

AUTOMATA THEORY & FORMAL

LANGUAGES

Tutorial 1: DFA, NFA

Pre Tutorial

1) Define NFA and DFA normally?

a) DFA:-

For each input symbol, one can determine the state to which the machine will move. As it has a finite number of states, the machine is called Deterministic Finite Machine / Deterministic Finite Automaton.

A DFA can be represented by a 5-tuple

$(Q, \Sigma, \delta, q_0, F)$ where

Q - finite set of states

Σ - finite set of symbols called alphabet

δ - transition function where $\delta: Q \times \Sigma \rightarrow Q$

q_0 - initial state ($q_0 \in Q$)

F - set of ~~final~~ final state/states of Q
($F \subseteq Q$)

NFA:-

The finite automata are called NFA when there exist many paths for specific

Input from the current state to next state
Each NFA can be translated into DFA but
every NFA is Non DFA (Here $d = Q \times \Sigma \rightarrow Q$)

The two exceptions are

- * It contains multiple next states
- * It contains ϵ transitions.

2) Construct a DFA that accepts the language
 $L = \{w \in \{0,1\}^* \mid w \text{ contains } 1001 \text{ or } 0110\}$

3) Write the steps for converting E-NFA to DFA and vice versa with an example for each?

A) Steps for converting NFA to DFA

step 1: Initially $Q' = \phi$

step 2: Add q_0 of NFA to Q' . Then find the transitions from this start state.

step 3: In Q' , find possible set of states for each input symbol, if this set of states is not in Q' , then add it to Q' .

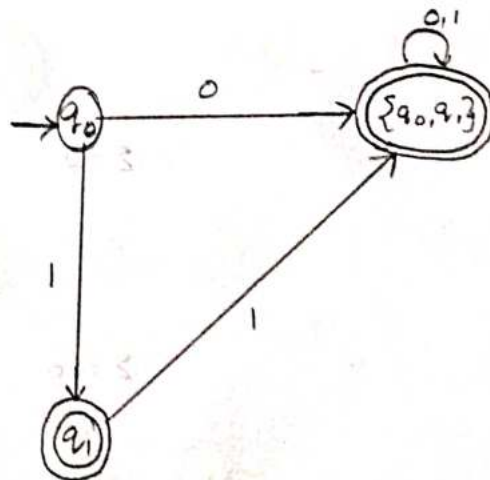
step 4: In DFA, the final state will be all states which contain F (Final states of NFA)

Ex:-



	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	ϕ	$\{q_0, q_1\}$

	0	1
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_1\}$
$*\{q_1\}$	ϕ	$\{q_0, q_1\}$
$*\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1\}$



Steps for converting DFA to NFA

step 1: Let's assume DFA D has state set

$$Q = \{q_0, q_1, \dots, q_n\}$$

step 2: Now we build NFA N as follows

(1) start with DFA D.

(2) Add an additional accepting state for NFA N, such that N will have $n+1$ total no. of states

Let's call new accepting state q_{n+1}

(3) Now, add an epsilon ϵ transition from all accepting states to new

accepting state q_{n+1} and make all

the original accepting states just normal states.

DONE

Ex:- $\Sigma = \{0, 1\}$ starts with '0'?

$$L = \{0, 00, 01, 000, 001, 010, 0010, 0101, \dots\}$$



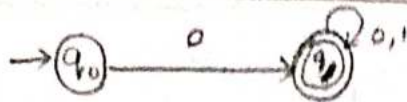
$\Sigma = 0$



$\Sigma = 00$



$\Sigma = 01$

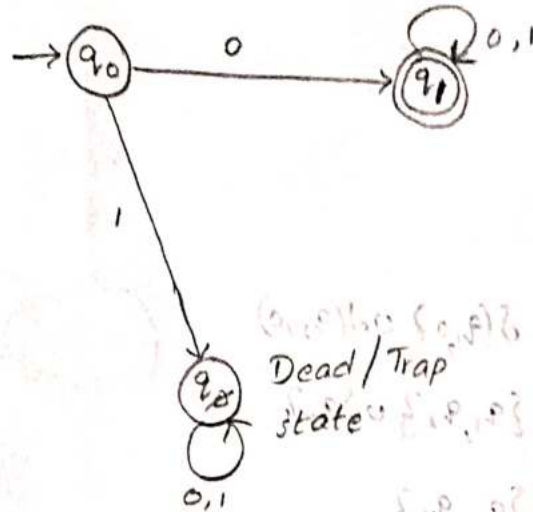


$\Sigma = 000$



$\Sigma = 001$ (we cannot draw)

