# Contents



# 1 Setting

## 1.1 vimrc

```
set nocp ai si nu et bs=2 mouse=a
set ts=4 sts=4 sw=4 hls showmatch
set ruler rulerformat=%17.(%l:%c%)
set noswapfile autoread wildmenu wildmode=list:longest
syntax on

map <F5> <ESC>:w<CR>:!g++ -g -Wall --std=c++0x -O2 %:r.cpp -o %:r && ./%:r < %:r
  .in > %:r.out<CR>
map <F6> <ESC>:w<CR>:!g++ -g -Wall --std=c++0x -O2 %:r.cpp -o %:r && ./%:r < %:r
  .in<CR>

map <C-t> :tabnew<CR>

command -nargs=1 PS :cd d:/ | :vi <args>.cpp | vs <args>.in | sp <args>.out
```

## 1.2 account

```
ID : team242
PW : zx6utcpf
```

# 2 Math

## 2.1 Basic Arithmetic

```
// calculate a*b % m
// x86-64 only
ll large_mod_mul(ll a, ll b, ll m) {
    return ll((__int128)a*(__int128)b%m);
}

// calculate a*b % m
// |m| < 2^62, x86 available
// O(logb)
ll large_mod_mul(ll a, ll b, ll m) {
    a %= m; b %= m; ll r = 0, v = a;
    while (b) {
        if (b & 1) {
            r = r + v;
            if (r >= m) r -= m;
        }
        b >>= 1;
        v <<= 1; if (v >= m) v -= m;
    }
    return r;
}

// calculate n^k % m
ll modpow(ll n, ll k, ll m) {
    ll ret = 1;
    n %= m;
    while (k) {
        if (k & 1) ret = large_mod_mul(ret, n, m);
        n = large_mod_mul(n, n, m);
        k /= 2;
    }
    return ret;
}

// find a pair (c, d) s.t. ac + bd = gcd(a, b)
pair<ll, ll> extended_gcd(ll a, ll b) {
    if (b == 0) return { 1, 0 };
    auto t = extended_gcd(b, a % b);
    return { t.second, t.first - t.second * (a / b) };
}

// find x in [0,m) s.t. ax === gcd(a, m) (mod m)
ll modinverse(ll a, ll m) {
    return (extended_gcd(a, m).first % m + m) % m;
}

// calculate modular inverse for 1 ~ n
void calc_range_modinv(int n, int mod, int ret[]) {
    ret[1] = 1;
    for (int i = 2; i <= n; ++i)
        ret[i] = (ll)(mod - mod/i) * ret[mod%i] % mod;
}
```

## 2.2 Sieve Methods : Prime, Divisor, Euler phi

```
// find prime numbers in 1 ~ n
// ret[x] = false  ->  x is prime
// O(n*loglogn)
void sieve(int n, bool ret[]) {
    for (int i = 2; i * i <= n; ++i)
        if (!ret[i])
            for (int j = i * i; j <= n; j += i)
                ret[i] = true;
}

// calculate number of divisors for 1 ~ n
// when you need to calculate sum, change += 1 to += i
// O(n*logn)
void num_of_divisors(int n, int ret[]) {
    for (int i = 1; i <= n; ++i)
        for (int j = i; j <= n; j += i)
            ret[j] += 1;
}

// calculate euler totient function for 1 ~ n
// phi(n) = number of x s.t. 0 < x < n && gcd(n, x) = 1
// O(n*loglogn)
void euler_phi(int n, int ret[]) {
    for (int i = 1; i <= n; ++i) ret[i] = i;
    for (int i = 2; i <= n; ++i)
        if (ret[i] == i)
            for (int j = i; j <= n; j += i)
                ret[j] -= ret[j] / i;
}
```

## 2.3 Primality Test

```
bool test_witness(ull a, ull n, ull s) {
    if (a >= n) a %= n;
    if (a <= 1) return true;
    ull d = n >> s;
    ull x = modpow(a, d, n);
    if (x == 1 || x == n-1) return true;
    while (s-- > 1) {
        x = large_mod_mul(x, x, n);
        if (x == 1) return false;
        if (x == n-1) return true;
    }
    return false;
}

// test whether n is prime
// based on miller-rabin test
// O(logn*logn)
bool is_prime(ull n) {
```

```
    if (n == 2) return true;
    if (n < 2 || n % 2 == 0) return false;

    ull d = n >> 1, s = 1;
    for(; (d&1) == 0; s++) d >>= 1;

#define T(a) test_witness(a##ull, n, s)
    if (n < 4759123141ull) return T(2) && T(7) && T(61);
    return T(2) && T(325) && T(9375) && T(28178)
        && T(450775) && T(9780504) && T(1795265022);
#undef T
}
```

## 2.4 Integer Factorization (Pollard's rho)

```
ll pollard_rho(ll n) {
    random_device rd;
    mt19937 gen(rd());
    uniform_int_distribution<ll> dis(1, n - 1);
    ll x = dis(gen);
    ll y = x;
    ll c = dis(gen);
    ll g = 1;
    while (g == 1) {
        x = (modmul(x, x, n) + c) % n;
        y = (modmul(y, y, n) + c) % n;
        y = (modmul(y, y, n) + c) % n;
        g = gcd(abs(x - y), n);
    }
    return g;
}

// integer factorization
// O(n^0.25 * logn)
void factorize(ll n, vector<ll>& fl) {
    if (n == 1) {
        return;
    }
    if (n % 2 == 0) {
        fl.push_back(2);
        factorize(n / 2, fl);
    }
    else if (is_prime(n)) {
        fl.push_back(n);
    }
    else {
        ll f = pollard_rho(n);
        factorize(f, fl);
        factorize(n / f, fl);
    }
}
```

## 2.5 Chinese Remainder Theorem

```
// find x s.t.  x === a[0] (mod n[0])
//              === a[1] (mod n[1])
//              ...
// assumption: gcd(n[i], n[j]) = 1
ll chinese_remainder(ll* a, ll* n, int size) {
    if (size == 1) return *a;
    ll tmp = modinverse(n[0], n[1]);
    ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
    ll ora = a[1];
    ll tgcd = gcd(n[0], n[1]);
    a[1] = a[0] + n[0] / tgcd * tmp2;
    n[1] *= n[0] / tgcd;
    ll ret = chinese_remainder(a + 1, n + 1, size - 1);
    n[1] /= n[0] / tgcd;
    a[1] = ora;
    return ret;
}
```

## 2.6 Modular Equation

$x \equiv a \pmod m$, $x \equiv b \pmod n$을 만족시키는 $x$를 구하는 방법.

$m$과 $n$을 소인수분해한 후 소수의 제곱꼴의 합동식들로 각각 쪼갠다. 이 때 특정 소수에 대하여 모순이 생기면 불가능한 경우고, 모든 소수에 대해서 모순이 생기지 않으면 전체 식을 CRT로 합치면 된다. 이제 $x \equiv x_1 \pmod{p^{k_1}}$과 $x \equiv x_2 \pmod{p^{k_2}}$가 모순이 생길 조건은 $k_1 \le k_2$라고 했을 때, $x_1 \not\equiv x_2 \pmod{p^{k_1}}$인 경우이다. 모순이 생기지 않았을 때 답을 구하려면 CRT로 합칠 때 $x \equiv x_2 \pmod{p^{k_2}}$만을 남기고 합쳐주면 된다.

## 2.7 Catalan number

다양한 문제의 답이 되는 수열이다.

- 길이가 $2n$인 올바른 괄호 수식의 수
- $n+1$개의 리프를 가진 풀 바이너리 트리의 수
- $n+2$각형을 $n$개의 삼각형으로 나누는 방법의 수

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n$$

## 2.8 Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..)해서 같은 경우들은 하나로 친다. 전체 경우의 수는?

- 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, "아무것도 하지 않는다"라는 operation도 있어야 함!)

- 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

## 2.9 Kirchoff's Theorem

그래프의 스패닝 트리의 개수를 구하는 정리.

무향 그래프의 Laplacian matrix $L$를 만든다. 이것은 (정점의 차수 대각 행렬) - (인접행렬)이다. $L$에서 행과 열을 하나씩 제거한 것을 $L'$라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 $det(L')$이다.

## 2.10 Lucas Theorem

```
// calculate nCm % p  when p is prime
int lucas_theorem(const char *n, const char *m, int p) {
    vector<int> np, mp;
    int i;
    for (i = 0; n[i]; i++) {
        if (n[i] == '0' && np.empty()) continue;
        np.push_back(n[i] - '0');
    }
    for (i = 0; m[i]; i++) {
        if (m[i] == '0' && mp.empty()) continue;
        mp.push_back(m[i] - '0');
    }

    int ret = 1;
    int ni = 0, mi = 0;
    while (ni < np.size() || mi < mp.size()) {
        int nmod = 0, mmod = 0;
        for (i = ni; i < np.size(); i++) {
            if (i + 1 < np.size())
                np[i + 1] += (np[i] % p) * 10;
            else
                nmod = np[i] % p;
            np[i] /= p;
        }
        for (i = mi; i < mp.size(); i++) {
            if (i + 1 < mp.size())
                mp[i + 1] += (mp[i] % p) * 10;
            else
                mmod = mp[i] % p;
            mp[i] /= p;
        }
        while (ni < np.size() && np[ni] == 0) ni++;
        while (mi < mp.size() && mp[mi] == 0) mi++;
        // implement binomial. binomial(m,n) = 0 if m < n
        ret = (ret * binomial(nmod, mmod)) % p;
    }
    return ret;
}
```

