



CFA INSTITUTE INVESTMENT SERIES

# DERIVATIVES



**Wendy L. Pirie, CFA**

Foreword by Mark Kritzman, CFA



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# DERIVATIVES



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Wendy L. Pirie, CFA

WILEY

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# FOREWORD

Since the breakthrough introduction of the Black–Scholes–Merton options pricing model in 1973, the field of financial derivatives has evolved into an extensive and highly scientific body of theoretical knowledge alongside a vast and vibrant market where economic producers, investors, finance professionals, and government regulators all interact to seek financial gains, manage risk, or promote price discovery. It is hard to imagine how even the most thoughtful and diligent practitioners can come to terms with such a broad and complex topic—until they read this book.

CFA Institute has compiled into a single book those parts of its curriculum that address this critically important topic. And it is apparent from reading this book that CFA Institute attracted preeminent scholars to develop its derivatives curriculum.

This book has several important virtues:

1. It is detailed, comprehensive, and exceptionally accessible.
2. It is efficiently organized in its coverage of topics.
3. It makes effective use of visualization with diagrams of transactions and strategy payoffs.
4. It includes numerous practice problems along with well-explained solutions.
5. And finally, unlike many academic textbooks, its focus is more practical than theoretical, although it does provide more-than-adequate treatment of the relevant theory.

The book begins by addressing the basics of derivatives, including definitions of the various types of derivatives and descriptions of the markets in which they trade.

It goes on to address the purpose of derivatives and the benefits they impart to society, including risk transfer, price discovery, and operational efficiency. It also discusses how derivatives can be misused to enable excessive speculation and how derivatives could contribute to the destabilization of financial markets.

The book provides comprehensive treatment of pricing and valuation with discussions of the law of one price, risk neutrality, the Black–Scholes–Merton options pricing model, and the binomial model. It also covers the pricing of futures and forward contracts as well as swaps.

The book then shifts to applications of derivatives. It discusses how derivatives can be used to create synthetic cash and equity positions along with several other positions. It relies heavily on numerical examples to illustrate these equivalencies.

It offers a comprehensive treatment of risk management with discussions of market risk, credit risk, liquidity risk, operational risk, and model risk, among others. It describes how to measure risk and, more importantly, how to manage it with the application of forward and futures contracts, swaps, and options.

This summary of topics is intended to provide a flavor of the book's contents. The contents of this book are far broader and deeper than I describe in this foreword.

Those who practice finance, as well as those who teach it, in my view, owe a huge debt of gratitude to CFA Institute—first, for assembling this extraordinary body of knowledge in its curriculum and, second, for organizing this knowledge with such cohesion and clarity. Anyone who wishes to acquire a solid knowledge of derivatives or to refresh and expand what they have learned about derivatives previously should certainly read this book.

Mark Kritzman

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# PREFACE

We are pleased to bring you *Derivatives*. The content was developed in partnership by a team of distinguished academics and practitioners, chosen for their acknowledged expertise in the field, and guided by CFA Institute. It is written specifically with the investment practitioner in mind and is replete with examples and practice problems that reinforce the learning outcomes and demonstrate real-world applicability.

The CFA Program Curriculum, from which the content of this book was drawn, is subjected to a rigorous review process to ensure that it is:

- Faithful to the findings of our ongoing industry practice analysis
- Valuable to members, employers, and investors
- Globally relevant
- Generalist (as opposed to specialist) in nature
- Replete with sufficient examples and practice opportunities
- Pedagogically sound

The accompanying workbook is a useful reference that provides Learning Outcome Statements, which describe exactly what readers will learn and be able to demonstrate after mastering the accompanying material. Additionally, the workbook has summary overviews and practice problems for each chapter.

We hope you will find this and other books in the CFA Institute Investment Series helpful in your efforts to grow your investment knowledge, whether you are a relatively new entrant or an experienced veteran striving to keep up to date in the ever-changing market environment. CFA Institute, as a long-term committed participant in the investment profession and a not-for-profit global membership association, is pleased to provide you with this opportunity.

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## THE CFA PROGRAM

If the subject matter of this book interests you, and you are not already a CFA charterholder, we hope you will consider registering for the CFA Program and progressing toward earning the Chartered Financial Analyst designation. The CFA designation is a globally recognized standard of excellence for measuring the competence and integrity of investment professionals. To earn the CFA charter, candidates must successfully complete the CFA Program, a global graduate-level self-study program that combines a broad curriculum with professional conduct requirements as preparation for a career as an investment professional.

Anchored by a practice-based curriculum, the CFA Program Body of Knowledge reflects the knowledge, skills, and abilities identified by professionals as essential to the investment decision-making process. This body of knowledge maintains its relevance through a regular,

extensive survey of practicing CFA charterholders across the globe. The curriculum covers 10 general topic areas, ranging from equity and fixed-income analysis to portfolio management to corporate finance—all with a heavy emphasis on the application of ethics in professional practice. Known for its rigor and breadth, the CFA Program curriculum highlights principles common to every market so that professionals who earn the CFA designation have a thoroughly global investment perspective and a profound understanding of the global marketplace.

## CFA INSTITUTE

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CFA Institute is the premier association for investment professionals around the world, with over 142,000 members in 159 countries. Since 1963 the organization has developed and administered the renowned Chartered Financial Analyst® Program. With a rich history of leading the investment profession, CFA Institute has set the highest standards in ethics, education, and professional excellence within the global investment community, and is the foremost authority on investment profession conduct and practice. Each book in the CFA Institute Investment Series is geared toward industry practitioners along with graduate-level finance students and covers the most important topics in the industry.

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We would like to thank the many individuals who played a role in the conception and production of this book. In addition to the authors, these include the following: Richard Applebach, CFA; Evan Ashcraft, CFA; Fredrik Axsater, CFA; Giuseppe Ballocchi, CFA; William Barker, CFA; Christoph Behr, CFA; Richard Bookstaber; James Bronson, CFA; Bolong Cao, CFA; Lachlan Christie, CFA; Scott Clifford, CFA; Veselina Dinova, CFA; Pamela Drake, CFA; Jane Farris, CFA; James Finnegan, CFA; Ioannis Georgiou, CFA; Darlene Halwas, CFA; Walter (Bud) Haslett, CFA; Jeffrey Heisler, CFA; Stanley Jacobs; Vahan Janjigian, CFA; Eric Jemetz, CFA; Oliver Kahl, CFA; Erin Lorenzen, CFA; Richard K.C. Mak, CFA; Doug Manz, CFA; Alessandra Panunzio, CFA; Ray Rath, CFA; Qudratullah Rehan, CFA; Gary Sanger, CFA; Adam Schwartz, CFA; Sandeep Singh, CFA; David Smith, CFA; Frank Smudde, CFA; Zhiyi Song, CFA; Peter Stimes, CFA; Ahmed Sule, CFA; Barbara Valbuzzi, CFA; Lavone Whitmer, CFA.

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# ABOUT THE CFA INSTITUTE INVESTMENT SERIES

CFA Institute is pleased to provide you with the CFA Institute Investment Series, which covers major areas in the field of investments. We provide this best-in-class series for the same reason we have been chartering investment professionals for more than 50 years: to lead the investment profession globally by setting the highest standards of ethics, education, and professional excellence.

The books in the CFA Institute Investment Series contain practical, globally relevant material. They are intended both for those contemplating entry into the extremely competitive field of investment management as well as for those seeking a means of keeping their knowledge fresh and up to date. This series was designed to be user friendly and highly relevant.

We hope you find this series helpful in your efforts to grow your investment knowledge, whether you are a relatively new entrant or an experienced veteran ethically bound to keep up to date in the ever-changing market environment. As a long-term, committed participant in the investment profession and a not-for-profit global membership association, CFA Institute is pleased to provide you with this opportunity.

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## THE TEXTS

*Corporate Finance: A Practical Approach* is a solid foundation for those looking to achieve lasting business growth. In today's competitive business environment, companies must find innovative ways to enable rapid and sustainable growth. This text equips readers with the foundational knowledge and tools for making smart business decisions and formulating strategies to maximize company value. It covers everything from managing relationships between stakeholders to evaluating merger and acquisition bids, as well as the companies behind them. Through extensive use of real-world examples, readers will gain critical perspective into interpreting corporate financial data, evaluating projects, and allocating funds in ways that increase corporate value. Readers will gain insights into the tools and strategies used in modern corporate financial management.

*Fixed Income Analysis* has been at the forefront of new concepts in recent years, and this particular text offers some of the most recent material for the seasoned professional who is not a fixed-income specialist. The application of option and derivative technology to the once staid province of fixed income has helped contribute to an explosion of thought in this area. Professionals have been challenged to stay up to speed with credit derivatives, swaptions, collateralized mortgage securities, mortgage-backed securities, and other vehicles, and this explosion of products has strained the world's financial markets and tested central banks to provide

sufficient oversight. Armed with a thorough grasp of the new exposures, the professional investor is much better able to anticipate and understand the challenges our central bankers and markets face.

*International Financial Statement Analysis* is designed to address the ever-increasing need for investment professionals and students to think about financial statement analysis from a global perspective. The text is a practically oriented introduction to financial statement analysis that is distinguished by its combination of a true international orientation, a structured presentation style, and abundant illustrations and tools covering concepts as they are introduced in the text. The authors cover this discipline comprehensively and with an eye to ensuring the reader's success at all levels in the complex world of financial statement analysis.

*Investments: Principles of Portfolio and Equity Analysis* provides an accessible yet rigorous introduction to portfolio and equity analysis. Portfolio planning and portfolio management are presented within a context of up-to-date, global coverage of security markets, trading, and market-related concepts and products. The essentials of equity analysis and valuation are explained in detail and profusely illustrated. The book includes coverage of practitioner-important but often neglected topics, such as industry analysis. Throughout, the focus is on the practical application of key concepts with examples drawn from both emerging and developed markets. Each chapter affords the reader many opportunities to self-check his or her understanding of topics.

One of the most prominent texts over the years in the investment management industry has been Maginn and Tuttle's *Managing Investment Portfolios: A Dynamic Process*. The third edition updates key concepts from the 1990 second edition. Some of the more experienced members of our community own the prior two editions and will add the third edition to their libraries. Not only does this seminal work take the concepts from the other readings and put them in a portfolio context, but it also updates the concepts of alternative investments, performance presentation standards, portfolio execution, and, very importantly, individual investor portfolio management. Focusing attention away from institutional portfolios and toward the individual investor makes this edition an important and timely work.

*Quantitative Investment Analysis* focuses on some key tools that are needed by today's professional investor. In addition to classic time value of money, discounted cash flow applications, and probability material, there are two aspects that can be of value over traditional thinking.

*The New Wealth Management: The Financial Advisor's Guide to Managing and Investing Client Assets* is an updated version of Harold Evensky's mainstay reference guide for wealth managers. Harold Evensky, Stephen Horan, and Thomas Robinson have updated the core text of the 1997 first edition and added an abundance of new material to fully reflect today's investment challenges. The text provides authoritative coverage across the full spectrum of wealth management and serves as a comprehensive guide for financial advisors. The book expertly blends investment theory and real-world applications and is written in the same thorough but highly accessible style as the first edition. The first involves the chapters dealing with correlation and regression that ultimately figure into the formation of hypotheses for purposes of testing. This gets to a critical skill that challenges many professionals: the ability to distinguish useful information from the overwhelming quantity of available data. Second, the final chapter of Quantitative Investment Analysis covers portfolio concepts and takes the reader beyond the

traditional capital asset pricing model (CAPM) type of tools and into the more practical world of multifactor models and arbitrage pricing theory.

All books in the CFA Institute Investment Series are available through all major book-sellers. And, all titles are available on the Wiley Custom Select platform at <http://customselect.wiley.com/> where individual chapters for all the books may be mixed and matched to create custom textbooks for the classroom.



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# DERIVATIVES



# CHAPTER 1

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## DERIVATIVE MARKETS AND INSTRUMENTS

Don M. Chance, PhD, CFA

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### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- define a derivative and distinguish between exchange-traded and over-the-counter derivatives;
- contrast forward commitments with contingent claims;
- define forward contracts, futures contracts, options (calls and puts), swaps, and credit derivatives and compare their basic characteristics;
- describe purposes of, and controversies related to, derivative markets;
- explain arbitrage and the role it plays in determining prices and promoting market efficiency.

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### 1. INTRODUCTION

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Equity, fixed-income, currency, and commodity markets are facilities for trading the basic assets of an economy. Equity and fixed-income securities are claims on the assets of a company. Currencies are the monetary units issued by a government or central bank. Commodities are natural resources, such as oil or gold. These underlying assets are said to trade in **cash markets** or **spot markets** and their prices are sometimes referred to as **cash prices** or **spot prices**, though we usually just refer to them as stock prices, bond prices, exchange rates, and commodity prices. These markets exist around the world and receive much attention in the financial and mainstream media. Hence, they are relatively familiar not only to financial experts but also to the general population.

Somewhat less familiar are the markets for **derivatives**, which are financial instruments that derive their values from the performance of these basic assets. This reading is an overview

of derivatives. Subsequent readings will explore many aspects of derivatives and their uses in depth. Among the questions that this first reading will address are the following:

- What are the defining characteristics of derivatives?
- What purposes do derivatives serve for financial market participants?
- What is the distinction between a forward commitment and a contingent claim?
- What are forward and futures contracts? In what ways are they alike and in what ways are they different?
- What are swaps?
- What are call and put options and how do they differ from forwards, futures, and swaps?
- What are credit derivatives and what are the various types of credit derivatives?
- What are the benefits of derivatives?
- What are some criticisms of derivatives and to what extent are they well founded?
- What is arbitrage and what role does it play in a well-functioning financial market?

This reading is organized as follows. Section 2 explores the definition and uses of derivatives and establishes some basic terminology. Section 3 describes derivatives markets. Section 4 categorizes and explains types of derivatives. Sections 5 and 6 discuss the benefits and criticisms of derivatives, respectively. Section 7 introduces the basic principles of derivative pricing and the concept of arbitrage. Section 8 provides a summary.

## 2. DERIVATIVES: DEFINITIONS AND USES

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The most common definition of a derivative reads approximately as follows:

*A derivative is a financial instrument that derives its performance from the performance of an underlying asset.*

This definition, despite being so widely quoted, can nonetheless be a bit troublesome. For example, it can also describe mutual funds and exchange-traded funds, which would never be viewed as derivatives even though they derive their values from the values of the underlying securities they hold. Perhaps the distinction that best characterizes derivatives is that they usually *transform* the performance of the underlying asset before paying it out in the derivatives transaction. In contrast, with the exception of expense deductions, mutual funds and exchange-traded funds simply pass through the returns of their underlying securities. This transformation of performance is typically understood or implicit in references to derivatives but rarely makes its way into the formal definition. In keeping with customary industry practice, this characteristic will be retained as an implied, albeit critical, factor distinguishing derivatives from mutual funds and exchange-traded funds and some other straight pass-through instruments. Also, note that the idea that derivatives take their *performance* from an underlying asset encompasses the fact that derivatives take their value and certain other characteristics from the underlying asset. Derivatives strategies perform in ways that are derived from the underlying and the specific features of derivatives.

Derivatives are similar to insurance in that both allow for the transfer of risk from one party to another. As everyone knows, insurance is a financial contract that provides protection against loss. The party bearing the risk purchases an insurance policy, which transfers the risk to the other party, the insurer, for a specified period of time. The risk itself does not change,

but the party bearing it does. Derivatives allow for this same type of transfer of risk. One type of derivative in particular, the put option, when combined with a position exposed to the risk, functions almost exactly like insurance, but all derivatives can be used to protect against loss. Of course, an insurance contract must specify the underlying risk, such as property, health, or life. Likewise, so do derivatives. As noted earlier, derivatives are associated with an underlying asset. As such, the so-called “underlying asset” is often simply referred to as the **underlying**, whose value is the source of risk.<sup>1</sup> In fact, the underlying need not even be an asset itself. Although common derivatives underlyings are equities, fixed-income securities, currencies, and commodities, other derivatives underlyings include interest rates, credit, energy, weather, and even other derivatives, all of which are not generally thought of as assets. Thus, like insurance, derivatives pay off on the basis of a source of risk, which is often, but not always, the value of an underlying asset. And like insurance, derivatives have a definite life span and expire on a specified date.

Derivatives are created in the form of legal contracts. They involve two parties—the buyer and the seller (sometimes known as the writer)—each of whom agrees to do something for the other, either now or later. The buyer, who purchases the derivative, is referred to as the **long** or the holder because he owns (holds) the derivative and holds a long position. The seller is referred to as the **short** because he holds a short position.<sup>2</sup>

A derivative contract always defines the rights and obligations of each party. These contracts are intended to be, and almost always are, recognized by the legal system as commercial contracts that each party expects to be upheld and supported in the legal system. Nonetheless, disputes sometimes arise, and lawyers, judges, and juries may be required to step in and resolve the matter.

There are two general classes of derivatives. Some provide the ability to lock in a price at which one might buy or sell the underlying. Because they force the two parties to transact in the future at a previously agreed-on price, these instruments are called **forward commitments**. The various types of forward commitments are called forward contracts, futures contracts, and swaps. Another class of derivatives provides *the right but not the obligation* to buy or sell the underlying at a pre-determined price. Because the choice of buying or selling versus doing nothing depends on a particular random outcome, these derivatives are called **contingent claims**. The primary contingent claim is called an **option**. The types of derivatives will be covered in more detail later in this reading and in considerably more depth later in the curriculum.

The existence of derivatives begs the obvious question of what purpose they serve. If one can participate in the success of a company by holding its equity, what reason can possibly explain why another instrument is required that takes its value from the performance of the equity? Although equity and other fundamental markets exist and usually perform reasonably well without derivative markets, it is possible that derivative markets can *improve* the performance of the markets for the underlyings. As you will see later in this reading, that is indeed true in practice.

<sup>1</sup>Unfortunately, English financial language often evolves without regard to the rules of proper usage. *Underlying* is typically an adjective and, therefore, a modifier, but the financial world has turned it into a noun.

<sup>2</sup>In the financial world, the *long* always benefits from an increase in the value of the instrument he owns, and the *short* always benefits from a decrease in the value of the instrument he has sold. Think of the long as having possession of something and the short as having incurred an obligation to deliver that something.

Derivative markets create beneficial opportunities that do not exist in their absence. Derivatives can be used to create strategies that cannot be implemented with the underlyings alone. For example, derivatives make it easier to go short, thereby benefiting from a decline in the value of the underlying. In addition, derivatives, in and of themselves, are characterized by a relatively high degree of leverage, meaning that participants in derivatives transactions usually have to invest only a small amount of their own capital relative to the value of the underlying. As such, small movements in the underlying can lead to fairly large movements in the amount of money made or lost on the derivative. Derivatives generally trade at lower transaction costs than comparable spot market transactions, are often more liquid than their underlyings, and offer a simple, effective, and low-cost way to transfer risk. For example, a shareholder of a company can reduce or even completely eliminate the market exposure by trading a derivative on the equity. Holders of fixed-income securities can use derivatives to reduce or completely eliminate interest rate risk, allowing them to focus on the credit risk. Alternatively, holders of fixed-income securities can reduce or eliminate the credit risk, focusing more on the interest rate risk. Derivatives permit such adjustments easily and quickly. These features of derivatives are covered in more detail later in this reading.

The types of performance transformations facilitated by derivatives allow market participants to practice more effective risk management. Indeed, the entire field of derivatives, which at one time was focused mostly on the instruments themselves, is now more concerned with the *uses* of the instruments. Just as a carpenter uses a hammer, nails, screws, a screwdriver, and a saw to build something useful or beautiful, a financial expert uses derivatives to manage risk. And just as it is critically important that a carpenter understands how to use these tools, an investment practitioner must understand how to properly use derivatives. In the case of the carpenter, the result is building something useful; in the case of the financial expert, the result is managing financial risk. Thus, like tools, derivatives serve a valuable purpose but like tools, they must be used carefully.

The practice of risk management has taken a prominent role in financial markets. Indeed, whenever companies announce large losses from trading, lending, or operations, stories abound about how poorly these companies managed risk. Such stories are great attention grabbers and a real boon for the media, but they often miss the point that risk management does not guarantee that large losses will not occur. Rather, *risk management is the process by which an organization or individual defines the level of risk it wishes to take, measures the level of risk it is taking, and adjusts the latter to equal the former*. Risk management never offers a guarantee that large losses will not occur, and it does not eliminate the possibility of total failure. To do so would typically require that the amount of risk taken be so small that the organization would be effectively constrained from pursuing its primary objectives. Risk taking is inherent in all forms of economic activity and life in general. The possibility of failure is never eliminated.

### EXAMPLE 1 Characteristics of Derivatives

1. Which of the following is the best example of a derivative?
  - A. A global equity mutual fund
  - B. A non-callable government bond
  - C. A contract to purchase Apple Computer at a fixed price

2. Which of the following is **not** a characteristic of a derivative?
  - A. An underlying
  - B. A low degree of leverage
  - C. Two parties—a buyer and a seller
3. Which of the following statements about derivatives is **not** true?
  - A. They are created in the spot market.
  - B. They are used in the practice of risk management.
  - C. They take their values from the value of something else.

*Solution to 1:* C is correct. Mutual funds and government bonds are not derivatives. A government bond is a fundamental asset on which derivatives might be created, but it is not a derivative itself. A mutual fund can technically meet the definition of a derivative, but as noted in the reading, derivatives transform the value of a payoff of an underlying asset. Mutual funds merely pass those payoffs through to their holders.

*Solution to 2:* B is correct. All derivatives have an underlying and must have a buyer and a seller. More importantly, derivatives have high degrees of leverage, not low degrees of leverage.

*Solution to 3:* A is correct. Derivatives are used to practice risk management and they take (derive) their values from the value of something else, the underlying. They are not created in the spot market, which is where the underlying trades.

Note also that risk management is a dynamic and ongoing process, reflecting the fact that the risk assumed can be difficult to measure and is constantly changing. As noted, derivatives are tools, indeed *the* tools that make it easier to manage risk. Although one can trade stocks and bonds (the underlyings) to adjust the level of risk, it is almost always more effective to trade derivatives.

Risk management is addressed more directly elsewhere in the CFA curriculum, but the study of derivatives necessarily entails the concept of risk management. In an explanation of derivatives, the focus is usually on the instruments and it is easy to forget the overriding objective of managing risk. Unfortunately, that would be like a carpenter obsessed with his hammer and nails, forgetting that he is building a piece of furniture. It is important to always try to keep an eye on the objective of managing risk.

### 3. THE STRUCTURE OF DERIVATIVE MARKETS

Having an understanding of equity, fixed-income, and currency markets is extremely beneficial—indeed, quite necessary—in understanding derivatives. One could hardly consider the wisdom of using derivatives on a share of stock if one did not understand the equity markets reasonably well. As you likely know, equities trade on organized exchanges as well as in over-the-counter (OTC) markets. These exchange-traded equity markets—such as the Deutsche Börse, the Tokyo Stock Exchange, and the New York Stock Exchange and its Eurex affiliate—are formal organizational structures that bring buyers and sellers together through market

makers, or dealers, to facilitate transactions. Exchanges have formal rule structures and are required to comply with all securities laws.

OTC securities markets operate in much the same manner, with similar rules, regulations, and organizational structures. At one time, the major difference between OTC and exchange markets for securities was that the latter brought buyers and sellers together in a physical location, whereas the former facilitated trading strictly in an electronic manner. Today, these distinctions are blurred because many organized securities exchanges have gone completely to electronic systems. Moreover, OTC securities markets can be formally organized structures, such as NASDAQ, or can merely refer to informal networks of parties who buy and sell with each other, such as the corporate and government bond markets in the United States.

The derivatives world also comprises organized exchanges and OTC markets. Although the derivatives world is also moving toward less distinction between these markets, there are clear differences that are important to understand.

### 3.1. Exchange-Traded Derivatives Markets

Derivative instruments are created and traded either on an exchange or on the OTC market. Exchange-traded derivatives are standardized, whereas OTC derivatives are customized. To standardize a derivative contract means that its terms and conditions are precisely specified by the exchange and there is very limited ability to alter those terms. For example, an exchange might offer trading in certain types of derivatives that expire only on the third Friday of March, June, September, and December. If a party wanted the derivative to expire on any other day, it would not be able to trade such a derivative on that exchange, nor would it be able to persuade the exchange to create it, at least not in the short run. If a party wanted a derivative on a particular entity, such as a specific stock, that party could trade it on that exchange only if the exchange had specified that such a derivative could trade. Even the magnitudes of the contracts are specified. If a party wanted a derivative to cover €150,000 and the exchange specified that contracts could trade only in increments of €100,000, the party could do nothing about it if it wanted to trade that derivative on that exchange.

This standardization of contract terms facilitates the creation of a more liquid market for derivatives. If all market participants know that derivatives on the euro trade in 100,000-unit lots and that they all expire only on certain days, the market functions more effectively than it would if there were derivatives with many different unit sizes and expiration days competing in the same market at the same time. This standardization makes it easier to provide liquidity. Through designated market makers, derivatives exchanges guarantee that derivatives can be bought and sold.<sup>3</sup>

The cornerstones of the exchange-traded derivatives market are the market makers (or dealers) and the speculators, both of whom typically own memberships on the exchange.<sup>4</sup> The

<sup>3</sup>It is important to understand that merely being able to buy and sell a derivative, or even a security, does not mean that liquidity is high and that the cost of liquidity is low. Derivatives exchanges guarantee that a derivative can be bought and sold, but they do not guarantee the price. The ask price (the price at which the market maker will sell) and the bid price (the price at which the market maker will buy) can be far apart, which they will be in a market with low liquidity. Hence, such a market can have liquidity, loosely defined, but the cost of liquidity can be quite high. The factors that can lead to low liquidity for derivatives are similar to those for securities: little trading interest and a high level of uncertainty.

<sup>4</sup>Exchanges are owned by their *members*, whose memberships convey the right to trade. In addition, some exchanges are themselves publicly traded corporations whose members are shareholders, and there are also non-member shareholders.

market makers stand ready to buy at one price and sell at a higher price. With standardization of terms and an active market, market makers are often able to buy and sell almost simultaneously at different prices, locking in small, short-term profits—a process commonly known as scalping. In some cases, however, they are unable to do so, thereby forcing them to either hold exposed positions or find other parties with whom they can trade and thus lay off (get rid of) the risk. This is when speculators come in. Although speculators are market participants who are willing to take risks, it is important to understand that being a speculator does not mean the reckless assumption of risk. Although speculators will take large losses at times, good speculators manage those risks by watching their exposures, absorbing market information, and observing the flow of orders in such a manner that they are able to survive and profit. Often, speculators will hedge their risks when they become uncomfortable.

Standardization also facilitates the creation of a clearing and settlement operation. **Clearing** refers to the process by which the exchange verifies the execution of a transaction and records the participants' identities. **Settlement** refers to the related process in which the exchange transfers money from one participant to the other or from a participant to the exchange or vice versa. This flow of money is a critical element of derivatives trading. Clearly, there would be no confidence in markets in which money is not efficiently collected and disbursed. Derivatives exchanges have done an excellent job of clearing and settlement, especially in comparison to securities exchanges. Derivatives exchanges clear and settle all contracts overnight, whereas most securities exchanges require two business days.

The clearing and settlement process of derivative transactions also provides a credit guarantee. If two parties engage in a derivative contract on an exchange, one party will ultimately make money and the other will lose money. Derivatives exchanges use their clearinghouses to provide a guarantee to the winning party that if the loser does not pay, the clearinghouse will pay the winning party. The clearinghouse is able to provide this credit guarantee by requiring a cash deposit, usually called the **margin bond** or **performance bond**, from the participants to the contract. Derivatives clearinghouses manage these deposits, occasionally requiring additional deposits, so effectively that they have never failed to pay in the nearly 100 years they have existed. We will say more about this process later and illustrate how it works.

Exchange markets are said to have **transparency**, which means that full information on all transactions is disclosed to exchanges and regulatory bodies. All transactions are centrally reported within the exchanges and their clearinghouses, and specific laws require that these markets be overseen by national regulators. Although this would seem a strong feature of exchange markets, there is a definite cost. Transparency means a loss of privacy: National regulators can see what transactions have been done. Standardization means a loss of flexibility: A participant can do only the transactions that are permitted on the exchange. Regulation means a loss of both privacy and flexibility. It is not that transparency or regulation is good and the other is bad. It is simply a trade-off.

Derivatives exchanges exist in virtually all developed (and some emerging market) countries around the world. Some exchanges specialize in derivatives and others are integrated with securities exchanges.

Although there have been attempts to create somewhat non-standardized derivatives for trading on an exchange, such attempts have not been particularly successful. Standardization is a critical element by which derivatives exchanges are able to provide their services. We will look at this point again when discussing the alternative to standardization: customized OTC derivatives.

### 3.2. Over-the-Counter Derivatives Markets

The OTC derivatives markets comprise an informal network of market participants that are willing to create and trade virtually any type of derivative that can legally exist. The backbone of these markets is the set of dealers, which are typically banks. Most of these banks are members of a group called the International Swaps and Derivatives Association (ISDA), a worldwide organization of financial institutions that engage in derivative transactions, primarily as dealers. As such, these markets are sometimes called *dealer markets*. Acting as principals, these dealers informally agree to buy and sell various derivatives. It is *informal* because the dealers are not obligated to do so. Their participation is based on a desire to profit, which they do by purchasing at one price and selling at a higher price. Although it might seem that a dealer who can “buy low, sell high” could make money easily, the process in practice is not that simple. Because OTC instruments are not standardized, a dealer cannot expect to buy a derivative at one price and simultaneously sell it to a different party who happens to want to buy the same derivative at the same time and at a higher price.

To manage the risk they assume by buying and selling customized derivatives, OTC derivatives dealers typically hedge their risks by engaging in alternative but similar transactions that pass the risk on to other parties. For example, if a company comes to a dealer to buy a derivative on the euro, the company would effectively be transferring the risk of the euro to the dealer. The dealer would then attempt to lay off (get rid of) that risk by engaging in an alternative but similar transaction that would transfer the risk to another party. This hedge might involve another derivative on the euro or it might simply be a transaction in the euro itself. Of course, that begs the question of why the company could not have laid off the risk itself and avoided the dealer. Indeed, some can and do, but laying off risk is not simple. Unable to find identical offsetting transactions, dealers usually have to find *similar* transactions with which they can lay off the risk. Hedging one derivative with a different kind of derivative on the same underlying is a similar but not identical transaction. It takes specialized knowledge and complex models to be able to do such transactions effectively, and dealers are more capable of doing so than are ordinary companies. Thus, one might think of a dealer as a middleman, a sort of financial wholesaler using its specialized knowledge and resources to facilitate the transfer of risk. In the same manner that one could theoretically purchase a consumer product from a manufacturer, a network of specialized middlemen and retailers is often a more effective method.

Because of the customization of OTC derivatives, there is a tendency to think that the OTC market is less liquid than the exchange market. That is not necessarily true. Many OTC instruments can easily be created and then essentially offset by doing the exact opposite transaction, often with the same party. For example, suppose Corporation A buys an OTC derivative from Dealer B. Before the expiration date, Corporation A wants to terminate the position. It can return to Dealer B and ask to sell a derivative with identical terms. Market conditions will have changed, of course, and the value of the derivative will not be the same, but the transaction can be conducted quite easily with either Corporation A or Dealer B netting a gain at the expense of the other. Alternatively, Corporation A could do this transaction with a different dealer, the result of which would remove exposure to the underlying risk but would leave two transactions open and some risk that one party would default to the other. In contrast to this type of OTC liquidity, some exchange-traded derivatives have very little trading interest and thus relatively low liquidity. Liquidity is always driven by trading interest, which can be strong or weak in both types of markets.

OTC derivative markets operate at a lower degree of regulation and oversight than do exchange-traded derivative markets. In fact, until around 2010, it could largely be said that the

OTC market was essentially unregulated. OTC transactions could be executed with only the minimal oversight provided through laws that regulated the parties themselves, not the specific instruments. Following the financial crisis that began in 2007, new regulations began to blur the distinction between OTC and exchange-listed markets. In both the United States (the Wall Street Reform and Consumer Protection Act of 2010, commonly known as the Dodd–Frank Act) and Europe (the Regulation of the European Parliament and of the Council on OTC Derivatives, Central Counterparties, and Trade Repositories), regulations are changing the characteristics of OTC markets.

When the full implementation of these new laws takes place, a number of OTC transactions will have to be cleared through central clearing agencies, information on most OTC transactions will need to be reported to regulators, and entities that operate in the OTC market will be more closely monitored. There are, however, quite a few exemptions that cover a significant percentage of derivative transactions. Clearly, the degree of OTC regulation, although increasing in recent years, is still lighter than that of exchange-listed market regulation. Many transactions in OTC markets will retain a degree of privacy with lower transparency, and most importantly, the OTC markets will remain considerably more flexible than the exchange-listed markets.

## EXAMPLE 2 Exchange-Traded versus Over-the-Counter Derivatives

1. Which of the following characteristics is **not** associated with exchange-traded derivatives?
  - A. Margin or performance bonds are required.
  - B. The exchange guarantees all payments in the event of default.
  - C. All terms except the price are customized to the parties' individual needs.
2. Which of the following characteristics is associated with over-the-counter derivatives?
  - A. Trading occurs in a central location.
  - B. They are more regulated than exchange-listed derivatives.
  - C. They are less transparent than exchange-listed derivatives.
3. Market makers earn a profit in both exchange and over-the-counter derivatives markets by:
  - A. charging a commission on each trade.
  - B. a combination of commissions and markups.
  - C. buying at one price, selling at a higher price, and hedging any risk.
4. Which of the following statements *most* accurately describes exchange-traded derivatives relative to over-the-counter derivatives? Exchange-traded derivatives are more likely to have:
  - A. greater credit risk.
  - B. standardized contract terms.
  - C. greater risk management uses.

*Solution to 1:* C is correct. Exchange-traded contracts are standardized, meaning that the exchange determines the terms of the contract except the price. The exchange guarantees against default and requires margins or performance bonds.

*Solution to 2:* C is correct. OTC derivatives have a lower degree of transparency than exchange-listed derivatives. Trading does not occur in a central location but, rather, is quite dispersed. Although new national securities laws are tightening the regulation of OTC derivatives, the degree of regulation is less than that of exchange-listed derivatives.

*Solution to 3:* C is correct. Market makers buy at one price (the bid), sell at a higher price (the ask), and hedge whatever risk they otherwise assume. Market makers do not charge a commission. Hence, A and B are both incorrect.

*Solution to 4:* B is correct. Standardization of contract terms is a characteristic of exchange-traded derivatives. A is incorrect because credit risk is well-controlled in exchange markets. C is incorrect because the risk management uses are not limited by being traded over the counter.

## 4. TYPES OF DERIVATIVES

As previously stated, derivatives fall into two general classifications: forward commitments and contingent claims. The factor that distinguishes forward commitments from contingent claims is that the former *oblige* the parties to engage in a transaction at a future date on terms agreed upon in advance, whereas the latter provide one party the *right but not the obligation* to engage in a future transaction on terms agreed upon in advance.

### 4.1. Forward Commitments

Forward commitments are contracts entered into at one point in time that require both parties to engage in a transaction at a later point in time (the expiration) on terms agreed upon at the start. The parties establish the identity and quantity of the underlying, the manner in which the contract will be executed or settled when it expires, and the fixed price at which the underlying will be exchanged. This fixed price is called the **forward price**.

As a hypothetical example of a forward contract, suppose that today Markus and Johannes enter into an agreement that Markus will sell his BMW to Johannes for a price of €30,000. The transaction will take place on a specified date, say, 180 days from today. At that time, Markus will deliver the vehicle to Johannes's home and Johannes will give Markus a bank-certified check for €30,000. There will be no recourse, so if the vehicle has problems later, Johannes cannot go back to Markus for compensation. It should be clear that both Markus and Johannes must do their due diligence and carefully consider the reliability of each other. The car could have serious quality issues and Johannes could have financial problems and be unable to pay the €30,000. Obviously, the transaction is essentially unregulated. Either party could renege on his obligation, in response to which the other party could go to court, provided a formal contract exists and is carefully written. Note finally that one of the two parties is likely to end up gaining and the other losing, depending on the secondary market price of this type of vehicle at expiration of the contract.

This example is quite simple but illustrates the essential elements of a forward contract. In the financial world, such contracts are very carefully written, with legal provisions that guard against fraud and require extensive credit checks. Now let us take a deeper look at the characteristics of forward contracts.

#### 4.1.1. Forward Contracts

The following is the formal definition of a forward contract:

*A forward contract is an over-the-counter derivative contract in which two parties agree that one party, the buyer, will purchase an underlying asset from the other party, the seller, at a later date at a fixed price they agree on when the contract is signed.*

In addition to agreeing on the price at which the underlying asset will be sold at a later date, the two parties also agree on several other matters, such as the specific identity of the underlying, the number of units of the underlying that will be delivered, and where the future delivery will occur. These are important points but relatively minor in this discussion, so they can be left out of the definition to keep it uncluttered.

As noted earlier, a forward contract is a commitment. Each party agrees that it will fulfill its responsibility at the designated future date. Failure to do so constitutes a default and the non-defaulting party can institute legal proceedings to enforce performance. It is important to recognize that although either party could default to the other, only one party at a time can default. The party owing the greater amount could default to the other, but the party owing the lesser amount cannot default because its claim on the other party is greater. The amount owed is always based on the net owed by one party to the other.

To gain a better understanding of forward contracts, it is necessary to examine their payoffs. As noted, forward contracts—and indeed all derivatives—take (derive) their payoffs from the performance of the underlying asset. To illustrate the payoff of a forward contract, start with the assumption that we are at time  $t = 0$  and that the forward contract expires at a later date, time  $t = T$ .<sup>5</sup> The spot price of the underlying asset at time 0 is  $S_0$  and at time  $T$  is  $S_T$ . Of course, when we initiate the contract at time 0, we do not know what  $S_T$  will ultimately be. Remember that the two parties, the buyer and the seller, are going long and short, respectively.

At time  $t = 0$ , the long and the short agree that the short will deliver the asset to the long at time  $T$  for a price of  $F_0(T)$ . The notation  $F_0(T)$  denotes that this value is established at time 0 and applies to a contract expiring at time  $T$ .  $F_0(T)$  is the forward price. Later, you will learn how the forward price is determined. It turns out that it is quite easy to do, but we do not need to know right now.<sup>6</sup>

So, let us assume that the buyer enters into the forward contract with the seller for a price of  $F_0(T)$ , with delivery of one unit of the underlying asset to occur at time  $T$ . Now, let us roll forward to time  $T$ , when the price of the underlying is  $S_T$ . The long is obligated to pay  $F_0(T)$ , for which he receives an asset worth  $S_T$ . If  $S_T > F_0(T)$ , it is clear that the transaction has worked out well for the long. He paid  $F_0(T)$  and receives something of greater value. Thus, the contract effectively pays off  $S_T - F_0(T)$  to the long, which is the value of the contract at expiration. The short has the mirror image of the long. He is required to deliver the asset worth  $S_T$  and accept a smaller amount,  $F_0(T)$ . The contract has a payoff for him of  $F_0(T) - S_T$ , which

<sup>5</sup>Such notations as  $t = 0$  and  $t = T$  are commonly used in explaining derivatives. To indicate that  $t = 0$  simply means that we initiate a contract at an imaginary time designated like a counter starting at zero. To indicate that the contract expires at  $t = T$  simply means that at some future time, designated as  $T$ , the contract expires. Time  $T$  could be a certain number of days from now or a fraction of a year later or  $T$  years later. We will be more specific in later readings that involve calculations. For now, just assume that  $t = 0$  and  $t = T$  are two dates—the initiation and the expiration—of the contract.

<sup>6</sup>This point is covered more fully elsewhere in the readings on derivatives, but we will see it briefly later in this reading.

is negative. Even if the asset's value,  $S_T$ , is less than the forward price,  $F_0(T)$ , the payoffs are still  $S_T - F_0(T)$  for the long and  $F_0(T) - S_T$  for the short. We can consolidate these results by writing the short's payoff as the negative of the long's,  $-(S_T - F_0(T))$ , which serves as a useful reminder that the long and the short are engaged in a zero-sum game, which is a type of competition in which one participant's gains are the other's losses. Although both lose a modest amount in the sense of both having some costs to engage in the transaction, these costs are relatively small and worth ignoring for our purposes at this time. In addition, it is worthwhile to note how derivatives transform the performance of the underlying. The gain from owning the underlying would be  $S_T - S_0$ , whereas the gain from owning the forward contract would be  $S_T - F_0(T)$ . Both figures are driven by  $S_T$ , the price of the underlying at expiration, but they are not the same.

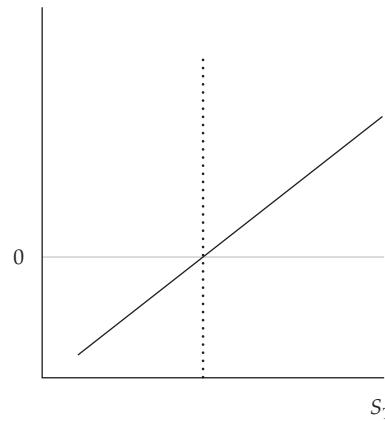
Exhibit 1 illustrates the payoffs from both buying and selling a forward contract.

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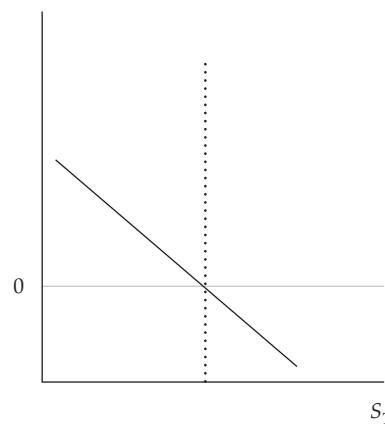
EXHIBIT 1 Payoffs from a Forward Contract

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**A. Payoff from Buying =  $S_T - F_0(T)$**   
 Payoff



**B. Payoff from Selling =  $-(S_T - F_0(T))$**   
 Payoff



The long hopes the price of the underlying will rise above the forward price,  $F_0(T)$ , whereas the short hopes the price of the underlying will fall below the forward price. Except in the extremely rare event that the underlying price at  $T$  equals the forward price, there will ultimately be a winner and a loser.

An important element of forward contracts is that no money changes hands between parties when the contract is initiated. Unlike in the purchase and sale of an asset, there is no value exchanged at the start. The buyer does not pay the seller some money and obtain something. In fact, forward contracts have zero value at the start. They are neither assets nor liabilities. As you will learn in later readings, their values will deviate from zero later as prices move. Forward contracts will almost always have non-zero values at expiration.

As noted previously, the primary purpose of derivatives is for risk management. Although the uses of forward contracts are covered in depth later in the curriculum, there are a few things to note here about the purposes of forward contracts. It should be apparent that locking in the future buying or selling price of an underlying asset can be extremely attractive for some parties. For example, an airline anticipating the purchase of jet fuel at a later date can enter into a forward contract to buy the fuel at a price agreed upon when the contract is initiated. In so doing, the airline has hedged its cost of fuel. Thus, forward contracts can be structured to create a perfect hedge, providing an assurance that the underlying asset can be bought or sold at a price known when the contract is initiated. Likewise, speculators, who ultimately assume the risk laid off by hedgers, can make bets on the direction of the underlying asset without having to invest the money to purchase the asset itself.

Finally, forward contracts need not specifically settle by delivery of the underlying asset. They can settle by an exchange of cash. These contracts—called **non-deliverable forwards** (NDFs), **cash-settled forwards**, or **contracts for differences**—have the same economic effect as do their delivery-based counterparts. For example, for a physical delivery contract, if the long pays  $F_0(T)$  and receives an asset worth  $S_T$ , the contract is worth  $S_T - F_0(T)$  to the long at expiration. A non-deliverable forward contract would have the short simply pay cash to the long in the amount of  $S_T - F_0(T)$ . The long would not take possession of the underlying asset, but if he wanted the asset, he could purchase it in the market for its current price of  $S_T$ . Because he received a cash settlement in the amount of  $S_T - F_0(T)$ , in buying the asset the long would have to pay out only  $S_T - [S_T - F_0(T)]$ , which equals  $F_0(T)$ . Thus, the long could acquire the asset, effectively paying  $F_0(T)$ , exactly as the contract promised. Transaction costs do make cash settlement different from physical delivery, but this point is relatively minor and can be disregarded for our purposes here.

As previously mentioned, forward contracts are OTC contracts. There is no formal forward contract exchange. Nonetheless, there are exchange-traded variants of forward contracts, which are called futures contracts or just futures.

#### 4.1.2. Futures

Futures contracts are specialized versions of forward contracts that have been standardized and that trade on a futures exchange. By standardizing these contracts and creating an organized market with rules, regulations, and a central clearing facility, the futures markets offer an element of liquidity and protection against loss by default.

Formally, a futures contract is defined as follows:

*A futures contract is a standardized derivative contract created and traded on a futures exchange in which two parties agree that one party, the buyer, will purchase an underlying*

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*asset from the other party, the seller, at a later date and at a price agreed on by the two parties when the contract is initiated and in which there is a daily settling of gains and losses and a credit guarantee by the futures exchange through its clearinghouse.*

First, let us review what standardization means. Recall that in forward contracts, the parties customize the contract by specifying the underlying asset, the time to expiration, the delivery and settlement conditions, and the quantity of the underlying, all according to whatever terms they agree on. These contracts are not traded on an exchange. As noted, the regulation of OTC derivatives markets is increasing, but these contracts are not subject to the traditionally high degree of regulation that applies to securities and futures markets. Futures contracts first require the existence of a futures exchange, a legally recognized entity that provides a market for trading these contracts. Futures exchanges are highly regulated at the national level in all countries. These exchanges specify that only certain contracts are authorized for trading. These contracts have specific underlying assets, times to expiration, delivery and settlement conditions, and quantities. The exchange offers a facility in the form of a physical location and/or an electronic system as well as liquidity provided by authorized market makers.

Probably the most important distinctive characteristic of futures contracts is the daily settlement of gains and losses and the associated credit guarantee provided by the exchange through its clearinghouse. When a party buys a futures contract, it commits to purchase the underlying asset at a later date and at a price agreed upon when the contract is initiated. The counterparty (the seller) makes the opposite commitment, an agreement to sell the underlying asset at a later date and at a price agreed upon when the contract is initiated. The agreed-upon price is called the **futures price**. Identical contracts trade on an ongoing basis at different prices, reflecting the passage of time and the arrival of new information to the market. Thus, as the futures price changes, the parties make and lose money. Rising (falling) prices, of course, benefit (hurt) the long and hurt (benefit) the short. At the end of each day, the clearinghouse engages in a practice called **mark to market**, also known as the **daily settlement**. The clearinghouse determines an average of the final futures trades of the day and designates that price as the **settlement price**. All contracts are then said to be *marked to the settlement price*. For example, if the long purchases the contract during the day at a futures price of £120 and the settlement price at the end of the day is £122, the long's account would be marked for a gain of £2. In other words, the long has made a profit of £2 and that amount is credited to his account, with the money coming from the account of the short, who has lost £2. Naturally, if the futures price decreases, the long loses money and is charged with that loss, and the money is transferred to the account of the short.<sup>7</sup>

The account is specifically referred to as a **margin** account. Of course, in equity markets, margin accounts are commonly used, but there are significant differences between futures margin accounts and equity margin accounts. Equity margin accounts involve the extension of credit. An investor deposits part of the cost of the stock and borrows the remainder at a rate of interest. With futures margin accounts, both parties deposit a required minimum sum of money, but the remainder of the price is not borrowed. This required margin is typically less

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<sup>7</sup>The actual amount of money charged and credited depends on the contract size and the number of contracts. A price of £120 might actually refer to a contract that has a standard size of £100,000. Thus, £120 might actually mean 120% of the standard size, or £120,000. In addition, the parties are likely to hold more than one contract. Hence, the gain of £2 referred to in the text might really mean £2,000 (122% minus 120% times the £100,000 standard size) times the number of contracts held by the party.

than 10% of the futures price, which is considerably less than in equity margin trading. In the example above, let us assume that the required margin is £10, which is referred to as the **initial margin**. Both the long and the short put that amount into their respective margin accounts. This money is deposited there to support the trade, not as a form of equity, with the remaining amount borrowed. There is no formal loan created as in equity markets. A futures margin is more of a performance bond or good faith deposit, terms that were previously mentioned. It is simply an amount of money put into an account that covers possible future losses.

Associated with each initial margin is another figure called the **maintenance margin**. The maintenance margin is the amount of money that each participant must maintain in the account after the trade is initiated, and it is always significantly lower than the initial margin. Let us assume that the maintenance margin in this example is £6. If the buyer's account is marked to market with a credit of £2, his margin balance moves to £12, while the seller's account is charged £2 and his balance moves to £8. The clearinghouse then compares each participant's balance with the maintenance margin. At this point, both participants more than meet the maintenance margin.

Let us say, however, that the price continues to move in the long's favor and, therefore, against the short. A few days later, assume that the short's balance falls to £4, which is below the maintenance margin requirement of £6. The short will then get a **margin call**, which is a request to deposit additional funds. The amount that the short has to deposit, however, is *not* the £2 that would bring his balance up to the maintenance margin. Instead, the short must deposit enough funds to bring the balance up to the initial margin. So, the short must come up with £6. The purpose of this rule is to get the party's position significantly above the minimum level and provide some breathing room. If the balance were brought up only to the maintenance level, there would likely be another margin call soon. A party can choose not to deposit additional funds, in which case the party would be required to close out the contract as soon as possible and would be responsible for any additional losses until the position is closed.

As with forward contracts, neither party pays any money to the other when the contract is initiated. Value accrues as the futures price changes, but at the end of each day, the mark-to-market process settles the gains and losses, effectively resetting the value for each party to zero.

The clearinghouse moves money between the participants, crediting gains to the winners and charging losses to the losers. By doing this on a daily basis, the gains and losses are typically quite small, and the margin balances help ensure that the clearinghouse will collect from the party losing money. As an extra precaution, in fast-moving markets, the clearinghouse can make margin calls during the day, not just at the end of the day. Yet there still remains the possibility that a party could default. A large loss could occur quickly and consume the entire margin balance, with additional money owed.<sup>8</sup> If the losing party cannot pay, the clearinghouse provides a guarantee that it will make up the loss, which it does by maintaining an insurance fund. If that fund were depleted, the clearinghouse could levy a tax on the other market participants, though that has never happened.

<sup>8</sup>For example, let us go back to when the short had a balance of £4, which is £2 below the maintenance margin and £6 below the initial margin. The short will get a margin call, but suppose he elects not to deposit additional funds and requests that his position be terminated. In a fast-moving market, the price might increase more than £4 before his broker can close his position. The remaining balance of £4 would then be depleted, and the short would be responsible for any additional losses.

Some futures contracts contain a provision limiting price changes. These rules, called **price limits**, establish a band relative to the previous day's settlement price, within which all trades must occur. If market participants wish to trade at a price above the upper band, trading stops, which is called **limit up**, until two parties agree on a trade at a price lower than the upper limit. Likewise, if market participants wish to trade at a price below the lower band, which is called **limit down**, no trade can take place until two parties agree to trade at a price above the lower limit. When the market hits these limits and trading stops, it is called **locked limit**. Typically, the exchange rules provide for an expansion of the limits the next day. These price limits, which may be somewhat objectionable to proponents of free markets, are important in helping the clearinghouse manage its credit exposure. Just because two parties wish to trade a futures contract at a price beyond the limits does not mean they should be allowed to do so. The clearinghouse is a third participant in the contract, guaranteeing to each party that it ensures against the other party defaulting. Therefore, the clearinghouse has a vested interest in the price and considerable exposure. Sharply moving prices make it more difficult for the clearinghouse to collect from the parties losing money.

Most participants in futures markets buy and sell contracts, collecting their profits and incurring their losses, with no ultimate intent to make or take delivery of the underlying asset. For example, the long may ultimately sell her position before expiration. When a party re-enters the market at a later date but before expiration and engages in the opposite transaction—a long selling her previously opened contract or a short buying her previously opened contract—the transaction is referred to as an offset. The clearinghouse marks the contract to the current price relative to the previous settlement price and closes out the participant's position.

At any given time, the number of outstanding contracts is called the **open interest**. Each contract counted in the open interest has a long and a corresponding short. The open interest figure changes daily as some parties open up new positions, while other parties offset their old positions. It is theoretically possible that all longs and shorts offset their positions before expiration, leaving no open interest when the contract expires, but in practice there is nearly always some open interest at expiration, at which time there is a final delivery or settlement.

When discussing forward contracts, we noted that a contract could be written such that the parties engage in physical delivery or cash settlement at expiration. In the futures markets, the exchange specifies whether physical delivery or cash settlement applies. In physical delivery contracts, the short is required to deliver the underlying asset at a designated location and the long is required to pay for it. Delivery replaces the mark-to-market process on the final day. It also ensures an important principle that you will use later: *The futures price converges to the spot price at expiration*. Because the short delivers the actual asset and the long pays the current spot price for it, the futures price at expiration has to be the spot price at that time. Alternatively, a futures contract initiated right at the instant of expiration is effectively a spot transaction and, therefore, the futures price at expiration must equal the spot price. Following this logic, in cash settlement contracts, there is a final mark to market, with the futures price formally set to the spot price, thereby ensuring automatic convergence.

In discussing forward contracts, we described the process by which they pay off as the spot price at expiration minus the forward price,  $S_T - F_0(T)$ , the former determined at expiration and the latter agreed upon when the contract is initiated. Futures contracts basically pay off the same way, but there is a slight difference. Let us say the contract is initiated on Day 0 and expires on Day  $T$ . The intervening days are designated Days 1, 2, ...,  $T$ . The initial futures price is designated  $f_0(T)$  and the daily settlement prices on Days 1, 2, ...,  $T$  are designated  $f_1(T), f_2(T), \dots, f_T(T)$ . There are, of course, futures prices within each trading day, but let us

focus only on the settlement prices for now. For simplicity, let us assume that the long buys at the settlement price on Day 0 and holds the position all the way to expiration. Through the mark-to-market process, the cash flows to the account of the long will be

$$\begin{aligned} f_1(T) - f_0(T) &\text{ on Day 1} \\ f_2(T) - f_1(T) &\text{ on Day 2} \\ f_3(T) - f_2(T) &\text{ on Day 3} \\ \dots \\ f_T(T) - f_{T-1}(T) &\text{ on Day } T \end{aligned}$$

These add up to

$$f_T(T) - f_0(T) \text{ on Day } T.$$

And because of the convergence of the final futures price to the spot price,

$$f_T(T) - f_0(T) = S_T - f_0(T),$$

which is the same as with forward contracts.<sup>9</sup> Note, however, that the timing of these profits is different from that of forwards. Forward contracts realize the full amount,  $S_T - f_0(T)$ , at expiration, whereas futures contracts realize this amount in parts on a day-to-day basis. Naturally, the time value of money principle says that these are not equivalent amounts of money. But the differences tend to be small, particularly in low-interest-rate environments, some of these amounts are gains and some are losses, and most futures contracts have maturities of less than a year.

But the near equivalence of the profits from a futures and a forward contract disguises an important distinction between these types of contracts. In a forward contact, with the entire payoff made at expiration, a loss by one party can be large enough to trigger a default. Hence, forward contracts are subject to default and require careful consideration of the credit quality of the counterparties. Because futures contracts settle gains and collect losses daily, the amounts that could be lost upon default are much smaller and naturally give the clearinghouse much greater flexibility to manage the credit risk it assumes.

Unlike forward markets, futures markets are highly regulated at the national level. National regulators are required to approve new futures exchanges and even new contracts proposed by existing exchanges as well as changes in margin requirements, price limits, and any significant changes in trading procedures. Violations of futures regulations can be subject to governmental prosecution. In addition, futures markets are far more transparent than forward markets. Futures prices, volume, and open interest are widely reported and easily obtained. Futures prices of nearby expiring contracts are often used as proxies for spot prices, particularly in decentralized spot markets, such as gold, which trades in spot markets all over the world.

In spite of the advantages of futures markets over forward markets, forward markets also have advantages over futures markets. Transparency is not always a good thing. Forward markets offer more privacy and fewer regulatory encumbrances. In addition, forward markets offer more flexibility. With the ability to tailor contracts to the specific needs of participants,

<sup>9</sup>Because of this equivalence, we will not specifically illustrate the profit graphs of futures contracts. You can generally treat them the same as those of forwards, which were shown in Exhibit 1.

forward contracts can be written exactly the way the parties want. In contrast, the standardization of futures contracts makes it more difficult for participants to get exactly what they want, even though they may get close substitutes. Yet, futures markets offer a valuable credit guarantee.

Like forward markets, futures markets can be used for hedging or speculation. For example, a jewelry manufacturer can buy gold futures, thereby hedging the price it will have to pay for one of its key inputs. Although it is more difficult to construct a futures strategy that hedges perfectly than to construct a forward strategy that does so, futures offer the benefit of the credit guarantee. It is not possible to argue that futures are better than forwards or vice versa. Market participants always trade off advantages against disadvantages. Some participants prefer futures, and some prefer forwards. Some prefer one over the other for certain risks and the other for other risks. Some might use one for a particular risk at a point in time and a different instrument for the same risk at another point in time. The choice is a matter of taste and constraints.

The third and final type of forward commitment we will cover is swaps. They go a step further in committing the parties to buy and sell something at a later date: They obligate the parties to a sequence of multiple purchases and sales.

#### 4.1.3. Swaps

The concept of a swap is that two parties exchange (swap) a series of cash flows. One set of cash flows is variable or floating and will be determined by the movement of an underlying asset or rate. The other set of cash flows can be variable and determined by a different underlying asset or rate, or it can be fixed. Formally, a swap is defined as follows:

*A swap is an over-the-counter derivative contract in which two parties agree to exchange a series of cash flows whereby one party pays a variable series that will be determined by an underlying asset or rate and the other party pays either (1) a variable series determined by a different underlying asset or rate or (2) a fixed series.*

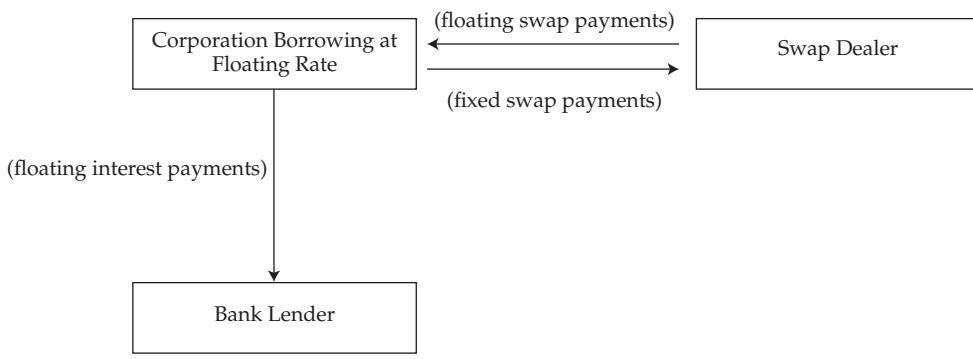
As with forward contracts, swap contracts also contain other terms—such as the identity of the underlying, the relevant payment dates, and the payment procedure—that are negotiated between the parties and written into the contract. A swap is a bit more like a forward contract than a futures contract in that it is an OTC contract, so it is privately negotiated and subject to default. Nonetheless, the similarities between futures and forwards apply to futures and swaps and, indeed, combinations of futures contracts expiring at different dates are often compared to swaps.

As with forward contracts, either party can default but only one party can default at a particular time. The money owed is always based on the net owed by one party to the other. Hence, the party owing the lesser amount cannot default to the party owing the greater amount. Only the latter can default, and the amount it owes is the net of what it owes and what is owed to it, which is also true with forwards.

Swaps are relatively young financial instruments, having been created only in the early 1980s. Thus, it may be somewhat surprising to learn that the swap is the most widely used derivative, a likely result of its simplicity and embracement by the corporate world. The most common swap is the **fixed-for-floating interest rate swap**. In fact, this type of swap is so common that it is often called a “plain vanilla swap” or just a “vanilla swap,” owing to the notion that vanilla ice cream is considered plain (albeit tasty).

Let us examine a scenario in which the vanilla interest rate swap is frequently used. Suppose a corporation borrows from a bank at a floating rate. It would prefer a fixed rate, which would enable it to better anticipate its cash flow needs in making its interest payments.<sup>10</sup> The corporation can effectively convert its floating-rate loan to a fixed-rate loan by adding a swap, as shown in Exhibit 2.

**EXHIBIT 2** Using an Interest Rate Swap to Convert a Floating-Rate Loan to a Fixed-Rate Loan



The interest payments on the loan are tied to a specific floating rate. For a dollar-based loan, that rate has typically been US dollar Libor.<sup>11</sup> The payments would be based on the rate from the Libor market on a specified reset date times the loan balance times a factor reflecting the number of days in the current interest calculation period. The actual payment is made at a later date. Thus, for a loan balance of, say, \$10 million with monthly payments, the rate might be based on Libor on the first business day of the month, with interest payable on the first business day of the next month, which is the next reset date, and calculated as \$10 million times the rate times 30/360. The 30/360 convention, an implicit assumption of 30 days in a month, is common but only one of many interest calculation conventions used in the financial world. Often, “30” is replaced by the exact number of days since the last interest payment. The use of a 360-day year is a common assumption in the financial world, which originated in the pre-calculator days when an interest rate could be multiplied by a number like 30/360, 60/360, 90/360, etc., more easily than if 365 were used.

Whatever the terms of the loan are, the terms of the swap are typically set to match those of the loan. Thus, a Libor-based loan with monthly payments based on the 30/360 convention would be matched with a swap with monthly payments based on Libor and the 30/360 convention and the same reset and payment dates. Although the loan has an actual balance (the amount owed by borrower to creditor), the swap does not have such a balance owed by

<sup>10</sup>Banks prefer to make floating-rate loans because their own funding is typically short term and at floating rates. Thus, their borrowing rates reset frequently, giving them a strong incentive to pass that risk on to their customers through floating-rate loans.

<sup>11</sup>Recall that US dollar Libor (London Interbank Offered Rate) is the estimated rate on a dollar-based loan made by one London bank to another. Such a loan takes the form of a time deposit known as a Eurodollar because it represents a dollar deposited in a European bank account. In fact, Libor is the same as the so-called Eurodollar rate. The banks involved can be British banks or British branches of non-British banks. The banks estimate their borrowing rates, and a single average rate is assembled and reported each day. That rate is then commonly used to set the rate on many derivative contracts.

one party to the other. Thus, it has no principal, but it does have a balance of sorts, called the **notional principal**, which ordinarily matches the loan balance. A loan with only one principal payment, the final one, will be matched with a swap with a fixed notional principal. An amortizing loan, which has a declining principal balance, will be matched with a swap with a pre-specified declining notional principal that matches the loan balance.

As with futures and forwards, no money changes hands at the start; thus, the value of a swap when initiated must be zero. The fixed rate on the swap is determined by a process that forces the value to zero, a procedure that will be covered later in the curriculum. As market conditions change, the value of a swap will deviate from zero, being positive to one party and negative to the other.

As with forward contracts, swaps are subject to default, but because the notional amount of a swap is not typically exchanged, the credit risk of a swap is much less than that of a loan.<sup>12</sup> The only money passing from one party to the other is the net difference between the fixed and floating interest payments. In fact, the parties do not even pay each other. Only one party pays the other, as determined by the net of the greater amount owed minus the lesser amount. This does not mean that swaps are not subject to a potentially large amount of credit risk. At a given point in time, one party could default, effectively owing the value of all remaining payments, which could substantially exceed the value that the non-defaulting party owes to the defaulting party. Thus, there is indeed credit risk in a swap. This risk must be managed by careful analysis before the transaction and by the potential use of such risk-mitigating measures as collateral.

There are also interest rate swaps in which one party pays on the basis of one interest rate and the other party pays on the basis of a different interest rate. For example, one party might make payments at Libor, whereas the other might make payments on the basis of the US Treasury bill rate. The difference between Libor and the T-bill rate, often called the TED spread (T-bills versus Eurodollar), is a measure of the credit risk premium of London banks, which have historically borrowed short term at Libor, versus that of the US government, which borrows short term at the T-bill rate. This transaction is called a basis swap. There are also swaps in which the floating rate is set as an average rate over the period, in accordance with the convention for many loans. Some swaps, called overnight indexed swaps, are tied to a Fed funds-type rate, reflecting the rate at which banks borrow overnight. As we will cover later, there are many other different types of swaps that are used for a variety of purposes. The plain vanilla swap is merely the simplest and most widely used.

Because swaps, forwards, and futures are forward commitments, they can all accomplish the same thing. One could create a series of forwards or futures expiring at a set of dates that would serve the same purpose as a swap. Although swaps are better suited for risks that involve multiple payments, at its most fundamental level, a swap is more or less just a series of forwards and, acknowledging the slight differences discussed above, more or less just a series of futures.

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<sup>12</sup>It is possible that the notional principal will be exchanged in a currency swap, whereby each party makes a series of payments to the other in different currencies. Whether the notional principal is exchanged depends on the purpose of the swap. This point will be covered later in the curriculum. At this time, you should see that it would be fruitless to exchange notional principals in an interest rate swap because that would mean each party would give the other the same amount of money when the transaction is initiated and re-exchange the same amount of money when the contract terminates.

**EXAMPLE 3** Forward Contracts, Futures Contracts, and Swaps

1. Which of the following characterizes forward contracts and swaps but **not** futures?
  - A. They are customized.
  - B. They are subject to daily price limits.
  - C. Their payoffs are received on a daily basis.
2. Which of the following distinguishes forwards from swaps?
  - A. Forwards are OTC instruments, whereas swaps are exchange traded.
  - B. Forwards are regulated as futures, whereas swaps are regulated as securities.
  - C. Swaps have multiple payments, whereas forwards have only a single payment.
3. Which of the following occurs in the daily settlement of futures contracts?
  - A. Initial margin deposits are refunded to the two parties.
  - B. Gains and losses are reported to other market participants.
  - C. Losses are charged to one party and gains credited to the other.

*Solution to 1:* A is correct. Forwards and swaps are OTC contracts and, therefore, are customized. Futures are exchange traded and, therefore, are standardized. Some futures contracts are subject to daily price limits and their payoffs are received daily, but these characteristics are not true for forwards and swaps.

*Solution to 2:* C is correct. Forwards and swaps are OTC instruments and both are regulated as such. Neither is regulated as a futures contract or a security. A swap is a series of multiple payments at scheduled dates, whereas a forward has only one payment, made at its expiration date.

*Solution to 3:* C is correct. Losses and gains are collected and distributed to the respective parties. There is no specific reporting of these gains and losses to anyone else. Initial margin deposits are not refunded and, in fact, additional deposits may be required.

This material completes our introduction to forward commitments. All forward commitments are firm contracts. The parties are required to fulfill the obligations they agreed to. The benefit of this rigidity is that neither party pays anything to the other when the contract is initiated. If one party needs some flexibility, however, it can get it by agreeing to pay the other party some money when the contract is initiated. When the contract expires, the party who paid at the start has some flexibility in deciding whether to buy the underlying asset at the fixed price. Thus, that party did not actually agree to do anything. It had a choice. This is the nature of contingent claims.

#### 4.2. Contingent Claims

A **contingent claim** is a derivative in which the outcome or payoff is dependent on the outcome or payoff of an underlying asset. Although this characteristic is also associated with forward commitments, a contingent claim has come to be associated with a *right*, but not an

*obligation*, to make a final payment contingent on the performance of the underlying. Given that the holder of the contingent claim has a choice, the term *contingent claim* has become synonymous with the term *option*. The holder has a choice of whether or not to exercise the option. This choice creates a payoff that transforms the underlying payoff in a more pronounced manner than does a forward, futures, or swap. Those instruments provide linear payoffs: As the underlying goes up (down), the derivative gains (loses). The further up (down) the underlying goes, the more the derivative gains (loses). Options are different in that they limit losses in one direction. In addition, options can pay off as the underlying goes down. Hence, they transform the payoffs of the underlying into something quite different.

#### 4.2.1. Options

We might say that an option, as a contingent claim, grants the right but not the obligation to buy an asset at a later date and at a price agreed on when the option is initiated. But there are so many variations of options that we cannot settle on this statement as a good formal definition. For one thing, options can also grant the right to sell instead of the right to buy. Moreover, they can grant the right to buy or sell earlier than at expiration. So, let us see whether we can combine these points into an all-encompassing definition of an option.

*An option is a derivative contract in which one party, the buyer, pays a sum of money to the other party, the seller or writer, and receives the right to either buy or sell an underlying asset at a fixed price either on a specific expiration date or at any time prior to the expiration date.*

Unfortunately, even that definition does not cover every unique aspect of options. For example, options can be created in the OTC market and customized to the terms of each party, or they can be created and traded on options exchanges and standardized. As with forward contracts and swaps, customized options are subject to default but are less regulated and relatively transparent. Exchange-traded options are protected against default by the clearinghouse of the options exchange and are relatively transparent and regulated at the national level. As noted in the definition above, options can be terminated early or at their expirations. When an option is terminated, either early or at expiration, the holder of the option chooses whether to exercise it. If he exercises it, he either buys or sells the underlying asset, but he does not have both rights. The right to buy is one type of option, referred to as a **call** or **call option**, whereas the right to sell is another type of option, referred to as a **put** or **put option**. With one very unusual and advanced exception that we do not cover, an option is either a call or a put, and that point is made clear in the contract.

An option is also designated as exercisable early (before expiration) or only at expiration. Options that can be exercised early are referred to as **American-style**. Options that can be exercised only at expiration are referred to as **European-style**. *It is extremely important that you do not associate these terms with where these options are traded.* Both types of options trade on all continents.<sup>13</sup>

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<sup>13</sup>For example, you do not associate French dressing with France. It is widely available and enjoyed worldwide. If you dig deeper into the world of options, you will find Asian options and Bermuda options. Geography is a common source of names for options as well as foods and in no way implies that the option or the food is available only in that geographical location.

As with forwards and futures, an option can be exercised by physical delivery or cash settlement, as written in the contract. For a call option with physical delivery, upon exercise the underlying asset is delivered to the call buyer, who pays the call seller the exercise price. For a put option with physical delivery, upon exercise the put buyer delivers the underlying asset to the put seller and receives the strike price. For a cash settlement option, exercise results in the seller paying the buyer the cash equivalent value as if the asset were delivered and paid for.

The fixed price at which the underlying asset can be purchased is called the **exercise price** (also called the “strike price,” the “strike,” or the “striking price”). This price is somewhat analogous to the forward price because it represents the price at which the underlying will be purchased or sold if the option is exercised. The forward price, however, is set in the pricing of the contract such that the contract value at the start is zero. The strike price of the option is chosen by the participants. The actual price or value of the option is an altogether different concept.

As noted, the buyer pays the writer a sum of money called the **option premium**, or just the “premium.” It represents a fair price of the option, and in a well-functioning market, it would be the value of the option. Consistent with everything we know about finance, it is the present value of the cash flows that are expected to be received by the holder of the option during the life of the option. At this point, we will not get into how this price is determined, but you will learn that later. For now, there are some fundamental concepts you need to understand, which form a basis for understanding how options are priced and why anyone would use an option.

Because the option buyer (the long) does not have to exercise the option, beyond the initial payment of the premium, there is no obligation of the long to the short. Thus, only the short can default, which would occur if the long exercises the option and the short fails to do what it is supposed to do. Thus, in contrast to forwards and swaps, in which either party could default to the other, default in options is possible only from the short to the long.

Ruling out the possibility of default for now, let us examine what happens when an option expires. Using the same notation used previously, let  $S_T$  be the price of the underlying at the expiration date,  $T$ , and  $X$  be the exercise price of the option. Remember that a call option allows the holder, or long, to pay  $X$  and receive the underlying. It should be obvious that the long would exercise the option at expiration if  $S_T$  is greater than  $X$ , meaning that the underlying value is greater than what he would pay to obtain the underlying. Otherwise, he would simply let the option expire. Thus, on the expiration date, the option is described as having a payoff of  $\text{Max}(0, S_T - X)$ . Because the holder of the option would be entitled to exercise it and claim this amount, it also represents the value of the option at expiration. Let us denote that value as  $c_T$ . Thus,

$$c_T = \text{Max}(0, S_T - X) \quad (\text{payoff to the call buyer}),$$

which is read as “take the maximum of either zero or  $S_T - X$ .” Thus, if the underlying value exceeds the exercise price ( $S_T > X$ ), then the option value is positive and equal to  $S_T - X$ . The call option is then said to be **in the money**. If the underlying value is less than the exercise price ( $S_T < X$ ), then  $S_T - X$  is negative; zero is greater than a negative number, so

the option value would be zero. When the underlying value is less than the exercise price, the call option is said to be **out of the money**. When  $S_T = X$ , the call option is said to be **at the money**, although at the money is, for all practical purposes, out of the money because the value is still zero.

This payoff amount is also the value of the option at expiration. It represents value because it is what the option is worth at that point. If the holder of the option sells it to someone else an instant before expiration, it should sell for that amount because the new owner would exercise it and capture that amount. To the seller, the value of the option at that point is  $-Max(0, S_T - X)$ , which is negative to the seller if the option is in the money and zero otherwise.

Using the payoff value and the price paid for the option, we can determine the profit from the strategy, which is denoted with the Greek symbol  $\Pi$ . Let us say the buyer paid  $c_0$  for the option at time 0. Then the profit is

$$\Pi = Max(0, S_T - X) - c_0 \quad (\text{profit to the call buyer}),$$

To the seller, who received the premium at the start, the payoff is

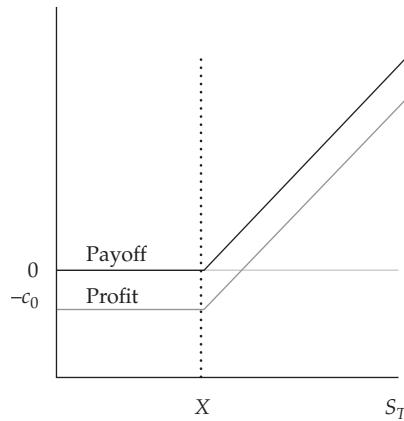
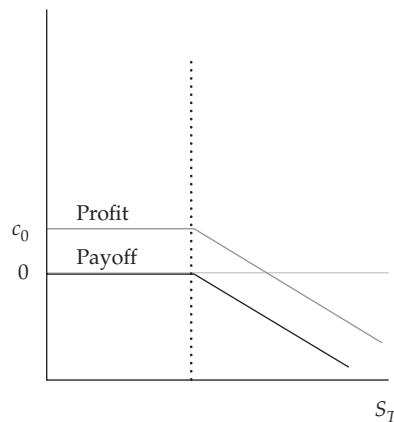
$$-c_T = -Max(0, S_T - X) \quad (\text{payoff to the call seller}),$$

The profit is

$$\Pi = -Max(0, S_T - X) + c_0 \quad (\text{profit to the call seller}).$$

Exhibit 3 illustrates the payoffs and profits to the call buyer and seller as graphical representations of these equations, with the payoff or value at expiration indicated by the dark line and the profit indicated by the light line. Note in Panel A that the buyer has no upper limit on the profit and has a fixed downside loss limit equal to the premium paid for the option. Such a condition, with limited loss and unlimited gain, is a temptation to many unsuspecting investors, but keep in mind that the graph does not indicate the frequency with which gains and losses will occur. Panel B is the mirror image of Panel A and shows that the seller has unlimited losses and limited gains. One might suspect that selling a call is, therefore, the worst investment strategy possible. Indeed, it is a risky strategy, but at this point these are only simple strategies. Other strategies can be added to mitigate the seller's risk to a substantial degree.

## EXHIBIT 3 Payoff and Profit from a Call Option

*A. Payoff and Profit from Buying*  
Payoff and Profit*B. Payoff and Profit from Selling*  
Payoff and Profit

Now let us consider put options. Recall that a put option allows its holder to sell the underlying asset at the exercise price. Thus, the holder should exercise the put at expiration if the underlying asset is worth less than the exercise price ( $S_T < X$ ). In that case, the put is said to be in the money. If the underlying asset is worth the same as the exercise price ( $S_T = X$ ), meaning the put is at the money, or more than the exercise price ( $S_T > X$ ), meaning the put is out of the money, the option holder would not exercise it and it would expire with zero value. Thus, the payoff to the put holder is

$$p_T = \text{Max}(0, X - S_T) \quad (\text{payoff to the put buyer}),$$

If the put buyer paid  $p_0$  for the put at time 0, the profit is

$$\Pi = \text{Max}(0, X - S_T) - p_0 \quad (\text{profit to the put buyer}),$$

And for the seller, the payoff is

$$-p_T = -\text{Max}(0, X - S_T) \quad (\text{payoff to the put seller}),$$

And the profit is

$$\Pi = -\text{Max}(0, X - S_T) + p_0 \quad (\text{profit to the put seller}).$$

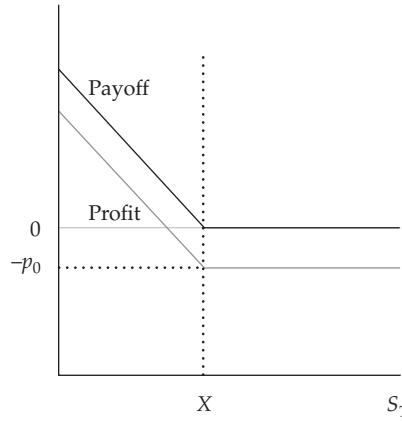
Exhibit 4 illustrates the payoffs and profits to the buyer and seller of a put.

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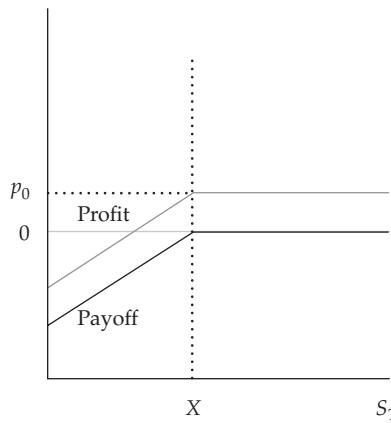
EXHIBIT 4 Payoff and Profit from a Put Option

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**A. Payoff and Profit from Buying**  
Payoff and Profit



**B. Payoff and Profit from Selling**  
Payoff and Profit




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The put buyer has a limited loss, and although the gain is limited by the fact that the underlying value cannot go below zero, the put buyer does gain more the lower the value of the underlying. In this manner, we see how a put option is like insurance. Bad outcomes for the

underlying trigger a payoff for both the insurance policy and the put, whereas good outcomes result only in loss of the premium. The put seller, like the insurer, has a limited gain and a loss that is larger the lower the value of the underlying. As with call options, these graphs must be considered carefully because they do not indicate the frequency with which gains and losses will occur. At this point, it should be apparent that buying a call option is consistent with a bullish point of view and buying a put option is consistent with a bearish point of view. Moreover, in contrast to forward commitments, which have payoffs that are linearly related to the payoffs of the underlying (note the straight lines in Exhibit 1), contingent claims have payoffs that are non-linear in relation to the underlying. There is linearity over a range—say, from 0 to  $X$  or from  $X$  upward or downward—but over the entire range of values for the underlying, the payoffs of contingent claims cannot be depicted with a single straight line.

We have seen only a snapshot of the payoff and profit graphs that can be created with options. Calls can be combined with puts, the underlying asset, and other calls or puts with different expirations and exercise prices to create a diverse set of payoff and profit graphs, some of which are covered later in the curriculum.

Before leaving options, let us again contrast the differences between options and forward commitments. With forward commitments, the parties agree to trade an underlying asset at a later date and at a price agreed upon when the contract is initiated. Neither party pays any cash to the other at the start. With options, the buyer pays cash to the seller at the start and receives the right, but not the obligation, to buy (if a call) or sell (if a put) the underlying asset at expiration at a price agreed upon (the exercise price) when the contract is initiated. In contrast to forwards, futures, and swaps, options do have value at the start: the premium paid by buyer to seller. That premium pays for the *right*, eliminating the *obligation*, to trade the underlying at a later date, as would be the case with a forward commitment.

Although there are numerous variations of options, most have the same essential features described here. There is, however, a distinctive family of contingent claims that emerged in the early 1990s and became widely used and, in some cases, heavily criticized. These instruments are known as credit derivatives.

#### 4.2.2. Credit Derivatives

Credit risk is surely one of the oldest risks known to mankind. Human beings have been lending things to each other for thousands of years, and even the most primitive human beings must have recognized the risk of lending some of their possessions to their comrades. Until the last 20 years or so, however, the management of credit risk was restricted to simply doing the best analysis possible before making a loan, monitoring the financial condition of the borrower during the loan, limiting the exposure to a given party, and requiring collateral. Some modest forms of insurance against credit risk have existed for a number of years, but insurance can be a slow and cumbersome way of protecting against credit loss. Insurance is typically highly regulated, and insurance laws are usually very consumer oriented. Thus, credit insurance as a financial product has met with only modest success.

In the early 1990s, however, the development of the swaps market led to the creation of derivatives that would hedge credit risk. These instruments came to be known as **credit derivatives**, and they avoided many of the regulatory constraints of the traditional insurance industry. Here is a formal definition:

*A credit derivative is a class of derivative contracts between two parties, a credit protection buyer and a credit protection seller, in which the latter provides protection to the former against a specific credit loss.*

One of the first credit derivatives was a **total return swap**, in which the underlying is typically a bond or loan, in contrast to, say, a stock or stock index. The credit protection buyer offers to pay the credit protection seller the total return on the underlying bond. This total return consists of all interest and principal paid by the borrower plus any changes in the bond's market value. In return, the credit protection seller typically pays the credit protection buyer either a fixed or a floating rate of interest. Thus, if the bond defaults, the credit protection seller must continue to make its promised payments, while receiving a very small return or virtually no return from the credit protection buyer. If the bond incurs a loss, as it surely will if it defaults, the credit protection seller effectively pays the credit protection buyer.

Another type of credit derivative is the **credit spread option**, in which the underlying is the credit (yield) spread on a bond, which is the difference between the bond's yield and the yield on a benchmark default-free bond. As you will learn in the fixed-income material, the credit spread is a reflection of investors' perception of credit risk. Because a credit spread option requires a credit spread as the underlying, this type of derivative works only with a traded bond that has a quoted price. The credit protection buyer selects the strike spread it desires and pays the option premium to the credit protection seller. At expiration, the parties determine whether the option is in the money by comparing the bond's yield spread with the strike chosen, and if it is, the credit protection seller pays the credit protection buyer the established payoff. Thus, this instrument is essentially a call option in which the underlying is the credit spread.

A third type of credit derivative is the **credit-linked note (CLN)**. With this derivative, the credit protection buyer holds a bond or loan that is subject to default risk (the underlying reference security) and issues its own security (the credit-linked note) with the condition that if the bond or loan it holds defaults, the principal payoff on the credit-linked note is reduced accordingly. Thus, the buyer of the credit-linked note effectively insures the credit risk of the underlying reference security.

These three types of credit derivatives have had limited success compared with the fourth type of credit derivative, the **credit default swap (CDS)**. The credit default swap, in particular, has achieved much success by capturing many of the essential features of insurance while avoiding the high degree of consumer regulations that are typically associated with traditional insurance products.

In a CDS, one party—the credit protection buyer, who is seeking credit protection against a third party—makes a series of regularly scheduled payments to the other party, the credit protection seller. The seller makes no payments until a credit event occurs. A declaration of bankruptcy is clearly a credit event, but there are other types of credit events, such as a failure to make a scheduled payment or an involuntary restructuring. The CDS contract specifies what constitutes a credit event, and the industry has a procedure for declaring credit events, though that does not guarantee the parties will not end up in court arguing over whether something was or was not a credit event.