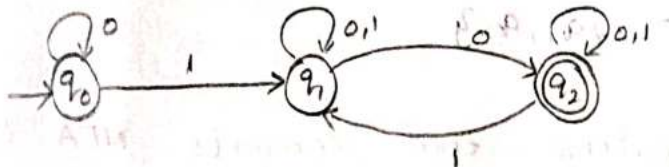
So,



Dead/Trap state

In-tutorial

1) Convert the following NFA to DFA

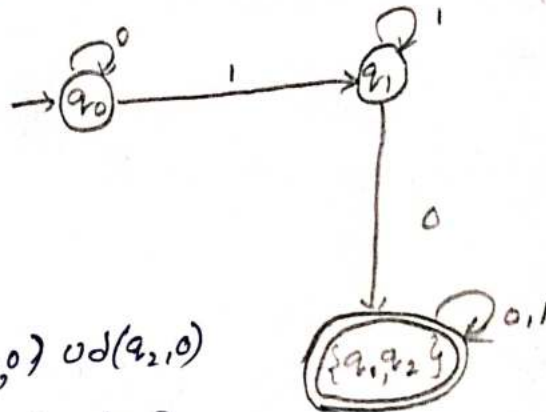


A)

	0	1
q_0	q_0	q_1
q_1	$\{q_1, q_2\}$	q_1
q_2	q_2	$\{q_1, q_2\}$

Now,

	0	1
$\rightarrow q_0$	q_0	q_1
q_1	$\{q_1, q_2\}$	q_1
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$



$$\begin{aligned}
 \delta(\{q_1, q_2\}, 0) &= \delta(q_1, 0) \cup \delta(q_2, 0) \\
 &= \{q_1, q_2\} \cup \{q_2\} \\
 &= \{q_1, q_2\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(\{q_1, q_2\}, 1) &= \delta(q_1, 1) \cup \delta(q_2, 1) \\
 &= \{q_1\} \cup \{q_1, q_2\} \\
 &= \{q_1, q_2\}
 \end{aligned}$$

2) Write the algorithm that converts NFA to DFA. Explain your algorithm works using the below NFA?

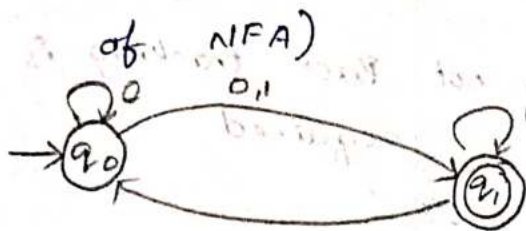
A) algorithm

step 1: Initially $Q' = \emptyset$

step 2: Add q_0 of NFA to Q' of DFA

step 3: In Q' , find possible states for each input symbol. If this set of states is not in Q' , then add it to Q'

step 4: In DFA, the final state will be all states which contain F (final state)

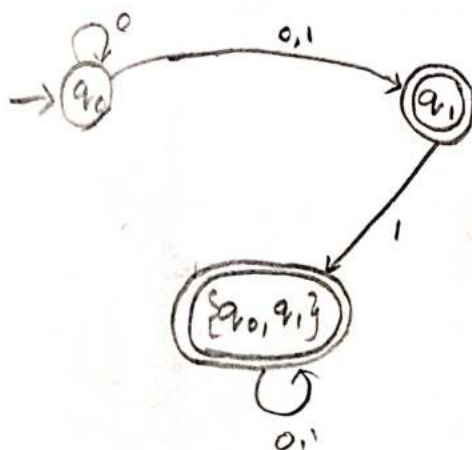


	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$* q_1$	\emptyset	$\{q_0, q_1\}$

	0	1
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	\emptyset	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1\}$

$$\begin{aligned} \delta(\{q_0, q_1\}, 0) &= \delta(q_0, 0) \cup \delta(q_1, 0) \\ &= \{q_0, q_1\} \cup \{\emptyset\} \\ &= \{q_0, q_1\} \end{aligned}$$

$$\begin{aligned} \delta(\{q_0, q_1\}, 1) &= \delta(q_0, 1) \cup \delta(q_1, 1) \\ &= \{q_1\} \cup \{q_0, q_1\} \\ &= \{q_0, q_1\} \end{aligned}$$



Post-Tutorial

1) Differentiate NFA and DFA

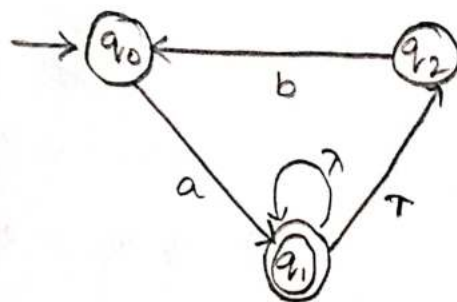
	NFA	DFA
1	$(Q, \Sigma, \delta, q_0, F)$ $\delta = Q \times \Sigma \rightarrow 2^Q$	$(Q, \Sigma, \delta, q_0, F)$ $\delta = Q \times \Sigma \rightarrow Q$
2	$Q = \{q_0, q_1\}, \Sigma = \{0, 1\}$ $q_0 \rightarrow 0 \quad q_1 \rightarrow 1$	$Q = \{q_0, q_1\}, \Sigma = \{0, 1\}$

3	Transition may leads to multiple states	Transition leads to unique state
4	Back tracking is not required	Back tracking is required
5	Practical implementation of DFA is feasible	Not feasible, convert NFA to DFA

