Homework 1.

- The file name of your homework (in PDF) should be in the format: “學號-作業編號.pdf”. For example: 00957999-hw1.pdf
- Please submit your homework to Tronclass **before 23:59, October 19, 2022.**

(可以用 word 檔寫完後轉成 pdf 檔上傳，或是手寫後拍照後存成 pdf 檔上傳)

1. (10%) Order the following function by growth rate in increasing order (由小到大排序):

   $5n^2$, 2^n, $n\ log(log\ n)$, $n\ log(n^2)$, n^n, n!

	1	2	3	4	5	6	7	8	9
$5n^2$	5	20	45	80	125	180	245	320	405
2^n	2	4	8	16	32	64	128	256	512
$n\ log(log\ n)$	log(log1)	2log(log2)	…	…	…	…	…	….	9log(log9)
$n\ log(n^2)$	log1	2log4	3log9	4log16	5log25	…	…	…	9log81
n^n	1	4	27	64	3125	46656	823543	16777216	387420489
n!	1	2	6	24	120	720	5040	40320	362880

Ans: $n\ \log(\log n) < n\ \log(n^2) < 5n^2 < 2^n < n! < n^n$

2. (20%) Show that the following statements are correct:

(a) $n! = O(3n^n)$

(b) $5n^2 - 20n = \Theta(n^2)$

(a): 存在 C, $n_0 > 0$, 當 $n >= n_0$ 時, $n! <= C3n^n$

$n! <= 3n! <= 3n^n <= C3n^n$ 當 C = 1 $n_0 = 1$ 不等式成立

(b): 證明 $5n^2 – 20n = O(n^2)$ &證明 $5n^2 – 20n = Omega(n^2)$

存在 C, $n_0 > 0$, 當 $n >= n_0$ 時, $5n^2 – 20n <= Cn$

$5n^2 – 20n <= 5n^2 <= Cn^2$ 當 C = 5, n = 1 時不等式成立

存在 C, $n_0 > 0$, 當 $n >= n_0$ 時, $5n^2 – 20n >= Cn^2$

$4n^2 + n^2 – 20n >= 4n^2$ (當 n>=20) $>= Cn^2$ 當 C = 2, $n_0 = 20$ 得證

3. (20%) Show that the following statements are incorrect:

(a) $10n^2 + 100n + 9 = O(n)$

(b) $3^n = O(2^n)$

(a): 不存在 C, $n_0 > 0$, 當 $n >= n_0$ 時, $10n^2 + 100n + 9 <= Cn$

反證:

假設 C, $n_0 > 0$ 存在 當 $n >= n_0$, $10n^2 + 100n + 9 <= Cn$

$Cn <= n^2 <= 10n^2 < 10n^2 + 100n + 9 <= Cn$

(當 n >= C 的時, 左式成立) 則 $n > max(n_0, C)$ 不等式成立, 矛盾

(b): 不存在 C, $n_0 > 0$, 當 $n >= n_0$ 時, $3^n <= C2^n$

反證:

假設 C, n0 > 0 存在 當 $n >= n_0$, $3^n <= C2^n$

取 log-> nlog3 <= (logC + nlog2)

logC + nlog2 < nlog3 <= (logC + nlog2)

當 C < n(log3 – log2) 時, 左式成立 矛盾

4. (20%) Fill the "accurate" step count table (yellow regions) for the following program:

Let MAX_SIZE = M

Statement	s/e	Frequency	Total Steps
void *mult*(**int** *a*[][*MAX_SIZE*],**int** *b*[][*MAX_SIZE*], **int** *c*[][*MAX_SIZE*])	0	0	0
{	0	0	0
int *i*, *j*, *k*;	0	0	0
for (*i* = 0; *i* < *MAX_SIZE*; *i*++)	1	$M + 1$	$M + 1$
for (*j* = 0; *j* < *MAX_SIZE*; *j*++) **{**	1	$M(M + 1)$	$M^2 + M$
c [*i*][*j*] = 0;	1	M^2	M^2
for (*k* = 0; *k* < *MAX_SIZE*; *k*++)	1	$M^2(M + 1)$	$M^3 + M^2$
c [*i*][*j*] += *a* [*i*][*k*] * *b* [*k*][*j*];	1	M^3	M^3
}	0	0	0
}	0	0	0
Total			$2M^3+3M^2+2M+1$

5. (10%) Given an four-dimensional array A[100][200][300][400], if the address of A[4][18][29][36] is α + X, what is the value of X ?

Note: (1) α is the address of A[0][0]

(2) Only consider row major here.

4*200*300*400 + 18*300*400 + 29*400 + 36 = 98171636

Ans: 98171636

6. (10%) 請說明如何使用課本 Program 2.6 的 padd 函式，計算多項式乘法 $A(x) * B(x)$，其中

$$A(x) = 3x^2 + 2x + 1$$
$$B(x) = 5x^3 + 4x - 1$$

```
int solve(int n, int k){
    int idx = 0;
    for(int i = 0; i < n; i++){
        for(int j = 0; j < k; j++){
            C[idx].coef = A[i].coef * B[j].coef;
            C[idx++].expon = A[i].expon + B[j].expon;
        }
    }
    int now = 0;
    for(int i = 0; i < idx; i++){
        if(C[i].vis) continue;
        ans[now].expon = C[i].expon;
        ans[now].coef = C[i].coef;
        for(int j = i + 1; j < idx; j++){
            if(C[i].expon == C[j].expon){
                ans[now].coef += C[j].coef;
                C[j].vis = 1;
            }
        }
        now++;
    }
    return now;
}
```

將 A 和 B 的所有項相乘

乘完以後同項合併

其中 now - 1 代表最高項次

7. (10%) Ackerman's function $A(m,n)$ is defined as:

$$A(m,n) = \begin{cases} n+1, & \text{if } m = 0 \\ A(m-1,1), & \text{if } n = 0 \\ A(m-1, A(m,n-1)), & \text{otherwise} \end{cases}$$

What is the value of $A(3,4)$?

A(m, n) = n + 1 when m = 0

n + 2 when m = 1

A(3, 4) = A(2, A(3, 3))

A(3, 3) = A(2, A(3, 2))

A(3, 2) = A(2, A(3, 1))

A(3, 1) = A(2, A(3, 0))

A(3, 0) = A(2, 1)

A(2, 1) = A(1, A(2, 0)) = A(1, 3) = 5

A(2, 0) = A(1, 1) = 3

A(2, 5) = A(1, A(2, 4)) = 13

A(2, 4) = A(1, A(2, 3)) = 11

A(2, 3) = A(1, A(2, 2)) = 9

A(2, 2) = A(1, A(2, 1)) = A(1, 5) = 7

由這幾項得出

→A(2, n) = 2(n + 3) – 3 此規律後再反推

A(3, 4) = A(2, A(3, 3))

A(3, 3) = A(2, A(3, 2))

A(3, 2) = A(2, A(3, 1))

A(3, 0) = A(2, 1) = 5

A(3, 1) = A(2, A(3, 0)) = A(2, 5) = 13

A(3, 2) = A(2, A(3, 1)) = A(2, 13) = 29

A(3, 3) = A(2, A(3, 2)) = A(2, 29) = 61

A(3, 4) = A(2, A(3, 3)) = A(2, 61) = 125

Ans: 125