Formally, a credit default swap is defined as follows:

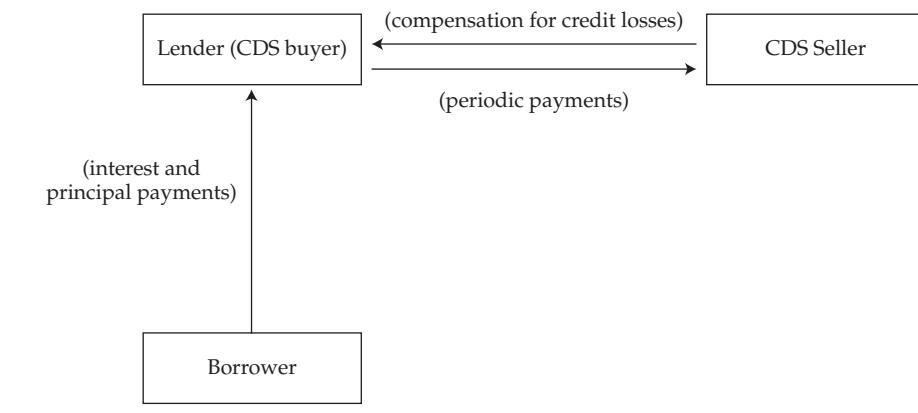
*A credit default swap is a derivative contract between two parties, a credit protection buyer and a credit protection seller, in which the buyer makes a series of cash payments to the seller and receives a promise of compensation for credit losses resulting from the default of a third party.*

A CDS is conceptually a form of insurance. Sellers of CDSs, oftentimes banks or insurance companies, collect periodic payments and are required to pay out if a loss occurs from the default of a third party. These payouts could take the form of restitution of the defaulted

amount or the party holding the defaulting asset could turn it over to the CDS seller and receive a fixed amount. The most common approach is for the payout to be determined by an auction to estimate the market value of the defaulting debt. Thus, CDSs effectively provide coverage against a loss in return for the protection buyer paying a premium to the protection seller, thereby taking the form of insurance against credit loss. Although insurance contracts have certain legal characteristics that are not found in credit default swaps, the two instruments serve similar purposes and operate in virtually the same way: payments made by one party in return for a promise to cover losses incurred by the other.

Exhibit 5 illustrates the typical use of a CDS by a lender. The lender is exposed to the risk of non-payment of principal and interest. The lender lays off this risk by purchasing a CDS from a CDS seller. The lender—now the CDS buyer—promises to make a series of periodic payments to the CDS seller, who then stands ready to compensate the CDS buyer for credit losses.

#### EXHIBIT 5 Using a Credit Default Swap to Hedge the Credit Risk of a Loan



Clearly, the CDS seller is betting on the borrower's not defaulting or—more generally, as insurance companies operate—that the total payouts it is responsible for are less than the total payments collected. Of course, most insurance companies are able to do this by having reliable actuarial statistics, diversifying their risk, and selling some of the risk to other insurance companies. Actuarial statistics are typically quite solid. Average claims for life, health, and casualty insurance are well documented, and insurers can normally set premiums to cover losses and operate at a reasonable profit. Although insurance companies try to manage some of their risks at the micro level (e.g., charging smokers more for life and health insurance), most of their risk management is at the macro level, wherein they attempt to make sure their risks are not concentrated. Thus, they avoid selling too much homeowners insurance to individuals in tornado-prone areas. If they have such an exposure, they can use the reinsurance market to sell some of the risk to other companies that are not overexposed to that risk. Insurance companies attempt to diversify their risks and rely on the principle of uncorrelated risks, which plays such an important role in portfolio management. A well-diversified insurance company, like a well-diversified portfolio, should be able to earn a return commensurate with its assumed risk in the long run.

Credit default swaps should operate the same way. Sellers of CDSs should recognize when their credit risk is too concentrated. When that happens, they become buyers of CDSs

from other parties or find other ways to lay off the risk. Unfortunately, during the financial crisis that began in 2007, many sellers of CDSs failed to recognize the high correlations among borrowers whose debt they had guaranteed. One well-known CDS seller, AIG, is a large and highly successful traditional insurance company that got into the business of selling CDSs. Many of these CDSs insured against mortgages. With the growth of the subprime mortgage market, many of these CDS-insured mortgages had a substantial amount of credit risk and were often poorly documented. AIG and many other CDS sellers were thus highly exposed to systemic credit contagion, a situation in which defaults in one area of an economy ripple into another, accompanied by bank weaknesses and failures, rapidly falling equity markets, rising credit risk premiums, and a general loss of confidence in the financial system and the economy. These presumably well-diversified risks guaranteed by CDS sellers, operating as though they were insurance companies, ultimately proved to be poorly diversified. Systemic financial risks can spread more rapidly than fire, health, and casualty risks. Virtually no other risks, except those originating from wars or epidemics, spread in the manner of systemic financial risks.

Thus, to understand and appreciate the importance of the CDS market, it is necessary to recognize how that market can fail. The ability to separate and trade risks is a valuable one. Banks can continue to make loans to their customers, thereby satisfying the customers' needs, while laying off the risk elsewhere. In short, parties not wanting to bear certain risks can sell them to parties wanting to assume certain risks. If all parties do their jobs correctly, the markets and the economy work more efficiently. If, as in the case of certain CDS sellers, not everyone does a good job of managing risk, there can be serious repercussions. In the case of AIG and some other companies, taxpayer bailouts were the ultimate price paid to keep these large institutions afloat so that they could continue to provide their other critical services to consumers. The rules proposed in the new OTC derivatives market regulations—which call for greater regulation and transparency of OTC derivatives and, in particular, CDSs—have important implications for the future of this market and these instruments.

#### EXAMPLE 4 Options and Credit Derivatives

1. An option provides which of the following?
  - A. Either the right to buy or the right to sell an underlying
  - B. The right to buy and sell, with the choice made at expiration
  - C. The obligation to buy or sell, which can be converted into the right to buy or sell
2. Which of the following is **not** a characteristic of a call option on a stock?
  - A. A guarantee that the stock will increase
  - B. A specified date on which the right to buy expires
  - C. A fixed price at which the call holder can buy the stock
3. A credit derivative is which of the following?
  - A. A derivative in which the premium is obtained on credit
  - B. A derivative in which the payoff is borrowed by the seller
  - C. A derivative in which the seller provides protection to the buyer against credit loss from a third party

*Solution to 1:* A is correct. An option is strictly the right to buy (a call) or the right to sell (a put). It does not provide both choices or the right to convert an obligation into a right.

*Solution to 2:* A is correct. A call option on a stock provides no guarantee of any change in the stock price. It has an expiration date, and it provides for a fixed price at which the holder can exercise the option, thereby purchasing the stock.

*Solution to 3:* C is correct. Credit derivatives provide a guarantee against loss caused by a third party's default. They do not involve borrowing the premium or the payoff.

#### 4.2.3. Asset-Backed Securities

Although these instruments are covered in more detail in the fixed-income material, we would be remiss if we failed to include them with derivatives. But we will give them only light coverage here.

As discussed earlier, derivatives take (derive) their value from the value of the underlying, as do mutual funds and exchange-traded funds (ETFs). A mutual fund or an ETF holding bonds is virtually identical to the investor holding the bonds directly. Asset-backed securities (ABSs) take this concept a step further by altering the payment streams. ABSs typically divide the payments into slices, called tranches, in which the priority of claims has been changed from equivalent to preferential. For example, in a bond mutual fund or an ETF, all investors in the fund have equal claims, and so the rate of return earned by each investor is exactly the same. If a portfolio of the same bonds were assembled into an ABS, some investors in the ABS would have claims that would supersede those of other investors. The differential nature of these claims becomes relevant when either prepayments or defaults occur.

Prepayments mostly affect only mortgages. When a portfolio of mortgages is assembled into an ABS, the resulting instrument is called a **collateralized mortgage obligation** (CMO). Commonly but not always, the credit risk has been reduced or eliminated, perhaps by a CDS, as discussed earlier. When homeowners pay off their mortgages early due to refinancing at lower rates, the holders of the mortgages suffer losses. They expected to receive a stream of returns that is now terminated. The funds that were previously earning a particular rate will now have to be invested to earn a lower rate. These losses are the mirror images of the gains homeowners make when they proudly proclaim that they refinanced their mortgages and substantially lowered their payments.

CMOs partition the claims against these mortgages into different tranches, which are typically called A, B, and C. Class C tranches bear the first wave of prepayments until that tranche has been completely repaid its full principal investment. At that point, the Class B tranche holders bear the next prepayments until they have been fully repaid. The Class A tranche holders then bear the next wave of prepayments.<sup>14</sup> Thus, the risk faced by the various tranche holders is different from that of a mutual fund or ETF, which would pass the returns directly through

<sup>14</sup>The reference to only three tranches is just a general statement. There are many more types of tranches. Our discussion of the three classes is for illustrative purposes only and serves to emphasize that there are high-priority claims, low-priority claims, and other claims somewhere in the middle.

such that investors would all receive the same rates of return. Therefore, the expected returns of CMO tranches vary and are commensurate with the prepayment risk they assume. Some CMOs are also characterized by credit risk, perhaps a substantial amount, from subprime mortgages.

When bonds or loans are assembled into ABSs, they are typically called **collateralized bond obligations** (CBOs) or **collateralized loan obligations** (CLOs). These instruments (known collectively as **collateralized debt obligations**, or CDOs) do not traditionally have much prepayment risk but they do have credit risk and oftentimes a great deal of it. The CDO structure allocates this risk to tranches that are called senior, mezzanine, or junior tranches (the last sometimes called equity tranches). When defaults occur, the junior tranches bear the risk first, followed by the mezzanine tranches, and then the senior tranches. The expected returns of the tranches vary according to the perceived credit risk, with the senior tranches having the highest credit quality and the junior the lowest. Thus, the senior tranches have the lowest expected returns and the junior tranches have the highest.

An asset-backed security is formally defined as follows:

*An asset-backed security is a derivative contract in which a portfolio of debt instruments is assembled and claims are issued on the portfolio in the form of tranches, which have different priorities of claims on the payments made by the debt securities such that pre-payments or credit losses are allocated to the most-junior tranches first and the most-senior tranches last.*

ABSs seem to have only an indirect and subtle resemblance to options, but they are indeed options. They promise to make a series of returns that are typically steady. These returns can be lowered if prepayments or defaults occur. Thus, they are contingent on prepayments and defaults. Take a look again at Exhibit 4, Panel B (the profit and payoff of a short put option). If all goes well, there is a fixed return. If something goes badly, the return can be lowered, and the worse the outcome, the lower the return. Thus, holders of ABSs have effectively written put options.

This completes the discussion of contingent claims. Having now covered forward commitments and contingent claims, the final category of derivative instruments is more or less just a catch-all category in case something was missed.

### 4.3. Hybrids

The instruments just covered encompass all the fundamental instruments that exist in the derivatives world. Yet, the derivatives world is truly much larger than implied by what has been covered here. We have not covered and will touch only lightly on the many hybrid instruments that combine derivatives, fixed-income securities, currencies, equities, and commodities. For example, options can be combined with bonds to form either callable bonds or convertible bonds. Swaps can be combined with options to form swap payments that have upper and lower limits. Options can be combined with futures to obtain options on futures. Options can be created with swaps as the underlying to form swaptions. Some of these instruments will be covered later. For now, you should just recognize that the possibilities are almost endless.

We will not address these hybrids directly, but some are covered elsewhere in the curriculum. The purpose of discussing them here is for you to realize that derivatives create possibilities

not otherwise available in their absence. This point will lead to a better understanding of why derivatives exist, a topic we will get to very shortly.

### EXAMPLE 5 Forward Commitments versus Contingent Claims

1. Which of the following is **not** a forward commitment?
  - A. An agreement to take out a loan at a future date at a specific rate
  - B. An offer of employment that must be accepted or rejected in two weeks
  - C. An agreement to lease a piece of machinery for one year with a series of fixed monthly payments
2. Which of the following statements is true about contingent claims?
  - A. Either party can default to the other.
  - B. The payoffs are linearly related to the performance of the underlying.
  - C. The most the long can lose is the amount paid for the contingent claim.

*Solution to 1:* B is correct. Both A and C are commitments to engage in transactions at future dates. In fact, C is like a swap because the party agrees to make a series of future payments and in return receives temporary use of an asset whose value could vary. B is a contingent claim. The party receiving the employment offer can accept it or reject it if there is a better alternative.

*Solution to 2:* C is correct. The maximum loss to the long is the premium. The payoffs of contingent claims are not linearly related to the underlying, and only one party, the short, can default.

## 4.4. Derivatives Underlyings

Before discussing the purposes and benefits of derivatives, we need to clarify some points that have been implied so far. We have alluded to certain underlying assets, this section will briefly discuss the underlyings more directly.

### 4.4.1. Equities

Equities are one of the most popular categories of underlyings on which derivatives are created. There are two types of equities on which derivatives exist: individual stocks and stock indices. Derivatives on individual stocks are primarily options. Forwards, futures, and swaps on individual stocks are not widely used. Index derivatives in the form of options, forwards, futures, and swaps are very popular. Index swaps, more often called equity swaps, are quite popular and permit investors to pay the return on one stock index and receive the return on another index or a fixed rate. They can be very useful in asset allocation strategies by allowing an equity manager to increase or reduce exposure to an equity market or sector without trading the individual securities.

In addition, options on stocks are frequently used by companies as compensation and incentives for their executives and employees. These options are granted to provide incentives to work toward driving the stock price up and can result in companies paying lower cash

compensation.<sup>15</sup> Some companies also issue warrants, which are options sold to the public that allow the holders to exercise them and buy shares directly from the companies.<sup>16</sup>

#### 4.4.2. Fixed-Income Instruments and Interest Rates

Options, forwards, futures, and swaps on bonds are widely used. The problem with creating derivatives on bonds, however, is that there are almost always many issues of bonds. A single issuer, whether it is a government or a private borrower, often has more than one bond issue outstanding. For futures contracts, with their standardization requirements, this problem is particularly challenging. What does it mean to say that a futures contract is on a German bund, a US Treasury note, or a UK gilt? The most common solution to this problem is to allow multiple issues to be delivered on a single futures contract. This feature adds some interesting twists to the pricing and trading strategies of these instruments.

Until now, we have referred to the underlying as an *asset*. Yet, one of the largest derivative underlyings is not an asset. It is simply an interest rate. An interest rate is not an asset. One cannot hold an interest rate or place it on a balance sheet as an asset. Although one can hold an instrument that pays an interest rate, the rate itself is not an asset. But there are derivatives in which the rate, not the instrument that pays the rate, is the underlying. In fact, we have already covered one of these derivatives: The plain vanilla interest rate swap in which Libor is the underlying.<sup>17</sup> Instead of a swap, an interest rate derivative could be an option. For example, a call option on 90-day Libor with a strike of 5% would pay off if at expiration Libor exceeds 5%. If Libor is below 5%, the option simply expires unexercised.

Interest rate derivatives are the most widely used derivatives. With that in mind, we will be careful in using the expression *underlying asset* and will use the more generic *underlying*.

#### 4.4.3. Currencies

Currency risk is a major factor in global financial markets, and the currency derivatives market is extremely large. Options, forwards, futures, and swaps are widely used. Currency derivatives can be complex, sometimes combining elements of other underlyings. For example, a currency swap involves two parties making a series of interest rate payments to each other in different currencies. Because interest rates and currencies are both subject to change, a currency swap has two sources of risk. Although this instrument may sound extremely complicated, it merely reflects the fact that companies operating across borders are subject to both interest rate risk and currency risk and currency swaps are commonly used to manage those risks.

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<sup>15</sup>Unfortunately, the industry has created some confusion with the terminology of these instruments. They are often referred to as *stock options*, and yet ordinary publicly traded options not granted to employees are sometimes referred to as stock options. The latter are also sometimes called *equity options*, whereas employee-granted options are almost never referred to as equity options. If the terms *executive stock options* and *employee stock options* were always used, there would be no problem. You should be aware of and careful about this confusion.

<sup>16</sup>A warrant is a type of option, similar to the employee stock option, written by the company on its own stock, in contrast to exchange-traded and OTC options, in which the company is not a party to the option contract. Also note that, unfortunately, the financial world uses the term *warrant* to refer to a number of other option-like instruments. Like a lot of words that have multiple meanings, one must understand the context to avoid confusion.

<sup>17</sup>As you will see later, there are also futures in which the underlying is an interest rate (Eurodollar futures) and forwards in which the underlying is an interest rate (forward rate agreements, or FRAs).

#### 4.4.4. Commodities

Commodities are resources, such as food, oil, and metals, that humans use to sustain life and support economic activity. Because of the economic principle of comparative advantage, countries often specialize in the production of certain resources. Thus, the commodities market is extremely large and subject to an almost unimaginable array of risks. One need only observe how the price of oil moves up as tension builds in the Middle East or how the price of orange juice rises on a forecast of cold weather in Florida.

Commodity derivatives are widely used to speculate in and manage the risk associated with commodity price movements. The primary commodity derivatives are futures, but forwards, swaps, and options are also used. The reason that futures are in the lead in the world of commodities is simply history. The first futures markets were futures on commodities. The first futures exchange, the Chicago Board of Trade, was created in 1848, and until the creation of currency futures in 1972, there were no futures on any underlying except commodities.

There has been a tendency to think of the commodities world as somewhat separate from the financial world. Commodity traders and financial traders were quite different groups. Since the creation of financial futures, however, commodity and financial traders have become relatively homogeneous. Moreover, commodities are increasingly viewed as an important asset class that should be included in investment strategies because of their ability to help diversify portfolios.

#### 4.4.5. Credit

As we previously discussed, credit is another underlying and quite obviously not an asset. Credit default swaps (CDSs) and collateralized debt obligations (CDOs) were discussed extensively in an earlier section. These instruments have clearly established that credit is a distinct underlying that has widespread interest from a trading and risk management perspective. In addition, to the credit of a single entity, credit derivatives are created on multiple entities. CDOs themselves are credit derivatives on portfolios of credit risks. In recent years, indices of CDOs have been created, and instruments based on the payoffs of these CDO indices are widely traded.

#### 4.4.6. Other

This category is included here to capture some of the really unusual underlyings. One in particular is weather. Although weather is hardly an asset, it is certainly a major force in how some entities perform. For example, a ski resort needs snow, farmers need an adequate but not excessive amount of rain, and public utilities experience strains on their capacity during temperature extremes. Derivatives exist in which the payoffs are measured as snowfall, rainfall, and temperature. Although these derivatives have not been widely used—because of some complexities in pricing, among other things—they continue to exist and may still have a future. In addition, there are derivatives on electricity, which is also not an asset. It cannot be held in the traditional sense because it is created and consumed almost instantaneously. Another unusual type of derivative is based on disasters in the form of insurance claims.

Financial institutions will continue to create derivatives on all types of risks and exposures. Most of these derivatives will fail because of little trading interest, but a few will succeed. If that speaks badly of derivatives, it must be remembered that most small businesses fail, most creative ideas fail, and most people who try to become professional entertainers or athletes fail. It is the sign of a healthy and competitive system that only the very best survive.

### The Size of the Derivatives Market

In case anyone thinks that the derivatives market is not large enough to justify studying, we should consider how big the market is. Unfortunately, gauging the size of the derivatives market is not a simple task. OTC derivatives contracts are private transactions. No reporting agency gathers data, and market size is not measured in traditional volume-based metrics, such as shares traded in the stock market. Complicating things further is the fact that derivatives underlyings include equities, fixed-income securities, interest rates, currencies, commodities, and a variety of other underlyings. All these underlyings have their own units of measurement. Hence, measuring how “big” the underlying derivatives markets are is like trying to measure how much fruit consumers purchase; the proverbial mixing of apples, oranges, bananas, and all other fruits.

The exchange-listed derivatives market reports its size in terms of volume, meaning the number of contracts traded. Exchange-listed volume, however, is an inconsistent number. For example, US Treasury bond futures contracts trade in units covering \$100,000 face value. Eurodollar futures contracts trade in units covering \$1,000,000 face value. Crude oil trades in 1,000-barrel (42 gallons each) units. Yet, one traded contract of each gets equal weighting in volume totals.

The March–April issue of the magazine *Futures Industry* (available to subscribers) reports the annual volume of the entire global futures and options industry. For 2011, that volume was more than 25 billion contracts.

OTC volume is even more difficult to measure. There is no count of the number of contracts that trade. In fact, *volume* is an almost meaningless concept in OTC markets because any notion of volume requires a standardized size. If a customer goes to a swaps dealer and enters into a swap to hedge a \$50 million loan, there is no measure of how much volume that transaction generated. The \$50 million swap’s notional principal, however, does provide a measure to some extent. Forwards, swaps, and OTC options all have notional principals, so they can be measured in that manner. Another measure of the size of the derivatives market is the market value of these contracts. As noted, forwards and swaps start with zero market value, but their market value changes as market conditions change. Options do not start with zero market value and almost always have a positive market value until expiration, when some options expire out of the money.

The OTC industry has taken both of these concepts—notional principal and market value—as measures of the size of the market. Notional principal is probably a more accurate measure. The amount of a contract’s notional principal is unambiguous: It is written into the contract and the two parties cannot disagree over it. Yet, notional principal terribly overstates the amount of money actually at risk. For example, a \$50 million notional principal swap will have nowhere near \$50 million at risk. The payments on such a swap are merely the net of two opposite series of interest payments on \$50 million. The market value of such a swap is the present value of one stream of payments minus the present value of the other. This market value figure will always be well below the notional principal. Thus, market value seems like a better measure except that, unlike notional principal, it is not unambiguous. Market value requires measurement, and two parties can disagree on the market value of the same transaction.

Notional principal and market value estimates for the global OTC derivatives market are collected semi-annually by the Bank for International Settlements of Basel, Switzerland, and published on its website (<http://www.bis.org/statistics/derstats.htm>). At

the end of 2011, notional principal was more than \$600 trillion and market value was about \$27 trillion. A figure of \$600 trillion is an almost unfathomable number and, as noted, is a misleading measure of the amount of money at risk.<sup>18</sup> The market value figure of \$27 trillion is a much more realistic measure, but as noted, it is less accurate, relying on estimates provided by banks.

Hence, the exchange-listed and OTC markets use different measures and each of those measures is subject to severe limitations. About all we can truly say for sure about the derivatives market is, "It is big."

## 5. THE PURPOSES AND BENEFITS OF DERIVATIVES

Economic historians know that derivatives markets have existed since at least the Middle Ages. It is unclear whether derivatives originated in the Asian rice markets or possibly in medieval trade fairs in Europe. We do know that the origin of modern futures markets is the creation of the Chicago Board of Trade in 1848. To understand why derivatives markets exist, it is useful to take a brief look at why the Chicago Board of Trade was formed.

In the middle of the 19th century, midwestern America was rapidly becoming the center of agricultural production in the United States. At the same time, Chicago was evolving into a major American city, a hub of transportation and commerce. Grain markets in Chicago were the central location to which midwestern farmers brought their wheat, corn, and soybeans to sell. Unfortunately, most of these products arrived at approximately the same time of the year, September through November. The storage facilities in Chicago were strained beyond capacity. As a result, prices would fall tremendously and some farmers reportedly found it more economical to dump their grains in the Chicago River rather than transport them back to the farm. At other times of the year, prices would rise steeply. A group of businessmen saw this situation as unnecessary volatility and a waste of valuable produce. To deal with this problem, they created the Chicago Board of Trade and a financial instrument called the "to-arrive" contract. A farmer could sell a to-arrive contract at any time during the year. This contract fixed the price of the farmer's grain on the basis of delivery in Chicago at a specified later date. Grain is highly storable, so farmers can hold on to the grain and deliver it at almost any later time. This plan substantially reduced seasonal market volatility and made the markets work much better for all parties.

The traders in Chicago began to trade these contracts, speculating on movements in grain prices. Soon, it became apparent that an important and fascinating market had developed. Widespread hedging and speculative interest resulted in substantial market growth, and about 80 years later, a clearinghouse and a performance guarantee were added, thus completing the evolution of the to-arrive contract into today's modern futures contract.

Many commodities and all financial assets that underlie derivatives contracts are not seasonally produced. Hence, this initial motivation for futures markets is only a minor advantage of derivatives markets today. But there are many reasons why derivative markets serve an important and useful purpose in contemporary finance.

<sup>18</sup>To put it in perspective, it would take 19 million years for a clock to tick off 600 trillion seconds!

### 5.1. Risk Allocation, Transfer, and Management

Until the advent of derivatives markets, risk management was quite cumbersome. Setting the actual level of risk to the desired level of risk required engaging in transactions in the underlyings. Such transactions typically had high transaction costs and were disruptive of portfolios. In many cases, it is quite difficult to fine-tune the level of risk to the desired level. From the perspective of a risk taker, it was quite costly to buy risk because a large amount of capital would be required.

Derivatives solve these problems in a very effective way: They allow trading the risk without trading the instrument itself. For example, consider a stockholder who wants to reduce exposure to a stock. In the pre-derivatives era, the only way to do so was to sell the stock. Now, the stockholder can sell futures, forwards, calls, or swaps, or buy put options, all while retaining the stock. For a company founder, these types of strategies can be particularly useful because the founder can retain ownership and probably board membership. Many other excellent examples of the use of derivatives to transfer risk are covered elsewhere in the curriculum. The objective at this point is to establish that derivatives provide an effective method of transferring risk from parties who do not want the risk to parties who do. In this sense, risk allocation is improved within markets and, indeed, the entire global economy.

The overall purpose of derivatives is to obtain more effective risk management within companies and the entire economy. Although some argue that derivatives do not serve this purpose very well (we will discuss this point in Section 6), for now you should understand that derivatives can improve the allocation of risk and facilitate more effective risk management for both companies and economies.

### 5.2. Information Discovery

One of the advantages of futures markets has been described as *price discovery*. A futures price has been characterized by some experts as a revelation of some information about the future. Thus, a futures price is sometimes thought of as predictive. This statement is not strictly correct because futures prices are not really forecasts of future spot prices. They provide only a little more information than do spot prices, but they do so in a very efficient manner. The markets for some underlyings are highly decentralized and not very efficient. For example, what is gold worth? It trades in markets around the world, but probably the best place to look is at the gold futures contract expiring soonest. What is the value of the S&P 500 Index when the US markets are not open? As it turns out, US futures markets open before the US stock market opens. The S&P 500 futures price is frequently viewed as an indication of where the stock market will open.

Derivative markets can, however, convey information not impounded in spot markets. By virtue of the fact that derivative markets require less capital, information can flow into the derivative markets before it gets into the spot market. The difference may well be only a matter of minutes or possibly seconds, but it can provide the edge to astute traders.

Finally, we should note that futures markets convey another simple piece of information: What price would one accept to avoid uncertainty? If you hold a stock worth \$40 and could hedge the next 12 months' uncertainty, what locked-in price should you expect to earn? As it turns out, it should be the price that guarantees the risk-free rate minus whatever dividends would be paid on the stock. Derivatives—specifically, futures, forwards, and swaps—reveal the price that the holder of an asset could take and avoid the risk.

What we have said until now applies to futures, forwards, and swaps. What about options? As you will learn later, given the underlying and the type of option (call or put), an option price reflects two characteristics of the option (exercise price and time to expiration), three characteristics of the underlying (price, volatility, and cash flows it might pay), and one general macroeconomic factor (risk-free rate). Only one of these factors, volatility, is not relatively easy to identify. But with the available models to price the option, we can infer what volatility people are using from the actual market prices at which they execute trades. That volatility, called **implied volatility**, measures the expected risk of the underlying. It reflects the volatility that investors use to determine the market price of the option. Knowing the expected risk of the underlying asset is an extremely useful piece of information. In fact, for options on broad-based market indices, such as the S&P 500, the implied volatility is a good measure of the general level of uncertainty in the market. Some experts have even called it a measure of fear. Thus, options provide information about what investors think of the uncertainty in the market, if not their fear of it.<sup>19</sup>

In addition, options allow the creation of trading strategies that cannot be done by using the underlying. As the exhibits on options explained, these strategies provide asymmetrical performance: limited movement in one direction and movement in the other direction that changes with movements in the underlying.

### 5.3. Operational Advantages

We noted earlier that derivatives have lower transaction costs than the underlying. The transaction costs of derivatives can be high relative to the value of the derivatives, but these costs are typically low relative to the value of the underlying. Thus, an investor who wants to take a position in, say, an equity market index would likely find it less costly to use the futures to get a given degree of exposure than to invest directly in the index to get that same exposure.

Derivative markets also typically have greater liquidity than the underlying spot markets, a result of the smaller amount of capital required to trade derivatives than to get the equivalent exposure directly in the underlying. Futures margin requirements and option premiums are quite low relative to the cost of the underlying.

One other extremely valuable operational advantage of derivative markets is the ease with which one can go short. With derivatives, it is nearly as easy to take a short position as to take a long position, whereas for the underlying asset, it is almost always much more difficult to go short than to go long. In fact, for many commodities, short selling is nearly impossible.

### 5.4. Market Efficiency

In the study of portfolio management, you learn that an efficient market is one in which no single investor can consistently earn returns in the long run in excess of those commensurate with the risk assumed. Of course, endless debates occur over whether equity markets are efficient. No need to resurrect that issue here, but let us proceed with the assumption that equity markets—and, in fact, most free and competitive financial markets—are reasonably efficient. This assumption does not mean that abnormal returns can never be earned, and indeed prices do get out of line with fundamental values. But competition, the relatively free flow of

<sup>19</sup>The Chicago Board Options Exchange publishes a measure of the implied volatility of the S&P 500 Index option, which is called the VIX (volatility index). The VIX is widely followed and is cited as a measure of investor uncertainty and sometimes fear.

information, and ease of trading tend to bring prices back in line with fundamental values. Derivatives can make this process work even more rapidly.

When prices deviate from fundamental values, derivative markets offer less costly ways to exploit the mispricing. As noted earlier, less capital is required, transaction costs are lower, and short selling is easier. We also noted that as a result of these features, it is possible, indeed likely, that fundamental value will be reflected in the derivatives markets before it is restored in the underlying market. Although this time difference could be only a matter of minutes, for a trader seeking abnormal returns, a few minutes can be a valuable opportunity.

All these advantages of derivatives markets make the financial markets in general function more effectively. Investors are far more willing to trade if they can more easily manage their risk, trade at lower cost and with less capital, and go short more easily. This increased willingness to trade increases the number of market participants, which makes the market more liquid. A very liquid market may not automatically be an efficient market, but it certainly has a better chance of being one.

Even if one does not accept the concept that financial markets are efficient, it is difficult to say that markets are not more effective and competitive with derivatives. Yet, many blame derivatives for problems in the market. Let us take a look at these arguments.

## 6. CRITICISMS AND MISUSES OF DERIVATIVES

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The history of financial markets is filled with extreme ups and downs, which are often called bubbles and crashes. Bubbles occur when prices rise for a long time and appear to exceed fundamental values. Crashes occur when prices fall rapidly. Although bubbles, if they truly exist, are troublesome, crashes are even more so because nearly everyone loses substantial wealth in a crash. A crash is then typically followed by a government study commissioned to find the causes of the crash. In the last 30 years, almost all such studies have implicated derivatives as having some role in causing the crash. Of course, because derivatives are widely used and involve a high degree of leverage, it is a given that they would be seen in a crash. It is unclear whether derivatives are the real culprit or just the proverbial smoking gun used by someone to do something wrong.

The two principal arguments against derivatives are that they are such speculative devices that they effectively permit legalized gambling and that they destabilize the financial system. Let us look at these points more closely.

### 6.1. Speculation and Gambling

As noted earlier, derivatives are frequently used to manage risk. In many contexts, this use involves hedging or laying off risk. Naturally, for hedging to work, there must be speculators. Someone must accept the risk. Derivatives markets are unquestionably attractive to speculators. All the benefits of derivatives draw speculators in large numbers, and indeed they should. The more speculators that participate in the market, the cheaper it is for hedgers to lay off risk. These speculators take the form of hedge funds and other professional traders who willingly accept risk that others need to shed. In recent years, the rapid growth of these types of investors has been alarming to some but almost surely has been beneficial for all investors.

Unfortunately, the general image of speculators is not a good one. Speculators are often thought to be short-term traders who attempt to exploit temporary inefficiencies, caring little about long-term fundamental values. The profits from short-term trading are almost always

taxed more heavily than the profits from long-term trading, clearly targeting and in some sense punishing speculators. Speculators are thought to engage in price manipulation and to trade at extreme prices.<sup>20</sup> All of this type of trading is viewed more or less as just a form of legalized gambling.

In most countries, gambling is a heavily regulated industry. In the United States, only certain states permit private industry to offer gambling. Many states operate gambling only through the public sector in the form of state-run lotteries. Many people view derivatives trading as merely a form of legalized and uncontrolled gambling.

Yet, there are notable differences between gambling and speculation. Gambling typically benefits only a limited number of participants and does not generally help society as a whole. But derivatives trading brings extensive benefits to financial markets, as explained earlier, and thus does benefit society as a whole. In short, the benefits of derivatives are broad, whereas the benefits of gambling are narrow.

Nonetheless, the argument that derivatives are a form of legalized gambling will continue to be made. Speculation and gambling are certainly both forms of financial risk taking, so these arguments are not completely off base. But insurance companies speculate on loss claims, mutual funds that invest in stocks speculate on the performance of companies, and entrepreneurs go up against tremendous odds to speculate on their own ability to create successful businesses. These so-called speculators are rarely criticized for engaging in a form of legalized gambling, and indeed entrepreneurs are praised as the backbone of the economy. Really, all investment is speculative. So, why is speculation viewed as such a bad thing by so many? The answer is unclear.

## 6.2. Destabilization and Systemic Risk

The arguments against speculation through derivatives often go a step further, claiming that it is not merely speculation or gambling *per se* but rather that it has destabilizing consequences. Opponents of derivatives claim that the very benefits of derivatives (low cost, low capital requirements, ease of going short) result in an excessive amount of speculative trading that brings instability to the market. They argue that speculators use large amounts of leverage, thereby subjecting themselves and their creditors to substantial risk if markets do not move in their hoped-for direction. Defaults by speculators can then lead to defaults by their creditors, their creditors' creditors, and so on. These effects can, therefore, be systemic and reflect an epidemic contagion whereby instability can spread throughout markets and an economy, if not the entire world. Given that governments often end up bailing out some banks and insurance companies, society has expressed concern that the risk managed with derivatives must be controlled.

This argument is not without merit. Such effects occurred in the Long-Term Capital Management fiasco of 1998 and again in the financial crisis of 2008, in which derivatives, particularly credit default swaps, were widely used by many of the problem entities. Responses to such events typically take the course of calling for more rules and regulations restricting the use of derivatives, requiring more collateral and credit mitigation measures, backing up banks with more capital, and encouraging, if not requiring, OTC derivatives to be centrally cleared like exchange-traded derivatives.

<sup>20</sup> Politicians and regulators have been especially critical of energy market speculators. Politicians, in particular, almost always blame rising oil prices on speculators, although credit is conspicuously absent for falling oil prices.

In response, however, we should note that financial crises—including the South Sea and Mississippi bubbles and the stock market crash of 1929, as well as a handful of economic calamities of the 19th and 20th centuries—have existed since the dawn of capitalism. Some of these events preceded the era of modern derivatives markets, and others were completely unrelated to the use of derivatives. Some organizations, such as Orange County, California, in 1994–1995, have proved that derivatives are not required to take on excessive leverage and nearly bring the entity to ruin. Proponents of derivatives argue that derivatives are but one of many mechanisms through which excessive risk can be taken. Derivatives may seem dangerous, and they can be if misused, but there are many ways to take on leverage that look far less harmful but can be just as risky.

Another criticism of derivatives is simply their complexity. Many derivatives are extremely complex and require a high-level understanding of mathematics. The financial industry employs many mathematicians, physicists, and computer scientists. This single fact has made many distrust derivatives and the people who work on them. It is unclear why this reason has tarnished the reputation of the derivatives industry. Scientists work on complex problems in medicine and engineering without public distrust. One explanation probably lies in the fact that scientists create models of markets by using scientific principles that often fail. To a physicist modeling the movements of celestial bodies, the science is reliable and the physicist is unlikely to misapply the science. The same science applied to financial markets is far less reliable. Financial markets are driven by the actions of people who are not as consistent as the movements of celestial bodies. When financial models fail to work as they should, the scientists are often blamed for either building models that are too complex and unable to accurately capture financial reality or misusing those models, such as using poor estimates of inputs. And derivatives, being so widely used and heavily leveraged, are frequently in the center of it all.

#### EXAMPLE 6 Purposes and Controversies of Derivative Markets

1. Which of the following is **not** an advantage of derivative markets?
  - A. They are less volatile than spot markets.
  - B. They facilitate the allocation of risk in the market.
  - C. They incur lower transaction costs than spot markets.
2. Which of the following pieces of information is **not** conveyed by at least one type of derivative?
  - A. The volatility of the underlying.
  - B. The most widely used strategy of the underlying.
  - C. The price at which uncertainty in the underlying can be eliminated.
3. Which of the following responds to the criticism that derivatives can be destabilizing to the underlying market?
  - A. Market crashes and panics have occurred since long before derivatives existed.
  - B. Derivatives are sufficiently regulated that they cannot destabilize the spot market.
  - C. The transaction costs of derivatives are high enough to keep their use at a minimum level.

*Solution to 1:* A is correct. Derivative markets are not by nature more or less volatile than spot markets. They facilitate risk allocation by making it easier and less costly to transfer risk, and their transaction costs are lower than those of spot markets.

*Solution to 2:* B is correct. Options do convey the volatility of the underlying, and futures, forwards, and swaps convey the price at which uncertainty in the underlying can be eliminated. Derivatives do not convey any information about the use of the underlying in strategies.

*Solution to 3:* A is correct. Derivatives regulation is not more and is arguably less than spot market regulation, and the transaction costs of derivatives are not a deterrent to their use; in fact, derivatives are widely used. Market crashes and panics have a very long history, much longer than that of derivatives.

An important element of understanding and using derivatives is having a healthy respect for their power. Every day, we use chemicals, electricity, and fire without thinking about their dangers. We consume water and drive automobiles, both of which are statistically quite dangerous. Perhaps these risks are underappreciated, but it is more likely the case that most adults learn how to safely use chemicals, electricity, fire, water, and automobiles. Of course, there are exceptions, many of which are foolish, and foolishness is no stranger to the derivatives industry. The lesson here is that derivatives can make our financial lives better, but like chemicals, electricity, and all the rest, we need to know how to use them safely, which is why they are an important part of the CFA curriculum.

Later in the curriculum, you will learn a great deal about how derivatives are priced. At this point, we introduce the pricing of derivatives. This material not only paves the way for a deeper understanding of derivatives but also complements earlier material by helping you understand how derivatives work.

## 7. ELEMENTARY PRINCIPLES OF DERIVATIVE PRICING

Pricing and valuation are fundamental elements of the CFA Program. The study of fixed-income and equity securities, as well as their application in portfolio management, is solidly grounded on the principle of valuation. In valuation, the question is simple: What is something worth? Without an answer to that question, one can hardly proceed to use that *something* wisely.

Determining what a derivative is worth is similar to determining what an asset is worth. As you learn in the fixed-income and equity readings, value is the present value of future cash flows, with discounting done at a rate that reflects both the opportunity cost of money and the risk. Derivatives valuation applies that same principle but in a somewhat different way.

Think of a derivative as *attached* to an underlying. We know that the derivative *derives* its value from the value of the underlying. If the underlying's value changes, so should the value of the derivative. The underlying takes its value from the discounted present value of the expected future cash flows it offers, with discounting done at a rate reflecting the investor's risk tolerance. But if the value of the underlying is embedded in the value of the derivative, it would be double counting to discount the derivative's expected future cash flows at a risky discount

rate. That effect has already been incorporated into the value of the underlying, which goes into the value of the derivative.

Derivatives usually take their values from the underlying by constructing a hypothetical combination of the derivatives and the underlyings that eliminates risk. This combination is typically called a **hedge portfolio**. With the risk eliminated, it follows that the hedge portfolio should earn the risk-free rate. A derivative's value is the price of the derivative that forces the hedge portfolio to earn the risk-free rate.

This principle of derivative valuation relies completely on the ability of an investor to hold or store the underlying asset. Let us take a look at what that means.

### 7.1. Storage

As noted previously, the first derivatives were agricultural commodities. Most of these commodities can be stored (i.e., held) for a period of time. Some extreme cases, such as oil and gold, which are storable for millions of years, are excellent examples of fully storable commodities. Grains, such as wheat and corn, can be stored for long but not infinite periods of time. Some commodities, such as bananas, are storable for relatively short periods of time. In the CFA Program, we are more interested in financial assets. Equities and currencies have perpetual storability, whereas bonds are storable until they mature.

Storage incurs costs. Commodity storage costs can be quite expensive. Imagine storing 1,000 kilograms of gold or a million barrels of oil. Financial assets, however, have relatively low storage costs. Some assets pay returns during storage. Stocks pay dividends and bonds pay interest. The net of payments offered minus storage costs plays a role in the valuation of derivatives.

An example earlier in this reading illustrates this point. Suppose an investor holds a dividend-paying stock and wants to eliminate the uncertainty of its selling price over a future period of time. Suppose further that the investor enters into a forward contract that commits him to deliver the stock at a later date, for which he will receive a fixed price. With uncertainty eliminated, the investor should earn the risk-free rate, but in fact, he does not. He earns more because while holding the stock, he collects dividends. Therefore, he should earn the risk-free rate *minus* the dividend yield, a concept known as the cost of carry, which will be covered in great detail in later readings. The cost of carry *plus* the dividends he earns effectively means that he makes the risk-free rate. Now, no one is claiming that this is a good way to earn the risk-free rate. There are many better ways to do that, but this strategy could be executed. There is one and only one forward price that guarantees that this strategy earns a return of the risk-free rate minus the dividend yield, or the risk-free rate after accounting for the dividends collected. If the forward price at which contracts are created does not equal this price, investors can take advantage of this discrepancy by engaging in arbitrage, which is discussed in the next section.

Forwards, futures, swaps, and options are all priced in this manner. Hence, they rely critically on the ability to store or hold the asset. Some underlyings are not storable. We previously mentioned electricity. It is produced and consumed almost instantaneously. Weather is also not storable. Fresh fish have very limited storability. Although this absence of storability may not be the reason, derivative markets in these types of underlyings have not been particularly successful, whereas those in underlyings that are more easily storable have often been successful.

The opposite of storability is the ability to go short—that is, to borrow the underlying, sell it, and buy it back later. We discussed earlier that short selling of some assets can be difficult. It is not easy to borrow oil or soybeans. There are ways around this constraint, but derivatives

valuation is generally much easier when the underlying can be shorted. This point is discussed in more depth later in the curriculum.

## 7.2. Arbitrage

What we have been describing is the foundation of the principle of **arbitrage**. In well-functioning markets with low transaction costs and a free flow of information, the same asset cannot sell for more than one price. If it did, someone would buy it in the cheaper market and sell it in the more expensive market, earning a riskless profit. The combined actions of all parties doing this would push up the lower price and push down the higher price until they converged. For this reason, arbitrage is often referred to as the **law of one price**. Of course, for arbitrage to be feasible, the ability to purchase and sell short the asset is important.

Obviously, this rule does not apply to all markets. The same consumer good can easily sell for different prices, which is one reason why people spend so much time shopping on the internet. The costs associated with purchasing the good in the cheaper market and selling it in the more expensive market can make the arbitrage not worthwhile. The absence of information on the very fact that different prices exist would also prevent the arbitrage from occurring. Although the internet and various price-comparing websites reduce these frictions and encourage all sellers to offer competitive prices, consumer goods are never likely to be arbitragable.<sup>21</sup>

Financial markets, of course, are a different matter. Information on securities prices around the world is quite accessible and relatively inexpensive. Most financial markets are fairly competitive because dealers, speculators, and brokers attempt to execute trades at the best prices. Arbitrage is considered a dependable rule in the financial markets. Nonetheless, there are people who purport to make a living as arbitrageurs. How could they exist? To figure that out, first consider some examples of arbitrage.

The simplest case of an arbitrage might be for the same stock to sell at different prices in two markets. If the stock were selling at \$52 in one market and \$50 in another, an arbitrageur would buy the stock at \$50 in the one market and sell it at \$52 in the other. This trade would net an immediate \$2 profit at no risk and would not require the commitment of any of the investor's capital. This outcome would be a strong motivation for all arbitrageurs, and their combined actions would force the lower price up and the higher price down until the prices converged.

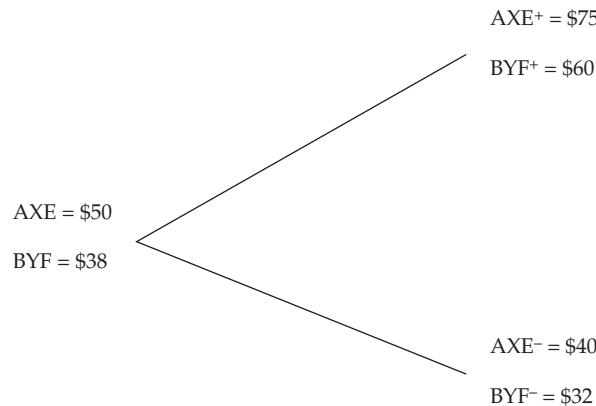
But what would be the final price? It is entirely possible that \$50 is the true fundamental value and \$52 is too high. Or \$52 could be the true fundamental value and \$50 is too low. Or the true fundamental value could lie somewhere between the two. Arbitrage does not tell us the true fundamental value. It is not an *absolute* valuation methodology, such as the discounted cash flow equity valuation model. It is a *relative* valuation methodology. It tells us the correct price of one asset or derivative *relative to* another asset or derivative.

Now, consider another situation, illustrated in Exhibit 6. Observe that we have one stock, AXE Electronics, that today is worth \$50 and one period later will be worth either \$75 or \$40. We will denote these prices as  $AXE = \$50$ ,  $AXE^+ = \$75$ , and  $AXE^- = \$40$ . Another stock, BYF Technology, is today worth \$38 and one period later will be worth \$60 or \$32. Thus,  $BYF = \$38$ ,

<sup>21</sup>If the same consumer good sells for different prices in markets with a relatively free flow of information (e.g., via price-comparing websites), it still may not be possible to truly arbitrage. Buying the good at a lower price and selling it at a higher price but less than the price of the most expensive seller may not be practical, but the most expensive seller may be driven out of business. When everyone knows what everyone else is charging, the same effect of arbitrage can still occur.

$BYF^+ = \$60$ , and  $BYF^- = \$32$ . Assume that the risk-free borrowing and lending rate is 4%. Also assume no dividends are paid on either stock during the period covered by this example.

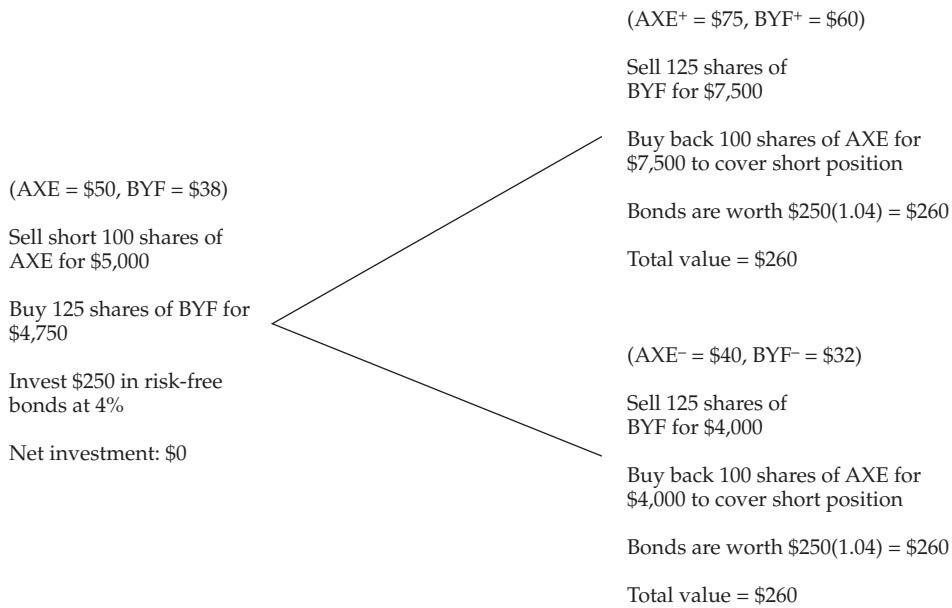
**EXHIBIT 6** Arbitrage Opportunity with Stock AXE, Stock BYF, and a Risk-Free Bond



Note: The risk-free rate is 4%.

The opportunity exists to make a profit at no risk without committing any of our funds, as demonstrated in Exhibit 7. Suppose we borrow 100 shares of stock AXE, which is selling for \$50, and sell them short, thereby receiving \$5,000. We take \$4,750 and purchase 125 shares of stock BYF ( $125 \times \$38 = \$4,750$ ). We invest the remaining \$250 in risk-free bonds at 4%. This transaction will not require us to use any funds of our own: The short sale will be sufficient to fund the investment in BYF and leave money to invest in risk-free bonds.

**EXHIBIT 7** Execution of Arbitrage Transaction with Stock AXE, Stock BYF, and a Risk-Free Bond



If the top outcome in Exhibit 7 occurs, we sell the 125 shares of BYF for  $125 \times \$60 = \$7,500$ . This amount is sufficient to buy back the 100 shares of AXE, which is selling for \$75. But we will also have the bonds, which are worth  $\$250 \times 1.04 = \$260$ . If the bottom outcome occurs, we sell the 125 shares of BYF for  $125 \times \$32 = \$4,000$ —enough money to buy back the 100 shares of AXE, which is selling for \$40. Again, we will have the risk-free bonds, worth \$260. Regardless of the outcome, we end up with \$260.

Recall that we invested no money of our own and end up with a sure \$260. It should be apparent that this transaction is extremely attractive, so everyone would do it. The combined actions of multiple investors would drive down the price of AXE and/or drive up the price of BYF until an equilibrium is reached, at which point this transaction would no longer be profitable. As noted earlier, we cannot be sure of the correct fundamental price, but let us assume that BYF's price remains constant. Then AXE would fall to \$47.50. Alternatively, if we assume that AXE's price remains constant, then the price of BYF would rise to \$40. These values are obtained by noting that the prices for both outcomes occur according to the ratio 1.25 ( $\$75/\$60 = 1.25$ ;  $\$40/\$32 = 1.25$ ). Thus, their initial prices should be consistent with that ratio. If BYF is \$38, AXE should be  $\$38 \times 1.25 = \$47.50$ . If AXE is \$50, BYF should be \$40.00 because  $\$40.00 \times 1.25 = \$50$ . Of course, the two prices could settle in between. Arbitrage is only a relative pricing method. It prices the two stocks in relation to each other but does not price either on the basis of its own fundamentals.

Of course, this example is extremely simplified. Clearly, a stock price can change to more than two other prices. Also, if a given stock is at one price, another stock may be at any other price. We have created a simple case here to illustrate a point. But as you will learn later in the curriculum, when derivatives are involved, the simplification here is relatively safe. As we know, the price of a derivative is determined by the price of the underlying. Hence, when the underlying is at one particular price, the derivative's price will be determined by that price. The two assets need not be two stocks; one can be a stock and the other can be a derivative on the stock.

To see that point, consider another type of arbitrage opportunity that involves a forward contract. Recall from the previous example that at the start, AXE sells for \$50. Suppose we borrow \$50 at 4% interest by issuing a risk-free bond, use the money to buy one share of stock AXE, and simultaneously enter into a forward contract to sell this share at a price of \$54 one period later. The stock will then move to either \$75 or \$40 in the next period. The forward contract requires that we deliver the stock and accept \$54 for it. And of course, we will owe  $\$50 \times 1.04 = \$52$  on the loan.

Now consider the two outcomes. Regardless of the outcome, the end result is the same. The forward contract fixes the delivery price of the stock at \$54:

**AXE goes to \$75**

Deliver stock to settle forward contract	+ \$54
Pay back loan	- \$52
Net	<u>+ \$2</u>

**AXE goes to \$40**

Deliver stock to settle forward contract	+ \$54
Pay back loan	- \$52
Net	<u>+ \$2</u>

In either case, we made \$2, free and clear. In fact, we can even accommodate the possibility of more than two future prices for AXE and we will always make \$2.<sup>22</sup> The key point is that we faced no risk and did not have to invest any of our own money, but ended up with \$2, which is clearly a good trade. The \$2 is an arbitrage profit. But where did it originate?

It turns out that the forward price, \$54, was an inappropriate price given current market conditions. In fact, it was just an arbitrary price made up to illustrate the point. To eliminate the opportunity to earn the \$2 profit, the forward price should be \$52, which is equal, not coincidentally, to the amount owed on the loan. It is also no coincidence that \$52 is the price of the asset increased by the rate of interest. We will cover this point later in the curriculum, but for now consider that you have just seen your first derivative pricing model.<sup>23</sup>

Of course, many market participants would do this transaction as long as it generated an arbitrage profit. These forces of arbitrage would either push the forward price down or the stock price up, or both, until an equilibrium is reached that eliminates the opportunity to profit at no risk with no commitment of one's own funds.

To summarize, the forces of arbitrage in financial markets assure us that the same asset cannot sell for different prices, nor can two equivalent combinations of assets that produce the same results sell for different prices. Realistically, some arbitrage opportunities can exist on a temporary basis, but they will be quickly exploited, bringing relative prices back in line with each other. Other apparent arbitrage opportunities will be too small to warrant exploiting.

Not to be naive, however, we must acknowledge that there is a large industry of people who call themselves arbitrageurs. So, how can such an industry exist if there are no opportunities for riskless profit? One explanation is that most of the arbitrage transactions are more complex than the simple examples used here. Many involve estimating information, which can result in differing opinions. Arbitrage involving options, for example, usually requires an estimate of a stock's volatility. Different participants have different opinions about the volatility. It is quite possible that the two counterparties trading with each other believe that each is arbitraging against the other.<sup>24</sup>

But more importantly, the absence of arbitrage opportunities is upheld, ironically, only if participants believe that arbitrage opportunities do exist. If traders believe that no opportunities exist to earn arbitrage profits, then traders will not follow market prices and compare those prices with what they ought to be. Thus, eliminating arbitrage opportunities requires that participants be alert in watching for arbitrage opportunities. In other words, strange as it may sound, disbelief and skepticism concerning the absence of arbitrage opportunities are required for the no-arbitrage rule to be upheld.

Markets in which arbitrage opportunities are either nonexistent or quickly eliminated are relatively efficient markets. Recall that efficient markets are those in which it is not possible to consistently earn returns in excess of those that would be fair compensation for the risk assumed. Although abnormal returns can be earned in a variety of ways, arbitrage profits are

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<sup>22</sup>A good study suggestion is to try this example with any future stock price. You should get the same result, a \$2 risk-free profit.

<sup>23</sup>This illustration is the quick look at forward pricing alluded to in Section 3.1.1.

<sup>24</sup>In reality, many of the transactions that arbitrageurs do are not really arbitrage. They are quite speculative. For example, many people call themselves arbitrageurs because they buy companies that are potential takeover targets and sell the companies they think will be the buyers. This transaction is not arbitrage by any stretch of the definition. Some transactions are called "risk arbitrage," but this term is an oxymoron. As an investment professional, you should simply be prepared for such misuses of words, which simply reflect the flexibility of language.

definitely examples of abnormal returns. Thus, they are the most egregious violations of the principle of market efficiency.

Throughout the derivatives component of the CFA curriculum, we will use the principle of arbitrage as a dominant theme and assume that arbitrage opportunities cannot exist for any significant length of time nor can any one investor consistently capture them. Thus, prices must conform to models that assume no arbitrage. But we do not want to take the absence of arbitrage opportunities so seriously that we give up and believe that arbitrage opportunities never exist. Otherwise, they will arise and someone else will take them. Consider the rule of arbitrage a law that will be broken from time to time but one that holds far more often than not and one that should be understood and respected.

### EXAMPLE 7 Arbitrage

1. Which of the following is a result of arbitrage?
  - A. The law of one price
  - B. The law of similar prices
  - C. The law of limited profitability
2. When an arbitrage opportunity exists, what happens in the market?
  - A. The combined actions of all arbitrageurs force the prices to converge.
  - B. The combined actions of arbitrageurs result in a locked-limit situation.
  - C. The combined actions of all arbitrageurs result in sustained profits to all.
3. Which of the following accurately defines arbitrage?
  - A. An opportunity to make a profit at no risk
  - B. An opportunity to make a profit at no risk and with the investment of no capital
  - C. An opportunity to earn a return in excess of the return appropriate for the risk assumed
4. Which of the following ways best describes how arbitrage contributes to market efficiency?
  - A. Arbitrage penalizes those who trade too rapidly.
  - B. Arbitrage equalizes the risks taken by all market participants.
  - C. Arbitrage improves the rate at which prices converge to their relative fair values.

*Solution to 1:* A is correct. Arbitrage forces equivalent assets to have a single price. There is nothing called the law of similar prices or the law of limited profitability.

*Solution to 2:* A is correct. Prices converge because of the heavy demand for the cheaper asset and the heavy supply of the more expensive asset. Profits are not sustained, and, in fact, they are eradicated as prices converge. Locked-limit is a condition in the futures market and has nothing to do with arbitrage.

*Solution to 3:* B is correct. An opportunity to profit at no risk could merely describe the purchase of a risk-free asset. An opportunity to earn a return in excess of the return appropriate for the risk assumed is a concept studied in portfolio management and is often referred to as an abnormal return. It is certainly desirable but is hardly an arbitrage because it requires the assumption of risk and the investment of capital. Arbitrage is risk

free and requires no capital because selling the overpriced asset produces the funds to buy the underpriced asset.

*Solution to 4:* C is correct. Arbitrage imposes no penalties on rapid trading; in fact, it tends to reward those who trade rapidly to take advantage of arbitrage opportunities. Arbitrage has no effect of equalizing risk among market participants. Arbitrage does result in an acceleration of price convergence to fair values relative to instruments with equivalent payoffs.

## 8. SUMMARY

This first reading on derivatives introduces you to the basic characteristics of derivatives, including the following points:

- A derivative is a financial instrument that derives its performance from the performance of an underlying asset.
- The underlying asset, called the underlying, trades in the cash or spot markets and its price is called the cash or spot price.
- Derivatives consist of two general classes: forward commitments and contingent claims.
- Derivatives can be created as standardized instruments on derivatives exchanges or as customized instruments in the over-the-counter market.
- Exchange-traded derivatives are standardized, highly regulated, and transparent transactions that are guaranteed against default through the clearinghouse of the derivatives exchange.
- Over-the-counter derivatives are customized, flexible, and more private and less regulated than exchange-traded derivatives, but are subject to a greater risk of default.
- A forward contract is an over-the-counter derivative contract in which two parties agree that one party, the buyer, will purchase an underlying asset from the other party, the seller, at a later date and at a fixed price they agree upon when the contract is signed.
- A futures contract is similar to a forward contract but is a standardized derivative contract created and traded on a futures exchange. In the contract, two parties agree that one party, the buyer, will purchase an underlying asset from the other party, the seller, at a later date and at a price agreed on by the two parties when the contract is initiated. In addition, there is a daily settling of gains and losses and a credit guarantee by the futures exchange through its clearinghouse.
- A swap is an over-the-counter derivative contract in which two parties agree to exchange a series of cash flows whereby one party pays a variable series that will be determined by an underlying asset or rate and the other party pays either a variable series determined by a different underlying asset or rate or a fixed series.
- An option is a derivative contract in which one party, the buyer, pays a sum of money to the other party, the seller or writer, and receives the right to either buy or sell an underlying asset at a fixed price either on a specific expiration date or at any time prior to the expiration date.
- A call is an option that provides the right to buy the underlying.
- A put is an option that provides the right to sell the underlying.
- Credit derivatives are a class of derivative contracts between two parties, the credit protection buyer and the credit protection seller, in which the latter provides protection to the former against a specific credit loss.

- A credit default swap is the most widely used credit derivative. It is a derivative contract between two parties, a credit protection buyer and a credit protection seller, in which the buyer makes a series of payments to the seller and receives a promise of compensation for credit losses resulting from the default of a third party.
- An asset-backed security is a derivative contract in which a portfolio of debt instruments is assembled and claims are issued on the portfolio in the form of tranches, which have different priorities of claims on the payments made by the debt securities such that prepayments or credit losses are allocated to the most-junior tranches first and the most-senior tranches last.
- Derivatives can be combined with other derivatives or underlying assets to form hybrids.
- Derivatives are issued on equities, fixed-income securities, interest rates, currencies, commodities, credit, and a variety of such diverse underlyings as weather, electricity, and disaster claims.
- Derivatives facilitate the transfer of risk, enable the creation of strategies and payoffs not otherwise possible with spot assets, provide information about the spot market, offer lower transaction costs, reduce the amount of capital required, are easier than the underlyings to go short, and improve the efficiency of spot markets.
- Derivatives are sometimes criticized for being a form of legalized gambling and for leading to destabilizing speculation, although these points can generally be refuted.
- Derivatives are typically priced by forming a hedge involving the underlying asset and a derivative such that the combination must pay the risk-free rate and do so for only one derivative price.
- Derivatives pricing relies heavily on the principle of storage, meaning the ability to hold or store the underlying asset. Storage can incur costs but can also generate cash, such as dividends and interest.
- Arbitrage is the condition that two equivalent assets or derivatives or combinations of assets and derivatives sell for different prices, leading to an opportunity to buy at the low price and sell at the high price, thereby earning a risk-free profit without committing any capital.
- The combined actions of arbitrageurs bring about a convergence of prices. Hence, arbitrage leads to the law of one price: Transactions that produce equivalent results must sell for equivalent prices.

## PROBLEMS

1. A derivative is *best* described as a financial instrument that derives its performance by:
  - passing through the returns of the underlying.
  - replicating the performance of the underlying.
  - transforming the performance of the underlying.
2. Compared with exchange-traded derivatives, over-the-counter derivatives would *most likely* be described as:
  - standardized.
  - less transparent.
  - more transparent.

3. Exchange-traded derivatives are:
  - A. largely unregulated.
  - B. traded through an informal network.
  - C. guaranteed by a clearinghouse against default.
4. Which of the following derivatives is classified as a contingent claim?
  - A. Futures contracts
  - B. Interest rate swaps
  - C. Credit default swaps
5. In contrast to contingent claims, forward commitments provide the:
  - A. right to buy or sell the underlying asset in the future.
  - B. obligation to buy or sell the underlying asset in the future.
  - C. promise to provide credit protection in the event of default.
6. Which of the following derivatives provide payoffs that are non-linearly related to the payoffs of the underlying?
  - A. Options
  - B. Forwards
  - C. Interest rate swaps
7. An interest rate swap is a derivative contract in which:
  - A. two parties agree to exchange a series of cash flows.
  - B. the credit seller provides protection to the credit buyer.
  - C. the buyer has the right to purchase the underlying from the seller.
8. Forward commitments subject to default are:
  - A. forwards and futures.
  - B. futures and interest rate swaps.
  - C. interest rate swaps and forwards.
9. Which of the following derivatives is *least likely* to have a value of zero at initiation of the contract?
  - A. Futures
  - B. Options
  - C. Forwards
10. A credit derivative is a derivative contract in which the:
  - A. clearinghouse provides a credit guarantee to both the buyer and the seller.
  - B. seller provides protection to the buyer against the credit risk of a third party.
  - C. the buyer and seller provide a performance bond at initiation of the contract.
11. Compared with the underlying spot market, derivative markets are *more likely* to have:
  - A. greater liquidity.
  - B. higher transaction costs.
  - C. higher capital requirements.
12. Which of the following characteristics is *least likely* to be a benefit associated with using derivatives?
  - A. More effective management of risk
  - B. Payoffs similar to those associated with the underlying
  - C. Greater opportunities to go short compared with the spot market
13. Which of the following is *most likely* to be a destabilizing consequence of speculation using derivatives?
  - A. Increased defaults by speculators and creditors
  - B. Market price swings resulting from arbitrage activities
  - C. The creation of trading strategies that result in asymmetric performance

14. The law of one price is *best* described as:
  - A. the true fundamental value of an asset.
  - B. earning a risk-free profit without committing any capital.
  - C. two assets that will produce the same cash flows in the future must sell for equivalent prices.
15. Arbitrage opportunities exist when:
  - A. two identical assets or derivatives sell for different prices.
  - B. combinations of the underlying asset and a derivative earn the risk-free rate.
  - C. arbitrageurs simultaneously buy takeover targets and sell takeover acquirers.



# CHAPTER 2

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## BASICS OF DERIVATIVE PRICING AND VALUATION

Don M. Chance, PhD, CFA

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### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- explain how the concepts of arbitrage, replication, and risk neutrality are used in pricing derivatives;
- distinguish between value and price of forward and futures contracts;
- explain how the value and price of a forward contract are determined at expiration, during the life of the contract, and at initiation;
- describe monetary and nonmonetary benefits and costs associated with holding the underlying asset and explain how they affect the value and price of a forward contract;
- define a forward rate agreement and describe its uses;
- explain why forward and futures prices differ;
- explain how swap contracts are similar to but different from a series of forward contracts;
- distinguish between the value and price of swaps;
- explain how the value of a European option is determined at expiration;
- explain the exercise value, time value, and moneyness of an option;
- identify the factors that determine the value of an option and explain how each factor affects the value of an option;
- explain put–call parity for European options;
- explain put–call–forward parity for European options;
- explain how the value of an option is determined using a one-period binomial model;
- explain under which circumstances the values of European and American options differ.

## 1. INTRODUCTION

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It is important to understand how prices of derivatives are determined. Whether one is on the buy side or the sell side, a solid understanding of pricing financial products is critical to effective investment decision making. After all, one can hardly determine what to offer or bid for a financial product, or any product for that matter, if one has no idea how its characteristics combine to create value.

Understanding the pricing of financial assets is important. Discounted cash flow methods and models, such as the capital asset pricing model and its variations, are useful for determining the prices of financial assets. The unique characteristics of derivatives, however, pose some complexities not associated with assets, such as equities and fixed-income instruments. Somewhat surprisingly, however, derivatives also have some simplifying characteristics. For example, as we will see in this reading, in well-functioning derivatives markets the need to determine risk premiums is obviated by the ability to construct a risk-free hedge. Correspondingly, the need to determine an investor's risk aversion is irrelevant for derivative pricing, although it is certainly relevant for pricing the underlying.

The purpose of this reading is to establish the foundations of derivative pricing on a basic conceptual level. The following topics are covered:

- How does the pricing of the underlying asset affect the pricing of derivatives?
- How are derivatives priced using the principle of arbitrage?
- How are the prices and values of forward contracts determined?
- How are futures contracts priced differently from forward contracts?
- How are the prices and values of swaps determined?
- How are the prices and values of European options determined?
- How does American option pricing differ from European option pricing?

This reading is organized as follows. Section 2 explores two related topics, the pricing of the underlying assets on which derivatives are created and the principle of arbitrage. Section 3 describes the pricing and valuation of forwards, futures, and swaps. Section 4 introduces the pricing and valuation of options. Section 5 provides a summary.

## 2. FUNDAMENTAL CONCEPTS OF DERIVATIVE PRICING

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In this section, we will briefly review the concepts associated with derivatives, the types of derivatives, and the pricing principles of the underlying assets. We will also look at arbitrage, a critical concept that links derivative pricing to the price of the underlying.

### 2.1. Basic Derivative Concepts

The definition of a derivative is as follows:

*A derivative is a financial instrument that derives its performance from the performance of an underlying asset.*

A derivative is created as a contract between two parties, the buyer and the seller. Derivatives trade in markets around the world, which include organized exchanges, where highly

standardized and regulated versions exist, and over-the-counter markets, where customized and more lightly regulated versions trade. The basic characteristics of derivatives that influence pricing are not particularly related to where the derivatives trade, but are critically dependent on the types of derivatives.

The two principal types of derivatives are forward commitments and contingent claims. A forward commitment is an obligation to engage in a transaction in the spot market at a future date at terms agreed upon today.<sup>1</sup> By entering into a forward commitment, a party locks in the terms of a transaction that he or she will conduct later. The word “commitment” is critical here. A forward contract is a firm obligation.

There are three types of forward commitments: forward contracts, futures contracts, and swap contracts. These contracts can be referred to more simply as forwards, futures, and swaps.

*A forward contract is an over-the-counter derivative contract in which two parties agree that one party, the buyer, will purchase an underlying asset from the other party, the seller, at a later date at a fixed price they agree upon when the contract is signed.*

*A futures contract is a standardized derivative contract created and traded on a futures exchange in which two parties agree that one party, the buyer, will purchase an underlying asset from the other party, the seller, at a later date at a price agreed upon by the two parties when the contract is initiated and in which there is a daily settling of gains and losses and a credit guarantee by the futures exchange through its clearinghouse.*

*A swap contract is an over-the-counter derivative contract in which two parties agree to exchange a series of cash flows whereby one party pays a variable series that will be determined by an underlying asset or rate and the other party pays either 1) a variable series determined by a different underlying asset or rate or 2) a fixed series.*

As these definitions illustrate, forwards and futures are similar. They both establish the terms of a spot transaction that will occur at a later date. Forwards are customized, less transparent, less regulated, and subject to higher counterparty default risk. Futures are standardized, more transparent, more regulated, and generally immune to counterparty default. A swap is equivalent to a series of forward contracts, a point that will be illustrated later.

A contingent claim is a derivative in which the outcome or payoff is determined by the outcome or payoff of an underlying asset, conditional on some event occurring. Contingent claims include options, credit derivatives, and asset-backed securities. Because credit derivatives and asset-backed securities are highly specialized, this reading will focus only on options.

Recall the definition of an option:

*An option is a derivative contract in which one party, the buyer, pays a sum of money to the other party, the seller or writer, and receives the right to either buy or sell an underlying asset at a fixed price either on a specific expiration date or at any time prior to the expiration date.*

<sup>1</sup>Remember that the term “spot market” refers to the market in which the underlying trades. A transaction in the spot market involves a buyer paying for an asset and receiving it right away or at least within a few days, given the normal time required to settle a financial transaction.

Options can be either customized over-the-counter contracts or standardized and traded on exchanges.

Because derivatives take their prices from the price of the underlying, it is important to first understand how the underlying is priced. We will approach the underlying from a slightly different angle, one that emphasizes the often-subtle costs of holding the underlying, which turn out to play a major role in derivative pricing.

## 2.2. Pricing the Underlying

The four main types of underlying on which derivatives are based are equities, fixed-income securities/interest rates, currencies, and commodities. Equities, fixed-income securities (but not interest rates), currencies, and commodities are all assets. An interest rate is not an asset, but it can be structured as the underlying of a derivative.<sup>2</sup>

Consider a generic underlying asset. This asset is something of value that you can own. Some assets are financial assets, such as equities, bonds, and currencies, and some are real assets, such as commodities (e.g., gold, oil, and agricultural products) and certain physical objects (e.g., houses, automobiles, and computers).

The price of a financial asset is often determined using a present value of future cash flows approach. The value of the financial asset is the expected future price plus any interim payments such as dividends or coupon interest discounted at a rate appropriate for the risk assumed. Such a definition presumes a period of time over which an investor anticipates holding an asset, known as the holding period. The investor forecasts the price expected to prevail at the end of the holding period as well as any cash flows that are expected to be earned over the holding period. He then takes that predicted future price and expected cash flows and finds their current value by discounting them to the present. Thereby, the investor arrives at a fundamental value for the asset and will compare that value with its current market price. Based on any differential relative to the cost of trading and his confidence in his valuation model, he will make a decision about whether to trade.

### 2.2.1. The Formation of Expectations

Let us first assume that the underlying does not pay interest or dividends, nor does it have any other cash flows attributable to holding the asset. Exhibit 1 illustrates the basic idea behind the valuation process. Using a probability distribution, the investor forecasts the future over a holding period spanning time 0 to time  $T$ . The center of the distribution is the expected price of the asset at time  $T$ , which we denote as  $E(S_T)$ , and represents the investor's prediction of the spot price at  $T$ . The investor knows there is risk, so this prediction is imperfect—hence the reason for the probability distribution. Nonetheless, at time 0 the investor makes her best prediction of the spot price at time  $T$ , which becomes the foundation for determining what she perceives to be the value of the asset.<sup>3</sup>

<sup>2</sup>This is a good example of why it is best not to use the term “underlying *asset*” when speaking of derivatives. Not all derivatives have underlying assets, but all have underlyings, some of which are not assets. Some other examples of non-asset underlyings used in derivatives are weather, insurance claims, and shipping rates. There are also some derivatives in which the underlying is another derivative.

<sup>3</sup>The distribution shown here is symmetrical and relatively similar to a normal distribution, but this characterization is for illustrative purposes only. We are making no assumptions about symmetry or normality at this point.

## EXHIBIT 1 The Formation of Expectations for an Asset



## 2.2.2. The Required Rate of Return on the Underlying Asset

To determine the value of the asset, this prediction must be converted into its price or present value. The specific procedure is to discount this expected future price, but that is the easy part. Determining the rate at which to discount the expected future price is the hard part. We use the symbol  $k$  to denote this currently unknown discount rate, which is often referred to as the required rate of return and sometimes the expected rate of return or just the expected return. At a minimum, that rate will include the risk-free rate of interest, which we denote as  $r$ . This rate represents the opportunity cost, or so-called time value of money, and reflects the price of giving up your money today in return for receiving more money later.

## 2.2.3. The Risk Aversion of the Investor

At this point, we must briefly discuss an important characteristic of investors: their degree of risk aversion. We can generally characterize three potential types of investors by how they feel about risk: risk averse, risk neutral, or risk seeking.

Risk-neutral investors are willing to engage in risky investments for which they expect to earn only the risk-free rate. Thus, they do not expect to earn a premium for bearing risk. For risk-averse investors, however, risk is undesirable, so they do not consider the risk-free rate an adequate return to compensate them for the risk. Thus, risk-averse investors require a risk premium, which is an increase in the expected return that is sufficient to justify the acceptance of risk. All things being equal, an investment with a higher risk premium will have a lower price. It is very important to understand, however, that risk premiums are not automatically earned. They are merely expectations. Actual outcomes can differ. Clearly stocks that decline in value did not earn risk premiums, even though someone obviously bought them with the expectation that they would. Nonetheless, risk premiums must exist in the long run or risk-averse investors would not accept the risk.

The third type of investor is one we must mention but do not treat as realistic. Risk seekers are those who prefer risk over certainty and will pay more to invest when there is risk, implying a negative risk premium. We almost always assume that investors prefer certainty over uncertainty, so we generally treat a risk-seeking investor as just a theoretical possibility and not a practical reality.<sup>4</sup>

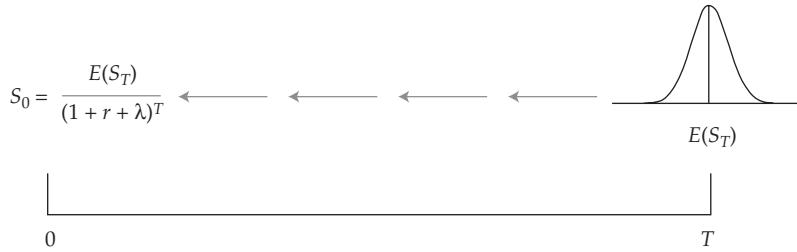
<sup>4</sup>People who gamble in casinos or play lotteries appear to be risk-seekers, given the advantage of the casino or the lottery organizer, but they are merely earning utility from the game itself, not necessarily from the expected financial outcome.

We will assume that investors are risk averse. To justify taking risk, risk-averse investors require a risk premium. We will use the Greek symbol  $\lambda$  (lambda) to denote the risk premium.<sup>5</sup>

#### 2.2.4. The Pricing of Risky Assets

Exhibit 2 illustrates the process by which an investor obtains the current price,  $S_0$ , by discounting the expected future price of an asset with no interim cash flows,  $E(S_T)$ , by  $r$  (the risk-free rate) plus  $\lambda$  (the risk premium) over the period from 0 to  $T$ .

EXHIBIT 2 Discounting the Expected Future Price to Obtain the Current Price



#### 2.2.5. Other Benefits and Costs of Holding an Asset

Many assets generate benefits and some incur costs to their owners. Some of these costs are monetary and others are nonmonetary. The dividends paid by companies and coupon interest paid by borrowers on their bonds represent obvious benefits to the holders of these securities. With currencies representing investments that earn the risk-free rate in a foreign country, they too generate benefits in the form of interest. Barring default, interest payments on bonds and currencies are relatively certain, so we will treat them as such. Dividend payments are not certain, but dividends do tend to be fairly predictable. As such, we will make an assumption common to most derivative models that dividends are certain.<sup>6</sup>

There is substantial evidence that some commodities generate a benefit that is somewhat opaque and difficult to measure. This benefit is called the **convenience yield**. It represents a nonmonetary advantage of holding the asset. For most financial assets, convenience yields are either nonexistent or extremely limited. Financial assets do not possess beauty that might make a person enjoy owning them just to look at them. Convenience yields are primarily associated with commodities and generally exist as a result of difficulty in either shorting the commodity or unusually tight supplies. For example, if a commodity cannot be sold short without great difficulty or cost, the holder of the commodity has an advantage if market conditions suggest that the commodity should be sold. Also, if a commodity is in short supply, the holders of the commodity can sometimes extract a price premium that is believed by some to be higher than what would be justified in well-functioning markets. The spot price of the commodity could even be above the market's expectation of its future price, a condition that would seem to imply a negative expected return. This scenario raises the question of why anyone would

<sup>5</sup>Although the risk-free rate is invariant with a country's economy, the risk premium varies with the amount of risk taken. Thus, while the risk-free rate is the same when applied to every investment, the risk premium is not the same for every investment.

<sup>6</sup>Some derivative models incorporate uncertain dividends and interest, but those are beyond the scope of this introductory reading.

want to hold the commodity if its expected return is negative. The convenience yield provides a possible explanation that attributes an implied but non-financial expected return to the advantage of holding a commodity in short supply. The holder of the commodity has the ability to sell it when market conditions suggest that selling is advisable and short selling is difficult.

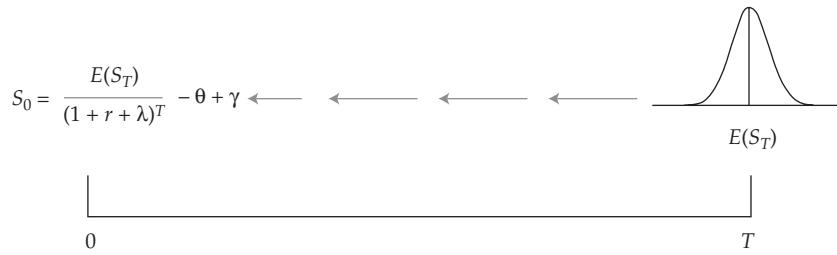
One cost incurred in owning commodities is the cost of storage. One could hardly own gold, oil, or wheat without incurring some costs in storing these assets. There are also costs incurred in protecting and insuring some commodities against theft or destruction. Depending on the commodity, these costs can be quite significant. For financial assets, however, the storage costs are so low that we can safely ignore them.

Finally, there is the opportunity cost of the money invested. If a person buys an asset, he forgoes interest on his money. The effect on this interest is reflected by compounding the price paid for the asset to a future value at the risk-free rate of interest. Thus, an investor who buys a stock that costs £50 in a market in which the risk-free rate is 4% will effectively have paid  $£50 \times 1.04 = £52$  a year later. Of course, the stock could be worth any value at that time, and any gain or loss should be determined in comparison to the effective price paid of £52.

As we described earlier, we determine the current price of an asset by discounting the expected future price by the sum of the risk-free rate ( $r$ ) plus the risk premium ( $\lambda$ ). When we introduce costs and benefits of holding the asset, we have to make an adjustment. With the exception of this opportunity cost of money, we will incorporate the effect of these costs and benefits by determining their value at the end of the holding period. Under the assumption that these costs and benefits are certain, we can then discount them at the risk-free rate to obtain their present value. There is a logic to doing it this way (i.e., finding their future value and discounting back to the present, as opposed to finding their present value directly). By finding their future value, we are effectively saying that the costs and benefits adjust the expected payoff at the end of the holding period. But because they are certain, we can discount their effects at the risk-free rate. So we have effectively just found their present value. The net effect is that the costs reduce the current price and the benefits increase the current price. We use the symbol  $\theta$  (theta) to denote the present value of the costs and  $\gamma$  (gamma) as the present value of any benefits.

The net of the costs and benefits is often referred to by the term **carry**, or sometimes **cost of carry**. The holding, storing, or “carrying” of an asset is said to incur a net cost that is essentially what it takes to “carry” an asset. Exhibit 3 illustrates the effect in which the carry adjusts the price of an asset in the valuation process.

EXHIBIT 3 Pricing an Asset That Incurs Costs and Generates Benefits



### EXAMPLE 1 Pricing the Spot Asset

1. Which of the following factors does **not** affect the spot price of an asset that has no interim costs or benefits?
  - A. The time value of money
  - B. The risk aversion of investors
  - C. The price recently paid by other investors
2. Which of the following does **not** represent a benefit of holding an asset?
  - A. The convenience yield
  - B. An optimistic expected outlook for the asset
  - C. Dividends if the asset is a stock or interest if the asset is a bond

*Solution to 1:* C is correct. The price recently paid by other investors is past information and does not affect the spot price. The time value of money and the risk aversion of investors determine the discount rate. Only current information is relevant as investors look ahead, not back.

*Solution to 2:* B is correct. An optimistic forecast for the asset is not a benefit of holding the asset, but it does appear in the valuation of the asset as a high expected price at the horizon date. Convenience yields and dividends and interest are benefits of holding the asset.

To recap, although the various underlyings differ with respect to the specifics of pricing, all of them are based on expectations, risk, and the costs and benefits of holding a specific underlying. Understanding how assets are priced in the spot market is critical to understanding how derivatives are priced. To understand derivative pricing, it is necessary to establish a linkage between the derivative market and the spot market. That linkage occurs through arbitrage.

### 2.3. The Principle of Arbitrage

Arbitrage is a type of transaction undertaken when two assets or portfolios produce identical results but sell for different prices. If a trader buys the asset or portfolio at the cheaper price and sells it at the more expensive price, she will generate a net inflow of funds at the start. Because the two assets or portfolios produce identical results, a long position in one and a short position in the other means that at the end of the holding period, the payoffs offset. Hence, no money is gained or lost at the end of the holding period, so there is no risk. The net effect is that the arbitrageur receives money at the start and never has to pay out any money later. Such a situation amounts to free money, like walking down the street, finding money on the ground, and never having to give it up. Exhibit 4 illustrates this process for assets A and B, which have no dividends or other benefits or costs and pay off identically but sell for different prices, with  $S_0^A < S_0^B$ .

## EXHIBIT 4 Executing an Arbitrage

Given: Assets A and B produce the same values at time  $T$  but at time 0, A is selling for less than B.

$S_0^A < S_0^B$ :  
 Buy A at  $S_0^A$   
 Sell B at  $S_0^B$   
 Cash flow =  $S_0^B - S_0^A (> 0)$

$S_T^A = S_T^B$ :  
 Sell A for  $S_T^A$   
 Buy B for  $S_T^B$   
 Cash flow =  $S_T^A - S_T^B (= 0)$



## 2.3.1. The (In)Frequency of Arbitrage Opportunities

When arbitrage opportunities exist, traders exploit them very quickly. The combined actions of many traders engaging in the same transaction of buying the low-priced asset or portfolio and selling the high-priced asset or portfolio results in increased demand and an increasing price for the former and decreased demand and a decreasing price for the latter. This market activity will continue until the prices converge. Assets that produce identical results can thus have only one true market price. This rule is called the “law of one price.” With virtually all market participants alert for the possibility of earning such profits at no risk, it should not be surprising that arbitrage opportunities are rare.

In practice, prices need not converge precisely, or even all that quickly, because the transaction cost of exploiting an opportunity could exceed the benefit. For example, say you are walking down the sidewalk of the Champs-Élysées in Paris and notice a €1 coin on the sidewalk. You have a bad back, and it would take some effort to bend over. The transaction cost of exploiting this opportunity without any risk could exceed the benefit of the money. Some arbitrage opportunities represent such small discrepancies that they are not worth exploiting because of transaction costs.

Significant arbitrage opportunities, however, will be exploited. A significant opportunity arises from a price differential large enough to overcome the transaction costs. Any such price differential will continue to be exploited until the opportunity disappears. Thus, if you find a €10 note on the Champs-Élysées sidewalk, there is a good chance you will find it worth picking up (even with your bad back), and even if you do not pick it up, it will probably not be there for long. With enough people alert for such opportunities, only a few will arise, and the ones that do will be quickly exploited and disappear. In this manner, arbitrage makes markets work much more efficiently.