## 2.11 Fast Fourier Transform

```
const double PI = acos(-1);

void fft(double *r, double *im, int N, bool f) {
    for (int i = 1, j = 0; i < N; i++) {
        int k; for (k = N >> 1; j >= k; k >>= 1) j -= k;
        j += k; if (i < j) swap(r[i], r[j]), swap(im[i], im[j]);
    }
    for (int i = 1; i < N; i <<= 1) {
        double w = PI / i; if (f) w = -w;
        double c = cos(w), s = sin(w);
        for (int j = 0; j < N; j += i << 1) {
            double yr = 1, yi = 0;
            for (int k = 0; k < i; k++) {
                double zr = r[i + j + k] * yr - im[i + j + k] * yi;
                double zi = r[i + j + k] * yi + im[i + j + k] * yr;
                r[i + j + k] = r[j + k] - zr;
                im[i + j + k] = im[j + k] - zi;
                r[j + k] += zr; im[j + k] += zi;
                tie(yr, yi) = make_pair(yr * c - yi * s, yr * s + yi * c);
            }
        }
    }
}

// Compute Poly(a)*Poly(b), write to r; Indexed from 0
// O(n*logn)
int mult(int *a, int n, int *b, int m, int *r) {
    const int maxn = 1048576;
    static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
    int fn = 1;
    while (fn < n + m) fn <<= 1; // n + m: interested length
    for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
    for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;
    for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
    for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
    fft(ra, ia, fn, false);
    fft(rb, ib, fn, false);
    for (int i = 0; i < fn; ++i) {
        double real = ra[i] * rb[i] - ia[i] * ib[i];
        double imag = ra[i] * ib[i] + rb[i] * ia[i];
        ra[i] = real, ia[i] = imag;
    }
    fft(ra, ia, fn, true);
    for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);
    return fn;
```

```
}
```

## 2.12 Number Theoretic FFT

$p = a \cdot 2^b + 1$꼴의 소수 $p$와 $p$의 원시근 $x$에 대하여, $n \le b$를 만족하는 모든 $2^n$ 크기의 배열에 대해 법 $p$로 FFT를 행할 수 있다. 다음은 위를 만족하는 충분히 큰 소수들 목록이다.

| $p$ | $a$ | $b$ | 원시근 | 덧셈 | 곱셈 |
|---|---|---|---|---|---|
| 3221225473 | 3 | 30 | 5 | 64-bit signed | 64-bit unsigned |
| 2281701377 | 17 | 27 | 3 | 64-bit signed | 64-bit signed |
| 2013265921 | 15 | 27 | 31 | 32-bit unsigned | 64-bit signed |
| 998244353 | 119 | 23 | 3 | 32-bit signed | 64-bit signed |
| 469762049 | 7 | 26 | 3 | 32-bit signed | 64-bit signed |

NTT 사용 시에 자료형에 유의하여, 덧셈 혹은 곱셈에서 Integer overflow가 나지 않도록 하라.

```
const int A = 7, B = 26, P = A << B | 1, R = 3;

int Pow(int x, int y) {
    int r = 1;
    while (y) {
        if (y & 1) r = r * 1ll * x % P;
        x = x * 1ll * x % P;
        y >>= 1;
    }
    return r;
}

void fft(int *a, int N, bool f) {
    for (int i = 1, j = 0; i < N; i++) {
        int k; for (k = N >> 1; j >= k; k >>= 1) j -= k;
        j += k; if (i < j) swap(a[i], a[j]);
    }
    for (int i = 1; i < N; i <<= 1) {
        int x = Pow(f ? Pow(R, P - 2) : R, P / i >> 1);
        for (int j = 0; j < N; j += i << 1) {
            int y = 1;
            for (int k = 0; k < i; k++) {
                int z = a[i + j + k] * 1ll * y % P;
                a[i + j + k] = a[j + k] - z;
                if (a[i + j + k] < P) a[i + j + k] += P;
                a[j + k] += z;
                if (a[j + k] >= P) a[j + k] -= P;
                y = y * 1ll * x % P;
            }
        }
    }
}
```

## 2.13 Example for FFT

```
string S;
int ai, bi, ri;
int A[MAXL], B[MAXL], R[MAXL];
int main(){
  cin>>S;
  for(auto it = S.rbegin(); it != S.rend(); it++) A[ai++] = *it - '0';
  cin>>S;
  for(auto it = S.rbegin(); it != S.rend(); it++) B[bi++] = *it - '0';
  mult(A, ai, B, bi, R);
  for(ri = 0; ri < ai + bi; ri++) R[ri + 1] += R[ri] / 10;
  while(!R[ri] && ri) ri--;
  while(ri >= 0) cout<<R[ri--] % 10;
  cout<<'\n';
  return 0;
}
```

## 2.14 Polynomial Division

```
vll get_inv(const vll& v, int deg){
    if (deg == 1) return vll(1, fastpow(v[0], MOD - 2));

    if (deg & 1){
        vll a = get_inv(v, deg - 1);
        ll c = 0;
        for (int i = 1; i < deg - 1; i++) c = (c + a[i] * v[deg - 1 - i]) % MOD;
        ll h1 = v[deg - 1];

        ll b = MOD - (h1 * a[0] + c) % MOD * a[0] % MOD;
        if (b == MOD) b = 0; a.push_back(b);
        return a;
    }

    vll a = get_inv(v, deg >> 1);
    vll h0(v.begin(), v.begin() + (deg >> 1));
    vll h1(v.begin() + (deg >> 1), v.begin() + deg);
    vll ah0 = mult(a, h0); ah0.push_back(0);
    vll c(ah0.begin() + (deg >> 1), ah0.begin() + deg);
    vll h1a = mult(h1, a);
    vll b_ = mult(a, add(h1a, c));

    vll b(b_.begin(), b_.begin() + (deg >> 1));
    for (ll e : b) a.push_back(e ? MOD - e : 0);
    return a;
}

vll divide(const vll& F, const vll& G, bool newg = false){
    static vll G_INV;
    const int N = (int)F.size() - 1, M = (int)G.size() - 1; // deg of F, G
    if (N < M) return vll();
    if (N == M) return vll(1, F.back()*fastpow(G.back(), MOD - 2) % MOD);

    vll f = F;
    if (G_INV.empty() || newg)
```

```
        vll g = G; reverse(g.begin(), g.end());
        while (g.size() < N - M + 1) g.push_back(0);
        G_INV = get_inv(g, N - M + 1);
    }

    reverse(f.begin(), f.end());
    vll ret = mult(f, G_INV);
    ret.resize(N - M + 1);
    reverse(ret.begin(), ret.end());

    return ret;
}
```

## 2.15 Gaussian Elimination

```
const double EPS = 1e-10;
typedef vector<vector<double>> VVD;

// Gauss-Jordan elimination with full pivoting.
// solving systems of linear equations (AX=B)
// INPUT:    a[][] = an n*n matrix
//           b[][] = an n*m matrix
// OUTPUT:   X      = an n*m matrix (stored in b[][])
//           A^{-1} = an n*n matrix (stored in a[][])
// O(n^3)
double gauss_jordan(VVD& a, VVD& b) {
    const int n = a.size();
    const int m = b[0].size();
    vector<int> irow(n), icol(n), ipiv(n);

    double det = 1;

    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk =
                  k; }
        if (fabs(a[pj][pk]) < EPS) return 0; // matrix is singular
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        irow[i] = pj;
        icol[i] = pk;

        double c = 1.0 / a[pk][pk];
        det *= a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
        }
    }
    for (int p = n - 1; p >= 0; p--) if (irow[p] != icol[p]) {
        for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
    }
    return det;
}
```

## 2.16 Simplex Algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
//     maximize     c^T x
//     subject to   Ax <= b
//                   x >= 0
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).
typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const double EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD& A, const VD& b, const VD& c) :
        m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j
          ];
        for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] =
          b[i]; }
        for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }

    void pivot(int r, int s) {
        double inv = 1.0 / D[r][s];
        for (int i = 0; i < m + 2; i++) if (i != r)
            for (int j = 0; j < n + 2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] * inv;
        for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
        for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool simplex(int phase) {
        int x = phase == 1 ? m + 1 : m;
```

```
        while (true) {
            int s = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] <
                   N[s]) s = j;
            }
            if (D[x][s] > -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] < EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                    (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] <
                       B[r]) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }

    double solve(VD& x) {
        int r = 0;
        for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
        if (D[r][n + 1] < -EPS) {
            pivot(r, n);
            if (!simplex(1) || D[m + 1][n + 1] < -EPS)
                return -numeric_limits<double>::infinity();
            for (int i = 0; i < m; i++) if (B[i] == -1) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                    if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[
                      j] < N[s]) s = j;
                pivot(i, s);
            }
        }
        if (!simplex(2))
            return numeric_limits<double>::infinity();
        x = VD(n);
        for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
        return D[m][n + 1];
    }
};
```

### 2.17 Nim Game

Nim Game의 해법 : 각 더미의 돌의 개수를 모두 XOR했을 때 0이 아니면 첫번째, 0이면 두번째 플레이어가 승리.

Grundy Number : 가능한 다음 state의 Grundy Number를 모두 모은 다음, 그 set에 포함되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러 개의 state들로 나뉠 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.

Subtraction Game : 한 번에 $k$개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를 $k+1$로 나눈 나머지를 XOR 합하여 판단한다.

Index-k Nim : 한 번에 최대 k개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을 $k+1$로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.

## 3 Data Structure

### 3.1 Order statistic tree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
#include <functional>
#include <iostream>
using namespace __gnu_pbds;
using namespace std;

// tree<key_type, value_type(set if null), comparator, ...>
using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update>;

int main()
{
    ordered_set X;
    for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
    cout << boolalpha;
    cout << *X.find_by_order(2) << endl; // 5
    cout << *X.find_by_order(4) << endl; // 9
    cout << (X.end() == X.find_by_order(5)) << endl; // true

    cout << X.order_of_key(-1) << endl; // 0
    cout << X.order_of_key(1) << endl; // 0
    cout << X.order_of_key(4) << endl; // 2
    X.erase(3);
    cout << X.order_of_key(4) << endl; // 1
    for (int t : X) printf("%d␣", t); // 1 5 7 9
}
```

