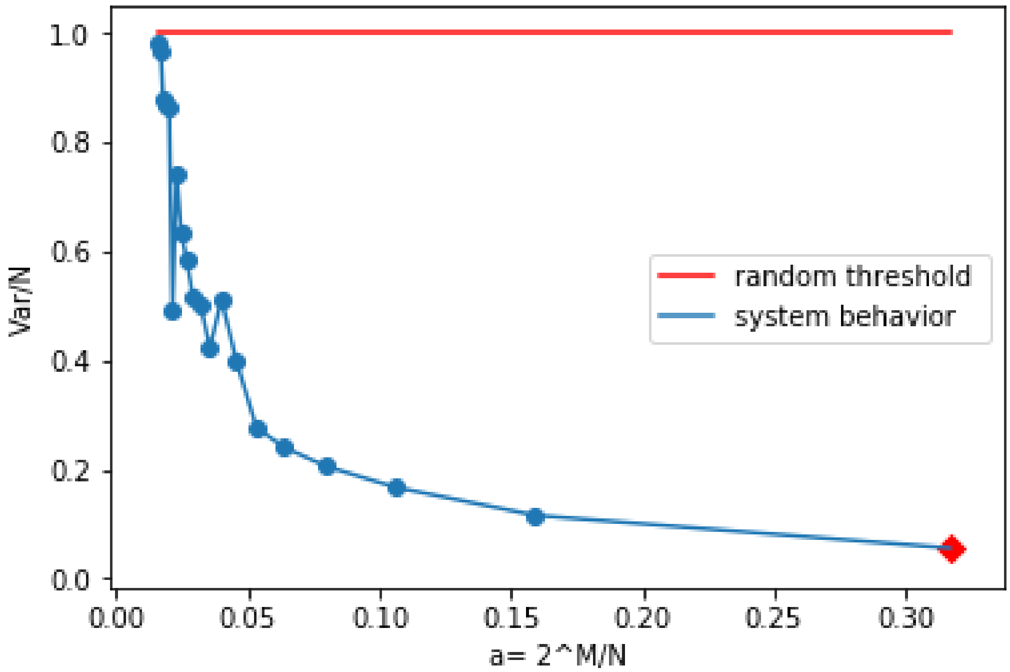
[5]:
```
S=2
simulations_M_stead_N_varying = []
N_LIST = [x for x in range(101,2101, 100)]
M=5
```

```
iterations_per_simulation = 1000
for N in N_LIST:
    simulations_M_stead_N_varying.append(
        experiment(
            N,
            M,
            S,
            iterations_per_simulation
        )
    )
```

2.0.3 Experiment 2

Although the first experiment has some bad characteristics the second one behaves more weird. Theoritically they should behave similarily instead it is like like the $f(x) = 1/x $, without converging to variance = 1 as it should.

[6]:
```
def visualize_S2(M, N_LIST,simulations_M_stead_N_varying):

    alpha_list = np.asarray([(2**M)/N for N in N_LIST])
    sigmasN = np.asarray([s.get_Variance_A() for s in␣
 ↪simulations_M_stead_N_varying]) / N

    # draw line of threshold at variance value 1
    plt.plot(alpha_list, [1 for _ in range(0, len(alpha_list))], 'r-')

    # draw data points and lines
    plt.scatter(alpha_list, sigmasN); plt.plot(alpha_list, sigmasN)

    # draw minimum point
    plt.scatter(x=alpha_list[np.argmin(min(sigmasN))], y=min(sigmasN), c='red',␣
 ↪marker='D', s=50)
    plt.xlabel("a= 2^M/N");    plt.ylabel("Var/N");    plt.legend(["random␣
 ↪threshold ", "system behavior"])
    plt.show()


visualize_S2(M, N_LIST,simulations_M_stead_N_varying)
# M, NS, simulations_M_stead_N_varying
```



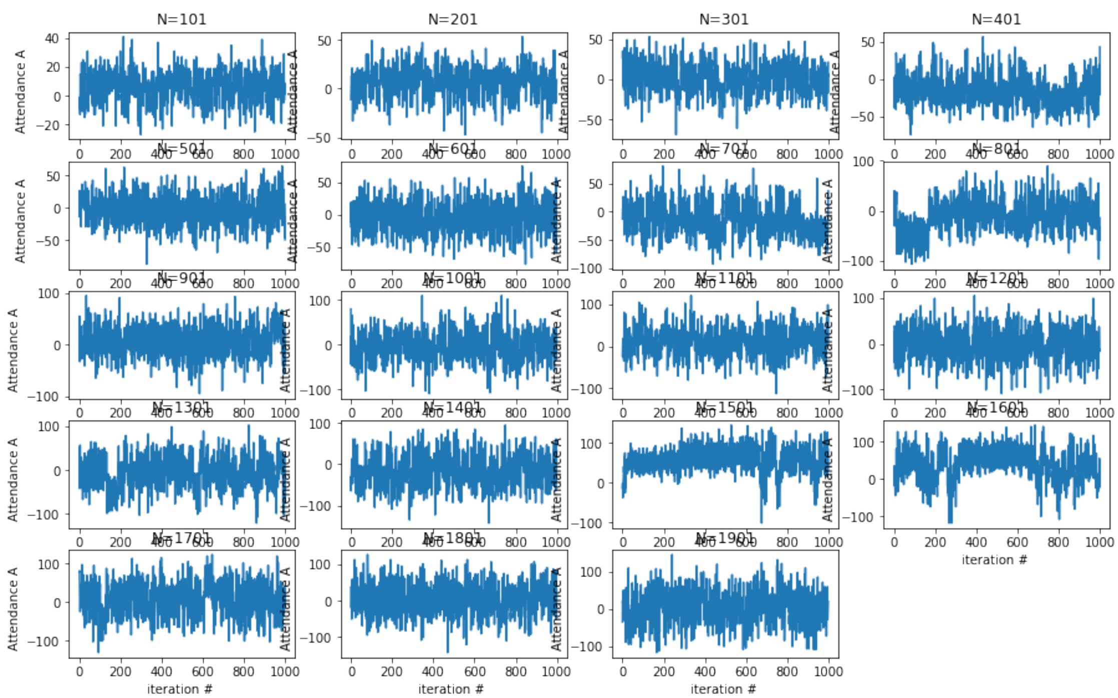
2.0.4 Attendance plots

I created some attendance plots to observe the behavior for every N in correlation with the attendance A.This way we can observer seperately the behavior for every differnet value of M. It is strange that variance has an increasing tendency.