## 2.3.2. Arbitrage and Derivatives

It may be difficult to conceive of many investments that would produce identical payoffs. Even similar companies such as McDonalds and Burger King, which are in the same line of business, do not perform identically. Their performance may be correlated, but each has its own unique characteristics. For equity securities and with no derivatives involved, about the only such situation that could exist in reality is a stock that trades simultaneously in two different markets, such as Royal Dutch Shell, which trades in Amsterdam and London but is a single company. Clearly there can be only one price. If those two markets operate in different currencies, the currency-adjusted prices should be the same. Bonds issued by the same borrower are also potentially arbitrageable. All bonds of an issuer will be priced off of the term structure of interest

rates. Because of this common factor, bonds of different maturities can be arbitAGED against each other. But in general, two securities are unlikely to perform identically.

The picture changes, however, if we introduce derivatives. For most derivatives, the pay-offs come (derive) directly from the value of the underlying at the expiration of the derivative. Although no one can predict with certainty the value of the underlying at expiration, as soon as that value is determined, the value of the derivative at expiration becomes certain. So, while the performance of McDonalds' stock may have a strong correlation to the performance of Burger King's stock, neither completely determines the other. But derivatives on McDonalds' stock and derivatives on Burger King's stock are completely determined by their respective stocks. All of the uncertainty in a derivative comes from the uncertainty in the underlying. As a result, the price of the derivative is tied to the price of the underlying. That being the case, the derivative can be used to hedge the underlying, or vice versa.

Exhibit 5 illustrates this point. When the underlying is combined with the derivative to produce a perfect hedge, all of the risk is eliminated and the position should earn the risk-free rate. If not, arbitrageurs begin to trade. If the position generates a return in excess of the risk-free rate, the arbitrageurs see an opportunity because the hedged position of the underlying and derivative earns more than the risk-free rate and a risk-free loan undertaken as a borrower incurs a cost equal to the risk-free rate. Therefore, going long the hedged position and borrowing at the risk-free rate earns a return in excess of the risk-free rate, incurs a cost of the risk-free rate, and has no risk. As a result, an investor can earn excess return at no risk without committing any capital. Arbitrageurs will execute this transaction in large volumes, continuing to exploit the pricing discrepancy until market forces push prices back in line such that both risk-free transactions earn the risk-free rate.

#### EXHIBIT 5 Hedging the Underlying with a Derivative (or Vice Versa)

Position in underlying + Opposite position in derivative	Underlying payoff - Derivative payoff = Risk-free return
0	$T$

Out of this process, one and only one price can exist for the derivative. Otherwise, there will be an arbitrage opportunity. We typically take the underlying price as given and infer the unique derivative price that prohibits any arbitrage opportunities. Most derivatives pricing models are established on this foundation. We simply assume that no arbitrage opportunities can exist and infer the derivative price that guarantees there are no arbitrage opportunities.

##### 2.3.3. Arbitrage and Replication

Because an asset and a derivative on the asset can be combined to produce a position equivalent to a risk-free bond, it follows that the asset and the risk-free asset can be combined to produce the derivative. Alternatively, the derivative and the risk-free asset can be combined to produce the asset. Exhibit 6 shows this process, referred to as **replication**. Replication is the creation of an asset or portfolio from another asset, portfolio, and/or derivative. Exhibit 6 shows first that an asset plus the derivative can replicate the risk-free asset. Second, an asset minus the risk-free asset (meaning to borrow at the risk-free rate) is equivalent to the opposite position

in the derivative, and third, a derivative minus the risk-free asset is equivalent to the opposite position in the asset.

#### EXHIBIT 6 Arbitrage, Replication, and Derivatives

$$\begin{array}{l}
 \boxed{\text{Asset}} + \boxed{\text{Derivative}} = \boxed{\text{Risk-free asset}} \\
 \boxed{\text{Asset}} - \boxed{\text{Risk-free asset}} = \boxed{-\text{Derivative}} \\
 \boxed{\text{Derivative}} - \boxed{\text{Risk-free asset}} = \boxed{-\text{Asset}}
 \end{array}$$

If all assets are correctly priced to prohibit arbitrage, however, the ability to replicate seems useless. Why would one replicate an asset or derivative if there is no cost advantage? Buying a government security to earn the risk-free rate is easier than buying the asset and selling a derivative to produce a risk-free position. At this point, that is certainly a reasonable question. As we progress through this material, however, we will relax the assumption that everything is always correctly priced and we will admit the possibility of occasional arbitrage opportunities. For example, it may be more profitable to hedge a portfolio with a derivative to produce a risk-free rate than to invest in the risk-free asset. In addition, we might find that replication can have lower transaction costs. For example, a derivative on a stock index combined with the risk-free asset can potentially replicate an index fund at lower transaction costs than buying all the securities in the index. Replication is the essence of arbitrage. The ability to replicate something with something else can be valuable to investors, either through pricing differentials, however temporary, or lower transaction costs.

#### 2.3.4. Risk Aversion, Risk Neutrality, and Arbitrage-Free Pricing

Most investors are risk averse. They do not accept risk without the expectation of a return commensurate with that risk. Thus, they require risk premiums to justify the risk. One might think that this point implies a method for pricing derivatives based on the application of a risk premium to the expected payoff of the derivative and its risk. As we will describe later, this methodology is not appropriate in the pricing of derivatives.

As previously described, a derivative can be combined with an asset to produce a risk-free position. This fact does not mean that one *should* create such a combination. It merely means that one *can* do so. The derivative price is the price that guarantees the risk-free combination of the derivative and the underlying produces a risk-free return. The derivative price can then be inferred from the characteristics of the underlying, the characteristics of the derivative, and the risk-free rate. The investor's risk aversion is not a factor in determining the derivative price. Because the risk aversion of the investor is not relevant to pricing the derivative, one can just as easily obtain the derivative price by assuming that the investor is risk neutral. That means that the expected payoff of the derivative can be discounted at the risk-free rate rather than the risk-free rate plus a risk premium. Virtually all derivative pricing models ultimately take this form: discounting the expected payoff of the derivative at the risk-free rate.

The entire process of pricing derivatives is not exactly as we have described it at this point. There is an intermediate step, which entails altering the probabilities of the outcomes from the true probabilities to something called risk-neutral probabilities. We will illustrate this process

later in this reading. The important point to understand is that while the risk aversion of investors is relevant to pricing assets, it is not relevant to pricing derivatives. As such, derivatives pricing is sometimes called **risk-neutral pricing**. Risk-neutral pricing uses the fact that arbitrage opportunities guarantee that a risk-free portfolio consisting of the underlying and the derivative must earn the risk-free rate. There is only one derivative price that meets that condition. Any mispricing of the derivative will lead to arbitrage transactions that drive the derivative price back to where it should be, the price that eliminates arbitrage opportunities.

The overall process of pricing derivatives by arbitrage and risk neutrality is called **arbitrage-free pricing**. We are effectively determining the price of a derivative by assuming that the market is free of arbitrage opportunities. This notion is also sometimes called the **principle of no arbitrage**. If there are no arbitrage opportunities, combinations of assets and/or derivatives that produce the same results must sell for the same price. The correct derivative price assures us that the market is free of arbitrage opportunities.

### 2.3.5. Limits to Arbitrage

As we previously described, there may be reasons to not pick up a coin lying on the ground. Likewise, some small arbitrage profits are never exploited. A bond selling for €1,000 might offer an arbitrage profit by trading a derivative on the bond and a risk-free asset at a total cost of €999, but the profit of €1 might be exceeded by the transaction costs. Such small differentials can easily remain essentially trapped within the bounds of transaction costs. In addition, arbitrage can require capital. Not everyone can borrow virtually unlimited amounts of money at what amounts to a risk-free rate. Moreover, some transactions can require additional capital to maintain positions. The corresponding gains from an offsetting position might not be liquid. Hence, on paper the position is hedged, but in practice, one position has a cash outflow while the other generates gains on paper that are realized only later. Borrowing against those future gains is not always easy.

Moreover, some apparent arbitrage transactions are not completely risk free. As you will learn later, option pricing requires knowledge of the volatility of the underlying asset, which is information that is not easy to obtain and subject to different opinions. Executing an arbitrage can entail risk if one lacks accurate information on the model inputs.

Some arbitrage positions require short-selling assets that can be difficult to short. Some securities are held only by investors who are unwilling to lend the securities and who, by policy, are not arbitrageurs themselves. Some commodities, in particular, can be difficult and costly to sell short. Hence, the arbitrage might exist in only one direction, which keeps the price from becoming seemingly too high or seemingly too low but permitting it to move virtually without limit in the opposite direction.

Arbitrage positions rely on the ultimate realization by other investors of the existence of the mispricing. For some investors, bearing these costs and risks until other investors drive the price back to its appropriate level can be nearly impossible.

The arbitrage principle is the essence of derivative pricing models. Yet, clearly there are limits to the ability of all investors to execute arbitrage transactions. In studying derivative pricing, it is important to accept the no-arbitrage rule as a paradigm, meaning a framework for analysis and understanding. Although no market experts think that arbitrage opportunities never occur, it is a common belief that finding and exploiting them is a challenging and highly competitive process that will not yield frequent success. But it is important that market participants stay alert for and exploit whatever arbitrage opportunities arise. In response, the market functions more efficiently.

### EXAMPLE 2 Arbitrage

1. Which of the following *best* describes an arbitrage opportunity? It is an opportunity to:
  - A. earn a risk premium in the short run.
  - B. buy an asset at less than its fundamental value.
  - C. make a profit at no risk with no capital invested.
2. What *most likely* happens when an arbitrage opportunity exists?
  - A. Investors trade quickly and prices adjust to eliminate the opportunity.
  - B. Risk premiums increase to compensate traders for the additional risk.
  - C. Markets cease operations to eliminate the possibility of profit at no risk.
3. Which of the following *best* describes how derivatives are priced?
  - A. A hedge portfolio is used that eliminates arbitrage opportunities.
  - B. The payoff of the underlying is adjusted downward by the derivative value.
  - C. The expected future payoff of the derivative is discounted at the risk-free rate plus a risk premium.
4. An investor who requires no premium to compensate for the assumption of risk is said to be which of the following?
  - A. Risk seeking
  - B. Risk averse
  - C. Risk neutral
5. Which of the following is a limit to arbitrage?
  - A. Clearinghouses restrict the transactions that can be arbitrated.
  - B. Pricing models do not show whether to buy or sell the derivative.
  - C. It may not always be possible to raise sufficient capital to engage in arbitrage.

*Solution to 1:* C is correct because it is the only answer that is based on the notion of when an arbitrage opportunity exists: when two identical assets or portfolios sell for different prices. A risk premium earned in the short run can easily have occurred through luck. Buying an asset at less than fair value might not even produce a profit.

*Solution to 2:* A is correct. The combined actions of traders push prices back in line to a level at which no arbitrage opportunities exist. Markets certainly do not shut down, and risk premiums do not adjust and, in fact, have no relevance to arbitrage profits.

*Solution to 3:* A is correct. A hedge portfolio is formed that eliminates arbitrage opportunities and implies a unique price for the derivative. The other answers are incorrect because the underlying payoff is not adjusted by the derivative value and the discount rate of the derivative does not include a risk premium.

*Solution to 4:* C is correct. Risk-seeking investors give away a risk premium because they enjoy taking risk. Risk-averse investors expect a risk premium to compensate for the risk. Risk-neutral investors neither give nor receive a risk premium because they have no feelings about risk.

*Solution to 5:* C is correct. It may not always be possible to raise sufficient capital to engage in arbitrage. Clearinghouses do not restrict arbitrage. Pricing models show what the price of the derivative should be.

Thus, comparison with the market price will indicate if the derivative is overpriced and should be sold or if it is underpriced and should be purchased.

## 2.4. The Concept of Pricing versus Valuation

In equity markets, analysis is undertaken with the objective of determining the value, sometimes called the fundamental value, of a stock. When a stock trades in the market for a price that differs from its fundamental value, investors will often buy or sell the stock based on the perceived mispricing. The fundamental value of a stock is typically determined by analyzing the company's financial statements, projecting its earnings and dividends, determining a discount rate based on the risk, and finding the present value of the future dividends. These steps make up the essence of dividend discount models. Other approaches include comparing the book value of a company to its market value, thereby using book value as a proxy for fundamental value, or by application of a price/earnings ratio to projected next-period earnings, or by discounting free cash flow. Each of these approaches purports to estimate the company's fundamental value, leading to the notion that a company is worth something that may or may not correspond to its price in the market.

In derivative markets, the notion of valuation as a representation of fundamental value is still a valid concept, but the terminology can be somewhat different and can lead to some confusion. Options are not a problem in this regard. They can be analyzed to determine their fundamental value, and the market price can be compared with the fundamental value. Any difference can then presumably be exploited via arbitrage. The combined actions of numerous investors should ultimately lead to the market price converging to its fundamental value, subject to the above limits to arbitrage.

The world of forwards, futures, and swaps, however, uses different terminology with respect to price and value. These contracts do not require the outlay of cash at the start the way an option, stock, or bond does. Forwards, futures, and swaps start off with values of zero. Then as the underlying moves, their values become either positive or negative. The forward, futures, or swap price is a concept that represents the fixed price or rate at which the underlying will be purchased at a later date. It is not an amount to be paid at the start. This fixed price or rate is embedded into the contract while the value will fluctuate as market conditions change. But more importantly, the value and price are not at all comparable with each other.

Consider a simple example. Suppose you own a stock priced at \$102. You have a short forward contract to sell the stock at a price of \$100 one year from now. The risk-free rate is 4%. Your position is riskless because you know that one year from now, you will sell the stock for \$100. Thus, you know you will get \$100 one year from now, which has a present value of  $\$100/(1.04) = \$96.15$ . Notice the discounting at the risk-free rate, which is appropriate because the position is riskless. Your overall position is that you own an asset worth \$102 and are short a contract worth something, and the two positions combine to have a value of \$96.15. Therefore, the forward contract must have a value of  $\$96.15 - \$102 = -\$5.85$ . Your

forward contract is thus worth  $-\$5.85$ . To the party on the opposite side, it is worth  $+\$5.85$ .<sup>7</sup> The price of the forward contract is still  $\$100$ , which was set when you created the contract at an earlier date. As you can see, the  $\$100$  forward price is not comparable to the  $\$5.85$  value of the contract.

Although the forward price is fixed, any new forward contract calling for delivery of the same asset at the same time will have a different price. We will cover that point in more detail later. For now, it is important to see that your contract has a price of  $\$100$  but a value of  $-\$5.85$ , which are two entirely different orders of magnitude. This information does not imply that the forward contract is mispriced. The value is the amount of wealth represented by owning the forward contract. The price is one of the terms the parties agreed on when they created the contract.<sup>8</sup> This idea applies in the same manner for futures and swaps.

### 3. PRICING AND VALUATION OF FORWARD COMMITMENTS

In this section, we will go into pricing forward commitments in a little more detail. Let us start by establishing that today, at time  $0$ , we create a forward commitment that expires at time  $T$ . The value of the underlying today is  $S_0$ . At expiration the underlying value is  $S_T$ , which is not known at the initiation of the contract.

#### 3.1. Pricing and Valuation of Forward Contracts

Previously, we noted that price and value are entirely different concepts for forward commitments. We gave an example of a forward contract with a price of  $\$100$  but a value of  $-\$5.85$  to the seller and  $+\$5.85$  to the buyer. In the next subsection, we will delve more deeply into understanding these concepts of pricing and valuation for forward contracts.

##### 3.1.1. Pricing and Valuation of Forward Contracts at Expiration

Recall that a forward contract specifies that one party agrees to buy the underlying from the other at the expiration date at a price agreed on at the start of the contract. Suppose that you enter into a contract with another party in which you will buy a used car from that party in one year at a price of  $\$10,000$ . Then  $\$10,000$  is the forward price. One year later, when the contract expires, you are committed to paying  $\$10,000$  and accepting delivery of the car. Let us say that at that time, you check the used car market and find that an identical car is worth  $\$10,800$ . How much is your forward contract worth to you at that time? It obligates you to pay  $\$10,000$  for a car that you would otherwise have to pay  $\$10,800$ . Thus, the contract benefits you by  $\$800$ , so its value is  $\$800$ . If you were on the opposite side of the transaction, its value would be  $-\$800$ . If the market price of the car were below  $\$10,000$ , the contract would have negative value to you and the mirror image positive value to the seller.

<sup>7</sup>This concept of the value of the forward contract as it evolves toward expiration is sometimes referred to as its mark-to-market value. The same notion is applicable to swaps. In futures, of course, contracts are automatically marked to market by the clearinghouse, and gains and losses are converted into actual cash flows from one party to the other.

<sup>8</sup>The forward price is more like the exercise price of the option. It is the price the two parties agree will be paid at a future date for the underlying. Of course, the option has the feature that the holder need not ever pay that price, which is the case if the holder chooses not to exercise the option.

This example leads us to our first important derivative pricing result. The forward price, established at the initiation date of contract is  $F_0(T)$ . Let us denote the value at expiration of the forward contract as  $V_T(T)$ . This value is formally stated as

$$V_T(T) = S_T - F_0(T) \quad (1)$$

In words,

*The value of a forward contract at expiration is the spot price of the underlying minus the forward price agreed to in the contract.*

In the financial world, we generally define value as the value to the long position, so the above definition is generally correct but would be adjusted if we look at the transaction from the point of view of the short party. In that case, we would multiply the value to the long party by  $-1$  to calculate the value to the short party. Alternatively, the value to the short party is the forward price minus the spot price at expiration.

If a forward contract could be initiated right at the instant of expiration, the forward price would clearly be the spot price. Such a contract would essentially be a spot transaction.

### 3.1.2. Pricing and Valuation at Initiation Date

In Exhibit 7, we see the nature of the problem of pricing a forward contract. We are situated at time 0, facing an uncertain future. At the horizon date, time  $T$ , the underlying price will be  $S_T$ . Of course, at time 0 we do not know what  $S_T$  will turn out to be. Yet at time 0, we need to establish the forward price,  $F_0(T)$ , which is the price we agree to pay at time  $T$  to purchase the asset.

#### EXHIBIT 7 The Time Horizon of Forward Contracts



When a forward contract is initiated, neither party pays anything to the other. It is a valueless contract, neither an asset nor a liability. Therefore, its value at initiation is zero:

$$V_0(T) = 0 \quad (2)$$

The forward price that the parties agree to at the initiation date of the contract is a special price that results in the contract having zero value and prohibiting arbitrage. This is our first important result:

*Because neither the long nor the short pays anything to the other at the initiation date of a forward contract, the value of a forward contract when initiated is zero.*

If this statement were not true and one party paid a sum of money to the other, the party receiving the money could find another party and engage in the opposite transaction, with

no money paid to the other on this second contract. The two transactions would completely offset, thereby eliminating the risk. Yet, the first party would have captured some cash from the second and consequently earned an arbitrage profit because his position is completely hedged. He would walk away with money and never have to worry about paying it back. The forward price is the price the two parties agree on that generates a value of zero at the initiation date. Finding that price is actually quite easy.

Consider a very simple asset price at  $S_0$  today that pays no dividends or interest, nor does it yield any nonfinancial benefits or incur any carrying costs. As described earlier, we can peer into the future, but at best we can make only a forecast of the price of this asset at our horizon date of time  $T$ . That forecast was previously referred to as the expected spot price at expiration,  $E(S_T)$ . On the surface, it might seem that pricing a forward contract would somehow involve a discounting of the expected spot price. As we said earlier, however, that is not how derivatives are priced—they are priced using arbitrage.

Suppose we hold the asset and enter into a forward contract to sell the asset at the price  $F_0(T)$ . It should be easy to see that we have constructed a risk-free position. We know that the asset, currently worth  $S_0$ , will be sold later at  $F_0(T)$  and that this price should guarantee a risk-free return. Thus, we should find the following relationship,

$$\frac{F_0(T)}{S_0} = (1 + r)^T \quad (3)$$

We can easily solve for the forward price to obtain

$$F_0(T) = S_0(1 + r)^T \quad (4)$$

Or, in words,

*The forward price is the spot price compounded at the risk-free rate over the life of the contract.*

There is a nice logic to this relationship. While the spot price is what someone would have to pay today to buy the asset, a forward contract locks in the purchase price at the horizon date. When that date arrives, the investor will own the asset. Instead of buying the asset today, suppose the investor uses the forward contract to guarantee that she will own the asset at the horizon date. By using the forward contract, the investor will not have committed the money,  $S_0$ , that would have forgone interest at the rate  $r$  for the period 0 to  $T$ . Notice how the risk premium on the asset does not directly appear in the pricing relationship. It does appear implicitly, because it determines the spot price paid to buy the asset. Knowing the spot price, however, eliminates the necessity of determining the risk premium. The derivatives market can simply let the spot market derive the risk premium.

As a simple example, let us say the underlying price,  $S_0$ , is £50, the risk-free rate,  $r$ , is 3%, and the contract expires in three months, meaning that  $T = 3/12 = 0.25$ . Then the forward price is  $£50(1.03)^{0.25} = £50.37$ . Thus, the two parties would agree that the buyer will pay £50.37 to the seller in three months, and the seller will deliver the underlying to the buyer at expiration.

Now suppose the asset generates cash payments and/or benefits and incurs storage costs. As we discussed, the net cost of carry consists of the benefits, denoted as  $\gamma$  (dividends or interest

plus convenience yield), minus the costs, denoted as  $\theta$ , both of which are in present value form. To put these concepts in future value form, we simply compound them at the risk-free rate,  $(\gamma - \theta)(1 + r)^T$ . Because this is their value at the expiration date of the contract, we can add them to  $F_0(T)$  in Equation 3, thereby restating that equation as

$$(1 + r)^T = \frac{F_0(T) + (\gamma - \theta)(1 + r)^T}{S_0}$$

The numerator is how much money we end up with at  $T$ . Rearranging, we obtain the forward price as

$$F_0(T) = (S_0 - \gamma + \theta)(1 + r)^T$$

or

$$F_0(T) = S_0(1 + r)^T - (\gamma - \theta)(1 + r)^T \quad (5)$$

We see that the forward price determined using Equation 4 is reduced by the future value of any benefits and increased by the future value of any costs. In other words,

*The forward price of an asset with benefits and/or costs is the spot price compounded at the risk-free rate over the life of the contract minus the future value of those benefits and costs.*

Again, the logic is straightforward. To acquire a position in the asset at time  $T$ , an investor could buy the asset today and hold it until time  $T$ . Alternatively, he could enter into a forward contract, committing him to buying the asset at  $T$  at the price  $F_0(T)$ . He would end up at  $T$  holding the asset, but the spot transaction would yield benefits and incur costs, whereas the forward transaction would forgo the benefits but avoid the costs.

Assume the benefits exceed the costs. Then the forward transaction would return less than the spot transaction. The formula adjusts the forward price downward by the expression  $-(\gamma - \theta)(1 + r)^T$  to reflect this net loss over the spot transaction. In other words, acquiring the asset in the forward market would be cheaper because it forgoes benefits that exceed the costs. That does not mean the forward strategy is better. It costs less but also produces less. Alternatively, if the costs exceeded the benefits, the forward price would be higher because the forward contract avoids the costs at the expense of the lesser benefits.

Returning to our simple example, suppose the present value of the benefits is  $\gamma = £3$  and the present value of the costs is  $\theta = £4$ . The forward price would be  $£50(1.03)^{0.25} - (£3 - £4)(1.03)^{0.25} = £51.38$ . The forward price, which was £50.37 without these costs and benefits, is now higher because the carrying costs exceed the benefits.

The value of the contract when initiated is zero provided the forward price conforms to the appropriate pricing formula. To keep the analysis as simple as possible, consider the case in which the asset yields no benefits and incurs no costs. Going long the forward contract or going long the asset produces the same position at  $T$ : ownership of the asset. Nonetheless, the strategies are not equivalent. Going long the forward contract enables the investor to avoid having to pay the price of the asset,  $S_0$ , so she would collect interest on the money. Thus, the forward strategy would have a value of  $S_0$ , reflecting the investment of that much cash invested in risk-free bonds, plus the value of the forward contract. The spot strategy would have a value of  $S_0$ , reflecting the investment in the asset. These two strategies must have equal values. Hence, the value of the forward contract must be zero.

Although a forward contract has zero value at the start, it will not have zero value during its life. We now take a look at what happens during the life of the contract.

### 3.1.3. Pricing and Valuation during the Life of the Contract

We previously worked an example in which a forward contract established with a price of \$100 later has a value of -\$5.85 to the seller and +\$5.85 to the buyer. Generally we would say the value is \$5.85. We explained that with the spot price at \$102, a party that is long the asset and short the forward contract would guarantee the sale of the asset priced at \$102 at a price of \$100 in one year. The present value of \$100 in one year at 4% is \$96.15. Thus, the party guarantees that his \$102 asset will be effectively sold at a present value of \$96.15, for a present value loss of \$5.85.

In general, we can say that

*The value of a forward contract is the spot price of the underlying asset minus the present value of the forward price.*

Again, the logic is simple. A forward contract provides a type of synthetic position in the asset, for which we promise to pay the forward price at expiration. Thus, the value of the forward contract is the value of the asset minus the present value of the forward price. Let us write out this relationship using  $V_t(T)$  as the value of the forward contract at time  $t$ , which is some point in time after the contract is initiated and before it expires:

$$V_t(T) = S_t - F_0(T)(1 + r)^{-(T-t)} \quad (6)$$

Note that we are working with the spot price at  $t$ , but the forward price was fixed when the contract was initiated.<sup>9</sup>

Now, recall the problem we worked in which the underlying had a price of £50 and the contract was initiated with a three-month life at a price of £50.37. Move one month later, so that the remaining time is two months:  $T - t = 2/12 = 0.167$ . Let the underlying price be £52. The value of the contract would be  $£52 - £50.37(1.03)^{-0.167} = £1.88$ .

If the asset has a cost of carry, we must make only a small adjustment:

$$V_t(T) = S_t - (\gamma - \theta)(1 + r)^t - F_0(T)(1 + r)^{-(T-t)} \quad (7)$$

Note how we adjust the formula by the net of benefits minus costs. The forward contract forgoes the benefits and avoids the costs of holding the asset. Consequently, we adjust the value downward to reflect the forgone benefits and upward to reflect the avoided costs. Remember that the costs ( $\theta$ ) and benefits ( $\gamma$ ) are expressed on a present value basis as of time 0. We need their value at time  $t$ . We could compound them from 0 to  $T$  and then discount them back to  $t$  by the period  $T - t$ , but a shorter route is to simply compound them from 0 to  $t$ . In the problem we previously worked, in which we priced the forward contract when the asset has costs and benefits, the benefits ( $\gamma$ ) were £3 and the costs ( $\theta$ ) were £4, giving us a forward price of £51.38. We have now moved one month ahead, so  $t = 1/12 = 0.0833$  and  $T - t = 2/12 = 0.167$ . Hence

<sup>9</sup>An alternative approach to valuing a forward contract during its life is to determine the price of a new forward contract that would offset the old one. The discounted difference between the new forward price and the original forward price will lead to the same value.

the value of the forward contract would be  $\text{£}52 - (\text{£}3 - \text{£}4)(1.03)^{0.0833} - \text{£}51.38(1.03)^{-0.167} = \text{£}1.88$ . In this case, the effect of the compounding of the net of costs and benefits (£1) over one month has no appreciable effect on the value, but that result is not a general rule.

It is important to note that although we say that Equation 7 holds during the life of the contract at some arbitrary time  $t$ , it also holds at the initiation date and at expiration. For the initiation date, we simply change  $t$  to 0 in Equation 7. Then we substitute Equation 5 for  $F_0(T)$  in Equation 7, obtaining  $V_0(T) = 0$ , confirming that the value of a forward contract at initiation is zero. At expiration, we let  $t = T$  in Equation 7 and obtain the spot price minus the forward price, as presented in Equation 1.<sup>10</sup>

### 3.1.4. A Word about Forward Contracts on Interest Rates

Forward contracts in which the underlying is an interest rate are called **forward rate agreements**, or FRAs. These instruments differ slightly from most other forward contracts in that the underlying is not an asset. Changes in interest rates, such as the value of an asset, are unpredictable. Moreover, virtually every company and organization is affected by the uncertainty of interest rates. Hence, FRAs are very useful devices for many companies. FRAs are forward contracts that allow participants to make a known interest payment at a later date and receive in return an unknown interest payment. In that way, a participant whose business will involve borrowing at a future date can lock in a fixed payment and receive a random payment that offsets the unknown interest payment it will make on its loan. Turning that argument around, a lender can also lock in a fixed rate on a loan it will make at a future date.

Even though FRAs do not involve an underlying asset, they can still be combined with an underlying asset to produce a hedged position, thereby leading to fairly straightforward pricing and valuation equations. The math is a little more complex than the math for forwards on assets, but the basic ideas are the same.

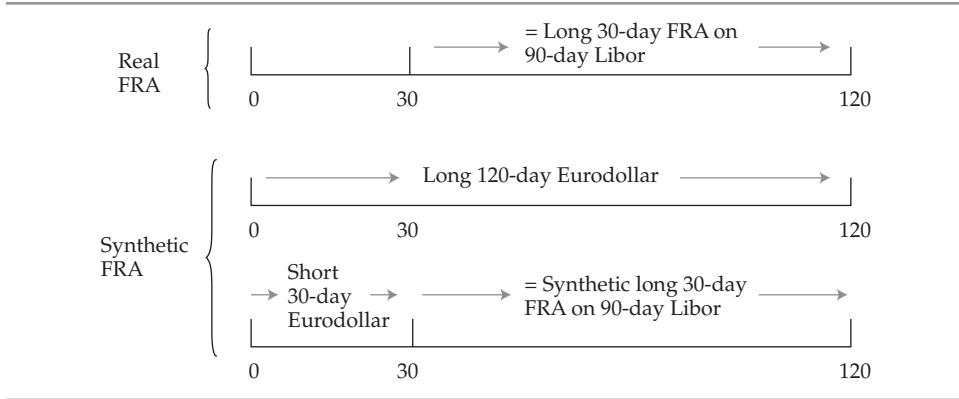
FRAs are often based on Libor, the London Interbank Offered Rate, which represents the rate on a Eurodollar time deposit, a loan in dollars from one London bank to another.<sup>11</sup> As an example, assume we are interested in going long a 30-day FRA in which the underlying is 90-day Libor. A long position means that in 30 days, we will make a known interest payment and receive an interest payment corresponding to 90-day Libor on that day. We can either enter into a 30-day FRA on 90-day Libor or create a synthetic FRA. To do the latter, we would go long a 120-day Eurodollar time deposit and short a 30-day Eurodollar time deposit. Exhibit 8 shows the structure of this strategy. We omit some of the details here, such as how much face value we should take on the two Eurodollar transactions as well as the size of the FRA. Those technical issues are covered in more advanced material. At this time, we focus on the fact that going long over the 120-day period and short over the 30-day period leaves an investor with no exposure over the 30-day period and then converts to a position that starts 30 days from now and matures 90 days later. This synthetic position corresponds to a 30-day FRA on 90-day Libor. Exhibit 8 illustrates this point.<sup>12</sup>

<sup>10</sup>You might be wondering whether the cost and benefit terms disappear when  $t = T$ . With the costs and benefits defined as those incurred over the period  $t$  to  $T$ , at expiration their value is zero by definition.

<sup>11</sup>Other rates such as Euribor (Euro Interbank Offered Rate) and Tibor (Tokyo Interbank Offered Rate) are also used.

<sup>12</sup>The real FRA we show appears to imply that an investor enters into a Eurodollar transaction in 30 days that matures 90 days later. This is not technically true. The investor does, however, engage in a cash settlement in 30 days that has the same value and economic form as such a transaction.

## EXHIBIT 8 Real FRA and Synthetic FRA (30-Day FRA on 90-Day Libor)



FRAs, and indeed all forward contracts relating to bonds and interest rates, are closely tied to the term structure of interest rates, a concept covered in virtually all treatments of fixed-income securities. Buying a 120-day zero-coupon bond and selling a 30-day zero-coupon bond produces a forward position in a 90-day zero-coupon bond that begins in 30 days. From that forward position, one can infer the forward rate. It would then be seen that the FRA rate *is* the forward rate, even though the derivative itself is not a forward contract on a bond.

## EXAMPLE 3 Forward Contract Pricing and Valuation

1. Which of the following *best* describes the difference between the price of a forward contract and its value?
  - The forward price is fixed at the start, and the value starts at zero and then changes.
  - The price determines the profit to the buyer, and the value determines the profit to the seller.
  - The forward contract value is a benchmark against which the price is compared for the purposes of determining whether a trade is advisable.
2. Which of the following *best* describes the value of the forward contract at expiration?  
The value is the price of the underlying:
  - minus the forward price.
  - divided by the forward price.
  - minus the compounded forward price.
3. Which of the following factors does *not* affect the forward price?
  - The costs of holding the underlying
  - Dividends or interest paid by the underlying
  - Whether the investor is risk averse, risk seeking, or risk neutral
4. Which of the following *best* describes the forward rate of an FRA?
  - The spot rate implied by the term structure
  - The forward rate implied by the term structure
  - The rate on a zero-coupon bond of maturity equal to that of the forward contract

*Solution to 1:* A is correct. The forward price is fixed at the start, whereas the value starts at zero and then changes. Both price and value are relevant in determining the profit for both parties. The forward contract value is not a benchmark for comparison with the price.

*Solution to 2:* A is correct because the holder of the contract gains the difference between the price of the underlying and the forward price. That value can, of course, be negative, which will occur if the holder is forced to buy the underlying at a price higher than the market price.

*Solution to 3:* C is correct. The costs of holding the underlying, known as carrying costs, and the dividends and interest paid by the underlying are extremely relevant to the forward price. How the investor feels about risk is irrelevant, because the forward price is determined by arbitrage.

*Solution to 4:* B is correct. FRAs are based on Libor, and they represent forward rates, not spot rates. Spot rates are needed to determine forward rates, but they are not equal to forward rates. The rate on a zero-coupon bond of maturity equal to that of the forward contract describes a spot rate.

As noted, we are not covering the details of derivative pricing but rather are focusing on the intuition. At this point, we have covered the intuition of pricing forward contracts. We now move to futures contracts.

### 3.2. Pricing and Valuation of Futures Contracts

Futures contracts differ from forward contracts in that they have standard terms, are traded on a futures exchange, and are more heavily regulated, whereas forward contracts are typically private, customized transactions. Perhaps the most important distinction is that they are marked to market on a daily basis, meaning that the accumulated gains and losses from the previous day's trading session are deducted from the accounts of those holding losing positions and transferred to the accounts of those holding winning positions. This daily settling of gains and losses enables the futures exchange to guarantee that a party that earns a profit from a futures transaction will not have to worry about collecting the money. Thus, futures exchanges provide a credit guarantee, which is facilitated through the use of a clearinghouse. The clearinghouse collects and disburses cash flows from the parties on a daily basis, thereby settling obligations quickly before they accumulate to much larger amounts. There is no absolute assurance that a clearinghouse will not fail, but none has ever done so since the first one was created in the 1920s.

The pattern of cash flows in a futures contract is quite similar to that in a forward contract. Suppose you enter into a forward contract two days before expiration in which you agree to buy an asset at €100, the forward price. Two days later, the asset is selling for

€103, and the contract expires. You therefore pay €100 and receive an asset worth €103, for a gain of €3. If the contract were cash settled, instead of involving physical delivery, you would receive €3 in cash, which you could use to defer a portion of the cost of the asset. The net effect is that you are buying the asset for €103, paying €100 plus the €3 profit on the forward contract.

Had you chosen a futures contract, the futures price at expiration would still converge to the spot price of €103. But now it would matter what the futures settlement price was on the next to last day. Let us assume that price was €99. That means on the next to last day, your account would be marked to market for a loss of €1, the price of €100 having fallen to €99. That is, you would be charged €1, with the money passed on to the opposite party. But then on the last day, your position would be marked from €99 to €103, a gain of €4. Your net would be €1 lost on the first day and €4 gained on the second for a total of €3. In both situations you gain €3, but with the forward contract, you gain it all at expiration, whereas with the futures contract, you gain it over two days. With this two-day example, the interest on the interim cash flow would be virtually irrelevant, but over longer periods and with sufficiently high interest rates, the difference in the amount of money you end up with could be noticeable.

The value of a futures contract is the accumulated gain or loss on a futures contract since its previous day's settlement. When that value is paid out in the daily settlement, the futures price is effectively reset to the settlement price and the value goes to zero. The different patterns of cash flows for forwards and futures can lead to differences in the pricing of forwards versus futures. But there are some conditions under which the pricing is the same. It turns out that if interest rates were constant, forwards and futures would have the same prices. The differential will vary with the volatility of interest rates. In addition, if futures prices and interest rates are uncorrelated, forwards and futures prices will be the same. If futures prices are positively correlated with interest rates, futures contracts are more desirable to holders of long positions than are forwards. The reason is because rising prices lead to futures profits that are reinvested in periods of rising interest rates, and falling prices leads to losses that occur in periods of falling interest rates. It is far better to receive cash flows in the interim than all at expiration under such conditions. This condition makes futures more attractive than forwards, and therefore their prices will be higher than forward prices. A negative correlation between futures prices and interest rates leads to the opposite interpretation, with forwards being more desirable than futures to the long position. The more desirable contract will tend to have the higher price.

The practical realities, however, are that the derivatives industry makes virtually no distinction between futures and forward prices.<sup>13</sup> Thus, we will make no distinction between futures and forward pricing, except possibly in noting some subtle issues that may arise from time to time.

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<sup>13</sup>At the time of this writing, many forwards (and swaps) are being processed through clearinghouses, a response to changes brought about by key legislation in several countries that was adopted following the financial crises of 2008. These OTC instruments are thus being effectively marked to market in a similar manner to the futures contracts described here. The full extent of this evolution of OTC trading through clearinghouses is not yet clear.

#### EXAMPLE 4 Futures Pricing and Valuation

1. Which of the following *best* describes how futures contract payoffs differ from forward contract payoffs?
  - A. Forward contract payoffs are larger.
  - B. They are equal, ignoring the time value of money.
  - C. Futures contract payoffs are larger if the underlying is a commodity.
2. Which of the following conditions will not make futures and forward prices equivalent?
  - A. Interest rates are constant.
  - B. Futures prices are uncorrelated with interest rates.
  - C. The volatility of the forward price is different from the volatility of the futures price.
3. With respect to the value of a futures contract, which of the following statements is *most* accurate? The value is the:
  - A. futures price minus the spot price.
  - B. present value of the expected payoff at expiration.
  - C. accumulated gain since the previous settlement, which resets to zero upon settlement.

*Solution to 1:* B is correct. Forward payoffs occur all at expiration, whereas futures payoffs occur on a day-to-day basis but would equal forward payoffs ignoring interest. Payoffs could differ, so forward payoffs are not always larger. The type of underlying is not relevant to the point of which payoff is larger.

*Solution to 2:* C is correct. Constant interest rates or the condition that futures prices are uncorrelated with interest rates will make forward and futures prices equivalent. The volatility of forward and futures prices has no relationship to any difference.

*Solution to 3:* C is correct. Value accumulates from the previous settlement and goes to zero when distributed.

### 3.3. Pricing and Valuation of Swap Contracts

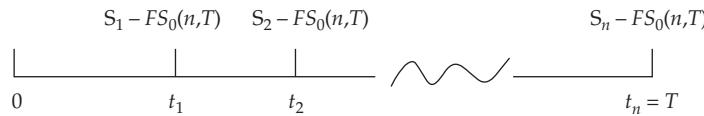
Recall the structure of a forward contract, as depicted in Exhibit 7. The investor is at time 0 and needs to determine the price,  $F_0(T)$ , that she will agree to pay at time  $T$  to purchase the asset. This price is set such that there is no value to the contract at that time. Value can arise later as prices change, but when initiated, the contract has zero value. Neither party pays anything to the other at the start.

Now consider a swap starting at time 0 and ending at time  $T$ . We will let this swap be the type that involves a fixed payment exchanged for a floating payment. The contract specifies that the two parties will make a series of  $n$  payments at times that we will designate as 1, 2, ...,  $n$ , with the last payment occurring at time  $T$ . On each of these payment dates, the owner

of the swap makes a payment of  $FS_0(n, T)$  and receives a payment based on the value of the underlying at the time of each respective payment,  $S_1, S_2, \dots, S_n$ . So from the point of view of the buyer, the sequence of cash flows from the swap is  $S_1 - FS_0(n, T), S_2 - FS_0(n, T), \dots, S_n - FS_0(n, T)$ . The notation  $FS_0(n, T)$  denotes the fixed payment established at time 0 for a swap consisting of  $n$  payments with the last payment at time  $T$ . We denote the time to each payment as  $t_1, t_2, \dots, t_n$ , where  $t_n = T$ . This structure is shown in Exhibit 9.

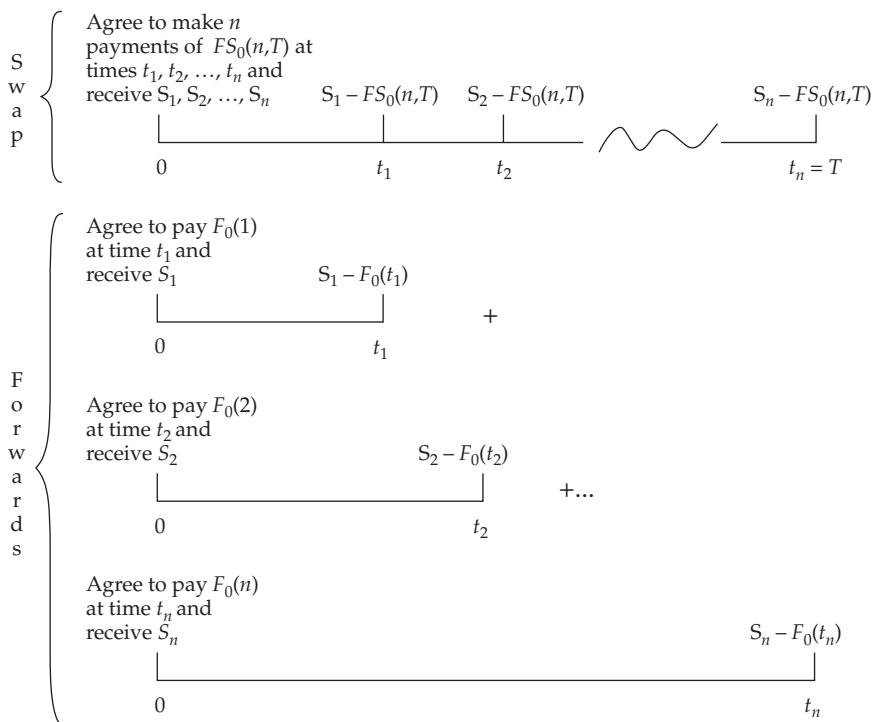
#### EXHIBIT 9 Structure of Cash Flows in a Swap

Agree to make  $n$   
payments of  
 $FS_0(n, T)$  at times  $t_1,$   
 $t_2, \dots, t_n$  and receive  
 $S_1, S_2, \dots, S_n$



Comparing Exhibit 7 with Exhibit 9 reveals some similarities. A swap is in some sense a series of forward contracts, specifically a set of contracts expiring at various times in which one party agrees to make a fixed payment and receive a variable payment. Now consider Exhibit 10, which breaks down a swap into a series of implicit forward contracts, with the expiration of each forward contract corresponding to a swap payment date.

#### EXHIBIT 10 A Swap as a Series of Forward Contracts



Recall from the material on forward contracts that the forward price is determined by the spot price and the net cost of carry (Equation 5), the latter being partially determined by the length of time of the contract. It should be obvious that a forward contract expiring at time  $t_1$  will not have the same price,  $F_0(t_1)$ , as a forward contract expiring at time  $t_2$ ,  $F_0(t_2)$ , and likewise for all of the implicit remaining forward contracts expiring up through time  $t_n$ . The cost of carrying an asset over different time periods will vary by the length of the time periods. In other words, the prices of the implicit forward contracts imbedded in a swap will not be equal:

$$F_0(t_1) \neq F_0(t_2) \neq \dots \neq F_0(t_n)$$

But for a swap, all the fixed payments are equal. So, how can we equate a swap to a series of forward contracts? It turns out that we can, and in doing so, we recall a valuable point about forward pricing.

Recall that the forward price is the price that produces a zero value of the contract at the start. Zero value is essential if there is no exchange of cash flows from one party to the other. And although no exchange of cash flows is customary, it is not mandatory. The parties could agree on any forward price at the start. If the zero-value forward price were \$30 and the parties agreed on a price of \$28, it should be apparent that the buyer would be getting a great price. The seller, being rational, would require that the buyer compensate him at the start. The seller should be getting \$30 at expiration and instead will get \$28. So the buyer should compensate the seller to the amount of the present value of \$2 at expiration. If the parties agree on a price greater than \$30, similar compensation would have to be paid from seller to buyer.

A forward transaction that starts with a nonzero value is called an off-market forward. There is generally no prohibition on the use of off-market forward contracts, so two parties can engage in a series of forward contracts at whatever fixed price they so desire. Assume they agree on the price  $FS_0(T)$ . That is, each forward contract will be created at the fixed price that corresponds to the fixed price of a swap of the same maturity with payments made at the same dates as the series of forward contracts. That means that some of the forward contracts would have positive values and some would have negative values, but their combined values would equal zero.

Now, it sounds like that price would be hard to find, but it is not. We would not, however, go about finding it by taking random guesses. Doing so would take seemingly forever. Along the way, we would notice that some of these implicit forward contracts would have positive values and some would have negative values. If the positives outweighed the negatives, then the overall swap value would be positive, which is too high. Likewise, we might plug in a number that would produce an overall negative value, with the implicit forward contract values tending to be predominantly negative, which is too low.

Not surprisingly, we can find that price easily by appealing to the principle of arbitrage. We said that the principle of arbitrage will guide us *all the way through* derivative pricing. We will omit the details, but here is the general idea.

Suppose we buy an asset that pays the amounts  $S_1, S_2, \dots, S_n$  at times  $t_1, t_2, \dots, t_n$ . These are unknown amounts. A simple example would be a floating-rate bond for which the  $S$  values represent the coupons that are unknown at the start but ultimately are determined by the evolution of interest rates. Then suppose we finance the purchase of that asset by borrowing money that we promise to repay with equal fixed payments of  $FS_0(T)$ . That strategy replicates the swap. As you have already learned, replication is the key to pricing.

Valuation of the swap during its life again appeals to replication and the principle of no arbitrage. We will find a way to reproduce the remaining payments on the swap with other transactions. The value of that strategy is the value of the swap.

To obtain the fixed rate on the swap or to value it later during its life, we will need information from the market for the underlying. As we previously noted, there are derivatives on bonds and interest rates, equities, currencies, and commodities. It is not possible to provide a general and simple statement of how to price swaps that covers all of these cases, but that topic is covered in advanced material.

### EXAMPLE 5 Swap Pricing and Valuation

1. A swap is equivalent to a series of:
  - A. forward contracts, each created at the swap price.
  - B. long forward contracts, matched with short futures contracts.
  - C. forward contracts, each created at their appropriate forward prices.
2. If the present value of the payments in a forward contract or swap is not zero, which of the following is most likely to be true?
  - A. The contract cannot legally be created.
  - B. The contract must be replicated by another contract with zero value.
  - C. The party whose stream of payments to be received is greater has to pay the other party the present value difference.

*Solution to 1:* A is correct. Each implicit forward contract is said to be off-market, because it is created at the swap price, not the appropriate forward price, which would be the price created in the forward market.

*Solution to 2:* C is correct. The party whose stream of payments to be received is greater has to pay the other party the present value difference. Such a contract can legally be created, but the party receiving the greater present value must compensate the other party with a cash payment at the start. Replication is never required.

## 4. PRICING AND VALUATION OF OPTIONS

Unlike a forward, futures, or swap contract, an option is clearly an asset to the holder and a liability to the seller. The buyer of an option pays a sum of money, called the premium, and receives the right to buy (a call) or sell (a put) the underlying. The seller receives the premium and undertakes a potential obligation because the buyer has the right, but not the obligation, to exercise the option. Options are, therefore, contingent claims. Pricing the option is the same as assigning its value. Some confusion from that terminology may still arise, in that an option could trade in the market for an amount that differs from its value.

As mentioned, there are two general types of options. Calls represent the right to buy, and puts represent the right to sell. There are also two important exercise characteristics of options. American options allow exercise at any time up to the expiration, while European options allow exercise only at expiration. It is important to understand that the terms “American” and “European” have no relationship to where the options are traded. Because the right to exercise

can be a complex feature of an option, European options are easier to understand, and we will focus on them first.

We will use the same notation used with forwards. We start by assuming that today is time 0, and the option expires at time  $T$ . The underlying is an asset currently priced at  $S_0$ , and at time  $T$ , its price is  $S_T$ . Of course, we do not know  $S_T$  until we get to the expiration. The option has an exercise or strike price of  $X$ . The symbols we use are as follows:

For calls,

$$\begin{aligned} c_0 &= \text{value (price) of European call today} \\ c_T &= \text{value (price) of European call at expiration} \\ C_0 &= \text{value (price) of American call today} \\ C_T &= \text{value (price) of American call at expiration} \end{aligned}$$

For puts,

$$\begin{aligned} p_0 &= \text{value (price) of European put today} \\ p_T &= \text{value (price) of European put at expiration} \\ P_0 &= \text{value (price) of American put today} \\ P_T &= \text{value (price) of American put at expiration} \end{aligned}$$

## 4.1. European Option Pricing

Recall that in studying forward contracts earlier in this reading, the first thing we learned is how a forward contract pays off at expiration. Then we backed up and determined how forward contracts are priced and valued prior to expiration. We follow that same approach for options.

### 4.1.1. Value of a European Option at Expiration

Recall that a European call option allows the holder to buy the underlying at expiration by paying the exercise price. Therefore, exercise is justified only if the value of the underlying exceeds the exercise price. Otherwise, the holder would simply let the call expire. So if the call is worth exercising ( $S_T > X$ ), the holder pays  $X$  and receives an asset worth  $S_T$ . Thus, the option is worth  $S_T - X$ . If the call is not worth exercising ( $S_T \leq X$ ), the option simply expires and is worth nothing at expiration.<sup>14</sup> Thus, the value of the option at expiration is the greater of either zero or the underlying price at expiration minus the exercise price, which is typically written as

$$c_T = \text{Max}(0, S_T - X) \quad (8)$$

This formula is also sometimes referred to as the **exercise value** or **intrinsic value**. In this reading, we will use the term exercise value.

Taking a simple example, if the exercise price is €40 and the underlying price is at expiration €43, the call is worth  $c_T = \text{Max}(0, €43 - €40) = \text{Max}(0, €3) = €3$ . If the underlying price at expiration is €39, the call is worth  $c_T = \text{Max}(0, €39 - €40) = \text{Max}(0, -€1) = €0$ .

<sup>14</sup>In all the remaining material, we identify conditions at expiration, such as  $S_T > X$  and  $S_T \leq X$ . Here we merged the equality case ( $S_T = X$ ) with the less-than case ( $<$ ). We could have done it the other way around ( $S_T < X$  and  $S_T \geq X$ ), which would have had no effect on our interpretations or any calculations of option value. For convenience, in some situations we will use one specification and in some the other.

For puts, the holder has the right to sell the underlying at  $X$ . If the underlying is worth less than  $X$  at expiration ( $X > S_T$ ), the put will be exercised and worth  $X - S_T$  because it allowed the holder to avoid the loss in value of the asset of that amount. If the underlying is equal to or worth more than the exercise price at expiration ( $S_T \geq X$ ), the put will simply expire with no value. So, the put is worth the greater of either zero or the exercise price minus the price of the underlying at expiration.

$$p_T = \text{Max}(0, X - S_T) \quad (9)$$

As discussed above, this formula is referred to as the exercise value or intrinsic value, and as noted, we will use the term exercise value.

Using the same example as with the call, if the underlying is €43 at expiration, the put is worth  $p_T = \text{Max}(0, €40 - €43) = \text{Max}(0, -€3) = €0$ . If the underlying is €39 at expiration, the put is worth  $p_T = \text{Max}(0, €40 - €39) = \text{Max}(€0, €1) = €1$ .

Thus, the holder of an option looks out into the future and sees these relationships as the payoff possibilities. That does not mean the holder knows what  $S_T$  will be, but the holder knows that all of the uncertainty of the option payoff is determined by the behavior of the underlying.

The results of this section can be restated as follows:

*The value of a European call at expiration is the exercise value, which is the greater of zero or the value of the underlying minus the exercise price.*

*The value of a European put at expiration is the exercise value, which is the greater of zero or the exercise price minus the value of the underlying.*

To understand option pricing, we have to work our way forward in a gradual manner. The next valuable steps involve using our intuition to identify some characteristics that will influence the value of the option. We might not be able to quantify their effects just yet, but we can rationalize why these factors affect the value of an option.

#### 4.1.2. Effect of the Value of the Underlying

The value of the underlying is obviously a critical element in determining the value of an option. It is the uncertainty of the underlying that provides the motivation for using options. It is easy to rationalize the direction of the effect of the underlying.

A call option can be viewed as a means of acquiring the underlying, whereas a put option can be viewed as a means of selling the underlying. Thus, a call option is logically worth more if the underlying is worth more, and a put option is logically worth more if the underlying is worth less.

The value of the underlying also forms one of the boundaries for calls. The value of a call option cannot exceed the value of the underlying. After all, a call option is only a means of acquiring the underlying. It can never give the holder more benefit than the underlying. Hence, the value of the underlying forms an upper boundary on what a call is worth. The underlying does not provide an upper or lower boundary for puts. That role is played by the exercise price, as we will see in the next section.

To recap what we learned here,

*The value of a European call option is directly related to the value of the underlying.*

*The value of a European put option is inversely related to the value of the underlying.*

#### 4.1.3. Effect of the Exercise Price

The exercise price is a critical factor in determining the value of an option. The exercise price is the hurdle beyond which the underlying must go to justify exercise. For a call, the underlying must rise above the exercise price, and for a put, the underlying must fall below the exercise price, to justify exercise. When the underlying is beyond the exercise price in the appropriate direction (higher for a call, lower for a put), the option is said to be **in-the-money**. When the underlying is precisely at the exercise price, the option is said to be **at-the-money**. When the underlying has not reached the exercise price (currently lower for a call, higher for a put), the option is said to be **out-of-the-money**. This characterization of whether the option is in-, at-, or out-of-the-money is referred to as the option's **moneyness**.

For a call option, a lower exercise price has two benefits. One is that there are more values of the underlying at expiration that are above the exercise price, meaning that there are more outcomes in which the call expires in-the-money. The other benefit is that assuming the call expires in-the-money, for any value of the underlying, the option value is greater the lower the exercise price. In other words, at expiration the underlying value  $S_T$  will be above the exercise price far more often, the lower is  $X$ . And if  $S_T$  is indeed higher than  $X$ , the payoff of  $S_T - X$  is greater, the lower is  $X$ .

For puts, the effect is just the opposite. To expire in-the-money, the value of the underlying must fall below the exercise price. The higher the exercise price, the better chance the underlying has of getting below it. Likewise, if the value of the underlying does fall below the exercise price, the higher the exercise price, the greater the payoff. So, if  $X$  is higher,  $S_T$  will be below it more often, and if  $S_T$  is less than  $X$ , the payoff of  $X - S_T$  is greater, the higher is  $X$  for whatever value of  $S_T$  occurs.

The exercise price also helps form an upper bound for the value of a European put. If you were holding a European put, the best outcome you could hope for is a zero value of the underlying. For equities, that would mean complete failure and dissolution of the company with shareholders receiving no final payment.<sup>15</sup> In that case, the put would pay  $X - S_T$ , but with  $S_T$  at zero, the put would pay  $X$ . If the underlying value goes to zero during the life of the European put, however, the holder cannot collect that payoff until expiration. Nonetheless, the holder would have a risk-free claim on a payoff of  $X$  at expiration. Thus, the most the put would be worth is the present value of  $X$ , meaning  $X$  discounted from expiration to the current day at the risk-free rate.<sup>16</sup> Although the holder cannot collect the payoff by exercising the option, he could sell it for the present value of  $X$ .

<sup>15</sup>You might think this point means that people who buy puts are hoping the company goes bankrupt, a seemingly morbid motivation. Yet, put buyers are often people who own the stock and buy the put for protection. This motivation is no different from owning a house and buying fire insurance. You do not want the house to burn down. If your sole motivation in buying the insurance were to make a profit on the insurance, you would want the house to burn down. This moral hazard problem illustrates why it is difficult, if not impossible, to buy insurance on a house you do not own. Likewise, executives are prohibited from owning puts on their companies' stock. Individual investors can own puts on stocks they do not own, because they cannot drive the stock price down.

<sup>16</sup>For the put holder to truly have a risk-free claim on  $X$  at expiration, given zero value of the underlying today, the underlying value must go to zero and have no possibility of any recovery. If there is any possibility of recovery, the underlying value would not go to zero, as is often observed when a legal filing for bankruptcy is undertaken. Many equities do recover. If there were some chance of recovery but the equity value was zero, demand for the stock would be infinite, which would push the price up.

To recap these results,

*The value of a European call option is inversely related to the exercise price.*

*The value of a European put option is directly related to the exercise price.*

#### 4.1.4. Effect of Time to Expiration

Logic suggests that longer-term options should be worth more than shorter-term options. That statement is usually true but not always. A call option unquestionably benefits from additional time. For example, the right to buy an asset for \$50 is worth a lot more if that right is available for two years instead of one. The additional time provides further opportunity for the underlying to rise above the exercise price. Although that means there is also additional time for the underlying to fall below the exercise price, it hardly matters to the holder of the call because the loss on the downside is limited to the premium paid.

For a European put option, the additional time still provides more opportunity for the underlying price to fall below the exercise price, but with the additional risk of it rising above the exercise price mitigated by the limited loss of the premium if the put expires out-of-the-money. Thus, it sounds as if puts benefit from longer time, but that is not necessarily true. There is a subtle penalty for this additional time. Put option holders are awaiting the sale of the underlying, for which they will receive the exercise price. The longer they have to wait, the lower the present value of the payoff. For some puts, this negative effect can dwarf the positive effect. This situation occurs with a put the longer the time to expiration, the higher the risk-free rate of interest, and the deeper it is in-the-money. The positive effect of time, however, is somewhat more dominant.

Note that we did not mention this effect for calls. For calls, the holder is waiting to pay out money at expiration. More time lowers the value of this possible outlay. Hence, a longer time period helps call option buyers in this regard.

To recap these results,

*The value of a European call option is directly related to the time to expiration.*

*The value of a European put option can be either directly or inversely related to the time to expiration. The direct effect is more common, but the inverse effect can prevail with a put the longer the time to expiration, the higher the risk-free rate, and the deeper it is in-the-money.*

#### 4.1.5. Effect of the Risk-Free Rate of Interest

We have already alluded to the effect of the risk-free rate. For call options, a longer time to expiration means that the present value of the outlay of the exercise price is lower. In other words, with a longer time to expiration, the call option holder continues to earn interest on the money that could be expended later in paying the exercise price upon exercise of the option. If the option is ultimately never exercised, this factor is irrelevant, but it remains at best a benefit and at worst has no effect. For puts, the opposite argument prevails. A longer time to expiration with a higher interest rate lowers the present value of the receipt of the exercise price upon exercise. Thus, the value today of what the put holder might receive at expiration is lower. If the put is ultimately never exercised, the risk-free rate has no effect. Thus, at best, a higher risk-free rate has no effect on the value of a put. At worst, it decreases the value of the put.

These results are summarized as follows:

*The value of a European call is directly related to the risk-free interest rate.*

*The value of a European put is inversely related to the risk-free interest rate.*

#### 4.1.6. Effect of Volatility of the Underlying

In studying the pricing of equities, we are conditioned to believe that volatility has a negative effect. After all, investors like return and dislike risk. Volatility is certainly an element of risk. Therefore, volatility is bad for investors, right? Well, partially right.

First, not all volatility is bad for investors. Unsystematic volatility should be irrelevant. Investors can hold diversified portfolios. Systematic volatility is clearly undesirable, but do not think that this means that volatility should be completely avoided where possible. If volatility were universally undesirable, no one would take risks. Clearly risks have to be taken to provide opportunity for reward.

With options, volatility of the underlying is, however, universally desirable. The greater the volatility of the underlying, the more an option is worth. This seemingly counterintuitive result is easy to understand with a little explanation.

First, let us make sure we know what volatility really means. In studying asset returns, we typically represent volatility with the standard deviation of the return, which measures the variation from the average return. The S&P 500 Index has an approximate long-run volatility of around 20%. Under the assumption of a normal distribution, a standard deviation of 20% implies that about 68% of the time, the returns will be within plus or minus one standard deviation of the average. About 95% of the time, they will be within plus or minus two standard deviations of the average. About 99% of the time, they will be within plus or minus three standard deviations of the average. When the distribution is non-normal, different interpretations apply, and in some extreme cases, the standard deviation can be nearly impossible to interpret.

Standard deviation is not the only notion of volatility, however, and it is not even needed at this point. You can proceed fairly safely with a measure as simple as the highest possible value minus the lowest, known as the range. The only requirement we need right now is that the concept of volatility reflects dispersion—how high and how low the underlying can go.

So, regardless of how we measure volatility, the following conditions will hold:

1. A call option will have a higher payoff the higher the underlying is at expiration.
2. A call option will have a zero payoff if it expires with the underlying below the exercise price.

If we could impose greater volatility on the underlying, we should be able to see that in Condition 1, the payoff has a better chance of being greater because the underlying has a greater possibility of large positive returns. In Condition 2, however, the zero payoff is unaffected if we impose greater volatility. Expiring more out-of-the-money is not worse than expiring less out-of-the-money, but expiring more in-the-money is better than expiring less-in-the-money.<sup>17</sup>

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<sup>17</sup>Think of an option expiring out-of-the-money as like it being dead. (Indeed, the option is dead.) Being “more dead” is not worse than being “less dead.”

For puts, we have

1. A put option will have a higher payoff the lower the underlying is at expiration.
2. A put option will have a zero payoff if it expires with the underlying above the exercise price.

If we could impose greater volatility, we would find that it would have a beneficial effect in (1) because a larger positive payoff would have a greater chance of occurring. In (2), the zero payoff is unaffected. The greater of the option expiring more out-of-the-money is irrelevant. Expiring more out-of-the-money is not worse than expiring less out-of-the-money.

Thus, we summarize our results in this section as

*The value of a European call is directly related to the volatility of the underlying.*

*The value of a European put is directly related to the volatility of the underlying.*

The combined effects of time and volatility give rise to the concept of the **time value** of an option. Time value of an option is not to be confused with the time value of money, which is the notion of money later being worth less than money today as a result of the combined effects of time and interest. The time value of an option reflects the value of the uncertainty that arises from the volatility of the underlying. Time value results in an option price being greater with volatility and time but declining as expiration approaches. At expiration, no time value remains and the option is worth only its exercise value. As such, an option price is said to decay over time, a process characterized as **time value decay**, which is covered in more advanced material.

#### 4.1.7. Effect of Payments on the Underlying and the Cost of Carry

We previously discussed how payments on the underlying and carrying costs enter into the determination of forward prices. They also affect option prices. Payments on the underlying refer to dividends on stocks and interest on bonds. In addition, some commodities offer a convenience yield benefit. Carrying costs include the actual physical costs of maintaining and/or storing an asset.

Let us first consider the effect of benefits. Payments of dividends and interest reduce the value of the underlying. Stocks and bonds fall in value as dividends and interest are paid. These benefits to holders of these securities do not flow to holders of options. For call option holders, this reduction is a negative factor. The price of the underlying is hurt by such payments, and call holders do not get to collect these payments. For put holders, the effect is the opposite. When the value of the underlying is reduced, put holders are helped.

Carrying costs have the opposite effect. They raise the effective cost of holding or shorting the asset. Holding call options enables an investor to participate in movements of the underlying without incurring these costs. Holding put options makes it more expensive to participate in movements in the underlying than by short selling because short sellers benefit from carrying costs, which are borne by owners of the asset.

To summarize the results from this section,

*A European call option is worth less the more benefits that are paid by the underlying and worth more the more costs that are incurred in holding the underlying.*

*A European put option is worth more the more benefits that are paid by the underlying and worth less the more costs that are incurred in holding the underlying.*

#### 4.1.8. Lowest Prices of Calls and Puts

What we have learned so far forms a framework for understanding how European options are priced. Let us now go a step further and establish a minimum price for these options.

First, we need to look at a call option as similar to the purchase of the underlying with a portion of the purchase price financed by borrowing. If the underlying is a stock, this transaction is usually called a margin transaction. Assume that the underlying is worth  $S_0$ . Also assume that you borrow cash in the amount of the present value of  $X$ , promising to pay  $X$  back  $T$  periods later at an interest rate of  $r$ . Thus,  $X/(1+r)^T$  is the amount borrowed, and  $X$  is the amount to be paid back. Now move forward to time  $T$  and observe the price of the underlying,  $S_T$ . Upon paying back the loan, the overall strategy will be worth  $S_T - X$ , which can be positive or negative.

Next, consider an alternative strategy of buying a call option expiring at  $T$  with an exercise price of  $X$ , the same value as the face value of the loan. We know that the option payoffs will be  $S_T - X$  if it expires in-the-money ( $S_T > X$ ) and zero if not ( $S_T \leq X$ ). Exhibit 11 compares these two strategies.<sup>18</sup>

EXHIBIT 11 Call Option vs. Leveraged (Margin) Transaction

	Outcome at $T$	
	Call Expires Out-of-the-Money ( $S_T \leq X$ )	Call Expires In-the-Money ( $S_T > X$ )
Call	0	$S_T - X$
<i>Leveraged transaction</i>		
Asset	$S_T$	$S_T$
Loan	$-X$	$-X$
Total	$S_T - X$	$S_T - X$

When the call expires in-the-money, both transactions produce identical payoffs. When the call expires out-of-the-money, the call value is zero, but the leveraged transaction is almost surely a loss. Its value  $S_T - X$  is negative or zero at best (if  $S_T$  is exactly equal to  $X$ ).

If two strategies are found to produce equivalent results in some outcomes but one produces a better result in all other outcomes, then one strategy dominates the former. Here we see that the call strategy dominates the leveraged strategy. Any strategy that dominates the other can never have a lower value at any time. Why would anyone pay more for one strategy than for another if the former will never produce a better result than the latter? Thus, the value of the call strategy,  $c_0$ , has to be worth at least the value of the leveraged transaction,  $S_0$  (the value of the asset), minus  $X/(1+r)^T$  (the value of the loan). Hence,  $c_0 \geq S_0 - X/(1+r)^T$ .

The inequality means that this statement provides the lowest price of the call, but there is one more thing we need to do. It can easily be true that  $X/(1+r)^T > S_0$ . In that case, we are saying that the lowest value is a negative number, but that statement is meaningless. A call can

<sup>18</sup>Note in Exhibit 11, and in others to come, that the inequality  $\leq$  is referred to as out-of-the-money. The case of equality is technically referred to as at-the-money but the verbiage is simplified if we continue to call it out-of-the-money. It is certainly not in-the-money and at-the-money is arguably the same as out-of-the-money. Regardless of one's preference, the equality case can be attached to either of the two outcomes with no effect on our conclusions.

never be worth less than zero, because its holder cannot be forced to exercise it. Thus, we tend to express this relationship as

$$c_0 \geq \text{Max}\left[0, S_0 - X/(1+r)^T\right] \quad (10)$$

which represents the greater of the value of zero or the underlying price minus the present value of the exercise price. This value becomes the lower limit of the call price.

Now consider an analogous result for puts. Suppose we want to profit from a declining price of the underlying. One way to do this is to sell the underlying short. Suppose we do that and invest cash equal to the present value of  $X$  into risk-free bonds that pay  $X$  at time  $T$ . At time  $T$ , given a price of the underlying of  $S_T$ , the short sale pays off  $-S_T$ , a reflection of the payment of  $S_T$  to cover the short sale. The bonds pay  $X$ . Hence, the total payoff is  $X - S_T$ .

Now, compare that result with the purchase of a put expiring at  $T$  with exercise price of  $X$ . If the put expires in-the-money ( $S_T < X$ ), it is worth  $X - S_T$ . If it expires out-of-the-money ( $S_T \geq X$ ), it is worth zero. Exhibit 12 illustrates the comparison of the put with the short sale and bond strategy. We see that for the in-the-money case, the put and short sale and bond strategies match each other. For the out-of-the-money case, however, the put performs better because the short sale and bond strategy pays  $X - S_T$ . With  $S_T \geq X$ , this payment amount is negative. With the put dominating the short sale and bond strategy, the put value cannot be less than the value of the short sale and bond strategy, meaning  $p_0 \geq X/(1+r)^T - S_0$ . But as with calls, the right-hand side can be negative, and it hardly helps us to say that a put must sell for more than a negative number. A put can never be worth less than zero, because its owner cannot be forced to exercise it. Thus, the overall result is expressed succinctly as

$$p_0 \geq \text{Max}\left[0, X/(1+r)^T - S_0\right] \quad (11)$$

EXHIBIT 12 Put vs. Short Sale and Bond Purchase

Outcome at $T$		
Put Expires	Put Expires	
In-the-Money	Out-of-the-Money	
( $S_T < X$ )	( $S_T \geq X$ )	
Put	$X - S_T$	0
<i>Short sale and bond purchase</i>		
Short sale	$-S_T$	$-S_T$
Bond	$X$	$X$
Total	$X - S_T$	$X - S_T$

Let us look at some basic examples. Assume the exercise price is €60, the risk-free rate is 4%, and the expiration is nine months, so  $T = 9/12 = 0.75$ . Consider two cases:

Underlying:  $S_0 = €70$

$$\text{Minimum call price} = \text{Max}[0, €70 - €60/(1.04)^{0.75}] = \text{Max}(0, €11.74) = €11.74$$

$$\text{Minimum put price} = \text{Max}[0, €60/(1.04)^{0.75} - €70] = \text{Max}(0, -€11.74) = €0.00$$

Underlying:  $S_0 = €50$

$$\text{Minimum call price} = \text{Max}[0, €50 - €60/(1.04)^{0.75}] = \text{Max}(0, -€8.26) = €0.00$$

$$\text{Minimum put price} = \text{Max}[0, €60/(1.04)^{0.75} - €50] = \text{Max}(0, €8.26) = €8.26$$

To recap, in this section we have established lower limits for call and put option values. Formally restating these results in words,

*The lowest value of a European call is the greater of zero or the value of the underlying minus the present value of the exercise price.*

*The lowest value of a European put is the greater of zero or the present value of the exercise price minus the value of the underlying.*

### EXAMPLE 6 Basic Principles of European Option Pricing

1. Which of the following factors does *not* affect the value of a European option?
  - A. The volatility of the underlying
  - B. Dividends or interest paid by the underlying
  - C. The percentage of the investor's assets invested in the option
2. Which of the following statements imply that a European call on a stock is worth more?
  - A. Less time to expiration
  - B. A higher stock price relative to the exercise price
  - C. Larger dividends paid by the stock during the life of the option
3. Why might a European put be worth less the longer the time to expiration?
  - A. The cost of waiting to receive the exercise price is higher.
  - B. The risk of the underlying is lower over a longer period of time.
  - C. The longer time to expiration means that the put is more likely to expire out-of-the-money.
4. The loss in value of an option as it moves closer to expiration is called what?
  - A. Time value decay
  - B. Volatility diminution
  - C. Time value of money
5. How does the minimum value of a call or put option differ from its exercise value?
  - A. The exercise price is adjusted for the time value of money.
  - B. The minimum value reflects the volatility of the underlying.
  - C. The underlying price is adjusted for the time value of money.

*Solution to 1:* C is correct. The investor's exposure to the option is not relevant to the price one should pay to buy or ask to sell the option. Volatility and dividends or interest paid by the underlying are highly relevant to the value of the option.

*Solution to 2:* B is correct. The higher the stock price and the lower the exercise price, the more valuable is the call. Less time to expiration and larger dividends reduce the value of the call.

*Solution to 3:* A is correct. Although the longer time benefits the holder of the option, it also has a cost in that exercise of a longer-term put comes much later. Therefore, the receipt of the exercise price is delayed. Longer time to expiration does not lower the risk of the underlying. The longer time also does not increase the likelihood of the option expiring out-of-the-money.

*Solution to 4:* A is correct. An option has time value that decays as the expiration approaches. There is no such concept as volatility diminution. Time value of money relates only to the value of money at one point in time versus another.

*Solution to 5:* A is correct. The minimum value formula is the greater of zero or the difference between the underlying price and the present value of the exercise price, whereas the exercise value is the maximum of zero and the appropriate difference between the underlying price and the exercise price. Volatility does not affect the minimum price. It does not make sense to adjust the underlying price for the time value of money for the simple reason that it is already adjusted for the time value of money.

#### 4.1.9. Put–Call Parity

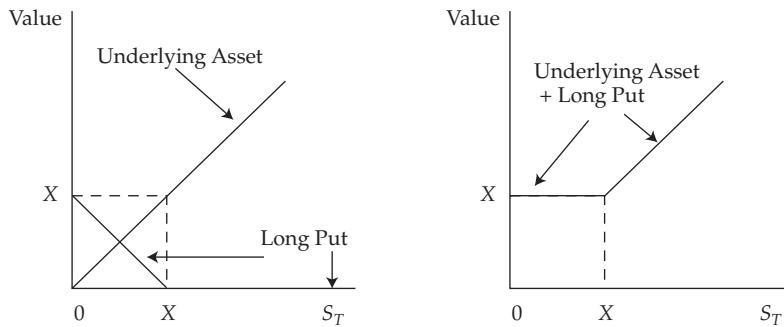
One of the first concepts that a trader learns in options is the parity relationship between puts and calls. Even though the word “parity” means “equivalence,” puts and calls are not equivalent. There is, however, a relationship between the call price and the price of its corresponding put, which we refer to as put–call parity.

Suppose Investor A owns an asset that has a current price of  $S_0$ . Assume the asset makes no cash payments and has no carrying costs. The end of the holding period is time  $T$ , at which point the asset will be worth  $S_T$ . Fearing the possibility that  $S_T$  will decline, Investor A buys a put option with an exercise price of  $X$ , which can be used to sell the asset for  $X$  at time  $T$ . This put option has a premium of  $p_0$ . Combined with the value of the asset, the investor’s current position is worth  $S_0 + p_0$ , which is the investor’s money at risk. This strategy of holding the asset and a put is sometimes called a **protective put**.

At expiration, the value of the asset is  $S_T$ . The value of the put will be either zero or  $X - S_T$ . If the asset increases in value such that  $S_T \geq X$ , then the overall position is worth  $S_T$ . The asset has performed well, and the investor will let the put expire. If the asset value declines to the point at which  $S_T < X$ , the asset is worth  $S_T$ , and the put is worth  $X - S_T$ , for a total of  $X$ . In other words, the investor would exercise the put, selling the asset for  $X$ , which exceeds the asset’s current value of  $S_T$ .

This strategy seems like a reasonable and possibly quite attractive investment. Investor A receives the benefit of unlimited upside potential, with the downside performance truncated at  $X$ . Exhibit 13 shows the performance of the protective put. The graph on the left illustrates the underlying asset and the put. The graph on the right shows their combined effects.

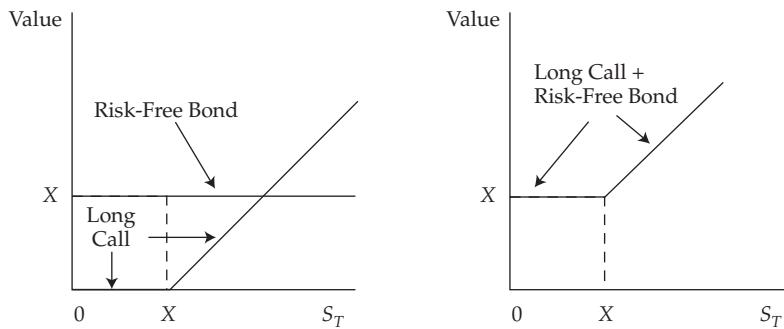
EXHIBIT 13 Protective Put (Asset Plus Long Put)



Consider Investor B, an options trader. At time 0, this investor buys a call option on this asset with an exercise price of  $X$  that expires at  $T$  and a risk-free zero-coupon bond with a face value of  $X$  that matures at  $T$ . The call costs  $c_0$ , and the bond costs the present value of  $X$ , which is  $X/(1+r)^T$ . Thus, Investor B has invested funds of  $c_0 + X/(1+r)^T$ . This strategy is sometimes known as a **fiduciary call**. If the underlying price exceeds the exercise price at expiration, the call will be worth  $S_T - X$ , and the bond will mature and pay a value of  $X$ . These values combine to equal  $S_T$ . If the underlying price does not exceed the exercise price at expiration, the call expires worthless and the bond is worth  $X$  for a combined value of  $X$ .

Exhibit 14 shows the performance of the fiduciary call. The graph on the left shows the call and bond, and the graph on the right shows the combined effects of the two strategies.

EXHIBIT 14 Fiduciary Call (Long Call Plus Risk-Free Bond)



Comparing Exhibit 13 with Exhibit 14 shows that a protective put and a fiduciary call produce the same result. Exhibit 15 shows this result more directly by identifying the payoffs in the various outcomes. Recall that Investor A committed funds of  $S_0 + p_0$ , while Investor B committed funds of  $c_0 + X/(1+r)^T$ . If both investors receive the same payoffs at time  $T$  regardless of the asset price at  $T$ , the amounts they invest at time 0 have to be the same. Thus, we require

$$S_0 + p_0 = c_0 + X/(1+r)^T \quad (12)$$

This relationship is known as **put-call parity**.

## EXHIBIT 15 Protective Put vs. Fiduciary Call

	Outcome at $T$	
	Put Expires In-the-Money ( $S_T < X$ )	Call Expires In-the-Money ( $S_T \geq X$ )
<i>Protective put</i>		
Asset	$S_T$	$S_T$
Long put	$X - S_T$	0
Total	$X$	$S_T$
<i>Fiduciary call</i>		
Long call	0	$S_T - X$
Risk-free bond	$X$	$X$
Total	$X$	$S_T$

For a simple example, assume call and put options with an exercise price of ¥100,000 in which the underlying is at ¥90,000 at time 0. The risk-free rate is 2% and the options expire in two months, so  $T = 2/12 = 0.167$ . To completely fill in the puzzle, we would need to know the put or call price, from which we could obtain the other. For now, let us write this relationship as

$$p_0 - c_0 = X/(1+r)^T - S_0$$

The right side would be  $¥100,000/(1.02)^{0.167} - ¥90,000 = ¥9,670$ . Thus, the put price should exceed the call price by ¥9,670. Thus, if the call were priced at ¥5,000, the put price would be ¥14,670. If we knew the put price, we could obtain the call price. Put–call parity does not tell us which price is correct, and it requires knowledge of one price to get the other. Alternatively, it can tell us the difference in the put and call prices.

Put–call parity must hold, at least within transaction costs, or arbitrage opportunities would arise. For example, suppose Investor C observes market prices and finds that the left-hand side of put–call parity,  $S_0 + p_0$ , is less than the right-hand side,  $c_0 + X/(1+r)^T$ . Thus, the put and the stock cost less than the call and the bond. Knowing that there should be equality (parity), Investor C executes an arbitrage transaction, selling the overpriced transactions (the call and the bond) and buying the underpriced transactions (the asset and the put).<sup>19</sup> By selling the higher priced side and buying the lower priced side, Investor C will take in more money than she will pay out, a net inflow of  $c_0 + X/(1+r)^T - (S_0 + p_0)$ . At expiration, the long put and long asset will offset the short call and bond, as shown in Exhibit 16.

<sup>19</sup> Selling the bond is equivalent to borrowing, meaning to issue a loan.

## EXHIBIT 16 Put–Call Parity Arbitrage

Transaction	Cash Flow at Time 0	Outcome at $T$	
		Put Expires In-the-Money ( $S_T < X$ )	Call Expires In-the-Money ( $S_T \geq X$ )
Buy asset	$-S_0$	$S_T$	$S_T$
Buy put	$-p_0$	$X - S_T$	0
Sell call	$+c_0$	0	$-(S_T - X)$
Borrow	$+X/(1+r)^T$	$-X$	$-X$
Total	$-S_0 - p_0 + c_0 + X/(1+r)^T > 0$	0	0

In simple terms, if  $S_T < X$ , the short call expires out-of-the-money and the put is exercised to sell the asset for  $X$ . This cash,  $X$ , is then used to pay off the loan. The net effect is that no money flows in or out at  $T$ . If  $S_T \geq X$ , the put expires out-of-the money, and the short call is exercised, meaning that Investor C must sell the asset for  $X$ . This cash,  $X$ , is then used to pay off the loan. Again, no money flows in or out. The net effect is a perfect hedge in which no money is paid out or received at  $T$ . But there was money taken in at time 0. Taking in money today and never having to pay it out is an arbitrage profit. Arbitrage opportunities like this, however, will be noticed by many investors who will engage in the same transactions. Prices will adjust until parity is restored, whereby  $S_0 + p_0 = c_0 + X/(1+r)^T$ .

Put–call parity provides tremendous insights into option pricing. Recall that we proved that going long the asset and long a put is equivalent to going long a call and long a risk-free bond. We can rearrange the put–call parity equation in the following ways:

$$\begin{aligned}
 S_0 + p_0 &= c_0 + X/(1+r)^T \\
 \Rightarrow \\
 p_0 &= c_0 - S_0 + X/(1+r)^T \\
 c_0 &= p_0 + S_0 - X/(1+r)^T \\
 S_0 &= c_0 - p_0 + X/(1+r)^T \\
 X/(1+r)^T &= S_0 + p_0 - c_0
 \end{aligned}$$

By using the symbols and the signs in these versions of put–call parity, we can see several important interpretations. In the equations below, plus signs mean long and minus signs mean short:

$$\begin{aligned}
 p_0 = c_0 - S_0 + X/(1+r)^T &\Rightarrow \text{long put} = \text{long call, short asset, long bond} \\
 c_0 = p_0 + S_0 - X/(1+r)^T &\Rightarrow \text{long call} = \text{long put, long asset, short bond} \\
 S_0 = c_0 - p_0 + X/(1+r)^T &\Rightarrow \text{long asset} = \text{long call, short put, long bond} \\
 X/(1+r)^T = S_0 + p_0 - c_0 &\Rightarrow \text{long bond} = \text{long asset, long put, short call}
 \end{aligned}$$

You should be able to convince yourself of any of these points by constructing a table similar to Exhibit 15.<sup>20</sup>

#### 4.1.10. Put–Call–Forward Parity

Recall that we demonstrated that one could create a risk-free position by going long the asset and selling a forward contract.<sup>21</sup> It follows that one can synthetically create a position in the asset by going long a forward contract and long a risk-free bond. Recall our put–call parity discussion and assume that Investor A creates his protective put in a slightly different manner. Instead of buying the asset, he buys a forward contract and a risk-free bond in which the face value is the forward price. Exhibit 17 shows that this strategy is a synthetic protective put. Because we showed that the fiduciary call is equivalent to the protective put, a fiduciary call has to be equivalent to a protective put with a forward contract. Exhibit 18 demonstrates this point.

EXHIBIT 17 Protective Put with Forward Contract vs. Protective Put with Asset

Outcome at $T$		
	Put Expires In-the-Money ( $S_T < X$ )	Put Expires Out-of-the-Money ( $S_T \geq X$ )
<i>Protective put with asset</i>		
Asset	$S_T$	$S_T$
Long put	$X - S_T$	0
Total	$X$	$S_T$
<i>Protective put with forward contract</i>		
Risk-free bond	$F_0(T)$	$F_0(T)$
Forward contract	$S_T - F_0(T)$	$S_T - F_0(T)$
Long put	$X - S_T$	0
Total	$X$	$S_T$

<sup>20</sup>As a further exercise, you might change the signs of each term in the above and provide the appropriate interpretations.

<sup>21</sup>You might wish to review Exhibit 6.