### 3.2 Segment Tree with Lazy Propagation

```
// example implementation of sum tree
const int TSIZE = 131072; // always 2^k form && n <= TSIZE
int segtree[TSIZE * 2], prop[TSIZE * 2];
void seg_init(int nod, int l, int r) {
    if (l == r) segtree[nod] = dat[l];
    else {
        int m = (l + r) >> 1;
        seg_init(nod << 1, l, m);
        seg_init(nod << 1 | 1, m + 1, r);
```

```
        segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
    }
}
void seg_relax(int nod, int l, int r) {
    if (prop[nod] == 0) return;
    if (l < r) {
        int m = (l + r) >> 1;
        segtree[nod << 1] += (m - l + 1) * prop[nod];
        prop[nod << 1] += prop[nod];
        segtree[nod << 1 | 1] += (r - m) * prop[nod];
        prop[nod << 1 | 1] += prop[nod];
    }
    prop[nod] = 0;
}
int seg_query(int nod, int l, int r, int s, int e) {
    if (r < s || e < l) return 0;
    if (s <= l && r <= e) return segtree[nod];
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    return seg_query(nod << 1, l, m, s, e) + seg_query(nod << 1 | 1, m + 1, r, s
      , e);
}
void seg_update(int nod, int l, int r, int s, int e, int val) {
    if (r < s || e < l) return;
    if (s <= l && r <= e) {
        segtree[nod] += (r - l + 1) * val;
        prop[nod] += val;
        return;
    }
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    seg_update(nod << 1, l, m, s, e, val);
    seg_update(nod << 1 | 1, m + 1, r, s, e, val);
    segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
}
// usage:
// seg_update(1, 0, n - 1, qs, qe, val);
// seg_query(1, 0, n - 1, qs, qe);
```

## 3.3 Persistent Segment Tree

```
// persistent segment tree impl: sum tree
// initial tree index is 0
namespace pstree {
    typedef int val_t;
    const int DEPTH = 18;
    const int TSIZE = 1 << 18;
    const int MAX_QUERY = 262144;

    struct node {
        val_t v;
        node *l, *r;
    } npoll[TSIZE * 2 + MAX_QUERY * (DEPTH + 1)], *head[MAX_QUERY + 1];

    int pptr, last_q;

    void init() {
        // zero-initialize, can be changed freely
        memset(&npoll[TSIZE - 1], 0, sizeof(node) * TSIZE);

        for (int i = TSIZE - 2; i >= 0; i--) {
            npoll[i].v = 0;
            npoll[i].l = &npoll[i*2+1];
            npoll[i].r = &npoll[i*2+2];
        }

        head[0] = &npoll[0];
        last_q = 0;
        pptr = 2 * TSIZE - 1;
    }

    // update val to pos
    // 0 <= pos < TSIZE
    // returns updated tree index
    int update(int pos, int val, int prev) {
        head[++last_q] = &npoll[pptr++];
        node *old = head[prev], *now = head[last_q];

        int flag = 1 << DEPTH;
        for (;;) {
            now->v = old->v + val;
            flag >>= 1;
            if (flag==0) {
                now->l = now->r = nullptr; break;
            }
            if (flag & pos) {
                now->l = old->l;
                now->r = &npoll[pptr++];
                now = now->r, old = old->r;
            } else {
                now->r = old->r;
                now->l = &npoll[pptr++];
                now = now->l, old = old->l;
            }
        }
        return last_q;
    }

    val_t query(int s, int e, int l, int r, node *n) {
        if (s == l && e == r) return n->v;
        int m = (l + r) / 2;
        if (m >= e) return query(s, e, l, m, n->l);
        else if (m < s) return query(s, e, m + 1, r, n->r);
        else return query(s, m, l, m, n->l) + query(m + 1, e, m + 1, r, n->r);
    }

    // query summation of [s, e] at time t
    val_t query(int s, int e, int t) {
        s = max(0, s); e = min(TSIZE - 1, e);
        if (s > e) return 0;
```

```
        return query(s, e, 0, TSIZE - 1, head[t]);
    }
}
```

## 3.4 Splay Tree

```
// example : https://www.acmicpc.net/problem/13159
struct node {
    node* l, * r, * p;
    int cnt, min, max, val;
    long long sum;
    bool inv;
    node(int _val) :
        cnt(1), sum(_val), min(_val), max(_val), val(_val), inv(false),
        l(nullptr), r(nullptr), p(nullptr) {
    }
};
node* root;

void update(node* x) {
    x->cnt = 1;
    x->sum = x->min = x->max = x->val;
    if (x->l) {
        x->cnt += x->l->cnt;
        x->sum += x->l->sum;
        x->min = min(x->min, x->l->min);
        x->max = max(x->max, x->l->max);
    }
    if (x->r) {
        x->cnt += x->r->cnt;
        x->sum += x->r->sum;
        x->min = min(x->min, x->r->min);
        x->max = max(x->max, x->r->max);
    }
}

void rotate(node* x) {
    node* p = x->p;
    node* b = nullptr;
    if (x == p->l) {
        p->l = b = x->r;
        x->r = p;
    }
    else {
        p->r = b = x->l;
        x->l = p;
    }
    x->p = p->p;
    p->p = x;
    if (b) b->p = p;
    x->p ? (p == x->p->l ? x->p->l : x->p->r) = x : (root = x);
    update(p);
    update(x);
}

// make x into root
void splay(node* x) {
    while (x->p) {
        node* p = x->p;
        node* g = p->p;
        if (g) rotate((x == p->l) == (p == g->l) ? p : x);
        rotate(x);
    }
}

void relax_lazy(node* x) {
    if (!x->inv) return;
    swap(x->l, x->r);
    x->inv = false;
    if (x->l) x->l->inv = !x->l->inv;
    if (x->r) x->r->inv = !x->r->inv;
}

// find kth node in splay tree
void find_kth(int k) {
    node* x = root;
    relax_lazy(x);
    while (true) {
        while (x->l && x->l->cnt > k) {
            x = x->l;
            relax_lazy(x);
        }
        if (x->l) k -= x->l->cnt;
        if (!k--) break;
        x = x->r;
        relax_lazy(x);
    }
    splay(x);
}

// collect [l, r] nodes into one subtree and return its root
node* interval(int l, int r) {
    find_kth(l - 1);
    node* x = root;
    root = x->r;
    root->p = nullptr;
    find_kth(r - l + 1);
    x->r = root;
    root->p = x;
    root = x;
    return root->r->l;
}

void traverse(node* x) {
    relax_lazy(x);
    if (x->l) {
        traverse(x->l);
    }
    // do something
    if (x->r) {
```

```
        traverse(x->r);
    }
}

void uptree(node* x) {
    if (x->p) {
        uptree(x->p);
    }
    relax_lazy(x);
}
```

### 3.5 Dynamic Connectivity with Example

```
#include <bits/stdc++.h>
using namespace std;

typedef long long lint;
typedef pair<int, int> pi;

vector<pi> tree[1050000];

void add(int s, int e, int ps, int pe, int p, pi v){
        if(e < ps || pe < s) return;
        if(s <= ps && pe <= e){
                tree[p].push_back(v);
                return;
        }
        int pm = (ps + pe) / 2;
        add(s, e, ps, pm, 2*p, v);
        add(s, e, pm+1, pe, 2*p+1, v);
}

vector<pi> tmp;

bool ok(pi a, pi b, pi c){
        return 1ll * (b.first - a.first) * (c.second - b.second) <= 1ll * (b.
          first - c.first) * (a.second - b.second);
}

void solve(int x){
        sort(tree[x].begin(), tree[x].end(), [&](const pi &a, const pi &b){
                return pi(a.first, -a.second) < pi(b.first, -b.second);
        });
        tmp.clear();
        int pv = -2e9;
        for(auto &i : tree[x]){
                if(i.first == pv) continue;
                pv = i.first;
                while(tmp.size() >= 2 && !ok(tmp[tmp.size()-2], tmp.back(), i)){
                        tmp.pop_back();
                }
                tmp.push_back(i);
        }
        tree[x] = tmp;
}

void dfs(int s, int e, int p){
        solve(p);
        if(s == e) return;
        int m = (s+e)/2;
        dfs(s, m, 2*p);
        dfs(m+1, e, 2*p+1);
}

lint nodequery(int p, int x){
        if(tree[p].empty()) return -5e18;
        auto func = [&](int q){
                return 1ll * tree[p][q].first * x + tree[p][q].second;
        };
        int s = 0, e = (int)tree[p].size() - 1;
        while(s != e){
                int m = (s+e)/2;
                if(func(m) < func(m+1)) s = m+1;
                else e = m;
        }
        return func(s);
}

lint query(int pos, int s, int e, int p, int x){
        lint ret = nodequery(p, x);
        if(s == e) return ret;
        int m = (s+e)/2;
        if(pos <= m) ret = max(ret, query(pos, s, m, 2*p, x));
        else ret = max(ret, query(pos, m+1, e, 2*p+1, x));
        return ret;
}

struct ins{
        int s, e, x, y;
};

int q;
vector<ins> inserts;
pi inslis[300005];
bool vis[300005];
int cnt[300005], qry[300005];
int N;
int main(){
        cin>>N;
        for(int i=1; i<=N; i++){
                int t;
        cin>>t;
                if(t == 1){
                        vis[i] = 1;
            cin>>inslis[i].first>>inslis[i].second;
                }
                if(t == 2){
                        int x;
            cin>>x;
                        inserts.push_back({cnt[x] + 1, cnt[i-1], inslis[x].first
```

```
                        , inslis[x].second});
                    vis[x] = 0;
                }
                if(t == 3){
            cin>>qry[i];
                    cnt[i]++;
                }
                cnt[i] += cnt[i-1];
        }
        if(cnt[N] == 0) return 0;
        for(int i=1; i<=N; i++){
                if(vis[i]){
                    inserts.push_back({cnt[i] + 1, cnt[N], inslis[i].first,
                      inslis[i].second});
                }
        }
        for(auto &i : inserts){
                add(i.s, i.e, 1, cnt[N], 1, pi(i.x, i.y));
        }
        dfs(1, cnt[N], 1);
        for(int i=1; i<=N; i++){
                if(cnt[i] != cnt[i-1]){
                    lint t = query(cnt[i], 1, cnt[N], 1, qry[i]);
                    if(t < -4e18) cout<<"EMPTY SET\n";
                    else cout<<t<<'\n';
                }
        }
}
```