2) Construct NFA for language $L = \{wc \mid w \in \{0,1\}^*\}$
 $|a^* + b^*|$

a)

3) Convert the following ϵ -NFA to DFA



A)

	a	b	λ
→ q ₀	q ₁	∅	∅
q ₁	∅	∅	{q ₁ , q ₂ }
q ₂	∅	q ₀	∅

	a	b	λ
→ q ₀	q ₁	∅	∅
q ₁	∅	∅	{q ₁ , q ₂ }
{q ₁ , q ₂ }	∅	q ₀	{q ₁ , q ₂ }

$$\delta(\{q_1, q_2\}, a)$$

$$= \delta(q_1, a) \cup \delta(q_2, a)$$

$$= \emptyset \cup \emptyset = \emptyset$$

$$\delta(\{q_1, q_2\}, b)$$

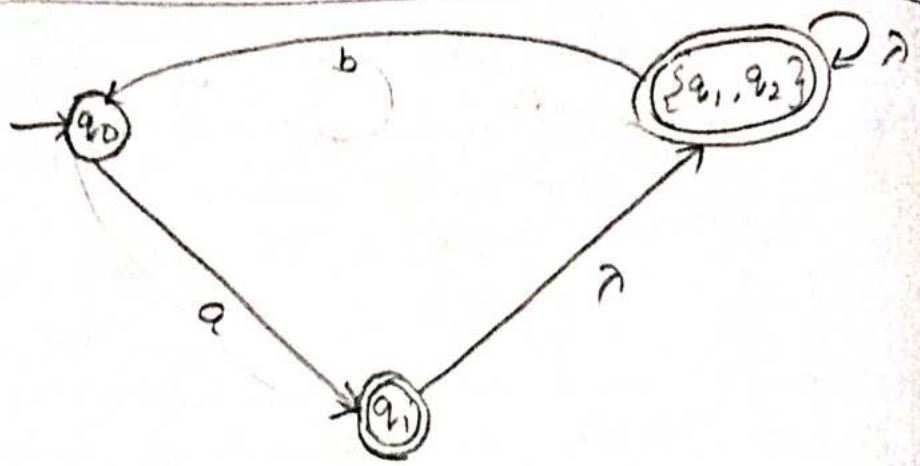
$$= \delta(q_1, b) \cup \delta(q_2, b)$$

$$= \emptyset \cup q_0 = q_0$$

$$\delta(\{q_1, q_2\}, \lambda)$$

$$= \delta(q_1, \lambda) \cup \delta(q_2, \lambda)$$

$$= \{q_1, q_2\} \cup \emptyset = \{q_1, q_2\}$$



Tutorial 2: Regular Expression.

Pre Tutorial

- 1) Explain regular expression and name some of identity rules for the regular expression?
Assume a, b and c are regular expressions in the identity rules.

A) Regular Expression:

It is used for representing certain sets of strings in algebraic function fashion

Identity rules:

$$\phi + r = r$$

$$\phi \cdot r = \phi \cdot \phi = \phi$$

$$\epsilon \cdot r = r \cdot \epsilon = r$$

$$r + r = r$$

$$r^* \cdot r^* = r^*$$

$$r^* r = r r^* = r^*$$

$$(r^*)^* = r^*$$

$$R R^* = R^* R = R^+$$

$$\epsilon + r^* r = \epsilon + r r^* = r^*$$

$$(ab)^* a = a (ba)^*$$

$$(a+b)^* = (a^* b^*)^* \\ = (a^* + b^*)^*$$

$$(a+b)c = ac+bc$$

$$\epsilon^* = \epsilon$$

$$\phi^* = \epsilon$$

1) Consider language L given by regular expression $(a+b)^* b(a+b)$ over the alphabet $\{a, b\}$

Design a DFA that accepts L .