## EXHIBIT 18 Protective Put with Forward Contract vs. Fiduciary Call

	Outcome at $T$	
	Put Expires In-the-Money ( $S_T < X$ )	Call Expires In-the-Money ( $S_T \geq X$ )
<i>Protective Put with Forward Contract</i>		
Risk-free bond	$F_0(T)$	$F_0(T)$
Forward contract	$S_T - F_0(T)$	$S_T - F_0(T)$
Long put	$X - S_T$	0
Total	$X$	$S_T$
<i>Fiduciary Call</i>		
Call	0	$S_T - X$
Risk-free bond	$X$	$X$
Total	$X$	$S_T$

It follows that the cost of the fiduciary call must equal the cost of the synthetic protective put, giving us what is referred to as **put–call–forward parity**,

$$F_0(T)/(1+r)^T + p_0 = c_0 + X/(1+r)^T \quad (13)$$

Returning to our put–call parity example, a forward contract on ¥90,000 expiring in two months with a 2% interest rate would have a price of  $¥90,000(1.02)^{0.167} = ¥90,298$ . Rearranging Equation 13, we have

$$p_0 - c_0 = [X - F_0(T)]/(1+r)^T$$

The right-hand side is  $(¥100,000 - ¥90,298)/(1.02)^{0.167} = ¥9,670$ , which is the same answer we obtained using the underlying asset rather than the forward contract. Naturally these two models give us the same answer. They are both based on the assumption that no arbitrage is possible within the spot, forward, and options markets.

So far we have learned only how to price options in relation to other options, such as a call versus a put or a call or a put versus a forward. We need a way to price options versus their underlying.

## EXAMPLE 7 Put–Call Parity

1. Which of the following statements *best* describes put–call parity?
  - A. The put price always equals the call price.
  - B. The put price equals the call price if the volatility is known.
  - C. The put price plus the underlying price equals the call price plus the present value of the exercise price.

2. From put–call parity, which of the following transactions is risk-free?
- Long asset, long put, short call
  - Long call, long put, short asset
  - Long asset, long call, short bond

*Solution to 1:* C is correct. The put and underlying make up a protective put, while the call and present value of the exercise price make up a fiduciary call. The put price equals the call price for certain combinations of interest rates, times to expiration, and option moneyness, but these are special cases. Volatility has no effect on put–call parity.

*Solution to 2:* A is correct. The combination of a long asset, long put, and short call is risk free because its payoffs produce a known cash flow of the value of the exercise price. The other two combinations do not produce risk-free positions. You should work through the payoffs of these three combinations in the form of Exhibit 12.

## 4.2. Binomial Valuation of Options

Because the option payoff is determined by the underlying, if we know the outcome of the underlying, we know the payoff of the option. That means that the price of the underlying is the only element of uncertainty. Moreover, the uncertainty is not so much the value of the underlying at expiration as it is whether the underlying is above or below the exercise price. If the underlying is above the exercise price at expiration, the payoff is  $S_T - X$  for calls and zero for puts. If the underlying is below the exercise price at expiration, the payoff is zero for calls and  $X - S_T$  for puts. In other words, the payoff of the option is straightforward and known, as soon as we know whether the option expires in- or out-of-the-money. Note that for forwards, futures, and swaps, there is no such added complexity. The payoff formula is the same regardless of whether the underlying is above or below the hurdle.

As a result of this characteristic of options, derivation of an option pricing model requires the specification of a model of a random process that describes movements in the underlying. Given the entirely different nature of the payoffs above and below the exercise price, it might seem difficult to derive the option price, even if we could model movements in the underlying. Fortunately, the process is less difficult than it first appears.

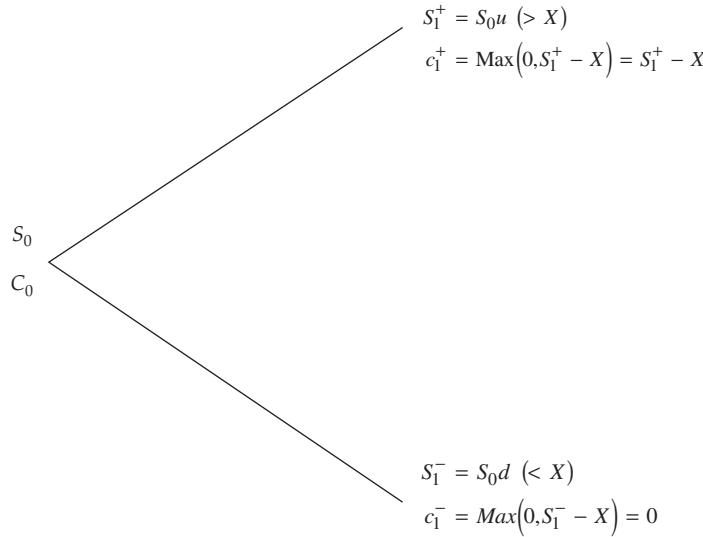
At this level of treatment, we will start with a very simple model that allows only two possible movements in the underlying—one going up and one going down from where it is now. This model with two possible outcomes is called the **binomial model**. Start with the underlying at  $S_0$ , and let it go up to  $S_1^+$  or down to  $S_1^-$ . We cannot arbitrarily set these values at just anything. We will be required to know the values of  $S_1^+$  and  $S_1^-$ . That does not mean we know which outcome will occur. It means that we know only what the possibilities are. In doing so, we effectively know the volatility. Assume the probability of the move to  $S_1^+$  is  $q$  and the probability of the move to  $S_1^-$  is  $1 - q$ . We specify the returns implied by these moves as up and down factors,  $u$  and  $d$ , where

$$u = \frac{S_1^+}{S_0}, \quad d = \frac{S_1^-}{S_0} \quad (14)$$

Now, consider a European call option that expires at time 1 and has an exercise price of  $X$ . Let the call prices be  $c_0$  today and  $c_1^+$  and  $c_1^-$  at expiration. Exhibit 19 illustrates the model.

Our objective is to determine the price of the option today, meaning to determine a formula for  $c_0$ . Knowing what we know about arbitrage and the pricing of forward contracts, it would seem we could construct a risk-free portfolio involving this option.

EXHIBIT 19 The Binomial Option Pricing Model



Because call options and the underlying move together, one possibility is that buying the underlying and selling a call could create a hedge. Indeed it does, but one unit of each is not the appropriate balance. Let us sell one call and hold  $h$  units of the underlying. The value  $h$  is unknown at the moment, but we will be able to determine its value. The value today of a combination of  $h$  units of the underlying and one short call is

$$V_0 = hS_0 - c_0$$

Think of  $V_0$  as the amount of money invested. Depending on which of the two paths is taken by the underlying, the value of this portfolio at time 1 will be

$$V_1^+ = hS_1^+ - c_1^+$$

or

$$V_1^- = hS_1^- - c_1^- \quad (15)$$

If the portfolio were hedged, then  $V_1^+$  would equal  $V_1^-$ . We can set  $V_1^+$  and  $V_1^-$  equal to each other and solve for the value of  $h$  that assures us that the portfolio is hedged:

$$\begin{aligned}
 V_1^+ &= V_1^- \\
 \Rightarrow hS_1^+ - c_1^+ &= hS_1^- - c_1^- \\
 \Rightarrow h &= \frac{c_1^+ - c_1^-}{S_1^+ - S_1^-}
 \end{aligned} \tag{16}$$

The values on the right-hand side are known, so we can easily calculate  $h$ . Thus, we can derive the number of units of the underlying that will perfectly hedge one unit of the short call.

We know that a perfectly hedged investment should earn the risk-free rate,  $r$ . Thus, the following statement must be true:

$$V_1^+ \text{ (or } V_1^-) = V_0(1+r)$$

We can substitute the value of  $V_1^+$  or  $V_1^-$  from Equation 15 into the above equation. Then we do a little algebra, which is not important to this discussion, and obtain the formula for the option price,

$$c_0 = \frac{\pi c_1^+ + (1-\pi)c_1^-}{1+r}$$

where

$$\pi = \frac{1+r-d}{u-d} \quad (17)$$

Equation 17 shows that the value of the call today is a weighted average of the next two possible call prices at expiration, where the weights,  $\pi$  and  $1 - \pi$ , are given by the second formula in Equation 17.

This formula sheds a great deal of light on option pricing. Notice the following:

- The volatility of the underlying, which is reflected in the difference between  $S_1^+$  and  $S_1^-$  and affects  $c_1^+$  and  $c_1^-$ , is an important factor in determining the value of the option.
- The probabilities of the up and down moves,  $q$  and  $1 - q$ , do not appear in the formula.<sup>22</sup>
- The values  $\pi$  and  $1 - \pi$  are similar to probabilities and are often called synthetic or pseudo probabilities. They produce a weighted average of the next two possible call values, a type of expected future value.
- The formula takes the form of an expected future value, the numerator, discounted at the risk-free rate.

On the first point, if volatility increases, the difference between  $S_1^+$  and  $S_1^-$  increases, which widens the range between  $c_1^+$  and  $c_1^-$ , leading to a higher option value. The upper payoff,  $c_1^+$ , will be larger and the lower payoff,  $c_1^-$ , will still be zero.<sup>23</sup> On the second point, the actual probabilities of the up and down moves do not matter. This result is because of our ability to construct a hedge and the rule of arbitrage. On the third point, the irrelevance of the actual probabilities is replaced by the relevance of a set of synthetic or pseudo probabilities,  $\pi$  and  $1 - \pi$ , which are called **risk-neutral probabilities**. On the fourth point, these risk-neutral probabilities are used to find a synthetic expected value, which is then discounted at the risk-free rate. Thus, the option is valued as though investors are risk neutral. As we discussed extensively earlier, that is not the same as assuming that investors are risk neutral.

If the option does not trade at the specified formula, Equation 17, investors can engage in arbitrage transactions. If the option is trading too high relative to the formula, investors can sell the call, buy  $h$  shares of the underlying, and earn a return in excess of the risk-free rate, while funding the transaction by borrowing at the risk-free rate. The combined actions

<sup>22</sup>We introduced them earlier to help make this point, but ultimately they serve no purpose.

<sup>23</sup>Although the lower payoff is zero in this example, that will not always be the case.

of arbitrageurs will result in downward pressure on the option price until it converges to the model price. If the option price is too low, buying the call, selling short  $h$  units of the asset, and investing the proceeds in risk-free bonds will generate risk-free cash that will earn more than the risk-free rate. The combined actions of arbitrageurs doing this will pressure the call price to rise until it reaches the price given by the model.

We will omit the details, but the hedge portfolio can also be constructed with puts.<sup>24</sup> Changing the  $c$ 's to  $p$ 's leads to the binomial put option pricing formula,

$$p_0 = \frac{\pi p_1^+ + (1 - \pi) p_1^-}{1 + r} \quad (18)$$

with the risk-neutral probability  $\pi$  determined by the same formula as for calls, as shown in Equation 17.

Let us construct a simple example. Let  $S_0$  be £40 and the risk-free rate be 5%. The up and down factors are  $u = 1.20$  and  $d = 0.75$ . Thus, the next two possible prices of the asset are  $S_1^+ = £40(1.20) = £48$  and  $S_1^- = £40(0.75) = £30$ . Consider a call and a put that have exercise prices of £38. Then the next two possible values of the call and put are

$$\begin{aligned} c_1^+ &= \text{Max}(0, £48 - £38) = £10 \\ c_1^- &= \text{Max}(0, £30 - £38) = £0 \\ p_1^+ &= \text{Max}(0, £38 - £48) = £0 \\ p_1^- &= \text{Max}(0, £38 - £30) = £8 \end{aligned}$$

Next we compute the risk-neutral probability,

$$\pi = \frac{1 + 0.05 - 0.75}{1.20 - 0.75} = 0.667$$

The values of the call and put are

$$c_0 = \frac{0.667(£10) + (1 - 0.667)£0}{1.05} = £6.35$$

and

$$p_0 = \frac{0.667(£0) + (1 - 0.667)£8}{1.05} = £2.54$$

The binomial model, as we see it here, is extremely simple. In reality, of course, there are more than two possible next-period prices for the underlying. As it turns out, we can extend the number of periods and subdivide an option's life into an increasing number of smaller time periods. In that case, we can obtain a more accurate and realistic model for option pricing, one that is widely used in practice. Given our objective in this reading of understanding the basic ideas behind derivative pricing, the one-period model is sufficient for the time being.

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<sup>24</sup>A long position in  $h$  units of the underlying would be hedged with one long put. The formula for  $h$  is the same as the one given here for calls, with call prices in the numerator instead of put prices.

### EXAMPLE 8 Binomial Valuation of Options

1. Which of the following terms directly represents the volatility of the underlying in the binomial model?
  - A. The standard deviation of the underlying
  - B. The difference between the up and down factors
  - C. The ratio of the underlying value to the exercise price
2. Which of the following is *not* a factor in pricing a call option in the binomial model?
  - A. The risk-free rate
  - B. The exercise price
  - C. The probability that the underlying will go up
3. Which of the following *best* describes the binomial option pricing formula?
  - A. The expected payoff is discounted at the risk-free rate plus a risk premium.
  - B. The spot price is compounded at the risk-free rate minus the volatility premium.
  - C. The expected payoff based on risk-neutral probabilities is discounted at the risk-free rate.

*Solution to 1:* B is correct. The up and down factors express how high and how low the underlying can go. Standard deviation does not appear directly in the binomial model, although it is implicit. The ratio of the underlying value to the exercise price expresses the moneyness of the option.

*Solution to 2:* C is correct. The actual probabilities of the up and down moves are irrelevant to pricing options. The risk-free and exercise price are, of course, highly relevant.

*Solution to 3:* C is correct. Risk-neutral probabilities are used, and discounting is at the risk-free rate. There is no risk premium incorporated into option pricing because of the use of arbitrage.

We have now seen how to obtain the price of a European option. Let us now consider what happens if the options are American, meaning they have the right to be exercised early.

#### 4.3. American Option Pricing

First, we will use upper case letters for American call and put prices:  $C_0$  and  $P_0$ . Second, we know that American options possess every characteristic of European options and one additional trait: They can be exercised at any time prior to expiration. Early exercise cannot be required, so the right to exercise early cannot have negative value. Thus, American options cannot sell for less than European options. Thus, we can state the following:

$$\begin{aligned} C_0 &\geq c_0 \\ P_0 &\geq p_0 \end{aligned} \tag{19}$$

Given the price of the underlying at  $S_0$ , the early-exercise feature means that we can exercise the option at any time. So, we can claim the value  $\text{Max}(0, S_0 - X)$  for calls and  $\text{Max}(0, X - S_0)$  for puts. These values establish new minimum prices for American calls and puts,

$$\begin{aligned} C_0 &= \text{Max}(0, S_0 - X) \\ P_0 &= \text{Max}(0, X - S_0) \end{aligned} \quad (20)$$

For call options, we previously learned that a European call has a minimum value given by Equation 10, which is restated here:

$$c_0 \geq \text{Max}\left[0, S_0 - X / (1+r)^T\right]$$

Comparing  $\text{Max}(0, S_0 - X)$  (the minimum for American calls) with  $\text{Max}[0, S_0 - X / (1+r)^T]$  (the minimum for European calls) reveals that the latter is either the same or higher. There are some circumstances in which both minima are zero, some in which the American minimum is zero and the European minimum is positive, and some in which both are positive, in which case  $S_0 - X / (1+r)^T$  is unquestionably more than  $S_0 - X$ . Given that an American call price cannot be less than a European call price, we have to reestablish the American call minimum as  $\text{Max}[0, S_0 - X / (1+r)^T]$ .

For put options, we previously learned that a European put has a minimum value given by Equation 11, which is restated here:

$$p_0 \geq \text{Max}\left[0, X / (1+r)^T - S_0\right]$$

Comparing  $\text{Max}(0, X - S_0)$  (the minimum for American puts) with  $\text{Max}[0, X / (1+r)^T - S_0]$  (the minimum for European puts) reveals that the former is never less. In some circumstances, they are both zero. In some,  $X - S_0$  is positive and  $X / (1+r)^T - S_0$  is negative, and in some cases both are positive but  $X - S_0$  is unquestionably more than  $X / (1+r)^T - S_0$ . Thus, the American put minimum value is the exercise value, which is  $\text{Max}(0, X - S_0)$ .

So, now we have new minimum prices for American calls and puts:

$$\begin{aligned} C_0 &\geq \text{Max}\left[0, S_0 - X / (1+r)^T\right] \\ P_0 &\geq \text{Max}(0, X - S_0) \end{aligned} \quad (21)$$

Thus, in the market these options will trade for at least these values.

Let us return to the previous examples for the minimum values. The exercise price is €60, the risk-free rate is 4%, and the expiration is  $T = 0.75$ . Consider the two cases below:

Underlying:  $S_0 = €70$

- The minimum European call price was previously calculated as €11.74. The exercise value of the American call is  $\text{Max}(0, €70 - €60) = €10$ . The American call has to sell for at least as much as the European call, so the minimum price of the American call is €11.74.
- The minimum European put price was €0.00. This is also the exercise value of the American put [ $\text{Max}(0, €60 - €70) = €0.00$ ], so the minimum price of the American put is still €0.00.

Underlying:  $S_0 = €50$

- The minimum European call price was previously calculated as €0.00. The exercise value of the American call is  $\text{Max}(0, €50 - €60) = €0.00$ , so €0.00 is still the minimum price of the American call.
- The minimum European put price was previously calculated as €8.26. The exercise value of the American put is  $\text{Max}(0, €60 - €50) = €10$ . So, €10 is the minimum price of the American put.

The call result leads us to a somewhat surprising conclusion. With the exception of what happens at expiration when American and European calls are effectively the same and both worth the exercise value, an American call is always worth more in the market than exercised. That means that an American call will never be exercised early. This result is probably not intuitive.

Consider a deep in-the-money call. One might think that if the holder expected the underlying to not increase any further, exercise might be justified. Yet, we said the call would sell for more in the market than its exercise value. What is the rationale? If the investor thinks the underlying will not go up any further and thus expects no further gains from the option, why would she prefer the underlying? Would the investor be happier holding the underlying, which she believes is not expected to increase? Moreover, she would tie up more funds exercising to acquire the underlying than if she just held on to the option or, better yet, sold it to another investor.

So far, however, we have left out a possible factor that can affect early exercise. Suppose the underlying is a stock and pays dividends. When a stock goes ex-dividend, its price instantaneously falls. Although we will omit the details, an investor holding a call option may find it worthwhile to exercise the call just before the stock goes ex-dividend. The capture of the dividend, thereby avoiding the ex-dividend drop in the price of the underlying, can make early exercise worthwhile. If the underlying is a bond, coupon interest can also motivate early exercise. But if there are significant carrying costs, the motivation for early exercise is weakened. Storage costs lend a preference for owning the option over owning the underlying.

Because the minimum value of an American put exceeds the minimum value of the European put, there is a much stronger motivation for early exercise. Suppose you owned an American put on a stock that is completely bankrupt, with a zero stock price and no possibility of recovery. You can either wait until expiration and capture its exercise value of  $\text{Max}(0, X - S_T) = \text{Max}(0, X - 0) = \text{Max}(0, X) = X$ , or you can capture that value by exercising now. Obviously now is better. As it turns out, however, the underlying does not need to go all the way to zero. There is a critical point at which a put is so deep in-the-money that early exercise is justified. This rationale works differently for a call. A deep in-the-money put has a limit to its ultimate value. It can get no deeper than when the underlying goes to zero. For a call, there is no limit to its moneyness because the underlying has no upper limit to its price.

Although dividends and coupon interest encourage early exercise for calls, they discourage early exercise for puts. The loss from the decline in the price of the underlying that is avoided by exercising a call just before the decline works to the benefit of a put holder. Therefore, if a put holder were considering exercising early, he would be better off waiting until right after the dividend or interest were paid. Carrying costs on the underlying, which discourage exercise for calls, encourage exercise for puts.

At this point, we cannot determine the critical prices at which American options are best exercised early. We require more knowledge and experience with option pricing models, which is covered in more advanced material.

### EXAMPLE 9 American Option Pricing

1. With respect to American calls, which of the following statements is *most* accurate?
  - A. American calls should be exercised early if the underlying has reached its expected maximum price.
  - B. American calls should be exercised early if the underlying has a lower expected return than the risk-free rate.
  - C. American calls should be exercised early only if there is a dividend or other cash payment on the underlying.
2. The effect of dividends on a stock on early exercise of a put is to:
  - A. make early exercise less likely.
  - B. have no effect on early exercise.
  - C. make early exercise more likely.

*Solution to 1:* C is correct. Cash payments on the underlying are the only reason to exercise American calls early. Interest rates, the expected return on the underlying, and any notion of a maximum price is irrelevant. But note that a dividend does not mean that early exercise should automatically be conducted. A dividend is only a necessary condition to justify early exercise for calls.

*Solution to 2:* A is correct. Dividends drive down the stock price when the dividend is paid. Thus, all else being equal, a stock paying dividends has a built-in force that drives down the stock price. This characteristic discourages early exercise, because stock price declines are beneficial to holders of puts.

## 5. SUMMARY

This reading on derivative pricing provides a foundation for understanding how derivatives are valued and traded. Key points include the following:

- The price of the underlying asset is equal to the expected future price discounted at the risk-free rate, plus a risk premium, plus the present value of any benefits, minus the present value of any costs associated with holding the asset.
- An arbitrage opportunity occurs when two identical assets or combinations of assets sell at different prices, leading to the possibility of buying the cheaper asset and selling the more expensive asset to produce a risk-free return without investing any capital.
- In well-functioning markets, arbitrage opportunities are quickly exploited, and the resulting increased buying of underpriced assets and increased selling of overpriced assets returns prices to equivalence.
- Derivatives are priced by creating a risk-free combination of the underlying and a derivative, leading to a unique derivative price that eliminates any possibility of arbitrage.

- Derivative pricing through arbitrage precludes any need for determining risk premiums or the risk aversion of the party trading the option and is referred to as risk-neutral pricing.
- The value of a forward contract at expiration is the value of the asset minus the forward price.
- The value of a forward contract prior to expiration is the value of the asset minus the present value of the forward price.
- The forward price, established when the contract is initiated, is the price agreed to by the two parties that produces a zero value at the start.
- Costs incurred and benefits received by holding the underlying affect the forward price by raising and lowering it, respectively.
- Futures prices can differ from forward prices because of the effect of interest rates on the interim cash flows from the daily settlement.
- Swaps can be priced as an implicit series of off-market forward contracts, whereby each contract is priced the same, resulting in some contracts being positively valued and some negatively valued but with their combined value equaling zero.
- At expiration, a European call or put is worth its exercise value, which for calls is the greater of zero or the underlying price minus the exercise price and for puts is the greater of zero and the exercise price minus the underlying price.
- European calls and puts are affected by the value of the underlying, the exercise price, the risk-free rate, the time to expiration, the volatility of the underlying, and any costs incurred or benefits received while holding the underlying.
- Option values experience time value decay, which is the loss in value due to the passage of time and the approach of expiration, plus the moneyness and the volatility.
- The minimum value of a European call is the maximum of zero and the underlying price minus the present value of the exercise price.
- The minimum value of a European put is the maximum of zero and the present value of the exercise price minus the price of the underlying.
- European put and call prices are related through put–call parity, which specifies that the put price plus the price of the underlying equals the call price plus the present value of the exercise price.
- European put and call prices are related through put–call–forward parity, which shows that the put price plus the value of a risk-free bond with face value equal to the forward price equals the call price plus the value of a risk-free bond with face value equal to the exercise price.
- The values of European options can be obtained using the binomial model, which specifies two possible prices of the asset one period later and enables the construction of a risk-free hedge consisting of the option and the underlying.
- American call prices can differ from European call prices only if there are cash flows on the underlying, such as dividends or interest; these cash flows are the only reason for early exercise of a call.
- American put prices can differ from European put prices, because the right to exercise early always has value for a put, which is because of a lower limit on the value of the underlying.

## PROBLEMS

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1. An arbitrage opportunity is *least likely* to be exploited when:
  - A. one position is illiquid.
  - B. the price differential between assets is large.
  - C. the investor can execute a transaction in large volumes.
2. An arbitrageur will *most likely* execute a trade when:
  - A. transaction costs are low.
  - B. costs of short-selling are high.
  - C. prices are consistent with the law of one price.
3. An arbitrage transaction generates a net inflow of funds:
  - A. throughout the holding period.
  - B. at the end of the holding period.
  - C. at the start of the holding period.
4. The price of a forward contract:
  - A. is the amount paid at initiation.
  - B. is the amount paid at expiration.
  - C. fluctuates over the term of the contract.
5. Assume an asset pays no dividends or interest, and also assume that the asset does not yield any non-financial benefits or incur any carrying cost. At initiation, the price of a forward contract on that asset is:
  - A. lower than the value of the contract.
  - B. equal to the value of the contract.
  - C. greater than the value of the contract.
6. With respect to a forward contract, as market conditions change:
  - A. only the price fluctuates.
  - B. only the value fluctuates.
  - C. both the price and the value fluctuate.
7. The value of a forward contract at expiration is:
  - A. positive to the long party if the spot price is higher than the forward price.
  - B. negative to the short party if the forward price is higher than the spot price.
  - C. positive to the short party if the spot price is higher than the forward price.
8. At the initiation of a forward contract on an asset that neither receives benefits nor incurs carrying costs during the term of the contract, the forward price is equal to the:
  - A. spot price.
  - B. future value of the spot price.
  - C. present value of the spot price.
9. Stocks BWQ and ZER are each currently priced at \$100 per share. Over the next year, stock BWQ is expected to generate significant benefits whereas stock ZER is not expected to generate any benefits. There are no carrying costs associated with holding either stock over the next year. Compared with ZER, the one-year forward price of BWQ is *most likely*:
  - A. lower.
  - B. the same.
  - C. higher.

10. If the net cost of carry of an asset is positive, then the price of a forward contract on that asset is *most likely*:
  - A. lower than if the net cost of carry was zero.
  - B. the same as if the net cost of carry was zero.
  - C. higher than if the net cost of carry was zero.
11. If the present value of storage costs exceeds the present value of its convenience yield, then the commodity's forward price is *most likely*:
  - A. less than the spot price compounded at the risk-free rate.
  - B. the same as the spot price compounded at the risk-free rate.
  - C. higher than the spot price compounded at the risk-free rate.
12. Which of the following factors *most likely* explains why the spot price of a commodity in short supply can be greater than its forward price?
  - A. Opportunity cost
  - B. Lack of dividends
  - C. Convenience yield
13. When interest rates are constant, futures prices are *most likely*:
  - A. less than forward prices.
  - B. equal to forward prices.
  - C. greater than forward prices.
14. In contrast to a forward contract, a futures contract:
  - A. trades over-the-counter.
  - B. is initiated at a zero value.
  - C. is marked-to-market daily.
15. To the holder of a long position, it is more desirable to own a forward contract than a futures contract when interest rates and futures prices are:
  - A. negatively correlated.
  - B. uncorrelated.
  - C. positively correlated.
16. The value of a swap typically:
  - A. is non-zero at initiation.
  - B. is obtained through replication.
  - C. does not fluctuate over the life of the contract.
17. The price of a swap typically:
  - A. is zero at initiation.
  - B. fluctuates over the life of the contract.
  - C. is obtained through a process of replication.
18. The value of a swap is equal to the present value of the:
  - A. fixed payments from the swap.
  - B. net cash flow payments from the swap.
  - C. underlying at the end of the contract.
19. A European call option and a European put option are written on the same underlying, and both options have the same expiration date and exercise price. At expiration, it is possible that both options will have:
  - A. negative values.
  - B. the same value.
  - C. positive values.

20. At expiration, a European put option will be valuable if the exercise price is:
  - A. less than the underlying price.
  - B. equal to the underlying price.
  - C. greater than the underlying price.
21. The value of a European call option at expiration is the greater of zero or the:
  - A. value of the underlying.
  - B. value of the underlying minus the exercise price.
  - C. exercise price minus the value of the underlying.
22. For a European call option with two months until expiration, if the spot price is below the exercise price, the call option will *most likely* have:
  - A. zero time value.
  - B. positive time value.
  - C. positive exercise value.
23. When the price of the underlying is below the exercise price, a put option is:
  - A. in-the-money.
  - B. at-the-money.
  - C. out-of-the-money.
24. If the risk-free rate increases, the value of an in-the-money European put option will *most likely*:
  - A. decrease.
  - B. remain the same.
  - C. increase.
25. The value of a European call option is inversely related to the:
  - A. exercise price.
  - B. time to expiration.
  - C. volatility of the underlying.
26. The table below shows three European call options on the same underlying:

	Time to Expiration	Exercise Price
Option 1	3 months	\$100
Option 2	6 months	\$100
Option 3	6 months	\$105

The option with the highest value is *most likely*:

- A. Option 1.
- B. Option 2.
- C. Option 3.
27. The value of a European put option can be either directly or inversely related to the:
  - A. exercise price.
  - B. time to expiration.
  - C. volatility of the underlying.
28. Prior to expiration, the lowest value of a European put option is the greater of zero or the:
  - A. exercise price minus the value of the underlying.
  - B. present value of the exercise price minus the value of the underlying.
  - C. value of the underlying minus the present value of the exercise price.

29. A European put option on a dividend-paying stock is *most likely* to increase if there is an increase in:
  - A. carrying costs.
  - B. the risk-free rate.
  - C. dividend payments.
30. Based on put-call parity, a trader who combines a long asset, a long put, and a short call will create a synthetic:
  - A. long bond.
  - B. fiduciary call.
  - C. protective put.
31. Which of the following transactions is the equivalent of a synthetic long call position?
  - A. Long asset, long put, short call
  - B. Long asset, long put, short bond
  - C. Short asset, long call, long bond
32. Which of the following is *least likely* to be required by the binomial option pricing model?
  - A. Spot price
  - B. Two possible prices one period later
  - C. Actual probabilities of the up and down moves
33. An at-the-money American call option on a stock that pays no dividends has three months remaining until expiration. The market value of the option will *most likely* be:
  - A. less than its exercise value.
  - B. equal to its exercise value.
  - C. greater than its exercise value.
34. At expiration, American call options are worth:
  - A. less than European call options.
  - B. the same as European call options.
  - C. more than European call options.
35. Which of the following circumstances will *most likely* affect the value of an American call option relative to a European call option?
  - A. Dividends are declared
  - B. Expiration date occurs
  - C. The risk-free rate changes
36. Combining a protective put with a forward contract generates equivalent outcomes at expiration to those of a:
  - A. fiduciary call.
  - B. long call combined with a short asset.
  - C. forward contract combined with a risk-free bond.



# CHAPTER 3

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## PRICING AND VALUATION OF FORWARD COMMITMENTS

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### LEARNING OUTCOMES

*After completing this chapter, you will be able to do the following:*

- describe and compare how equity, interest rate, fixed-income, and currency forward and futures contracts are priced and valued;
- calculate and interpret the no-arbitrage value of equity, interest rate, fixed-income, and currency forward and futures contracts;
- describe and compare how interest rate, currency, and equity swaps are priced and valued;
- calculate and interpret the no-arbitrage value of interest rate, currency, and equity swaps.

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### 1. INTRODUCTION

Forward commitments cover forwards, futures, and swaps. Pricing and valuation of forward commitments will be introduced here. A forward commitment is a derivative instrument in the form of a contract that provides the ability to lock in a price or rate at which one can buy or sell the underlying instrument at some future date or exchange an agreed-upon amount of money at a series of dates. As many investments can be viewed as a portfolio of forward commitments, this material is important to the practice of investment management.

The reading is organized as follows. Section 2 introduces the principles of the no-arbitrage approach to pricing and valuation of forward commitments. Section 3 presents the pricing and valuation of forwards and futures. Subsections address the cases of equities, interest rates, fixed-income instruments, and currencies as underlyings of forward commitments. Section 4 presents the pricing and valuation of swaps, addressing interest rate, currency, and equity swaps.

## 2. PRINCIPLES OF ARBITRAGE-FREE PRICING AND VALUATION OF FORWARD COMMITMENTS

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In this section, we examine arbitrage-free pricing and valuation of forward commitments—also known as the no-arbitrage approach to pricing and valuing such instruments. We introduce some guiding principles that heavily influence the activities of arbitrageurs who are price setters in forward commitment markets.

There is a distinction between the pricing and the valuation of forward commitments. Forward commitment pricing involves determining the appropriate forward commitment price or rate when initiating the forward commitment contract. Forward commitment valuation involves determining the appropriate value of the forward commitment, typically after it has been initiated.

Our approach to pricing and valuation is based on the assumption that prices adjust to not allow arbitrage profits. Hence, the material will be covered from an arbitrageur's perspective. Key to understanding this material is to think like an arbitrageur. Specifically, like most people, the arbitrageur would rather have more money today than less. The arbitrageur abides by two fundamental rules:

Rule #1 Do not use your own money.

Rule #2 Do not take any price risk.

The arbitrageur often needs to borrow or lend money to satisfy Rule #1. If we buy the underlying, we borrow the money. If we sell the underlying, we lend the money. These transactions will synthetically create the identical cash flows to a particular forward commitment, but they will be opposite and, therefore, offsetting, which satisfies Rule #2. Note that for Rule #2, the concern is only market price risk related to the underlying and the derivatives used, as explained in detail later. Clearly, if we can generate positive cash flows today and abide by both rules, we have a great business; such is the life of an arbitrageur.

In an effort to demonstrate various pricing and valuation results based on the no-arbitrage approach, we will rely heavily on tables showing cash flows at Times 0 and T. From an arbitrage perspective, if an initial investment requires 100 euros, then we will present it as a -100 euro cash flow. Cash inflows to the arbitrageur have a positive sign, and outflows are negative.

Pricing and valuation tasks based on the no-arbitrage approach imply an inability to create a portfolio with no future liabilities and a positive cash flow today. In other words, if cash and forward markets are priced correctly with respect to each other, we cannot create such a portfolio. That is, we cannot create money today with no risk or future liability. This approach is built on the **law of one price**, which states that if two investments have the same or equivalent future cash flows regardless of what will happen in the future, then these two investments should have the same current price. Alternatively, if the law of one price is violated, someone could buy the cheaper asset and sell the more expensive, resulting in a gain at no risk and with no commitment of capital. The law of one price is built on the value additivity principle, which states that the value of a portfolio is simply the sum of the values of each instrument held in the portfolio.

Throughout this reading, the following key assumptions are made: (1) Replicating instruments are identifiable and investable, (2) market frictions are nil, (3) short selling is allowed with full use of proceeds, and (4) borrowing and lending are available at a known risk-free rate.

Analyses in this reading will rely on the **carry arbitrage model**, a no-arbitrage approach in which the underlying instrument is either bought or sold along with a forward position—

hence the term “carry.” Carry arbitrage models are also known as cost-of-carry arbitrage models or cash-and-carry arbitrage models. Typically, each type of forward commitment will result in a different model, but common elements will be observed. Carry arbitrage models are a great first approximation to explaining observed forward commitment prices in many markets.

The central theme here is that forward commitments are generally priced so as to preclude arbitrage profits. Section 3 demonstrates how to price and value equity, interest rate, fixed-income, and currency forward contracts. We also explain how these results apply to futures contracts.

### 3. PRICING AND VALUING FORWARD AND FUTURES CONTRACTS

In this section, we examine the pricing of forward and futures contracts based on the no-arbitrage approach. The resulting carry arbitrage models are based on the replication of the forward contract payoff with a position in the underlying that is financed through an external source. Although the margin requirements, mark-to-market features, and centralized clearing in futures markets result in material differences between forward and futures markets in some cases, we focus mainly on cases in which the particular carry arbitrage model can be used in both markets.

We start with a very simple setup to arrive at the primary insight that the current forward or futures price of a non-cash-paying instrument is simply equal to the price of the underlying adjusted upward for the amount that would be earned over the term of the contract by compounding the initial underlying price at the rate that incorporates costs and benefits related to the underlying instrument. Initially, we adopt a simplified approach in which we determine the forward price by compounding the underlying price at the risk-free rate. We then turn to examining the particular nuances of equity, interest rate, fixed-income, and currency forward and futures contracts. Mastery of the simple setup will make understanding the unique nuances in each market easier to comprehend. First, we examine selected introductory material.

#### 3.1. Our Notation

In the following, notations are established for forward and futures contracts that will allow us to express concisely the key pricing and valuation relationships. **Forward price or futures price** refers to the price that is negotiated between the parties in the forward or futures contract. The market value of the forward or futures contract, termed **forward value or futures value** and sometimes just value, refers to the monetary value of an existing forward or futures contract. When the forward or futures contract is established, the price is negotiated so that the value of the contract on the initiation date is zero. Subsequent to the initiation date, the value can be significantly positive or negative.

Let  $S_t$  denote the price of the underlying instrument observed at Time  $t$ , where  $t$  is the time since the initiation of the forward contract and is expressed as a fraction of years.<sup>1</sup> Consider  $T$  as the initial time to expiration, expressed as a fraction of years.  $S_0$  denotes the underlying price observed when the forward contract is initiated, and  $S_T$  denotes the underlying price observed when the forward contract expires. Also, let  $F_0(T)$  denote the forward price

<sup>1</sup>Note that  $t$  can be greater than a year—for example  $t = 1.25$ . The variable  $t$  is expressed in years, not days or months, because interest rates, dividend yields, and most financial returns are expressed as yearly rates.

established at the initiation date, 0, and expiring at date  $T$ , where  $T$  represents a period of time later. For example, suppose that on the initiation date ( $t = 0$ ) a forward contract is negotiated for which  $F_0(0.25) = €350$ . Then the forward price for the forward contract is €350, with the contract expiration  $T = 0.25$  years later. Similarly, let  $f_0(T)$  denote the futures price for a contract established at the initiation date, 0, that expires at date  $T$ . Therefore, uppercase “F” denotes the forward price, whereas lowercase “f” denotes the futures price. Similarly, we let uppercase “V” denote the forward value, whereas lowercase “v” denotes the futures value. Many concepts in this reading apply equally to pricing and valuation of both forwards and futures. When they differ, we will emphasize the distinctions.

A key observation, to which we will return in greater detail, is that as a result of the no-arbitrage approach, when the forward contract is established, the forward price is negotiated so that the market value of the forward contract on the initiation date is zero. Most forward contracts are structured this way and are referred to as **at market**. No money changes hands, meaning that the initial value is zero. The forward contract value when initiated is expressed as  $V_0(T) = v_0(T) = 0$ . Again, we assume no margin requirements. Subsequent to the initiation date, the forward value can be significantly positive or negative.

At expiration, both the forward contract and the futures contract are equivalent to a spot transaction in the underlying. In fact, forward and futures contracts negotiated at Time  $T$  for delivery at Time  $T$  are by definition equivalent to a spot transaction at Time  $T$ . This property is often called **convergence**, and it implies that at Time  $T$ , both the forward price and the futures price are equivalent to the spot price—that is,  $F_T(T) = f_T(T) = S_T$ .

Let us define  $V_t(T)$  as the forward contract value at Time  $t$  during the life of the futures contract. At expiration,  $T$ ,

The market value of a long position in a forward contract value is  $V_T(T) = S_T - F_0(T)$ .

The market value of a short position in a forward contract value is  $V_T(T) = F_0(T) - S_T$ .

Let us define  $v_t(T)$  as the futures contract value at Time  $t$  during the life of the futures contract. Note that as a result of marking to market, the value of a futures contract at expiration is simply the difference in the futures price from the previous day. Our time subscript is expressed in a fraction of a year; hence, we use  $(t-)$  to denote the fraction of the year that the previous trading day represents. At expiration,  $T$ :

The market value of a long position in a futures contract value before marking to market is  $v_t(T) = f_t(T) - f_{t-}(T)$ .

The market value of a short position in a futures contract value before marking to market is  $v_t(T) = f_{t-}(T) - f_t(T)$ .

The futures contract value after daily settlement is  $v_t(T) = 0$ .

As illustrated later, in this reading we adopt a simplified approach in which the valuation of forward and futures contracts is treated as the same, whereas the forward value and the futures value will be different because of futures contracts being marked to market and forward contracts not being marked to market.<sup>2</sup>

Exhibit 1 shows a forward contract at initiation and expiration. A long position in a forward contract will have a positive value at expiration if the underlying is above the initial

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<sup>2</sup>There are specific cases when  $f_t(T) \neq F_t(T)$ , but they are beyond the scope of this reading.

forward price, whereas a short position in a forward contract will have a positive value at expiration if the underlying is below the initial forward price.

EXHIBIT 1 Value of a Forward Contract at Initiation and Expiration

Contract Initiation	Contract Expiration
0	T
$V_0(T) = 0$	$V_T(T) = S_T - F_0(T)$ (Long) $V_T(T) = F_0(T) - S_T$ (Short)

We turn now to focus on generic forward contracts.

### 3.2. No-Arbitrage Forward Contracts

We first consider a generic forward contract, meaning that we do not specify the underlying as anything more than just an asset. As we move through this section, we will continue to address specific additional factors to bring each carry arbitrage model closer to real markets. Thus, we will develop several different carry arbitrage models, each one applicable to specific forward commitment contracts.

#### 3.2.1. Carry Arbitrage Model When There Are No Underlying Cash Flows

Carry arbitrage models receive their name from the literal interpretation of carrying the underlying over the life of the forward contract. If an arbitrageur enters a forward contract to sell an underlying instrument for delivery at Time T, then to hedge this exposure, one strategy is to buy the underlying instrument at Time 0 with borrowed funds and carry it to the forward expiration date so it can be sold under the terms of the forward contract as illustrated in Exhibit 2.

EXHIBIT 2 Cash Flows Related to Carrying the Underlying through Calendar Time

Underlying Purchased	Underlying Sold
0	T
Underlying: $-S_0$	$+S_T$
Borrow: $+S_0$	$-FV(S_0)$
Forward: $0$	$F_0(T) - S_T$
Net: $0$	$F_0(T) - FV(S_0)$

For now, we will keep the significant technical issues to a minimum. When possible, we will just use FV and PV to denote the future value and present value, respectively. We are not concerned now about compounding conventions, day count conventions, or even the appropriate risk-free interest rate proxy. We will address these complexities only when necessary.

Carry arbitrage models rest on the no-arbitrage assumptions given earlier. To understand carry arbitrage models, it is helpful to think like an arbitrageur. The arbitrageur seeks to exploit any pricing discrepancy between the futures or forward price and the underlying spot price. The arbitrageur is assumed to prefer more money compared to less money, assuming everything else is the same. We now expand on the two fundamental rules for the arbitrageur.

- Rule #1 Do not use our own money. Specifically, the arbitrageur does not use his or her own money to acquire positions but borrows to purchase the underlying. Also, the arbitrageur does not spend proceeds from short selling transactions but invests them at the risk-free interest rate.
- Rule #2 Do not take any price risk. In our discussion, the arbitrageur focuses here only on market price risk related to the underlying and the derivatives used. We do not consider other risks, such as liquidity risk and counterparty credit risk. These topics are covered in more advanced treatments.

Consider the following strategy in which an arbitrageur purchases the underlying instrument with borrowed money in the spot market at price  $S_0$  at Time 0 and later, at Time T, contemporaneously sells the underlying at a price of  $S_T$  and repays the loan. The cash flow from this strategy evaluated at Time T is the proceeds from the sales of the underlying,  $S_T$ , less  $FV_{0,T}(S_0)$  or, more simply,  $FV(S_0)$ , the price of the underlying purchased at Time 0 grossed up by the finance cost, assumed to be the risk-free interest rate. In other words, the arbitrageur borrows the money to buy the asset, so he will pay back  $FV(S_0)$  at Time T, based on the risk-free rate.

Clearly, when  $S_T$  is below  $FV(S_0)$ , this transaction will suffer a loss. Note that breakeven will occur when the underlying value at T exactly equals the future price of the underlying at 0 grossed up by the finance cost or  $S_T = FV(S_0)$ . If we assume continuous compounding ( $r_c$ ), then  $FV(S_0) = S_0 e^{r_c T}$ . If we assume annual compounding ( $r$ ), then  $FV(S_0) = S_0 (1 + r)^T$ . Note that in practice, observed interest rates are derived from market prices; it is not the other way around. Significant errors can occur if the quoted interest rate is used with the wrong compounding convention.<sup>3</sup> When possible, we just use the generic present value and future value to minimize confusion.

To help clarify, Exhibit 3 shows the cash flows from carrying the underlying, say, stock, assuming  $S_0 = 100$ ,  $r = 5\%$ ,  $T = 1$ , and  $S_T = 90$  or  $110$ .<sup>4</sup> Each step consists of transactions that generate the cash flows shown at times 0 and T. Each row of cash flows in tables such as the one below are termed “steps,” and they will involve a wide array of cash flow producing items from market transactions, bank transactions, and other events. The set of transactions is executed simultaneously in practice, not sequentially.

- Step 1 Purchase one unit of the underlying at Time 0.
- Step 2 Borrow the purchase price. Recall that cash flow is the opposite of investment. An investment of 100 implies a negative cash flow of 100—that is,  $-100$ . We assume the interest rate is quoted on an annual compounding basis and time is expressed in fractions of a year.

<sup>3</sup>For many quantitative finance tasks, it is easier to do the analysis with continuous compounding even though the underlying rate quotation conventions are based on another method.

<sup>4</sup>Note that  $S_T$  can take on any value, but in the table we present just two values, one representing an up move and one representing a down move.

## EXHIBIT 3 Cash Flows for Financed Position in the Underlying Instrument

Steps	Cash Flows at Time 0	Cash Flows at Time T
1. Purchase underlying at 0 and sell at T	$-S_0 = -100$	$+S_T = 90$ or $+S_T = 110$
2. Borrow funds at 0 and repay with interest at T	$+S_0 = 100$	$-FV(S_0) = -100(1 + 0.05)^1 = -105$
Net cash flow	0	$+S_T - FV(S_0) = 90 - 105 = -15$ or $= 110 - 105 = 5$

Because the two outcomes are not the same, the strategy at this point fails to satisfy the arbitrageur's Rule #2: Do not take any price risk. Thus, to satisfy Rule #2, consider a third transaction that allows one to lock in the value of the underlying at Time T. This result can be achieved by selling, at Time 0, a forward contract on the underlying at price  $F_0(T)$ , where the underlying will be delivered at Time T.<sup>5</sup> Recall that the value of the forward contract at expiration will simply be the difference between the underlying,  $S_T$ , and the initial forward price,  $F_0(T)$ .

As seen in Exhibit 4, we add two additional steps, again executed simultaneously:

- Step 3 Sell a forward contract. As we are seeking to determine the equilibrium forward price, we do not assume that the forward price is initially at market, meaning that the value is zero. Thus, the forward contract value at Time 0,  $V_0(T)$ , may be non-zero. We illustrate selected numerical values for clarity.
- Step 4 Borrow the arbitrage profit in order to capture it today. If the transaction leads to an arbitrage profit at the Time T expiration, you borrow against it. In other words, suppose that in setting up the transaction, you know that it will produce an arbitrage profit of €5. Then you could borrow the present value of €5 and pay it back at expiration with the arbitrage profit. In effect, you are pre-capturing your arbitrage profit by bringing it to the present so as to receive it at Time 0. The amount you borrow will be the forward price minus the future value of the spot price when compounded at the risk-free rate. As we will see shortly, if the forward contract is priced correctly, there will be no arbitrage profit and, hence, no Step 4. Note also that we exclude the case of lending, because it would occur only if you executed a strategy to capture a certain loss, which we presume no one would do.

In this exhibit, the forward price is assumed to be trading at 105.

<sup>5</sup>Note that when an arbitrageur needs to sell the underlying, it must be assumed that she does not hold it in inventory and thus must short sell it. When the transaction calls for selling a derivative instrument, such as a forward contract, it is always just selling—technically, not short selling.

**EXHIBIT 4** Cash Flows for Financed Position in the Underlying Instrument Combined with a Forward Contract

Steps	Cash Flows at Time 0	Cash Flows at Time T
1. Purchase underlying at 0 and sell at T	$-S_0 = -100$	$+S_T = 90$ or $+S_T = 110$
2. Borrow funds at 0 and repay with interest at T	$+S_0 = 100$	$-FV(S_0) = -S_0(1 + r)^T$ $= -100(1 + 0.05)^1 = -105$
3. Sell forward contract at 0 when $F_0(T) = 105$	$+V_0(T)$	$V_T(T) = F_0(T) - S_T = 105 - 90 = 15$ or $V_T(T) = F_0(T) - S_T = 105 - 110 = -5$
4. Borrow arbitrage profit	$+PV[F_0(T) - FV(S_0)]$	$-[F_0(T) - FV(S_0)]$ $= -[105 - 100(1 + 0.05)] = 0$
Net cash flow	$+V_0(T)$ $+ PV[F_0(T) - FV(S_0)]$	$+S_T - FV(S_0) + F_0(T) - S_T$ $- [F_0(T) - FV(S_0)] = 0$ (For every underlying value)

Notice that at expiration the underlying is worth 90 or 110 and the forward contract is worth either 15 or -5. The combination of the underlying and the forward value is  $90 + 15 = 105$  or  $110 - 5 = 105$ , and that 105 is precisely the amount necessary to pay off the loan. So, there is zero cash flow at expiration under any and all circumstances.

Based on the no-arbitrage approach, a portfolio offering zero cash flow in the future is expected to be valued at zero at Time 0. That is, based on Exhibit 4, the net cash flow at Time 0 can be expressed as  $V_0(T) + PV[F_0(T) - FV(S_0)] = 0$ . With this perspective, the value of a given short forward contract is, therefore,  $V_0(T) = -PV[F_0(T) - FV(S_0)]$ , which can be rearranged and denoted  $V_0(T) = S_0 - PV[F_0(T)]$ . Based on this result, we see that the no-arbitrage forward price is simply the future value of the underlying, or

$$F_0(T) = \text{Future value of underlying} = FV(S_0) \quad (1)$$

In our example,  $F_0(T) = FV(S_0) = 105$ . In fact, with annual compounding and  $T = 1$ , we have simply  $F_0(1) = S_0(1 + r)^T = 100(1 + 0.05)^1$ . The future value refers to the amount of money equal to the spot price invested at the compound risk-free interest rate during the time period. It is not to be confused with or mistaken for the mathematical expectation of the spot price at Time T.

To better understand the arbitrage mechanics, suppose we observe that  $F_0(1) = 106$ . Based on the prior information, we observe that the forward price is higher than that determined by the carry arbitrage model (recall  $F_0(T) = FV(S_0) = 105$ ). Because the model value is lower than the market forward price, we conclude that the market forward price is too high and should be sold. An arbitrage opportunity exists, and it will involve selling the forward contract at 106. Because of Rule #2—the arbitrageur should not take any market price risk—the second transaction is to purchase the underlying instrument so that gains (or losses) on the underlying will be offset by losses (or gains) on the forward contract. Finally, because of Rule #1—the arbitrageur does not use his or her own money—the third transaction involves borrowing the purchase price of the underlying security. Based on a desire by the arbitrageur to receive future arbitrage profits today, the fourth transaction involves borrowing the known terminal profits. Note that all four transactions are

done simultaneously. To summarize, the arbitrage transactions can be represented in the following four steps:

- Step 1 Sell the forward contract on the underlying.
- Step 2 Purchase the underlying.
- Step 3 Borrow the funds for the underlying purchase.
- Step 4 Borrow the arbitrage profit.<sup>6</sup>

Exhibit 5 shows the resulting cash flows from these transactions. This strategy is known as carry arbitrage because we are carrying—that is, we are long—the underlying instrument. Note that if the forward price were 106, the value of the forward contract would be 0.9524 at Time 0. In fact,  $V_0(T) = PV[F_0(T) - FV(S_0)] = (106 - 105)/(1 + 0.05) = 0.9524$ . But if the counterparty enters a long position in the forward contract at a forward price of 106, valuing it incorrectly, then the forward contract seller has the opportunity to receive the 0.9524 with no liability in the future. In Step 4, the arbitrageur borrows this amount. At Time T, the arbitrage profit of 1 will exactly offset the repayment of this loan. This opportunity represents a portfolio that will be pursued aggressively. It is a clear arbitrage opportunity.

EXHIBIT 5 Cash Flows with Forward Contract Market Price Too High Relative to Carry Arbitrage Model

Steps	Cash Flows at Time 0	Cash Flows at Time T
1. Sell forward contract on underlying at $F_0(T) = 106$	$V_0(T) = 0$	$V_T(T) = F_0(T) - S_T = 106 - 90 = 16$ or $V_T(T) = F_0(T) - S_T = 106 - 110 = -4$
2. Purchase underlying at 0 and sell at T	$-S_0 = -100$	$+S_T = 90$ or $+S_T = 110$
3. Borrow funds for underlying purchase	$+S_0 = 100$	$-FV(S_0) = -100(1 + 0.05) = -105$
4. Borrow arbitrage profit	$+PV[F_0(T) - FV(S_0)]$ $= (106 - 105)/$ $(1+0.05) = 0.9524$	$-[F_0(T) - FV(S_0)]$ $= -[106 - 100(1+0.05)] = -1$
Net cash flow	0.9524	$16 + 90 - 105 - 1$ or $-4 + 110 - 105 - 1$ $= 0$

Suppose instead we observe a lower forward price of  $F_0(T) = 104$ . Based on the prior information, we conclude that the forward price is too low when compared to the forward price determined by the carry arbitrage model. In fact, the carry arbitrage model forward price is again  $F_0(T) = FV(S_0) = 105$ . Thus, Step 1 here is to buy a forward contract, and the value at T is  $S_T - F_0(T)$ . Because of Rule #2—the arbitrageur not taking any risk—Step 2 is to sell short the underlying instrument. Because of Rule #1—the arbitrageur not using her own money, or technically here spending another entity's money—Step 3 involves lending the short sale

<sup>6</sup>Remember that you are bringing the arbitrage profit from the future, time T, to the present, time 0, by borrowing against it and paying back the loan at T with the arbitrage profit. We exclude the case of lending, because it involves an arbitrage loss and would mean that the arbitrageur invests some of his own money at time 0 and pays out its value at T to cover the arbitrage loss.

proceeds. Finally, to capture the arbitrage profit today, you borrow its present value. Again, to summarize, the arbitrage transactions involve the following four steps:

- Step 1 Buy the forward contract on the underlying.
- Step 2 Sell the underlying short.
- Step 3 Lend the short sale proceeds.
- Step 4 Borrow the arbitrage profit.

Note that this set of transactions is the exact opposite of the prior case in Exhibit 5. This strategy is known as **reverse carry arbitrage** because we are doing the opposite of carrying the underlying instrument; that is, we are short selling the underlying instrument.

Therefore, unless  $F_0(T) = FV(S_0)$ , there is an arbitrage opportunity. Notice that if  $F_0(T) > FV(S_0)$ , then the forward contract is sold and the underlying is purchased. Thus, arbitrageurs drive down the forward price and drive up the underlying price until  $F_0(T) = FV(S_0)$  and a risk-free positive cash flow today no longer exists. Further, if  $F_0(T) < FV(S_0)$ , then the forward contract is purchased and the underlying is sold short. In this case, the forward price is driven up and the underlying price is driven down. Arbitrageurs' market activities will drive forward prices to equal the future value of the underlying, bringing the law of one price into effect once again. Most importantly, if the forward contract is priced at its equilibrium price, there will be no arbitrage profit and thus no Step 4.

### EXAMPLE 1 Forward Contract Price

An Australian stock paying no dividends is trading in Australian dollars for A\$63.31, and the annual Australian interest rate is 2.75% with annual compounding.

1. Based on the current stock price and the no-arbitrage approach, which of the following values is *closest* to the equilibrium three-month forward price?
  - A. A\$63.31
  - B. A\$63.74
  - C. A\$65.05
2. If the interest rate immediately falls 50 bps to 2.25%, the three-month forward price will:
  - A. decrease.
  - B. increase.
  - C. be unchanged.

*Solution to 1:* B is correct. Based on the information given, we know  $S_0 = \text{A\$63.31}$ ,  $r = 2.75\%$  (annual compounding), and  $T = 0.25$ . Therefore,

$$F_0(T) = FV_{0,T}(S_0) = 63.31(1 + 0.0275)^{0.25} = \text{A\$63.7408}.$$

*Solution to 2:* A is correct, and we know this is true because the forward price is directly related to the interest rate. Specifically,

$$F_0(T) = FV_{0,T}(S_0) = 63.31(1 + 0.0225)^{0.25} = A\$63.6632.$$

Therefore, we see in this case a fall in interest rates resulted in a decrease in the forward price. This relationship between forward prices and interest rates will generally hold so long as the underlying is not also influenced by interest rates.

As we see here, remember that one significant implication of this arbitrage activity is that the quoted forward price does not directly reflect expectations of future underlying prices. The only factors that matter are the interest rate and time to expiration. Other factors will be included later as we make the carry arbitrage model more realistic, but we will not be including expectations of future underlying prices. So, in other words, an opinion that the underlying will increase in value, perhaps even substantially, has no bearing on the forward price.

We now turn to the task of understanding the value of an existing forward contract. There are many circumstances in which, once a forward contract has been entered, one wants to know the contract's fair value. The goal is to calculate the position's value at current market prices. It may be due to market-based accounting, in which the accounting statements need to reflect the current fair value of various instruments. Finally, it is simply important to know whether a position in a forward contract is making money or losing money.

The forward value, based on arbitrage, can best be understood by referring to Exhibit 6. Suppose the first transaction involves buying a forward contract with a price of  $F_0(T)$  at Time 0 with expiration of Time T. Now consider selling a new forward contract with price  $F_t(T)$  at Time t again with expiration of Time T. Exhibit 6 shows the potential cash flows. Remember the equivalence at expiration between the forward price, the futures price, and the underlying price, meaning  $F_T(T) = f_T(T) = S_T$ . Note that the column labeled "Value at Time t" represents the value of the forward contracts. Note that we are seeking the forward value; hence, this transaction would result in cash flows only if it is actually executed. We need not actually execute the transaction; we just need to see what it would produce if we did. This point is analogous to the fact that if holding a liquid asset, we need not sell it to determine its value; we can simply observe its market price, which gives us an estimate of the price at which we could sell it.

EXHIBIT 6 Cash Flows for the Valuation of a Long Forward Position

Steps	Cash Flow at Time 0	Value at Time t	Cash Flow at Time T
1. Buy forward contract at 0 at $F_0(T)$	0	$V_t(T)$	$V_T(0, T) = S_T - F_0(T)$
2. Sell forward contract at t at $F_t(T)$	NA	0	$V_T(t, T) = F_t(T) - S_T$
Net cash flows/Value	0	$V_t(T)$	$+F_t(T) - F_0(T)$

There are now three different points in time to consider: Time 0, Time t, and Time T. For clarity, we explicitly state the period for present value,  $PV_{t,T}()$  rather than  $PV()$ , which means the present value at point t of an amount paid in  $T - t$  years, and for future value,  $FV_{t,T}()$  rather than  $FV()$ , which means the future value in  $T - t$  years of an amount paid at point t.

Note that once the offsetting forward is entered, the net position is not subject to market risk in that the cash flow at Time T is not influenced by what happens to the spot price. The position is completely hedged. Therefore, the value observed at Time t of the original forward contract initiated at Time 0 and expiring at Time T is simply the present value of the difference in the forward prices,  $PV_{t,T}[F_t(T) - F_0(T)]$ . Based on Exhibit 6, the forward value at Time t for a long position in the forward contract entered at Time 0 is the present value of the difference in forward prices, or

$$\begin{aligned} V_t(T) &= \text{Present value of difference in forward prices} \\ &= PV_{t,T}[F_t(T) - F_0(T)] \end{aligned} \quad (2)$$

Thus, there is the old forward price, which is the price the participants agreed on when the contract was started, and now there is also the new forward price, which is the price at which any two participants would agree to deliver the underlying at the same date as in the original contract. Of course, now the spot price has changed and some time has elapsed, so the new forward price will likely not equal the old forward price. The value of the contract is simply the present value of the difference in these two prices, with the present value calculated over the remaining life of the contract.

Alternatively,  $V_t(T) = S_t - PV_{t,T}[F_0(T)]$ .<sup>7</sup> Thus, the long forward contract value can be viewed as the present value, determined using the given interest rate, of the difference in forward prices—the original one and a new one that is priced at the point of valuation. If we know the underlying price at Time t,  $S_t$ , then we can estimate the forward price,  $F_t(T) = FV_{t,T}(S_t)$ . Based on Equation 2, we then solve for the forward value. Note that the short position is simply the negative value of Equation 2.

### EXAMPLE 2 Forward Contract Value

Assume that at Time 0 we entered into a one-year forward contract with price  $F_0(T) = 105$ . Nine months later, at Time  $t = 0.75$ , the observed price of the stock is  $S_{0.75} = 110$  and the interest rate is 5%. The value of the existing forward contract expiring in three months will be *closest* to:

- A. -6.34.
- B. 6.27.
- C. 6.34.

*Solution:* B is correct. Note that, based on  $F_0(T) = 105$ ,  $S_{0.75} = 110$ ,  $r = 5\%$ , and  $T - t = 0.25$ , the three-month forward price at Time t is equal to  $F_t(T) = FV_{t,T}(S_t) = 110(1 + 0.05)^{0.25} = 111.3499$ . Therefore, we find that the value of the existing forward entered at Time 0 valued at Time t using the difference method is

$$V_t(T) = PV_{t,T}[F_t(T) - F_0(T)] = (111.3499 - 105)/(1 + 0.05)^{0.25} = 6.2729.$$

<sup>7</sup>From Equation 1 and assuming annual compounding,  $F_t(T) = S_t(1 + r)^{(T-t)}$ , so  $PV_{t,T}[F_t(T)] = PV_{t,T}[S_t(1 + r)^{(T-t)}] = S_t$ .

Now that we have the basics of forward pricing and forward valuation, we introduce some other realistic carrying costs that influence pricing and valuation.

### 3.2.2. Carry Arbitrage Model When Underlying Has Cash Flows

We have seen that forward pricing and valuation is driven by arbitrageurs seeking to exploit mispricing by either carrying or reverse carrying the underlying instrument. Carry arbitrage requires paying the interest cost, whereas reverse carry arbitrage results in receiving the interest benefit. For many instruments, there are other significant carry costs and benefits. We will now incorporate into forward pricing various costs and benefits related to the underlying instrument. For this reason, we need to introduce some notation.

Let  $\gamma$  (Greek lowercase gamma) denote the **carry benefits** (for example, dividends, foreign interest, and bond coupon payments that would arise from certain underlyings). Let  $\gamma_T = FV_{0,T}(\gamma_0)$  denote the future value of underlying carry benefits and  $\gamma_0 = PV_{0,T}(\gamma_T)$  denote the present value of underlying carry benefits. Let  $\theta$  (Greek lowercase theta) denote the **carry costs**. For financial instruments, these costs are essentially zero. For commodities, these costs include such factors as waste, storage, and insurance. Let  $\theta_T = FV_{0,T}(\theta_0)$  denote the future value of underlying costs and  $\theta_0 = PV_{0,T}(\theta_T)$  denote the present value of underlying costs. We do not cover commodities in this reading, but you should be aware of this cost. Moreover, you should note that carry costs are similar to financing costs. Holding a financial asset does not generate direct carry costs, but it does result in the opportunity cost of the interest that could be earned on the money tied up in the asset. Thus, the financing costs that come from the rate of interest and the carry costs that are common to physical assets are equivalent concepts.

The key forward pricing equation, based on these notations, can be expressed as

$$\begin{aligned} F_0(T) &= \text{Future value of underlying adjusted for carry cash flows} \\ &= FV_{0,T}(S_0 + \theta_0 - \gamma_0) \end{aligned} \quad (3)$$

Thus, the forward price is the future value of the underlying adjusted for carry cash flows. Carry costs, like the rate of interest, increase the burden of carrying the underlying instrument through time; hence, these costs are added in the forward pricing equation. Alternatively, carry benefits decrease the burden of carrying the underlying instrument through time; hence, these benefits are subtracted in the forward pricing equation.

In the following discussion, we follow the arbitrage procedure discussed previously, but now we also consider that the underlying pays some form of benefit during the life of the forward contract. Because of the types of instruments considered here, underlying benefits will be our focus. Note, however, that costs are handled in exactly the same way except there is a sign change.

The arbitrageur purchases the underlying with borrowed money at Time 0 and then sells it at Time T. Notice that any benefits from owning the underlying are placed in a risk-free investment. The risk again is that the underlying value ( $S_T$ ) will decrease between 0 and T, when the position is unwound. Note that breakeven will occur when the underlying value at T exactly equals the future value of the underlying at 0 adjusted for any benefits, or  $S_T = FV(S_0) - \gamma_T = FV(S_0 - \gamma_0)$ . Thus, based on this breakeven expression, the underlying benefits ( $\gamma$ ) have the effect of lowering the cost of carrying the underlying, and therefore, the forward price is lower.

To help clarify, we illustrate in Exhibit 7 the same example as before in which  $S_0 = 100$ ,  $r = 5\%$ ,  $T = 1$ , and  $S_T = 90$  or  $110$ . We now assume the underlying is known to distribute  $2.9277$  at Time  $t = 0.5$ :  $\gamma_t = 2.9277$ . Thus, the time until the distribution of  $2.9277$  is  $t$ , and hence,

the present value is  $\gamma_0 = 2.9277/(1 + 0.05)^{0.5} = 2.8571$ . The time between the distribution and the forward expiration is  $T - t = 0.5$ , and thus, the future value is  $\gamma_T = 2.9277(1 + 0.05)^{0.5} = 3$ .

Remember that the steps in these tables simply refer to cash flow producing events and are initiated simultaneously.

- Step 1 Purchase the underlying at Time 0, receive the dividend at Time  $t = 0.5$ , and sell the underlying at Time T.
- Step 2 Reinvest the dividend received at Time  $t = 0.5$  at the risk-free interest rate until Time T.
- Step 3 Borrow the initial cost of the underlying. The strategy again at this point fails to satisfy Rule #2 of the arbitrageur: Do not take any price risk. If the underlying falls in value, then there is price risk.
- Step 4 Sell a forward contract. This transaction addresses Rule #2. Specifically, we sell a forward contract at Time 0 and the underlying will be delivered at Time T.
- Step 5 Borrow the arbitrage profit.

EXHIBIT 7 Cash Flows for Financed Position in the Underlying with Forward

Steps	Cash Flow at Time 0	Cash Flow at Time $t$	Cash Flow at Time T
1. Purchase underlying at 0, sell at T	$-S_0 = -100$	$+\gamma_t = 2.9277$	$+S_T = 90$ or $+S_T = 110$
2. Reinvest distribution		$-\gamma_t = -2.9277$	$+\gamma_T = 2.9277(1 + 0.05)^{0.5} = 3$
3. Borrow funds	$+S_0 = 100$		$-FV(S_0) = -100(1 + 0.05)^1 = -105$
4. Sell forward contract	$V_0(T)$		$V_T(T) = F_0(T) - S_T = 102 - 90 = 12$ or $102 - 110 = -8$
5. Borrow arbitrage profit	$+PV[F_0(T) + \gamma_T - FV(S_0)]$		$-[F_0(T) + \gamma_T - FV(S_0)]$
Net cash flows	$V_0(T) + PV[F_0(T) + \gamma_T - FV(S_0)]$	0	$+S_T + \gamma_T - FV(S_0)$ $+ F_0(T) - S_T$ $- [F_0(T) + \gamma_T - FV(S_0)] = 0$

We know in equilibrium the value of the cash flow at Time 0 is zero, or  $V_0(T) + PV[F_0(T) + \gamma_T - FV(S_0)] = 0$ , and thus  $V_0(T) = -PV[F_0(T) + \gamma_T - FV(S_0)]$ . If the forward contract has zero value, then the forward price is simply the future value of the underlying less the future value of carry benefits, or

$$\begin{aligned} F_0(T) &= \text{Future value of underlying} - \text{Future value of carry benefits} \\ &= FV(S_0) - \gamma_T \end{aligned}$$

As the carry benefits increase, the forward price decreases. In short, benefits reduce the cost of carrying the asset, and that reduces the forward price. In this example, the equilibrium forward price is  $FV_{0,T}(S_0) - \gamma_T = 105 - 3 = 102$ . This is the rationale for the carry arbitrage model adjusted for underlying benefits paid, or  $F_0(T) = FV_{0,T}(S_0) - \gamma_T$ . Note that because  $\gamma_T = FV_{0,T}(\gamma_0)$ , we can also express the carry benefit adjusted model as  $F_0(T) = FV_{0,T}(S_0 - \gamma_0)$ . In words, the initial forward price is equal to the future value of the underlying minus the value of any ownership benefits at expiration. Carry benefits lower the carry burden of the arbitrageur.

In effect, because the underlying benefits reduce the burden of carrying the underlying, the forward price is lower. We see that the cost of carrying the underlying is now  $F_0(T) = 102$ , which is lower than the previous example in which  $F_0(T) = 105$ .

The forward value for a long position when the underlying has carry benefits or carry costs is found in the same way as described previously except that the new forward price, as well as the old, is adjusted to account for these benefits and costs. Specifically,

$$\begin{aligned} V_t(T) &= \text{Present value of difference in forward prices} \\ &= PV_{t,T}[F_t(T) - F_0(T)] \end{aligned} \quad (4)$$

The forward value is equal to the present value of the difference in forward prices. The benefits and costs are reflected in this valuation equation because they are incorporated in the forward price:  $F_t(T) = FV_{t,T}(S_t + \theta_t - \gamma_t)$ . Again, the forward value is simply the present value of the difference in forward prices.

Before examining equity, interest rate, fixed-income bond, and currency underlyings, we review an important technical issue related to compounding convention. Assume the underlying is a common stock quoted in euros (€) with an initial price of €100 ( $S_0 = €100$ ), the European risk-free interest rate is 5% ( $r = 0.05$ , annual compounding),  $T = 1$  year, and the known dividend payment in  $t = 0.5$  years is  $\gamma_t = €2.9277$  or in future value terms is  $\gamma_T = €3.0$ . As illustrated previously, the no-arbitrage forward price is €102, which is determined as follows:

$$\begin{aligned} F_0(T) &= FV_{0,T}(S_0 + \theta_0 - \gamma_0) \\ &= [100 + 0 - 2.9277/(1 + 0.05)^{0.5}](1 + 0.05)^1 \\ &= 105 - 3 = €102 \end{aligned}$$

Recall that  $\gamma_0$  denotes the present value of carry benefits. In this case, the carry benefits are not paid until  $t = 0.5$ ; hence, discounting is required. Thus,  $\gamma_0 = 2.9277/(1 + 0.05)^{0.5} = 2.8571$ .

Now let us consider stock indexes, such as the EURO STOXX 50 or the US Russell 3000. With stock indexes, it is difficult to account for the numerous dividend payments paid by underlying stocks that vary in timing and amount. Dividend index point is a measure of the quantity of dividends attributable to a particular index. It is a useful measure of the amount of dividends paid; a very useful number for arbitrage trading. To simplify the problem, a continuous dividend yield is often assumed. What this means is that it is assumed that dividends accrue continuously over the period in question rather than on specific discrete dates, which is not an unreasonable assumption for an index with a large number of component stocks.

Before turning to this carry arbitrage model variation, we will review continuous compounding in general, based on the previous example, because it is a perennial source of confusion. The equivalence between annual compounding and continuous compounding can be expressed as  $(1 + r)^T = e^{r_c T}$  or  $r_c = \ln[(1 + r)^T]/T = \ln(1 + r)$ .<sup>8</sup> “ln” refers to the natural log of the function. Note that in the marketplace, zero coupon bond prices or bank deposit amounts are the underlying instrument and interest rates are derived from prices. Though we often refer to these instruments in terms of quoted rates, ultimately investors are concerned with the resulting cash flows. Therefore, if the quoted interest rate is 5% based on annual compounding as shown in the previous example, then we can solve for the implied interest rate based on

<sup>8</sup>Recall that  $\ln(a^x) = x \ln(a)$ . Thus,  $\ln[(1 + r)^T]/T = \ln(1 + r)$  and time to maturity does not influence this conversion from annual to continuous rates.

continuous compounding, or  $r_c = \ln(1 + r) = \ln(1 + 0.05) = 0.0488$ , or 4.88%. In most cases, the context makes clear when the rate being used is continuous; hence, we use the subscript  $c$  only when clarity is required.

We see that compounding continuously results in a lower quoted rate. What this implies is that a cash flow compounded at 5% annually is equivalent to being compounded at 4.88% continuously. Based on the information in the previous example, the implied dividend yield can be derived. Specifically, the carry arbitrage model with continuous compounding is again the future value of the underlying adjusted for carry and can be expressed as

$$F_0(T) = S_0 e^{(r_c + \theta - \gamma)T} \text{ (Future value of the underlying adjusted for carry)}$$

Note that in this context  $r_c$ ,  $\theta$ , and  $\gamma$  are continuously compounded rates.

The carry arbitrage model can also be used when the underlying requires storage costs, needs to be insured, and suffers from spoilage. In these cases, rather than lowering the carrying burden, these costs make it more costly to carry and hence the forward price is higher.

We now apply these results to equity forward and futures contracts.

### 3.3. Equity Forward and Futures Contracts

Although we alluded to equity forward pricing and valuation in the last section, we illustrate with concrete examples the application of carry arbitrage models to equity forward and futures contracts. Remember that here we assume that forward contracts and futures contracts are priced in the same way. It is vital to treat the compounding convention of interest rates appropriately.

If the underlying is a stock, then the carry benefit is the dividend payments as illustrated in the next two examples.

#### EXAMPLE 3 Equity Futures Contract Price with Continuously Compounded Interest Rates

The continuously compounded dividend yield on the EURO STOXX 50 is 3%, and the current stock index level is 3,500. The continuously compounded annual interest rate is 0.15%. Based on the carry arbitrage model, the three-month futures price will be *closest* to:

- A. 3,473.85.
- B. 3,475.15.
- C. 3,525.03.

*Solution:* B is correct. Based on the carry arbitrage model, the forward price is  $F_0(T) = S_0 e^{(r_c - \gamma)T}$ . The future value of the underlying adjusted for carry, i.e., the dividend payments, over the next year would be  $3,500 e^{(0.0015 - 0.03)(3/12)} = 3,475.15$ .

#### EXAMPLE 4 Equity Forward Pricing and Forward Valuation with Discrete Dividends

Suppose Nestlé common stock is trading for CHF70 and pays a CHF2.20 dividend in one month. Further, assume the Swiss one-month risk-free rate is 1.0%, quoted on an annual compounding basis. Assume that the stock goes ex-dividend the same day the single stock forward contract expires. Thus, the single stock forward contract expires in one month.

1. The one-month forward price for Nestlé common stock will be *closest* to:
  - A. CHF67.80.
  - B. CHF67.86.
  - C. CHF69.94.
2. An increase in which of the following parameters would result in an increase in the forward price?
  - A. Dividends
  - B. Risk-free interest rate
  - C. Expected future stock price

*Solution to 1:* B is correct. In this case, we have  $S_0 = 70$ ,  $r = 1.0\%$ ,  $T = 1/12$ , and  $\gamma_T = 2.2$ . Therefore,  $F_0(T) = FV_{0,T}(S_0 + \theta_0 - \gamma_0) = FV_{0,T}(S_0) + FV_{0,T}(\theta_0) - FV_{0,T}(\gamma_0) = 70(1 + 0.01)^{1/12} + 0 - 2.2 = \text{CHF}67.86$ .

*Solution to 2:* B is correct. The forward price is not influenced by the expected spot price. It solely reflects carry costs and carry benefits. Being a carry benefit, the increase in dividends reduces the forward price. Thus, in the answers above, only an increase in the risk-free rate will result in an increase in the forward price.

The value of an equity forward contract entered earlier is simply the present value of the difference in the initial forward price and the current forward price as illustrated in the next example.

#### EXAMPLE 5 Equity Forward Valuation

Suppose we bought a one-year forward contract at 102 and there are now three months to expiration. The underlying is currently trading for 110, and interest rates are 5% on an annual compounding basis.

1. If there are no other carry cash flows, the forward value of the existing contract will be *closest* to:
  - A. -10.00.
  - B. 9.24.
  - C. 10.35.

2. If a dividend payment is announced between the forward's valuation and expiration dates, assuming the news announcement does not change the current underlying price, the forward value will *most likely*:
- decrease.
  - increase.
  - be the same.

Suppose that instead of buying a forward contract, we buy a one-year *futures* contract at 102 and there are now three months to expiration. Today's futures price is 112.35. There are no other carry cash flows.

- After marking to market, the futures value of the existing contract will be *closest* to:
  - 10.35.
  - 0.00.
  - 10.35.
- Compared to the value of a forward contract, the value of a futures contract is *most likely*:
  - lower.
  - higher.
  - the same.

*Solution to 1:* B is correct. For this case, we have  $F_0(T) = 102$ ,  $S_{0.75} = 110$ ,  $r = 5\%$ , and  $T - t = 0.25$ . Note that the new forward price at  $t$  is simply  $F_t(T) = FV_{t,T}(S_t) = 110(1 + 0.05)^{0.25} = 111.3499$ . Therefore, we have

$$V_t(T) = PV_{t,T}[F_t(T) - F_0(T)] = (111.3499 - 102)/(1 + 0.05)^{0.25} = 9.2365.$$

Thus, we see that the current forward value is greater than the difference between the current underlying value of 110 and the initial forward price of 102 as a result of interest costs resulting in the new forward price being 111.35.

*Solution to 2:* A is correct. The old forward price is fixed. The discounted difference in the new forward price and the old forward price is the value. If we impose a new dividend, it would lower the new forward price and thus lower the value of the old forward contract.

*Solution to 3:* B is correct. Futures contracts are marked to market daily, which implies that the market value, resulting in profits and losses, is received or paid at each daily settlement. Hence, the equity futures value is zero each day after settlement has occurred.

*Solution to 4:* A is correct. After marking to market, the futures contract value is zero because profits and losses are taken daily. Thus, because we are long the futures or forward contract and the price has risen, the futures value will be lower than the forward value.

We turn now to the widely used interest rate forward and futures contracts.

### 3.4. Interest Rate Forward and Futures Contracts

Libor, which stands for London Interbank Offered Rate, is a widely used interest rate that serves as the underlying for many derivative instruments. It represents the rate at which London banks can borrow from other London banks. When these loans are in dollars, they are known as Eurodollar time deposits, with the rate referred to as dollar Libor. There are, however, Libor rates for all major non-dollar currencies. Average Libor rates are derived and posted each day at 11:30 a.m. London time. Lenders and participants in the interest rate derivatives market use these posted Libor rates to determine the interest payments on loans and the pay-offs of various derivatives.<sup>9</sup> In addition to this London spot market, there are active forward and futures markets for derivatives based on Libor. Our focus will be on forward markets, as represented by forward rate agreements. In order to understand the forward market, however, let us first look at the Libor spot market. Assume the following notation:

$L_i(m)$  = Libor on an  $m$ -day deposit observed on day  $i$

NA = notional amount, quantity of funds initially deposited

NTD = number of total days in a year, used for interest calculations (always 360 in the Libor market)

$t_m$  = accrual period, fraction of year for  $m$ -day deposit— $t_m = m/NTD$

TA = terminal amount, quantity of funds repaid when the Libor deposit is withdrawn

For example, suppose day  $i$  is designated as Time 0, and we are considering a 90-day Eurodollar deposit ( $m = 90$ ). Dollar Libor is quoted at 2%; thus,  $L_i(m) = L_0(90) = 0.02$ . If \$50,000 is initially deposited, then NA = \$50,000. Libor is stated on an actual over 360-day count basis (often denoted ACT/360) with interest paid on an add-on basis.<sup>10</sup> Hence,  $t_m = 90/360 = 0.25$ . Accordingly, the terminal amount can be expressed as  $TA = NA[1 + L_0(m)t_m]$ , and the interest paid is thus  $TA - NA = NA[L_0(m)t_m]$ . In this example,  $TA = \$50,000[1 + 0.02(90/360)] = \$50,250$  and the interest is  $\$50,250 - \$50,000 = \$250$ .

Now let us turn to the forward market for Libor. A **forward rate agreement** (FRA) is an over-the-counter (OTC) forward contract in which the underlying is an interest rate on a deposit. An FRA involves two counterparties: the fixed receiver (short) and the floating receiver (long). Thus, being long the FRA means that you gain when Libor rises. The fixed receiver counterparty receives an interest payment based on a fixed rate and makes an interest payment based on a floating rate. The floating receiver counterparty receives an interest payment based on a floating rate and makes an interest payment based on a fixed rate. If we are the fixed receiver, then it is understood without saying that we also are the floating payer, and vice versa. Because there is no initial exchange of cash flows, to eliminate arbitrage opportunities, the FRA price is the fixed interest rate such that the FRA value is zero on the initiation date.

<sup>9</sup>In 2008, financial regulators and many market participants began to suspect that the daily quoted Libor, which was compiled by the British Bankers Association (BBA), was being manipulated by certain banks that submitted their rates to the BBA for use in determining this average. In 2014, the BBA ceded control of the daily Libor reporting process to the Intercontinental Exchange.

<sup>10</sup>The add-on basis is one way to quote interest rates and the convention in the Libor market. The idea is that the interest is added on at the end—in contrast, for example, to the discount basis, in which the current price is discounted based on the amount paid at maturity.

FRAs are identified in the form of “ $X \times Y$ ,” where  $X$  and  $Y$  are months and the multiplication symbol,  $\times$ , is read as “by.” To grasp this concept and the notion of exactly what is the underlying in an FRA, consider a  $3 \times 9$  FRA, which is pronounced “3 by 9.” The 3 indicates that the FRA expires in three months. The underlying is implied by the difference in the 3 and the 9. That is, the payoff of the FRA is determined by six-month Libor when the FRA expires in three months. The notation  $3 \times 9$  is market convention, though it can seem confusing at first. We will see shortly that the rate on the FRA will be determined by the relationship between the spot rate on a nine-month Libor deposit and the spot rate on a three-month deposit when the FRA is initiated. A short (long) FRA will effectively replicate going short (long) a nine-month Libor deposit and long (short) a three-month FRA deposit. And although market convention quotes the time periods as months, the calculations use days based on the assumption of 30 days in a month.

The contract established between the two counterparties settles in cash the difference between a fixed interest payment established on the initiation date and a floating interest payment established on the FRA expiration date. The underlying of an FRA is neither a financial asset nor even a financial instrument; it is just an interest payment. It is also important to understand that the parties to an FRA are not necessarily engaged in a Libor deposit in the spot market. The Libor spot market is simply the benchmark from which the payoff of the FRA is determined. Although a party may use an FRA in conjunction with a Libor deposit, it does not have to do so any more than a party that uses a forward or futures on a stock index has to have a position in the stock index.

In Exhibit 8, we illustrate the key time points in an FRA transaction. The FRA is created and priced at Time 0, the initiation date, and expires  $h$  days later. The underlying instrument has  $m$  days to maturity as of the FRA expiration date. Thus, the FRA is on  $m$ -day Libor. We assume there is a point during the life of the FRA, day  $g$ , at which we wish to determine the value of the FRA. So, for example, a 30-day FRA on 90-day Libor would have  $h = 30$ ,  $m = 90$ , and  $h + m = 120$ . If we wanted to value the FRA prior to expiration,  $g$  could be any day between 0 and 30. The FRA value is the market value on the evaluation date and reflects the fair value of the original position.

EXHIBIT 8 Important FRA Dates, Expressed in Days from Initiation

Initiation Date	Evaluation Date	FRA Expires	Underlying Matures
0	$g$	$h$	$h + m$

Using the notation in Exhibit 8, let  $\text{FRA}(0,h,m)$  denote the fixed forward rate set at Time 0 that expires at Time  $h$  wherein the underlying Libor deposit has  $m$  days to maturity at expiration of the FRA. Thus, the rate set at initiation of a contract expiring in 30 days in which the underlying is 90-day Libor is denoted  $\text{FRA}(0,30,90)$  and will be a number, such as 1% or 2.5%. Like all standard forward contracts, no money changes hands when an FRA is initiated, so our objective is to price the FRA, meaning to determine the fixed rate  $[\text{FRA}(0,30,90)]$  such that the value is zero on the initiation date.

When any interest rate derivative expires, there are technically two ways to settle at expiration: “advanced set, settled in arrears” and “advanced set, advanced settled.” FRAs are typically settled based on advanced set, advanced settled, whereas swaps and interest rate options are

normally based on advanced set, settled in arrears. Let us look at both approaches, because they are both used in the interest rate derivatives markets.