# 4 DP

## 4.1 Convex Hull Optimization

$O(n^2) \to O(n \log n)$

DP 점화식 꼴

$D[i] = \max_{j<i}(D[j] + b[j] * a[i])$ $(b[k] \le b[k+1])$

$D[i] = \min_{j<i}(D[j] + b[j] * a[i])$ $(b[k] \ge b[k+1])$

특수조건) $a[i] \le a[i+1]$ 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이 필요없어지기 때문에 amortized $O(n)$ 에 해결할 수 있음

```
struct CHTLinear {
    struct Line {
        long long a, b;
        long long y(long long x) const { return a * x + b; }
    };
    vector<Line> stk;
    int qpt;
    CHTLinear() : qpt(0) { }
    // when you need maximum :  (previous l).a < (now l).a
    // when you need minimum :  (previous l).a > (now l).a
    void pushLine(const Line& l) {
        while (stk.size() > 1) {
            Line& l0 = stk[stk.size() - 1];
            Line& l1 = stk[stk.size() - 2];
            if ((l0.b - l.b) * (l0.a - l1.a) > (l1.b - l0.b) * (l.a - l0.a))
              break;
            stk.pop_back();
        }
        stk.push_back(l);
    }
    // (previous x) <= (current x)
    // it calculates max/min at x
    long long query(long long x) {
        while (qpt + 1 < stk.size()) {
            Line& l0 = stk[qpt];
            Line& l1 = stk[qpt + 1];
            if (l1.a - l0.a > 0 && (l0.b - l1.b) > x * (l1.a - l0.a)) break;
            if (l1.a - l0.a < 0 && (l0.b - l1.b) < x * (l1.a - l0.a)) break;
            ++qpt;
        }
        return stk[qpt].y(x);
    }
};
```

## 4.2 Divide & Conquer Optimization

$O(kn^2) \to O(kn \log n)$

조건 1) DP 점화식 꼴

$D[t][i] = \min_{j<i}(D[t-1][j] + C[j][i])$

조건 2) $A[t][i]$는 $D[t][i]$의 답이 되는 최소의 $j$라 할 때, 아래의 부등식을 만족해야 함

$A[t][i] \le A[t][i+1]$

조건 2-1) 비용$C$가 다음의 사각부등식을 만족하는 경우도 조건 2)를 만족하게 됨

$C[a][c] + C[b][d] \le C[a][d] + C[b][c]$ $(a \le b \le c \le d)$

## 4.3 Knuth Optimization

$O(n^3) \to O(n^2)$

조건 1) DP 점화식 꼴

$D[i][j] = \min_{i<k<j}(D[i][k] + D[k][j]) + C[i][j]$

조건 2) 사각 부등식

$C[a][c] + C[b][d] \le C[a][d] + C[b][c]$ $(a \le b \le c \le d)$

조건 3) 단조성

$C[b][c] \le C[a][d]\ \ (a \le b \le c \le d)$

결론) 조건 2, 3을 만족한다면 $A[i][j]$를 $D[i][j]$의 답이 되는 최소의 $k$라 할 때, 아래의 부등식을 만족하게 됨

$A[i][j-1] \le A[i][j] \le A[i+1][j]$

3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 $O(n^2)$ 이 됨

# 5 Graph

## 5.1 SCC

```
const int MAXN = 100;
vector<int> graph[MAXN];
int up[MAXN], visit[MAXN], vtime;
vector<int> stk;
int scc_idx[MAXN], scc_cnt;

void dfs(int nod) {
    up[nod] = visit[nod] = ++vtime;
    stk.push_back(nod);
    for (int next : graph[nod]) {
        if (visit[next] == 0) {
            dfs(next);
            up[nod] = min(up[nod], up[next]);
        }
        else if (scc_idx[next] == 0)
            up[nod] = min(up[nod], visit[next]);
    }
    if (up[nod] == visit[nod]) {
        ++scc_cnt;
        int t;
        do {
            t = stk.back();
            stk.pop_back();
            scc_idx[t] = scc_cnt;
        } while (!stk.empty() && t != nod);
    }
}

// find SCCs in given directed graph
// O(V+E)
// the order of scc_idx constitutes a reverse topological sort
void get_scc() {
    vtime = 0;
    memset(visit, 0, sizeof(visit));
    scc_cnt = 0;
    memset(scc_idx, 0, sizeof(scc_idx));
    for (int i = 0; i < n; ++i)
        if (visit[i] == 0) dfs(i);
}
```

## 5.2 BCC, Cut vertex, Bridge

```
const int MAXN = 100;
vector<pair<int, int>> graph[MAXN];  // { next vertex id, edge id }
int up[MAXN], visit[MAXN], vtime;
vector<pair<int, int>> stk;

int is_cut[MAXN];           // v is cut vertex if  is_cut[v] > 0
vector<int> bridge;         // list of edge ids
vector<int> bcc_idx[MAXN];  // list of bccids for vertex i
int bcc_cnt;

void dfs(int nod, int par_edge) {
    up[nod] = visit[nod] = ++vtime;
    int child = 0;
    for (const auto& e : graph[nod]) {
        int next = e.first, edge_id = e.second;
        if (edge_id == par_edge) continue;
        if (visit[next] == 0) {
            stk.push_back({ nod, next });
            ++child;
            dfs(next, edge_id);
            if (up[next] == visit[next]) bridge.push_back(edge_id);
            if (up[next] >= visit[nod]) {
                ++bcc_cnt;
                do {
                    auto last = stk.back();
                    stk.pop_back();
                    bcc_idx[last.second].push_back(bcc_cnt);
                    if (last == pair<int, int>{ nod, next }) break;
                } while (!stk.empty());
                bcc_idx[nod].push_back(bcc_cnt);
                is_cut[nod]++;
            }
            up[nod] = min(up[nod], up[next]);
        }
        else
            up[nod] = min(up[nod], visit[next]);
    }
    if (par_edge == -1 && is_cut[nod] == 1)
        is_cut[nod] = 0;
}

// find BCCs & cut vertexs & bridges in undirected graph
// O(V+E)
void get_bcc() {
    vtime = 0;
    memset(visit, 0, sizeof(visit));
    memset(is_cut, 0, sizeof(is_cut));
    bridge.clear();
    for (int i = 0; i < n; ++i) bcc_idx[i].clear();
    bcc_cnt = 0;
    for (int i = 0; i < n; ++i) {
```

```
        if (visit[i] == 0)
            dfs(i, -1);
    }
}
```

## 5.3 Heavy-Light Decomposition

```
// heavy-light decomposition
//
// hld h;
// insert edges to tree[0~n-1];
// h.init(n, root);
// h.decompose(root);
// h.hldquery(u, v); // edges from u to v
struct hld {
    static const int MAXLN = 18;
    static const int MAXN = 1 << (MAXLN - 1);
    vector<int> tree[MAXN];
    int subsize[MAXN], depth[MAXN], pa[MAXLN][MAXN];

    int chead[MAXN], cidx[MAXN];
    int lchain;
    int flatpos[MAXN + 1], fptr;

    void dfs(int u, int par) {
        pa[0][u] = par;
        subsize[u] = 1;
        for (int v : tree[u]) {
            if (v == pa[0][u]) continue;
            depth[v] = depth[u] + 1;
            dfs(v, u);
            subsize[u] += subsize[v];
        }
    }

    void init(int size, int root)
    {
        lchain = fptr = 0;
        dfs(root, -1);
        memset(chead, -1, sizeof(chead));

        for (int i = 1; i < MAXLN; i++) {
            for (int j = 0; j < size; j++) {
                if (pa[i - 1][j] != -1) {
                    pa[i][j] = pa[i - 1][pa[i - 1][j]];
                }
            }
        }
    }

    void decompose(int u) {
        if (chead[lchain] == -1) chead[lchain] = u;
        cidx[u] = lchain;
        flatpos[u] = ++fptr;
        int maxchd = -1;
        for (int v : tree[u]) {
            if (v == pa[0][u]) continue;
            if (maxchd == -1 || subsize[maxchd] < subsize[v]) maxchd = v;
        }
        if (maxchd != -1) decompose(maxchd);

        for (int v : tree[u]) {
            if (v == pa[0][u] || v == maxchd) continue;
            ++lchain; decompose(v);
        }
    }

    int lca(int u, int v) {
        if (depth[u] < depth[v]) swap(u, v);

        int logu;
        for (logu = 1; 1 << logu <= depth[u]; logu++);
        logu--;

        int diff = depth[u] - depth[v];
        for (int i = logu; i >= 0; --i) {
            if ((diff >> i) & 1) u = pa[i][u];
        }
        if (u == v) return u;

        for (int i = logu; i >= 0; --i) {
            if (pa[i][u] != pa[i][v]) {
                u = pa[i][u];
                v = pa[i][v];
            }
        }
        return pa[0][u];
    }

    // TODO: implement query functions
    inline int query(int s, int e) {
        return 0;
    }

    int subquery(int u, int v) {
        int uchain, vchain = cidx[v];
        int ret = 0;
        for (;;) {
            uchain = cidx[u];
            if (uchain == vchain) {
                ret += query(flatpos[v], flatpos[u]);
                break;
            }

            ret += query(flatpos[chead[uchain]], flatpos[u]);
            u = pa[0][chead[uchain]];
        }
        return ret;
    }
```

```
    inline int hldquery(int u, int v) {
        int p = lca(u, v);
        return subquery(u, p) + subquery(v, p) - query(flatpos[p], flatpos[p]);
    }
};
```