* Convert RE into NFA and then find out DFA from NFA

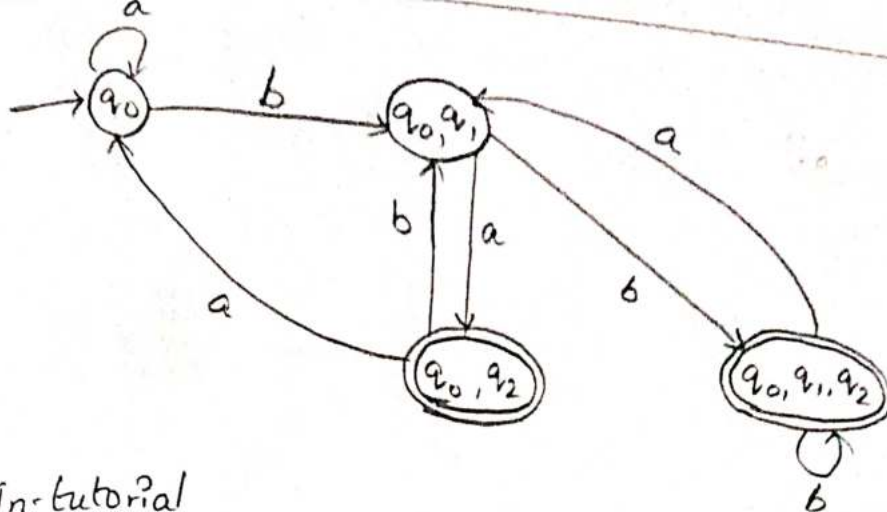
$$RE = (a+b)^* b(a+b)$$



	a	b
$\rightarrow q_0$	q_0	$\{q_0, q_1\}$
q_1	q_2	q_2
q_2	\emptyset	\emptyset

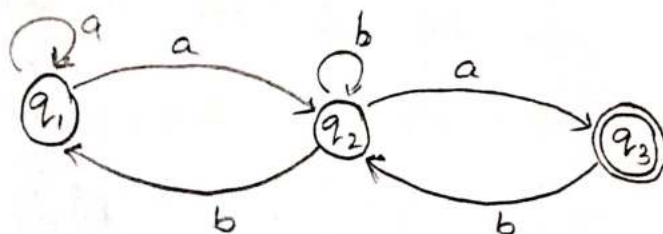
DFA table from NFA

	a	b
$\rightarrow q_0$	q_0	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$	q_0	$\{q_0, q_1\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$



In-tutorial

- 1) Provide algorithm to convert NFA into RE
Create a RE for following NFA



$$q_1 = q_1 a + q_2 b \quad \text{---}$$

A) $q_2 = q_2 b + q_1 a$

$$q_3 = q_2 a$$

Now,

$$q_1 = q_1 a + q_2 b + \epsilon \rightarrow \textcircled{1}$$

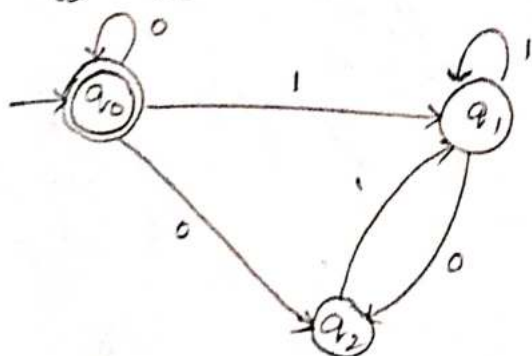
$$q_2 = q_1 a + q_2 b \rightarrow \textcircled{2}$$

$$q_3 = q_2 a \rightarrow \textcircled{3}$$

Substitute $\textcircled{2}$ in $\textcircled{1}$

$$q_1 = q_2 + \epsilon$$

a) Explain Arden's theorem to convert FA to RE. Use algorithm to convert following DFA to RE



A) Arden's Theorem

If P and Q are 2 RE over Σ and if P doesn't contain ϵ then the following equation in R by $R = Q + RP$ has a unique solution i.e., $R = QP^*$

steps:-

1) For each state q_1, q_2, q_3, \dots all exists that comes into state written in equation format

2) Add epsilon to initial state

3) Calculate all equations

4) Result is value of final state

solution for example

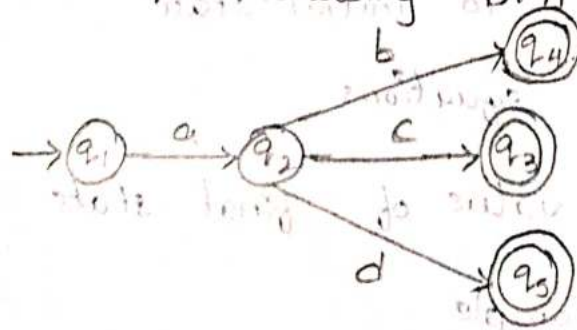
$$q_0 = q_0 0 + \epsilon \rightarrow \textcircled{1}$$

$$q_1 = q_0 1 + q_1 1 \rightarrow \textcircled{2}$$

$$q_2 = q_0 0 + q_1 0 \rightarrow \textcircled{3}$$

Post Tutorial

B) Find RE for following DFA



A)

$$① \leftarrow 0,1P + 0,0P = 0P$$

$$② \leftarrow 1,0P + 1,0P = 1P$$

$$③ \leftarrow 0,1P + 0,0P = 0P$$

2) For $\Sigma = \{a, b\}$ let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$

What could be the minimum pumping length (the constant guaranteed by the pumping lemma) for L ?

1) $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$

$$L = \{a^1, a^2, a^3, a^4, \dots \cup b^{10}, b^{20}, b^{30}, \dots\}$$

Pumping Lemma:

Let L be an infinite RL. Then there exists some positive integer m such that any $w \in L$ with $|w| \geq m$ can be decomposed as

$$w = xyz$$

with $|xy| \leq m$ such that $w_i = xy^i z$

is also L for all $i = 0, 1, 2, \dots$

\therefore minimum pumping length should be 11,

because string with length 10 ($w = b^{10}$) does not repeat anything, but string with length 11 (i.e., $w = b^{11}$) will repeat states

length of pumping lemma is 24

Tutorial - 3

Context Free Grammar, left most and Right most deviation.