In the earlier example of a Libor deposit of \$50,000 for 90 days at 2%, the rate was set when the money was deposited, interest accrued over the life of the deposit, and the interest was paid and the principal of \$50,250 was repaid at maturity, 90 days later. The term **advanced set** is used because the reference interest rate is set at the time the money is deposited. The advanced set convention is almost always used, because most issuers and buyers of financial instruments want to know the rate on the instrument while they have a position in it.

In an FRA, the term “advanced” refers to the fact that the interest rate is set at Time  $h$ , the FRA expiration date, which is the time the underlying deposit starts. The term **settled in arrears** is used when the interest payment is made at Time  $h + m$ , the maturity of the underlying instrument. Thus, an FRA with advanced set, settled in arrears works the same way as a typical bank deposit as described in the previous example. At Time  $h$ , the interest rate is set, and the interest payment is made at Time  $h + m$ . Alternatively, when **advanced settled** is used, the settlement is made at Time  $h$ . Thus, in a FRA with the advanced set, advanced settled feature, the FRA expires and settles at the same time. Advanced set, advanced settled is almost always used in FRAs, though we will see advanced set, settled in arrears when we cover interest rate swaps, and it is also used in interest rate options. From this point forward in this reading, all FRAs will be advanced set, advanced settled, as they are in practice.

Mathematically, the settlement amounts for advanced set, advanced settled are determined in the following manner:

Settlement amount at  $h$  for receive-floating:

$$NA\{[L_h(m) - FRA(0, h, m)]t_m\} / [1 + D_h(m)t_m]$$

Settlement amount at  $h$  for receive-fixed:

$$NA\{[FRA(0, h, m) - L_h(m)]t_m\} / [1 + D_h(m)t_m]$$

Note the divisor,  $1 + D_h(m)t_m$ . This term is a discount factor applied to the FRA payoff. It reflects the fact that the rate on which the payoff is determined,  $L_h(m)$ , is obtained on day  $h$  from the Libor spot market, which uses settled in arrears. In the Libor spot market, this rate assumes that a Libor deposit has been made on day  $h$  at this rate with interest to be paid on day  $h + m$ —that is, settled in arrears. In the FRA market, the payment convention is advanced settle. The discount factor is, therefore, appropriately applied to the FRA payment because the payment is received in advance, not in arrears. Often it is assumed that  $D_h(m) = L_h(m)$  and we will commonly do so here, but it can be different.<sup>11</sup>

Again, it is important to not be confused by the role played by the Libor spot market in an FRA. In the spot market, Libor deposits are made by various parties that are lending to banks.

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<sup>11</sup>For example, there is a current debate on whether the overnight index swap (OIS) rate is the appropriate discount rate for financial derivatives. Because Libor and the OIS rate are different, we need the capacity to incorporate different rates for the reference rate for settlement and the discount rate for valuation. We do not seek to resolve this debate here. Historically, there have been several candidate discount rates offered, and the popularity of each rate changes over time.

These rates are used as the benchmark for determining the payoffs of FRAs. The two parties to an FRA do not necessarily engage in any Libor spot transactions. Moreover, Libor spot deposits are settled in arrears, whereas FRA payoffs are settled in advance—hence the discounting.

### EXAMPLE 6 Calculating Interest on Libor Spot and FRA Payments

In 30 days, a UK company expects to make a bank deposit of £10,000,000 for a period of 90 days at 90-day Libor set 30 days from today. The company is concerned about a possible decrease in interest rates. Its financial adviser suggests that it negotiate today, at Time 0, a  $1 \times 4$  FRA, an instrument that expires in 30 days and is based on 90-day Libor. The company enters into a £10,000,000 notional amount  $1 \times 4$  receive-fixed FRA that is advanced set, advanced settled. The appropriate discount rate for the FRA settlement cash flows is 0.40%. After 30 days, 90-day Libor in British pounds is 0.55%.

1. The interest actually paid at maturity on the UK company's bank deposit will be closest to:
  - A. £10,000.
  - B. £13,750.
  - C. £27,500.
2. If the FRA was initially priced at 0.60%, the payment received to settle it will be closest to:
  - A. -£2,448.75.
  - B. £1,248.75.
  - C. £1,250.00.
3. If the FRA was initially priced at 0.50%, the payment received to settle it will be closest to:
  - A. -£1,248.75.
  - B. £1,248.75.
  - C. £1,250.00.

*Solution to 1:* B is correct. This is a simple Libor deposit of £10,000,000 for 90 days at 0.55%. Therefore,  $TA = 10,000,000[1 + 0.0055(0.25)] = £10,013,750$ . So the interest paid at maturity is £13,750.

*Solution to 2:* B is correct. In this example,  $m = 90$  (number of days in the deposit),  $t_m = 90/360$  (fraction of year until deposit matures observed at the FRA expiration date), and  $h = 30$  (number of days initially in the FRA). The settlement amount of the  $1 \times 4$  FRA at  $h$  for receive-fixed is

$$\begin{aligned} NA\{[FRA(0,h,m) - L_h(m)]t_m\}/[1 + D_h(m)t_m] \\ = [10,000,000(0.0060 - 0.0055)(0.25)]/[1 + 0.0040(0.25)] = £1,248.75. \end{aligned}$$

Because the FRA involves paying floating, its value benefited from a decline in rates.

*Solution to 3:* A is correct. The data are similar to those in the previous question, but the initial FRA rate was 0.50% and not 0.60%. Thus, the settlement amount of the  $1 \times 4$  FRA at  $h$  for receive-fixed is

$$\begin{aligned} \text{NA}\{[\text{FRA}(0,h,m) - L_h(m)t_m]/[1 + D_h(m)t_m] \\ = [10,000,000(0.0050 - 0.0055)(0.25)]/[1 + 0.0040(0.25)] = -\text{£}1,248.75 \end{aligned}$$

The FRA suffered from a rise in rates because it is again paying floating.

With this background, we turn to FRA pricing by illustrating the appropriate  $\text{FRA}(0,h,m)$  rate that makes the value of the FRA equal to zero on the initiation date. For our purposes, we assume that borrowing and lending can be done at Libor. Also, the notional amount is assumed to be one unit of the designated currency:  $\text{NA} = 1$ . Finally, we will assume that the discount rate on the FRA settlement is the FRA rate at that point in time.

Consider the following no-arbitrage strategy, depicted in Exhibit 9, in which numerical values are also provided as an aid to understanding the concepts. We illustrate a  $3 \times 6$  FRA for which  $\text{NA} = 1$ ,  $h = 90$ ,  $m = 90$ ,  $t_h = 90/360$ ,  $L_0(h) = L_0(90) = 1.5\%$ ,  $t_{h+m} = 180/360$ ,  $L_0(h+m) = L_0(180) = 2.0\%$ , and  $t_m = 90/360$ . That is, today 90-day Libor is 1.5% and 180-day Libor is 2%. First, consider the following three arbitrage-related transactions all done at Time 0:

- Step 1 Deposit funds for  $h + m$  days: At Time 0, deposit an amount equal to  $1/[1 + L_0(h)t_h]$ , the present value of 1 maturing in  $h$  days, in a bank for  $h + m$  days at an agreed upon rate of  $L_0(h + m)$ . After  $h + m$  days, withdraw an amount equal to  $[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h]$ . Based on the data provided, the deposit amount is  $1/[1 + 0.015(90/360)] = 0.996264$ . After  $h + m$  days, the withdrawn amount is equal to  $0.996264[1 + 0.02(180/360)] = 1.006227$ . In other words, deposit 0.996264 for 180 days at 2%. One hundred eighty days later, withdraw 1.006227.
- Step 2 Borrow funds for  $h$  days: At Time 0, borrow 0.996264, corresponding to  $\{1/[1 + L_0(h)t_h]\}$ , for  $h$  days so that the net cash flow at Time 0 is zero. In  $h$  days, this borrowing will be worth 1. In other words, borrow 0.996264 for 90 days at 1.5%. In 90 days, pay back 1.
- Step 3 At Time  $h$ , roll over the maturing loan in Step 2 by borrowing funds for  $m$  days at the rate  $L_h(m)$ . Assume rates rise and  $L_h(m) = 3.0\%$ . Then at the end of  $m$  days, we will owe  $[1 + L_h(m)t_m] = [1 + 0.03(90/360)] = 1.0075$ .

Recall the two rules of the arbitrageur: Rule #1: Do not use our own money. Rule #2: Do not take any price risk. In the transactions above, Rule #1 is satisfied. Unfortunately, Rule #2 is not satisfied because the future value at Time  $h + m$  of the borrowed cash flows may be more than the asset cash flows. Note that the risk is that the rate  $L_h(m)$  will cause us to roll over the loan in Step 2 at a higher rate than offsets the gain from the loan we make in Step 1. This is the case here, because we will owe 1.0075 at the end of period  $m$  (Step 3) but will receive only 1.006227 from Step 1 if interest rates go up at Time  $h$  to 3%.

This risk can be eliminated by entering a receive-floating FRA on  $m$ -day Libor that expires at Time  $h$  and has the rate set at  $\text{FRA}(0,h,m)$ . Now assume we roll the FRA payoff forward

from  $h$  to  $h + m$  by investing any gain or borrowing to cover any loss at the rate  $L_h(m)$ . Let us assume the discount factor in the FRA payoff formula is  $1 + L_h(m)t_m$ . We see in Exhibit 9 that the following transaction enables us to satisfy Rule #2.

Step 4 Enter a receive-floating FRA and roll the payoff at  $h$  to  $h + m$  at the rate  $L_h(m)$ .

The payoff at Time  $h$  will be  $([L_h(m) - FRA(0,h,m)]t_m)/(1 + L_h(m)t_m)$ . There will be no cash flow from this FRA at Time  $h$  because this amount will be rolled forward at the rate  $L_h(m)t_m$ . Therefore, the value realized at Time  $h + m$  will be  $[L_h(m) - FRA(0,h,m)]t_m$ .

EXHIBIT 9 Cash Flow Table for Deposit and Lending Strategy with FRA

Steps	Cash Flow at Time 0	Cash Flow at Time $h$	Cash Flow at Time $h + m$
1. Make deposit for $h + m$ days	$-1/[1 + L_0(h)t_h]$ $= -0.996264$	0	$+[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h]$ $= 1.006227$
2. Borrow funds for $h$ days	$+1/[1 + L_0(h)t_h]$ $= +0.996264$	-1	
3. Borrow funds for $m$ days initiated at $h$		+1	$-[1 + L_h(m)t_m] = -1.0075$
4. Receive-floating FRA and roll payoff at $L_h(m)$ rate from $h$ to $h + m$	0	0	$+[L_h(m) - FRA(0, h, m)]t_m$ $= [0.03 - FRA(0,h,m)](90/360)$
Net cash flows	0	0	$+[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h]$ $t_h] - [1 + L_h(m)t_m]$ $+[L_h(m) - FRA(0,h,m)]t_m$

Recall that the goal is to identify the appropriate  $FRA(0,h,m)$  rate that makes the value of the FRA equal to zero on the initiation date. The terminal cash flows as expressed in the table can be used to solve for the FRA fixed rate. Because the transaction starts off with no initial investment or receipt of cash, the net cash flows at Time  $h + m$  should equal zero; thus,

$$+[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h] - [1 + L_h(m)t_m] + [L_h(m) - FRA(0,h,m)]t_m = 0$$

Solving for the FRA fixed rate, we have

$$FRA(0,h,m) = \{[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h] - 1\}/t_m \quad (5)$$

This equation looks complex, but it is really quite simple. In fact, it may well be quite familiar. It is essentially the compound value of \$1 invested at the longer-term Libor for  $h + m$  days divided by the compound value of \$1 invested at the shorter-term Libor for  $h$  days minus 1 and then annualized. The result is simply *the forward rate in the Libor term structure*. Recall that with simple interest, a one-period forward rate is found by solving the expression  $[1 + y(1)][1 + f(1)] = [1 + y(2)]^2$ , where  $y$  denotes the one- and two-period yield to maturity and  $f$  denotes the forward rate in the next period. The equation above is similar but simply

addresses the unique features of add-on interest rate calculations. Based on the numbers used in the previous two tables, we note

$$\begin{aligned} \text{FRA}(0,90,90) &= \{[1 + L_0(180)t_{180}]/[1 + L_0(90)t_{90}] - 1\}/t_{90} \\ &= \{[1 + 0.02(180/360)]/[1 + 0.015(90/360)] - 1\}/(90/360) \\ &= 0.024907 \text{ or } 2.49\%.^{12} \end{aligned}$$

### EXAMPLE 7 FRA Fixed Rate

Based on market quotes on Canadian dollar (C\$) Libor, the six-month C\$ Libor and the nine-month C\$ Libor are presently at 1.5% and 1.75%, respectively. Assume a 30/360-day count convention. The  $6 \times 9$  FRA fixed rate will be *closest* to:

- A. 2.00%.
- B. 2.23%.
- C. 2.25%.

*Solution:* B is correct. Based on the information given, we know  $L(180) = 1.5\%$  and  $L(270) = 1.75\%$ . The  $6 \times 9$  FRA rate is thus

$$\begin{aligned} \text{FRA}(0,h,m) &= \{[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h] - 1\}/t_m \\ \text{FRA}(0,180,90) &= \{[1 + 0.0175(270/360)]/[1 + 0.015(180/360)] - 1\}/(90/360) \\ \text{FRA}(0,180,90) &= [(1.013125/1.0075) - 1]4 = 0.0223325, \text{ or } 2.23\% \end{aligned}$$

We can now value an existing FRA using the same general approach as we did with the forward contracts previously covered; specifically, we can enter into an offsetting transaction at the new rate that would be set on an FRA that expires at the same time as our original FRA. By taking the opposite position, the new FRA offsets the old one. That is, if we are long the old FRA, we will receive the rate  $L_h(m)$  at  $h$ . We will go short a new FRA that will force us to pay  $L_h(m)$  at  $h$ . Consider the following strategy, illustrated in Exhibit 10, in which we again assume that  $NA = 1$ . Let us assume that we initiate an FRA that expires in 90 days and is based on 90-day Libor. The fixed rate at initiation is 2.49%. Thus,  $t_m = 90/360$ , and  $\text{FRA}(0,h,m) = \text{FRA}(0,90,90) = 2.49\%$ . When the FRA expires and makes its payoff, assume that we do not

<sup>12</sup>The result given in this example can be compared with the result from a simple approximation technique. Notice that for this FRA, 90 is half of 180. Thus, we can use the simple arithmetic average equation—here,  $(1/2)1.5\% + (1/2)X = 2.0\%$ —and solve for the missing variable  $X$ :  $X = 2.5\%$ . Knowing this approximation will always be biased slightly high, we know we are looking for an answer that is a little less than 2.5%. This is a nice way to check your final answer.

collect or pay the payoff; instead, we roll it forward by lending it (if a gain) or borrowing it (if a loss) from period  $h$  to period  $h + m$  at the rate  $L_h(m)$ . We then collect or pay the rolled forward value at  $h + m$ . Thus, there is no cash realized at Time  $h$ .

Now having entered into the long FRA with the intention of rolling the payoff forward, let us now position ourselves 30 days later, at Time  $g$ , at which there are 60 days remaining in the life of the FRA. Assume that at this point, the rate on an FRA based on 90-day Libor that expires in 60 days is 2.59%. Thus,  $FRA(g,h-g,m) = FRA(30,60,90) = 2.59\%$ . We go short this FRA, and as with the long FRA, we roll forward its payoff from Time  $h$  to  $h + m$ . Therefore, there is no cash realized from this FRA at Time  $h$ . This strategy is illustrated in Exhibit 10.

EXHIBIT 10 Cash Flows for FRA Valuation

Steps	Cash Flow at Time $g$	Cash Flow at Time $h$	Cash Flow at Time $h + m$
1. Receive-floating FRA (settled in arrears) at Time 0; roll forward at Rate $L_h(m)$ from $h$ to $h + m$		0	$+ \{[L_h(m) - FRA(0,h,m)]t_m\}$ $= +(L_h(m) - 0.0249)(90/360)$
2. Receive-fixed FRA (settled in arrears) at Time $g$ ; roll forward at Rate $L_h(m)$ from $h$ to $h + m$	0	0	$+ [FRA(g,h-g,m) - L_h(m)]t_m$ $= +[0.0259 - L_h(m)](90/360)$
Net cash flows	0	0	$+ [FRA(g,h-g,m) - FRA(0,h,m)]t_m$ $= +(0.0259 - 0.0249)(90/360)$ $= 0.00025$

To recap, the original FRA that we wish to value had its fixed rate set at 2.49% when it was initiated. Now, 30 days later, a new offsetting FRA can be created at 2.59%. The value of the offset position is 10 bps (2.59% – 2.49%) times 90/360 paid at Time  $h + m$ , assuming we roll the FRA payoffs forward. We will receive this amount at  $h + m$ , so it must be discounted back to Time  $g$  in order to obtain the value.

Because the cash flows at  $h + m$  are now known with certainty at  $g$ , this offsetting transaction at Time  $g$  has completely eliminated all of the risk at Time  $h + m$ . Our task, however, is to determine the fair value of the original FRA at Time  $g$ . Therefore, we need the present value of this Time  $h + m$  cash flow at Time  $g$ . That is, the value of the old FRA is the present value of the difference in the new FRA rate and the old FRA rate. Specifically, we let  $V_g(0,h,m)$  be the value of the FRA at Time  $g$  that was initiated at Time 0, expires at Time  $h$ , and is based on  $m$ -day Libor. Note that discounting will be over the period  $h + m - g$ . With  $D_g(h + m - g)$  as the discount rate, the value is

$$V_g(0,h,m) = \{[FRA(g,h-g,m) - FRA(0,h,m)]t_m\} / [1 + D_g(h + m - g)t_{h+m-g}] \quad (6)$$

where the new FRA rate is the formula we previously learned, simply applied to this new offsetting transaction:

$$FRA(g,h-g,m) = \{[1 + L_g(h + m - g)t_{h+m-g}] / [1 + L_g(h - g)t_{h-g}] - 1\} / t_m$$

Thus, the date  $g$  value of the receive-floating FRA initiated at date 0 is merely the present value of the difference in FRA rates, one entered on date  $g$  and one entered on date 0.

Traditionally, it is assumed that the discount rate,  $D_g(h + m - g)$ , is equal to the underlying floating rate,  $L_g(h + m - g)$ , but that is not necessary.<sup>13</sup> Let us assume a 60-day rate of 3% on day  $g$ . Thus,  $L_g(h - g) = L_{30}(60) = 3\%$ . Then the value of the FRA would be

$$V_g(0,h,m) = V_{30}(0,90,90) = 0.00025/[1 + 0.03(60/360)] = 0.000249.$$

And of course, this amount is per notional of 1. Thus, the answer found here must be multiplied by the actual notional amount as demonstrated in the following example.

### EXAMPLE 8 FRA Valuation

Suppose we entered a receive-floating  $6 \times 9$  FRA at a rate of 0.86%, with notional amount of C\$10,000,000 at Time 0. The six-month spot Canadian dollar (C\$) Libor was 0.628%, and the nine-month C\$ Libor was 0.712%. Also, assume the  $6 \times 9$  FRA rate is quoted in the market at 0.86%. After 90 days have passed, the three-month C\$ Libor is 1.25% and the six-month C\$ Libor is 1.35%, which we will use as the discount rate to determine the value at  $g$ . We have  $h = 180$  and  $m = 90$ .

Assuming the appropriate discount rate is C\$ Libor, the value of the original receive-floating  $6 \times 9$  FRA will be *closest* to:

- A. C\$14,500.
- B. C\$14,625.
- C. C\$14,651.

*Solution:* C is correct. Initially, we have  $L_0(h) = L_0(180) = 0.628\%$ ,  $L_0(h + m) = L_0(270) = 0.712\%$ , and  $FRA(0,180,90) = 0.86\%$ . After 90 days ( $g = 90$ ), we have  $L_g(h - g) = L_{90}(90) = 1.25\%$  and  $L_g(h + m - g) = L_{90}(180) = 1.35\%$ . Interest rates rose during this period; hence, the FRA likely has gained value because the position is receive-floating. First, we compute the new FRA rate at Time  $g$  and then estimate the fair FRA value as the discounted difference in the new and old FRA rates. The new FRA rate at Time  $g$ , denoted  $FRA(g,h - g,m) = FRA(90,90,90)$ , is the rate on day 90 of an FRA to expire in 90 days in which the underlying is 90-day Libor. That rate is found as

$$\begin{aligned} FRA(g,h - g,m) &= FRA(90,90,90) \\ &= \{[1 + L_g(h + m - g)t_{h+m-g}]/[1 + L_g(h - g)t_{h-g}] - 1\}/t_m, \end{aligned}$$

and based on the information in this example, we have

$$\begin{aligned} FRA(90,90,90) &= \{[1 + L_{90}(180 + 90 - 90)(180/360)]/[1 + L_{90}(180 - 90)(90/360)] \\ &\quad - 1\}/(90/360). \end{aligned}$$

<sup>13</sup>Again, there is a current debate on whether the OIS rate should be used for discounting; hence, we may have a different discount rate, but in any case, that rate would be known at time  $g$ .

Substituting the values given in this problem, we find

$$\begin{aligned} \text{FRA}(90,90,90) &= \{[1 + 0.0135(180/360)]/[1 + 0.0125(90/360)] - 1\}/(90/360) \\ &= [(1.00675/1.003125) - 1]4 = 0.0145, \text{ or } 1.45\%. \end{aligned}$$

Therefore,

$$\begin{aligned} V_g(0,h,m) &= V_{90}(0,180,90) \\ &= 10,000,000[(0.0145 - 0.0086)(90/360)]/[1 + 0.0135(180/360)] \\ &= 14,651. \end{aligned}$$

Again, floating rates rose during this period; hence, the FRA enjoyed a gain. Notice that the FRA rate rose by roughly 59 bps ( $= 145 - 86$ ), and 1 bp for 90-day money and a 1,000,000 notional amount is 25. Thus, we can also estimate the terminal value as  $10 \times 25 \times 59 = 14,750$ . As with all fixed-income strategies, understanding the value of a basis point is often helpful when estimating profits and losses and managing the risks of FRAs.

We now turn to the specific features of various forward and futures markets. The same general principles will apply, but the specifics will be different.

### 3.5. Fixed-Income Forward and Futures Contracts

Fixed-income forward and futures contracts have several unique issues that influence the specifics of the carry arbitrage model. First, in some countries the prices of fixed-income securities (termed “bonds” here) are quoted without the interest that has accrued since the last coupon date. The quoted price is sometimes known as the clean price. Naturally, when buying a bond, one must pay the full price, which is sometimes called the dirty price, so the accrued interest is included. Nonetheless, it is necessary to understand how the quoted bond price and accrued interest compose the true bond price and the effect this convention has on derivative pricing. The quote convention for futures contracts, whether based on clean or dirty prices, usually corresponds to the quote convention in the respective bond market. In this section, we will largely treat forwards and futures the same, except in certain places where noted.

In general, accrued interest is computed based on the following linear interpolation formula:

$$\text{Accrued interest} = \text{Accrual period} \times \text{Periodic coupon amount, or}$$

$$AI = (NAD/NTD) \times (C/n)$$

where NAD denotes the number of accrued days since the last coupon payment, NTD denotes the number of total days during the coupon payment period, n denotes the number of coupon payments per year, and C is the stated annual coupon amount. For example, after two months (60 days), a 3% semi-annual coupon bond with par of 1,000 would have accrued interest of  $AI = (60/180) \times (30/2) = 5$ . Note that accrued interest is expressed in currency (not percent) and the number of total days (NTD) depends on the coupon payment frequency (semi-annual on 30/360 day count convention would be 180).

Second, fixed-income futures contracts often have more than one bond that can be delivered by the seller. Because bonds trade at different prices based on maturity and stated coupon, an adjustment known as the conversion factor is used in an effort to make all deliverable bonds roughly equal in price.

Third, when multiple bonds can be delivered for a particular maturity of a futures contract, a cheapest-to-deliver bond typically emerges after adjusting for the conversion factor. The conversion factor is a mathematical adjustment to the amount required when settling a futures contract that is supposed to make all eligible bonds equal the same amount. For example, the conversion factor may seek to adjust each bond to an equivalent 6% coupon bond. The conversion factor adjustment, however, is not precise. Thus, the seller will deliver the bond that is least expensive.

For bond markets in which the quoted price includes the accrued interest and in which futures or forward prices assume accrued interest is in the bond price quote, the futures or forward price simply conforms to the general formula we have previously discussed. Recall that the futures or forward price is simply the future value of the underlying in which finance costs, carry costs, and carry benefits are all incorporated or

$$\begin{aligned} F_0(T) &= \text{Future value of underlying adjusted for carry cash flows} \\ &= FV_{0,T}(S_0 + \theta_0 - \gamma_0) \end{aligned}$$

Again, Time 0 is the forward contract trade initiation date, and Time T is the contract expiration date. For the fixed-income bond, let  $T + Y$  denote the underlying instrument's current time to maturity. Therefore,  $Y$  is the time to maturity of the underlying bond at Time T, when the contract expires. Let  $B_0(T + Y)$  denote the quoted price observed at Time 0 of a fixed-rate bond that matures at Time  $T + Y$  and pays a fixed coupon rate. For bonds quoted without accrued interest, let  $AI_0$  denote the accrued interest at Time 0. The carry benefits are the bond's fixed coupon payments,  $\gamma_0 = PVCI_{0,T}$ , meaning the present value of all coupon interest paid over the forward contract horizon from Time 0 to Time T. The corresponding future value of these coupons is  $\gamma_T = FVCI_{0,T}$ . Finally, there are no carry costs, and thus  $\theta_0 = 0$ . To be consistent with prior notation, we have

$$S_0 = \text{Quoted bond price} + \text{Accrued interest} = B_0(T + Y) + AI_0$$

We could just insert this price into the previous equation, letting  $\gamma_0 = PVCI_{0,T}$ , and thereby obtain the futures price the simple and traditional way. But fixed-income futures contracts often permit delivery of more than one bond and use a conversion factor system to provide this flexibility. In these markets, the futures price,  $F_0(T)$ , is defined here as the quoted futures price,  $QF_0(T)$ , times the conversion factor,  $CF(T)$ . In fact, the futures contract settles against the quoted bond price without accrued interest. Thus, the total profit or loss on a long futures position is  $B_T(T + Y) - F_0(T)$ . Based on our notation above, we can represent this profit or loss as  $(S_T - AI_T) - F_0(T)$ . Therefore, the fixed-income forward or futures price including the conversion factor, termed the “adjusted price,” can be expressed as<sup>14</sup>

$$\begin{aligned} F_0(T) &= QF_0(T)CF(T) \\ &= \text{Future value of underlying adjusted for carry cash flows} \\ &= FV_{0,T}[S_0 - PVCI_{0,T}] = FV_{0,T}[B_0(T + Y) + AI_0 - PVCI_{0,T}] \end{aligned} \tag{7}$$

<sup>14</sup>In this section, we will use the letter F to denote either the forward price or the futures price times the conversion factor.

In other words, the actual futures price is  $F_0(T)$ , but in the market, the availability of multiple deliverable bonds gives rise to the adjustment factor. Hence, the price you would see quoted is  $QF_0$ .

Recall that the bracketed term  $B_0(T + Y) + AI_0 - PVCI_{0,T}$  is just the full spot price minus the present value of the coupons over the life of the forward or futures contract. The fixed-income forward or futures price is thus the future value of the quoted bond price plus accrued interest less any coupon payments made during the life of the contract. Again, the quoted bond price plus the accrued interest is the spot price: It is in fact the price you would have to pay to buy the bond. Market conventions in some countries just happen to break this price out into the quoted price plus the accrued interest.

Now let us explore carry arbitrage in the bond market, assuming that accrued interest is broken out and that multiple bonds are deliverable, thereby requiring the use of the conversion factor. Consider the following transactions:

- Step 1 Buy the underlying bond, requiring  $S_0$  cash flow.
- Step 2 Borrow an amount equivalent to the cost of the underlying bond,  $S_0$ .
- Step 3 Sell the futures contract at  $F_0(T)$ .
- Step 4 Borrow the arbitrage profit.

Exhibit 11 shows the cash flow consequences for this portfolio in which the futures price is not in equilibrium. Note that  $FVCI_{0,T}$  denotes the future value as of Time T of any coupons paid during the life of the futures contract. Again, for illustration purposes, we provide a numerical example: Suppose  $T = 0.25$ ,  $CF(T) = 0.8$ ,  $B_0(T + Y) = 107$  (the quoted price),  $FVCI_{0,T} = 0.0$  (meaning no coupon payments over the life of the contract),  $AI_0 = 0.07$  (the accrued interest at Time 0),  $AI_T = 0.20$  (the accrued interest at Time T),  $QF_0(T) = 135$  (the quoted futures price), and  $r = 0.2\%$ . Thus,  $S_0 = B_0(T + Y) + AI_0 = 107 + 0.07 = 107.07$  (the full or spot price), and  $F_0(T) = CF(T)QF_0(T) = 0.8(135) = 108$  (the adjusted price). At Time T, suppose  $B_T(T + Y) = 110$  and thus  $S_T = B_T(T + Y) + AI_T = 110 + 0.20 = 110.20$ . Because  $FVCI_{0,T} = 0.0$ , there are no coupons paid over the life of the futures. Note that the adjusted price,  $F_0(T)$ , is 108 whereas the future value adjusted for carry cash flows (Equation 7) is  $(107 + 0.07)(1.002)^{0.25} = 107.12$ . Adding the accrued interest at expiration ( $AI_T = 0.20$ ) to the adjusted futures price gives 108.20. The difference between 108.20 and 107.12 is 1.08, which means that the futures contract is overpriced by 1.08. Thus, the arbitrage will involve borrowing the arbitrage profit, which is the present value of 1.08, or 1.0795—that is,  $108(1.002)^{-0.25}$ .

#### EXHIBIT 11 Cash Flows for Fixed-Rate Coupon Bond Futures Pricing

Steps	Cash Flow at Time 0	Cash Flow at Time T
1. Buy bond	$-S_0 = -[B_0(T + Y) + AI_0]$ $= -[107 + 0.07]$ $= -107.07$	$S_T + FVCI_{0,T}$ $= 110.20 + 0.0$ $= 110.20$
2. Borrow	$+S_0 = 107.07$	$-FV_{0,T}(S_0)$ $= -(1+0.002)^{0.25}(107.07)$ $= -107.12$
3. Sell futures	0	$F_0(T) - B_T(T + Y)$ $= 108 - 110$ $= -2$

## EXHIBIT 11 (Continued)

Steps	Cash Flow at Time 0	Cash Flow at Time T
4. Borrow arbitrage profit	$+PV_{0,T}[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}]$ $=(1 + 0.002)^{-0.25}[108 - 107.12 + 0.20 + 0.0]$ $= 1.0795$	$-[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}]$ $=-[108 - 107.12 + 0.20 + 0.0]$ $= -1.08$
Net cash flows	$+PV_{0,T}[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}]$ $=1.0795$	0

Thus, the value of the Time 0 cash flows should be zero or else there is an arbitrage opportunity. The numerical example provided shows a 1.0795 cash flow at Time 0 per bond. If the value in the Time 0 column for net cash flows is positive, then conduct the carry arbitrage of buy bond, borrow, and sell futures (again, termed *carry arbitrage* because the underlying is “carried”). If the Time 0 column is negative, then conduct the reverse carry arbitrage of short sell bond, lend, and buy futures (termed *reverse carry arbitrage* because the underlying is not carried but is sold short).

Thus, in equilibrium, to eliminate an arbitrage opportunity, we expect

$$PV_{0,T}[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}] = 0$$

or

$$F_0(T) = FV_{0,T}(S_0) - AI_T - FVCI_{0,T}$$

For clarity, substituting for  $F_0(T)$  and  $S_0$  and solving for the quoted futures price, we have

$$\begin{aligned} QF_0(T) &= \text{Conversion factor adjusted future} \\ &\quad \text{value of underlying adjusted for carry} \\ &= [1 / CF(T)]\{FV_{0,T}[B_0(T + Y) + AI_0] - AI_T - FVCI_{0,T}\} \end{aligned} \tag{8}$$

In the example above, we have

$$\begin{aligned} QF_0(T) &= [1 / CF(T)]\{FV_{0,T}[B_0(T + Y) + AI_0] - AI_T - FVCI_{0,T}\} \\ &= (1 / 0.8)[(1 + 0.002)^{0.25}(107 + 0.07) - 0.20 - 0.0] = 133.65 \end{aligned}$$

Note that the futures price of 135 used for calculations in Exhibit 11 was higher than the equilibrium futures price of 133.65; hence, the arbitrage transaction of selling the futures contract resulted in a riskless positive cash flow.

### EXAMPLE 9 Estimating the Euro-Bund Futures Price

Euro-bund futures have a contract value of €100,000, and the underlying consists of long-term German debt instruments with 8.5 to 10.5 years to maturity. They are traded on the Eurex. Suppose the underlying 2% German bund is quoted at €108 and has accrued interest of €0.083 (one-half of a month since last coupon). The euro-bund futures contract matures in one month. At contract expiration, the underlying bond will have

accrued interest of €0.25, there are no coupon payments due until after the futures contract expires, and the current one-month risk-free rate is 0.1%. The conversion factor is 0.729535. In this case, we have  $T = 1/12$ ,  $CF(T) = 0.729535$ ,  $B_0(T + Y) = 108$ ,  $FVCI_{0,T} = 0$ ,  $AI_0 = 0.5(2/12) = €0.083$ ,  $AI_T = 1.5(2/12) = 0.25$ , and  $r = 0.1\%$ . The equilibrium euro-bund futures price based on the carry arbitrage model will be *closest* to:

- A. €147.57.
- B. €147.82.
- C. €148.15.

*Solution:* B is correct. The carry arbitrage model for forwards and futures is simply the future value of the underlying with adjustments for unique carry features. With bond futures, the unique features include the conversion factor, accrued interest, and any coupon payments. Thus, the equilibrium euro-bund futures price can be found using the carry arbitrage model in which

$$F_0(T) = FV_{0,T}(S_0) - AI_T - FVCI_{0,T}$$

or

$$QF_0(T) = [1/CF(T)]\{FV_{0,T}[B_0(T + Y) + AI_0] - AI_T - FVCI_{0,T}\}$$

Thus, we have

$$QF_0(T) = [1/0.729535][(1 + 0.001)^{1/12}(108 + 0.083) - 0.25 - 0] = 147.82$$

In equilibrium, the euro-bund futures price should be approximately €147.82 based on the carry arbitrage model.

Because of the mark-to-market settlement procedure, the value of a bond futures is essentially the price change since the previous day's settlement. That value is captured at the settlement at the end of the day, at which time the value of a bond futures contract, like other futures contracts, is zero.

We now turn to the task of estimating the fair value of the bond forward contract at a point in time during its life. Forwards are not settled daily, so the value is not formally realized until expiration. Suppose the first transaction is buying an at-market bond forward contract at Time 0 with expiration of Time T. Now consider selling a new bond forward contract at Time t again with expiration of Time T. Exhibit 12 shows the potential cash flows. Because this is a bond forward contract, we assume either no conversion factor or effectively a conversion factor of 1. Suppose now  $B_T(T + Y) = 108$ ,  $F_0(T) = 107.12$ , and  $F_t(T) = 107.92$ .

## EXHIBIT 12 Cash Flows for Offsetting a Long Forward Position

Steps	Cash Flow at Time 0	Cash Flow at Time t	Cash Flow at Time T
1. Buy bond forward contract at 0	0	$V_t(T)$	$V_T(0, T) = B_T(T + Y) - F_0(T)$ $= 108 - 107.12 = 0.88$
2. Sell bond forward contract at t	NA	0	$V_T(t, T) = F_t(T) - B_T(T + Y)$ $= 107.92 - 108 = -0.08$
Net cash flows	0	$V_t(T)$	$F_t(T) - F_0(T)$ $= 107.92 - 107.12 = 0.8$

Note that the net position from these bond forward transactions is risk free. It is independent of the underlying bond value,  $B_T(T + Y)$ . Therefore, the forward value observed at Time t of a Time T maturity bond forward contract is simply the present value—denoted  $PV_{t,T}()$ —of the difference in forward prices. That is,

$$V_t(T) = \text{Present value of difference in forward prices} = PV_{t,T}[F_t(T) - F_0(T)]$$

Based on our example in the table and assuming  $T - t = 0.1$  and  $r = 0.15\%$ , we have  $V_t(T) = (107.92 - 107.12)/(1 + 0.0015)^{0.1} = 0.79988$ . Note that this is the same result as the generic case with a simple conversion factor adjustment. Recall that the conversion factor is an adjustment to make all bonds roughly equal in value.

## EXAMPLE 10 Estimating the Value of a Euro-Bund Forward Position

Suppose that one month ago, we purchased five euro-bund forward contracts with two months to expiration and a contract notional of €100,000 each at a price of 145 (quoted as a percentage of par). The euro-bund forward contract now has one month to expiration. Again, assume the underlying is a 2% German bund quoted at 108 and has accrued interest of 0.0833 (one-half of a month since last coupon). At the contract expiration, the underlying bund will have accrued interest of 0.25, there are no coupon payments due until after the forward contract expires, and the current annualized one-month risk-free rate is 0.1%.

Based on the current forward price of 148, the value of the euro-bund forward position will be *closest* to:

- A. €2,190.
- B. €14,998.
- C. €15,000.

*Solution:* B is correct. Because we are given both forward prices, the solution is simply

$$V_t(T) = PV_{t,T}[F_t(T) - F_0(T)] = (148 - 145)/(1 + 0.001)^{1/12} = 2.9997$$

which is 2.9997 per €100 par value because this forward price was quoted as a percentage of par. Because five contracts each with €100,000 par were entered, we have  $0.029997(\€100,000)5 = \€14,998.50$ . Note that when interest rates are so low and the forward contract has a short maturity, then the present value effect is minimal.

### 3.6. Currency Forward and Futures Contracts

Currency derivative contracts require careful attention to the unit of value. For example, if we are discussing bond futures, then the underlying is perceived in currency per unit of par value. If we are trading gold futures, then the quotation will be in currency per troy ounce. If trading a common stock, then it will be in currency per share. When trading currency itself, great care must be taken to know which currency is the base currency. When quoting an exchange rate, we will say that the foreign currency is trading for a certain number of units of domestic currency. For example, we could say, “The euro is trading for \$1.30,” meaning that €1 is worth \$1.30. We use the shorthand notation of DC/FC to refer to the price of one unit of foreign currency expressed in terms of domestic currency units when embedded in an equation.<sup>15</sup> With currency, perspective makes a significant difference. Thus, when pricing and valuing currency forwards and futures contracts, a clear perspective requires considerable care. The carry arbitrage model with foreign exchange presented here is also known as **covered interest rate parity** and sometimes just **interest rate parity**.

Recall that currency forward contracts are agreements to exchange one currency for another on a future date at an exchange rate the counterparties agree on today. One approach to pricing is based on a forward exchange rate satisfying an arbitrage relationship that equates the investment return on two alternative but equivalent investment strategies. We illustrate these two strategies assuming the domestic currency is British pounds (£) and the foreign currency is the euro (€).

#### Strategy #1:

We simply invest one currency unit in a domestic risk-free bond. Thus, at Time T, we have the original investment grossed up at the domestic interest rate or the future value of 1DC, denoted  $FV(1DC)$ . For example, the future value at Time T of this strategy can be expressed as  $FV_{\text{£},T}(1)$ , given British pounds as the domestic currency.

#### Strategy #2:

We engage in three simultaneous transactions termed *steps* here. In Step 1, the domestic currency is converted at the current spot exchange rate,  $S_0(\text{FC}/\text{DC})$ , into the foreign currency (FC). At this point, 1 domestic currency unit is being converted to the foreign currency;

<sup>15</sup>Some practitioners prefer to express the discussion here as FC/DC, contradicting normal mathematics as well as contradicting standard market quotations, such as \$ per bushel of wheat or \$ per ounce of gold.

hence, we use  $S_0(\text{FC}/\text{DC})$  generically or  $S_0(\text{€}/\text{£})$  in our example. Note that the final answer will express the spot exchange rate as the reciprocal  $1/S_0(\text{FC}/\text{DC}) = S_0(\text{DC}/\text{FC})$ . In Step 2, FC is invested at the foreign risk-free rate until Time T. For example, the future value at Time T of this strategy can be expressed as  $\text{FV}_{\text{€},\text{T}}(1)$ , given that the euro is the foreign currency. In Step 3, a forward foreign exchange contract is entered to sell the foreign currency at Time T in exchange for domestic currency with the forward rate denoted  $F_0(\text{DC}/\text{FC},\text{T})$ . So, for example,  $F_0(\text{£}/\text{€},\text{T})$  is the rate on a forward commitment at Time 0 to sell one euro for British pounds at Time T. This transaction can be looked at as being short the euro in pound terms or being long the pound in euro terms for delivery at Time T.

We are examining two ways to invest British pounds at Time 0, and both strategies should result in the same value in domestic currency units at Time T. If not, then an arbitrage opportunity exists. Remember that the current spot exchange rate,  $S_0(\text{£}/\text{€})$ , is the number of British pounds for one euro. Again, in our example,  $\text{FV}_{\text{£},\text{T}}(1)$  denotes the future value of one British pound and  $\text{FV}_{\text{€},\text{T}}(1)$  denotes the future value of one euro.<sup>16</sup> Based on the two strategies, the value at Time T follows:

Strategy 1. Future value at Time T of investing £1:  $\text{FV}_{\text{£},\text{T}}(1)$

Strategy 2. Future value at Time T of investing £1:  $F_0(\text{£}/\text{€},\text{T})\text{FV}_{\text{€},\text{T}}(1)S_0(\text{€}/\text{£})$

Assuming both strategies lead to the same number of British pounds at Time T, we have  $\text{FV}_{\text{£},\text{T}}(1) = F_0(\text{£}/\text{€})\text{FV}_{\text{€},\text{T}}(1)S_0(\text{€}/\text{£})$ . Note that  $S_0(\text{£}/\text{€}) = 1/S_0(\text{€}/\text{£})$ , simply reflecting the reciprocal of the exchange rate. Thus, solving for the forward foreign exchange rate, the forward rate can be expressed as

$$\begin{aligned} F_0(\text{£}/\text{€},\text{T}) &= \text{Future value of spot exchange rate adjusted for foreign rate} \\ &= \text{FV}_{\text{£},\text{T}}(1) / [\text{FV}_{\text{€},\text{T}}(1)S_0(\text{€}/\text{£})] = S_0(\text{£}/\text{€})\text{FV}_{\text{£},\text{T}}(1) / \text{FV}_{\text{€},\text{T}}(1) \end{aligned} \quad (9)$$

The carry adjustment, though it looks different, is similar to what we did in other carry models. In the numerator, we have simply the future value of the spot exchange rate. Rather than subtracting the carry benefit of foreign interest—the euro here—we divide by the future value of one euro, based on the euro interest rate. The effect is similar: The higher the foreign interest rate, the greater the benefit, and hence, the lower the forward or futures price will be.

If the two strategies result in different values at Time T, then the arbitrageur would buy the strategy offering the higher value at Time T and sell the strategy offering the lower value at Time T. This arbitrage activity would result in no cash flow today and positive cash flow at expiration. As with previous examples, we could borrow the arbitrage profit today and pay the loan back when the profit is captured at T.

<sup>16</sup>Note that the interest could be compounded annually, continuously, or by any other method at this point; hence, we use the generic future value specification.

### EXAMPLE 11 Pricing Forward Foreign Exchange Contracts

Suppose the current spot exchange rate,  $S_0(\text{£}/\text{€})$ , is £0.792 (what 1€ is trading for in £). Further assume that the annual compounded annualized risk-free rates are 1% for the British pound and 0.3% for the euro.

1. The arbitrage-free one-year foreign exchange forward rate,  $F_0(\text{£}/\text{€}, T)$  (expressed as the number of £ per 1€), will be *closest* to:
  - A. 0.792.
  - B. 0.794.
  - C. 0.798.
2. Now suppose the foreign exchange forward rate,  $F_0(\text{£}/\text{€}, T)$ , is observed to be below the foreign exchange spot rate,  $S_0(\text{£}/\text{€})$ . Based on the carry arbitrage model, compared to British interest rates, the eurozone interest rate will *most likely* be:
  - A. lower.
  - B. higher.
  - C. the same.

*Solution to 1:* C is correct. Based on the information given, we have  $S_0(\text{£}/\text{€}) = 0.792$ ,  $T = 1$  year,  $r_{\text{£}} = 1.0\%$ , and  $r_{\text{€}} = 0.3\%$  (both with annual compounding). Therefore,

$$F_0(\text{£}/\text{€}, 1) = S_0(\text{£}/\text{€}) \text{FV}_{\text{£}, 1}(1) / \text{FV}_{\text{€}, 1}(1) = 0.792(1 + 0.01)^1 / (1 + 0.003)^1 = 0.798,$$

or £0.798/€.

*Solution to 2:* B is correct. Note that if we observe that  $F_0(\text{£}/\text{€}, T)$  is smaller than  $S_0(\text{£}/\text{€})$ , then the carry arbitrage model provides a simple explanation: The British interest rate is lower than the eurozone interest rate. Based on the carry arbitrage model, foreign exchange forward rates solely reflect interest-related carry costs. Specifically,  $F_0(\text{£}/\text{€}, T) < S_0(\text{£}/\text{€})$  if and only if  $r_{\text{£}} < r_{\text{€}}$ .

Note that the future value expressions in Equation 9 are in the same pattern as the spot exchange rate. If the spot exchange rate is expressed as 1€ is trading for £—denoted  $S_0(\text{£}/\text{€})$  and  $F_0(\text{£}/\text{€}, T)$ —then the future value ratio is  $\text{FV}_{\text{£}, T}(1) / \text{FV}_{\text{€}, T}(1)$ . If we assume annual compounding and denote the risk-free rates  $r_{\text{£}}$  and  $r_{\text{€}}$ , respectively, we have

$$F_0(\text{£}/\text{€}, T) = S_0(\text{£}/\text{€}) (1 + r_{\text{£}})^T / (1 + r_{\text{€}})^T \text{ (Annually compounded version)}$$

If we assume continuous compounding and denote these risk-free rates in domestic (UK) and eurozone as  $r_{\text{£}, c}$  and  $r_{\text{€}, c}$ , respectively, we have

$$F_0(\text{£}/\text{€}, T) = S_0(\text{£}/\text{€}) e^{(r_{\text{£}, c} - r_{\text{€}, c})T} \text{ (Continuously compounded version)}$$

To summarize, we identify several ways we get tripped up in understanding currency forward and futures contracts. First, if we let DC denote generically domestic currency and FC

denote generically foreign currency, then there are two representations of the carry arbitrage model based on  $S_0(FC/DC) = 1/S_0(DC/FC)$  and  $F_0(FC/DC) = 1/F_0(DC/FC)$ . If we assume annual compounding, we have either

$$F_0(DC/FC, T) = S_0(DC/FC) \frac{(1+r_{DC})^T}{(1+r_{FC})^T} \text{ or } F_0(FC/DC, T) = S_0(FC/DC) \frac{(1+r_{FC})^T}{(1+r_{DC})^T}$$

A good way to remember this relationship is that the interest rate in the numerator should be the rate for the country whose currency is specified in the spot rate quote. Thus, if the spot rate quote is in euros, the numerator should be the euro interest rate. Then the interest rate in the denominator is the rate in the other country.

Second, interest rates can be quoted in a wide variety of ways, including annual compounding (previous equation) and continuous compounding (following equation).

$$F_0(DC/FC, T) = S_0(DC/FC) e^{(r_{DC,c} - r_{FC,c})T} \text{ or } F_0(FC/DC, T) = S_0(FC/DC) e^{(r_{FC,c} - r_{DC,c})T}$$

Here, likewise, the currency quote should match the first interest rate. Thus, if the spot rate is quoted in euros, then the first interest rate in the exponential will be the euro rate.

In equilibrium,  $F_0(\mathcal{E}/\mathcal{E}, T) = S_0(\mathcal{E}/\mathcal{E}) FV_{\mathcal{E}}(1)/FV_{\mathcal{E}}(1)$ ; otherwise, positive future cash flow can be generated with no initial investment, which is an arbitrage profit.

We now turn to the task of estimating the fair value of the foreign exchange forward contract. The forward value, based on arbitrage, can best be understood by referring to Exhibit 13. Suppose the first transaction is buying a foreign exchange forward contract at Time 0 with expiration of Time T. Now consider selling a new foreign exchange forward contract at Time t also with expiration of Time T. Exhibit 13 shows the potential cash flows again using British pounds ( $\mathcal{E}$ ) as the domestic currency and euros ( $\mathcal{E}$ ) as the foreign currency. Suppose  $T = 1$ ,  $T - t = 0.5$ ,  $F_0(\mathcal{E}/\mathcal{E}, T) = 0.804$ ,  $F_t(\mathcal{E}/\mathcal{E}, T) = 0.901$ ,  $S_T(\mathcal{E}/\mathcal{E}) = 1.2$ , and  $r_{\mathcal{E},t} = 1.2\%$ . In other words, six months ago we bought a forward contract at 0.804, and the new forward price is 0.901.

EXHIBIT 13 Cash Flows for Offsetting a Long Forward Position

Steps	Cash Flow at Time 0	Cash Flow at Time t	Cash Flow at Time T
1. Buy forward contract at 0	0	$V_t(T)$	$V_T(0, T) = S_T(\mathcal{E}/\mathcal{E}) - F_0(\mathcal{E}/\mathcal{E}, T) = 1.2 - 0.804 = 0.396$
2. Sell forward contract at t	NA	0	$V_T(t, T) = F_t(\mathcal{E}/\mathcal{E}, T) - S_T(\mathcal{E}/\mathcal{E}) = 0.901 - 1.2 = -0.299$
Net cash flows	0	$V_t(T)$	$+F_t(\mathcal{E}/\mathcal{E}, T) - F_0(\mathcal{E}/\mathcal{E}, T) = 0.901 - 0.804 = 0.097$

Note that the net position is again risk free. Therefore, the forward value observed at t of a T maturity forward contract is simply the present value of the difference in foreign exchange forward prices. That is,

$$V_t(T) = \text{Present value of the difference in forward prices} \\ = PV_{\mathcal{E},t,T} [F_t(\mathcal{E}/\mathcal{E}, T) - F_0(\mathcal{E}/\mathcal{E}, T)] \quad (10)$$

Based on our numerical example, we have  $V_t(T) = (0.901 - 0.804)/(1 + 0.012)^{0.5} = \mathcal{E}0.0964/\mathcal{E}$ .

### EXAMPLE 12 Computing the Foreign Exchange Forward Contract Value

A corporation sold €10,000,000 against a British pound forward at a forward rate of £0.8000 for €1 at Time 0. The current spot market at Time  $t$  is such that €1 is worth £0.7500, and the annually compounded risk-free rates are 0.80% for the British pound and 0.40% for the euro. Assume at Time  $t$  there are three months until the forward contract expiration.

1. The forward price  $F_t(\text{£}/\text{€}, T)$  at Time  $t$  will be *closest* to:
  - A. 0.72.
  - B. 0.74.
  - C. 0.75.
2. The value of the foreign exchange forward contract at Time  $t$  will be *closest* to:
  - A. £492,000.
  - B. £495,000.
  - C. £500,000.

*Solution to 1:* C is correct. Note that the forward price at Time  $t$  is

$$\begin{aligned} F_t(\text{£}/\text{€}, T) &= S_t(\text{£}/\text{€}) FV_{\text{£}, t, T}(1) / FV_{\text{€}, t, T}(1) \\ &= 0.75 (1 + 0.008)^{0.25} / (1 + 0.004)^{0.25} \\ &= 0.7507. \end{aligned}$$

*Solution to 2:* A is correct. The value per euro to the seller of the foreign exchange futures contract at Time  $t$  is simply the present value of the difference between the initial forward price and the £/€ forward price at Time  $t$  or

$$\begin{aligned} V_t(T) &= PV_{\text{£}, t, T} [F_0(\text{£}/\text{€}, T) - F_t(\text{£}/\text{€}, T)] \\ &= (0.8000 - 0.7507) / (1 + 0.008)^{0.25} \\ &= £0.0492 \text{ per euro.} \end{aligned}$$

Note that the corporation has an initial short position, so the short position of a €10,000,000 notional amount has a positive value of €10,000,000(£0.0492/€) = £492,000 for the corporation because the forward rate fell between Time 0 and Time  $t$ .

We conclude this section with observations on the similarities and differences between forward and futures contracts.

#### 3.7. Comparing Forward and Futures Contracts

For every market considered here, the carry arbitrage model provides an approach for both pricing and valuing forward contracts. Recall the two generic expressions:

$$F_0(T) = FV_{0, T}(S_0 + \theta_0 - \gamma_0) \text{ (Forward pricing)}$$

$$V_t(T) = PV_{t, T}[F_t(T) - F_0(T)] \text{ (Forward valuation)}$$

Carry costs ( $\theta_0$ ) increase the forward price, and carry benefits ( $\gamma_0$ ) decrease the forward price. The arbitrageur is carrying the underlying, and costs increase the burden whereas benefits decrease the burden. The forward value can be expressed as either the present value of the difference in forward prices or as a function of the current underlying price adjusted for carry cash flows and the present value of the initial forward price.

Futures prices are generally found using the same model, but futures values are different because of the daily marking to market. Recall that the futures values are zero at the end of each day because profits and losses are taken daily.

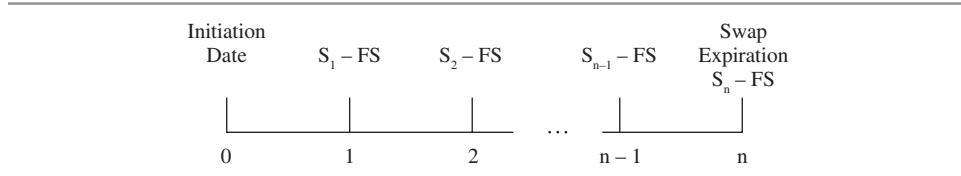
In summary, the carry arbitrage model provides a compelling way to price and value forward and futures contracts. In short, the forward or futures price is simply the future value of the underlying adjusted for any carry cash flows. The forward value is simply the present value of the difference in forward prices at an intermediate time in the contract. The futures value is zero after marking to market. We turn now to pricing and valuing swaps.

#### 4. PRICING AND VALUING SWAP CONTRACTS

Based on the foundational materials in the last section on using the carry arbitrage model for pricing and valuing forward and futures contracts, we now apply this approach to pricing and valuing swap contracts. Swap contracts can be synthetically created by either a portfolio of underlying instruments or a portfolio of forward contracts. We focus here solely on the portfolio of underlying instruments approach.

We consider a receive-floating and pay-fixed interest rate swap. The swap will involve a series of  $n$  future cash flows at points in time represented simply here as 1, 2, ...,  $n$ . Let  $S_i$  denote the generic floating interest rate cash flow based on some underlying, and let  $FS$  denote the cash flow based on some fixed interest rate. We assume that the last cash flow occurs at the swap expiration. Exhibit 14 shows the cash flows of a generic swap. Later we will let  $S_i$  denote the floating cash flows tied to currency movements or equity movements.

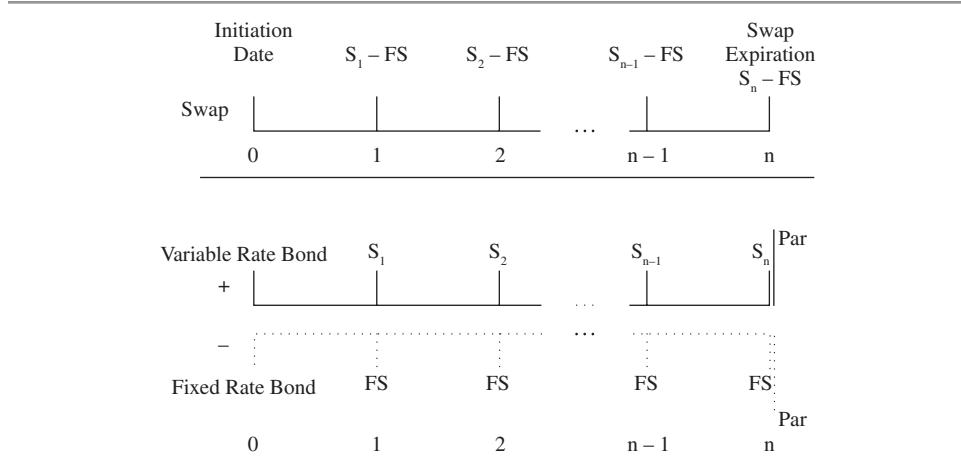
EXHIBIT 14 Generic Swap Cash Flows: Receive-Floating, Pay-Fixed



We again will rely on the arbitrage approach for determining the pricing of a swap. This procedure involves finding the fixed swap rate such that the value of the swap at initiation is zero. Recall that the goal of the arbitrageur is to generate positive cash flows with no risk and no investment of one's own capital. Thus, it is helpful to be able to synthetically create a swap with a portfolio of other instruments. A receive-floating, pay-fixed swap is equivalent to being long a floating-rate bond and short a fixed-rate bond. Assuming both bonds were purchased at par, the initial cash flows are zero and the par payments at the end offset each other. Thus, the fixed bond payment should be equivalent to the fixed swap payment. Exhibit 15 shows the view of a swap as a pair of bonds. Note that the coupon dates on the

bonds match the settlement dates on the swap and the maturity date matches the expiration date of the swap.<sup>17</sup>

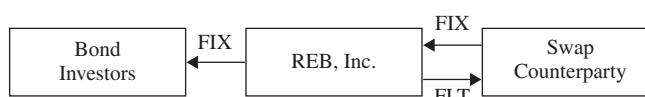
EXHIBIT 15 Receive-Floating, Pay-Fixed as a Portfolio of Bonds



As futures contracts can be viewed as marketable forward contracts, swaps can also be viewed as a portfolio of futures contracts.<sup>18</sup> In addition, because a single forward contract can be viewed as a portfolio of a call and a put option, a swap can also be viewed as a portfolio of options.<sup>19</sup>

Market participants often use swaps to transform one series of cash flows into another. For example, suppose that because of the relative ease of issuance, REB, Inc. sells a fixed-rate bond to investors. Based on careful analysis of the interest rate sensitivity of the company's assets, REB's leadership deems a Libor-based variable rate bond to be more appropriate. By entering a receive-fixed, pay-floating interest rate swap, REB can create a synthetic floating-rate bond, as illustrated in Exhibit 16. REB issues fixed-rate bonds and thus must make periodic fixed-rate-based payments, denoted FIX. REB then enters a receive-fixed (FIX) and pay-floating (FLT) interest rate swap. The two fixed rate payments cancel, leaving on net the floating-rate payments. Thus, we say that REB has created a synthetic floating-rate loan.

EXHIBIT 16 REB's Synthetic Floating-Rate Bond Based on Fixed-Rate Bond Issuance with Receive-Fixed Swap



<sup>17</sup>As with all derivative instruments, there are numerous technical details that have been simplified here. We will explore some of these details shortly.

<sup>18</sup>In practice, futures have standardized characteristics, so there is rarely a set of futures contracts that can perfectly replicate a swap.

<sup>19</sup>For example, a long forward contract is equivalent to a long call and a short put with the strike price equal to the forward price.

The example in Exhibit 16 is for a swap in which the underlying is an interest rate. There are also currency swaps and equity swaps. Currency swaps can be used in a similar fashion, but the risks being addressed are both interest rate and currency exposures. Equity swaps can also be used in a similar fashion, but the risk being addressed is equity exposure.

Swaps have several technical nuances that can have a significant influence on pricing and valuation. Differences in payment frequency and day count methods often have a material impact on pricing and valuation. Another difficult issue is identifying the appropriate discount rate to apply to the future cash flows. We turn now to examining three types of swap contracts—interest rate, currency, and equity—with a focus on pricing and valuation.

#### 4.1. Interest Rate Swap Contracts

One approach to pricing and valuing interest rate swaps is based on a pair of bonds. We first need to introduce some basic notation and typical structures. It is important to understand that because they are OTC products in which the characteristics are agreed upon by the counterparties, swaps can be designed with an infinite number of variations. For example, a plain vanilla Libor-based interest rate swap can involve different frequencies of cash flow settlements and day count conventions. In fact, a swap can have both semi-annual payments and quarterly payments, as well as actual day counts and day counts based on 30 days per month. Also, the notional amount can vary across the maturities, such as would occur when aligning a swap with an amortizing loan. Thus, it is important to build in our models the flexibility to handle these variations and issues. Unless stated otherwise, we will assume the notional amounts are all equal to one ( $NA = 1$ ); hence, we do not consider amortizing swaps here. Swap values per 1 notional amount can be simply multiplied by the actual notional amount to arrive at the swap's fair market value.

Interest rate swaps have two legs, typically a floating leg (FLT) and a fixed leg (FIX). The floating leg cash flow (denoted  $S_i$  to be consistent with other underlying instruments) can be expressed as

$$S_i = CF_{FLT,i} = AP_{FLT,i}r_{FLT,i} = \left( \frac{NAD_{FLT,i}}{NTD_{FLT,i}} \right) r_{FLT,i}$$

and the fixed leg cash flow (denoted FS) can be expressed as

$$FS = CF_{FIX,i} = AP_{FIX,i}r_{FIX} = \left( \frac{NAD_{FIX,i}}{NTD_{FIX,i}} \right) r_{FIX}$$

where  $CF_i$  simply reminds us that our focus is on cash flows,  $AP_i$  denotes the accrual period,  $r_{FLT,i}$  denotes the observed floating rate appropriate for Time  $i$ ,  $NAD_i$  denotes the number of accrued days during the payment period,  $NTD_i$  denotes the total number of days during the year applicable to cash flow  $i$ , and  $r_{FIX}$  denotes the fixed swap rate. The accrual period accounts for the payment frequency and day count methods. The two most popular day count methods are known as 30/360 and ACT/ACT. As the name suggests, 30/360 treats each month as having 30 days, and thus a year has 360 days. ACT/ACT treats the accrual period as having the actual number of days divided by the actual number of days in the year (365 or 366). Finally,

the convention in the swap market is that the floating interest rate is assumed to be advanced set and settled in arrears; thus,  $r_{FLT,i}$  is set at the beginning of period  $i$  and paid at the end.<sup>20</sup> If we assume constant accrual periods, the receive-fixed, pay-floating net cash flow can be expressed as

$$FS - S_i = AP(r_{FIX} - r_{FLT,i})$$

and the receive-floating, pay-fixed net cash flow can be expressed as

$$S_i - FS = AP(r_{FLT,i} - r_{FIX})$$

As a simple example, if the fixed rate is 5%, the floating rate is 5.2%, and the accrual period is 30 days based on a 360 day year, the payment of a receive-fixed, pay-floating swap is calculated as  $(30/360)(0.05 - 0.052) = -0.000167$  per notional of 1. Because the floating rate exceeds the fixed rate, the party that pays floating (and receives fixed) would pay this amount to the party that receives floating (and pays fixed). In other words, there is only a single payment made from one party to the other.

We now turn to swap pricing. Exhibit 17 shows the cash flows for an interest rate swap along with a pair of bonds each with the same par amount.<sup>21</sup> Suppose the arbitrageur enters a receive-fixed, pay-floating interest rate swap with some initial value  $V$ . Because we are exploring the equilibrium fixed swap rate, we do not first assume the swap value is in fact zero or in equilibrium. Because this swap will lose value when floating rates rise, the arbitrageur purchases a variable rate bond whose value is denoted  $VB$ —satisfying Rule #2 of not taking any risk. Note that the terms of the variable rate bond are selected to match exactly the floating payments of the swap. To satisfy Rule #1 of not spending money, a fixed-rate bond is sold short—equivalent to borrowing funds—with terms to match exactly the fixed payments of the swap.

EXHIBIT 17 Cash Flows for Receive-Fixed Swap Hedge with Bonds

Steps	Time 0	Time 1	Time 2	...	Time n
1. Receive fixed swap	-V	+FS - $S_1$	+FS - $S_2$	...	+FS - $S_n$
2. Buy floating-rate bond	-VB	+ $S_1$	+ $S_2$	...	+ $S_n$ + Par
3. Short sell fixed-rate bond	+FB	-FS	-FS	...	-(FS + Par)
Net cash flows	-V - VB + FB	0	0	0	0

<sup>20</sup>Often, interest rate swaps are used to convert floating-rate loans to synthetic fixed rate loans. These floating-rate loans are advanced set, settled in arrears. Otherwise, while interest is accruing, we have no idea what rate is being applied until the end. Thus, with advanced set, settled in arrears, the interest begins accruing at a known rate and then the interest is paid at the end of the period, whereupon the interest rate is reset once again.

<sup>21</sup>The underlying bonds have a designated par value on which their interest payments are based, whereas swaps are based on a notional amount that is never paid. The notional amount determines the size of the swap interest payments. Thus, a swap is like an offsetting pair of bonds with interest payments but no principal payments. In general, the notional amount of the swap will equal the par value of the underlying bonds.

Thus, the fixed coupon such that the floating-rate bond price equals the fixed-rate bond price is the equilibrium fixed swap rate. That is, in equilibrium we must have  $-V - VB + FB = 0$  or else there is an arbitrage opportunity. For a receiver of a fixed rate and payer of a floating rate, the value of the swap is

$$V = \text{Value of fixed bond} - \text{Value of floating bond} = FB - VB \quad (11)$$

The value of a receive-fixed, pay-floating interest rate swap is simply the value of buying a fixed-rate bond and issuing a floating-rate bond.<sup>22</sup> If we further stipulate that pricing the swap means to determine the fixed rate such that the value of the swap at initiation is zero, then the value of the fixed bond must equal the value of the floating bond.

The value of a floating-rate bond, assuming we are on a reset date and the interest payment matches the discount rate, is par, assumed to be 1 here. The value of a fixed bond is as follows:

$$\text{Fixed bond rate: } FB = C \sum_{i=1}^n PV_{0,t_i}(1) + PV_{0,t_n}(1) \quad (12)$$

where  $C$  denotes the coupon amount for the fixed-rate bond and  $PV_{0,t_i}(1)$  is the appropriate present value factor for the  $i^{\text{th}}$  fixed cash flow.

Based on the value of these bonds and noting that the fixed coupon amount is equivalent to the fixed swap rate,  $r_{\text{FIX}}$ , we obtain the swap pricing equation:

$$\text{Swap pricing equation: } r_{\text{FIX}} = \frac{1 - PV_{0,t_n}(1)}{\sum_{i=1}^n PV_{0,t_i}(1)} \quad (13)$$

The fixed swap rate is simply one minus the final present value term divided by the sum of present values. Therefore, one interpretation of the fixed swap rate is that it should be equal to the fixed rate on a par bond, which is the ratio one minus the present value of the final cash flow all divided by an annuity.<sup>23</sup>

The fixed swap leg cash flow for a unit of notional amount is simply the fixed swap rate adjusted for the accrual period, or  $FS_i = AP_{\text{FIX},i}r_{\text{FIX}}$ . Alternatively, the annualized fixed swap rate is equal to the fixed swap leg cash flow divided by the fixed rate accrual period, or  $r_{\text{FIX},i} = FS_i/AP_{\text{FIX},i}$ . Note that if the accrual period varies across the swap payments, then the fixed swap payment will also vary. Thus, when relevant, a subscript  $i$  will be used. Often the fixed leg accrual period is constant; hence, the subscript can be safely omitted.

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<sup>22</sup>In Exhibit 17, the trades illustrated in Steps 2 and 3 are synthetically creating an offsetting position; hence, the floating bond is purchased and the fixed bond is short sold.

<sup>23</sup>The denominator of Equation 13 is simply the sum of the present values of receiving one currency unit on each payment date or an annuity.

### EXAMPLE 13 Solving for the Fixed Swap Rate Based on Present Value Factors

Suppose we are pricing a five-year Libor-based interest rate swap with annual resets (30/360 day count). The estimated present value factors,  $PV_{0,t_i}(1)$ , are given in the following table.

Maturity (years)	Present Value Factors
1	0.990099
2	0.977876
3	0.965136
4	0.951529
5	0.937467

The fixed rate of the swap will be *closest* to:

- A. 1.0%.
- B. 1.3%.
- C. 1.6%.

*Solution:* B is correct. Note that the sum of present values is

$$\sum_{i=1}^n PV_{0,t_i}(1) = 0.990099 + 0.977876 + 0.965136 + 0.951529 + 0.937467 \\ = 4.822107$$

Therefore, the solution for the fixed swap rate is

$$r_{\text{FIX}} = \frac{1 - PV_{0,t_n}(1)}{\sum_{i=1}^n PV_{0,t_i}(1)} = \frac{1 - 0.937467}{4.822107} = 0.012968, \text{ or } 1.2968\%$$

We now turn to interest rate swap valuation. Following a similar pattern as forward contracts, Exhibit 18 shows the cash flows for a receive-fixed interest rate swap initiated at Time 0 but that needs to be valued at Time  $t$  expressed per unit of the underlying currency. We achieve this valuation through entering an offsetting swap—receive-floating, pay-fixed. The floating sides offset, leaving only the difference in the fixed rates. We assume  $n'$  remaining cash flows. At Time  $t$ , the swap value is represented as the funds need to generate the appropriate future cash flows.

EXHIBIT 18 Cash Flows for Receive-Fixed Swap Valued at Time  $t$ 

Steps	Time $t$	Time 1	Time 2	...	Time $n'$
1. Receive fixed swap (Time 0)	$-V$	$+FS_0 - S_1$	$+FS_0 - S_2$	...	$+FS_0 - S_{n'}$
2. Receive floating swap (Time $t$ )	0	$S_1 - FS_t$	$S_2 - FS_t$	...	$S_{n'} - FS_t$
Net cash flows	$-V$	$FS_0 - FS_t$	$FS_0 - FS_t$	...	$FS_0 - FS_t$

Thus, the value of a fixed rate swap at some future point in Time  $t$  is simply the sum of the present value of the difference in fixed swap rates times the stated notional amount (denoted NA), or

$$V = NA(FS_0 - FS_t) \sum_{i=1}^{n'} PV_{t,t_i} \quad (14)$$

It is important to be clear on which side this value applies. The rate  $FS_0$  is the fixed rate established at the start of the swap and goes to the party receiving fixed. Thus, when Equation 14 with  $FS_0$  having a positive sign is used, it provides the value to the party receiving fixed. The negative of this amount is the value to the fixed rate payer.

The examples illustrated here show swap valuation only on a payment date. If a swap is being valued between payment dates, some adjustments are necessary. We do not pursue this topic here.

## EXAMPLE 14 Solving for the Swap Value Based on Present Value Factors

Suppose two years ago we entered a €100,000,000 seven-year receive-fixed Libor-based interest rate swap with annual resets (30/360 day count). The fixed rate in the swap contract entered two years ago was 2%. Again, the estimated present value factors,  $PV_{0,t_i}(1)$ , are repeated from the previous example.

Maturity (years)	Present Value Factors
1	0.990099
2	0.977876
3	0.965136
4	0.951529
5	0.937467

From the previous example, we know the current equilibrium fixed swap rate is 1.3% (two years after the swap was originally entered).

1. The value (in thousands) for the party receiving the fixed rate will be *closest* to:
  - A. -€5,000.
  - B. €3,375.
  - C. €4,822.
2. The value (in thousands) for the party in the swap receiving the floating rate will be *closest* to:
  - A. -€4,822.
  - B. -€3,375.
  - C. €5,000.

*Solution to 1:* B is correct. Recall the sum of present values is 4.822107. Thus, the swap value per dollar notional is

$$\begin{aligned}
 V &= (FS_0 - FS_t) \sum_{i=1}^{n'} PV_{t,t_i} \\
 &= (0.02 - 0.013)4.822107 \\
 &= 0.03375
 \end{aligned}$$

Thus, the swap value is €3,375,000.

*Solution to 2:* B is correct. The equivalent receive-floating swap value is simply the negative of the receive-fixed swap value.

## 4.2. Currency Swap Contracts

A currency swap is a contract in which two counterparties agree to exchange future interest payments in different currencies. These interest payments can be based on either a fixed interest rate or a floating interest rate. Thus, with the addition of day count options and payment frequencies, there are many different ways to set up a currency swap. There are four major types of currency swaps: fixed-for-fixed, floating-for-fixed, fixed-for-floating, and floating-for-floating.

Currency swaps come in a wide array of types and structures. We review a few key features. First, currency swaps often but not always involve an exchange of notional amounts at both the initiation of the swap and at the expiration of the swap. Second, the payment on each leg of the swap is in a different currency unit, such as euros and Japanese yen, and the payments are not netted. Third, each leg of the swap can be either fixed or floating. To understand the pricing and valuation of currency swaps, we need a general approach that is flexible enough to handle each of these situations. We first focus on the fixed-for-fixed currency swaps with a very simple structure and only then consider other variations.

Currency swap pricing has three key variables: two fixed interest rates and one notional amount. Pricing a currency swap involves solving for the appropriate notional amount in one currency, given the notional amount in the other currency, as well as two fixed interest rates such that the currency swap value is zero at initiation. Because one notional amount is given, there are three swap pricing variables.

Because we are focused on fixed-for-fixed currency swaps, we need notation that reflects the different generic currency units. Thus, we let  $k = a$  and  $b$  to reflect two different currency units, such as euros and yen. Letters are used rather than numbers to avoid confusion with calendar time. The value of a fixed-rate bond in Currency  $k$  can be expressed generically as

$$FB_k = C_k \sum_{i=1}^n PV_{0,t_i,k}(1) + PV_{0,t_n,k}(Par_k)$$

where  $k = a$  or  $b$ ,  $C_k$  denotes the periodic fixed coupon amount in Currency  $k$ ,  $\sum_{i=1}^n PV_{0,t_i,k}(1)$

denotes the present value from Time 0 to Time  $t_i$  discounting at the Currency  $k$  risk-free rate, and  $Par_k$  denotes the  $k$  currency unit par value. We do not assume par equals 1 because the notional amounts are typically different in each currency within the currency swap.

Exhibit 19 shows the cash flows for a fixed-for-fixed currency swap along with an offsetting pair of fixed-rate bonds. In this case, notice that the two bonds are in different currencies.<sup>24</sup> We assume the arbitrage cash flows will be evaluated in currency unit  $a$ . Therefore, all cash flows are converted to Currency  $a$  in the cash flow table based on the exchange rate denoted  $S_i$ —expressed as the number of units of Currency  $a$  for one unit of Currency  $b$  at Time  $i$ . We again ignore the technical nuances and assume the same accrual periods on both legs of the swap. Note that all the future cash flows, expressed in Currency  $a$ , are zero because the coupon rates on the fixed-rate bonds were selected to equal the fixed swap rates. Because we are demonstrating swap pricing, we do not assume the currency swap is initially valued correctly; hence,  $V$  can be either positive or negative. We initially use a negative sign, because an investment usually involves negative cash flows. We assume the par value of each bond is the same as the notional amount of each leg of the swap. From the arbitrageur's perspective, whether there is an exchange of notional amounts on the initiation date is not relevant because this exchange will be done at the current foreign exchange rate, and hence, it will have a fair value of zero. It is important, however, that this exchange of notional amounts is done at expiration. Because the swap notional amounts differ between the two currencies, it would be confusing to express these results per unit of Currency  $a$ . Therefore, each leg of the swap is assumed to have different notional amounts, but  $Par_a = NA_a$  and  $Par_b = NA_b$  in order to achieve zero cash flow at Time  $n$ .

EXHIBIT 19 Cash Flows for Currency Swap Hedged with Bonds

Steps	Time 0	Time 1	Time 2	...	Time $n$
1. Enter currency swap	$-V_a$	$+FS_a - S_1 FS_b$	$+FS_a - S_2 FS_b$	...	$+FS_a + NA_a$ $- S_n (FS_b + NA_b)$
2. Short sell bond in Currency $a$	$+FB_a (C_a = FS_a)$	$-FS_a$	$-FS_a$	...	$-(FS_a + Par_a)$
3. Buy bond in Currency $b$	$-S_0 FB_b (C_b = FS_b)$	$+S_1 FS_b$	$+S_2 FS_b$	...	$+S_n (FS_b + Par_b)$
Net cash flows	$-V_a + FB_a - S_0 FB_b$	0	0	0	0

Based on this table, in equilibrium we must have

$$-V_a + FB_a - S_0 FB_b = 0$$

<sup>24</sup>Technically, we build these portfolios such that the initial value in each currency is par.

and the fixed-for-fixed currency swap value is

$$V_a = FB_a - S_0 FB_b$$

or else there is an arbitrage opportunity. Notice that the two-bond approach allows the arbitrageur to avoid having to convert one currency into another in the future. This approach mitigates all future currency exposure and basically identifies the current exchange rate that makes the value of the two bonds equal. Remember that the exchange rate  $S_0$  is the number of Currency a units for one unit of Currency b at Time 0; thus,  $S_0 FB_b$  is expressed in Currency a units.

Exhibit 20 provides a simple illustration of an at-market 10-year receive-fixed US\$ and pay-fixed € swap, for which the annual reset coupon amount in US dollars is US\$10 with par of US\$1,300 and the annual reset coupon amount in euros is €9 with par of €1,000. Both bonds are assumed to be trading at par and have a 10-year maturity. This exhibit assumes a current spot exchange rate ( $S_0$ ) at which €1 trades for US\$1.3, and selected future spot exchange rates are  $S_1 = \$1.5$ ,  $S_2 = \$1.1$ , and  $S_{10} = \$1.2$ . These future spot exchange rates are used to illustrate the conversion of future euro cash flows into US dollars, but notice that the cash flows are all zero regardless of the future spot exchange rates. In other words, we could have used any numbers for  $S_1$ ,  $S_2$ , and  $S_{10}$ .

EXHIBIT 20 Numerical Example of Currency Swap Hedged with Bonds

Steps	Time 0	Time 1	Time 2	...	Time 10
1. Enter currency swap	0	+\$10 – (\$1.5/€)€9 = -\$3.5	+\$10 – (\$1.1/€)€9 = \$0.1	...	+\$10 + \$1,300 –(\$1.2/€)(€9 + €1,000) = \$99.2
2. Short sell US dollar bond	+\$1,300	-\$10	-\$10	...	–(\$10 + \$1,300)
3. Buy euro bond	–(\$1.3/€)€1,000	+\$(\$1.5/€)€9	+\$(\$1.1/€)€9	...	+\$(\$1.2/€)(€9 + €1,000)
Net cash flows	0	0	0	0	0

Clearly, if the initial swap value is not at market or zero, then there are arbitrage opportunities. If the initial swap value is positive, then this set of transactions would be implemented. If the initial swap value is negative, then the opposite set of transactions would be implemented. Specifically, enter a pay-US dollar, receive-euro swap, buy Currency a bonds, and short sell Currency b bonds. As before, the swap value after initiation is a simple variation of the expression above—specifically,

$$V_a = FB_a - S_0 FB_b = 1,300 - 1.3(1,000) = 0$$

Note further that  $C_a = FS_a$  and  $C_b = FS_b$  are fixed swap payment amounts stipulated in the currency swap. One way to find the equilibrium currency swap price (that is, the two fixed rates) is to identify the initial coupon rates ( $C_{0,a}$  and  $C_{0,b}$ ) such that the two bonds trade at par—specifically,

$$FB_a(C_{0,a} Par_a) = Par_a$$

and

$$FB_b(C_{0,b}, Par_b) = Par_b$$

In equilibrium, the notional amounts of the two legs of the currency swap are  $NA_b = Par_b$  and  $NA_a = Par_a = S_0 Par_b$ . That is, one first decides the par value desired in one currency and then solves for the implied notional amount in the other currency.

The goal is to determine the fixed rates of the swap such that the current swap value is zero; then we have

$$FB_a(C_{0,a}, Par_a) = S_0 FB_b(C_{0,b}, Par_b)$$

Because the fixed swap rate does not depend on the notional amounts, the fixed swap rates are found in exactly the same manner as the fixed interest rate swap rate. For emphasis, we repeat the equilibrium fixed swap rate equations for each currency:

$$r_{FIX,a} = \frac{1 - PV_{0,t_n,a}(1)}{\sum_{i=1}^n PV_{0,t_i,a}(1)}$$

and

$$r_{FIX,b} = \frac{1 - PV_{0,t_n,b}(1)}{\sum_{i=1}^n PV_{0,t_i,b}(1)} \quad (15)$$

Again, the fixed swap rate in each currency is simply one minus the final present value term divided by the sum of present values. We need to be sure that the present value terms are expressed on the basis of the appropriate currency.

We illustrate currency swap pricing with spot rates by way of an example.

### EXAMPLE 15 Currency Swap Pricing with Spot Rates

A US company needs to borrow 100 million Australian dollars (A\$) for one year for its Australian subsidiary. The company decides to issue US-denominated bonds in an amount equivalent to A\$100 million. Then the company enters into a one-year currency swap with quarterly reset (30/360 day count) and the exchange of notional amounts at initiation and at maturity. At the swap's initiation, the US company receives the notional amount in Australian dollars and pays to the counterparty the notional amount in US dollars. At the swap's expiration, the US company pays the notional amount in Australian dollars and receives from the counterparty the notional amount in US dollars. Based on interbank rates, we observe the following spot rates today, at Time 0:

Days to Maturity	A\$ Spot Interest Rates (%)	US\$ Spot Interest Rates (%)
90	2.50	0.10
180	2.60	0.15
270	2.70	0.20
360	2.80	0.25

Assume that the counterparties in the currency swap agree to an A\$/US\$ spot exchange rate of 1.140 (expressed as number of Australian dollars for US\$1).

1. The annual fixed swap rates for Australian dollars and US dollars, respectively, will be *closest* to:
  - A. 2.80% and 0.10%.
  - B. 2.77% and 0.25%.
  - C. 2.65% and 0.175%.
2. The notional amount (in US\$ millions) will be *closest* to:
  - A. 88.
  - B. 100.
  - C. 114.
3. The fixed swap quarterly payments in the currency swap will be *closest* to:
  - A. A\$692,000 and US\$55,000.
  - B. A\$220,000 and US\$173,000.
  - C. A\$720,000 and US\$220,000.

*Solution to 1:* B is correct. We first find the PV factors and then solve for the fixed swap rates. The present value expression based on spot rates (not forward rates) is

$$PV_{0,t_i}(1) = \frac{1}{1 + r_{\text{Spot}_i} \left( \frac{NAD_i}{NTD} \right)}.$$

Spot rates cover the entire period from 0 to  $t_i$ , unlike

forward rates, which cover incremental periods. Based on the data given, we construct the following present value data table. The calculations are shown to the sixth decimal place in an effort to minimize rounding error. Rounding differences may occur in the solutions.

Days to Maturity	A\$ Spot Interest Rates (%)	Present Value (A\$1)	US\$ Spot Interest Rates (%)	Present Value (US\$1)
90	2.50	0.993789 <sup>a</sup>	0.10	0.999750
180	2.60	0.987167	0.15	0.999251 <sup>b</sup>
270	2.70	0.980152	0.20	0.998502
360	2.80	0.972763	0.25	0.997506
	<i>Sum:</i>	<i>3.933870</i>	<i>Sum:</i>	<i>3.995009</i>

<sup>a</sup>A\$0.993789 = 1/[1 + 0.0250(90/360)].

<sup>b</sup>US\$0.999251 = 1/[1 + 0.00150(180/360)].

Therefore, the Australian dollar periodic rate is

$$r_{\text{FIX.AUD}} = \frac{1 - PV_{0,t_4, \text{AUD}}(1)}{\sum_{i=1}^4 PV_{0,t_i, \text{AUD}}(1)} = \frac{1 - 0.972763}{3.933870} \\ = 0.00692381 \text{ or } 0.692381\%$$

and the US dollar periodic rate is

$$r_{\text{FIX.USD}} = \frac{1 - PV_{0,t_4, \text{USD}}(1)}{\sum_{i=1}^4 PV_{0,t_i, \text{USD}}(1)} = \frac{1 - 0.997506}{3.995009} \\ = 0.00062422 \text{ or } 0.062422\%$$

The annualized rate is simply  $(360/90)$  times the period results: 2.7695% for Australian dollars and 0.2497% for US dollars.