## 5.4 Bipartite Matching (Hopcroft-Karp)

```
// in: n, m, graph
// out: match, matched
// vertex cover: (reached[0][left_node] == 0) || (reached[1][right_node] == 1)
// O(E*sqrt(V))
struct BipartiteMatching {
    int n, m;
    vector<vector<int>> graph;
    vector<int> matched, match, edgeview, level;
    vector<int> reached[2];
    BipartiteMatching(int n, int m) : n(n), m(m), graph(n), matched(m, -1),
      match(n, -1) {}

    bool assignLevel() {
        bool reachable = false;
        level.assign(n, -1);
        reached[0].assign(n, 0);
        reached[1].assign(m, 0);
        queue<int> q;
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                level[i] = 0;
                reached[0][i] = 1;
                q.push(i);
            }
        }
        while (!q.empty()) {
            auto cur = q.front(); q.pop();
            for (auto adj : graph[cur]) {
                reached[1][adj] = 1;
                auto next = matched[adj];
                if (next == -1) {
                    reachable = true;
                }
                else if (level[next] == -1) {
                    level[next] = level[cur] + 1;
                    reached[0][next] = 1;
                    q.push(next);
                }
            }
        }
        return reachable;
    }

    int findpath(int nod) {
        for (int &i = edgeview[nod]; i < graph[nod].size(); i++) {
            int adj = graph[nod][i];
            int next = matched[adj];
            if (next >= 0 && level[next] != level[nod] + 1) continue;
            if (next == -1 || findpath(next)) {
                match[nod] = adj;
                matched[adj] = nod;
                return 1;
            }
        }
        return 0;
    }

    int solve() {
        int ans = 0;
        while (assignLevel()) {
            edgeview.assign(n, 0);
            for (int i = 0; i < n; i++)
                if (match[i] == -1)
                    ans += findpath(i);
        }
        return ans;
    }
};
```

## 5.5 Maximum Flow (Dinic)

```
// usage:
// MaxFlowDinic::init(n);
// MaxFlowDinic::add_edge(0, 1, 100, 100); // for bidirectional edge
// MaxFlowDinic::add_edge(1, 2, 100); // directional edge
// result = MaxFlowDinic::solve(0, 2); // source -> sink
// graph[i][edgeIndex].res -> residual
//
// in order to find out the minimum cut, use `l'.
// if l[i] == 0, i is unrechable.
//
// O(V*V*E)
// with unit capacities, O(min(V^(2/3), E^(1/2)) * E)
struct MaxFlowDinic {
    typedef int flow_t;
    struct Edge {
        int next;
        size_t inv; /* inverse edge index */
        flow_t res; /* residual */
    };
    int n;
    vector<vector<Edge>> graph;
    vector<int> q, l, start;

    void init(int _n) {
        n = _n;
        graph.resize(n);
        for (int i = 0; i < n; i++) graph[i].clear();
    }
    void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
        Edge forward{ e, graph[e].size(), cap };
```

```
        Edge reverse{ s, graph[s].size(), caprev };
        graph[s].push_back(forward);
        graph[e].push_back(reverse);
    }
    bool assign_level(int source, int sink) {
        int t = 0;
        memset(&l[0], 0, sizeof(l[0]) * l.size());
        l[source] = 1;
        q[t++] = source;
        for (int h = 0; h < t && !l[sink]; h++) {
            int cur = q[h];
            for (const auto& e : graph[cur]) {
                if (l[e.next] || e.res == 0) continue;
                l[e.next] = l[cur] + 1;
                q[t++] = e.next;
            }
        }
        return l[sink] != 0;
    }
    flow_t block_flow(int cur, int sink, flow_t current) {
        if (cur == sink) return current;
        for (int& i = start[cur]; i < graph[cur].size(); i++) {
            auto& e = graph[cur][i];
            if (e.res == 0 || l[e.next] != l[cur] + 1) continue;
            if (flow_t res = block_flow(e.next, sink, min(e.res, current))) {
                e.res -= res;
                graph[e.next][e.inv].res += res;
                return res;
            }
        }
        return 0;
    }
    flow_t solve(int source, int sink) {
        q.resize(n);
        l.resize(n);
        start.resize(n);
        flow_t ans = 0;
        while (assign_level(source, sink)) {
            memset(&start[0], 0, sizeof(start[0]) * n);
            while (flow_t flow = block_flow(source, sink, numeric_limits<flow_t
              >::max()))
                ans += flow;
        }
        return ans;
    }
};
```

## 5.6 Maximum Flow with Edge Demands

그래프 $G = (V, E)$ 가 있고 source $s$와 sink $t$가 있다. 각 간선마다 $d(e) \le f(e) \le c(e)$ 를 만족하도록 flow $f(e)$를 흘려야 한다. 이 때의 maximum flow를 구하는 문제다.

먼저 모든 demand를 합한 값 $D$를 아래와 같이 정의한다.

$$D = \sum_{(u \to v) \in E} d(u \to v)$$

이제 $G$ 에 몇개의 정점과 간선을 추가하여 새로운 그래프 $G' = (V', E')$ 을 만들 것이다. 먼저 새로운 source $s'$ 과 새로운 sink $t'$ 을 추가한다. 그리고 $s'$에서 $V$의 모든 점마다 간선을 이어주고, $V$의 모든 점에서 $t'$로 간선을 이어준다.

새로운 capacity function $c'$을 아래와 같이 정의한다.

1. $V$의 점 $v$에 대해 $c'(s' \to v) = \sum_{u \in V} d(u \to v)$ , $c'(v \to t') = \sum_{w \in V} d(v \to w)$
2. $E$의 간선 $u \to v$에 대해 $c'(u \to v) = c(u \to v) - d(u \to v)$
3. $c'(t \to s) = \infty$

이렇게 만든 새로운 그래프 $G'$에서 maximum flow를 구했을 때 그 값이 $D$라면 원래 문제의 해가 존재하고, 그 값이 $D$가 아니라면 원래 문제의 해는 존재하지 않는다.

위에서 maximum flow를 구하고 난 상태의 residual graph 에서 $s'$과 $t'$을 떼버리고 $s$에서 $t$사이의 augument path 를 계속 찾으면 원래 문제의 해를 구할 수 있다.

```
struct MaxFlowEdgeDemands
{
    MaxFlowDinic mf;
    using flow_t = MaxFlowDinic::flow_t;

    vector<flow_t> ind, outd;
    flow_t D; int n;

    void init(int _n) {
        n = _n; D = 0; mf.init(n + 2);
        ind.clear(); outd.clear();
        ind.resize(n, 0); outd.resize(n, 0);
    }

    void add_edge(int s, int e, flow_t cap, flow_t demands = 0) {
        mf.add_edge(s, e, cap - demands);
        D += demands; ind[e] += demands; outd[s] += demands;
    }

    // returns { false, 0 } if infeasible
    // { true, maxflow } if feasible
    pair<bool, flow_t> solve(int source, int sink) {
        mf.add_edge(sink, source, numeric_limits<flow_t>::max());

        for (int i = 0; i < n; i++) {
            if (ind[i]) mf.add_edge(n, i, ind[i]);
            if (outd[i]) mf.add_edge(i, n + 1, outd[i]);
        }

        if (mf.solve(n, n + 1) != D) return{ false, 0 };

        for (int i = 0; i < n; i++) {
```

```
            if (ind[i]) mf.graph[i].pop_back();
            if (outd[i]) mf.graph[i].pop_back();
        }

        return{ true, mf.solve(source, sink) };
    }
};
```