Pre Tutorial

1) What is context free grammar. Explain with an example?

A) Context Free Grammar:

It is a formal grammar which is used to generate all the possible patterns of strings in a given formal language.

$$G = (V, T, P, S)$$

$G \rightarrow$ grammar, which consists of set of production rules

$T \rightarrow$ lower case letter, final set of terminal symbols

$V \rightarrow$ capital letter, final set of non-terminal symbols

$P \rightarrow$ set of production rules, used for replacing non terminal symbols in (with a) a string with other terminals

$S \rightarrow$ start symbol used to derive string

Ex - Construct CFG of language having any 2 number of a's over set $\Sigma = \{a, b\}$

Sol:- $RE = (a^* + b)^* a (a+b)^* a (a+b)^*$

$S \rightarrow \bullet T a T a T$

$T \rightarrow a T / b T / \epsilon$

Now, let's take ~~another~~ $V = \{S, T\}$

$\Sigma = \{a, b\}$

$S = \{S\}$

$S \Rightarrow T a T a T$

↓

$b T a T a T$

↓

$b \epsilon a T a T$

$b a T a T$

↓

$b a b T a T$

↓

$b a b \epsilon a T$

$b a b a T$

↓

$b a b a b T$

↓

$b a b a b \epsilon$

$b a b a b$

In Tutorial

Construct a CFG for a language $L = \{w c w^R \mid w \in (a, b)^*\}$

A) $L = \{wcw^R \mid c \in (a,b)^* \}$

$S \rightarrow aSa$ rule 1

$S \rightarrow bSb$ rule 2

$S \rightarrow c$ rule 3

$s = abbcbbba$

$S \rightarrow aSa$

$S \rightarrow absba$ from rule 2

$S \rightarrow abbsbba$ from rule 2

$S \rightarrow abbcbbba$ from rule 3

2) Derive string "aabbabba" for left most derivation and right most derivation using a CFG

A) $S \rightarrow aB \mid bA$

$A \rightarrow a \mid as \mid bAA$

$B \rightarrow b \mid bs \mid aBB$

Leftmost derivation \Rightarrow Right most Derivation

Leftmost derivation:
 S
 \downarrow
 aB
 \downarrow
 $aaBB$
 \downarrow
 $aaBbs$
 \downarrow
 $aaBbba$
 \downarrow

Right most Derivation:
 S
 \downarrow
 aB
 \downarrow
 $aaBB$
 \downarrow
 $aa bB$
 \downarrow

$aaBbba$
 \downarrow
 $aa b S bba$
 \downarrow
 $aa b b A bba$
 \downarrow
 $aa b b a bba$

$aa b b S$
 \downarrow
 $aa b b a B$
 \downarrow
 $aa b b a b S$
 \downarrow
 $aa b b a b b A$
 \downarrow
 $aa b b a b b a$

Post Tutorial

D) Generate CFG for language $L = \{0^i 1^j 0^k \mid j > i+k\}$

A) $L = \{0^i 1^j 0^k \mid j > i+k\}$

let $i=1, k=1$
 $j=3$

$L = \{0^1 1^3 0^1, \dots\}$

$\Rightarrow 01110$

$S \rightarrow x y z$

$x \rightarrow 0 x 1 \mid 01$

$y \rightarrow 1 y \mid 1$

$z \rightarrow 1 z 0 \mid 10$

$S \rightarrow x y z$

\downarrow
 $0 x 1 y z$

\downarrow
 $0 0 1 1 y z$

\downarrow
 $0 0 1 1 1 y z$

\downarrow
 $0 0 1 1 1 1 z$

\downarrow
 $0 0 1 1 1 1 1 z 0$

~~$0 0 1 1 1 1 1 1 z 0$~~

\downarrow
 $0 0 1 1 1 1 1 0 0$

\downarrow
 $0 0 1 1 1 1 1 0 0 0$

(or)

$x \rightarrow 0 x 1 \mid \epsilon$

$z \rightarrow 1 z 0 \mid \epsilon$

also can use ϵ as start

001111100

$i=2 \quad j=5 \quad k=1$

$j > i+k$

Tutorial 4: Parse Tree, Ambiguity In CFG

Pre Tutorial

1) Differentiate ambiguous and unambiguous grammar?

