A Kernel Test for Three-Variable Interactions with Random Processes

Abstract

Explain what this is all about, and the main contributions:

- Applied Wild Bootstrap to Lancaster test statistic
- Main theoretical challenge was to show that the conditions required to apply WB are satisfied by Lancaster
- This was done in a novel way - rather than using the Hoeffding decomposition, we come up with a new method which is simpler, (but requires an extra condition on the timeseries?)
- We also show that the power of the Lancaster test described in Arthur's original paper can be improved - we show that they used conservative p-values

1. Introduction

- Describe three variable interaction. It is particularly useful for cases in which any pairwise interaction is weak, but that the three variables interact strongly together.

- Test consists of two parts - calculating the test statistic, and bootstrapping the statistic to sample from the null in order to calculate the p-value threshold.

- When using time series, the difficult part is the bootstrapping because shuffling the indices breaks the temporal dependence structure.

- In [Leucht], they give a method for bootstrapping a certain class of statistics.

- The main contributions of this paper are the folllowing:
  - To show that the Lancaster test statistic is such a statistic
  - This is done using a new style of technique which in particular gives a significantly simpler proof that HSIC is also such a statistic (and thus simplifies the proofs used in [HSIC+time series])
  - To show that the multiple testing corrections used in [Lancaster] are too conservative, and therefore that we can improve test power by using a more relaxed correction.

This work combines the works of [HSIC + time series] and [Lancaster interaction] to give a non-parametric test for three variable interactions in which the samples are drawn from random processes.

2. Background

- Kernel mean embedding

- Lancaster

- Time series
  - τ-mixing
  - β-mixing
  - Lemma that sub-processes of β-mixing processes are β-mixing

- V-statistics

- Hilbert space valued random variable central limit theorem

3. Lancaster Interaction for Random Processes

- Statement of Wild Bootstrap theorem (maybe in background though?)

- Proof that Lancaster satisfies WB theorem hypothesis

- ...

- Multiple testing correction (maybe in next section though?)

Preliminary work. Under review by the International Conference on Machine Learning (ICML). Do not distribute.

4. p-values for Lancaster test

- In [Lancaster], they use the Holm-Bonferroni correction. Show here that this isn't actually necessary - that the 'naive' correction works and is therefore more powerful as we use $[\alpha, \alpha, \alpha]$ as the thresholds rather than $[\alpha/3, \alpha/2, \alpha]$ or whatever.

5. Experiments

5.1. Artificial data

5.2. Real data

Maybe check this out for some data? https://stat.duke.edu/~mw/ts_data_sets.html

6. Proofs

Acknowledgments

cheers!