## 5.7 Min-cost Maximum Flow

```
// precondition: there is no negative cycle.
// usage:
// MinCostFlow mcf(n);
// for(each edges) mcf.addEdge(from, to, cost, capacity);
// mcf.solve(source, sink); // min cost max flow
// mcf.solve(source, sink, 0); // min cost flow
// mcf.solve(source, sink, goal_flow); // min cost flow with total_flow >=
  goal_flow if possible
struct MinCostFlow {
    typedef int cap_t;
    typedef int cost_t;

    bool iszerocap(cap_t cap) { return cap == 0; }

    struct edge {
        int target;
        cost_t cost;
        cap_t residual_capacity;
        cap_t orig_capacity;
        size_t revid;
    };

    int n;
    vector<vector<edge>> graph;

    MinCostFlow(int n) : graph(n), n(n) {}

    void addEdge(int s, int e, cost_t cost, cap_t cap) {
        if (s == e) return;
        edge forward{ e, cost, cap, cap, graph[e].size() };
        edge backward{ s, -cost, 0, 0, graph[s].size() };
        graph[s].emplace_back(forward);
        graph[e].emplace_back(backward);
    }

    pair<cost_t, cap_t> augmentShortest(int s, int e, cap_t flow_limit) {
        auto infinite_cost = numeric_limits<cost_t>::max();
        auto infinite_flow = numeric_limits<cap_t>::max();
        vector<pair<cost_t, cap_t>> dist(n, make_pair(infinite_cost, 0));
        vector<int> from(n, -1), v(n);

        dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
        queue<int> q;
        v[s] = 1; q.push(s);

        while(!q.empty()) {
            int cur = q.front();
            v[cur] = 0; q.pop();
            for (const auto& e : graph[cur]) {
                if (iszerocap(e.residual_capacity)) continue;
                auto next = e.target;
                auto ncost = dist[cur].first + e.cost;
                auto nflow = min(dist[cur].second, e.residual_capacity);
                if (dist[next].first > ncost) {
                    dist[next] = make_pair(ncost, nflow);
                    from[next] = e.revid;
                    if (v[next]) continue;
                    v[next] = 1; q.push(next);
                }
            }
        }

        auto p = e;
        auto pathcost = dist[p].first;
        auto flow = dist[p].second;
        if (iszerocap(flow)|| (flow_limit <= 0 && pathcost >= 0)) return pair<
          cost_t, cap_t>(0, 0);
        if (flow_limit > 0) flow = min(flow, flow_limit);

        while (from[p] != -1) {
            auto nedge = from[p];
            auto np = graph[p][nedge].target;
            auto fedge = graph[p][nedge].revid;
            graph[p][nedge].residual_capacity += flow;
            graph[np][fedge].residual_capacity -= flow;
            p = np;
        }
        return make_pair(pathcost * flow, flow);
    }

    pair<cost_t,cap_t> solve(int s, int e, cap_t flow_minimum = numeric_limits<
      cap_t>::max()) {
        cost_t total_cost = 0;
        cap_t total_flow = 0;
        for(;;) {
            auto res = augmentShortest(s, e, flow_minimum - total_flow);
            if (res.second <= 0) break;
            total_cost += res.first;
            total_flow += res.second;
        }
        return make_pair(total_cost, total_flow);
    }
};
```

## 5.8 General Min-cut (Stoer-Wagner)

```
// implementation of Stoer-Wagner algorithm
// O(V^3)
//usage
// MinCut mc;
```

```
// mc.init(n);
// for (each edge) mc.addEdge(a,b,weight);
// mincut = mc.solve();
// mc.cut = {0,1}^n describing which side the vertex belongs to.
struct MinCutMatrix
{
    typedef int cap_t;
    int n;
    vector<vector<cap_t>> graph;

    void init(int _n) {
        n = _n;
        graph = vector<vector<cap_t>>(n, vector<cap_t>(n, 0));
    }
    void addEdge(int a, int b, cap_t w) {
        if (a == b) return;
        graph[a][b] += w;
        graph[b][a] += w;
    }

    pair<cap_t, pair<int, int>> stMinCut(vector<int> &active) {
        vector<cap_t> key(n);
        vector<int> v(n);
        int s = -1, t = -1;
        for (int i = 0; i < active.size(); i++) {
            cap_t maxv = -1;
            int cur = -1;
            for (auto j : active) {
                if (v[j] == 0 && maxv < key[j]) {
                    maxv = key[j];
                    cur = j;
                }
            }
            t = s; s = cur;
            v[cur] = 1;
            for (auto j : active) key[j] += graph[cur][j];
        }
        return make_pair(key[s], make_pair(s, t));
    }

    vector<int> cut;

    cap_t solve() {
        cap_t res = numeric_limits<cap_t>::max();
        vector<vector<int>> grps;
        vector<int> active;
        cut.resize(n);
        for (int i = 0; i < n; i++) grps.emplace_back(1, i);
        for (int i = 0; i < n; i++) active.push_back(i);
        while (active.size() >= 2) {
            auto stcut = stMinCut(active);
            if (stcut.first < res) {
                res = stcut.first;
                fill(cut.begin(), cut.end(), 0);
                for (auto v : grps[stcut.second.first]) cut[v] = 1;
            }

            int s = stcut.second.first, t = stcut.second.second;
            if (grps[s].size() < grps[t].size()) swap(s, t);

            active.erase(find(active.begin(), active.end(), t));
            grps[s].insert(grps[s].end(), grps[t].begin(), grps[t].end());
            for (int i = 0; i < n; i++) { graph[i][s] += graph[i][t]; graph[i][t
              ] = 0; }
            for (int i = 0; i < n; i++) { graph[s][i] += graph[t][i]; graph[t][i
              ] = 0; }
            graph[s][s] = 0;
        }
        return res;
    }
};
```

## 5.9 General Max Matching

```
struct DisjointSet
{
    vector<int> parent, rnk;

    DisjointSet(int n = 0) : rnk(n) {
        parent.reserve(n);
        for (int i = 0; i < n; i++) parent.push_back(i);
    }

    void reset(int n) {
        parent.clear(); rnk.assign(n, 0);
        for (int i = 0; i < n; i++) parent.push_back(i);
    }

    void increase(int n) {
        int base = parent.size();
        for (int i = base; i < base + n; i++) {
            parent.push_back(i);
            rnk.push_back(0);
        }
    }

    int find(int p) {
        return parent[p] == p ? p : parent[p] = find(parent[p]);
    }

    void merge(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return;
        if (rnk[a] < rnk[b]) swap(a, b);
        else if (rnk[a] == rnk[b]) ++rnk[a];
        parent[b] = a;
    }
};

struct MaxMatching
```

```
{
    int n;
    vector<vector<int>> gnext;
    vector<int> matched;

    int vcnt;

    MaxMatching(int n) : n(n), gnext(n), matched(n, -1) {}

    void AddEdge(int a, int b) {
        gnext[a].push_back(b);
        gnext[b].push_back(a);
    }

    int Match() {
        int ans = 0;
        while (findAugment()) ++ans;
        return ans;
    }

    vector<int> parent; // shrunken -> real
    vector<int> forest;
    vector<int> level;
    vector<pair<int,int>> bridge;
    queue<int> q;
    DisjointSet blossomSet;
    vector<int> origin; // blossomSet number to -> origin vertex
    vector<int> ancestorChecker;
    int ancestorCheckerValue;


    vector<int> marker;
    void markBlossomPath(int vv, pair<int, int> vu, int ancestor){
        int p = vv;
        marker.clear();
        while (p != ancestor) {
            int np = origin[blossomSet.find(parent[p])];
            marker.push_back(p); p = np;

            np = origin[blossomSet.find(parent[p])];
            marker.push_back(p);
            bridge[p] = vu; // need original vertex number
            q.push(p); // odd level edges were not considered
            p = np;
        }
        for (auto x : marker) blossomSet.merge(ancestor, x);
        origin[blossomSet.find(ancestor)] = ancestor;
    }

    void mergeBlossom(int vv, int uu, int v, int u){
        if (uu == vv) return;
        ++ancestorCheckerValue;
        int p1 = uu, p2 = vv;
        int ancestor = -1;
        for (;;) {
            if (p1 >= 0) {
                if (ancestorChecker[p1] == ancestorCheckerValue) {
                    ancestor = p1;
                    break;
                }
                ancestorChecker[p1] = ancestorCheckerValue;
                if (parent[p1] >= 0) p1 = origin[blossomSet.find(parent[p1])];
                  else p1 = -1;
            }
            if (p2 >= 0) {
                if (ancestorChecker[p2] == ancestorCheckerValue) {
                    ancestor = p2;
                    break;
                }
                ancestorChecker[p2] = ancestorCheckerValue;
                if (parent[p2] >= 0) p2 = origin[blossomSet.find(parent[p2])];
                  else p2 = -1;
            }
        }
        markBlossomPath(uu, make_pair(u, v), ancestor);
        markBlossomPath(vv, make_pair(v, u), ancestor);
    }

    vector<int> augmentPathLink;

    void getRootPath(int v, int w, bool reversed){
        if (v == w) return;
        if (level[v] & 1) {
            // odd. use bridge
            int x, y, mate = matched[v];
            tie(x,y) = tie(bridge[v].first, bridge[v].second);
            getRootPath(x, mate, !reversed);
            getRootPath(y, w, reversed);
            if (reversed) {
                augmentPathLink[y] = x;
                augmentPathLink[mate] = v;
            } else {
                augmentPathLink[v] = mate;
                augmentPathLink[x] = y;
            }
        } else {
            // even
            int mate = matched[v];
            getRootPath(parent[mate], w, reversed);
            if (reversed) {
                augmentPathLink[parent[mate]] = mate;
                augmentPathLink[mate] = v;
            } else {
                augmentPathLink[v] = mate;
                augmentPathLink[mate] = parent[mate];
            }
        }
    }

    void augmentPath(int v, int w) {
```

```
        augmentPathLink = vector<int>(n,-1);
        int x = forest[v];
        int y = forest[w];
        getRootPath(v,x,true);
        getRootPath(w,y,false);
        augmentPathLink[v] = w;
        int p = x;
        for(;;) {
            int q = augmentPathLink[p];
            matched[p] = q;
            matched[q] = p;
            if (q == y) break;
            p = augmentPathLink[q];
        }
    }

    bool findAugment() {
        parent = vector<int>(n,-1);
        forest = vector<int>(n,-1);
        level = vector<int>(n);
        bridge = vector<pair<int,int>>(n,make_pair(-1,-1));
        q = queue<int>();
        blossomSet.reset(n);
        origin = vector<int>(n);
        ancestorChecker = vector<int>(n);
        ancestorCheckerValue = 0;

        for(int i = 0; i < n; i++) {
            origin[i] = i;
            if (matched[i] == -1) {
                forest[i] = i;
                q.push(i);
            }
        }
        bool foundPath = false;
        while(!q.empty() && !foundPath) {
            int v = q.front(); q.pop();
            for(auto u : gnext[v]) {
                int vv = origin[blossomSet.find(v)];
                int uu = origin[blossomSet.find(u)];
                if (forest[uu] == -1) {
                    // assert(u == uu)
                    parent[uu] = v;
                    forest[uu] = forest[vv];
                    level [uu] = level [vv] + 1;
                    parent[matched[uu]] = uu;
                    forest[matched[uu]] = forest[vv];
                    level [matched[uu]] = level [vv] + 2;
                    q.push(matched[uu]);
                } else if (level[uu]&1) {
                    // odd level
                } else if (forest[uu] != forest[vv]){
                    // found path. both are even level
                    foundPath = true;
                    augmentPath(v,u);
                    break;
                } else {
                    // blossom formed
                    mergeBlossom(vv, uu, v, u);
                }
            }
        }

        return foundPath;
    }
};
```