SNO	Ambiguous Grammar	Unambiguous Grammar
1	It generates more than one parse tree	It generates exactly one parse tree
2	The leftmost & Rightmost derivation represents different parse tree	The leftmost & Rightmost derivations represents same parse tree
3	Contains a smaller number of non-terminals	Contains a greater number of non-terminals
4	length is less	length is large
5	Example $S \rightarrow S + S / S * S / id$	Example $S \rightarrow S + E / E$ $E \rightarrow E * F / F$ $F \rightarrow id$

Intutorial

1) Consider following grammar?

$$S \rightarrow A S B | c$$

$$A \rightarrow \epsilon | a A$$

$$B \rightarrow \epsilon | b B$$

$$L = \{a^n, n \geq 0\} \text{ or } a^*$$

i) Find leftmost derivation and Right most derivation

ii) Also, prove all the strings generated from this grammar have their leftmost and rightmost ^{derivation} exactly same. Draw the parse tree for the same

A)

$$L = \{a^n, n \geq 0\} \text{ or } a^*$$

$$L = \{ \epsilon, a, aa, aaa, aaaa, \dots \}$$

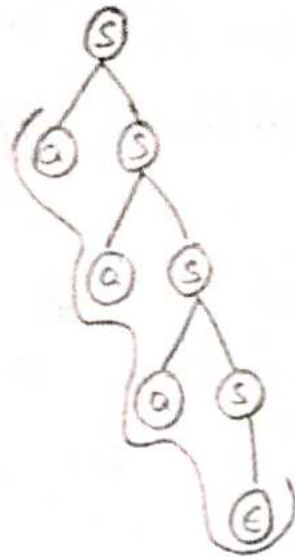
$$S \rightarrow as/\epsilon$$

Leftmost Derivation	Rightmost Derivation
$ \begin{array}{c} S \rightarrow as \\ \downarrow \\ aas \\ \downarrow \\ aaas \\ \downarrow \\ aa\epsilon \\ \text{aaa} \end{array} $	$ \begin{array}{c} S \rightarrow as \\ \downarrow \\ aas \\ \downarrow \\ aaas \\ \downarrow \\ aa\epsilon \\ \text{aaa} \end{array} $

Leftmost = Rightmost
derivation derivation

Similar is the case for all other strings

Parse Tree is



• Post-Tutorial

1) Consider the following grammar -

$$S \rightarrow sas | b$$

Is it ambiguous grammar? Generate the string "babab" from this grammar to prove your point.

A)

Chapter 10: The Role of the Nurse
10.1 Introduction

The role of the nurse is to provide care and support to patients. This involves assessing the patient's needs, planning care, and evaluating the effectiveness of the care provided. The nurse also plays a key role in educating patients and their families about health and illness.

The nurse's role is to ensure that the patient receives the best possible care. This involves working closely with the doctor and other members of the healthcare team. The nurse also provides emotional support to the patient and their family, helping them to cope with the challenges of illness and hospitalization.

The nurse's role is to ensure that the patient is safe and comfortable. This involves monitoring the patient's vital signs, administering medication, and providing basic nursing care. The nurse also ensures that the patient's privacy and dignity are maintained at all times.

The nurse's role is to ensure that the patient is involved in their care. This involves providing information about the patient's condition and treatment options, and encouraging the patient to participate in decisions about their care.

The nurse's role is to ensure that the patient is satisfied with their care. This involves listening to the patient's concerns and feedback, and making changes to the care plan as needed. The nurse also provides emotional support to the patient and their family, helping them to cope with the challenges of illness and hospitalization.

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TUTORIAL-5: Simplification of CFG, Normal forms

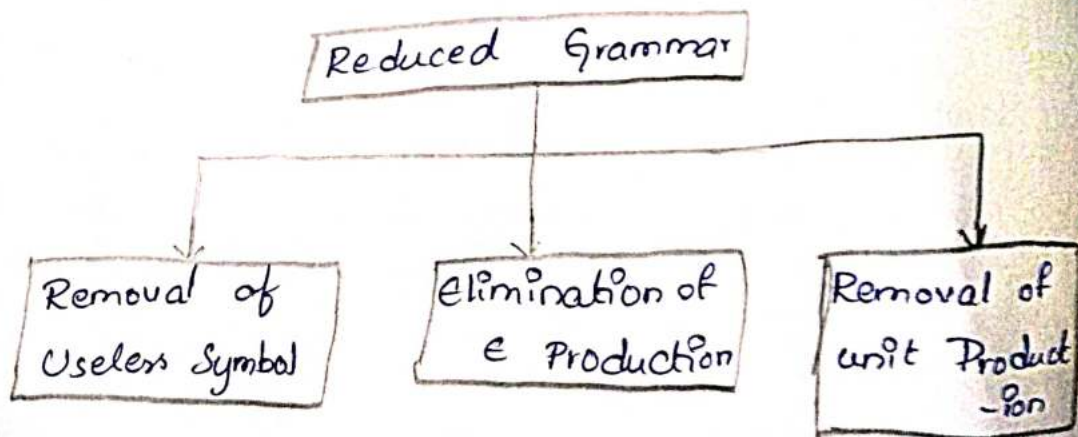
Pore Tutorial

- 1) Explain simplification of grammar? Mention its use? Elaborate the steps that are followed in simplification process?

A) Simplification of grammar:

It means reduction of grammar by removing useless symbols.