*Solution to 2:* A is correct. The US dollar notional amount is calculated as A\$100 million divided by the current spot exchange rate at which US\$1 dollar trades for A\$1.1400. This exchange is equal to US\$87,719,298 ( $= \text{A\$}100,000,000/1.14$ ).

*Solution to 3:* A is correct. The fixed swap payments in currency units equal the periodic swap rate times the appropriate notional amounts. From the answers to 1 and 2, we have

$$FS_{\text{A\$}} = NA_{\text{A\$}}(\text{AP})r_{\text{FIX,A\$}} \\ = \text{A\$}100,000,000(90/360)(0.027695) \\ = \text{A\$}692,375$$

and

$$FS_{\text{US\$}} = NA_{\text{US\$}}(\text{AP})r_{\text{FIX,US\$}} \\ = \text{US\$}87,719,298(90/360)(0.002497) \\ = \text{US\$}54,759.$$

Therefore, one approach to pricing currency swaps is to view the swap as a pair of fixed-rate bonds. The main advantage of this approach is that all foreign exchange considerations are moved to the initial exchange rate. We do not need to address future foreign currency transactions. Also, note that a fixed-for-floating currency swap is simply a fixed-for-fixed currency swap paired with a floating-for-fixed interest rate swap. Also, we do not technically “price” a floating-rate swap, because we do not designate a single coupon rate, and the value

of such a swap is par on any reset date. Thus, we have the capacity to price any variation of currency swaps.

We now turn to currency swap valuation. Recall that with currency swaps, there are two main sources of risk: interest rates and exchange rates. Exhibit 21 shows the cash flows from three transactions. Note this exhibit is similar to the currency swap pricing exhibit, but the currency swap was initiated at Time 0 and here we are evaluating it at Time  $t$ . Step 1 shows the cash flows for a fixed-for-fixed currency swap expressed in units of Currency a. Step 2 is borrowing or short selling a bond in Currency a to generate sufficient funds to exactly offset the currency swap cash flows that are in units of Currency a. Step 3 is lending or buying a bond in Currency b to generate sufficient funds to exactly offset the currency swap cash flows that are in units of Currency b. The net cash flows at each future point in time are zero. Recall that  $S_i$  denotes the spot exchange rate in units of Currency a for each unit of Currency b at Time  $t_i$ . Thus,  $S_t FS_{b,0}$  is the value of the Currency b fixed cash flow expressed in Currency a at Time  $t$ . From a value perspective,  $FS_{b,0}$  is equivalent in value in Currency b to  $S_t FS_{b,0}$  in Currency a. Hence, the future net cash flows are all zero.

EXHIBIT 21 Cash Flows for Currency Swap Hedged with Bonds

Steps	Time $t$	Time 1	Time 2	...	Time $n'$
1. Currency swap	$-V_a$	$+FS_{a,0} - S_1 FS_{b,0}$	$+FS_{a,0} - S_2 FS_{b,0}$	...	$+FS_{a,0} + NA_{a,0}$ $- S_n'(FS_{b,0} + NA_{b,0})$
2. Short sell bond (a)	$+FB_a$	$-FS_{a,0}$	$-FS_{a,0}$	...	$-(FS_{a,0} + NA_{a,0})$
3. Buy bond (b)	$-S_t FB_b$	$+S_1 FS_{b,0}$	$+S_2 FS_{b,0}$	...	$-S_n'(FS_{b,0} + NA_{b,0})$
Net cash flows	0	0	0		0

The value of a fixed-for-fixed currency swap at some future point in time, Time  $t$ , is simply the difference in a pair of fixed-rate bonds, one expressed in Currency a and one expressed in Currency b. To express the bonds in the same currency units, we convert the Currency b bond into units of Currency a through a spot foreign exchange transaction. Hence, we have

$$V_a = FB_a - S_0 FB_b \\ = FS_{a,0} \sum_{i=1}^{n'} PV_{t,t_i,a} + NA_{a,0} PV_{t,t_{n'},a} - S_t \left( FS_{b,0} \sum_{i=1}^{n'} PV_{t,t_i,b} + NA_{b,0} PV_{t,t_{n'},b} \right)$$

Note that the fixed swap amount (FS) is the per-period fixed swap rate times the notional amount. Therefore, the currency swap valuation equation can be expressed as

$$V_a = NA_{a,0} \left( r_{FIX,a,0} \sum_{i=1}^{n'} PV_{t,t_i,a} + PV_{t,t_{n'},a} \right) - S_t NA_{b,0} \left( r_{FIX,b,0} \sum_{i=1}^{n'} PV_{t,t_i,b} + PV_{t,t_{n'},b} \right) \quad (16)$$

### EXAMPLE 16 Currency Swap Valuation with Spot Rates

This example builds on the previous example addressing currency swap pricing. Recall that a US company needed to borrow 100 million Australian dollars (A\$) for one year for its Australian subsidiary. The company decided to borrow in US dollars (US\$) an amount equivalent to A\$100 million by issuing US-denominated bonds. The company entered into a one-year currency swap with quarterly reset (30/360 day count) and exchange of notional amounts at initiation and at maturity. At the swap's expiration, the US company pays the notional amount in Australian dollars and receives from the counterparty the notional amount in US dollars. The fixed rates were 2.7695% for Australian dollars and 0.2497% for US dollars. The notional amount in US dollars was US\$87,719,298.

Assume 60 days have passed and we observe the following market information:

Days to Maturity	A\$ Spot		US\$ Spot	
	Interest Rates (%)	Present Value (A\$1)	Interest Rates (%)	Present Value (US\$1)
30	2.00	0.998336	0.50	0.999584
120	1.90	0.993707	0.40	0.998668
210	1.80	0.989609	0.30	0.998253
300	1.70	0.986031	0.20	0.998336
	<i>Sum:</i>	3.967683	<i>Sum:</i>	3.994841

The currency spot exchange rate is now A\$1.13 for US\$1.

The current value to the US company of the currency swap entered into 60 days ago will be *closest* to:

- A. -AD\$2,000,000.
- B. AD\$2,000,000.
- C. -AD\$2,145,200.

*Solution:* C is correct. Based on the data given, the currency swap value to the US company is

$$\begin{aligned}
 V_a &= -NA_{a,0} \left( r_{\text{FIX},a,0} \sum_{i=1}^{n'} PV_{t,t_i,a} + PV_{t,t_{n'},a} \right) + S_0 NA_{b,0} \left( r_{\text{FIX},b,0} \sum_{i=1}^{n'} PV_{t,t_i,b} + PV_{t,t_{n'},b} \right) \\
 &= -100,000,000 [0.00692381(3.967683) + 0.986031] + 1.13(87,719,298) \\
 &\quad [0.00062422(3.994841) + 0.998336] \\
 &= -2,145,200
 \end{aligned}$$

In other words, the value of the payments to be made exceeds the value of the payments to be received by this amount. This value incorporates the change in the exchange rate and changes in interest rates in both countries since the start of the swap.

### 4.3. Equity Swap Contracts

Drawing on our prior definition of a swap, we define an equity swap in the following manner: An **equity swap** is an OTC derivative contract in which two parties agree to exchange a series of cash flows whereby one party pays a variable series that will be determined by an equity and the other party pays either (1) a variable series determined by a different equity or rate or (2) a fixed series. An equity swap is used to convert the returns from an equity investment into another series of returns, which, as noted, either can be derived from another equity series or can be a fixed rate. Equity swaps are widely used in equity portfolio investment management to modify returns and risks.

We examine three types of equity swaps: receive-equity return, pay-fixed; receive-equity return, pay-floating; and receive-equity return, pay-another equity return. Like interest rate swaps and currency swaps, there are several unique nuances for equity swaps. We highlight just a few. First, the underlying reference instrument for the equity leg of an equity swap can be an individual stock, a published stock index, or a custom portfolio. Second, the equity leg cash flow can be with or without dividends. Third, all the interest rate swap nuances exist with equity swaps that have a fixed or floating interest rate leg.

We focus here on viewing an equity swap as a portfolio of an equity position and a bond. The equity swap cash flows can be expressed as follows:

NA(Equity return – Fixed rate) (for receive-equity, pay-fixed),  
 NA(Equity return – Floating rate) (for receive-equity, pay-floating), and  
 NA(Equity return<sub>a</sub> – Equity return<sub>b</sub>) (for receive-equity, pay-equity),

where a and b denote different equities. Note that an equity-for-equity swap can be viewed simply as a receive-equity a, pay-fixed swap combined with a pay-equity b, receive-fixed swap. The fixed payments cancel out, and we have synthetically created an equity-for-equity swap.

#### EXAMPLE 17 Equity Swap Cash Flows

Suppose we entered into a receive-equity index and pay-fixed swap. It is quarterly reset, 30/360 day count, €5,000,000 notional amount, pay-fixed (1.6% annualized, quarterly pay, or 0.4% per quarter).

1. If the equity index return was 4.0% for the quarter (not annualized), the equity swap cash flow will be *closest* to:
  - A. –€220,000.
  - B. –€180,000.
  - C. €180,000.
2. If the equity index return was –6.0% for the quarter (not annualized), the equity swap cash flow will be closest to:
  - A. –€320,000.
  - B. –€180,000.
  - C. €180,000.

*Solution to 1:* C is correct. Note that the equity index return is reported on a quarterly basis. It is not an annualized number. The fixed leg is often reported on an annual basis. Thus, one must carefully interpret the different return conventions. In this case, receive-equity index counterparty cash flows are as follows:

$$\text{€}5,000,000(0.04 - 0.004) = \text{€}180,000 \text{ (Receive 4%, pay 0.4% for the quarter)}$$

*Solution to 2:* A is correct. Similar to 1, we have

$$\text{€}5,000,000(-0.06 - 0.004) = -\text{€}320,000 \text{ (Receive -6%, pay 0.4% for the quarter)}$$

When the equity leg of the swap is negative, then the receive-equity counterparty must pay both the equity return as well as the fixed rate (or whatever the payment terms are). Note, also, that equity swaps may cause liquidity problems. As seen here, if the equity return is negative, then the receive-equity return, pay-floating or pay-fixed swap may result in a large negative cash flow.

The cash flows for the equity leg of an equity swap can be expressed as

$$S_i = NA_E R_{E_i}$$

where  $R_{E_i}$  denotes the periodic return of the equity either with or without dividends as specified in the swap contract and  $NA_E$  denotes the notional amount. The cash flows for the fixed interest rate leg of the equity swap are the same as those of an interest rate swap, or

$$FS = NA_E AP_{FIX} r_{FIX}$$

where  $AP_{FIX}$  denotes the accrual period for the fixed leg for which we assume the accrual period is constant and  $r_{FIX}$  here denotes the fixed rate on the equity swap.

For equity swaps, the equity position could be a wide variety of claims, including the return on a stock index with or without dividends and the return on an individual stock with or without dividends. For our objectives here, we ignore the influence of dividends by assuming the equity swap leg assumes all dividends are reinvested in the equity position.<sup>25</sup> The equity leg of the swap is produced by selling the equity position on a reset date and reinvesting the original equity notional amount, leaving a remaining balance that is the cash flow required of the equity swap leg.<sup>26</sup> Exhibit 22 shows the cash flows from an equity swap arbitrage transaction.

<sup>25</sup>The arbitrage transactions for an equity swap when dividends are not included are extremely complex and beyond our objectives.

<sup>26</sup>Technically, we just sell off any equity value in excess of  $NA_E$  or purchase additional shares to return the equity value to  $NA_E$ , effectively generating  $S_i$ .

EXHIBIT 22 Cash Flows for Receive-Fixed Equity Swap Hedged with Equity and Bond

Steps	Time 0	Time 1	Time 2	...	Time n
1. Enter equity swap	-V	+FS - S <sub>1</sub>	+FS - S <sub>2</sub>	...	+FS - S <sub>n</sub>
2. Buy NA <sub>E</sub> equity	-NA <sub>E</sub>	+S <sub>1</sub>	+S <sub>2</sub>	...	+S <sub>n</sub> + NA <sub>E</sub>
3. Short sell fixed-rate bond	+FB(C = FS)	-FS	-FS	...	-(FS + Par)
4. Borrow arbitrage profit	-PV(Par - NA <sub>E</sub> )				Par - NA <sub>E</sub>
Net cash flows	-V - NA <sub>E</sub> + FB - PV(Par - NA <sub>E</sub> )	0	0	0	0

Let us examine the Time 1 cash flow. The equity swap is receive-fixed, pay-equity. For Step 1, if the equity-related cash flow  $S_1$  is less than the fixed-leg cash flow, then the swap generates a positive cash flow to this counterparty. For Step 2, the cash flow is simply the cash flow related to the equity movement and dividends, if applicable. Essentially, if the position value is greater than NA<sub>E</sub>, then the excess value is sold off, but if the position value is less than NA<sub>E</sub>, then an additional equity position is acquired. For Step 3, the short bond position requires the payment of coupons. Note that these coupons, by construction, equal the fixed leg cash flows. The sum of these three transactions is always zero.

Note the final cash flow for the long position in the equity includes the final sale of the underlying equity position. The final periodic return on the equity plus the original equity value will equal the proceeds from the final sale of the underlying equity position. Note that for the terminal cash flows to equal zero, we must either set the bond par value to equal the initial equity position or finance this difference. In this case, the bond par value could be different from the notional amount of equity. Therefore, in equilibrium, we have  $-V - NA_E + FB - PV(Par - NA_E) = 0$ , and hence, the equity swap value is  $V = -NA_E + FB - PV(Par - NA_E)$ .

The fixed swap rate can be expressed as the  $r_{FIX}$  rate such that  $FB_0 = NA_E + PV(Par - NA_E)$ . Note that assuming  $NA_E = Par = 1$ ,

$$r_{FIX} = \frac{1 - PV_{0,t_n}(1)}{\sum_{i=1}^n PV_{0,t_i}(1)}$$

You should recognize that the pricing of an equity swap is identical to the pricing of a comparable interest rate swap even though the future cash flows are dramatically different. If the swap required a floating payment, there would be no need to price the swap, as the floating side effectively prices itself at par automatically at the start. If the swap involves paying one equity return against another, there would also be no need to price it. You could effectively view this arrangement as paying equity a and receiving a fixed rate as specified above and receiving equity b and paying the same fixed rate. The fixed rates would cancel.

Valuing an equity swap after the swap is initiated ( $V_t$ ) is similar to valuing an interest rate swap except that rather than adjust the floating-rate bond for the last floating rate observed (remember, advanced set), we adjust the value of the notional amount of equity, or

$$V_t = FB_t(C_0) - (S_t/S_{t-})NA_E - PV(Par - NA_E) \quad (17)$$

where  $FB_t(C_0)$  denotes the Time  $t$  value of a fixed-rate bond initiated with coupon  $C_0$  at Time 0,  $S_t$  denotes the current equity price,  $S_{t-}$  denotes the equity price observed at the last reset date, and  $PV()$  denotes the present value function from Time  $t$  to the swap maturity time.

### EXAMPLE 18 Equity Swap Pricing

In Examples 13 and 14 related to interest rate swaps, we considered a five-year, annual reset, 30/360 day count, Libor-based swap. The following table provides the present values per €1.

Maturity (years)	Present Value Factors
1	0.990099
2	0.977876
3	0.965136
4	0.951529
5	0.937467

Assume an annual reset Libor floating-rate bond trading at par. The fixed rate was previously found to be 1.2968%. Given these same data, the fixed interest rate in the EURO STOXX 50 equity swap is *closest* to:

- A. 0.0%.
- B. 1.1%.
- C. 1.3%.

*Solution:* C is correct. The fixed rate on an equity swap is the same as that on an interest rate swap or 1.2968% as in Example 13. That is, the fixed rate on an equity swap is simply the fixed rate on a comparable interest rate swap.

### EXAMPLE 19 Equity Swap Valuation

Suppose six months ago we entered a receive-fixed, pay-equity five-year annual reset swap in which the fixed leg is based on a 30/360 day count. At the time the swap was entered, the fixed swap rate was 1.5%, the equity was trading at 100, and the notional amount was 10,000,000. Now all spot interest rates have fallen to 1.2% (a flat term structure), and the equity is trading for 105.

1. The fair value of this equity swap is *closest* to:
  - A. -300,000.
  - B. -500,000.
  - C. 500,000.

2. The value of the equity swap will be *closest* to zero if the stock price is:
- 100.
  - 102.
  - 105.

*Solution to 1:* A is correct. Because we have not yet passed the first reset date, there are five remaining cash flows for this equity swap. The fair value of this swap is found by solving for the fair value of the implied fixed-rate bond. We then adjust for the equity value. The fixed rate of 1.5% results in fixed cash flows of 150,000 at each settlement. Applying the respective present value factors, which are based on the new spot rates of 1.2%, gives us the following:

Date (in years)	Present Value Factors (PV)	Fixed Cash Flow	PV(Fixed Cash Flow)*
0.5	0.994036	150,000	149,105
1.5	0.982318	150,000	147,348
2.5	0.970874	150,000	145,631
3.5	0.959693	150,000	143,954
4.5	0.948767	10,150,000	9,629,981
		Total:	10,216,019

\* Answers may differ due to rounding.

Therefore, the fair value of this equity swap is 10,216,019 less 10,500,000 [ $= (105/100)10,000,000$ ], or a loss of 283,981.

*Solution to 2:* B is correct. The stock price at which this equity swap's fair value is zero would require (Par =  $NA_E$  in this case)

$$V_t = FB_t(C_0) - (S_t/S_{t-})NA_E$$

The value of the fixed leg is now approximately 102% of par; a stock price of 102 will result in a value of zero,

$$V_t = 102 - (S_t/100)100 = 0$$

where  $S_t$  is 102.

## 5. SUMMARY

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This reading on forward commitment pricing and valuation provides a foundation for understanding how forwards, futures, and swaps are both priced and valued.

Key points include the following:

- The arbitrageur would rather have more money than less and abides by two fundamental rules: Do not use your own money, and do not take any price risk.
- The no-arbitrage approach is used for the pricing and valuation of forward commitments and is built on the key concept of the law of one price, which states that if two investments have the same future cash flows, regardless of what happens in the future, these two investments should have the same current price.
- Throughout this reading, the following key assumptions are made:
  - Replicating instruments are identifiable and investable.
  - Market frictions are nil.
  - Short selling is allowed with full use of proceeds.
  - Borrowing and lending is available at a known risk-free rate.
- Carry arbitrage models used for forward commitment pricing and valuation are based on the no-arbitrage approach.
- With forward commitments, there is a distinct difference between pricing and valuation; pricing involves the determination of the appropriate fixed price or rate, and valuation involves the determination of the contract's current value expressed in currency units.
- Forward commitment pricing results in determining a price or rate such that the forward contract value is equal to zero.
- The price of a forward commitment is a function of the price of the underlying instrument, financing costs, and other carry costs and benefits.
- With equities, currencies, and fixed-income securities, the forward price is determined such that the initial forward value is zero.
- With forward rate agreements, the fixed interest rate is determined such that the initial value of the FRA is zero.
- Futures contract pricing here can essentially be treated the same as forward contract pricing.
- Because of daily marking to market, futures contract values are zero after each daily settlement.
- The general approach to pricing and valuing swaps as covered here is using a replicating or hedge portfolio of comparable instruments.
- With a basic understanding of pricing and valuing a simple interest rate swap, it is a straightforward extension to pricing and valuing currency swaps and equity swaps.
- With interest rate swaps and some equity swaps, pricing involves solving for the fixed interest rate.
- With currency swaps, pricing involves solving for the two fixed rates as well as the notional amounts in each currency.

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## PROBLEMS

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### **The following information relates to Questions 1–7**

Donald Troubadour is a derivatives trader for Southern Shores Investments. The firm seeks arbitrage opportunities in the forward and futures markets using the carry arbitrage model.

Troubadour identifies an arbitrage opportunity relating to a fixed-income futures contract and its underlying bond. Current data on the futures contract and underlying bond are presented in Exhibit 1. The current annual compounded risk-free rate is 0.30%.

**EXHIBIT 1 Current Data for Futures and Underlying Bond**

Futures Contract		Underlying Bond	
Quoted futures price	125.00	Quoted bond price	112.00
Conversion factor	0.90	Accrued interest since last coupon payment	0.08
Time remaining to contract expiration	Three months	Accrued interest at futures contract expiration	0.20
Accrued interest over life of futures contract	0.00		

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Troubadour next gathers information on three existing positions.

*Position 1 (Nikkei 225 Futures Contract):*

Troubadour holds a long position in a Nikkei 225 futures contract that has a remaining maturity of three months. The continuously compounded dividend yield on the Nikkei 225 Stock Index is 1.1%, and the current stock index level is 16,080. The continuously compounded annual interest rate is 0.2996%.

*Position 2 (Euro/JGB Forward Contract):*

One month ago, Troubadour purchased euro/yen forward contracts with three months to expiration at a quoted price of 100.20 (quoted as a percentage of par). The contract notional amount is ¥100,000,000. The current forward price is 100.05.

*Position 3 (JPY/USD Currency Forward Contract):*

Troubadour holds a short position in a yen/US dollar forward contract with a notional value of \$1,000,000. At contract initiation, the forward rate was ¥112.10 per \$1. The forward contract expires in three months. The current spot exchange rate is ¥112.00 per \$1, and the annually compounded risk-free rates are –0.20% for the yen and 0.30% for the US dollar. The current quoted price of the forward contract is equal to the no-arbitrage price.

Troubadour next considers an equity forward contract for Texas Steel, Inc. (TSI). Information regarding TSI common shares and a TSI equity forward contract is presented in Exhibit 2.

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#### EXHIBIT 2 Selected Information for TSI

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- TSI has historically paid dividends every six months.
  - The price per share of TSI's common shares is \$250.
  - The forward price per share for a nine-month TSI equity forward contract is \$250.562289.
  - Assume annual compounding.
- 

Troubadour takes a short position in the TSI equity forward contract. His supervisor asks, "Under which scenario would our position experience a loss?"

Three months after contract initiation, Troubadour gathers information on TSI and the risk-free rate, which is presented in Exhibit 3.

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#### EXHIBIT 3 Selected Data on TSI and the Risk-Free Rate

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- The price per share of TSI's common shares is \$245.
  - The risk-free rate is 0.325% (quoted on an annual compounding basis).
  - TSI recently announced its regular semiannual dividend of \$1.50 per share that will be paid exactly three months before contract expiration.
  - The market price of the TSI equity forward contract is equal to the no-arbitrage forward price.
- 

1. Based on Exhibit 2 and assuming annual compounding, the arbitrage profit on the bond futures contract is *closest* to:
  - 0.4158.
  - 0.5356.
  - 0.6195.
2. The current no-arbitrage futures price of the Nikkei 225 futures contract (Position 1) is *closest* to:
  - 15,951.81.
  - 16,047.86.
  - 16,112.21.
3. The value of Position 2 is *closest* to:
  - −¥149,925.
  - −¥150,000.
  - −¥150,075.
4. The value of Position 3 is *closest* to:
  - −¥40,020.
  - ¥139,913.
  - ¥239,963.

5. Based on Exhibit 2, Troubadour should find that an arbitrage opportunity relating to TSI shares is
  - A. not available.
  - B. available based on carry arbitrage.
  - C. available based on reverse carry arbitrage.
6. The *most appropriate* response to Troubadour's supervisor's question regarding the TSI forward contract is:
  - A. a decrease in TSI's share price, all else equal.
  - B. an increase in the risk-free rate, all else equal
  - C. a decrease in the market price of the forward contract, all else equal.
7. Based on Exhibits 2 and 3, and assuming annual compounding, the per share value of Troubadour's short position in the TSI forward contract three months after contract initiation is *closest* to:
  - A. \$1.6549.
  - B. \$5.1561.
  - C. \$6.6549.

### The following information relates to Questions 8–16

Sonal Johnson is a risk manager for a bank. She manages the bank's risks using a combination of swaps and forward rate agreements (FRAs).

Johnson prices a three-year Libor-based interest rate swap with annual resets using the present value factors presented in Exhibit 1.

EXHIBIT 1 Present Value Factors	
Maturity (years)	Present Value Factors
1	0.990099
2	0.977876
3	0.965136

Johnson also uses the present value factors in Exhibit 1 to value an interest rate swap that the bank entered into one year ago as the receive-floating party. Selected data for the swap are presented in Exhibit 2. Johnson notes that the current equilibrium two-year fixed swap rate is 1.00%.

EXHIBIT 2 Selected Data on Fixed for Floating Interest Rate Swap	
Swap notional amount	\$50,000,000
Original swap term	Three years, with annual resets
Fixed swap rate (since initiation)	3.00%

One of the bank's investments is exposed to movements in the Japanese yen, and Johnson desires to hedge the currency exposure. She prices a one-year fixed-for-fixed currency swap involving yen and US dollars, with a quarterly reset. Johnson uses the interest rate data presented in Exhibit 3 to price the currency swap.

## EXHIBIT 3 Selected Japanese and US Interest Rate Data

Days to Maturity	Yen Spot Interest Rates	US Dollar Spot Interest Rates
90	0.05%	0.20%
180	0.10%	0.40%
270	0.15%	0.55%
360	0.25%	0.70%

Johnson next reviews an equity swap with an annual reset that the bank entered into six months ago as the receive-fixed, pay-equity party. Selected data regarding the equity swap, which is linked to an equity index, are presented in Exhibit 4. At the time of initiation, the underlying equity index was trading at 100.00.

## EXHIBIT 4 Selected Data on Equity Swap

Swap notional amount	\$20,000,000
Original swap term	Five years, with annual resets
Fixed swap rate	2.00%

The equity index is currently trading at 103.00, and relevant US spot rates, along with their associated present value factors, are presented in Exhibit 5.

## EXHIBIT 5 Selected US Spot Rates and Present Value Factors

Maturity (years)	Spot Rate	Present Value Factors
0.5	0.40%	0.998004
1.5	1.00%	0.985222
2.5	1.20%	0.970874
3.5	2.00%	0.934579
4.5	2.60%	0.895255

Johnson reviews a  $6 \times 9$  FRA that the bank entered into 90 days ago as the pay-fixed/receive-floating party. Selected data for the FRA are presented in Exhibit 6, and current Libor data are presented in Exhibit 7. Based on her interest rate forecast, Johnson also considers whether the bank should enter into new positions in  $1 \times 4$  and  $2 \times 5$  FRAs.

EXHIBIT 6  $6 \times 9$  FRA Data

FRA term	$6 \times 9$
FRA rate	0.70%
FRA notional amount	US\$20,000,000
FRA settlement terms	Advanced set, advanced settle

## EXHIBIT 7 Current Libor

30-day Libor	0.75%
60-day Libor	0.82%
90-day Libor	0.90%
120-day Libor	0.92%
150-day Libor	0.94%
180-day Libor	0.95%
210-day Libor	0.97%
270-day Libor	1.00%

Three months later, the  $6 \times 9$  FRA in Exhibit 6 reaches expiration, at which time the three-month US dollar Libor is 1.10% and the six-month US dollar Libor is 1.20%. Johnson determines that the appropriate discount rate for the FRA settlement cash flows is 1.10%.

8. Based on Exhibit 1, Johnson should price the three-year Libor-based interest rate swap at a fixed rate *closest* to:
  - A. 0.34%.
  - B. 1.16%.
  - C. 1.19%.
9. From the bank's perspective, using data from Exhibit 1, the current value of the swap described in Exhibit 2 is *closest* to:
  - A. -\$2,951,963.
  - B. -\$1,967,975.
  - C. -\$1,943,000.
10. Based on Exhibit 3, Johnson should determine that the annualized equilibrium fixed swap rate for Japanese yen is *closest* to:
  - A. 0.0624%.
  - B. 0.1375%.
  - C. 0.2496%.
11. From the bank's perspective, using data from Exhibits 4 and 5, the fair value of the equity swap is *closest* to:
  - A. -\$1,139,425.
  - B. -\$781,323.
  - C. -\$181,323.
12. Based on Exhibit 5, the current value of the equity swap described in Exhibit 4 would be zero if the equity index was currently trading the *closest* to:
  - A. 97.30.
  - B. 99.09.
  - C. 100.00.
13. From the bank's perspective, based on Exhibits 6 and 7, the value of the  $6 \times 9$  FRA 90 days after inception is *closest* to:
  - A. \$14,817.
  - B. \$19,647.
  - C. \$29,635.

14. Based on Exhibit 7, the no-arbitrage fixed rate on a new  $1 \times 4$  FRA is *closest* to:
  - A. 0.65%.
  - B. 0.73%.
  - C. 0.98%.
15. Based on Exhibit 7, the fixed rate on a new  $2 \times 5$  FRA is *closest* to:
  - A. 0.61%.
  - B. 1.02%.
  - C. 1.71%.
16. Based on Exhibit 6 and the three-month US dollar Libor at expiration, the payment amount that the bank will receive to settle the  $6 \times 9$  FRA is *closest* to:
  - A. \$19,945.
  - B. \$24,925.
  - C. \$39,781.



# CHAPTER 4

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## VALUATION OF CONTINGENT CLAIMS

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David Maurice Gentle, MEC, BSc, CFA

### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- describe and interpret the binomial option valuation model and its component terms;
- calculate the no-arbitrage values of European and American options using a two-period binomial model;
- identify an arbitrage opportunity involving options and describe the related arbitrage;
- describe how interest rate options are valued using a two-period binomial model;
- calculate and interpret the value of an interest rate option using a two-period binomial model;
- describe how the value of a European option can be analyzed as the present value of the option's expected payoff at expiration;
- identify assumptions of the Black–Scholes–Merton option valuation model;
- interpret the components of the Black–Scholes–Merton model as applied to call options in terms of a leveraged position in the underlying;
- describe how the Black–Scholes–Merton model is used to value European options on equities and currencies;
- describe how the Black model is used to value European options on futures;
- describe how the Black model is used to value European interest rate options and European swaptions;
- interpret each of the option Greeks;
- describe how a delta hedge is executed;
- describe the role of gamma risk in options trading;
- define implied volatility and explain how it is used in options trading.

## 1. INTRODUCTION

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A contingent claim is a derivative instrument that provides its owner a right but not an obligation to a payoff determined by an underlying asset, rate, or other derivative. Contingent claims include options, the valuation of which is the objective of this reading. Because many investments contain embedded options, understanding this material is vital for investment management.

Our primary purpose is to understand how the values of options are determined. Option values, as with the values of all financial instruments, are typically obtained using valuation models. Any financial valuation model takes certain inputs and turns them into an output that tells us the fair value or price. Option valuation models, like their counterparts in the forward, futures, and swaps markets, are based on the principle of no arbitrage, meaning that the appropriate price of an option is the one that makes it impossible for any party to earn an arbitrage profit at the expense of any other party. The price that precludes arbitrage profits is the value of the option. Using that concept, we then proceed to introduce option valuation models using two approaches. The first approach is the binomial model, which is based on discrete time, and the second is the Black–Scholes–Merton (BSM) model, which is based on continuous time.

The reading is organized as follows. Section 2 introduces the principles of the no-arbitrage approach to pricing and valuation of options. In Section 3, the binomial option valuation model is explored, and in Section 4, the BSM model is covered. In Section 5, the Black model, being a variation of the BSM model, is applied to futures options, interest rate options, and swaptions. Finally, in Section 6, the Greeks are reviewed along with implied volatility. Section 7 provides a summary.

## 2. PRINCIPLES OF A NO-ARBITRAGE APPROACH TO VALUATION

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Our approach is based on the concept of arbitrage. Hence, the material will be covered from an arbitrageur's perspective. Key to understanding this material is to think like an arbitrageur. Specifically, like most people, the arbitrageur would rather have more money than less. The arbitrageur, as will be detailed later, follows two fundamental rules:

Rule #1 Do not use your own money.

Rule #2 Do not take any price risk.

Clearly, if we can generate positive cash flows today and abide by both rules, we have a great business—such is the life of an arbitrageur. If traders could create a portfolio with no future liabilities and positive cash flow today, then it would essentially be a money machine that would be attractive to anyone who prefers more cash to less. In the pursuit of these positive cash flows today, the arbitrageur often needs to borrow to satisfy Rule #1. In effect, the arbitrageur borrows the arbitrage profit to capture it today and, if necessary, may borrow to purchase the underlying. Specifically, the arbitrageur will build portfolios using the underlying instrument to synthetically replicate the cash flows of an option. The underlying instrument is the financial instrument whose later value will be referenced to determine the option value. Examples of underlying instruments include shares, indexes, currencies, and interest rates. As we will see, with options we will often rely on a specific trading strategy that changes over time based on the underlying price behavior.

Based on the concept of comparability, the no-arbitrage valuation approach taken here is built on the concept that if two investments have the same future cash flows regardless of what happens, then these two investments should have the same current price. This principle is known as the **law of one price**. In establishing these foundations of option valuation, the following key assumptions are made: (1) Replicating instruments are identifiable and investable. (2) There are no market frictions, such as transaction costs and taxes. (3) Short selling is allowed with full use of proceeds. (4) The underlying instrument follows a known statistical distribution. (5) Borrowing and lending at a risk-free interest rate is available. When we develop the models in this reading, we will be more specific about what these assumptions mean, in particular what we mean by a known statistical distribution.

In an effort to demonstrate various valuation results based on the absence of arbitrage, we will rely heavily on cash flow tables, which are a representation of the cash flows that occur during the life of an option. For example, if an initial investment requires €100, then from an arbitrageur's perspective, we will present it as a –€100 cash flow. If an option pays off ¥1,000, we will represent it as a +¥1,000 cash flow. That is, cash outflows are treated as negative and inflows as positive.

We first demonstrate how to value options based on a two-period binomial model. The option payoffs can be replicated with a dynamic portfolio of the underlying instrument and financing. A dynamic portfolio is one whose composition changes over time. These changes are important elements of the replicating procedure. Based on the binomial framework, we then turn to exploring interest rate options using a binomial tree. Although more complex, the general approach is shown to be the same.

The multiperiod binomial model is a natural transition to the BSM option valuation model. The BSM model is based on the key assumption that the value of the underlying instrument follows a statistical process called geometric Brownian motion. This characterization is a reasonable way to capture the randomness of financial instrument prices while incorporating a pre-specified expected return and volatility of return. Geometric Brownian motion implies a lognormal distribution of the return, which implies that the continuously compounded return on the underlying is normally distributed.

We also explore the role of carry benefits, meaning the reward or cost of holding the underlying itself instead of holding the derivative on the underlying.

Next we turn to Fischer Black's futures option valuation model (Black model) and note that the model difference, versus the BSM model, is related to the underlying futures contract having no carry costs or benefits. Interest rate options and swaptions are valued based on simple modifications of the Black model.

Finally, we explore the Greeks, otherwise known as delta, gamma, theta, vega, and rho. The Greeks are representations of the sensitivity of the option value to changes in the factors that determine the option value. They provide comparative information essential in managing portfolios containing options. The Greeks are calculated based on an option valuation model, such as the binomial model, BSM model, or the Black model. This information is model dependent, so managers need to carefully select the model best suited for their particular situation. In the last section, we cover implied volatility, which is a measure derived from a market option price and can be interpreted as reflecting what investors believe is the volatility of the underlying.

The models presented here are useful first approximations for explaining observed option prices in many markets. The central theme is that options are generally priced to preclude arbitrage profits, which is not only a reasonable theoretical assumption but is sufficiently accurate in practice.

We turn now to option valuation based on the binomial option valuation model.

### 3. BINOMIAL OPTION VALUATION MODEL

The binomial model is a valuable tool for financial analysts. It is particularly useful as a heuristic device to understand the unique valuation approach used with options. This model is extensively used to value path-dependent options, which are options whose values depend not only on the value of the underlying at expiration but also how it got there. The path-dependency feature distinguishes this model from the Black–Scholes–Merton option valuation model (BSM model) presented in the next section. The BSM model values only path-independent options, such as European options, which depend on only the values of their respective underlyings at expiration. One particular type of path-dependent option that we are interested in is American options, which are those that can be exercised prior to expiration. In this section, we introduce the general framework for developing the binomial option valuation models for both European and American options.

The binomial option valuation model is based on the no-arbitrage approach to valuation. Hence, understanding the valuation of options improves if one can understand how an arbitrageur approaches financial markets. An arbitrageur engages in financial transactions in pursuit of an initial positive cash flow with no possibility of a negative cash flow in the future. As it appears, it is a great business if you can find it.<sup>1</sup>

To understand option valuation models, it is helpful to think like an arbitrageur. The arbitrageur seeks to exploit any pricing discrepancy between the option price and the underlying spot price. The arbitrageur is assumed to prefer more money compared with less money, assuming everything else is the same. As mentioned earlier, there are two fundamental rules for the arbitrageur.

- Rule #1 Do not use your own money. Specifically, the arbitrageur does not use his or her own money to acquire positions. Also, the arbitrageur does not spend proceeds from short selling transactions on activities unrelated to the transaction at hand.
- Rule #2 Do not take any price risk. The focus here is only on market price risk related to the underlying and the derivatives used. We do not consider other risks, such as liquidity risk and counterparty credit risk.

We will rely heavily on these two rules when developing option valuation models. Remember, these rules are general in nature, and as with many things in finance, there are nuances.

In Exhibit 1, the two key dates are the option contract initiation date (identified as Time 0) and the option contract expiration date (identified as Time T). Based on the no-arbitrage approach, the option value from the initiation date onward will be estimated with an option valuation model.

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<sup>1</sup>There is not a one-to-one correspondence between arbitrage and great investment opportunities. An arbitrage is certainly a great investment opportunity because it produces a risk-free profit with no investment of capital, but all great investment opportunities are not arbitrage. For example, an opportunity to invest €1 today in return for a 99% chance of receiving €1,000,000 tomorrow or a 1% chance of receiving €0 might appear to be a truly great investment opportunity, but it is not arbitrage because it is not risk free and requires the investment of capital.

## EXHIBIT 1 Illustration of Option Contract Initiation and Expiration



Let  $S_t$  denote the underlying instrument price observed at Time  $t$ , where  $t$  is expressed as a fraction of a year. Similarly,  $S_T$  denotes the underlying instrument price observed at the option expiration date,  $T$ . For example, suppose a call option had 90 days to expiration when purchased ( $T = 90/365$ ), but now only has 35 days to expiration ( $t = 35/365$ ). Further, let  $c_t$  denote a European-style call price at Time  $t$  and with expiration on Date  $t = T$ , where both  $t$  and  $T$  are expressed in years. Similarly, let  $C_t$  denote an American-style call price. At the initiation date, the subscripts are omitted, thus  $c = c_0$ . We follow similar notation with a put, using the letter  $p$ , in place of  $c$ . Let  $X$  denote the exercise price.<sup>2</sup>

For example, suppose on 15 April a 90-day European-style call option contract with a 14 July expiration is initiated with a call price of  $c = €2.50$  and  $T = 90/365 = 0.246575$ .

At expiration, the call and put values will be equal to their intrinsic value or exercise value. These **exercise values** can be expressed as

$$C_T = \text{Max}(0, S_T - X) \text{ and} \\ P_T = \text{Max}(0, X - S_T),$$

respectively. If the option values deviate from these expressions, then there will be arbitrage profits available. The option is expiring, there is no uncertainty remaining, and the price must equal the market value obtained from exercising it or letting it expire.

Technically, European options do not have exercise values prior to expiration because they cannot be exercised until expiration. Nonetheless, the notion of the value of the option if it could be exercised,  $\text{Max}(0, S_t - X)$  for a call and  $\text{Max}(0, X - S_t)$  for a put, forms a basis for understanding the notion that the value of an option declines with the passage of time. Specifically, option values contain an element known as time value, which is just the market valuation of the potential for higher exercise value relative to the potential for lower exercise value. The time value is always non-negative because of the asymmetry of option payoffs at expiration. For example, for a call, the upside is unlimited, whereas the downside is limited to zero. At expiration, time value is zero.

Although option prices are influenced by a variety of factors, the underlying instrument has a particularly significant influence. At this point, the underlying is assumed to be the only uncertain factor affecting the option price. We now look in detail at the one-period binomial option valuation model. The one-period binomial model is foundational for the material that follows.

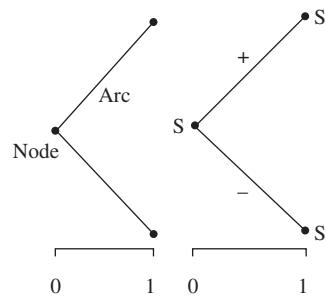
### 3.1. One-Period Binomial Model

Exhibit 2 illustrates the one-period binomial process for an asset priced at  $S$ . In the figure on the left, each dot represents a particular outcome at a particular point in time in the binomial lattice. The dots are termed nodes. At the Time 0 node, there are only two possible future paths in the

<sup>2</sup>In financial markets, the exercise price is also commonly called the strike price.

binomial process, an up move and a down move, termed arcs. The figure on the right illustrates the underlying price at each node. At Time 1, there are only two possible outcomes:  $S^+$  denotes the outcome when the underlying goes up, and  $S^-$  denotes the outcome when the underlying goes down.

EXHIBIT 2 One-Period Binomial Lattice with Underlying Distribution Illustrated



At Time 1, there are only two possible outcomes and two resulting values of the underlying,  $S^+$  (up occurs) and  $S^-$  (down occurs). Although the one-period binomial model is clearly unrealistic, it will provide key insights into the more realistic multiperiod binomial as well as the BSM model.

We further define the total returns implied by the underlying movements as

$$u = \frac{S^+}{S} \text{ (up factor) and}$$

$$d = \frac{S^-}{S} \text{ (down factor).}$$

The up factors and down factors are the total returns; that is, one plus the rate of return. The magnitudes of the up and down factors are based on the volatility of the underlying. In general, higher volatility will result in higher up values and lower down values.

We briefly review option valuation within a one-period binomial tree. With this review, we can move quickly to option valuation within a two-period binomial lattice by performing the one-period exercise three times.

We consider the fair value of a two-period call option value measured at Time 1 when an up move occurs, that is  $c^+$ . Based on arbitrage forces, we know this option value at expiration is either

$$c^{++} = \text{Max}(0, S^{++} - X) = \text{Max}(0, u^2 S - X), \text{ or}$$

$$c^+ = \text{Max}(0, S^+ - X) = \text{Max}(0, uS - X).$$

At this point, we assume that there are no costs or benefits from owning the underlying instrument. Now consider the transactions illustrated in Exhibit 3. These transactions are presented as cash flows. Thus, if we write a call option, we receive money at Time Step 0 and may have to pay out money at Time Step 1. Suppose the first trade is to write or sell one call option within the single-period binomial model. The value of a call option is positively related to the value of the underlying. That is, they both move up or down together. Hence, by writing a call option, the trader will lose money if the underlying goes up and make money if the underlying falls. Therefore, to execute a hedge, the trader will need a position that will make money if

the underlying goes up. Thus, the second trade needs to be a long position in the underlying. Specifically, the trader buys a certain number of units,  $h$ , of the underlying. The symbol  $h$  is used because it represents a hedge ratio.

Note that with these first two trades, neither arbitrage rule is satisfied. The future cash flow could be either  $-c^- + hS^-$  or  $-c^+ + hS^+$  and can be positive or negative. Thus, the cash flows at the Time Step 1 could result in the arbitrageur having to pay out money if one of these values is less than zero. To resolve both of these issues, we set the Time Step 1 cash flows equal to each other—that is,  $-c^+ + hS^+ = -c^- + hS^-$ —and solve for the appropriate hedge ratio:

$$h = \frac{c^+ - c^-}{S^+ - S^-} \geq 0 \quad (1)$$

We determine the hedge ratio such that we are indifferent to the underlying going up or down. Thus, we are hedged against moves in the underlying. A simple rule for remembering this formula is that the hedge ratio is the value of the call if the underlying goes up minus the value of the call if the underlying goes down divided by the value of the underlying if it goes up minus the value of the underlying if it goes down. The up and down patterns are the same in the numerator and denominator, but the numerator contains the option and the denominator contains the underlying.

Because call prices are positively related to changes in the underlying price, we know that  $h$  is non-negative. As shown in Exhibit 3, we will buy  $h$  underlying units as depicted in the second trade, and we will finance the present value of the net cash flows as depicted in the third trade. If we assume  $r$  denotes the per period risk-free interest rate, then the present value calculation, denoted as  $PV$ , is equal to  $1/(1+r)$ . We need to borrow or lend an amount such that the future net cash flows are equal to zero. Therefore, we finance today the present value of  $-hS^- + c^-$  which also equals  $-hS^+ + c^+$ . At this point we do not know if the finance term is positive or negative, thus we may be either borrowing or lending, which will depend on  $c$ ,  $h$ , and  $S$ .

EXHIBIT 3 Writing One Call Hedge with  $h$  Units of the Underlying and Finance

Strategy	Time Step 0	Time Step 1 Down Occurs	Time Step 1 Up Occurs
1. Write one call option	$+c$	$-c^-$	$-c^+$
2. Buy $h$ underlying units	$-hS$	$+hS^-$	$+hS^+$
3. Borrow or lend	$-PV(-hS^- + c^-)$ $= -PV(-hS^+ + c^+)$	$-hS^- + c^-$	$-hS^+ + c^+$
Net Cash Flow	$+c - hS$ $-PV(-hS^- + c^-)$	0	0

The value of the net portfolio at Time Step 0 should be zero or there is an arbitrage opportunity. If the net portfolio has positive value, then arbitrageurs will engage in this strategy, which will push the call price down and the underlying price up until the net is no longer positive. We assume the size of the borrowing will not influence interest rates. If the net portfolio has negative value, then arbitrageurs will engage in the opposite strategy—buy calls, short sell the underlying, and lend—pushing the call price up and the underlying price down until the net cash flow at Time 0 is no longer positive. Therefore, within the single-period binomial model, we have

$$+c - hS - PV(-hS^- + c^-) = 0$$

or, equivalently,

$$+c - hS - PV(-hS^+ + c^+) = 0.$$

Therefore, the **no-arbitrage approach** leads to the following single-period call option valuation equation:

$$c = hS + PV(-hS^- + c^-) \quad (2)$$

or, equivalently,  $c = hS + PV(-hS^+ + c^+)$ . In words, long a call option is equal to owning  $h$  shares of stock partially financed, where the financed amount is  $PV(-hS^- + c^-)$ , or using the per period rate,  $(-hS^- + c^-)/(1 + r)$ .<sup>3</sup>

We will refer to Equation 2 as the no-arbitrage single-period binomial option valuation model. This equation is foundational to understanding the two-period binomial as well as other option valuation models. The option can be replicated with the underlying and financing, a point illustrated in the following example.

### EXAMPLE 1 Long Call Option Replicated with Underlying and Financing

Identify the trading strategy that will generate the payoffs of taking a long position in a call option within a single-period binomial framework.

- A. Buy  $h = (c^+ + c^-)/(S^+ + S^-)$  units of the underlying and financing of  $-PV(-hS^- + c^-)$
- B. Buy  $h = (c^+ - c^-)/(S^+ - S^-)$  units of the underlying and financing of  $-PV(-hS^- + c^-)$
- C. Short sell  $h = (c^+ - c^-)/(S^+ - S^-)$  units of the underlying and financing of  $+PV(-hS^- + c^-)$

*Solution:* B is correct. The following table shows the terminal payoffs to be identical between a call option and buying the underlying with financing.

Strategy	Time Step 0	Time Step 1	Time Step 1
		Down Occurs	Up Occurs
Buy 1 call option	$-c$	$+c^-$	$+c^+$
OR A REPLICATING PORTFOLIO			
Buy $h$ underlying units	$-hS$	$+hS^-$	$+hS^+$
Borrow or lend	$-PV(-hS^- + c^-)$ $= -PV(-hS^+ + c^+)$	$-hS^- + c^-$	$-hS^+ + c^+$
Net	$-hS - PV(-hS^- + c^-)$	$+c^-$	$+c^+$

Recall that by design,  $h$  is selected such that  $-hS^- + c^- = -hS^+ + c^+$  or  $h = (c^+ - c^-)/(S^+ - S^-)$ . Therefore, a call option can be replicated with the underlying and financing. Specifically, the call option is equivalent to a leveraged position in the underlying.

<sup>3</sup>Or, by the same logic,  $PV(-hS^+ + c^+)$ , which is  $(-hS^+ + c^+)/(1 + r)$ .

Thus, the no-arbitrage approach is a replicating strategy: A call option is synthetically replicated with the underlying and financing. Following a similar strategy with puts, the no-arbitrage approach leads to the following no-arbitrage single-period put option valuation equation:

$$p = hS + PV(-hS^- + p^-) \quad (3)$$

or, equivalently,  $p = hS + PV(-hS^+ + p^+)$  where

$$h = \frac{p^+ - p^-}{S^+ - S^-} \leq 0 \quad (4)$$

Because  $p^+$  is less than  $p^-$ , the hedge ratio is negative. Hence, to replicate a long put position, the arbitrageur will short sell the underlying and lend a portion of the proceeds. Note that a long put position would be replicated by trading  $h$  units of the underlying. With  $h$  negative, this trade is a short sale, and because  $-h$  is positive, the value  $-hS$  results in a positive cash flow at Time Step 0.

### EXAMPLE 2 Long Put Option Replicated with Underlying and Financing

Identify the trading strategy that will generate the payoffs of taking a long position in a put option within a single-period binomial framework.

- A. Short sell  $-h = -(p^+ - p^-)/(S^+ - S^-)$  units of the underlying and financing of  $-PV(-hS^- + p^-)$
- B. Buy  $-h = (p^+ - p^-)/(S^+ - S^-)$  units of the underlying and financing of  $-PV(-hS^- + p^-)$
- C. Short sell  $h = (p^+ - p^-)/(S^+ - S^-)$  units of the underlying and financing of  $+PV(-hS^- + p^-)$

*Solution:* A is correct. Before illustrating the replicating portfolio, we make a few observations regarding the hedge ratio. Note that by design,  $h$  is selected such that  $-hS^- + p^- = -hS^+ + p^+$  or  $h = (p^+ - p^-)/(S^+ - S^-)$ . Unlike calls, the put hedge ratio is not positive (note that  $p^+ < p^-$  but  $S^+ > S^-$ ). Remember that taking a position in  $-h$  units of the underlying is actually short selling the underlying rather than buying it. The following table shows the terminal payoffs to be identical between a put option and a position in the underlying with financing.

Strategy	Time Step 0	Time Step 1 Down Occurs	Time Step 1 Up Occurs
Buy 1 Put Option	$-p$	$+p^-$	$+p^+$
OR A REPLICATING PORTFOLIO			
Short sell $-h$ Underlying Units	$-hS$	$+hS^-$	$+hS^+$
Borrow or Lend	$-PV(-hS^- + p^-)$ $= -PV(-hS^+ + p^+)$	$-hS^- + p^-$	$-hS^+ + p^+$
Net	$-hS - PV(-hS^- + p^-)$	$+p^-$	$+p^+$

Therefore, a put option can be replicated with the underlying and financing. Specifically, the put option is simply equivalent to a short position in the underlying with financing in the form of lending.

What we have shown to this point is the no-arbitrage approach. Before turning to the expectations approach, we mention, for the sake of completeness, that the transactions for writing options are the reverse for those of buying them. Thus, for writing a call option, the writer will be selling stock short and investing proceeds, whereas for a put, the writer will be purchasing stock on margin. Once again, we see the powerful result that the same basic conceptual structure is used for puts and calls, whether written or purchased. Only the exercise and expiration conditions vary.

The no-arbitrage results that have been presented can be expressed as the present value of a unique expectation of the option payoffs.<sup>4</sup> Specifically, the **expectations approach** results in an identical value as the no-arbitrage approach, but it is usually easier to compute. The formulas are viewed as follows:

$$c = PV[\pi c^+ + (1 - \pi)c^-] \text{ and} \quad (5)$$

$$p = PV[\pi p^+ + (1 - \pi)p^-] \quad (6)$$

where the probability of an up move is

$$\pi = [FV(1) - d]/(u - d)$$

Recall the future value is simply the reciprocal of the present value or  $FV(1) = 1/PV(1)$ . Thus, if  $PV(1) = 1/(1 + r)$ , then  $FV(1) = (1 + r)$ . Note that the option values are simply the present value of the expected terminal option payoffs. The expected terminal option payoffs can be expressed as

$$E(c_1) = \pi c^+ + (1 - \pi)c^- \text{ and}$$

$$E(p_1) = \pi p^+ + (1 - \pi)p^-$$

where  $c_1$  and  $p_1$  are the values of the options at Time 1. The present value and future value calculations are based on the risk-free rate, denoted  $r$ .<sup>5</sup> Thus, the option values based on the expectations approach can be written and remembered concisely as

$$c = PV_r[E(c_1)] \text{ and}$$

$$p = PV_r[E(p_1)]$$

<sup>4</sup>It takes a bit of algebra to move from the no-arbitrage expression to the present value of the expected future payoffs, but the important point is that both expressions yield exactly the same result.

<sup>5</sup>We will suppress “ $r$ ” most of the time and simply denote the calculation as  $PV$ . The “ $r$ ” will be used at times to reinforce that the present value calculation is based on the risk-free interest rate.

The expectations approach to option valuation differs in two significant ways from the discounted cash flow approach to securities valuation. First, the expectation is not based on the investor's beliefs regarding the future course of the underlying. That is, the probability,  $\pi$ , is objectively determined and not based on the investor's personal view. This probability has taken several different names, including risk-neutral (RN) probability. Importantly, we did not make any assumption regarding the arbitrageur's risk preferences: The expectations approach is a result of this arbitrage process, not an assumption regarding risk preferences. Hence, they are called risk-neutral probabilities. Although we called them probabilities from the very start, they are not the true probabilities of up and down moves.

Second, the discount rate is *not* risk adjusted. The discount rate is simply based on the estimated risk-free interest rate. The expectations approach here is often viewed as superior to the discounted cash flow approach because both the subjective future expectation as well as the subjective risk-adjusted discount rate have been replaced with more objective measures.

### EXAMPLE 3 Single-Period Binomial Call Value

A non-dividend-paying stock is currently trading at €100. A call option has one year to mature, the periodically compounded risk-free interest rate is 5.15%, and the exercise price is €100. Assume a single-period binomial option valuation model, where  $u = 1.35$  and  $d = 0.74$ .

1. The optimal hedge ratio will be *closest* to:
  - A. 0.57.
  - B. 0.60.
  - C. 0.65.
2. The call option value will be *closest* to:
  - A. €13.
  - B. €15.
  - C. €17.

*Solution to 1:* A is correct. Given the information provided, we know the following:

$$S^+ = uS = 1.35(100) = 135$$

$$S^- = dS = 0.74(100) = 74$$

$$c^+ = \text{Max}(0, uS - X) = \text{Max}(0, 135 - 100) = 35$$

$$c^- = \text{Max}(0, dS - X) = \text{Max}(0, 74 - 100) = 0$$

With this information, we can compute both the hedge ratio as well as the call option value. The hedge ratio is:

$$h = \frac{c^+ - c^-}{S^+ - S^-} = \frac{35 - 0}{135 - 74} = 0.573770$$

*Solution to 2:* C is correct. The risk-neutral probability of an up move is

$$\pi = [FV(1) - d]/(u - d) = (1.0515 - 0.74)/(1.35 - 0.74) = 0.510656,$$

where  $FV(1) = (1 + r) = 1.0515$ .

Thus the call value by the expectations approach is

$$c = PV[\pi c^+ + (1 - \pi)c^-] = 0.951022[(0.510656)35 + (1 - 0.510656)0] = €16.998,$$

where  $PV(1) = 1/(1 + r) = 1/(1.0515) = 0.951022$ .

Note that the call value by the no-arbitrage approach yields the same answer:

$$\begin{aligned} c &= hS + PV(-hS^- + c^-) = 0.573770(100) + 0.951022[-0.573770(74) + 0] \\ &= €16.998. \end{aligned}$$

The value of a put option can also be found based on put–call parity. Put–call parity can be remembered as simply two versions of portfolio insurance, long stock and long put or lend and long call, where the exercise prices for the put and call are identical. Put–call parity with symbols is

$$S + p = PV(X) + c \quad (7)$$

Put–call parity holds regardless of the particular valuation model being used. Depending on the context, this equation can be rearranged. For example, a call option can be expressed as a position in a stock, financing, and a put, or

$$c = S - PV(X) + p$$

#### EXAMPLE 4 Single-Period Binomial Put Value

You again observe a €100 price for a non-dividend-paying stock with the same inputs as the previous box. That is, the call option has one year to mature, the periodically compounded risk-free interest rate is 5.15%, the exercise price is €100,  $u = 1.35$ , and  $d = 0.74$ . The put option value will be *closest* to:

- A. €12.00.
- B. €12.10.
- C. €12.20.

*Solution:* B is correct. For puts, we know the following:

$$p^+ = \text{Max}(0, 100 - uS) = \text{Max}(0, 100 - 135) = 0$$

$$p^- = \text{Max}(0, 100 - dS) = \text{Max}(0, 100 - 74) = 26$$

With this information, we can compute the put option value based on risk-neutral probability from the previous example or [recall that  $PV(1) = 0.951022$ ]

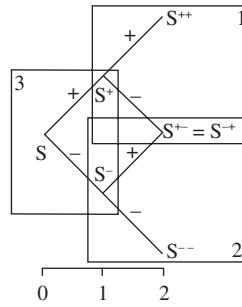
$$p = PV[\pi p^+ + (1 - \pi)p^-] = 0.951022[(0.510656)0 + (1 - 0.510656)26] = €12.10$$

Therefore, in summary, option values can be expressed either in terms of replicating portfolios or as the present value of the expected future cash flows. Both expressions yield the same valuations.

### 3.2. Two-Period Binomial Model

The two-period binomial lattice can be viewed as three one-period binomial lattices, as illustrated in Exhibit 4. Clearly, if we understand the one-period model, then the process can be repeated three times. First, we analyze Box 1 and Box 2. Finally, based on the results of Box 1 and Box 2, we analyze Box 3.

EXHIBIT 4 Two-Period Binomial Lattice as Three One-Period Binomial Lattices



At Time 2, there are only three values of the underlying,  $S^{++}$  (an up move occurs twice),  $S^{--}$  (a down move occurs twice), and  $S^{+-} = S^{-+}$  (either an up move occurs and then a down move or a down move occurs and then an up move). For computational reasons, it is extremely helpful that the lattice recombines—that is,  $S^{+-} = S^{-+}$ , meaning that if the underlying goes up and then down, it ends up at the same price as if it goes down and then up. A recombining binomial lattice will always have just one more ending node in the final period than the number of time steps. In contrast, a non-recombining lattice of  $n$  time steps will have  $2^n$  ending nodes, which poses a tremendous computational challenge even for powerful computers.

For our purposes here, we assume the up and down factors are constant throughout the lattice, ensuring that the lattice recombines—that is  $S^{+-} = S^{-+}$ . For example, assume  $u = 1.25$ ,  $d = 0.8$ , and  $S_0 = 100$ . Note that  $S^{+-} = 1.25(0.8)100 = 100$  and  $S^{-+} = 0.8(1.25)100 = 100$ . So the middle node at Time 2 is 100 and can be reached from either of two paths.

The two-period binomial option valuation model illustrates two important concepts, self-financing and dynamic replication. Self-financing implies that the replicating portfolio will not require any additional funds from the arbitrageur during the life of this dynamically rebalanced portfolio. If additional funds are needed, then they are financed externally.

Dynamic replication means that the payoffs from the option can be exactly replicated through a planned trading strategy. Option valuation relies on self-financing, dynamic replication.

Mathematically, the no-arbitrage approach for the two-period binomial model is best understood as working backward through the binomial tree. At Time 2, the payoffs are driven by the option's exercise value.

For calls:

$$c^{++} = \max(0, S^{++} - X) = \max(0, u^2 S - X),$$

$$c^{+-} = \max(0, S^{+-} - X) = \max(0, u d S - X), \text{ and}$$

$$c^{-+} = \max(0, S^{-+} - X) = \max(0, d^2 S - X)$$

For puts:

$$p^{++} = \max(0, X - S^{++}) = \max(0, X - u^2 S),$$

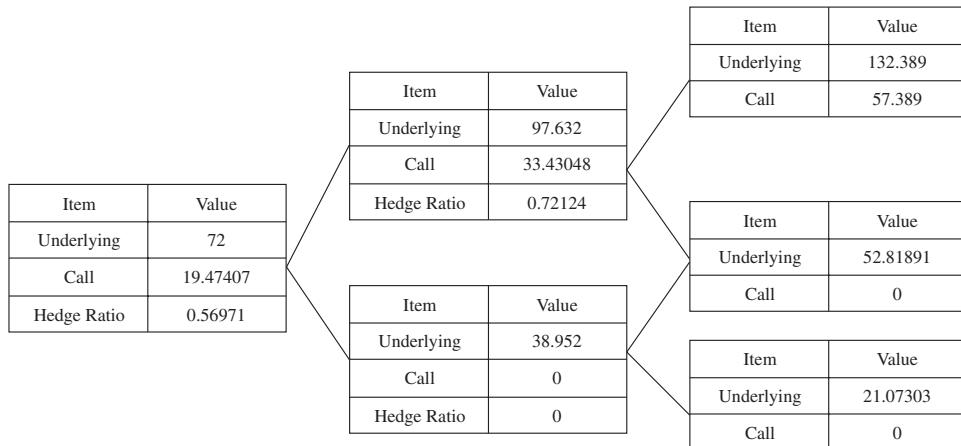
$$p^{+-} = \max(0, X - S^{+-}) = \max(0, X - u d S), \text{ and}$$

$$p^{-+} = \max(0, X - S^{-+}) = \max(0, X - d^2 S)$$

At Time 1, the option values are driven by the arbitrage transactions that synthetically replicate the payoffs at Time 2. We can compute the option values at Time 1 based on the option values at Time 2 using the no-arbitrage approach based on Equations 1 and 2. At Time 0, the option values are driven by the arbitrage transactions that synthetically replicate the value of the options at Time 1 (again based on Equations 1 and 2).

We illustrate the no-arbitrage approach for solving the two-period binomial call value. Suppose the annual interest rate is 3%, the underlying stock is  $S = 72$ ,  $u = 1.356$ ,  $d = 0.541$ , and the exercise price is  $X = 75$ . The stock does not pay dividends. Exhibit 5 illustrates the results.

EXHIBIT 5 Two-Period Binomial Tree with Call Values and Hedge Ratios



We now verify selected values reported in Exhibit 5. At Time Step 2 and assuming up occurs twice, the underlying stock value is  $u^2 S = (1.356)^2 72 = 132.389$ , and hence, the call value is 57.389 [ $= \max(0, 132.389 - 75)$ ]. The hedge ratio at Time Step 1, assuming up occurs once, is

$$h^+ = \frac{c^{++} - c^{+-}}{S^{++} - S^{+-}} = \frac{57.389 - 0}{132.389 - 52.819} = 0.72124$$

The RN probability of an up move throughout this tree is

$$\pi = [FV(1) - d]/(u - d) = (1.03 - 0.541)/(1.356 - 0.541) = 0.6$$

With this information, we can compute the call price at Time 1 when an up move occurs as

$$c = PV[\pi c^{++} + (1 - \pi)c^{+-}] = (1/1.03)[(0.6)57.389 + (1 - 0.6)0] = 33.43048$$

and at Time Step 0,

$$h = \frac{c^+ - c^-}{S^+ - S^-} = \frac{33.43048 - 0}{97.632 - 38.952} = 0.56971$$

Thus, the call price at the start is

$$c = PV[\pi c^+ + (1 - \pi)c^-] = (1/1.03)[(0.6)33.43048 + (1 - 0.6)0] = 19.47$$

From the no-arbitrage approach, the call payoffs can be replicated by purchasing  $h$  shares of the underlying and financing  $-PV(-hS^- + c^-)$ . Therefore, we purchase 0.56971 shares of stock for 41.019 [= 0.56971(72)] and borrow 21.545 {or in cash flow terms,  $-21.545 = (1/1.03)[-0.56971(38.952) + 0]$ }, replicating the call values at Time 1. We then illustrate Time 1 assuming that an up move occurs. The stock position will now be worth 55.622 [= 0.56971(97.632)], and the borrowing must be repaid with interest or 22.191 [= 1.03(21.545)]. Note that the portfolio is worth 33.431 (55.622 - 22.191), the same value as the call except for a small rounding error. Therefore, the portfolio of stock and the financing dynamically replicates the value of the call option.

The final task is to demonstrate that the portfolio is self-financing. Self-financing can be shown by observing that the new portfolio at Time 1, assuming an up move occurs, is equal to the old portfolio that was formed at Time 0 and liquidated at Time 1. Notice that the hedge ratio rose from 0.56971 to 0.72124 as we moved from Time 0 to Time 1, assuming an up move occurs, requiring the purchase of additional shares. These additional shares will be financed with additional borrowing. The total borrowing is 36.98554 {=  $-PV(-hS^+ + c^+) = -(1/1.03)[-0.72124(52.81891) + 0]$ }. The borrowing at Time 0 that is due at Time 1 is 22.191. The funds borrowed at Time 1 grew to 36.98554. Therefore, the strategy is self-financing.

The two-period binomial model can also be represented as the present value of an expectation of future cash flows. Based on the one-period results, it follows by repeated substitutions that

$$c = PV[\pi^2 c^{++} + 2\pi(1 - \pi)c^{+-} + (1 - \pi)^2 c^{--}] \quad (8)$$

and

$$p = PV[\pi^2 p^{++} + 2\pi(1 - \pi)p^{+-} + (1 - \pi)^2 p^{--}] \quad (9)$$

Therefore, the two-period binomial model is again simply the present value of the expected future cash flows based on the RN probability. Again, the option values are simply the present

value of the expected terminal option payoffs. The expected terminal option payoffs can be expressed as

$$E(c_2) = \pi^2 c^{++} + 2\pi(1 - \pi)c^{+-} + (1 - \pi)^2 c^{--}$$

and

$$E(p_2) = \pi^2 p^{++} + 2\pi(1 - \pi)p^{+-} + (1 - \pi)^2 p^{--}$$

Thus, the two-period binomial option values based on the expectations approach can be written and remembered concisely as

$$c = PV_r[E\pi(c_2)] \text{ and}$$

$$p = PV_r[E\pi(p_2)]$$

It is vital to remember that this present value is over two periods, so the discount factor with discrete rates is  $PV = [1/(1 + r)^2]$ . Recall the subscript “r” just emphasizes the present value calculation and is based on the risk-free interest rate.

### EXAMPLE 5 Two-Period Binomial Model Call Valuation

You observe a €50 price for a non-dividend-paying stock. The call option has two years to mature, the periodically compounded risk-free interest rate is 5%, the exercise price is €50,  $u = 1.356$ , and  $d = 0.744$ . Assume the call option is European-style.

1. The probability of an up move based on the risk-neutral probability is *closest* to:
  - A. 30%.
  - B. 40%.
  - C. 50%.
2. The current call option value is *closest* to:
  - A. €9.53.
  - B. €9.71.
  - C. €9.87.
3. The current put option value is *closest* to:
  - A. €5.06.
  - B. €5.33.
  - C. €5.94.

*Solution to 1:* C is correct. Based on the RN probability equation, we have:

$$\pi = [FV(1 - d)/(u - d)] = [(1 + 0.05) - 0.744]/(1.356 - 0.744) = 0.5 \text{ or } 50\%$$

*Solution to 2:* B is correct. The current call option value calculations are as follows:

$$c^{++} = \text{Max}(0, u^2 S - X) = \text{Max}[0, 1.356^2(50) - 50] = 41.9368$$

$$c^{+-} = c^- = \text{Max}(0, u d S - X) = \text{Max}[0, 1.356(0.744)(50) - 50] = 0.44320$$

$$c^{--} = \text{Max}(0, d^2 S - X) = \text{Max}[0, 0.744^2(50) - 50] = 0.0$$

With this information, we can compute the call option value:

$$\begin{aligned} c &= \text{PV}[\text{E}(c_2)] = \text{PV}[\pi^2 c^{++} + 2\pi(1 - \pi)c^{+-} + (1 - \pi)^2 c^{--}] \\ &= [1/(1 + 0.05)]^2[0.5^2 41.9368 + 2(0.5)(1 - 0.5)0.44320 + (1 - 0.5)^2 0.0] \\ &= 9.71 \end{aligned}$$

It is vital to remember that the present value is over two periods, hence the single-period PV is squared. Thus, the current call price is €9.71.

*Solution to 3:* A is correct. The put option value can be computed simply by applying put-call parity or  $p = c + \text{PV}(X) - S = 9.71 + [1/(1 + 0.05)]^2 50 - 50 = 5.06$ . Thus, the current put price is €5.06.

We now turn to consider American-style options. It is well-known that non-dividend-paying call options on stock will not be exercised early because the minimum price of the option exceeds its exercise value. To illustrate by example, consider a call on a US\$100 stock, with an exercise price of US\$10 (that is, very deep in the money). Suppose the call is worth its exercise value of only US\$90. To get stock exposure, one could fund and pay US\$100 to buy the stock, or fund and pay only US\$90 for the call and pay the last US\$10 at expiration only if the stock is at or above US\$100 at that time. Because the latter choice is preferable, the call must be worth more than the US\$90 exercise value. Another way of looking at it is that it would make no sense to exercise this call because you do not believe the stock can go any higher and you would thus simply be obtaining a stock that you believe would go no higher. Moreover, the stock would require that you pay far more money than you have tied up in the call. It is always better to just sell the call in this situation because it will be trading for more than the exercise value.

The same is not true for put options. By early exercise of a put, particularly a deep in-the-money put, the sale proceeds can be invested at the risk-free rate and earn interest worth more than the time value of the put. Thus, we will examine how early exercise influences the value of an American-style put option. As we will see, when early exercise has value, the no-arbitrage approach is the only way to value American-style options.

Suppose the periodically compounded interest rate is 3%, the non-dividend-paying underlying stock is currently trading at 72, the exercise price is 75,  $u = 1.356$ ,  $d = 0.541$ , and the put option expires in two years. Exhibit 6 shows the results for a European-style put option.

EXHIBIT 6 Two-Period Binomial Model for a European-Style Put Option

Item	Value
Underlying	72
Put	18.16876
Hedge Ratio	-0.43029

Item	Value
Underlying	97.632
Put	8.61401
Hedge Ratio	-0.27876

Item	Value
Underlying	38.952
Put	33.86353
Hedge Ratio	-1

Item	Value
Underlying	52.81891
Put	22.18109
Underlying	21.07303
Put	53.92697

The Time 1 down move is of particular interest. The exercise value for this put option is 36.048 [=Max(0.75 – 38.952)]. Therefore, the exercise value is higher than the put value. So, if this same option were American-style, then the option would be worth more exercised than not exercised. Thus, the put option should be exercised. Exhibit 7 illustrates how the analysis changes if this put option were American-style. Clearly, the right to exercise early translates into a higher value.

EXHIBIT 7 Two-Period Binomial Model for an American-Style Put Option

Item	Value
Underlying	72
Put	18.16876 19.01710
Hedge Ratio	-0.43029 -0.46752

Item	Value
Underlying	97.632
Put	8.61401
Hedge Ratio	-0.27876

Item	Value
Underlying	38.952
Put	33.86353 36.04800
Hedge Ratio	-1

Item	Value
Underlying	52.81891
Put	22.18109
Underlying	21.07303
Put	53.92697

American-style option valuation requires that one work backward through the binomial tree and address whether early exercise is optimal at each step. In Exhibit 7, the early exercise premium at Time 1 when a down move occurs is 2.18447 (36.048 – 33.86353). Also, if we replace 33.86353 with 36.048—in bold below for emphasis—in the Time 0 calculation, we obtain a put value of

$$p = PV[\pi p^+ + (1 - \pi)p^-] = (1/1.03)[(0.6)8.61401 + (1 - 0.6)\mathbf{36.048}] = 19.02$$

Thus, the early exercise premium at Time 0 is 0.85 ( $19.02 - 18.17$ ). From this illustration, we see clearly that in a multiperiod setting, American-style put options cannot be valued simply as the present value of the expected future option payouts, as shown in Equation 9. American-style put options can be valued as the present value of the expected future option payout in a single-period setting. Hence, when early exercise is a consideration, we must address the possibility of early exercise as we work backward through the binomial tree.

### EXAMPLE 6 Two-Period Binomial American-Style Put Option Valuation

Suppose you are given the following information:  $S_0 = 26$ ,  $X = 25$ ,  $u = 1.466$ ,  $d = 0.656$ ,  $n = 2$  (time steps),  $r = 2.05\%$  (per period), and no dividends. The tree is provided in Exhibit 8.

#### EXHIBIT 8 Two-Period Binomial American-Style Put Option

Item	Value
Underlying	26
Put	4.01174
Hedge Ratio	-0.35345

Item	Value
Underlying	38.116
Put	0
Hedge Ratio	0

Item	Value
Underlying	17.056
Put	7.44360
Hedge Ratio	-0.99970

Item	Value
Underlying	55.87806
Put	0

Item	Value
Underlying	25.00410
Put	0

Item	Value
Underlying	11.18874
Put	13.81126

The early exercise premium of the above American-style put option is *closest* to:

- A. 0.27.
- B. 0.30.
- C. 0.35.

*Solution:* A is correct. The exercise value at Time 1 with a down move is 7.944 [ $= \text{Max}(0, 25 - 17.056)$ ]. Thus, we replace this value in the binomial tree and compute the hedge ratio at Time 0. The resulting put option value at Time 0 is thus 4.28143 (see Exhibit 9).

## EXHIBIT 9 Solution

Item	Value
Underlying	26
Put	4.01174 4.28143
Hedge Ratio	-0.35345 -0.37721
Underlying	38.116
Put	0
Hedge Ratio	0
Underlying	17.056
Put	7.44360 7.94400
Hedge Ratio	-0.99970
Underlying	55.87806
Put	0
Underlying	25.00410
Put	0
Underlying	11.18874
Put	13.81126

In Exhibit 9, the early exercise premium at Time 1 when a down move occurs is 0.5004 ( $7.944 - 7.44360$ ). Thus, if we replace 7.44360 with 7.944—in bold below for emphasis—in the Time 0 calculation, we have the put value of

$$p = PV[\pi p^+ + (1 - \pi)p^-] = (1/1.0205)[(0.45)0 + (1 - 0.45)\mathbf{7.944}] = 4.28$$

Thus, the early exercise premium at Time 0 when a down move occurs is 0.27 ( $= 4.28 - 4.01$ ).

We now briefly introduce the role of dividend payments within the binomial model. Our approach here is known as the escrow method. Because dividends lower the value of the stock, a call option holder is hurt. Although it is possible to adjust the option terms to offset this effect, most option contracts do not provide protection against dividends. Thus, dividends affect the value of an option. We assume dividends are perfectly predictable; hence, we split the underlying instrument into two components: the underlying instrument without the known dividends and the known dividends. For example, the current value of the underlying instrument without dividends can be expressed as

$$\hat{S} = S - \gamma$$

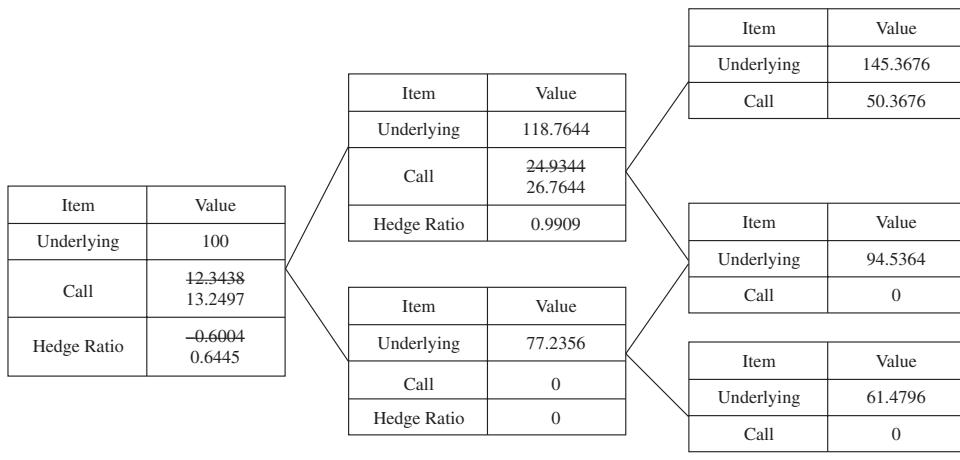
where  $\gamma$  denotes the present value of dividend payments. We use the  $\wedge$  symbol to denote the underlying instrument without dividends. In this case, we model the uncertainty of the stock based on  $\hat{S}$  and not  $S$ . At expiration, the underlying instrument value is the same,  $\hat{S}_T = S_T$ , because we assume any dividends have already been paid. The value of an investment in the stock, however, would be  $S_T + \gamma_T$ , which assumes the dividend payments are reinvested at the risk-free rate.

To illustrate by example, consider a call on a US\$100 stock with exercise price of US\$95. The periodically compounded interest rate is 1.0%, the stock will pay a US\$3 dividend at Time Step 1,  $u = 1.224$ ,  $d = 0.796$ , and the call option expires in two years. Exhibit 10 shows some results for an American-style call option. The computations in Exhibit 10 involve several

technical nuances that are beyond the scope of our objectives. The key objective here is to see how dividend-motivated early exercise influences American options.

The Time 1 up move is particularly interesting. At Time 0, the present value of the US\$3 dividend payment is US\$2.970297 ( $= 3/1.01$ ). Therefore,  $118.7644 = (100 - 2.970297)1.224$  is the stock value without dividends at Time 1, assuming an up move occurs. The exercise value for this call option, including dividends, is 26.7644 [ $= \text{Max}(0, 118.7644 + 3 - 95)$ ], whereas the value of the call option per the binomial model is 24.9344. In other words, the stock price just before it goes ex-dividend is  $118.7644 + 3 = 121.7644$ , so the option can be exercised for  $121.7644 - 95 = 26.7644$ . If not exercised, the stock drops as it goes ex-dividend and the option becomes worth 24.9344 at the ex-dividend price. Thus, by exercising early, the call buyer acquires the stock just before it goes ex-dividend and thus is able to capture the dividend. If the call is not exercised, the call buyer will not receive this dividend. The American-style call option is worth more than the European-style call option because at Time Step 1 when an up move occurs, the call is exercised early, capturing additional value.

#### EXHIBIT 10 Two-Period Binomial Model for an American-Style Call Option with Dividends



We now provide a comprehensive binomial option valuation example. In this example, we contrast European-style exercise with American-style exercise.

#### EXAMPLE 7 Comprehensive Two-Period Binomial Option Valuation Model Exercise

Suppose you observe a non-dividend-paying Australian equity trading for A\$7.35. The call and put options have two years to mature, the periodically compounded risk-free interest rate is 4.35%, and the exercise price is A\$8.0. Based on an analysis of this equity, the estimates for the up and down moves are  $u = 1.445$  and  $d = 0.715$ , respectively.

1. Calculate the European-style call and put option values at Time Step 0 and Time Step 1. Describe and interpret your results.

2. Calculate the European-style call and put option hedge ratios at Time Step 0 and Time Step 1. Based on these hedge ratios, interpret the component terms of the binomial option valuation model.
3. Calculate the American-style call and put option values and hedge ratios at Time Step 0 and Time Step 1. Explain how your results differ from the European-style results.

*Solution to 1:* The expectations approach requires the following preliminary calculations:

$$\begin{aligned}
 \text{RN probability: } \pi &= [FV(1) - d]/(u - d) \\
 &= [(1 + 0.0435) - 0.715]/(1.445 - 0.715) = 0.45 \\
 c^{++} &= \text{Max}(0, u^2 S - X) \\
 &= \text{Max}[0, 1.445^2(7.35) - 8.0] = 7.347 \\
 c^{+-} &= \text{Max}(0, u d S - X) \\
 &= \text{Max}[0, 1.445(0.715)7.35 - 8.0] = 0 \\
 c^{-+} &= \text{Max}(0, d^2 S - X) \\
 &= \text{Max}[0, 0.715^2(7.35) - 8.0] = 0 \\
 p^{++} &= \text{Max}(0, X - u^2 S) \\
 &= \text{Max}[0, 8.0 - 1.445^2(7.35)] = 0.406 \\
 p^{+-} &= \text{Max}(0, X - u d S) \\
 &= \text{Max}[0, 8.0 - 1.445(0.715)7.35] = 0.406 \\
 p^{-+} &= \text{Max}(0, X - d^2 S) \\
 &= \text{Max}[0, 8.0 - 0.715^2(7.35)] = 4.24
 \end{aligned}$$

Therefore, at Time Step 1, we have (note that  $c_2|_1^+$  is read as the call value expiring at Time Step 2 observed at Time Step 1, assuming an up move occurs)

$$\begin{aligned}
 E(c_2|_1^+) &= \pi c^{++} + (1 - \pi)c^{+-} = 0.45(7.347) + (1 - 0.45)0 = 3.31 \\
 E(c_2|_1^-) &= \pi c^{-+} + (1 - \pi)c^{-+} = 0.45(0.0) + (1 - 0.45)0.0 = 0.0 \\
 E(p_2|_1^+) &= \pi p^{++} + (1 - \pi)p^{+-} = 0.45(0.0) + (1 - 0.45)0.406 = 0.2233 \\
 E(p_2|_1^-) &= \pi p^{-+} + (1 - \pi)p^{-+} = 0.45(0.406) + (1 - 0.45)4.24 = 2.51
 \end{aligned}$$

Thus, because  $PV_{1,2}(1) = 1/(1 + 0.0435) = 0.958313$ , we have the Time Step 1 option values of

$$\begin{aligned}
 c^+ &= PV_{1,2} \left[ E(c_2|_1^+) \right] = 0.958313(3.31) = 3.17 \\
 c^- &= PV_{1,2} \left[ E(c_2|_1^-) \right] = 0.958313(0.0) = 0.0 \\
 p^+ &= PV_{1,2} \left[ E(p_2|_1^+) \right] = 0.958313(0.2233) = 0.214 \\
 p^- &= PV_{1,2} \left[ E(p_2|_1^-) \right] = 0.958313(2.51) = 2.41
 \end{aligned}$$

At Time Step 0, we have

$$\begin{aligned} E(c_2|_0) &= \pi^2 c^{++} + 2\pi(1-\pi)c^{+-} + (1-\pi)^2 c^{--} \\ &= 0.45^2(7.347) + 2(0.45)(1-0.45)0 + (1-0.45)^2 0 = 1.488 \\ E(p_2|_0) &= \pi^2 p^{++} + 2\pi(1-\pi)p^{+-} + (1-\pi)^2 p^{--} \\ &= 0.45^2(0) + 2(0.45)(1-0.45)0.406 + (1-0.45)^2 4.24 = 1.484 \end{aligned}$$

Thus,

$$c = PV_{rf,0,2} \left[ E(c_2|_0) \right] = 0.91836(1.488) = 1.37 \text{ and}$$

$$p = PV_{rf,0,2} \left[ E(p_2|_0) \right] = 0.91836(1.484) = 1.36$$

With the two-period binomial model, the call and put values based on the expectations approach are simply the present values of the expected payoffs. The present value of the expected payoffs is based on the risk-free interest rate and the expectations approach is based on the risk-neutral probability. The parameters in this example were selected so that the European-style put and call would have approximately the same value. Notice that the stock price is less than the exercise price by roughly the present value factor or  $7.35 = 8.0/1.0435^2$ . One intuitive explanation is put-call parity, which can be expressed as  $c - p = S - PV(X)$ . Thus, if  $S = PV(X)$ , then  $c = p$ .