## 5.10 Hungarian Algorithm

```
int n, m;
int mat[MAX_N + 1][MAX_M + 1];

// hungarian method : bipartite min-weighted matching
// O(n^3) or O(m*n^2)
// http://e-maxx.ru/algo/assignment_hungary
// mat[1][1] ~ mat[n][m]
// matched[i] : matched column of row i
int hungarian(vector<int>& matched) {
    vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1), minv(m + 1);
    vector<char> used(m + 1);
    for (int i = 1; i <= n; ++i) {
        p[0] = i;
        int j0 = 0;
        fill(minv.begin(), minv.end(), INF);
        fill(used.begin(), used.end(), false);
        do {
            used[j0] = true;
            int i0 = p[j0], delta = INF, j1;
            for (int j = 1; j <= m; ++j) {
                if (!used[j]) {
                    int cur = mat[i0][j] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta) delta = minv[j], j1 = j;
                }
            }
            for (int j = 0; j <= m; ++j) {
                if (used[j])
                    u[p[j]] += delta, v[j] -= delta;
                else
                    minv[j] -= delta;
            }
            j0 = j1;
        } while (p[j0] != 0);
        do {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
    }
    for (int j = 1; j <= m; ++j) matched[p[j]] = j;
```

```
    return -v[0];
}
```

# 6 Geometry

## 6.1 Basic Operations

```
const double eps = 1e-9;

inline int diff(double lhs, double rhs) {
    if (lhs - eps < rhs && rhs < lhs + eps) return 0;
    return (lhs < rhs) ? -1 : 1;
}

inline bool is_between(double check, double a, double b) {
    if (a < b)
        return (a - eps < check && check < b + eps);
    else
        return (b - eps < check && check < a + eps);
}

struct Point {
    double x, y;
    bool operator==(const Point& rhs) const {
        return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
    }
    Point operator+(const Point& rhs) const {
        return Point{ x + rhs.x, y + rhs.y };
    }
    Point operator-(const Point& rhs) const {
        return Point{ x - rhs.x, y - rhs.y };
    }
    Point operator*(double t) const {
        return Point{ x * t, y * t };
    }
};

struct Circle {
    Point center;
    double r;
};

struct Line {
    Point pos, dir;
};

inline double inner(const Point& a, const Point& b) {
    return a.x * b.x + a.y * b.y;
}

inline double outer(const Point& a, const Point& b) {
    return a.x * b.y - a.y * b.x;
}

inline int ccw_line(const Line& line, const Point& point) {
    return diff(outer(line.dir, point - line.pos), 0);
}

inline int ccw(const Point& a, const Point& b, const Point& c) {
    return diff(outer(b - a, c - a), 0);
}

inline double dist(const Point& a, const Point& b) {
    return sqrt(inner(a - b, a - b));
}

inline double dist2(const Point &a, const Point &b) {
    return inner(a - b, a - b);
}

inline double dist(const Line& line, const Point& point, bool segment = false) {
    double c1 = inner(point - line.pos, line.dir);
    if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);
    double c2 = inner(line.dir, line.dir);
    if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);
    return dist(line.pos + line.dir * (c1 / c2), point);
}

bool get_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    ret = b.pos + b.dir * t2;
    return true;
}

bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    if (!is_between(t1, 0, 1) || !is_between(t2, 0, 1)) return false;
    ret = b.pos + b.dir * t2;
    return true;
}

Point inner_center(const Point &a, const Point &b, const Point &c) {
    double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
    double w = wa + wb + wc;
    return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.y + wb * b.y +
      wc * c.y) / w };
}

Point outer_center(const Point &a, const Point &b, const Point &c) {
    Point d1 = b - a, d2 = c - a;
    double area = outer(d1, d2);
    double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
        + d1.y * d2.y * (d1.y - d2.y);
    double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
```

```
        + d1.x * d2.x * (d1.x - d2.y);
    return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
}

vector<Point> circle_line(const Circle& circle, const Line& line) {
    vector<Point> result;
    double a = 2 * inner(line.dir, line.dir);
    double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
        + line.dir.y * (line.pos.y - circle.center.y));
    double c = inner(line.pos - circle.center, line.pos - circle.center)
        - circle.r * circle.r;
    double det = b * b - 2 * a * c;
    int pred = diff(det, 0);
    if (pred == 0)
        result.push_back(line.pos + line.dir * (-b / a));
    else if (pred > 0) {
        det = sqrt(det);
        result.push_back(line.pos + line.dir * ((-b + det) / a));
        result.push_back(line.pos + line.dir * ((-b - det) / a));
    }
    return result;
}

vector<Point> circle_circle(const Circle& a, const Circle& b) {
    vector<Point> result;
    int pred = diff(dist(a.center, b.center), a.r + b.r);
    if (pred > 0) return result;
    if (pred == 0) {
        result.push_back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r)));
        return result;
    }
    double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
    double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
    double tmp = (bb - aa) / 2.0;
    Point cdiff = b.center - a.center;
    if (diff(cdiff.x, 0) == 0) {
        if (diff(cdiff.y, 0) == 0)
            return result; // if (diff(a.r, b.r) == 0): same circle
        return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 } });
    }
    return circle_line(a,
        Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
}

Circle circle_from_3pts(const Point& a, const Point& b, const Point& c) {
    Point ba = b - a, cb = c - b;
    Line p{ (a + b) * 0.5, Point{ ba.y, -ba.x } };
    Line q{ (b + c) * 0.5, Point{ cb.y, -cb.x } };
    Circle circle;
    if (!get_cross(p, q, circle.center))
        circle.r = -1;
    else
        circle.r = dist(circle.center, a);
    return circle;
}

Circle circle_from_2pts_rad(const Point& a, const Point& b, double r) {
    double det = r * r / dist2(a, b) - 0.25;
    Circle circle;
    if (det < 0)
        circle.r = -1;
    else {
        double h = sqrt(det);
        // center is to the left of a->b
        circle.center = (a + b) * 0.5 + Point{ a.y - b.y, b.x - a.x } * h;
        circle.r = r;
    }
    return circle;
}
```

## 6.2 Compare angles

```
int ccw(pair<int, int> p1, pair<int, int> p2) {
    auto ret = p1.first * 1ll * p2.second - p2.first * 1ll * p1.second;
    return ret > 0 ? 1 : (ret < 0 ? -1 : 0);
}

bool upper(pair<int, int> p) {
    return tie(p.second, p.first) > tuple<int, int>();
}

// sorting criterion: [0 ~ 2 * pi)
sort(dat.begin(), dat.end(), [](pair<int, int> a, pair<int, int> b){
    if (upper(a) != upper(b)) return upper(a) > upper(b);
    if (ccw(a, b)) return ccw(a, b) > 0;

    // optional: closest to farthest
    return hypot(a.first, a.second) < hypot(b.first, b.second);
});
```

## 6.3 Convex Hull

```
// find convex hull
// O(n*logn)
vector<Point> convex_hull(vector<Point>& dat) {
    if (dat.size() <= 3) return dat;
    vector<Point> upper, lower;
    sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x) ? a.y < b.y : a.x < b.x;
    });
    for (const auto& p : dat) {
        while (upper.size() >= 2 && ccw(*++upper.rbegin(), *upper.rbegin(), p)
          >= 0) upper.pop_back();
        while (lower.size() >= 2 && ccw(*++lower.rbegin(), *lower.rbegin(), p)
          <= 0) lower.pop_back();
        upper.emplace_back(p);
        lower.emplace_back(p);
    }
    upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
```

```
    return upper;
}
```

## 6.4 Rotating Calipers

```
// get all antipodal pairs
// O(n)
void antipodal_pairs(vector<Point>& pt) {
    // calculate convex hull
    sort(pt.begin(), pt.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x) ? a.y < b.y : a.x < b.x;
    });
    vector<Point> up, lo;
    for (const auto& p : pt) {
        while (up.size() >= 2 && ccw(*++up.rbegin(), *up.rbegin(), p) >= 0) up.
          pop_back();
        while (lo.size() >= 2 && ccw(*++lo.rbegin(), *lo.rbegin(), p) <= 0) lo.
          pop_back();
        up.emplace_back(p);
        lo.emplace_back(p);
    }

    for (int i = 0, j = (int)lo.size() - 1; i + 1 < up.size() || j > 0; ) {
        get_pair(up[i], lo[j]);  // DO WHAT YOU WANT
        if (i + 1 == up.size()) --j;
        else if (j == 0) ++i;
        else if ((long long)(up[i + 1].y - up[i].y) * (lo[j].x - lo[j - 1].x)
            > (long long)(up[i + 1].x - up[i].x) * (lo[j].y - lo[j - 1].y)) ++i;
        else --j;
    }
}
```