- * Each variable and each terminal of G appears in the derivation of some word in L
- * There should not be any production as $X \rightarrow Y$ where X and Y are non-terminal
- * If ϵ is not in language L there need not be production $X \rightarrow \epsilon$



Removal of Useless Symbol:

A variable can be useless, if it does not

take part in derivation of any string.

$$E_1: T \rightarrow aAB / aBA / aAT$$

$$A \rightarrow aA$$

$$B \rightarrow ab/d$$

$$C \rightarrow ad$$

Here $C \rightarrow ad$ is useless, $A \rightarrow aA$ is useless

To remove $A \rightarrow aA$, we will first find all variables which will never lead to a terminal string such as variable 'A'. Then we will remove all productions in which the variable 'A' occurs.

Elimination of ϵ (empty) Production:

$S \rightarrow \epsilon$ are called ϵ productions

step 1: Find out all nullable non-terminal variable which derives ϵ

step 2: For each production $A \rightarrow a$ construct all production $A \rightarrow x$, where x is obtained from a by removing one or more non-terminal from step 1.

step 3: Now combine the result of step 2 with original production and remove ϵ productions

Ex:- $S \rightarrow XYX$

$$X \rightarrow OX / \epsilon$$

$$Y \rightarrow IY / \epsilon$$

$$S \rightarrow x y x$$

↓

$$\epsilon y x$$

$$y x$$

↓

$$\epsilon x$$

Sol:

let us take

$$S \rightarrow x y x$$

↓

$$\epsilon y x$$

$$y x$$

$$S \rightarrow x y x$$

↓

$$x y \epsilon$$

$$x y$$

$$S \rightarrow x y x$$

↓

$$x \epsilon x$$

$$x x$$

if y and x are ϵ

$$S \rightarrow x$$

if both x are ϵ

$$S \rightarrow y$$

Now,

$$S \rightarrow x y \mid y x \mid x x \mid x \mid y$$

Consider $x \rightarrow 0 x$

replace ϵ at RHS $y x$ then

$$x \rightarrow 0$$

$$x \rightarrow 0 x \mid 0$$

Similarly $y \rightarrow 1 y \mid 1$

Rewrite the CFG as

$$S \rightarrow xy | yx | xy | x | y$$

$$X \rightarrow 0x$$

$$Y \rightarrow 1y | 1$$

Removing unit Productions:

These are productions in which one non-terminal gives another non-terminal.

Step 1: To remove $X \rightarrow Y$ add production

$X \rightarrow a$ to grammar rule whenever

$Y \rightarrow a$ occurs in this grammar

Step 2: Now delete $X \rightarrow Y$ from grammar

Step 3: Repeat step 1 and step 2 until all unit productions are removed

Ex: $S \rightarrow 0A | 1B | C$

$$A \rightarrow 0S | 00$$

$$B \rightarrow 1 | A$$

$$C \rightarrow 01$$

Sol:

$S \rightarrow C$ is unit production, By removing

$S \rightarrow C$, add a rule to S

$$S \rightarrow 0A | 1B | 01$$

$B \rightarrow A$ is also a unit production

$$B \rightarrow 1 | 0S | 00$$

Rewrite CFG

$$S \rightarrow OA / IB / OI$$

$$A \rightarrow OS / OO$$

$$B \rightarrow I / OS / OO$$

$$C \rightarrow OI$$

- 2) Find a reduced grammar equivalent to the grammar G , having production rules

$$S \rightarrow AC / B$$

$$A \rightarrow a$$

$$C \rightarrow c / BC$$

$$E \rightarrow aA / e$$

A) Phase 1 -

$$T = \{a, c, e\}$$

$$W_1 = \{A, C, E\} \text{ From rules } A \rightarrow a, C \rightarrow c, E \rightarrow aA$$

$$W_2 = \{A, C, E\} \cup \{S\} \text{ from rule } S \rightarrow AC$$

$$W_3 = \{A, C, E, S\} \cup \phi$$

Since $W_2 = W_3$, we can derive G' as-

$$G' = \{ \{A, C, E, S\}, \{a, c, e\}, P, \{S\} \}$$

where $P: S \rightarrow AC, A \rightarrow a, C \rightarrow c, E \rightarrow aA / e$

Phase 2 -

$$Y_1 = \{s\}$$

$$Y_2 = \{s, A, c\} \quad S \rightarrow AC$$

$$Y_3 = \{s, A, c, a, c\} \quad A \rightarrow a \text{ \& } C \rightarrow c$$

$$Y_4 = \{s, A, c, a, c\}$$

Since $Y_3 = Y_4$, we can derive G''

$$G'' = \{ \{A, c, s\}, \{a, c\}, p, \{s\} \}$$

where $P: S \rightarrow AC, A \rightarrow a, C \rightarrow c$

In Tutorial

1) Remove unit production from following grammar

$$S \rightarrow AC$$

$$A \rightarrow a$$

$$C \rightarrow x \mid b$$

$$x \rightarrow y$$

$$y \rightarrow z$$

$$z \rightarrow a$$

2) There are 3 unit productions in the grammar -

$$C \rightarrow x, x \rightarrow y \text{ and } y \rightarrow z$$

At first remove $y \rightarrow z$

As $z \rightarrow a$, we add $y \rightarrow a$ & $y \rightarrow z$ is removed

So,

$$S \rightarrow AC, A \rightarrow a, C \rightarrow x|b, x \rightarrow y, y \rightarrow a, z \rightarrow a$$