*Solution to 2:* The computation of the hedge ratios at Time Step 1 and Time Step 0 will require the option values at Time Step 1 and Time Step 2. The terminal values of the options are given in Solution 1.

$$\begin{aligned} S^{++} &= u^2 S = 1.445^2(7.35) = 15.347 \\ S^{+-} &= u d S = 1.445(0.715)7.35 = 7.594 \\ S^{--} &= d^2 S = 0.715^2(7.35) = 3.758 \\ S^+ &= u S = 1.445(7.35) = 10.621 \\ S^- &= d S = 0.715(7.35) = 5.255 \end{aligned}$$

Therefore, the hedge ratios at Time 1 are

$$h_c^+ = \frac{c^{++} - c^{+-}}{S^{++} - S^{+-}} = \frac{7.347 - 0.0}{15.347 - 7.594} = 0.9476$$

$$h_c^- = \frac{c^{+-} - c^{--}}{S^{+-} - S^{--}} = \frac{0.0 - 0.0}{7.594 - 3.758} = 0.0$$

$$h_p^+ = \frac{p^{++} - p^{+-}}{S^{++} - S^{+-}} = \frac{0.0 - 0.406}{15.347 - 7.594} = -0.05237$$

$$h_p^- = \frac{p^{+-} - p^{--}}{S^{+-} - S^{--}} = \frac{0.406 - 4.24}{7.594 - 3.758} = -1.0$$

In the last hedge ratio calculation, both put options are in the money ( $p^+$  and  $p^-$ ). In this case, the hedge ratio will be  $-1$ , subject to a rounding error. We now turn to interpreting the model's component terms. Based on the no-arbitrage approach, we have for the call price, assuming an up move has occurred, at Time Step 1,

$$\begin{aligned} c^+ &= h_c^+ S^+ + PV_{1,2}(-h_c^+ S^{++} + c^{++}) \\ &= 0.9476(10.621) + (1/1.0435)[-0.9476(7.594) + 0.0] = 3.1684 \end{aligned}$$

Thus, the call option can be interpreted as a leveraged position in the stock. Specifically, long 0.9476 shares for a cost of 10.0645 [= 0.9476(10.621)] partially financed with a 6.8961 {= (1/1.0435)[-0.9476(7.594) + 0.0]} loan. Note that the loan amount can be found simply as the cost of the position in shares less the option value [6.8961 = 0.9476(10.621) - 3.1684]. Similarly, we have

$$\begin{aligned} c^- &= h_c^- S^- + PV_{1,2}(-h_c^- S^{--} + c^{--}) \\ &= 0.0(5.255) + (1/1.0435)[-0.0(3.758) + 0.0] = 0.0 \end{aligned}$$

Specifically, long 0.0 shares for a cost of 0.0 [= 0.0(5.255)] with no financing. For put options, the interpretation is different. Specifically, we have

$$\begin{aligned} p^+ &= PV_{1,2}(-h_p^+ S^{++} + p^{++}) + h_p^+ S^+ \\ &= (1/1.0435)[-(-0.05237)15.347 + 0.0] + (-0.05237)10.621 = 0.2140 \end{aligned}$$

Thus, the put option can be interpreted as lending that is partially financed with a short position in shares. Specifically, short 0.05237 shares for a cost of 0.55622 [= (-0.05237)10.621] with financing of 0.77022 {= (1/1.0435)[-(-0.05237)15.347 + 0.0]}. Note that the lending amount can be found simply as the proceeds from the short sale of shares plus the option value [0.77022 = (0.05237)10.621 + 0.2140]. Again, we have

$$\begin{aligned} p^- &= PV_{1,2}(-h_p^- S^{--} + p^{--}) + h_p^- S^- \\ &= (1/1.0435)[-(-1.0)7.594 + 0.406] + (-1.0)5.255 = 2.4115 \end{aligned}$$

Here, we short 1.0 shares for a cost of 5.255 [= (-1.0)5.255] with financing of 7.6665 [= (1/1.0435)[-(-1.0)7.594 + 0.406]]. Again, the lending amount can be found simply as the proceeds from the short sale of shares plus the option value [7.6665 = (1.0)5.255 + 2.4115].

Finally, we have at Time Step 0

$$h_c = \frac{c^+ - c^-}{S^+ - S^-} = \frac{3.1684 - 0}{10.621 - 5.255} = 0.5905$$

$$h_p = \frac{p^+ - p^-}{S^+ - S^-} = \frac{0.2140 - 2.4115}{10.621 - 5.255} = -0.4095$$

The interpretations remain the same at Time Step 0:

$$\begin{aligned} c &= h_c S + PV_{0,1}(-h_c S^- + c^-) \\ &= 0.5905(7.35) + (1/1.0435)[-0.5905(5.255) + 0.0] = 1.37 \end{aligned}$$

Here, we are long 0.5905 shares for a cost of 4.3402 [=0.5905(7.35)] partially financed with a 2.97 {= (1/1.0435)[-0.5905(5.255) + 0.0] or = 0.5905(7.35) – 1.37} loan.

$$\begin{aligned} p &= PV_{0,1}(-h_p S^+ + p^+) + h_p S \\ &= (1/1.0435)\{-[-0.4095(10.621)] + 0.214\} + (-0.4095)7.35 = 1.36 \end{aligned}$$

Here, we short 0.4095 shares for a cost of 3.01 [=(-0.4095)7.35] with financing of 4.37 {= (1/1.0435)\{-[-0.4095(10.621)] + 0.214\} or = (0.4095)7.35 + 1.36}.

*Solution to 3:* We know that American-style call options on non-dividend-paying stocks are worth the same as European-style call options because early exercise will not occur. Thus, as previously computed,  $c^+ = 3.17$ ,  $c^- = 0.0$ , and  $c = 1.37$ . Recall that the call exercise value (denoted with EV) is simply the maximum of zero or the stock price minus the exercise price. We note that the EVs are less than or equal to the call model values; that is,

$$\begin{aligned} c_{EV}^+ &= \text{Max}(0, S^+ - X) = \text{Max}(0, 10.621 - 8.0) = 2.621 (< 3.1684) \\ c_{EV}^- &= \text{Max}(0, S^- - X) = \text{Max}(0, 5.255 - 8.0) = 0.0 (= 0.0) \\ c_{EV} &= \text{Max}(0, S - X) = \text{Max}(0, 7.35 - 8.0) = 0.0 (< 1.37) \end{aligned}$$

Therefore, the American-style feature for non-dividend-paying stocks has no effect on either the hedge ratio or the option value. The binomial model for American-style calls on non-dividend-paying stocks can be described and interpreted the same as a similar European-style call. This point is consistent with what we said earlier. If there are no dividends, an American-style call will not be exercised early.

This result is not true for puts. We know that American-style put options on non-dividend-paying stocks may be worth more than the analogous European-style put options. The hedge ratios at Time Step 1 will be the same as European-style puts because there is only one period left. Therefore, as previously shown,  $p^+ = 0.214$  and  $p^- = 2.41$ .

The put exercise values are

$$\begin{aligned} p_{EV}^+ &= \text{Max}(0, X - S^+) = \text{Max}(0, 8.0 - 10.621) = 0 (< 0.214) \\ p_{EV}^- &= \text{Max}(0, X - S^-) = \text{Max}(0, 8.0 - 5.255) = 2.745 (> 2.41) \end{aligned}$$

Because the exercise value for the put at Time Step 1, assuming a down move occurred, is greater than the model value, we replace the model value with the exercise value. Hence,

$$p^- = 2.745$$

and the hedge ratio at Time Step 0 will be affected. Specifically, we now have

$$h_p = \frac{p^+ - p^-}{S^+ - S^-} = \frac{0.2140 - 2.745}{10.621 - 5.255} = -0.4717$$

and thus the put model value is

$$p = (1/1.0435)[0.45(0.214) + 0.55(2.745)] = 1.54$$

Clearly, the early exercise feature has a significant impact on both the hedge ratio and the put option value in this case. The hedge ratio goes from  $-0.4095$  to  $-0.4717$ . The put value is raised from  $1.36$  to  $1.54$ .

We see through the simple two-period binomial model that an option can be viewed as a position in the underlying with financing. Furthermore, this valuation model can be expressed as the present value of the expected future cash flows, where the expectation is taken under the RN probability and the discounting is at the risk-free rate.

Up to this point, we have focused on equity options. The binomial model can be applied to any underlying instrument though often requiring some modifications. For example, currency options would require incorporating the foreign interest rate. Futures options would require a binomial lattice of the futures prices. Interest rate options, however, require somewhat different tools that we now examine.

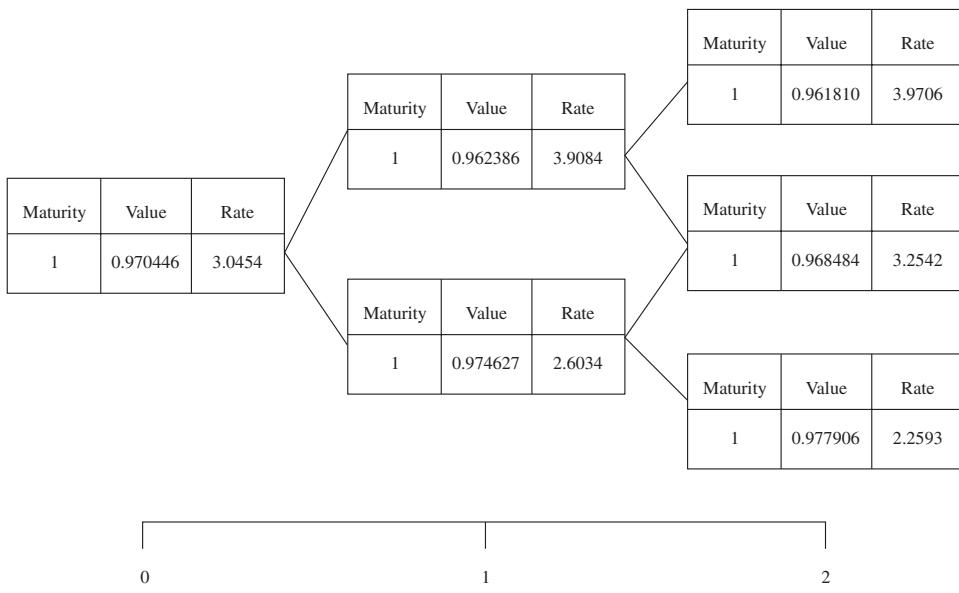
### 3.3. Interest Rate Options

In this section, we will briefly illustrate how to value interest rate options. There are a wide variety of approaches to valuing interest rate options. We do not delve into how arbitrage-free interest rate trees are generated. The particular approach used here assumes the RN probability of an up move at each node is 50%.

Exhibit 11 presents a binomial lattice of interest rates covering two years along with the corresponding zero-coupon bond values. The rates are expressed in annual compounding. Therefore, at Time 0, the spot rate is  $(1.0/0.970446) - 1$  or 3.04540%.<sup>6</sup> Note that at Time 1, the value in the column labeled "Maturity" reflects time to maturity not calendar time. The lattice shows the rates on one-period bonds, so all bonds have a maturity of 1. The column labeled "Value" is the value of a zero-coupon bond with the stated maturity based on the rates provided.

<sup>6</sup>The values in the first box from the left are observed at  $t = 0$ . The values in the remainder of the lattice are derived by using a technique that is outside the scope of this reading.

EXHIBIT 11 Two-Year Binomial Interest Rate Lattice by Year



The underlying instrument for interest rate options here is the spot rate. A call option on interest rates will be in the money when the current spot rate is above the exercise rate. A put option on interest rates will be in the money when the current spot rate is below the exercise rate. Thus, based on the notation in the previous section, the current spot rate is denoted  $S$ . Option valuation follows the expectations approach discussed in the previous section but taken only one period at a time. The procedure is illustrated with an example.

### EXAMPLE 8 Option on Interest Rates

This example is based on Exhibit 11. Suppose we seek to value two-year European-style call and put options on the periodically compounded one-year spot interest rate (the underlying). Assume the notional amount of the options is US\$1,000,000 and the call and put exercise rate is 3.25% of par. Assume the RN probability is 50% and these options cash settle at Time 2 based on the observed rates.

*Solution:* Using the expectations approach introduced in the last section, we have (per US\$1) at Time Step 2

$$\begin{aligned}
 c^{++} &= \text{Max}(0, S^{++} - X) = \text{Max}[0, 0.039706 - 0.0325] = 0.007206 \\
 c^{+-} &= \text{Max}(0, S^{+-} - X) = \text{Max}[0, 0.032542 - 0.0325] = 0.000042 \\
 c^{-+} &= \text{Max}(0, S^{-+} - X) = \text{Max}[0, 0.022593 - 0.0325] = 0.0 \\
 p^{++} &= \text{Max}(0, X - S^{++}) = \text{Max}[0, 0.0325 - 0.039706] = 0.0 \\
 p^{+-} &= \text{Max}(0, X - S^{+-}) = \text{Max}[0, 0.0325 - 0.032542] = 0.0 \\
 p^{-+} &= \text{Max}(0, X - S^{-+}) = \text{Max}[0, 0.0325 - 0.022593] = 0.009907
 \end{aligned}$$

At Time Step 1, we have

$$\begin{aligned}
 c^+ &= PV_{1,2}[\pi c^{++} + (1 - \pi)c^+] \\
 &= 0.962386[0.5(0.007206) + (1 - 0.5)0.000042] \\
 &= 0.003488 \\
 c^- &= PV_{1,2}[\pi c^+ + (1 - \pi)c^-] \\
 &= 0.974627[0.5(0.000042) + (1 - 0.5)0.0] \\
 &= 0.00002 \\
 p^+ &= PV_{1,2}[\pi p^{++} + (1 - \pi)p^+] \\
 &= 0.962386[0.5(0.0) + (1 - 0.5)0.0] \\
 &= 0.0 \\
 p^- &= PV_{1,2}[\pi p^+ + (1 - \pi)p^-] \\
 &= 0.974627[0.5(0.0) + (1 - 0.5)0.009907] \\
 &= 0.004828
 \end{aligned}$$

Notice how the present value factors are different for the up and down moves. At Time Step 1 in the + outcome, we discount by a factor of 0.962386, and in the - outcome, we discount by the factor 0.974627. Because this is an option on interest rates, it should not be surprising that we have to allow the interest rate to vary.

Therefore, at Time Step 0, we have

$$\begin{aligned}
 c &= PV_{rf,0,1}[\pi c^+ + (1 - \pi)c^-] \\
 &= 0.970446[0.5(0.003488) + (1 - 0.5)0.00002] \\
 &= 0.00170216 \\
 p &= PV_{rf,0,1}[\pi p^+ + (1 - \pi)p^-] \\
 &= 0.970446[0.5(0.0) + (1 - 0.5)0.004828] \\
 &= 0.00234266
 \end{aligned}$$

Because the notional amount is US\$1,000,000, the call value is US\$1,702.16 [= US\$1,000,000(0.00170216)] and the put value is US\$2,342.66 [= US\$1,000,000(0.00234266)]. The key insight is to just work a two-period binomial model as three one-period binomial models.

We turn now to briefly generalize the binomial model as it leads naturally to the Black–Scholes–Merton option valuation model.

### 3.4. Multiperiod Model

The multiperiod binomial model provides a natural bridge to the Black–Scholes–Merton option valuation model presented in the next section. The idea is to take the option's expiration and slice it up into smaller and smaller periods. The two-period model divides the expiration into two periods. The three-period model divides expiration into three periods and so forth. The process continues until you have a large number of time steps. The key feature is that each time step is of equal length. Thus, with a maturity of  $T$ , if there are  $n$  time steps, then each time step is  $T/n$  in length.

For American-style options, we must also test at each node whether the option is worth more exercised or not exercised. As in the two-period case, we work backward through the binomial tree testing the model value against the exercise value and always choosing the higher one.

The binomial model is an important and useful methodology for valuing options. The expectations approach can be applied to European-style options and will lead naturally to the BSM model in the next section. This approach simply values the option as the present value of the expected future payoffs, where the expectation is taken under the risk-neutral probability and the discounting is based on the risk-free rate. The no-arbitrage approach can be applied to either European-style or American-style options because it provides the intuition for the fair value of options.

## 4. BLACK–SCHOLES–MERTON OPTION VALUATION MODEL

The BSM model, although very complex in its derivation, is rather simple to use and interpret. The objective here is to illustrate several facets of the BSM model with the objective of highlighting its practical usefulness. After a brief introduction, we examine the assumptions of the BSM model and then delve into the model itself.

### 4.1. Introductory Material

Louis Bachelier published the first known mathematically rigorous option valuation model in 1900. By the late 1960s, there were several published quantitative option models. Fischer Black, Myron Scholes, and Robert Merton introduced the BSM model in 1973 in two published papers, one by Black and Scholes and the other by Merton. The innovation of the BSM model is essentially the no-arbitrage approach introduced in the previous section but applied with a continuous time process, which is equivalent to a binomial model in which the length of the time step essentially approaches zero. It is also consistent with the basic statistical fact that the binomial process with a “large” number of steps converges to the standard normal distribution. Myron Scholes and Robert Merton won the 1997 Nobel Prize in Economics based, in part, on their work related to the BSM model.<sup>7</sup> Let us now examine the BSM model assumptions.

### 4.2. Assumptions of the BSM Model

The key assumption for option valuation models is how to model the random nature of the underlying instrument. This characteristic of how an asset evolves randomly is called a stochastic process. Many financial instruments enjoy limited liability; hence, the values of instruments cannot be negative, but they certainly can be zero. In 1900, Bachelier proposed the normal distribution. The key advantages of the normal distribution are that zero is possible, meaning that bankruptcy is allowable, it is symmetric, it is relatively easy to manipulate, and it is additive (which means that sums of normal distributions are normally distributed). The key disadvantage is that negative stock values are theoretically possible, which violates the limited liability principal of stock ownership. Based on research on stock prices in the 1950s

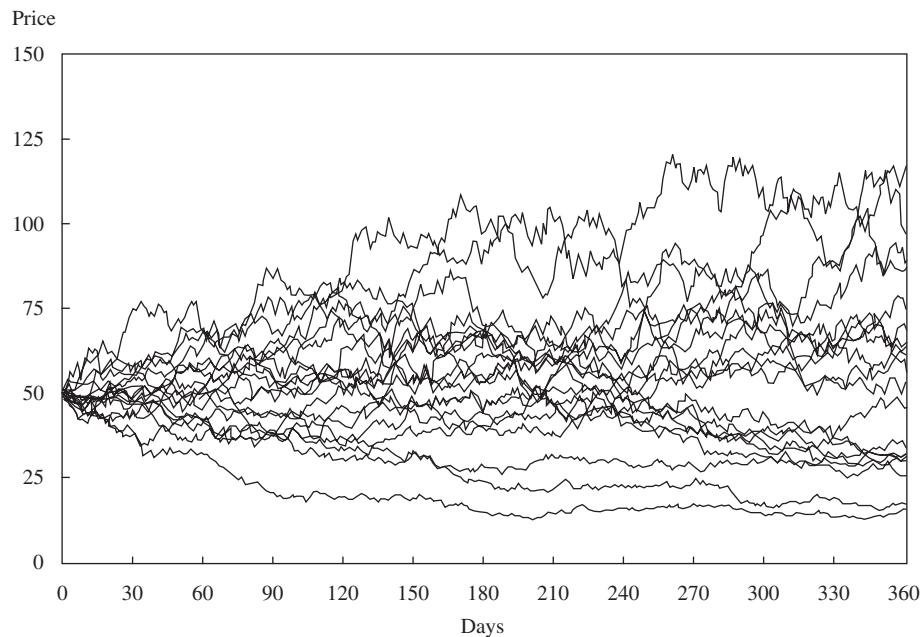
<sup>7</sup>Fischer Black passed away in 1995 and the Nobel Prize is not awarded posthumously.

and 1960s, a preference emerged for the lognormal distribution, which means that log returns are distributed normally. Black, Scholes, and Merton chose to use the lognormal distribution.

Recall that the no-arbitrage approach requires self-financing and dynamic replication; we need more than just an assumption regarding the terminal distribution of the underlying instrument. We need to model the value of the instrument as it evolves over time, which is what we mean by a stochastic process. The stochastic process chosen by Black, Scholes, and Merton is called geometric Brownian motion (GBM).

Exhibit 12 illustrates GBM, assuming the initial stock price is  $S = 50$ . We assume the stock will grow at 3% ( $\mu = 3\%$  annually, geometrically compounded rate). This GBM process also reflects a random component that is determined by a volatility ( $\sigma$ ) of 45%. This volatility is the annualized standard deviation of continuously compounded percentage change in the underlying, or in other words, the log return. Note that as a particular sample path drifts upward, we observe more variability on an absolute basis, whereas when the particular sample path drifts downward, we observe less variability on an absolute basis. For example, examine the highest and lowest lines shown in Exhibit 12. The highest line is much more erratic than the lowest line. Recall that a 10% move in a stock with a price of 100 is 10 whereas a 10% move in a stock with a price of 10 is only 1. Thus, GBM can never hit zero nor go below it. This property is appealing because many financial instruments enjoy limited liability and cannot be negative. Finally, note that although the stock movements are rather erratic, there are no large jumps—a common feature with marketable financial instruments.

EXHIBIT 12 Geometric Brownian Motion Simulation ( $S = 50$ ,  $\mu = 3\%$ ,  $\sigma = 45\%$ )



Within the BSM model framework, it is assumed that all investors agree on the distributional characteristics of GBM except the assumed growth rate of the underlying. This growth rate depends on a number of factors, including other instruments and time. The standard BSM model assumes a constant growth rate and constant volatility.

The specific assumptions of the BSM model are as follows:

- The underlying follows a statistical process called geometric Brownian motion, which implies a lognormal distribution of the return, meaning that the logarithmic return, which is the continuously compounded return, is normally distributed.
- Geometric Brownian motion implies continuous prices, meaning that the price of the underlying instrument does not jump from one value to another; rather, it moves smoothly from value to value.
- The underlying instrument is liquid, meaning that it can be easily bought and sold.
- Continuous trading is available, meaning that in the strictest sense one must be able to trade at every instant.
- Short selling of the underlying instrument with full use of the proceeds is permitted.
- There are no market frictions, such as transaction costs, regulatory constraints, or taxes.
- No-arbitrage opportunities are available in the marketplace.
- The options are European-style, meaning that early exercise is not allowed.
- The continuously compounded risk-free interest rate is known and constant; borrowing and lending is allowed at the risk-free rate.
- The volatility of the return on the underlying is known and constant.
- If the underlying instrument pays a yield, it is expressed as a continuous known and constant yield at an annualized rate.

Naturally, the foregoing assumptions are not absolutely consistent with real financial markets, but, as in all financial models, the question is whether they produce models that are tractable and useful in practice, which they do.

### EXAMPLE 9 BSM Model Assumptions

Which is the *correct* pair of statements? The BSM model assumes:

- A. the return on the underlying has a normal distribution. The price of the underlying can jump abruptly to another price.
- B. brokerage costs are factored into the BSM model. It is impossible to trade continuously.
- C. volatility can be predicted with certainty. Arbitrage is non-existent in the marketplace.

*Solution:* C is correct. All four of the statements in A and B are incorrect within the BSM model paradigm.

We turn now to a careful examination of the BSM model.

### 4.3. BSM Model

The BSM model is a continuous time version of the discrete time binomial model. Given that the BSM model is based on continuous time, it is customary to use a continuously compounded interest rate rather than some discretely compounded alternative. Thus, when an interest rate is used here, denoted simply as  $r$ , we mean solely the annualized continuously compounded rate.<sup>8</sup> The volatility, denoted as  $\sigma$ , is also expressed in annualized percentage terms. Initially, we focus on a non-dividend-paying stock. The BSM model, with some adjustments, applies to other underlying instruments, which will be examined later.

The BSM model for stocks can be expressed as

$$c = SN(d_1) - e^{-rT}XN(d_2) \quad (10)$$

and

$$p = e^{-rT}XN(-d_2) - SN(-d_1) \quad (11)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$N(x)$  denotes the standard normal cumulative distribution function, which is the probability of obtaining a value of less than  $x$  based on a standard normal distribution. In our context,  $x$  will have the value of  $d_1$  or  $d_2$ .  $N(x)$  reflects the likelihood of observing values less than  $x$  from a random sample of observations taken from the standard normal distribution.

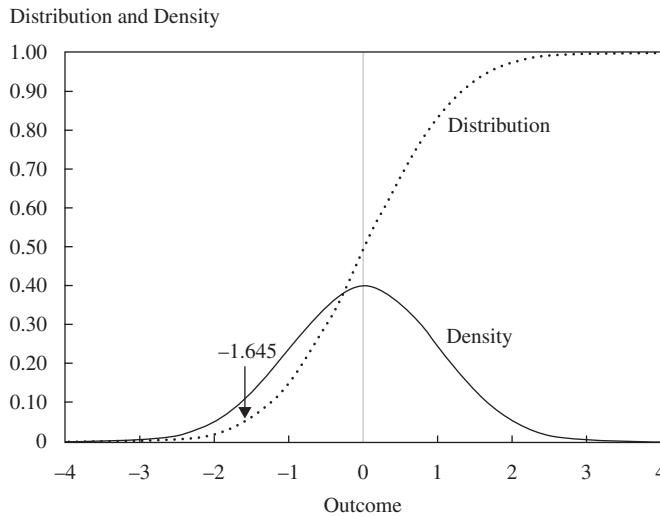
Although the BSM model appears very complicated, it has straightforward interpretations that will be explained.  $N(x)$  can be estimated by a computer program or a spreadsheet or approximated from a lookup table. The normal distribution is a symmetric distribution with two parameters, the mean and standard deviation. The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.

Exhibit 13 illustrates the standard normal probability density function (the standard bell curve) and the cumulative distribution function (the accumulated probability and range of 0 to 1). Note that even though GBM is lognormally distributed, the  $N(x)$  functions in the BSM model are based on the standard normal distribution. In Exhibit 13, we see that if  $x = -1.645$ , then  $N(x) = N(-1.645) = 0.05$ . Thus, if the model value of  $d$  is  $-1.645$ , the corresponding probability is 5%. Clearly, values of  $d$  that are less than 0 imply values of  $N(x)$  that are less than 0.5. As a result of the symmetry of the normal distribution, we note that  $N(-x) = 1 - N(x)$ .

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<sup>8</sup>Note  $e^r = 1 + r_d$ , where  $r_d$  is the annually compounded rate.

## EXHIBIT 13 Standard Normal Distribution



The BSM model can be described as the present value of the expected option payoff at expiration. Specifically, we can express the BSM model for calls as  $c = PV_r[E(c_T)]$  and for puts as  $p = PV_r[E(p_T)]$ , where  $E(c_T) = Se^{rT}N(d_1) - XN(d_2)$  and  $E(p_T) = XN(-d_2) - Se^{rT}N(-d_1)$ . The present value term in this context is simply  $e^{-rT}$ . As with most valuation tasks in finance, the value today is simply the present value of the expected future cash flows. It is important to note that the expectation is based on the risk-neutral probability measure defined in Section 3.1. The expectation is not based on the investor's subjective beliefs, which reflect an aversion to risk. Also, the present value function is based on the risk-free interest rate not on the investor's required return on invested capital, which of course is a function of risk.

Alternatively, the BSM model can be described as having two components: a stock component and a bond component. For call options, the stock component is  $SN(d_1)$  and the bond component is  $e^{-rT}XN(d_2)$ . The BSM model call value is the stock component minus the bond component. For put options, the stock component is  $SN(-d_1)$  and the bond component is  $e^{-rT}XN(-d_2)$ . The BSM model put value is the bond component minus the stock component.

The BSM model can be interpreted as a dynamically managed portfolio of the stock and zero-coupon bonds.<sup>9</sup> The goal is to replicate the option payoffs with stocks and bonds. For both call and put options, we can represent the initial cost of this replicating strategy as

$$\text{Replicating strategy cost} = n_S S + n_B B$$

where the equivalent number of underlying shares is  $n_S = N(d_1) > 0$  for calls and  $n_S = -N(-d_1) < 0$  for puts. The equivalent number of bonds is  $n_B = -N(d_2) < 0$  for calls and  $n_B = N(-d_2) > 0$  for puts. The price of the zero-coupon bond is  $B = e^{-rT}X$ . Note, if  $n$  is positive, we are

<sup>9</sup>When covering the binomial model, the bond component was generically termed financing. This component is typically handled with bank borrowing or lending. With the BSM model, it is easier to understand as either buying or short selling a risk-free zero-coupon bond.

buying the underlying and if  $n$  is negative we are selling (short selling) the underlying. The cost of the portfolio will exactly equal either the BSM model call value or the BSM model put value.

For calls, we are simply buying stock with borrowed money because  $n_S > 0$  and  $n_B < 0$ . Again the cost of this portfolio will equal the BSM model call value, and if appropriately rebalanced, then this portfolio will replicate the payoff of the call option. Therefore, a call option can be viewed as a leveraged position in the stock.

Similarly, for put options, we are simply buying bonds with the proceeds from short selling the underlying because  $n_S < 0$  and  $n_B > 0$ . The cost of this portfolio will equal the BSM model put value, and if appropriately rebalanced, then this portfolio will replicate the payoff of the put option. Note that a short position in a put will result in receiving money today and  $n_S > 0$  and  $n_B < 0$ . Therefore, a short put can be viewed as an over-leveraged or over-geared position in the stock because the borrowing exceeds 100% of the cost of the underlying.

Exhibit 14 illustrates the direct comparison between the no-arbitrage approach to the single-period binomial option valuation model and the BSM option valuation model. The parallel between the  $h$  term in the binomial model and  $N(d_1)$  is easy to see. Recall that the term hedge ratio was used with the binomial model because we were creating a no-arbitrage portfolio. Note for call options,  $-N(d_2)$  implies borrowing money or short selling  $N(d_2)$  shares of a zero-coupon bond trading at  $e^{-rT}X$ . For put options,  $N(-d_2)$  implies lending money or buying  $N(-d_2)$  shares of a zero-coupon bond trading at  $e^{-rT}X$ .

EXHIBIT 14 BSM and Binomial Option Valuation Model Comparison

Option Valuation Model Terms	Call Option		Put Option	
	Underlying	Financing	Underlying	Financing
Binomial Model	$hS$	$PV(-hS^- + c^-)$	$hS$	$PV(-hS^- + p^-)$
BSM Model	$N(d_1)S$	$-N(d_2)e^{-rT}X$	$-N(-d_1)S$	$N(-d_2)e^{-rT}X$

If the value of the underlying,  $S$ , increases, then the value of  $N(d_1)$  also increases because  $S$  has a positive effect on  $d_1$ . Thus, the replicating strategy for calls requires continually buying shares in a rising market and selling shares in a falling market.

Within the BSM model theory, the aggregate losses from this “buy high/sell low” strategy, over the life of the option, adds up exactly to the BSM model option premium received for the option at inception.<sup>10</sup> This result must be the case; otherwise there would be arbitrage profits available. Because transaction costs are not, in fact, zero, the frequent rebalancing by buying and selling the underlying adds significant costs for the hedger. Also, markets can often move discontinuously, contrary to the BSM model’s assumption that prices move continuously, thus allowing for continuous hedging adjustments. Hence, in reality, hedges are imperfect. For example, if a company announces a merger, then the company’s stock price may jump substantially higher, contrary to the BSM model’s assumption.

In addition, volatility cannot be known in advance. For these reasons, options are typically more expensive than they would be as predicted by the BSM model theory. In order to continue using the BSM model, the volatility parameter used in the formula is usually higher

<sup>10</sup>The validity of this claim does not rest on the validity of the BSM model assumptions; rather the validity depends only on whether the BSM model accurately predicts the replication cost.

(by, say, 1% or 2%, but this can vary a lot) than the volatility of the stock actually expected by market participants. We will ignore this point for now, however, as we focus on the mechanics of the model.

#### EXAMPLE 10 Illustration of BSM Model Component Interpretation

Suppose we are given the following information on call and put options on a stock:  $S = 100$ ,  $X = 100$ ,  $r = 5\%$ ,  $T = 1.0$ , and  $\sigma = 30\%$ . Thus, based on the BSM model, it can be demonstrated that  $PV(X) = 95.123$ ,  $d_1 = 0.317$ ,  $d_2 = 0.017$ ,  $N(d_1) = 0.624$ ,  $N(d_2) = 0.507$ ,  $N(-d_1) = 0.376$ ,  $N(-d_2) = 0.493$ ,  $c = 14.23$ , and  $p = 9.35$ .

1. The initial trading strategy required by the no-arbitrage approach to replicate the call option payoffs for a buyer of the option is:
  - A. buy 0.317 shares of stock and short sell  $-0.017$  shares of zero-coupon bonds.
  - B. buy 0.624 shares of stock and short sell 0.507 shares of zero-coupon bonds.
  - C. short sell 0.317 shares of stock and buy 0.017 shares of zero-coupon bonds.
2. Identify the initial trading strategy required by the no-arbitrage approach to replicate the put option payoffs for a buyer of the put.
  - A. Buy 0.317 shares of stock and short sell  $-0.017$  shares of zero-coupon bonds.
  - B. Buy 0.624 shares of stock and short sell 0.507 shares of zero-coupon bonds.
  - C. Short sell 0.376 shares of stock and buy 0.493 shares of zero-coupon bonds.

*Solution to 1:* B is correct. The no-arbitrage approach to replicating the call option involves purchasing  $n_S = N(d_1) = 0.624$  shares of stock partially financed with  $n_B = -N(d_2) = -0.507$  shares of zero-coupon bonds priced at  $B = Xe^{-rT} = 95.123$  per bond. Note that by definition the cost of this replicating strategy is the BSM call model value or  $n_S S + n_B B = 0.624(100) + (-0.507)95.123 = 14.17$ . Without rounding errors, the option value is 14.23.

*Solution to 2:* C is correct. The no-arbitrage approach to replicating the put option is similar. In this case, we trade  $n_S = -N(-d_1) = -0.376$  shares of stock—specifically, short sell 0.376 shares—and buy  $n_B = N(-d_2) = 0.493$  shares of zero-coupon bonds. Again, the cost of the replicating strategy is  $n_S S + n_B B = -0.376(100) + (0.493)95.123 = 9.30$ . Without rounding errors, the option value is 9.35. Thus, to replicate a call option based on the BSM model, we buy stock on margin. To replicate a put option, we short sell stock and lend part of the proceeds.

Note that the  $N(d_2)$  term has an additional important interpretation. It is a unique measure of the probability that the call option expires in the money, and correspondingly,  $1 - N(d_2) = N(-d_2)$  is the probability that the put option expires in the money. Specifically, the probability based on the RN probability of being in the money, not one's own estimate of the probability of being in the money nor the market's estimate. That is,  $N(d_2) = \text{Prob}(S_T > X)$  based on the unique RN probability.

We now turn to incorporating various carry benefits into the BSM model. Carry benefits include dividends for stock options, foreign interest rates for currency options, and coupon payments for bond options. For other underlying instruments, there are carry costs that can easily be treated as negative carry benefits, such as storage and insurance costs for agricultural products. Because the BSM model is established in continuous time, it is common to model these carry benefits as a continuous yield, denoted generically here as  $\gamma$  or simply  $\gamma$ .

The BSM model requires a few adjustments to accommodate carry benefits. The carry benefit-adjusted BSM model is

$$c = Se^{-\gamma T}N(d_1) - e^{-rT}XN(d_2) \quad (12)$$

and

$$p = e^{-rT}XN(-d_2) - Se^{-\gamma T}N(-d_1) \quad (13)$$

where

$$d_1 = \frac{\ln(S/X) + (r - \gamma + \sigma^2/2)T}{\sigma\sqrt{T}}$$

Note that  $d_2$  can be expressed again simply as  $d_2 = d_1 - \sigma\sqrt{T}$ . The value of a put option can also be found based on the carry benefit-adjusted put-call parity:

$$p + Se^{-\gamma T} = c + e^{-rT}X \quad (14)$$

The carry benefit-adjusted BSM model can again be described as the present value of the expected option payoff at expiration. Now, however,  $E(c_T) = Se^{(r-\gamma)T}N(d_1) - XN(d_2)$  and  $E(p_T) = XN(-d_2) - Se^{(r-\gamma)T}N(-d_1)$ . The present value term remains simply  $e^{-rT}$ . Carry benefits will have the effect of lowering the expected future value of the underlying.

Again, the carry benefit adjusted BSM model can be described as having two components, a stock component and a bond component. For call options, the stock component is  $Se^{-\gamma T}N(d_1)$  and the bond component is again  $e^{-rT}XN(d_2)$ . For put options, the stock component is  $Se^{-\gamma T}N(-d_1)$  and the bond component is again  $e^{-rT}XN(-d_2)$ . Although both  $d_1$  and  $d_2$  are reduced by carry benefits, the general approach to valuation remains the same. An increase in carry benefits will lower the value of the call option and raise the value of the put option.

Note that  $N(d_2)$  term continues to be interpreted as the RN probability of a call option being in the money. The existence of carry benefits has the effect of lowering  $d_1$  and  $d_2$ , hence the probability of being in the money with call options declines as the carry benefit rises. This RN probability is an important element to describing how the BSM model is used in various valuation tasks.

For stock options,  $\gamma = \delta$ , which is the continuously compounded dividend yield. The dividend-yield BSM model can again be interpreted as a dynamically managed portfolio of the stock and zero coupon bonds. Based on the call model above applied to a dividend yielding stock, the equivalent number of units of stock is now  $n_S = e^{-\delta T}N(d_1) > 0$  and the equivalent number of units of bonds remains  $n_B = -N(d_2) < 0$ . Similarly with puts, the equivalent number of units of stock is now  $n_S = -e^{-\delta T}N(-d_1) < 0$  and the equivalent number of units of bonds again remains  $n_B = N(-d_2) > 0$ .

With dividend paying stocks, the arbitrageur is able to receive the benefits of dividend payments when long the stock and has to pay dividends when short the stock. Thus, the

burden of carrying the stock is diminished for a long position. The key insight is that dividends influence the dynamically managed portfolio by lowering the number of shares to buy for calls and lowering the number of shares to short sell for puts. Higher dividends will lower the value of  $d_1$ , thus lowering  $N(d_1)$ . Also, higher dividends will lower the number of bonds to short sell for calls and lower the number of bonds to buy for puts.

### EXAMPLE 11 BSM Model Applied to Equities

Suppose we are given the following information on an underlying stock and options:  $S = 60$ ,  $X = 60$ ,  $r = 2\%$ ,  $T = 0.5$ ,  $\delta = 2\%$ , and  $\sigma = 45\%$ . Assume we are examining European-style options.

1. Which answer *best* describes how the BSM model is used to value a call option with the parameters given?
  - A. The BSM model call value is the exercise price times  $N(d_1)$  less the present value of the stock price times  $N(d_2)$ .
  - B. The BSM model call value is the stock price times  $e^{-\delta T}N(d_1)$  less the exercise price times  $e^{-rT}N(d_2)$ .
  - C. The BSM model call value is the stock price times  $e^{-\delta T}N(-d_1)$  less the present value of the exercise price times  $e^{-rT}N(-d_2)$ .
2. Which answer *best* describes how the BSM model is used to value a put option with the parameters given?
  - A. The BSM model put value is the exercise price times  $N(d_1)$  less the present value of the stock price times  $N(d_2)$ .
  - B. The BSM model put value is the exercise price times  $e^{-\delta T}N(-d_2)$  less the stock price times  $e^{-rT}N(-d_2)$ .
  - C. The BSM model put value is the exercise price times  $e^{-rT}N(-d_2)$  less the stock price times  $e^{-\delta T}N(-d_1)$ .
3. Suppose now that the stock does not pay a dividend—that is,  $\delta = 0\%$ . Identify the correct statement.
  - A. The BSM model option value is the same as the previous problems because options are not dividend adjusted.
  - B. The BSM model option values will be different because there is an adjustment term applied to the exercise price, that is  $e^{-\delta T}$ , which will influence the option values.
  - C. The BSM model option value will be different because  $d_1$ ,  $d_2$ , and the stock component are all adjusted for dividends.

*Solution to 1:* B is correct. The BSM call model for a dividend-paying stock can be expressed as  $Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$ .

*Solution to 2:* C is correct. The BSM put model for a dividend-paying stock can be expressed as  $Xe^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$ .

*Solution to 3:* C is correct. The BSM model option value will be different because  $d_1$ ,  $d_2$ , and the stock component are all adjusted for dividends.

### EXAMPLE 12 How the BSM Model Is Used to Value Stock Options

Suppose that we have some Bank of China shares that are currently trading on the Hong Kong Stock Exchange at HKD4.41. Our view is that the Bank of China's stock price will be steady for the next three months, so we decide to sell some three-month out-of-the-money calls with exercise price at 4.60 in order to enhance our returns by receiving the option premium. Risk-free government securities are paying 1.60% and the stock is yielding HKD 0.24%. The stock volatility is 28%. We use the BSM model to value the calls.

Which statement is correct? The BSM model inputs (underlying, exercise, expiration, risk-free rate, dividend yield, and volatility) are:

- A. 4.60, 4.41, 3, 0.0160, 0.0024, and 0.28.
- B. 4.41, 4.60, 0.25, 0.0160, 0.0024, and 0.28.
- C. 4.41, 4.41, 0.3, 0.0160, 0.0024, and 0.28.

*Solution:* B is correct. The spot price of the underlying is HKD4.41. The exercise price is HKD4.60. The expiration is 0.25 years (three months). The risk-free rate is 0.016. The dividend yield is 0.0024. The volatility is 0.28.

For foreign exchange options,  $\gamma = r^f$ , which is the continuously compounded foreign risk-free interest rate. When quoting an exchange rate, we will give the value of the domestic currency per unit of the foreign currency. For example, Japanese yen (¥) per unit of the euro (€) will be expressed as the euro trading for ¥135 or succinctly 135¥/€. This is called the foreign exchange spot rate. Thus, the foreign currency, the euro, is expressed in terms of the Japanese yen, which is in this case the domestic currency. This is logical, for example, when a Japanese firm would want to express its foreign euro holdings in terms of its domestic currency, Japanese yen.

With currency options, the underlying instrument is the foreign exchange spot rate. Again, the carry benefit is the interest rate in the foreign country because the foreign currency could be invested in the foreign country's risk-free instrument. Also, with currency options, the underlying and the exercise price must be quoted in the same currency unit. Lastly, the volatility in the model is the volatility of the log return of the spot exchange rate. Each currency option is for a certain quantity of foreign currency, termed the notional amount, a concept analogous to the number of shares of stock covered in an option contract. The total cost of the option would be obtained by multiplying the formula value by the notional amount in the same way that one would multiply the formula value of an option on a stock by the number of shares the option contract covers.

The BSM model applied to currencies can be described as having two components, a foreign exchange component and a bond component. For call options, the foreign exchange component is  $Se^{-r^f T}N(d_1)$  and the bond component is  $e^{-rT}XN(d_2)$ , where  $r$  is the domestic risk-free rate. The BSM call model applied to currencies is simply the foreign exchange component minus the bond component. For put options, the foreign exchange component is  $Se^{-r^f T}N(-d_1)$  and the bond component is  $e^{-rT}XN(-d_2)$ . The BSM put model applied to currencies is simply the bond component minus the foreign exchange component. Remember that the underlying is expressed in terms of the domestic currency.

### EXAMPLE 13 BSM Model Applied to Value Options on Currency

A Japanese camera exporter to Europe has contracted to receive fixed euro (€) amounts each quarter for his goods. The spot price of the currency pair is 135¥/€. If the exchange rate falls to, say, 130¥/€, then the yen will have strengthened because it will take fewer yen to buy one euro. The exporter is concerned that the yen will strengthen because in this case, his forthcoming fixed euro will buy fewer yen. Hence, the exporter is considering buying an at-the-money spot euro put option to protect against this fall; this in essence is a call on yen. The Japanese risk-free rate is 0.25% and the European risk-free rate is 1.00%.

1. What are the underlying and exercise prices to use in the BSM model to get the euro put option value?
  - A. 1/135; 1/135
  - B. 135; 135
  - C. 135; 130
2. What are the risk-free rate and the carry rate to use in the BSM model to get the euro put option value?
  - A. 0.25%; 1.00%
  - B. 0.25%; 0.00%
  - C. 1.00%; 0.25%

*Solution to 1:* B is correct. The underlying is the spot FX price of 135 ¥/€. Because the put is at-the-money spot, the exercise price equals the spot price.

*Solution to 2:* A is correct. The risk-free rate to use is the Japanese rate because Japan is the domestic economy. The carry rate is the foreign currency's risk-free rate, which is the European rate.

We turn now to examine a modification of the BSM model when the underlying is a forward or futures contract.

## 5. BLACK OPTION VALUATION MODEL

In 1976, Fischer Black introduced a modified version of the BSM model approach that is applicable to options on underlying instruments that are costless to carry, such as options on futures contracts—for example, equity index futures—and options on forward contracts. The latter include interest rate-based options, such as caps, floors, and swaptions.

### 5.1. European Options on Futures

We assume that the futures price also follows geometric Brownian motion. We ignore issues like margin requirements and marking to market. Black proposed the following model for European-style futures options:

$$c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)] \quad (15)$$

and

$$p = e^{-rT}[XN(-d_2) - F_0(T)N(-d_1)] \quad (16)$$

where

$$d_1 = \frac{\ln[F_0(T)/X] + (\sigma^2/2)T}{\sigma\sqrt{T}} \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Note that  $F_0(T)$  denotes the futures price at Time 0 that expires at Time  $T$ , and  $\sigma$  denotes the volatility related to the futures price. The other terms are as previously defined. Black's model is simply the BSM model in which the futures contract is assumed to reflect the carry arbitrage model. Futures option put-call parity can be expressed as

$$c = e^{-rT}[F_0(T) - X] + p \quad (17)$$

As we have seen before, put-call parity is a useful tool for describing the valuation relationship between call and put values within various option valuation models.

The Black model can be described in a similar way to the BSM model. The Black model has two components, a futures component and a bond component. For call options, the futures component is  $F_0(T)e^{-rT}N(d_1)$  and the bond component is again  $e^{-rT}XN(d_2)$ . The Black call model is simply the futures component minus the bond component. For put options, the futures component is  $F_0(T)e^{-rT}N(-d_1)$  and the bond component is again  $e^{-rT}XN(-d_2)$ . The Black put model is simply the bond component minus the futures component.

Alternatively, futures option valuation, based on the Black model, is simply computing the present value of the difference between the futures price and the exercise price. The futures price and exercise price are appropriately adjusted by the  $N(d)$  functions. For call options, the futures price is adjusted by  $N(d_1)$  and the exercise price is adjusted by  $-N(d_2)$  to arrive at the difference. For put options, the futures price is adjusted by  $-N(-d_1)$  and the exercise price is adjusted by  $+N(-d_2)$ .

#### EXAMPLE 14 European Options on Futures Index

The S&P 500 Index (a spot index) is presently at 1,860 and the 0.25 expiration futures contract is trading at 1,851.65. Suppose further that the exercise price is 1,860, the continuously compounded risk-free rate is 0.2%, time to expiration is 0.25, volatility is 15%, and the dividend yield is 2.0%. Based on this information, the following results are obtained for options on the futures contract.<sup>11</sup>

<sup>11</sup>We ignore the effect of the multiplier. As of this writing, the S&P 500 futures option contract has a multiplier of 250. The prices reported here have not been scaled up by this amount. In practice, the option cost would be 250 times the option value.

Options on Futures	
Calls	Puts
$N(d_1) = 0.491$	$N(-d_1) = 0.509$
$N(d_2) = 0.461$	$N(-d_2) = 0.539$
$c = \text{US\$}51.41$	$p = \text{US\$}59.76$

1. Identify the statement that *best* describes how the Black model is used to value a European call option on the futures contract just described.
  - The call value is the present value of the difference between the exercise price times 0.461 and the current futures price times 0.539.
  - The call value is the present value of the difference between the current futures price times 0.491 and the exercise price times 0.461.
  - The call value is the present value of the difference between the current spot price times 0.491 and the exercise price times 0.461.
2. Which statement *best* describes how the Black model is used to value a European put option on the futures contract just described?
  - The put value is the present value of the difference between the exercise price times 0.539 and the current futures price times 0.509.
  - The put value is the present value of the difference between the current futures price times 0.491 and the exercise price times 0.461.
  - The put value is the present value of the difference between the current spot price times 0.491 and the exercise price times 0.461.
3. What are the underlying and exercise prices to use in the Black futures option model?
  - 1,851.65; 1,860
  - 1,860; 1,860
  - 1,860; 1,851.65

*Solution to 1:* B is correct. Recall Black's model for call options can be expressed as  $c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)]$ .

*Solution to 2:* A is correct. Recall Black's model for put options can be expressed as  $p = e^{-rT}[XN(-d_2) - F_0(T)N(-d_1)]$ .

*Solution to 3:* A is correct. The underlying is the futures price of 1,851.65 and the exercise price was given as 1,860.

## 5.2. Interest Rate Options

With interest rate options, the underlying instrument is a reference interest rate, such as three-month Libor. An interest rate call option gains when the reference interest rate rises and an interest rate put option gains when the reference interest rate falls. Interest rate options are the building blocks of many other instruments.

For an interest rate call option on three-month Libor with one year to expiration, the underlying interest rate is a forward rate agreement (FRA) rate that expires in one year. This FRA

is observed today and is the underlying rate used in the Black model. The underlying rate of the FRA is a 3-month Libor deposit that is investable in 12 months and matures in 15 months. Thus, in one year, the FRA rate typically converges to the three-month spot Libor.

Interest rates are typically set in advance, but interest payments are made in arrears, which is referred to as advanced set, settled in arrears. For example, with a bank deposit, the interest rate is usually set when the deposit is made, say  $t_{j-1}$ , but the interest payment is made when the deposit is withdrawn, say  $t_j$ . The deposit, therefore, has  $t_m = t_j - t_{j-1}$  time until maturity. Thus, the rate is advanced set, but the payment is settled in arrears. Likewise with a floating rate loan, the rate is usually set and the interest accrues at this known rate, but the payment is made later. Similarly, with some interest rate options, the time to option expiration ( $t_{j-1}$ ) when the interest rate is set does not correspond to the option settlement ( $t_j$ ) when the cash payment is made, if any. For example, if an interest rate option payment based on three-month Libor is US\$5,000 determined on January 15th, the actual payment of the US\$5,000 would occur on April 15.

Interest rates are quoted on an annual basis, but the underlying implied deposit is often less than a year. Thus, the annual rates must be adjusted for the accrual period. Recall that the accrual period for a quarterly reset 30/360 day count FRA is 0.25 (= 90/360). If the day count is on an actual (ACT) number of days divided by 360 (ACT/360), then the accrual period may be something like 0.252778 (= 91/360), assuming 91 days in the period. Typically, the accrual period in FRAs is based on 30/360 whereas the accrual period based on the option is actual number of days in the contract divided by the actual number of days in the year (identified as ACT/ACT or ACT/365).

The model presented here is known as the standard market model and is a variation of Black's futures option valuation model. Again, let  $t_{j-1}$  denote the time to option expiration (ACT/365), whereas let  $t_j$  denote the time to the maturity date of the underlying FRA. Note that the interest accrual on the underlying begins at the option expiration (Time  $t_{j-1}$ ). Let  $FRA(0, t_{j-1}, t_m)$  denote the fixed rate on a FRA at Time 0 that expires at Time  $t_{j-1}$ , where the underlying matures at Time  $t_j$  ( $= t_{j-1} + t_m$ ), with all times expressed on an annual basis. We assume the FRA is 30/360 day count. For example,  $FRA(0, 0.25, 0.5) = 2\%$  denotes the 2% fixed rate on a forward rate agreement that expires in 0.25 years with settlement amount being paid in 0.75 (= 0.25 + 0.5) years.<sup>12</sup> Let  $R_X$  denote the exercise rate expressed on an annual basis. Finally, let  $\sigma$  denote the interest rate volatility. Specifically,  $\sigma$  is the annualized standard deviation of the continuously compounded percentage change in the underlying FRA rate.

Interest rate options give option buyers the right to certain cash payments based on observed interest rates. For example, an interest rate call option gives the call buyer the right to a certain cash payment when the underlying interest rate exceeds the exercise rate. An interest rate put option gives the put buyer the right to a certain cash payment when the underlying interest rate is below the exercise rate.

With the standard market model, the prices of interest rate call and put options can be expressed as

$$c = (AP) e^{-r(t_{j-1} + t_m)} \left[ FRA(0, t_{j-1}, t_m) N(d_1) - R_X N(d_2) \right] \quad (18)$$

<sup>12</sup>Note that in other contexts the time periods are expressed in months. For example with months, this FRA would be expressed as FRA(0,3,6). Note that the third term in parentheses denotes the maturity of the underlying deposit from the expiration of the FRA.

and

$$p = (AP)e^{-r(t_{j-1}+t_m)} \left[ R_X N(-d_2) - FRA(0, t_{j-1}, t_m) N(-d_1) \right] \quad (19)$$

where

AP denotes the accrual period in years

$$d_1 = \frac{\ln \left[ FRA(0, t_{j-1}, t_m) / R_X \right] + (\sigma^2 / 2) t_{j-1}}{\sigma \sqrt{t_{j-1}}}$$

$$d_2 = d_1 - \sigma \sqrt{t_{j-1}}$$

The formulas here give the value of the option for a notional amount of 1. In practice, the notional would be more than one, so the full cost of the option is obtained by multiplying these formula amounts by the notional amount. Of course, this point is just the same as finding the value of an option on a single share of stock and then multiplying that value by the number of shares covered by the option contract.

Immediately, we note that the standard market model requires an adjustment when compared with the Black model for the accrual period. In other words, a value such as  $FRA(0, t_{j-1}, t_m)$  or the strike rate,  $R_X$ , as appearing in the formula given earlier, is stated on an annual basis, as are interest rates in general. The actual option premium would have to be adjusted for the accrual period. After accounting for this adjustment, this model looks very similar to the Black model, but there are important but subtle differences. First, the discount factor,  $e^{-r(t_{j-1}+t_m)}$ , does not apply to the option expiration,  $t_{j-1}$ . Rather, the discount factor is applied to the maturity date of the FRA or  $t_j$  ( $= t_{j-1} + t_m$ ). We express this maturity as  $(t_{j-1} + t_m)$  rather than  $t_j$  to emphasize the settlement in arrears nature of this option. Second, rather than the underlying being a futures price, the underlying is an interest rate, specifically a forward rate based on a forward rate agreement or  $FRA(0, t_{j-1}, t_m)$ . Third, the exercise price is really a rate and reflects an interest rate, not a price. Fourth, the time to the option expiration,  $t_{j-1}$ , is used in the calculation of  $d_1$  and  $d_2$ . Finally, both the forward rate and the exercise rate should be expressed in decimal form and not as percent (for example, 0.02 and not 2.0). Alternatively, if expressed as a percent, then the notional amount adjustment could be divided by 100.

As with other option models, the standard market model can be described as simply the present value of the expected option payoff at expiration. Specifically, we can express the standard market model for calls as  $c = PV[E(c_{t_j})]$  and for puts as  $p = PV[E(p_{t_j})]$ , where  $E(c_{t_j}) = (AP) [FRA(0, t_{j-1}, t_m) N(d_1) - R_X N(d_2)]$  and  $E(p_{t_j}) = (AP) [R_X N(-d_2) - FRA(0, t_{j-1}, t_m) N(-d_1)]$ . The present value term in this context is simply  $e^{-rt_j} = e^{-r(t_{j-1}+t_m)}$ . Again, note we discount from Time  $t_j$ , the time when the cash flows are settled on the FRA.

There are several interesting and useful combinations that can be created with interest rate options. We focus on a few that will prove useful for understanding swaptions in the next section. First, if the exercise rate is selected so as to equal the current FRA rate, then long an interest rate call option and short an interest rate put option is equivalent to a receive-floating, pay-fixed FRA.

Second, if the exercise rate is again selected so it is equal to the current FRA rate, then long an interest rate put option and short an interest rate call option is equivalent to a receive-fixed, pay-floating FRA. Note that FRAs are the building blocks of interest rate swaps.

Third, an interest rate cap is a portfolio or strip of interest rate call options in which the expiration of the first underlying corresponds to the expiration of the second option and so forth. The underlying interest rate call options are termed caplets. Thus, a set of floating-rate loan payments can be hedged with a long position in an interest rate cap encompassing a series of interest rate call options.

Fourth, an interest rate floor is a portfolio or strip of interest rate put options in which the expiration of the first underlying corresponds with the expiration of the second option and so forth. The underlying interest rate put options are termed floorlets. Thus, a floating-rate bond investment or any other floating-rate lending situation can be hedged with an interest rate floor encompassing a series of interest rate put options.

Fifth, applying put-call parity as discussed earlier, long an interest rate cap and short an interest rate floor with the exercise prices set at the swap rate is equivalent to a receive-floating, pay-fixed swap. On a settlement date, when the underlying rate is above the strike, both the cap and the swap pay off to the party. When the underlying rate is below the strike on a settlement date, the party must make a payment on the short floor, just as the case with a swap. For the opposite position, long an interest rate floor and short an interest rate cap result in the party making a payment when the underlying rate is above the strike and receiving one when the underlying rate is below the strike, just as is the case for a pay-floating, receive-fixed swap.

Finally, if the exercise rate is set equal to the swap rate, then the value of the cap must be equal to the value of the floor at the start. When an interest rate swap is initiated, its current value is zero and is known as an at-market swap. When an exercise rate is selected such that the cap value equals the floor value, then the initial cost of being long a cap and short the floor is also zero. This occurs when the cap and floor strike are equal to the swap rate.

### EXAMPLE 15 European Interest Rate Options

Suppose you are a speculative investor in Singapore. On 15 May, you anticipate that some regulatory changes will be enacted, and you want to profit from this forecast. On 15 June, you intend to borrow 10,000,000 Singapore dollars to fund the purchase of an asset, which you expect to resell at a profit three months after purchase, say on 15 September. The current three-month Sibor (that is, Singapore Libor) is 0.55%. The appropriate FRA rate over the period of 15 June to 15 September is currently 0.68%. You are concerned that rates will rise, so you want to hedge your borrowing risk by purchasing an interest rate call option with an exercise rate of 0.60%.

1. In using the Black model to value this interest rate call option, what would the underlying rate be?
  - A. 0.55%
  - B. 0.68%
  - C. 0.60%
2. The discount factor used in pricing this option would be over what period of time?
  - A. 15 May–15 June
  - B. 15 June–15 September
  - C. 15 May–15 September

*Solution to 1:* B is correct. In using the Black model, a forward or futures price is used as the underlying. This approach is unlike the BSM model in which a spot price is used as the underlying.

*Solution to 2:* C is correct. You are pricing the option on 15 May. An option expiring 15 June when the underlying is three-month Sibor will have its payoff determined on 15 June, but the payment will be made on 15 September. Thus, the expected payment must be discounted back from 15 September to 15 May.

Interest rate option values are linked in an important way with interest rate swap values through caps and floors. As we will see in the next section, an interest rate swap serves as the underlying for swaptions. Thus, once again, we see that important links exist between interest rate options, swaps, and swaptions.

### 5.3. Swaptions

A swap option or swaption is simply an option on a swap. It gives the holder the right, but not the obligation, to enter a swap at the pre-agreed swap rate—the exercise rate. Interest rate swaps can be either receive fixed, pay floating or receive floating, pay fixed. A payer swaption is an option on a swap to pay fixed, receive floating. A receiver swaption is an option on a swap to receive fixed, pay floating. Note that the terms “call” and “put” are often avoided because of potential confusion over the nature of the underlying. Notice also that the terminology focuses on the fixed swap rate.

A payer swaption buyer hopes the fixed rate goes up before the swaption expires. When exercised, the payer swaption buyer is able to enter into a pay-fixed, receive-floating swap at the predetermined exercise rate,  $R_X$ . The buyer can then immediately enter an offsetting at-market receive-fixed, pay-floating swap at the current fixed swap rate. The floating legs of both swaps will offset, leaving the payer swaption buyer with an annuity of the difference between the current fixed swap rate and the swaption exercise rate. Thus, swaption valuation will reflect an annuity.

Swap payments are advanced set, settled in arrears. Let the swap reset dates be expressed as  $t_0, t_1, t_2, \dots, t_n$ . Let  $R_{\text{FIX}}$  denote the fixed swap rate starting when the swaption expires, denoted as before with  $T$ , quoted on an annual basis, and  $R_X$  denote the exercise rate starting at Time  $T$ , again quoted on an annual basis. As before, we will assume a notional amount of 1.

Because swap rates are quoted on an annual basis, let  $AP$  denote the accrual period. Finally, we need some measure of uncertainty. Let  $\sigma$  denote the volatility of the forward swap rate. More precisely,  $\sigma$  denotes annualized, standard deviation of the continuously compounded percentage changes in the forward swap rate.

The swaption model presented here is a modification of the Black model. Let the present value of an annuity matching the forward swap payment be expressed as

$$PVA = \sum_{j=1}^n PV_{0,t_j} (1)$$

This term is equivalent to what is sometimes referred to as an annuity discount factor. It applies here because a swaption creates a series of equal payments of the difference in the market swap rate at expiration and the chosen exercise rate. Therefore, the payer swaption valuation model is

$$\text{PAY}_{\text{SWN}} = (\text{AP})\text{PVA}[\text{R}_{\text{FIX}}\text{N}(\text{d}_1) - \text{R}_{\text{X}}\text{N}(\text{d}_2)] \quad (20)$$

and the receiver swaption valuation model

$$\text{REC}_{\text{SWN}} = (\text{AP})\text{PVA}[\text{R}_{\text{X}}\text{N}(-\text{d}_2) - \text{R}_{\text{FIX}}\text{N}(-\text{d}_1)] \quad (21)$$

where

$$\text{d}_1 = \frac{\ln(\text{R}_{\text{FIX}}/\text{R}_{\text{X}}) + (\sigma^2/2)\text{T}}{\sigma\sqrt{\text{T}}}, \text{ and as always,}$$

$$\text{d}_2 = \text{d}_1 - \sigma\sqrt{\text{T}}$$

As noted with interest rate options, the actual premium would need to be scaled by the notional amount. Once again, we can see the similarities to the Black model. We note that the swaption model requires two adjustments, one for the accrual period and one for the present value of an annuity. After accounting for these adjustments, this model looks very similar to the Black model but there are important subtle differences. First, the discount factor is absent. The payoff is not a single payment but a series of payments. Thus, the present value of an annuity used here embeds the option-related discount factor. Second, rather than the underlying being a futures price, the underlying is the fixed rate on a forward interest rate swap. Third, the exercise price is really expressed as an interest rate. Finally, both the forward swap rate and the exercise rate should be expressed in decimal form and not as percent (for example, 0.02 and not 2.0).

As with other option models, the swaption model can be described as simply the present value of the expected option payoff at expiration. Specifically, we can express the payer swaption model value as

$$\text{PAY}_{\text{SWN}} = \text{PV}[\text{E}(\text{PAY}_{\text{SWN},\text{T}})]$$

and the receiver swaption model value as

$$\text{REC}_{\text{SWN}} = \text{PV}[\text{E}(\text{REC}_{\text{SWN},\text{T}})],$$

where

$$\begin{aligned} \text{E}(\text{PAY}_{\text{SWN},\text{T}}) &= e^{-r\text{T}}\text{PAY}_{\text{SWN}} \text{ and} \\ \text{E}(\text{REC}_{\text{SWN},\text{T}}) &= e^{-r\text{T}}\text{REC}_{\text{SWN}}. \end{aligned}$$

The present value term in this context is simply  $e^{-r\text{T}}$ . Because the annuity term embedded the discounting over the swaption life, the expected swaption values are the current swaption values grossed up by the current risk-free interest rate.

Alternatively, the swaption model can be described as having two components, a swap component and a bond component. For payer swaptions, the swap component is  $(\text{AP})\text{PVA}(\text{R}_{\text{FIX}})\text{N}(\text{d}_1)$  and the bond component is  $(\text{AP})\text{PVA}(\text{R}_{\text{X}})\text{N}(\text{d}_2)$ . The payer

swaption model value is simply the swap component minus the bond component. For receiver swaptions, the swap component is  $(AP)PVA(R_{FIX})N(-d_1)$  and the bond component is  $(AP)PVA(R_X)N(-d_2)$ . The receiver swaption model value is simply the bond component minus the swap component.

As with nearly all derivative instruments, there are many useful equivalence relationships. Recall that long an interest rate cap and short an interest rate floor with the same exercise rate is equal to a receive-floating, pay-fixed interest rate swap. Also, short an interest rate cap and long an interest rate floor with the same exercise rate is equal to a pay-floating, receive-fixed interest rate swap. There are also equivalence relationships with swaptions. In a similar way, long a receiver swaption and short a payer swaption with the same exercise rate is equivalent to entering a receive-fixed, pay-floating forward swap. Long a payer swaption and short a receiver swaption with the same exercise rate is equivalent to entering a receive-floating, pay-fixed forward swap. Note that if the exercise rate is selected such that the receiver and payer swaptions have the same value, then the exercise rate is equal to the at-market forward swap rate. Thus, there is again a put–call parity relationship important for valuation.

In addition, being long a callable fixed-rate bond can be viewed as being long a straight fixed-rate bond and short a receiver swaption. A receiver swaption gives the buyer the right to receive a fixed rate. Hence, the seller will have to pay the fixed rate when this right is exercised in a lower rate environment. Recall that the bond issuer has the right to call the bonds. If the bond issuer sells a receiver swaption with similar terms, then the bond issuer has essentially converted the callable bond into a straight bond. The bond issuer will now pay the fixed rate on the underlying swap and the floating rate received will be offset by the floating-rate loan created when the bond was refinanced. Specifically, the receiver swaption buyer will benefit when rates fall and the swaption is exercised. Thus, the embedded call feature is similar to a receiver swaption.

### EXAMPLE 16 European Swaptions

Suppose you are an Australian company and have ongoing floating-rate debt. You have profited for some time by paying at a floating rate because rates have been falling steadily for the last few years. Now, however, you are concerned that within three months the Australian central bank may tighten its monetary policy and your debt costs will thus increase. Rather than lock in your borrowing via a swap, you prefer to hedge by buying a swaption expiring in three months, whereby you will have the choice, but not the obligation, to enter a five-year swap locking in your borrowing costs. The current three-month forward, five-year swap rate is 2.65%. The current five-year swap rate is 2.55%. The current three-month risk-free rate is 2.25%.

With reference to the Black model to value the swaption, which statement is correct?

- A. The underlying is the three-month forward, five-year swap rate.
- B. The discount rate to use is 2.55%.
- C. The swaption time to expiration,  $T$ , is five years.

*Solution:* A is correct. The current five-year swap rate is not used as a discount rate with swaptions. The swaption time to expiration is 0.25, not the life of the swap.

## 6. OPTION GREEKS AND IMPLIED VOLATILITY

With option valuation models, such as the binomial model, BSM model, and Black's model, we are able to estimate a wide array of comparative information, such as how much the option value will change for a small change in a particular parameter.<sup>13</sup> We will explore this derived information as well as implied volatility in this section. These topics are essential for those managing option positions and in general in obtaining a solid understanding of how option prices change. Our discussion will be based on stock options, though the material covered in this section applies to all types of options.

The measures examined here are known as the Greeks and include, delta, gamma, theta, vega, and rho. With these calculations, we seek to address how much a particular portfolio will change for a given small change in the appropriate parameter. These measures are sometimes referred to as static risk measures in that they capture movements in the option value for a movement in one of the factors that affect the option value, while holding all other factors constant.

Our focus here is on European stock options in which the underlying stock is assumed to pay a dividend yield (denoted  $\delta$ ). Note that for non-dividend-paying stocks,  $\delta = 0$ .

### 6.1. Delta

**Delta** is defined as the change in a given instrument for a given small change in the value of the stock, holding everything else constant. Thus, the delta of long one share of stock is by definition +1.0, and the delta of short one share of stock is by definition -1.0. The concept of the option delta is similarly the change in an option value for a given small change in the value of the underlying stock, holding everything else constant. The option deltas for calls and puts are, respectively,

$$\text{Delta}_c = e^{-\delta T} N(d_1) \quad (22)$$

and

$$\text{Delta}_p = -e^{-\delta T} N(-d_1) \quad (23)$$

Note that the deltas are a simple function of  $N(d_1)$ . The delta of an option answers the question of how much the option will change for a given change in the stock, holding everything else constant. Therefore, delta is a static risk measure. It does not address how likely this particular change would be. Recall that  $N(d_1)$  is a value taken from the cumulative distribution function of a standard normal distribution. As such, the range of values is between 0 and 1. Thus, the range of call delta is 0 and  $e^{-\delta T}$  and the range of put delta is  $-e^{-\delta T}$  and 0. As the stock price increases, the call option goes deeper in the money and the value of  $N(d_1)$  is moving toward 1. As the stock price decreases, the call option goes deeper out of the money and the value of  $N(d_1)$  is moving toward zero. When the option gets closer to maturity, the delta will drift either toward 0 if it is out of the money or drift toward 1 if it is in the money. Clearly, as the stock price changes and as time to maturity changes, the deltas are also changing.

<sup>13</sup>Parameters in the BSM model, for example, include the stock price, exercise price, volatility, time to expiration, and the risk-free interest rate.

Delta hedging an option is the process of establishing a position in the underlying stock of a quantity that is prescribed by the option delta so as to have no exposure to very small moves up or down in the stock price. Hence, to execute a single option delta hedge, we first calculate the option delta and then buy or sell delta units of stock. In practice, rarely does one have only one option position to manage. Thus, in general, delta hedging refers to manipulating the underlying portfolio delta by appropriately changing the positions in the portfolio. A delta neutral portfolio refers to setting the portfolio delta all the way to zero. In theory, the delta neutral portfolio will not change in value for small changes in the stock instrument. Let  $N_H$  denote the number of units of the hedging instrument and  $\Delta_{H_i}$  denote the delta of the hedging instrument, which could be the underlying stock, call options, or put options. Delta neutral implies the portfolio delta plus  $N_H \Delta_{H_i}$  is equal to zero. The optimal number of hedging units,  $N_H$ , is

$$N_H = -\frac{\text{Portfolio delta}}{\Delta_{H_i}}$$

Note that if  $N_H$  is negative, then one must short the hedging instrument, and if  $N_H$  is positive, then one must go long the hedging instrument. Clearly, if the portfolio is options and the hedging instrument is stock, then we will buy or sell shares to offset the portfolio position. For example, if the portfolio consists of 100,000 shares of stock at US\$10 per share, then the portfolio delta is 100,000. The delta of the hedging instrument, stock, is +1. Thus, the optimal number of hedging units,  $N_H$ , is  $-100,000$  ( $= -100,000/1$ ) or short 100,000 shares. Alternatively, if the portfolio delta is 5,000 and a particular call option with delta of 0.5 is used as the hedging instrument, then to arrive at a delta neutral portfolio, one must sell 10,000 call options ( $= -5,000/0.5$ ). Alternatively, if a portfolio of options has a delta of  $-1,500$ , then one must buy 1,500 shares of stock to be delta neutral [ $= -(-1,500)/1$ ]. If the hedging instrument is stock, then the delta is +1 per share.

### EXAMPLE 17 Delta Hedging

Apple stock is trading at US\$125. We write calls (that is, we sell calls) on 1,000 Apple shares and now are exposed to an increase in the price of the Apple stock. That is, if Apple rises, we will lose money because the calls we sold will go up in value, so our liability will increase. Correspondingly, if Apple falls, we will make money. We want to neutralize our exposure to Apple. Say the call delta is 0.50, which means that if Apple goes up by US\$0.10, a call on one Apple share will go up US\$0.05. We need to trade in such a way as to make money if Apple goes up, to offset our exposure. Hence, we buy 500 Apple shares to hedge. Now, if Apple goes up US\$0.10, the sold calls will go up US\$50 (our liability goes up), but our long 500 Apple hedge will profit by US\$50. Hence, we are delta hedged.

Identify the *incorrect* statement:

- If we sell Apple puts, we need to buy Apple stock to delta hedge.
- Call delta is non-negative ( $\geq 0$ ); put delta is non-positive ( $\leq 0$ ).
- Delta hedging is the process of neutralizing exposure to the underlying.

*Solution:* A is the correct answer because statement A is incorrect. If we sell puts, we need to short sell stock to delta hedge.

One final interpretation of option delta is related to forecasting changes in option prices. Let  $\hat{c}$ ,  $\hat{p}$ , and  $\hat{S}$  denote some new value for the call, put, and stock. Based on an approximation method, the change in the option price can be estimated with a concept known as a delta approximation or

$$\hat{c} - c \approx \text{Delta}_c (\hat{S} - S) \text{ for calls and}$$

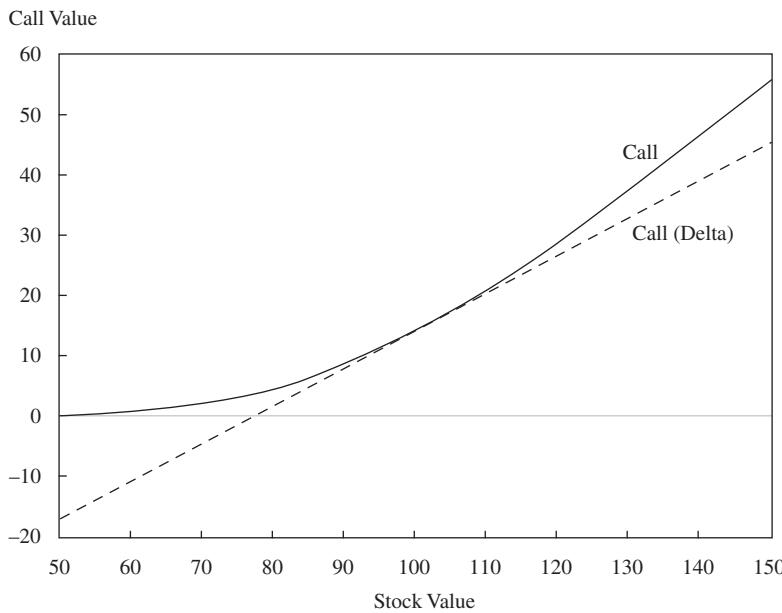
$$\hat{p} - p \approx \text{Delta}_p (\hat{S} - S) \text{ for puts.}^{14}$$

We can now illustrate the actual call values as well as the estimated call values based on delta. Exhibit 15 illustrates the call value based on the BSM model and the call value based on the delta approximation,

$$\hat{c} = c + \text{Delta}_c (\hat{S} - S).$$

Notice for very small changes in the stock, the delta approximation is fairly accurate. For example, if the stock value rises from 100 to 101, notice that both the call line and the call (delta) estimated line are almost the same value. If, however, the stock value rises from 100 to 150, the call line is now significantly above the call (delta) estimated line. Thus, we see that as the change in the stock increases, the estimation error also increases. The delta approximation is biased low for both a down move and an up move.

EXHIBIT 15 Call Values and Delta Estimated Call Values ( $S = 100 = X$ ,  $r = 5\%$ ,  $\sigma = 30\%$ ,  $\delta = 0$ )



<sup>14</sup>The symbol  $\approx$  denotes approximately. The approximation method is known as a Taylor series. Also note that the put delta is non-positive ( $\leq 0$ ).

We see that delta hedging is imperfect and gets worse as the underlying moves further away from its original value of 100. Based on the graph, the BSM model assumption of continuous trading is essential to avoid hedging risk. This hedging risk is related to the difference between these two lines and the degree to which the underlying price experiences large changes.

### EXAMPLE 18 Delta Hedging

Suppose we know  $S = 100$ ,  $X = 100$ ,  $r = 5\%$ ,  $T = 1.0$ ,  $\sigma = 30\%$ , and  $\delta = 5\%$ . We have a short position in put options on 10,000 shares of stock. Based on this information, we note  $\Delta_c = 0.532$ , and  $\Delta_p = -0.419$ . Assume each stock option contract is for one share of stock.

1. The appropriate delta hedge, assuming the hedging instrument is stock, is executed by which of the following transactions? Select the *closest* answer.
  - A. Buy 5,320 shares of stock.
  - B. Short sell 4,190 shares of stock.
  - C. Buy 4,190 shares of stock.
2. The appropriate delta hedge, assuming the hedging instrument is calls, is executed by which of the following transactions? Select the *closest* answer.
  - A. Sell 7,876 call options.
  - B. Sell 4,190 call options.
  - C. Buy 4,190 call options.
3. Identify the correct interpretation of an option delta.
  - A. Option delta measures the curvature in the option price with respect to the stock price.
  - B. Option delta is the change in an option value for a given small change in the stock's value, holding everything else constant.
  - C. Option delta is the probability of the option expiring in the money.

*Solution to 1:* B is correct. Recall that  $N_H = -\frac{\text{Portfolio delta}}{\Delta_H}$ . The put delta is given as  $-0.419$ , thus the short put delta is  $0.419$ . In this case, Portfolio delta =  $10,000(0.419) = 4,190$  and  $\Delta_H = 1.0$ . Thus, the number of hedging units is  $-4,190 [= -(4,190/1)]$  or short sell 4,190 shares of stock.

*Solution to 2:* A is correct. Again the Portfolio delta =  $4,190$  but now  $\Delta_H = 0.532$ . Thus, the number of hedging units is  $-7,875.9 [= -(4,190/0.532)]$  or sell 7,876 call options.

*Solution to 3:* B is correct. Delta is defined as the change in a given portfolio for a given small change in the stock's value, holding everything else constant. Option delta is defined as the change in an option value for a given small change in the stock's value, holding everything else constant.

## 6.2. Gamma

Recall that delta is a good approximation of how an option price will change for a small change in the stock. For larger changes in the stock, we need better accuracy. **Gamma** is defined as the change in a given instrument's delta for a given small change in the stock's value, holding everything else constant. Option gamma is similarly defined as the change in a given option delta for a given small change in the stock's value, holding everything else constant. Option gamma is a measure of the curvature in the option price in relationship to the stock price. Thus, the gamma of a long or short position in one share of stock is zero because the delta of a share of stock never changes. A stock always moves one-for-one with itself. Thus, its delta is always +1 and, of course, -1 for a short position in the stock. The gamma for a call and put option are the same and can be expressed as

$$\text{Gamma}_c = \text{Gamma}_p = \frac{e^{-\delta T}}{S\sigma\sqrt{T}} n(d_1) \quad (24)$$

where  $n(d_1)$  is the standard normal probability density function. The lowercase "n" is distinguished from the cumulative normal distribution—which the density function generates—and that we have used elsewhere in this reading denoted by uppercase "N". The gamma of a call equals the gamma of a similar put based on put-call parity or  $c - p = S_0 - e^{-rT}X$ . Note that neither  $S_0$  nor  $e^{-rT}X$  is a direct function of delta. Hence, the right-hand side of put-call parity has a delta of 1. Thus, the right-hand side delta is not sensitive to changes in the underlying. Therefore, the gamma of a call must equal the gamma of a put.

Gamma is always non-negative. Gamma takes on its largest value near at the money. Options deltas do not change much for small changes in the stock price if the option is either deep in or deep out of the money. Also, as the stock price changes and as time to expiration changes, the gamma is also changing.

Gamma measures the rate of change of delta as the stock changes. Gamma approximates the estimation error in delta for options because the option price with respect to the stock is non-linear and delta is a linear approximation. Thus, gamma is a risk measure; specifically, gamma measures the non-linearity risk or the risk that remains once the portfolio is delta neutral. A gamma neutral portfolio implies the gamma is zero. For example, gamma can be managed to an acceptable level first and then delta is neutralized as a second step. This hedging approach is feasible because options have gamma but a stock does not. Thus, in order to modify gamma, one has to include additional option trades in the portfolio. Once the revised portfolio, including any new option trades, has the desired level of gamma, then the trader can get the portfolio delta to its desired level as step two. To alter the portfolio delta, the trader simply buys or sells stock. Because stock has a positive delta, but zero gamma, the portfolio delta can be brought to its desired level with no impact on the portfolio gamma.

One final interpretation of gamma is related to improving the forecasted changes in option prices. Again, let  $\hat{c}$ ,  $\hat{p}$ , and  $\hat{S}$  denote new values for the call, put, and stock. Again based on an approximation method, the change in the option price can be estimated by a delta-plus-gamma approximation or

$$\hat{c} - c \approx \text{Delta}_c (\hat{S} - S) + \frac{\text{Gamma}_c}{2} (\hat{S} - S)^2 \text{ for calls and}$$

$$\hat{p} - p \approx \text{Delta}_p (\hat{S} - S) + \frac{\text{Gamma}_p}{2} (\hat{S} - S)^2 \text{ for puts.}$$

Exhibit 16 illustrates the call value based on the BSM model; the call value based on the delta approximation,

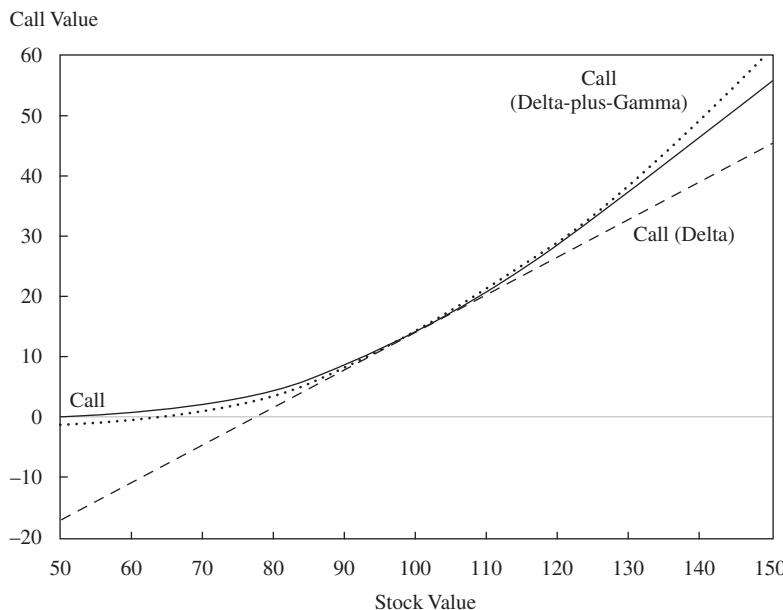
$$\hat{c} = c + \text{Delta}_c (\hat{S} - S)$$

and the call value based on the delta-plus-gamma approximation,

$$\hat{c} = c + \text{Delta}_c (\hat{S} - S) + \frac{\text{Gamma}_c}{2} (\hat{S} - S)^2$$

Notice again that for very small changes in the stock, the delta approximation and the delta-plus-gamma approximations are fairly accurate. If the stock value rises from 100 to 150, the call line is again significantly above the delta estimated line but is below the delta-plus-gamma estimated line. Importantly, the call delta-plus-gamma estimated line is significantly closer to the BSM model call values. Thus, we see that even for fairly large changes in the stock, the estimation error also increases. From Exhibit 16, we see the delta-plus-gamma approximation is biased low for a down move but biased high for an up move. Thus, when estimating how the call price changes when the underlying changes, we see how the delta-plus-gamma approximation is an improvement when compared with using the delta approximation on its own.

EXHIBIT 16 Call Values, Delta Estimated Call Values, and Delta-Plus-Gamma Estimated Call Values ( $S = 100 = X$ ,  $r = 5\%$ ,  $\sigma = 30\%$ ,  $\delta = 0$ )



If the BSM model assumptions hold, then we would have no risk in managing option positions. In reality, however, stock prices often jump rather than move continuously and smoothly, which creates “gamma risk.” Gamma risk is so-called because gamma measures the risk of stock prices jumping when hedging an option position, and thus leaving us suddenly unhedged.

### EXAMPLE 19 Gamma Risk in Option Trading

Suppose we are options traders and have only one option position—a short call option. We also hold some stock such that we are delta hedged. Which one of the following statements is true?

- A. We are gamma neutral.
- B. Buying a call will increase our overall gamma.
- C. Our overall position is a positive gamma, which will make large moves profitable for us, whether up or down.

*Solution:* B is correct. Buying options (calls or puts) will always increase net gamma. A is incorrect because we are short gamma, not gamma neutral. C is also incorrect because we are short gamma. We can only become gamma neutral from a short gamma position by purchasing options.

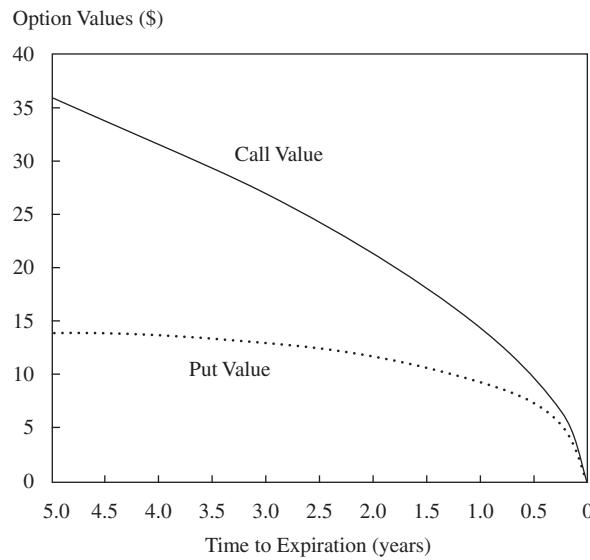
### 6.3. Theta

**Theta** is defined as the change in a portfolio for a given small change in calendar time, holding everything else constant. Option theta is similarly defined as the change in an option value for a given small change in calendar time, holding everything else constant. Option theta is the rate at which the option time value declines as the option approaches expiration. To understand theta, it is important to remember the “holding everything else constant” assumption. Specifically, the theta calculation assumes nothing changes except calendar time. Clearly, if calendar time passes, then time to expiration declines. Because stocks do not have an expiration date, the stock theta is zero. Like gamma, theta cannot be adjusted with stock trades.