## 6.5 Point in Polygon Test

```
typedef double coord_t;

inline coord_t is_left(Point p0, Point p1, Point p2) {
    return (p1.x - p0.x) * (p2.y - p0.y) - (p2.x - p0.x) * (p1.y - p0.y);
}

// point in polygon test
// http://geomalgorithms.com/a03-_inclusion.html
bool is_in_polygon(Point p, vector<Point>& poly) {
    int wn = 0;
    for (int i = 0; i < poly.size(); ++i) {
        int ni = (i + 1 == poly.size()) ? 0 : i + 1;
        if (poly[i].y <= p.y) {
            if (poly[ni].y > p.y) {
                if (is_left(poly[i], poly[ni], p) > 0) {
                    ++wn;
                }
            }
        }
        else {
            if (poly[ni].y <= p.y) {
                if (is_left(poly[i], poly[ni], p) < 0) {
                    --wn;
                }
            }
        }
    }
    return wn != 0;
}
```

## 6.6 Polygon Cut

```
// left side of a->b
vector<Point> cut_polygon(const vector<Point>& polygon, Line line) {
    if (!polygon.size()) return polygon;
    typedef vector<Point>::const_iterator piter;
    piter la, lan, fi, fip, i, j;
    la = lan = fi = fip = polygon.end();
    i = polygon.end() - 1;
    bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
    for (j = polygon.begin() ; j != polygon.end() ; j++) {
        bool thisin = diff(ccw_line(line, *j), 0) > 0;
        if (lastin && !thisin) {
            la = i;
            lan = j;
        }
        if (!lastin && thisin) {
            fi = j;
            fip = i;
        }
        i = j;
        lastin = thisin;
    }
    if (fi == polygon.end()) {
        if (!lastin) return vector<Point>();
        return polygon;
    }
    vector<Point> result;
    for (i = fi ; i != lan ; i++) {
        if (i == polygon.end()) {
            i = polygon.begin();
            if (i == lan) break;
        }
        result.push_back(*i);
    }
    Point lc, fc;
    get_cross(Line{ *la, *lan - *la }, line, lc);
    get_cross(Line{ *fip, *fi - *fip }, line, fc);
    result.push_back(lc);
    if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
    return result;
}
```

### 6.7 Pick's theorem

격자점으로 구성된 simple polygon이 주어짐. $i$는 polygon 내부의 격자점 수, $b$는 polygon 선분 위 격자점 수, $A$는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.

$A = i + \frac{b}{2} - 1$

## 7 String

### 7.1 KMP

```
typedef vector<int> seq_t;

void calculate_pi(vector<int>& pi, const seq_t& str) {
    pi[0] = -1;
    for (int i = 1, j = -1; i < str.size(); i++) {
        while (j >= 0 && str[i] != str[j + 1]) j = pi[j];
        if (str[i] == str[j + 1])
            pi[i] = ++j;
        else
            pi[i] = -1;
    }
}

// returns all positions matched
// O(|text|+|pattern|)
vector<int> kmp(const seq_t& text, const seq_t& pattern) {
    vector<int> pi(pattern.size()), ans;
    if (pattern.size() == 0) return ans;
    calculate_pi(pi, pattern);
    for (int i = 0, j = -1; i < text.size(); i++) {
        while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
        if (text[i] == pattern[j + 1]) {
            j++;
            if (j + 1 == pattern.size()) {
                ans.push_back(i - j);
                j = pi[j];
            }
        }
    }
    return ans;
}
```

### 7.2 Z Algorithm

```
// Z[i] : maximum common prefix length of &s[0] and &s[i]
// O(|s|)
using seq_t = string;
vector<int> z_func(const seq_t &s) {
    vector<int> z(s.size());
    z[0] = s.size();
    int l = 0, r = 0;
    for (int i = 1; i < s.size(); i++) {
        if (i > r) {
            int j;
            for (j = 0; i + j < s.size() && s[i + j] == s[j]; j++) ;
            z[i] = j; l = i; r = i + j - 1;
        } else if (z[i - l] < r - i + 1) {
            z[i] = z[i - l];
        } else {
            int j;
            for (j = 1; r + j < s.size() && s[r + j] == s[r - i + j]; j++) ;
            z[i] = r - i + j; l = i; r += j - 1;
        }
    }

    return z;
}
```

### 7.3 Aho-Corasick

```
struct AhoCorasick
{
    const int alphabet;
    struct node {
        node() {}
        explicit node(int alphabet) : next(alphabet) {}
        vector<int> next, report;
        int back = 0, output_link = 0;
    };
    int maxid = 0;
    vector<node> dfa;
    explicit AhoCorasick(int alphabet) : alphabet(alphabet), dfa(1, node(
      alphabet)) { }
    template<typename InIt, typename Fn> void add(int id, InIt first, InIt last,
       Fn func) {
        int cur = 0;
        for ( ; first != last; ++first) {
            auto s = func(*first);
            if (auto next = dfa[cur].next[s]) cur = next;
            else {
                cur = dfa[cur].next[s] = (int)dfa.size();
                dfa.emplace_back(alphabet);
            }
        }
        dfa[cur].report.push_back(id);
        maxid = max(maxid, id);
    }
    void build() {
        queue<int> q;
        vector<char> visit(dfa.size());
        visit[0] = 1;
        q.push(0);
        while(!q.empty()) {
            auto cur = q.front(); q.pop();
            dfa[cur].output_link = dfa[cur].back;
```

```
            if (dfa[dfa[cur].back].report.empty())
                dfa[cur].output_link = dfa[dfa[cur].back].output_link;
            for (int s = 0; s < alphabet; s++) {
                auto &next = dfa[cur].next[s];
                if (next == 0) next = dfa[dfa[cur].back].next[s];
                if (visit[next]) continue;
                if (cur) dfa[next].back = dfa[dfa[cur].back].next[s];
                visit[next] = 1;
                q.push(next);
            }
        }
    }
    template<typename InIt, typename Fn> vector<int> countMatch(InIt first, InIt
       last, Fn func) {
        int cur = 0;
        vector<int> ret(maxid+1);
        for (; first != last; ++first) {
            cur = dfa[cur].next[func(*first)];
            for (int p = cur; p; p = dfa[p].output_link)
                for (auto id : dfa[p].report) ret[id]++;
        }
        return ret;
    }
};
```

## 7.4 Suffix Array with LCP

```
typedef char T;

// calculates suffix array.
// O(n*logn)
vector<int> suffix_array(const vector<T>& in) {
    int n = (int)in.size(), c = 0;
    vector<int> temp(n), pos2bckt(n), bckt(n), bpos(n), out(n);
    for (int i = 0; i < n; i++) out[i] = i;
    sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });
    for (int i = 0; i < n; i++) {
        bckt[i] = c;
        if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
    }
    for (int h = 1; h < n && c < n; h <<= 1) {
        for (int i = 0; i < n; i++) pos2bckt[out[i]] = bckt[i];
        for (int i = n - 1; i >= 0; i--) bpos[bckt[i]] = i;
        for (int i = 0; i < n; i++)
            if (out[i] >= n - h) temp[bpos[bckt[i]]++] = out[i];
        for (int i = 0; i < n; i++)
            if (out[i] >= h) temp[bpos[pos2bckt[out[i] - h]]++] = out[i] - h;
        c = 0;
        for (int i = 0; i + 1 < n; i++) {
            int a = (bckt[i] != bckt[i + 1]) || (temp[i] >= n - h)
                    || (pos2bckt[temp[i + 1] + h] != pos2bckt[temp[i] + h]);
            bckt[i] = c;
            c += a;
        }
        bckt[n - 1] = c++;
        temp.swap(out);
    }
    return out;
}

// calculates lcp array. it needs suffix array & original sequence.
// O(n)
vector<int> lcp(const vector<T>& in, const vector<int>& sa) {
    int n = (int)in.size();
    if (n == 0) return vector<int>();
    vector<int> rank(n), height(n - 1);
    for (int i = 0; i < n; i++) rank[sa[i]] = i;
    for (int i = 0, h = 0; i < n; i++) {
        if (rank[i] == 0) continue;
        int j = sa[rank[i] - 1];
        while (i + h < n && j + h < n && in[i + h] == in[j + h]) h++;
        height[rank[i] - 1] = h;
        if (h > 0) h--;
    }
    return height;
}
```

## 7.5 Manacher's Algorithm

```
// find longest palindromic span for each element in str
// O(|str|)
void manacher(const string& str, int plen[]) {
    int r = -1, p = -1;
    for (int i = 0; i < str.length(); ++i) {
        if (i <= r)
            plen[i] = min((2 * p - i >= 0) ? plen[2 * p - i] : 0, r - i);
        else
            plen[i] = 0;
        while (i - plen[i] - 1 >= 0 && i + plen[i] + 1 < str.length()
               && str[i - plen[i] - 1] == str[i + plen[i] + 1]) {
            plen[i] += 1;
        }
        if (i + plen[i] > r) {
            r = i + plen[i];
            p = i;
        }
    }
}
```

# 8 Miscellaneous

## 8.1 Fast I/O

```
namespace fio {
    const int BSIZE = 524288;
    char buffer[BSIZE];
    int p = BSIZE;
    inline char readChar() {
```

```
        if(p == BSIZE) {
            fread(buffer, 1, BSIZE, stdin);
            p = 0;
        }
        return buffer[p++];
    }
    int readInt() {
        char c = readChar();
        while ((c < '0' || c > '9') && c != '-') {
            c = readChar();
        }
        int ret = 0; bool neg = c == '-';
        if (neg) c = readChar();
        while (c >= '0' && c <= '9') {
            ret = ret * 10 + c - '0';
            c = readChar();
        }
        return neg ? -ret : ret;
    }
}
```

## 8.2 Magic Numbers

소수 : 10 007 , 10 009 , 10 111 , 31 567 , 70 001 , 1 000 003 , 1 000 033 , 4 000 037 , 99 999 989 , 999 999 937 , 1 000 000 007 , 1 000 000 009 , 9 999 999 967 , 99 999 999 977