Now remove $x \rightarrow y$

As $y \rightarrow a$ we add $x \rightarrow a$ and $x \rightarrow y$ is removed

So,

$$S \rightarrow AC, A \rightarrow a, C \rightarrow x|b, x \rightarrow a, y \rightarrow a, z \rightarrow a$$

Now remove $C \rightarrow x$

As $x \rightarrow a$ we add $C \rightarrow a$ and $C \rightarrow x$ is removed

$$S \rightarrow AC, A \rightarrow a, C \rightarrow a|b, x \rightarrow a, y \rightarrow a, z \rightarrow a$$

Now, x, y, z are unreachable, hence we can remove those

The final CFG is unit production free

$$S \rightarrow AC, A \rightarrow a, C \rightarrow a|b$$

2) A grammar G is defined with rules

$$S \rightarrow xA|yB, B \rightarrow b|zB, x \rightarrow b, A \rightarrow a$$

Write productions obtained after normalizing G.

A)

$$S \rightarrow XA \mid BB$$

$$B \rightarrow b \mid SB$$

$$X \rightarrow b$$

$$A \rightarrow a$$

step 1: Convert grammar into CNF

step 2: If grammar exists left recursion, eliminate it

step 3: convert production rule into GNF form in the grammar

Sol: step 1 and step 2 already exist in question, so skipping it.

step 3:

~~$B \rightarrow SB$~~ is not in 'GNF', so we

substitute $S \rightarrow XA \mid BB$ in production

rule $B \rightarrow SB$ as

$$S \rightarrow XA \mid BB$$

$$B \rightarrow b \mid XAB \mid BBB$$

$$A \rightarrow a$$

$$X \rightarrow b$$

$S \rightarrow XA$ and $A \rightarrow XAB$ is not in GNF

so substitute $X \rightarrow b$ in production rule

$S \rightarrow XA$ and $A \rightarrow XAB$ as:

$$S \rightarrow bA \mid BB$$

$$B \rightarrow b \mid bAB \mid BBB$$

$$A \rightarrow a$$

$$X \rightarrow a$$

Now, remove left recursion ($B \rightarrow BBB$)

$$S \rightarrow bA \mid BB$$

$$B \rightarrow bC \mid bABC$$

$$C \rightarrow BBC \mid \epsilon$$

$$A \rightarrow a$$

$$X \rightarrow b$$

Now remove null production $C \rightarrow \epsilon$

$$S \rightarrow bA \mid BB$$

$$B \rightarrow bC \mid bABC \mid b \mid bAB$$

$$C \rightarrow BBC \mid BB$$

$$A \rightarrow a$$

$$X \rightarrow b$$

$S \rightarrow BB$ is not in GNF, substitute $B \rightarrow$

$bC \mid bABC \mid b \mid bAB$ in production rule $S \rightarrow BB$

$$S \rightarrow bA \mid bCB \mid bABCB \mid bA \mid bABB$$

$$B \rightarrow bC \mid bABC \mid b \mid bAB$$

$$C \rightarrow BBC$$

$$C \rightarrow bCB \mid bABCb \mid bB \mid bABB$$

$$A \rightarrow a$$

$$X \rightarrow b$$

$C \rightarrow BBC$ is not in GNF, substitute $B \rightarrow bc \mid bABC \mid b \mid bAB$ in production rule

$$C \rightarrow BBC \text{ as}$$

$$S \rightarrow bA \mid bCB \mid bABCb \mid bB \mid bABB$$

$$B \rightarrow bc \mid bABC \mid b \mid bAB$$

$$C \rightarrow bCB \mid bABCb \mid bB \mid bABB$$

$$A \rightarrow a$$

$$X \rightarrow b$$

Hence, this is GNF form for grammar G.

Post Tutorial

1) Convert the following CFG into CNF

$$S \rightarrow ASA \mid AB, \quad A \rightarrow B \mid S, \quad B \rightarrow b \mid \epsilon$$

A) $SI \rightarrow S$

$$S \rightarrow ASA \mid AB$$

$$A \rightarrow B \mid S$$

$$B \rightarrow b \mid \epsilon$$

Now, ϵ is removed

$$S \rightarrow SA \mid AS \mid ABA \mid BAA \mid \dots$$

$$S \rightarrow ASA \mid aB$$

$$A \rightarrow B \mid S$$

$$B \rightarrow b \text{ and } a \text{ are the only symbols in the alphabet}$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

hence, this is the grammar

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

hence, this is the grammar

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

$$S \rightarrow ASA \mid aB \mid bA \mid \dots$$

hence, this is the grammar

2) Write steps for removing null productions and unreachable symbols? Explain with an example of your own.

A) Same answer in 1st Question of Pre Tutorial