

Report

Assignment 4

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Question 1 (Maximum-Flow Problem)

Pseudo Code of augment(f,p)

```
Let b = bottleneck (P , f )
For each edge (u, v) ∈ P
    If e = (u, v) is a forward edge then
        increase f (e) in G by b
    Else ( (u, v) is a backward edge, and let e = (v, u) )
        decrease f (e) in G by b
    Endif
Endfor
Return( f )
```

Pseudo Code of Max-Flow(G,s,t)

```
Initially f (e) = 0 for all e in G
While there is an s - t path in the residual graph Gf
    Let P be a simple s - t path in Gf
    f' = augment ( f , P )
    Update f to be f'
    Update the residual graph Gf to be Gf'
Endwhile
Return f
```

Observations

- The result of augment (f, P) is a new flow f' in G , obtained by increasing and decreasing the flow values on edges of P .
- Algorithm implemented above is also called *Ford-Fulkerson Algorithm*.
- Time complexity of *Ford-Fulkerson algorithm* is $O(EC)$, where C is the max_flow .
- To find an s - t path in G_f , we use breadth-first search.

Question 2 (0-1 Knapsack Problem)

Pseudo Code Knapsack(n, W)

```
Array M[0 . . . n, 0 . . . W]
Initialize M[0, w] = 0 for each w = 0, 1, . . . , W
For i = 1, 2, . . . , n
    For w = 0, . . . , W
        Use the recurrence
            If  $w < w_i$  then  $\text{OPT}(i, w) = \text{OPT}(i - 1, w)$ . Otherwise
             $\text{OPT}(i, w) = \max(\text{OPT}(i - 1, w), v_i + \text{OPT}(i - 1, w - w_i))$ .
        to compute M[i, w]
    Endfor
Endfor
Return M[n, W]
```

Observations

- The Knapsack(n, W) Algorithm correctly computes the optimal value of the problem, and runs in $O(nW)$ time.
- Dynamic Programming is used in above algorithm.

Question 3 (Dijkstra's algorithm)

Pseudo Code Dijkstra(Graph, Source)

```
create vertex set Q
for each vertex  $v$  in  $Graph$ :
     $dist[v] \leftarrow INFINITY$ 
     $prev[v] \leftarrow UNDEFINED$ 
    add  $v$  to  $Q$ 

 $dist[source] \leftarrow 0$ 
while  $Q$  is not empty:
     $u \leftarrow$  vertex in  $Q$  with min  $dist[u]$ 
    remove  $u$  from  $Q$ 
    for each neighbor  $v$  of  $u$ :
         $alt \leftarrow dist[u] + length(u, v)$ 
        if  $alt < dist[v]$ :
             $dist[v] \leftarrow alt$ 
             $prev[v] \leftarrow u$ 

return  $dist[], prev[]$ 
```

Observations

- Time complexity of above algorithm is $O(E \log V)$.
- The code finds shortest distances from source to all vertices.
- Dijkstra's algorithm doesn't work for graphs with negative weight edges.
- It is a greedy algorithm.