[7]:
```
def showPlots(plots, titles):
    # TODO I tried to apply padding among figures but couldn't manage it :/
    plt.tight_layout(pad=2., w_pad=1., h_pad=2.0)
    fig=plt.figure(figsize=(15, 15))
    columns = 4
    rows = 8
    k=1
    for i in range(1, len(plots)):
        fig.add_subplot(rows, columns, i)
        plt.plot(*plots[k-1])
        plt.xlabel("iteration # ")
        plt.ylabel("Attendance A  ")

        plt.title(titles[k-1])
        k+= 1
    plt.show()

Attendances = np.asarray([s.get_A() for s in simulations_M_stead_N_varying])
plots=[]
```

```
titles = []
for A in range(len(Attendances)):
            titles.append("N="+ str(N_LIST[A]))
            Iterations = np.asarray([x for x in␣
 ↪range(0,iterations_per_simulation)])
            plots.append((Iterations, Attendances[A]))
showPlots(plots[:31], titles[:31])
len(plots), len(titles)
```

```
<Figure size 432x288 with 0 Axes>
```



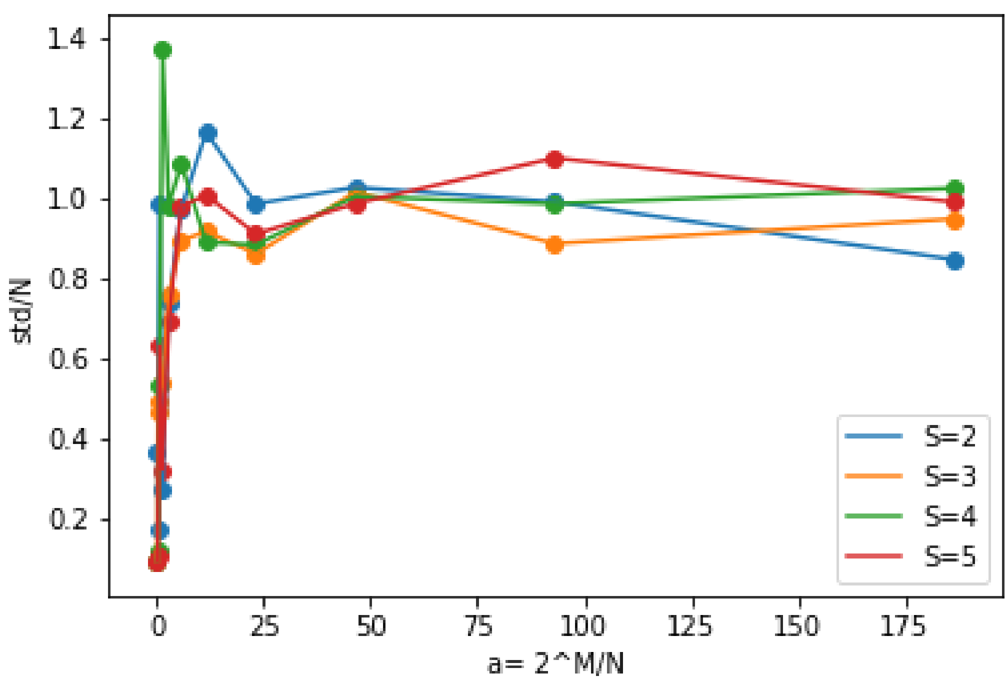
[7]:
```
(20, 20)
```

2.1 Multiple strategies experimentations

Finally I experiment with : - various agents number - for sizes of history M - with a prefixed set of strategies per agent and on which every agent will be choosing one at every iteration - with the meta-strategy (how who choose strategies)

[8]:
```
N = 11
S_num = range(2,6)
iterations_per_simulation=300
MS= range(1,12)

for S in S_num:
```

```
    simulations_S = []

    for M in MS:
        simulations_S.append(
                experiment(
                    N,
                    M,
                    S,
                    iterations_per_simulation
                )
        )

    simulations_S = [s.get_Variance_A() for s in simulations_S]
    sigmas = simulations_S
    sigmasN = np.asarray(sigmas)/ N
    aN = np.asarray([2**M for M in MS])/N

    plt.plot(aN, sigmasN)
    plt.scatter(aN, sigmasN)
    plt.legend(["S="+str(s) for s in S_num])
    plt.xlabel("a= 2^M/N")
    plt.ylabel("std/N")
plt.show()
```



Again we notice the concergence of the variance towards 1 in the assymetric phase (after a_c) but the symmetric phase is not clear. The influence of strategy is minimal and doesnt change the overall behavior of the graph which is correct.

3 Conclusions

Although at first glance a simple problem, I quickly got problems with calcuation times of M. I tried to try to approach metaheuristically having studies some papers about the behavior of the problem. The behavior of the system should be strongly corelated in M and N parameters and although the ideal corelation was not derived an influence is visible.

The a_c although doubtfully appearing on the M=1 it correctly converges towards variance 1.