The gain or loss of an option portfolio in response to the mere passage of calendar time is known as time decay. Particularly with long options positions, often the mere passage of time without any change in other variables, such as the stock, will result in significant losses in value. Therefore, investment managers with significant option positions carefully monitor theta and their exposure to time decay. Time decay is essentially the measure of profit and loss of an option position as time passes, holding everything else constant.

Note that theta is fundamentally different from delta and gamma in the sense that the passage of time does not involve any uncertainty. There is no chance that time will go backward. Time marches on, but it is important to understand how your investment position will change with the mere passage of time.

Typically, theta is negative for options. That is, as calendar time passes, expiration time declines and the option value also declines. Exhibit 17 illustrates the option value with respect to time to expiration. Remember, as calendar time passes, the time to expiration declines. Both the call and the put option are at the money and eventually are worthless if the stock does not change. Notice, however, how the speed of the option value decline increases as time to expiration decreases.

EXHIBIT 17 Option Values and Time to Expiration ( $S = 100 = X$ ,  $r = 5\%$ ,  $\sigma = 30\%$ ,  $\delta = 0$ )

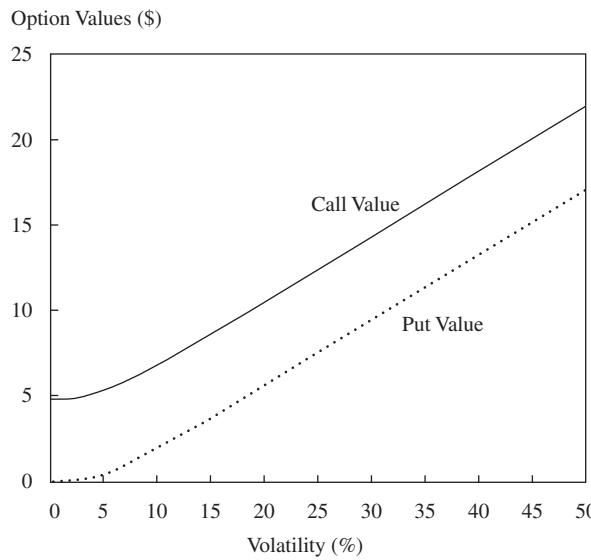
#### 6.4. Vega

**Vega** is defined as the change in a given portfolio for a given small change in volatility, holding everything else constant. Vega measures the sensitivity of a given portfolio to volatility. The vega of an option is positive. An increase in volatility results in an increase in the option value for both calls and puts.

The vega of a call equals the vega of a similar put based on put–call parity or  $c - p = S_0 - e^{-rT}X$ . Note that neither  $S_0$  nor  $e^{-rT}X$  is a direct function of volatility. Therefore, the vega of a call must offset the vega of a put so that the vega of the right-hand side is zero.

Unlike the Greeks we have already discussed, vega is based on an unobservable parameter, future volatility. Although historical volatility can be calculated, there is no objective measure of future volatility. Similar to the concept of expected value, future volatility is subjective. Thus, vega measures the sensitivity of a portfolio to changes in the volatility used in the option valuation model. Option values are generally quite sensitive to volatility. In fact, of the five variables in the BSM, an option's value is most sensitive to volatility changes.

At extremely low volatility, the option values tend toward their lower bounds. The lower bound of a European-style call option is zero or the stock less the present value of the exercise price, whichever is greater. The lower bound of a European-style put option is zero or the present value of the exercise price less the stock, whichever is greater. Exhibit 18 illustrates the option values with respect to volatility. In this case, the call lower bound is 4.88 and the put lower bound is 0. The difference between the call and put can be explained by put–call parity.

EXHIBIT 18 Option Values and Volatility ( $S = 100 = X, r = 5\%, T = 1, \delta = 0$ )

Vega is very important in managing an options portfolio because option values can be very sensitive to volatility changes. Vega is high when options are at or near the money and are short dated. Volatility is usually only hedged with other options and volatility itself can be quite volatile. Volatility is sometimes considered a separate asset class or a separate risk factor. Because it is rather exotic and potentially dangerous, exposure to volatility needs to be managed, bearing in mind that risk managers, board members, and clients may not understand or appreciate losses if volatility is the source.

## 6.5. Rho

**Rho** is defined as the change in a given portfolio for a given small change in the risk-free interest rate, holding everything else constant. Thus, rho measures the sensitivity of the portfolio to the risk-free interest rate.

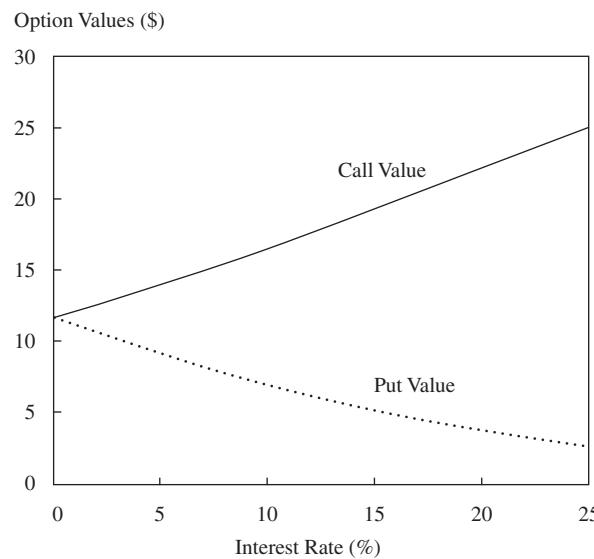
The rho of a call is positive. Intuitively, buying an option avoids the financing costs involved with purchasing the stock. In other words, purchasing a call option allows an investor to earn interest on the money that otherwise would have gone to purchasing the stock. The higher the interest rate, the higher the call value.

The rho of a put is negative. Intuitively, the option to sell the stock delays the opportunity to earn interest on the proceeds from the sale. For example, purchasing a put option rather than selling the stock deprives an investor of the potential interest that would have been earned from the proceeds of selling the stock. The higher the interest rate, the lower the put value.

When interest rates are zero, the call and put option values are the same for at-the-money options. Recall that with put-call parity, we have  $c - p = S_0 - e^{-rT}X$ , and when interest rates are zero, then the present value function has no effect. As interest rates rise, the difference between call and put options increases as illustrated in Exhibit 19. The impact on option prices

when interest rates change is relatively small when compared with that for volatility changes and that for changes in the stock. Hence, the influence of interest rates is generally not a major concern.<sup>15</sup>

EXHIBIT 19 Option Values and Interest Rates ( $S = 100 = X, r = 5\%, T = 1, \delta = 0$ )



## 6.6. Implied Volatility

As we have already touched on in Section 6.4, for most options, the value is particularly sensitive to volatility. Unlike the price of the underlying, however, volatility is not an observable value in the marketplace. Volatility can be, and often is estimated, based on a sample of historical data. For example, for a three-month option, we might look back over the last three months and calculate the actual historical stock volatility. We can then use this figure as an estimate of volatility over the next three months. The volatility parameter in the BSM model, however, is the *future* volatility. As we know, history is a very frail guide of the future, so the option may appear to be “mispriced” with respect to the actual future volatility experienced. Different investors will have different views of the future volatility. The one with the most accurate forecast will have the most accurate assessment of the option value.

Much like yield to maturity with bonds, volatility can be inferred from option prices. This inferred volatility is called the **implied volatility**. Thus, one important use of the BSM

<sup>15</sup>An exception to this rule is that with interest rate options, the interest rate is not constant and serves as the underlying. The relationship between the option value and the underlying interest rate is, therefore, captured by the delta, not the rho. Rho is really more generally the relationship between the option value and the rate used to discount cash flows.

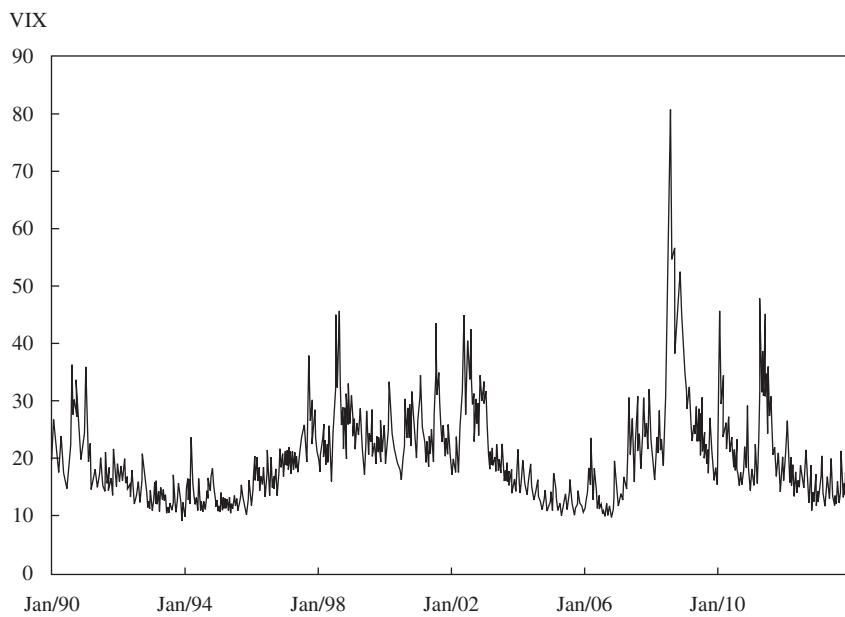
model is to invert the model and estimate implied volatility. The key advantage is that implied volatility provides information regarding the perceived uncertainty going forward and thereby allows us to gain an understanding of the collective opinions of investors on the volatility of the underlying and the demand for options. If the demand for options increases and the no-arbitrage approach is not perfectly reflected in market prices—for example, because of transaction costs—then the preference for buying options will drive option prices up, and hence, the observed implied volatility. This kind of information is of great value to traders in options.

Recall that one assumption of the BSM model is that all investors agree on the value of volatility and that this volatility is non-stochastic. Note that the original BSM model assumes the underlying instrument volatility is constant in our context. That is, when we calculate option values, we have assumed a single volatility number, like 30%. In practice, it is very common to observe different implied volatilities for different exercise prices and observe different implied volatilities for calls and puts with the same terms. Implied volatility also varies across time to expiration as well as across exercise prices. The implied volatility with respect to time to expiration is known as the term structure of volatility, whereas the implied volatility with respect to the exercise price is known as the volatility smile or sometimes skew depending on the particular shape. It is common to construct a three dimensional plot of the implied volatility with respect to both expiration time and exercise prices, a visualization known as the volatility surface. If the BSM model assumptions were true, then one would expect to find the volatility surface flat.

Implied volatility is also not constant through calendar time. As implied volatility increases, market participants are communicating an increased market price of risk. For example, if the implied volatility of a put increases, it is more expensive to buy downside protection with a put. Hence, the market price of hedging is rising. With index options, various volatility indexes have been created, and these indexes measure the collective opinions of investors on the volatility in the market. Investors can now trade futures and options on various volatility indexes in an effort to manage their vega exposure in other options.

Exhibit 20 provides a look at a couple of decades of one such volatility index, the Chicago Board Options Exchange S&P 500 Volatility Index, known as the VIX. The VIX is quoted as a percent and is intended to approximate the implied volatility of the S&P 500 over the next 30 days. VIX is often termed the fear index because it is viewed as a measure of market uncertainty. Thus, an increase in the VIX index is regarded as greater investor uncertainty. From this figure, we see that the implied volatility of the S&P 500 is not constant and goes through periods when the VIX is low and periods when the VIX is high. In the recent financial crisis, the VIX was extremely high, indicating great fear and uncertainty in the equity market. Remember that implied volatility reflects both beliefs regarding future volatility as well as a preference for risk mitigating products like options. Thus, during the crisis, the higher implied volatility reflected both higher expected future volatility as well as increased preference for buying rather than selling options.

## EXHIBIT 20 VIX Daily Values, 2 January 1990–18 July 2014



Implied volatility has several uses in option trading. An understanding of implied volatility is essential in managing an options portfolio. This reading explains the valuation of options as a function of the value of the underlying, the exercise price, the expiration date, the risk-free rate, dividends or other benefits paid by the underlying, and the volatility of the underlying. Note that each of these parameters is observable except the volatility of the underlying over the option term looking ahead. This volatility has to be estimated in some manner, such as by calculating historical volatility. But as noted, historical volatility involves looking back in time. There are, however, a vast number of liquid options traded on exchanges around the world so that a wide variety of option prices are observable. Because we know the price and all the parameters except the volatility, we can back out the volatility needed by the option valuation model to get the known price. This volatility is the implied volatility.

Hence, implied volatility can be interpreted as the market's view of how to value options. In the option markets, participants use volatility as the medium in which to quote options. The price is simply calculated by the use of an agreed model with the quoted volatility. For example, rather than quote a particular call option as trading for €14.23, it may be quoted as 30.00, where 30.00 denotes in percentage points the implied volatility based on a €14.23 option price. Note that there is a one-to-one relationship between the implied volatility and the option price, ignoring rounding errors.

The benefit of quoting via implied volatility (or simply volatility), rather than price, is that it allows volatility to be traded in its own right. Volatility is the “guess factor” in option pricing. All other inputs—value of the underlying, exercise price, expiration, risk-free rate, and dividend yield—are agreed.<sup>16</sup> Volatility is often the same order of magnitude across exercise prices

<sup>16</sup>The risk-free rate and dividend yield may not be entirely agreed, but the impact of variations to these parameters is generally very small compared with the other inputs.

and expiration dates. This means that traders can compare the values of two options, which may have markedly different exercise prices and expiration dates, and therefore, markedly different prices in a common unit of measure, specifically implied volatility.

### EXAMPLE 20 Implied Volatility in Option Trading within One Market

Suppose we hold a portfolio of options all tied to FTSE 100 futures contracts. Let the current futures price be 6,850. A client calls to request our offer prices on out-of-the-money puts and at-the-money puts, both with the same agreed expiration date. We calculate the prices to be respectively, 190 and 280 futures points. The client wants these prices quoted in implied volatility as well as in futures points because she wants to compare prices by comparing the quoted implied volatilities. The implied volatilities are 16% for the out-of-the-money puts and 15.2% for the at-the-money puts. Why does the client want the quotes in implied volatility?

- A. Because she can better compare the two options for value—that is, she can better decide which is cheap and which is expensive.
- B. Because she can assess where implied volatility is trading at that time, and thus consider revaluing her options portfolio at the current market implied volatilities for the FTSE 100.
- C. Both A and B are valid reasons for quoting options in volatility units.

*Solution:* C is correct. Implied volatility can be used to assess the relative value of different options, neutralizing the moneyness and time to expiration effects. Also, implied volatility is useful for revaluing existing positions over time.

### EXAMPLE 21 Implied Volatility in Option Trading Across Markets

Suppose an options dealer offers to sell a three-month at-the-money call on the FTSE index option at 19% implied volatility and a one-month in-the-money put on Vodafone (VOD) at 24%. An option trader believes that based on the current outlook, FTSE volatility should be closer to 25% and VOD volatility should be closer to 20%. What actions might the trader take to benefit from her views?

- A. Buy the FTSE call and the VOD put.
- B. Buy the FTSE call and sell the VOD put.
- C. Sell the FTSE call and sell the VOD puts.

*Solution:* B is correct. The trader believes that the FTSE call volatility is understated by the dealer and that the VOD put volatility is overstated. Thus, the trader would expect FTSE volatility to rise and VOD volatility to fall. As a result, the FTSE call would be expected to increase in value and the VOD put would be expected to decrease in value. The trader would take the positions as indicated in B.

Regulators, banks, compliance officers, and most option traders use implied volatilities to communicate information related to options portfolios. This is because implied volatilities, together with standard pricing models, give the “market consensus” valuation, in the same way that other assets are valued using market prices.

In summary, as long as all market participants agree on the underlying option model and how other parameters are calculated, then implied volatility can be used as a quoting mechanism. Recall that there are calls and puts, various exercise prices, various maturities, American and European, and exchange-traded and OTC options. Thus, it is difficult to conceptualize all these different prices. For example, if two call options on the same stock had different prices, but one had a longer expiration and lower exercise price and the other had a shorter expiration and higher exercise, which should be the higher priced option? It is impossible to tell on the surface. But if one option implied a higher volatility than the other, we know that after taking into account the effects of time and exercise, one option is more expensive than the other. Thus, by converting the quoted price to implied volatility, it is easier to understand the current market price of various risk exposures.

## 7. SUMMARY

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This reading on the valuation of contingent claims provides a foundation for understanding how a variety of different options are valued. Key points include the following:

- The arbitrageur would rather have more money than less and abides by two fundamental rules: Do not use your own money and do not take any price risk.
- The no-arbitrage approach is used for option valuation and is built on the key concept of the law of one price, which says that if two investments have the same future cash flows regardless of what happens in the future, then these two investments should have the same current price.
- Throughout this reading, the following key assumptions are made:
  - Replicating instruments are identifiable and investable.
  - Market frictions are nil.
  - Short selling is allowed with full use of proceeds.
  - The underlying instrument price follows a known distribution.
  - Borrowing and lending is available at a known risk-free rate.
- The two-period binomial model can be viewed as three one-period binomial models, one positioned at Time 0 and two positioned at Time 1.
- In general, European-style options can be valued based on the expectations approach in which the option value is determined as the present value of the expected future option payouts, where the discount rate is the risk-free rate and the expectation is taken based on the risk-neutral probability measure.
- Both American-style options and European-style options can be valued based on the no-arbitrage approach, which provides clear interpretations of the component terms; the option value is determined by working backward through the binomial tree to arrive at the correct current value.
- For American-style options, early exercise influences the option values and hedge ratios as one works backward through the binomial tree.
- Interest rate option valuation requires the specification of an entire term structure of interest rates, so valuation is often estimated via a binomial tree.

- A key assumption of the Black–Scholes–Merton option valuation model is that the return of the underlying instrument follows geometric Brownian motion, implying a lognormal distribution of the return.
- The BSM model can be interpreted as a dynamically managed portfolio of the underlying instrument and zero-coupon bonds.
- BSM model interpretations related to  $N(d_1)$  are that it is the basis for the number of units of underlying instrument to replicate an option, that it is the primary determinant of delta, and that it answers the question of how much the option value will change for a small change in the underlying.
- BSM model interpretations related to  $N(d_2)$  are that it is the basis for the number of zero-coupon bonds to acquire to replicate an option and that it is the basis for estimating the risk-neutral probability of an option expiring in the money.
- The Black futures option model assumes the underlying is a futures or a forward contract.
- Interest rate options can be valued based on a modified Black futures option model in which the underlying is a forward rate agreement (FRA), there is an accrual period adjustment as well as an underlying notional amount, and that care must be given to day-count conventions.
- An interest rate cap is a portfolio of interest rate call options termed caplets, each with the same exercise rate and with sequential maturities.
- An interest rate floor is a portfolio of interest rate put options termed floorlets, each with the same exercise rate and with sequential maturities.
- A swaption is an option on a swap.
- A payer swaption is an option on a swap to pay fixed and receive floating.
- A receiver swaption is an option on a swap to receive fixed and pay floating.
- Long a callable fixed-rate bond can be viewed as long a straight fixed-rate bond and short a receiver swaption.
- Delta is a static risk measure defined as the change in a given portfolio for a given small change in the value of the underlying instrument, holding everything else constant.
- Delta hedging refers to managing the portfolio delta by entering additional positions into the portfolio.
- A delta neutral portfolio is one in which the portfolio delta is set and maintained at zero.
- A change in the option price can be estimated with a delta approximation.
- Because delta is used to make a linear approximation of the non-linear relationship that exists between the option price and the underlying price, there is an error that can be estimated by gamma.
- Gamma is a static risk measure defined as the change in a given portfolio delta for a given small change in the value of the underlying instrument, holding everything else constant.
- Gamma captures the non-linearity risk or the risk—via exposure to the underlying—that remains once the portfolio is delta neutral.
- A gamma neutral portfolio is one in which the portfolio gamma is maintained at zero.
- The change in the option price can be better estimated by a delta-plus-gamma approximation compared with just a delta approximation.
- Theta is a static risk measure defined as the change in the value of an option given a small change in calendar time, holding everything else constant.
- Vega is a static risk measure defined as the change in a given portfolio for a given small change in volatility, holding everything else constant.
- Rho is a static risk measure defined as the change in a given portfolio for a given small change in the risk-free interest rate, holding everything else constant.

- Although historical volatility can be estimated, there is no objective measure of future volatility.
- Implied volatility is the BSM model volatility that yields the market option price.
- Implied volatility is a measure of future volatility, whereas historical volatility is a measure of past volatility.
- Option prices reflect the beliefs of option market participant about the future volatility of the underlying.
- The volatility smile is a two dimensional plot of the implied volatility with respect to the exercise price.
- The volatility surface is a three dimensional plot of the implied volatility with respect to both expiration time and exercise prices.
- If the BSM model assumptions were true, then one would expect to find the volatility surface flat, but in practice, the volatility surface is not flat.

## PROBLEMS

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### **The following information relates to Questions 1–9**

Bruno Sousa has been hired recently to work with senior analyst Camila Rocha. Rocha gives him three option valuation tasks.

#### Alpha Company

Sousa's first task is to illustrate how to value a call option on Alpha Company with a one-period binomial option pricing model. It is a non-dividend-paying stock, and the inputs are as follows.

- The current stock price is 50, and the call option exercise price is 50.
- In one period, the stock price will either rise to 56 or decline to 46.
- The risk-free rate of return is 5% per period.

Based on the model, Rocha asks Sousa to estimate the hedge ratio, the risk-neutral probability of an up move, and the price of the call option. In the illustration, Sousa is also asked to describe related arbitrage positions to use if the call option is overpriced relative to the model.

#### Beta Company

Next, Sousa uses the two-period binomial model to estimate the value of a European-style call option on Beta Company's common shares. The inputs are as follows.

- The current stock price is 38, and the call option exercise price is 40.
- The up factor ( $u$ ) is 1.300, and the down factor ( $d$ ) is 0.800.
- The risk-free rate of return is 3% per period.

Sousa then analyzes a put option on the same stock. All of the inputs, including the exercise price, are the same as for the call option. He estimates that the value of a European-style put option is 4.53. Exhibit 1 summarizes his analysis. Sousa next must determine whether an American-style put option would have the same value.

## EXHIBIT 1 Two-Period Binomial European-Style Put Option on Beta Company

Item	Value
Underlying	38
Put	4.5346
Hedge Ratio	-0.4307

Item	Value
Underlying	49.4
Put	0.2517
Hedge Ratio	-0.01943

Item	Value
Underlying	30.4
Put	8.4350
Hedge Ratio	-1

Item	Value
Underlying	64.22
Put	0

Time = 0

Time = 1

Time = 2

Sousa makes two statements with regard to the valuation of a European-style option under the expectations approach.

Statement 1 The calculation involves discounting at the risk-free rate.

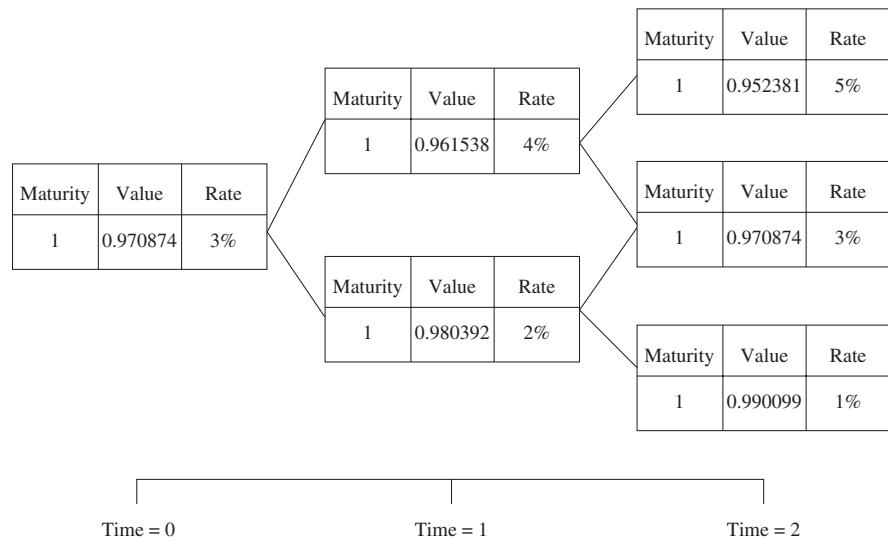
Statement 2 The calculation uses risk-neutral probabilities instead of true probabilities.

Rocha asks Sousa whether it is ever profitable to exercise American options prior to maturity. Sousa answers, "I can think of two possible cases. The first case is the early exercise of an American call option on a dividend-paying stock. The second case is the early exercise of an American put option."

### Interest Rate Option

The final option valuation task involves an interest rate option. Sousa must value a two-year, European-style call option on a one-year spot rate. The notional value of the option is 1 million, and the exercise rate is 2.75%. The risk-neutral probability of an up move is 0.50. The current and expected one-year interest rates are shown in Exhibit 2, along with the values of a one-year zero-coupon bond of 1 notional value for each interest rate.

## EXHIBIT 2 Two-Year Interest Rate Lattice for an Interest Rate Option



Rocha asks Sousa why the value of a similar in-the-money interest rate call option decreases if the exercise price is higher. Sousa provides two reasons.

Reason 1 The exercise value of the call option is lower

Reason 2 The risk-neutral probabilities are changed

1. The optimal hedge ratio for the Alpha Company call option using the one-period binomial model is *closest* to:
    - A. 0.60.
    - B. 0.67.
    - C. 1.67.
  2. The risk-neutral probability of the up move for the Alpha Company stock is *closest* to:
    - A. 0.06.
    - B. 0.40.
    - C. 0.65.
  3. The value of the Alpha Company call option is *closest* to:
    - A. 3.71.
    - B. 5.71.
    - C. 6.19.
  4. For the Alpha Company option, the positions to take advantage of the arbitrage opportunity are to write the call and:
    - A. short shares of Alpha stock and lend.
    - B. buy shares of Alpha stock and borrow.
    - C. short shares of Alpha stock and borrow.

5. The value of the European-style call option on Beta Company shares is *closest* to:
  - A. 4.83.
  - B. 5.12.
  - C. 7.61.
6. The value of the American-style put option on Beta Company shares is *closest* to:
  - A. 4.53.
  - B. 5.15.
  - C. 9.32.
7. Which of Sousa's statements about binomial models is correct?
  - A. Statement 1 only
  - B. Statement 2 only
  - C. Both Statement 1 and Statement 2
8. Based on Exhibit 2 and the parameters used by Sousa, the value of the interest rate option is *closest* to:
  - A. 5,251.
  - B. 6,236.
  - C. 6,429.
9. Which of Sousa's reasons for the decrease in the value of the interest rate option is correct?
  - A. Reason 1 only
  - B. Reason 2 only
  - C. Both Reason 1 and Reason 2

### The following information relates to Questions 10–18

Trident Advisory Group manages assets for high-net-worth individuals and family trusts.

Alice Lee, chief investment officer, is meeting with a client, Noah Solomon, to discuss risk management strategies for his portfolio. Solomon is concerned about recent volatility and has asked Lee to explain options valuation and the use of options in risk management.

#### Options on Stock

Lee begins: “We use the Black–Scholes–Merton (BSM) model for option valuation. To fully understand the BSM model valuation, one needs to understand the assumptions of the model. These assumptions include normally distributed stock returns, constant volatility of return on the underlying, constant interest rates, and continuous prices.” Lee uses the BSM model to price TCB, which is one of Solomon’s holdings. Exhibit 1 provides the current stock price ( $S$ ), exercise price ( $X$ ), risk-free interest rate ( $r$ ), volatility ( $\sigma$ ), and time to expiration ( $T$ ) in years as well as selected outputs from the BSM model. TCB does not pay a dividend.

#### EXHIBIT 1 BSM Model for European Options on TCB

BSM Inputs				
$S$	$X$	$r$	$\Sigma$	$T$
\$57.03	55	0.22%	32%	0.25

BSM Outputs					
$d_1$	$N(d_1)$	$d_2$	$N(d_2)$	BSM Call Price	BSM Put Price
0.3100	0.6217	0.1500	0.5596	\$4.695	\$2.634

## Options on Futures

The Black model valuation and selected outputs for options on another of Solomon's holdings, the GPX 500 Index (GPX), are shown in Exhibit 2. The spot index level for the GPX is 187.95, and the index is assumed to pay a continuous dividend at a rate of 2.2% ( $\delta$ ) over the life of the options being valued, which expire in 0.36 years. A futures contract on the GPX also expiring in 0.36 years is currently priced at 186.73.

EXHIBIT 2 Black Model for European Options on the GPX Index

Black Model Inputs					
GPX Index	$X$	$r$	$\sigma$	$T$	$\delta$ Yield
187.95	180	0.39%	24%	0.36	2.2%
Black Model Call Value		Black Model Put Value	Market Call Price	Market Put Price	
\$14.2089		\$7.4890	\$14.26	\$7.20	
Option Greeks					
Delta (call)	Delta (put)	Gamma (call or put)	Theta (call) daily	Rho (call) per %	Vega per % (call or put)
0.6232	-0.3689	0.0139	-0.0327	0.3705	0.4231

After reviewing Exhibit 2, Solomon asks Lee which option Greek letter best describes the changes in an option's value as time to expiration declines.

Solomon observes that the market price of the put option in Exhibit 2 is \$7.20. Lee responds that she used the historical volatility of the GPX of 24% as an input to the BSM model, and she explains the implications for the implied volatility for the GPX.

## Options on Interest Rates

Solomon forecasts the three-month Libor will exceed 0.85% in six months and is considering using options to reduce the risk of rising rates. He asks Lee to value an interest rate call with a strike price of 0.85%. The current three-month Libor is 0.60%, and an FRA for a three-month Libor loan beginning in six months is currently 0.75%.

## Hedging Strategy for the Equity Index

Solomon's portfolio currently holds 10,000 shares of an exchange-traded fund (ETF) that tracks the GPX. He is worried the index will decline. He remarks to Lee, "You have told me how the BSM model can provide useful information for reducing the risk of my GPX position." Lee suggests a delta hedge as a strategy to protect against small moves in the GPX Index.

Lee also indicates that a long position in puts could be used to hedge larger moves in the GPX. She notes that although hedging with either puts or calls can result in a delta-neutral position, they would need to consider the resulting gamma.

10. Lee's statement about the assumptions of the BSM model is accurate with regard to:
  - A. interest rates but not continuous prices.
  - B. continuous prices but not the return distribution.
  - C. the stock return distribution but not the volatility.
11. Based on Exhibit 1 and the BSM valuation approach, the initial portfolio required to replicate the long call option payoff is:
  - A. long 0.3100 shares of TCB stock and short 0.5596 shares of a zero-coupon bond.
  - B. long 0.6217 shares of TCB stock and short 0.1500 shares of a zero-coupon bond.
  - C. long 0.6217 shares of TCB stock and short 0.5596 shares of a zero-coupon bond.
12. To determine the long put option value on TCB stock in Exhibit 1, the correct BSM valuation approach is to compute:
  - A. 0.4404 times the present value of the exercise price minus 0.6217 times the price of TCB stock.
  - B. 0.4404 times the present value of the exercise price minus 0.3783 times the price of TCB stock.
  - C. 0.5596 times the present value of the exercise price minus 0.6217 times the price of TCB stock.
13. What are the correct spot value ( $S$ ) and the risk-free rate ( $r$ ) that Lee should use as inputs for the Black model?
  - A. 186.73 and 0.39%, respectively
  - B. 186.73 and 2.20%, respectively
  - C. 187.95 and 2.20%, respectively
14. Which of the following is the correct answer to Solomon's question regarding the option Greek letter?
  - A. Vega
  - B. Theta
  - C. Gamma
15. Based on Solomon's observation about the model price and market price for the put option in Exhibit 2, the implied volatility for the GPX is *most likely*:
  - A. less than the historical volatility.
  - B. equal to the historical volatility.
  - C. greater than the historical volatility.
16. The valuation inputs used by Lee to price a call reflecting Solomon's interest rate views should include an underlying FRA rate of:
  - A. 0.60% with six months to expiration.
  - B. 0.75% with nine months to expiration.
  - C. 0.75% with six months to expiration.
17. The strategy suggested by Lee for hedging small moves in Solomon's ETF position would *most likely* involve:
  - A. selling put options.
  - B. selling call options.
  - C. buying call options.
18. Lee's put-based hedge strategy for Solomon's ETF position would *most likely* result in a portfolio gamma that is:
  - A. negative.
  - B. neutral.
  - C. positive.

# CHAPTER 5

## DERIVATIVES STRATEGIES

Robert A. Strong, PhD, CFA  
Russell A. Rhoads, CFA

### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- describe how interest rate, currency, and equity swaps, futures, and forwards can be used to modify risk and return;
- describe how to replicate an asset by using options and by using cash plus forwards or futures;
- describe the investment objectives, structure, payoff, and risk(s) of a covered call position;
- describe the investment objectives, structure, payoff, and risks(s) of a protective put position;
- calculate and interpret the value at expiration, profit, maximum profit, maximum loss, and breakeven underlying price at expiration for covered calls and protective puts;
- contrast protective put and covered call positions to being long an asset and short a forward on the asset;
- describe the investment objective(s), structure, payoffs, and risks of the following option strategies: bull spread, bear spread, collar, and straddle;
- calculate and interpret the value at expiration, profit, maximum profit, maximum loss, and breakeven underlying price at expiration of the following option strategies: bull spread, bear spread, collar, and straddle;
- describe uses of calendar spreads;
- identify and evaluate appropriate derivatives strategies consistent with given investment objectives.

## 1. INTRODUCTION

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There are many ways in which investors and financial managers routinely use put and call options, futures, forward contracts, and various types of swap contracts to modify their investment positions or to implement market strategies. Some derivative strategies are purely speculative, designed to profit if a particular market change occurs. Other strategies are defensive, providing protection against an adverse event or removing the uncertainty around future events.

The purpose of this chapter is to illustrate ways in which derivatives might be used in typical investment situations. Few financial managers or individual investors will ever use all of the strategies described here. That does not mean that some strategies are not important. An informed investment professional should still be aware of them and understand the associated risk–return trade-off. Although part of the medical school curriculum, many physicians will never treat a patient for frostbite or malaria. Regardless, patients have a right to expect that their doctors have an idea about how these conditions might be treated or, better yet, prevented. Someone who is travelling to an area where malaria is prevalent wants to acquire protection before the trip, and we expect doctors to know how to prescribe something for that even if they have never done it before. The doctor may also choose to refer the patient to a specialist, just as an investment manager may need to confer with a derivatives specialist.

An investment manager may not currently use derivatives but should be sufficiently familiar with them that he or she can answer questions about them and explain how they might logically be used to benefit a corporate treasury or protect an investment portfolio. When a financial adviser deals with a client's money, the adviser is dealing with the long-term financial health of the client. Physicians are expected to be current and knowledgeable, and investors have a right to expect the same of their financial advisers.

Section 2 of this chapter shows how swaps, futures, and forwards can be used to change the risk exposure of an existing position. Section 3 shows how certain combinations of securities are equivalent to others. Section 4 is a discussion of two of the most widely used derivative strategies, covered calls and protective puts. In Section 5, we look at popular option strategies used by investors. Section 6 demonstrates a series of applications showing ways in which a money manager might solve a problem with derivatives.

## 2. CHANGING RISK EXPOSURES WITH SWAPS, FUTURES, AND FORWARDS

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Financial managers use the derivatives markets to quickly and efficiently alter the underlying risk exposure of their asset portfolios or forthcoming business transactions. This section covers a variety of common examples that use various derivative products.

### 2.1. Interest Rate Swap/Futures Examples

Interest rate swaps and futures can be used to modify the risk and return of a fixed-income portfolio and can also be used in conjunction with an equity portfolio, as we will show later in this chapter. Both interest rate swaps and futures are interest-sensitive instruments, and by adding them to a portfolio, either as long or short positions, they can increase or decrease the exposure of the portfolio to interest rates.

### 2.1.1. Interest Rate Swap

Interest rate swaps are an indispensable tool for many corporate treasurers, especially at a financial institution. In an interest rate swap, the two parties involved agree that on specified payment dates they will exchange cash flows, one based on a *variable* (floating) interest rate and the other based on a *fixed* rate determined at the time the swap is initiated. This fixed rate is called the swap rate.<sup>1</sup> Both interest rates are applied to the swap's notional value to determine the size of the payment. The period of time over which the payments are exchanged is called the swap tenor. At the end of this period, the swap is said to expire. The notional value is needed to determine the size of the payments, but the notional value does not actually change hands, nor is it borrowed or lent between the parties. Typically no funds change hands when the parties enter into the agreement. Normally, the resulting two payments (one fixed, one floating) will not be equal, so they are typically netted, with the party owing the greater amount sending the difference to the other party.

Let us examine how swaps can be used by looking at an application in fixed-income portfolio management. One way to measure interest rate risk is by using the concept of duration.<sup>2</sup> Consider a portfolio manager with an investment portfolio containing \$500 million of fixed-rate US Treasury bonds with an average duration of five years. Suppose the manager wants to reduce this duration to three over the next year but does not want to sell any of the securities.

One way to do this would be with a pay-fixed interest rate swap. Because the portfolio is currently receiving a fixed rate, the manager will want to exchange part of this fixed-rate income stream for a floating-rate stream in order to lower the overall duration. This approach means the appropriate swap would pay the fixed rate and receive the floating rate.

Suppose the duration of the swap used by the manager is 1.5.<sup>3</sup> This duration is less than the existing portfolio duration, so adding the swap to the portfolio will reduce the overall average duration. By properly choosing the notional value of the swap, the portfolio manager can achieve a combination of the existing portfolio duration and the swap duration that sets the overall duration to the target duration.

### 2.1.2. Interest Rate Futures

A swap is an over-the-counter derivative that is subject to counterparty risk on the interest payment dates, although as stated previously, the notional value is not at risk. Because one party will owe money to the other, there is the possibility of default by one side of the trade. An alternative would be to use exchange-traded interest rate futures contracts, which are guaranteed by a clearinghouse and are virtually free of concerns about counterparty risk. These contracts are also sometimes referred to as bond futures because the underlying asset is often a bond. Because there are usually many different bonds that could be used to satisfy the delivery requirement, the hedge calculation can be complicated. Here we will provide a general description of how interest rate futures are used in adjusting portfolio duration.

Because most interest rate futures contracts are futures contracts in which the underlying is a bond, this type of contract would have a duration that is consistent with the forward behavior of the underlying deliverable bond. That is, the interest rate futures price will move as

<sup>1</sup>The swap price is also sometimes called the swap rate.

<sup>2</sup>Although there are various duration measures, the most important is modified duration, which is an approximate measure of how a bond price changes given a small change in the level of interest rates, adjusted for the level of interest rates.

<sup>3</sup>The duration of an interest rate swap is the duration of the fixed-rate component minus the duration of the floating-rate component.

though it was the forward price of the bond on the expiration date. The price sensitivity of the futures will, therefore, reflect a type of forward duration that is based on the underlying bond. Thus, the futures price will move fairly consistently and proportionately with the yield that drives the underlying bond. Continuing with the example earlier in which the manager whose portfolio has a duration of five years wants to lower the duration to three years, the general principle is the same: By selling bond futures, a portfolio manager “adds negative duration” to the portfolio, and if done in the right quantity, the overall duration can be reduced to the target level.<sup>4</sup>

In general, anything done with an interest rate *futures* contract could also be done with an interest rate *forward* contract. Forwards, like swaps, have counterparty risk and can be customized. Futures are standardized and come with greater regulatory oversight and with a clearinghouse that makes counterparty risk virtually zero.<sup>5</sup>

## 2.2. Currency Swap/Futures Examples

Currency swaps and futures can be used effectively to alter risk exposures. We provide examples of how it is done in the next two sub-sections.

### 2.2.1. Currency Swap

A currency swap is different from an interest rate swap in two ways: 1) The interest rates are associated with different currencies, and 2) the notional value may be exchanged at the beginning and end of the swap’s life.<sup>6</sup> As an example, suppose Assicurazioni Generali, the largest insurance company in Italy, wants to fund its operations in Switzerland, and for that, it needs Swiss francs. But it discovers that it can borrow cheaper in the euro market. The company decides to fund itself in euros and swap the cash flow into Swiss francs. Assicurazioni contacts a dealer and requests a quote on a pay-fixed €50 million three-year swap with semi-annual interest payments. The swap agreement provides that both parties pay a fixed rate. After entering into the swap agreement the dealer will hedge the swap with another party, possibly through a trade in the futures markets.

Exhibit 1 shows the direction of the cash flows. With the “block and arrow” diagram, it is easy to see how the cash flows net out. For instance, at origination Assicurazioni receives euros from the eurozone lender and passes them on to the swap dealer, so they are a pass through cash flow. Assicurazioni receives Swiss francs from the dealer just as if the firm had borrowed them. At each payment date, Assicurazioni receives a fixed-rate euro payment and passes it on to the eurozone lender.<sup>7</sup> The firm is left with a net outflow in fixed Swiss francs, which is the firm’s preferred payment option. At maturity Assicurazioni returns the Swiss francs notional amount to the dealer and in return receives the notional amount in euros, which it uses to pay off its creditor.

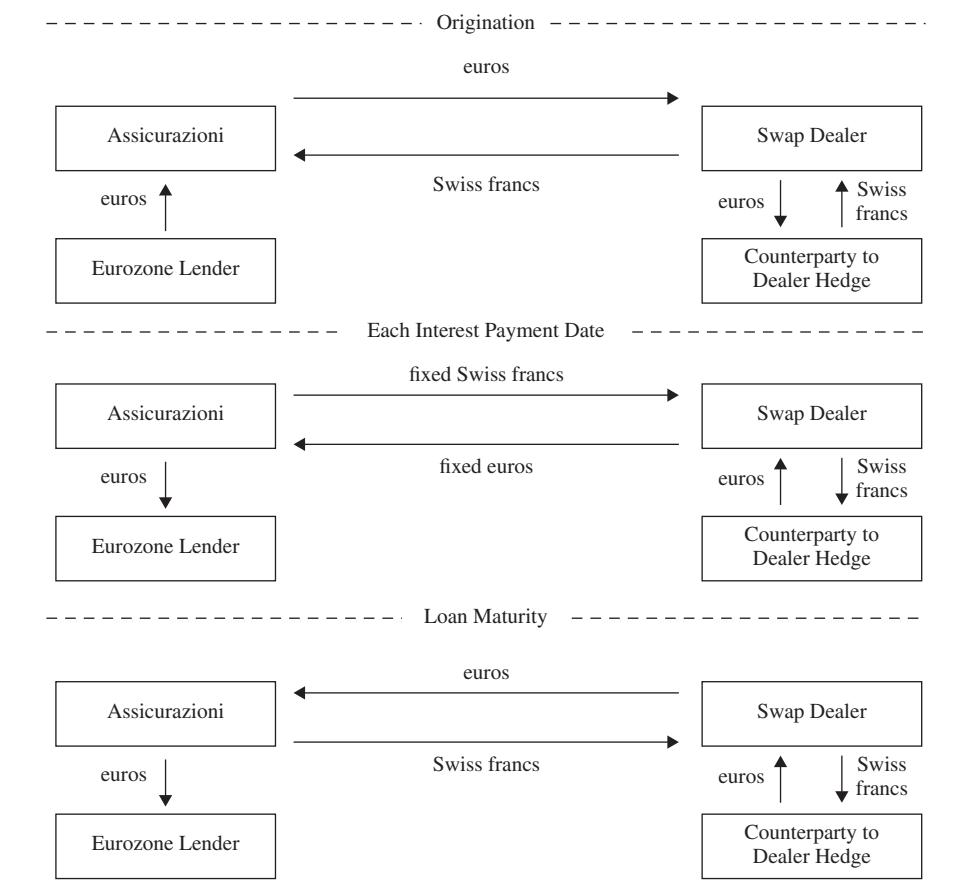
<sup>4</sup>It is important to note that the portfolio manager would sell the futures in this example because he wants to reduce the duration. If he bought futures, he would be increasing duration.

<sup>5</sup>Regulatory changes in global markets are moving both over-the-counter swaps and forward contracts toward a clearing process as well.

<sup>6</sup>Although an exchange of notional is often the case, the parties may agree not to do this. Some types of hedge transactions are designed to hedge only foreign cash flows and not principal payments, so a principal exchange on a currency swap would not be necessary.

<sup>7</sup>The rate paid to the creditors on the euro loan will not exactly equal the rate paid to the firm on the swap. Also, when a company uses a swap, it can elect to receive either a fixed or a floating rate and will want to align the cash receipt with the floating or fixed rate on the loan.

## EXHIBIT 1 Currency Swap Cash Flows



## 2.2.2. Currency Futures

Foreign currency futures are another useful tool in risk management.<sup>8</sup> For example, Swiss Re, a reinsurance company, might have a €25.18 million liability due to be paid in 15 months. If the euro were to appreciate versus the Swiss franc before the payment date, Swiss Re's effective borrowing cost would be higher. The firm could largely eliminate the foreign exchange risk associated with this debt by fixing the price of the euros now. One way to do this would be via a futures contract in which Swiss Re promises to buy the euros in 15 months at a Swiss franc price determined now. In 15 months, the firm needs the euros to pay the bill, so it wants to *buy* the euros and promises to pay for them in Swiss francs. At a futures exchange, contracts are standardized by size, so Swiss Re would determine the number of contracts needed to fund the liability. If, for instance, the contract size is €125,000, the firm would need

$$\frac{\text{€25,180,000}}{\text{€125,000/contract}} = 201.44 \text{ contracts}$$

<sup>8</sup>Many firms prefer to hedge with currency *forward* contracts rather than futures because the forward contract trades in the over-the-counter market and can be customized to any size and any payment date. Exchange-traded futures are somewhat limited by standardized expiration dates and fixed notional sizes.

Fractional contracts are not allowed, so to fully hedge the risk the firm would buy either 201 or 202 contracts. In the over-the-counter market, customized products, such as forward agreements, can be used to specify a contract size exactly. The contract size problem is not the sole determinant of whether exchanged-listed or over-the-counter derivatives are chosen.

### 2.3. Equity Swap/Futures Examples

Equity swaps can be used to modify risk exposures, as we demonstrate in the next two subsections.

#### 2.3.1. Equity Swap

In an **equity swap**, one party agrees to trade the return on a stock portfolio for the return on another asset.<sup>9</sup> This other asset might be another portfolio or a market index, or the swap might specify a set interest rate, such as Libor. The swap has a set tenor and may provide for one single payment at the end of the swap's life, although in most cases a series of periodic payments would be arranged. Equity swaps are created in the over-the-counter market, so they can be customized as desired. Consider an institutional investor that would like to temporarily remove \$100 million in equities from its current equity exposure. The firm could enter into a six-month equity swap with one payment at termination, exchanging the total return on the underlying portfolio for six-month Libor, assumed to be 0.50%.<sup>10</sup> We will consider two scenarios: In the first, in six months the underlying portfolio is up 1%, and in the second, it is down 1%. Exhibit 2 shows the two situations.

#### EXHIBIT 2 Six-Month Equity Swap

Pay the return on a \$100 million equity portfolio  
Receive six-month Libor, assumed to be 0.50%

##### Scenario 1: Equity portfolio rises 1%

Pay: \$100 million $\times$ 1% =	\$1,000,000
Receive: \$100 million $\times$ 0.50% $\times$ 0.50 =	250,000
Net payment =	\$750,000

##### Scenario 2: Equity portfolio declines 1%

Pay: \$100 million $\times$ -1% =	(\$1,000,000)
Receive: \$100 million $\times$ 0.50% $\times$ 0.50 =	250,000
Net receipt =	\$1,250,000

<sup>9</sup>The same principles discussed in this section apply to assets other than equities. When applied to bonds, loans, or an equity index, this arrangement may be called a “total return” swap.

<sup>10</sup>This rate is stated on an annual basis, so for six months, one would earn half this amount.

In the first scenario, the institutional investor would have an obligation to pay  $1\% \times \$100 \text{ million}$ , or  $\$1 \text{ million}$ . On the Libor portion of the swap the investor would receive  $0.50\% \times 0.50 \times \$100 \text{ million}$ , or  $\$250,000$ . The two parties would net the payments and provide for a single payment of  $\$750,000$ , which the institutional investor would pay. The second scenario is slightly more complicated because the return the institutional investor must pay is *negative*, which means it will receive money both from “paying” a negative return and from the Libor rate. It would receive  $\$1 \text{ million}$  from the “negative payment” and  $\$250,000$  from Libor, for a total of  $\$1.25 \text{ million}$ . At the end of the swap the institutional investor is in the same position in which it started, with the equity portfolio fully invested and again subject to market risk.

### 2.3.2. Stock Index Futures

Stock index futures are one of the most successful financial innovations of all time. The contract on the S&P 500’s stock index began trading in 1982, and today this product is an indispensable tool for many money managers. Stock index futures differ from most other futures contracts in that they are **cash settled** at expiration. That is, it would not be feasible to actually transfer the underlying securities from one party to another, so instead the cash value changes hands. Rather than using the equity swap in the previous example, the institutional investor could temporarily remove market risk by selling stock index futures. If done in the correct quantity, this “short” position will largely offset the institution’s long position.

One S&P 500 stock index futures contract is standardized as  $\$250$  times the index level. Assume that a one-month futures contract trades at  $2,000.00$  and that the portfolio carries average market risk, meaning a beta of  $1.0$ .<sup>11</sup> To fully hedge the  $\$100,000,000$  portfolio, the portfolio manager would want to sell 200 contracts:

$$\frac{\$100,000,000}{(\$250 \times 2000.00)} = 200 \text{ contracts}$$

Suppose the S&P 500 stock index *rises* by  $0.5\%$  such that at the futures delivery time, the index is at  $2,010.00$ . The institution sold the futures at  $2,000.00$  and cash settles the contract at  $2,010.00$ . There is a “loss” of 10 index points, each point being worth  $\$250$ , on 200 contracts for a total cash outflow of  $\$500,000$ :

$$-10 \text{ points per contract} \times \$250 \text{ per point} \times 200 \text{ contracts} = \$500,000 \text{ loss}$$

If the stock index rises by  $0.5\%$ , the portfolio would also be expected to rise by this amount:

$$\$100,000,000 \times 0.5\% = \$500,000 \text{ gain}$$

If instead the market *fell* by  $0.5\%$ , the numbers would be the same, but the signs would change; there would be a  $\$500,000$  gain on the futures contract and a corresponding loss on the stock portfolio. Either way, the portfolio value is essentially “fixed” by selling the futures contracts. The market risk is hedged away.

<sup>11</sup>If the portfolio carried above market risk, say with a beta of  $1.10$ , the number of contracts needed to hedge would increase by this factor. Similarly, a lower-risk portfolio would require proportionately fewer contracts.

### EXAMPLE 1 Using Swaps to Manage Risk

1. A US bond portfolio manager who wants to hedge a bond portfolio against a potential rise in domestic interest rates could *best* hedge by:
  - A. buying Treasury bond futures.
  - B. paying a fixed rate in an interest rate swap.
  - C. selling foreign currency futures on the home currency.
2. A typical currency swap used to hedge a bond portfolio differs from an interest rate swap with respect to the:
  - A. tenor of the swap.
  - B. size of the initial cash flows.
  - C. presence of counterparty risk.
3. A stock portfolio manager who enters into a Libor-based equity swap and pays the equity return would owe money to the counterparty under which of the following conditions?
  - A. Portfolio return > Libor
  - B. Portfolio return = Libor
  - C. Portfolio return < minus Libor

*Solution to 1:* B is correct. In an interest rate swap, if someone pays the fixed rate he or she would receive the floating rate. A floating-rate asset would most likely have a lower duration than a fixed-rate asset, and duration is a direct measure of interest rate risk. The swap would lower the portfolio duration.

*Solution to 2:* B is correct. In an interest rate swap, there is no initial cash flow; the notional value is not exchanged. In a currency swap hedging a bond portfolio, both currencies change hands at the initiation and termination of the swap.

*Solution to 3:* A is correct. The portfolio manager is agreeing to exchange the return on the portfolio for the Libor rate. If the portfolio earns more than Libor, the manager must remit the difference.

### 3. POSITION EQUIVALENCIES

It is useful to think of derivatives as building blocks that can be combined to create a particular end product with the desired risk exposure, much in the same way that a cook combines ingredients in preparing a meal. The risk manager's "spice rack" has puts and calls, each with many different expirations and exercise prices, as well as futures, forwards, and swaps. Often there is more than one way to create the desired risk–return exposure. Stated another way, certain collections of derivatives are economically equivalent to other assets or asset portfolios. A few of these relationships are especially important and are covered in the following pages.

### 3.1. Synthetic Long Asset

An outright long position in an asset is a common situation and easily understood. Suppose an investor buys shares of stock at a price of 50: She makes money when prices go up and is hurt when prices decline. What happens if she buys a call and writes a put, and both options have the same expiration date and the same exercise price of, say, 50?<sup>12</sup> Exhibit 3 shows the values of these two options and the overall position at expiration in comparison with the value of the stock at that same time. Remember that at expiration, a call is worth the greater of zero or the stock price minus the exercise price, and a put is worth the greater of zero or the exercise price minus the stock price. This combination of options is equivalent to a long position in the stock and is often called a **synthetic long position**.<sup>13</sup>

EXHIBIT 3 Synthetic Long Position

Stock price at expiration:	0	20	40	50	60	80	100
<b>Alternative 1:</b>							
Long 50-strike call payoff	0	0	0	0	10	30	50
Short 50-strike put payoff	-50	-30	-10	0	0	0	0
Total value	-50	-30	-10	0	10	30	50
<b>Alternative 2:</b> Long stock at 50							
Value	-50	-30	-10	0	10	30	50

The reason a long call and a short put synthetically replicate a long position in the underlying is that the long call creates the upside and the short put creates the downside of the underlying. The call exercises when the underlying is higher than the strike and turns into a synthetic position in the upside of the underlying. A short put obligates the writer to compensate the put buyer for downside moves by purchasing the stock at a higher price than its value. Thus, as a result of exercise by the buyer, a put writer incurs losses on the downside. These combined effects synthetically replicate the payoffs of the underlying.

### 3.2. Synthetic Short Asset

If instead of buying the call and writing the same-strike put to get a synthetic long stock position the investor did the opposite, logically the resulting position would be a **synthetic short position**. Indeed, that is exactly what happens, as Exhibit 4 shows. The explanation is the exact opposite of the explanation for the synthetic long.

<sup>12</sup>The exercise price is the price at which the option holder has the right to buy (with a call) or sell (with a put) and is also commonly called the *strike* or *striking* price. We will occasionally use terminology such as a “50-strike call” meaning a call option with an exercise price of 50.

<sup>13</sup>Technically, the position is not precisely equal to a long position in a stock. From put-call parity, a long position in a stock equals a long call, short put, and a long position in a risk-free zero-coupon bond. With the options strategy, you must pay the exercise price at expiration, whereas with the stock strategy, the analog is the value of the stock price when the options are put in place. Adding a zero-coupon bond to the options strategy or setting the present value of the exercise price to the stock price would make it precisely the same as the stock strategy.

## EXHIBIT 4 Synthetic Short Position

Stock price at expiration:	0	20	40	50	60	80	100
<b>Alternative 1:</b>							
Long 50-strike put payoff	50	30	10	0	0	0	0
Short 50-strike call payoff	0	0	0	0	-10	-30	-50
Total payoff	50	30	10	0	-10	-30	-50
<b>Alternative 2:</b> Short stock at 50							
Payoff	50	30	10	0	-10	-30	-50

## 3.3. Synthetic Assets with Futures/Forwards

One very common use of futures or forward contracts is to eliminate future price risk. Consider an investor who owns a dividend-paying stock and wants to lock in a future selling price. The investor might enter into a forward or futures contract requiring him to deliver the shares at a future date in exchange for a cash amount determined today. Because the initial and final stock prices are known, this investment should earn the risk-free rate. What actually happens is that the dividends earned on the stock plus the return from the forward or futures contract together equal the risk-free rate. Thinking of this as an equation, Long stock + Short futures = Risk-free rate, or rearranging the equation, Stock – Futures = Risk-free rate.<sup>14</sup> This strategy is sometimes called a “synthetic risk-free rate,” or occasionally “synthetic cash.”

This result also means that someone can create a synthetic long position by investing in the risk-free asset and using the remaining funds to margin a long futures position: Stock = Risk-free rate + Futures.

## 3.4. Synthetic Put

When the Chicago Board Options Exchange opened in 1973, there were exchange-traded calls but no puts. Regulators were concerned about approving a financial instrument that benefited from falling prices. Informed market participants, however, knew that a put could easily be created by combining a short stock position with a long call. Suppose, for instance, an investor wants a put with an exercise price of 50. Exhibit 5 shows the payoffs from the put at various stock prices at option expiration, along with the payoffs from a stock shorted at a price of 50 and held simultaneously with a long call with an exercise price of 50. Regardless of the stock price at option expiration, the two alternatives have the same economic characteristics.<sup>15</sup>

<sup>14</sup>When the underlying asset on the futures contract is an equity security, the asset may pay dividends. If the dividend yield on the stock is higher than the interest rate, the futures price will be less than the spot price. The futures price will rise to converge at the spot price at the end of the futures contract, resulting in a loss on the short futures position. This loss offsets part of the gain from the dividend yield. Regardless, if held to the delivery date, the long stock/short futures position nets the risk-free interest rate.

<sup>15</sup>As discussed in an earlier footnote, the addition of a risk-free bond is technically required to make the alternatives precisely equal, but they do have the same economic characteristics.

## EXHIBIT 5 Synthetic Put

Stock price at expiration:	0	20	40	50	60	80	100
<b>Alternative 1:</b> Long 50-strike put							
Payoff	50	30	10	0	0	0	0
<b>Alternative 2:</b> Short stock at 50; buy 50-strike call							
Profit from short stock	50	30	10	0	-10	-30	-50
Payoff from long call	0	0	0	0	10	30	50
Net payoff	50	30	10	0	0	0	0

The creation of synthetic puts was widespread in the early days of option trading, and in 1975 the regulatory authorities allowed a pilot project authorizing puts on 25 different underlying stock issues. These puts were actively traded, no major problems were caused, and the exchanges gradually were able to increase the number of listed puts. Nonetheless, it is important to know how to synthetically replicate a put because mispricing may make a replicated put cheaper or more expensive than a direct put.

## 3.5. Synthetic Call

In similar fashion, an investor can replicate a call from a long stock position combined with a long put. See Exhibit 6. The long put eliminates much of the downside risk whereas the long stock leaves the profit potential unlimited. As will be shown shortly, the popular “protective put” strategy has a profit and loss diagram similar to that of a long call.

## EXHIBIT 6 Synthetic Call

Stock price at expiration:	0	20	40	50	60	80	100
<b>Alternative 1:</b> Long 50-strike call							
Payoff	0	0	0	0	10	30	50
<b>Alternative 2:</b> Long stock at 50; buy 50-strike put							
Profit from long stock	-50	-30	-10	0	10	30	50
Payoff from long put	50	30	10	0	0	0	0
Net payoff	0	0	0	0	10	30	50

## 3.6. Foreign Currency Options

Suppose the treasurer of a multinational corporation with the euro as the home currency has an obligation to pay ¥145 million in one month. This obligation might arise, for example, from a commitment to purchase some Japanese products. The treasurer wants to protect against an appreciation of the yen relative to the euro, but at the same time does not want to lock in the exchange rate because of a belief that the yen may depreciate. Because the obligation is denominated in yen, however, the nature of the problem requires careful attention. Suppose the spot exchange rate is EUR/JPY = 136.99.<sup>16</sup> If the yen appreciates, this number will go down, and if the yen depreciates, this number will go up.

<sup>16</sup>This quote convention means that one euro is worth 136.99 yen.

An appreciating yen means the euro is weakening and thus the home currency, the euro, would buy fewer yen. A futures contract or a forward contract could eliminate the risk but would also eliminate the potential for gains if the yen depreciates. With forwards or futures, the returns are symmetrical around the fixed price, but with options, only one side of the return distribution is affected. If the treasurer wants to benefit from an appreciating yen but not be locked in to a fixed rate, as with a futures or forward, he might buy a one-month call option on yen. Because the spot rate is quoted in yen, the strike will typically be quoted in yen.

Now, perhaps seemingly strange, we have a situation in which the treasurer would exercise the call option when the spot rate at expiration is *below* the strike. For example, at the strike rate of ¥136.99, exercise would require that the treasurer deliver ¥145 million/¥136.99 = €1,058,471 to obtain the necessary yen. If the spot rate at expiration rises to ¥140, a weakening of the yen (strengthening of the euro), he could deliver ¥145 million/¥140 = €1,035,714 to obtain the yen cheaper in the spot market—that is, without exercising the call. If the spot rate at expiration falls to ¥130, a strengthening of the yen (weakening of the euro), he would have to deliver ¥145 million/¥130 = €1,115,385 to obtain the necessary yen in the market. Clearly it would be cheaper to exercise the option, thereby paying €1,058,471 to get the yen.

Note that the treasurer would exercise the foreign currency call when the yen was below the strike, but that is because the yen is quoted in relation to the euro in terms of yen per euro. This fact connects us to an important point about foreign currency options. A foreign currency call option always has a put option that is an identical twin. Instead of the yen call, suppose the treasurer had purchased an at-the-money put on €1,058,471 with the same expiration and exercise price as the call. This gives him the right to deliver euros in exchange for yen. This put option provides for exactly the same cash flows as the call option in the same circumstances. If he exercises his right to sell euros, he would deliver €1,058,471 and receive ¥145,000,000.

In either case, after receiving the ¥145,000,000 from the option counterparty the treasurer would use it to cover the company's obligation. With a set exercise price, the right to deliver euros in exchange for yen is exactly the same as the right to buy yen in exchange for euros.<sup>17</sup>

## EXAMPLE 2 Option Position Equivalencies

1. Which of the following is *most* similar to a long put position?
  - A. Buy stock, write call
  - B. Short stock, buy call
  - C. Short stock, write call
2. Which of the following is *most* similar to a long call position?
  - A. Buy stock, buy put
  - B. Buy stock, write put
  - C. Short stock, write put

<sup>17</sup>This mirror-image nature of puts and calls is actually true for any underlying asset. A call giving the holder the right to buy stock can also be thought of as a put giving the right to sell a currency in exchange for stock. Such an arrangement would imply thinking of that currency as being worth a certain number of shares of stock. For example, we could say that a stock priced at €50 implies that €1 = 1/50 = 0.02 share. This is an unconventional way of thinking about stock, bonds, or commodities, but is perfectly natural for currencies.

3. Which option portfolio with the same exercise price for both options is *most* similar to a long stock position?
- Short call, long put
  - Long call, short put
  - Short call, short put

*Solution to 1:* B is correct. The long call “cuts off” the unlimited losses from the short stock position.

*Solution to 2:* A is correct. The long put provides a floor value to the position, making the maximum loss flat below the exercise price. The profit and loss diagram is the same shape as a long call.

*Solution to 3:* B is correct. When both options have the same exercise price, a short put and long call produce a profit and loss diagram that is the same as a long stock position.

#### 4. COVERED CALLS AND PROTECTIVE PUTS

Writing a **covered call** is a very common option strategy used by both individual and institutional investors. In this strategy, someone who already owns shares sells a call option giving someone else the right to buy their shares at the exercise price.<sup>18</sup> The investor owns the shares and has taken on the potential obligation to deliver the shares to the option buyer and accept the exercise price as the price at which he sells the shares. For his willingness to do this, the investor receives the premium on the option.

When someone simultaneously holds a long position in an asset and a long position in a put option on that asset, the put is often called a **protective put**. The name comes from the fact that the put protects against losses in the value of the underlying asset.

The examples that follow use the convention of identifying an option by the underlying asset, expiration, exercise price, and option type. For example, in Exhibit 7, the PBR October 16 call option sells for 1.42. The underlying asset is Petróleo Brasileiro (PBR) common stock, the expiration is October, the exercise price is 16, the option is a call, and the call price is 1.42. It is important to note that even though we will refer to this as the October 16 option, it does not expire on 16 October; 16 is the price at which the call owner has the right to buy, otherwise known as the exercise price or strike.

PBR	October	16	Call
<i>Underlying asset</i>	<i>expiration</i>	<i>exercise price</i>	<i>option type</i>

On some exchanges, certain options may have weekly expirations in addition to a monthly expiration, which means investors need to be careful in specifying the option of interest. For a given underlying asset and exercise price, there may be several weekly and one monthly option expiring in October. The examples that follow all assume a single monthly expiration.

<sup>18</sup>If someone creates a call without owning the underlying asset, it is a *naked call*.

#### 4.1. Investment Objectives of Covered Calls

Consider the option data in Exhibit 7. Suppose there is one month until the September expiration. By convention, option listings show data for a single call or put, but in practice the smallest trading unit for an exchange-traded option is one contract covering 100 shares. Consider three different market participants that might logically use covered calls.

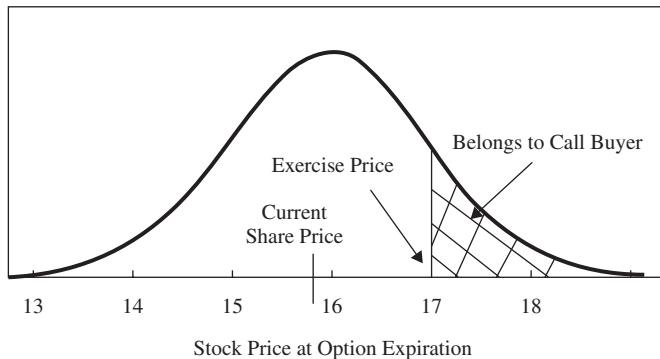
EXHIBIT 7 PBR Option Premiums: Current PBR share price = 15.84

Calls			Exercise Price	Puts		
SEP	OCT	NOV		SEP	OCT	NOV
1.64	1.95	2.44	15	0.65	0.99	1.46
0.97	1.42	1.90	16	1.14	1.48	1.96
0.51	1.02	1.44	17	1.76	2.09	2.59

##### 4.1.1. Market Participant #1: Income Generation

The most common motivation for writing covered calls is income generation. When someone writes an option they keep the option premium regardless of what happens in the future. Some investors view this premium as an additional source of income in the same way they view cash dividends. It is important to recognize that when someone writes a call option, however, he is essentially selling part of the return distribution associated with the underlying asset. See the illustration of the return distribution in Exhibit 8. The largest returns are those on the far right of the distribution. Someone who writes a call with an exercise price of 17 is giving up all returns above the strike price in exchange for the option premium. So, in Exhibit 8, the portion of the underlying return distribution that lies above a stock price of 17 belongs to the call buyer.

EXHIBIT 8 Covered Calls and the Return Distribution: With stock at 15.84, write 17-strike call



Perhaps an individual investor owns PBR and believes the stock price is likely to remain relatively flat over the next few months. With the stock currently trading at just under 16, the investor might think it unlikely that the stock will rise above 17. Exhibit 7 shows that the premium for a call option expiring in September with an exercise price of 17, referred to as the SEP 17 call, is 0.51. He could write that call and receive this premium. Alternatively, he could write a different call, say the NOV 17 call and receive 1.44. There is a clear trade-off between the size of the option premium and the likelihood of option exercise. The option writer would get more income from writing the longer-term option, but there is a greater chance that the option would move in the money, resulting in the option being exercised by the buyer and, therefore, the stock being called away from the writer.

Although it may be acceptable to think of the option premium as income, it is important to remember that the call writer has given up the most desirable part of the expected return distribution. For someone who believes significant price increases in the underlying are unlikely over the option's life, however, writing covered calls can be very attractive.

#### 4.1.2. Market Participant #2: Improving on the Market

Next, consider Sofia Porto, a retail portfolio manager with a portfolio that has become overweight in energy companies. She wants to reduce this imbalance. She holds 5,000 shares of PBR, an energy company, and she expects the price of this stock to remain relatively stable over the next month. Porto might decide to write 10 exchange-traded PBR SEP 15 call contracts. This means she is creating 10 option contracts, each of which covers 100 shares. These 10 contracts give the buyer the right to purchase 1,000 shares of PBR stock at 15 per share anytime between the time of purchase and expiration in September. In exchange for this contingent claim, she receives the option premium of  $1.64/\text{call} \times 100 \text{ calls/contract} \times 10 \text{ contracts} = 1,640$ . This income remains in her account regardless of what happens to the future PBR stock price or whether or not the option is exercised by its holder. Because the current PBR stock price (15.84) is above the exercise price of 15, the options she wrote are in the money. Given her expectation that the stock price will be stable over the next month, there is a high likelihood that the option will be exercised: The holder of the right to buy at 15 will certainly exercise this right if the option is in the money at the September option expiration. Because Porto wants to reduce the overweighting in energy stocks, this outcome is desirable. If the option is exercised, she has effectively sold the stock at 16.64. She gets 1.64 when she writes the option, and she gets 15 when the option is exercised. She could have simply sold the shares at their original price of 15.84, but in this specific situation the option strategy gave her an additional 800, or 5%, in a month's time.<sup>19</sup>

The option premium is composed of two parts: exercise value (also called intrinsic value) and time value.<sup>20</sup> Exercise value is the price advantage the option gives the buyer. In this case, the right to buy at 15 when the stock price is 15.84 is clearly worth 0.84. The option premium is 1.64, which is 0.80 more than the exercise value. This difference of 0.80 is called time value. Someone who writes covered calls to improve on the market is capturing the time value, which augments the stock selling price. Remember, though, that giving up part of the return distribution would result in an opportunity loss if the underlying goes up.

#### 4.1.3. Market Participant #3: Target Price Realization

A third popular use of options is really a hybrid of the first two objectives. This strategy involves writing calls with an exercise price near the target price for the stock. Suppose a bank trust department holds PBR in many of its accounts and that its research team believes the stock would be properly priced at 16/share, which is just slightly higher than its current price. In those accounts for which the investment policy statement permits option activity, the manager might choose to write near-term calls with an exercise price near the target price, 16 in this case. Suppose an account holds 500 shares of PBR. Writing 5 SEP 16 call contracts at 0.97 brings in 485 in cash. If the stock is above 16 in a month, the stock will be sold at its target price, with the option premium adding an additional 6% positive return to the account.<sup>21</sup>

<sup>19</sup>Her effective selling price of 16.64 is 0.80 higher than the original price of 15.84:  $0.80/15.84 = 5.05\%$ .

<sup>20</sup>Although the term "intrinsic value" is widely used among option practitioners, it is an unfortunate word choice. Those familiar with option pricing know that in the absence of arbitrage even an out-of-the-money option must have a certain positive value, but in practice that is not called intrinsic value. In addition to exercise value, some use the term "economic value" for intrinsic value because it is the value of the option if the investor were to exercise it at this very moment and trade out of the stock position.

<sup>21</sup>Relative to a stock price of 16, the option premium of 0.97 is  $0.97/16 = 6.06\%$ .

If PBR fails to rise to 16, the manager might write a new OCT expiration call with the same objective in mind.

Although popular, the investor should not view this strategy as a source of free money. The stock is currently very close to the target price, and the manager could simply sell it and be satisfied. Although the covered call writing program potentially adds to the return, there is also the chance that the stock could experience bad news or the overall market might pull back, resulting in an opportunity loss relative to the outright sale of the stock. The investor also would have an opportunity loss if the stock rises sharply above the exercise price and it was called away at a lower-than-market price.

#### 4.1.4. Profit and Loss at Expiration

In the process of learning option strategies, it is always helpful to look at a graphical display of the profit and loss possibilities at the option expiration. Suppose an investor owns PBR, currently trading at 15.84, and chooses to write the NOV 17 call at 1.44. If the stock is above 17 at expiration, the option holder will exercise the call option and the investor will deliver the shares in exchange for the exercise price of 17. The maximum gain with a covered call is the appreciation to the exercise price plus the option premium.<sup>22</sup>

Some symbols will be helpful in learning these relationships:

$S_0$  = Stock price when option position opened

$S_T$  = Stock price at option expiration

$X$  = Option exercise price

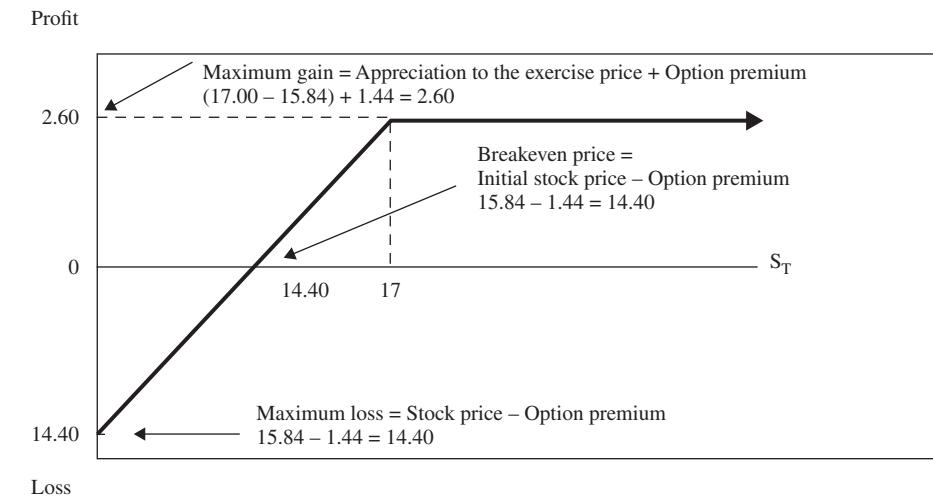
$c_0$  = Call premium received or paid

The maximum gain =  $(X - S_0) + c_0$ . With a starting price of 15.84, a sale price of 17 results in 1.16 of price appreciation. The option writer would keep the option premium of 1.44 for a total gain of  $1.16 + 1.44 = 2.60$ . This is the maximum gain from this strategy because all price appreciation above 17 belongs to the call holder. The call writer keeps the option premium regardless of what the stock does, so if it were to drop, the overall loss is reduced by the option premium received. Exhibit 9 shows the situation. The breakeven price for a covered call is the stock price minus the premium, or  $S_0 - c_0$ . In other words, the breakeven point occurs when the stock falls by the premium received; in this example,  $15.84 - 1.44 = 14.40$ . The maximum loss would occur if the stock became worthless; it equals the original stock price minus the option premium received, or  $S_0 - c_0$ .<sup>23</sup> In this single unlikely scenario, the investor would lose 15.84 on the stock position, but still keep the premium of 1.44 for a total loss of 14.40.

<sup>22</sup>If someone writes an in-the-money covered call, there is “depreciation” to the exercise price, so the difference would be subtracted. For instance, if the stock price is 50 and a 45 call sells for 7, the maximum gain is  $(45 - 50) + 7 = 2$ .

<sup>23</sup>Note that with a covered call, the breakeven price and the maximum loss are the same value.

EXHIBIT 9 Covered Call Profit and Loss Diagram: With stock at 15.84, write 17 call at 1.44



At option expiration, the *value* of the covered call position is the stock price minus the exercise value of the call. Any appreciation beyond the exercise price belongs to the option buyer, so the covered call writer does not earn any gains beyond that point. Symbolically,

$$\text{Covered call expiration value} = S_T - \text{Max}[(S_T - X), 0]$$

The *profit* at option expiration is the covered call value plus the option premium received minus the original price of the stock:

$$\text{Covered call profit at expiration} = S_T - \text{Max}[(S_T - X), 0] + c_0 - S_0$$

In summary:

$$\text{Maximum gain} = (X - S_0) + c_0$$

$$\text{Maximum loss} = S_0 - c_0$$

$$\text{Breakeven point} = S_0 - c_0$$

$$\text{Expiration value} = S_T - \text{Max}[(S_T - X), 0]$$

$$\text{Profit at expiration} = S_T - \text{Max}[(S_T - X), 0] + c_0 - S_0$$

It is important to remember that these profit and loss diagrams depict the situation only at the end of the option's life.<sup>24</sup> Most equity covered call writing occurs with exchange-traded options, so the call writer always has the ability to buy the option back before expiration. If, for instance, the PBR stock price were to decline by one shortly after writing the covered call, the call value would most likely also decline. If this investor believed the decline was temporary, he might buy the call back at the new lower option premium, making a profit on that trade, and then write the option again after the share price recovered.

<sup>24</sup>It is also important to note that the general shape of the profit and loss diagram for a covered call is the same as that of writing a put. Covered call writing is the most common use of options by individual investors, whereas writing puts is the least common.

### EXAMPLE 3 Characteristics of Covered Calls

$S_0$  = Stock price when option position opened = 25.00

$X$  = Option exercise price = 30.00

$S_T$  = Stock price at option expiration = 31.33

$c_0$  = Call premium received = 1.55

1. Which of the following correctly calculates the maximum profit from writing a covered call?
  - A.  $(S_T - X) + c_0 = 31.33 - 30.00 + 1.55 = 2.88$
  - B.  $(S_T - S_0) - c_0 = 31.33 - 25.00 - 1.55 = 4.78$
  - C.  $(X - S_0) + c_0 = 30.00 - 25.00 + 1.55 = 6.55$
2. Which of the following correctly calculates the breakeven stock price from writing a covered call?
  - A.  $S_0 - c_0 = 25.00 - 1.55 = 23.45$
  - B.  $S_T - c_0 = 31.33 - 1.55 = 29.78$
  - C.  $X + c_0 = 30.00 + 1.55 = 31.55$
3. Which of the following correctly calculates the maximum loss from writing a covered call?
  - A.  $S_0 - c_0 = 25.00 - 1.55 = 23.45$
  - B.  $S_T - c_0 = 31.33 - 1.55 = 29.78$
  - C.  $S_T - X + c_0 = 31.33 - 25.00 + 1.55 = 7.88$

*Solution to 1:* C is correct. The covered call writer participates in gains up to the exercise price, after which further appreciation is lost to the call buyer. That is,  $X - S_0 = 30.00 - 25.00 = 5.00$ . The call writer also keeps  $c_0$ , the option premium, which is 1.55. So, total maximum profit is  $5.00 + 1.55 = 6.55$ .

*Solution to 2:* A is correct. The call premium of 1.55 offsets a decline in the stock price by the amount of the premium received:  $25.00 - 1.55 = 23.45$ .

*Solution to 3:* A is correct. The stock price can fall to zero, causing a loss of the entire investment, but the option writer still gets to keep the option premium received:  $25.00 - 1.55 = 23.45$ .

## 4.2. Investment Objective of Protective Puts

The primary motivation for the purchase of a protective put is to protect against loss when the asset falls in value. A protective put is similar to buying insurance, as we shall see in the next section.

### 4.2.1. Protecting Profits

Suppose a portfolio manager has a client with a 50,000 share position in PBR. His research suggests there may be a negative shock to the stock price in the next four to six weeks, and he wants to guard against a price decline. Buying a put while owning the stock is analogous to buying insurance. See Exhibit 10.

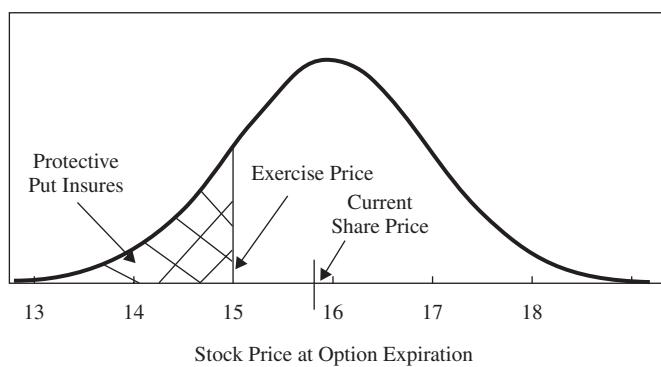
## EXHIBIT 10 Protective Put and Insurance Analogies

Insurance Policy	Put Option
Premium	Time value
Value of asset	Price of stock
Face value	Exercise price
Term of policy	Time until option expiration
Likelihood of loss	Volatility of stock

As with insurance policies, a put implies a deductible, which is the amount of the loss the insured is willing to bear. In this analogy, the stock price minus the exercise price is the deductible.<sup>25</sup>

With PBR stock at 15.84, Exhibit 7 indicates the portfolio manager could buy a one-month (SEP) 15-strike put for 0.65. This option is out of the money, so the premium is entirely time value. In this example, the deductible is  $15.84 - 15.00 = 0.84$ . In other words, the stock can fall by 0.84 and the stock holder suffers this loss, but once the stock falls to 15.00 the put becomes valuable and offsets further losses on the stock. See Exhibit 11, which shows how the put cuts off the lower part of the return distribution. The protective put insures against the portion of the underlying return distribution that is below 15. Alternatively, the portfolio manager could buy a two-month option, paying 0.99 for an OCT 15 put, or he could buy a three-month option, paying 1.46 for a NOV 15 put. Note that there is not a linear relationship between the put value and its time until expiration. A two-month option does not sell for twice the price of a one-month option, nor does a three-month option sell for three times the price of a one-month option. The portfolio manager can also reduce the cost of insurance by increasing the size of the deductible, perhaps by using a put option with a 14 exercise price. A put option with an exercise price of 14 would have a lower premium, but would not protect against losses in the stock until it falls to 14.00 per share. The option price is cheaper, but on a 50,000 share position, the deductible would be 50,000 more than if the exercise price of 15 were selected.

## EXHIBIT 11 Protective Puts and the Return Distribution



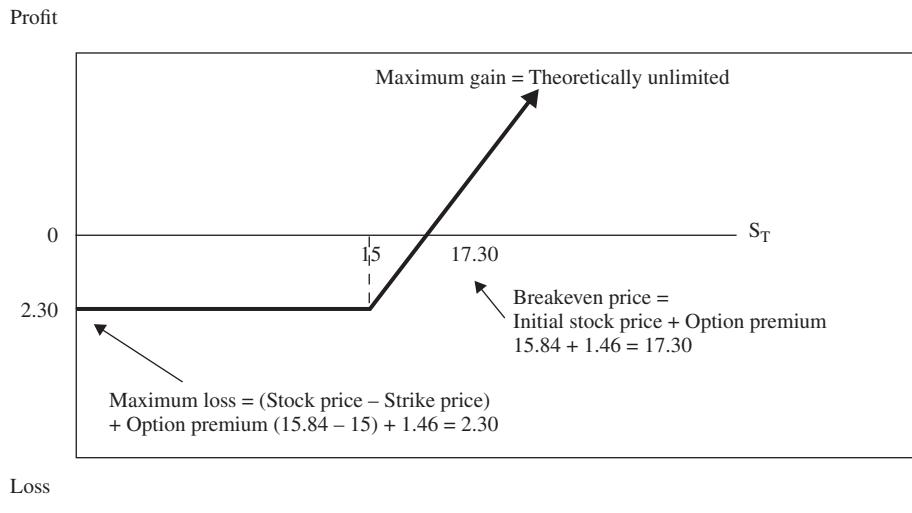
<sup>25</sup>Unlike typical insurance policies, however, it is possible to use a put to insure an asset for more than its current value. One simply sets the exercise price higher than the current stock price. Such a put will have a higher price.

Because of the uncertainty about the timing of the “shock event” he anticipates, the manager might decide to buy the NOV 15 put. Here is why. If he were to buy the cheaper SEP put, there is a good chance this option would expire before the anticipated stock price shock occurred. Given the four- to six-week time horizon, the OCT put would be appropriate, but there is still the potential to lose the premium without realizing any benefit. With a 0.99 premium for the OCT 15 put and 50,000 shares to protect, the cost to the account would be almost 50,000. The advantage of the NOV option is that although it is more expensive, it has the greater likelihood of not having expired before the news hits. As the stock drops, the value of the put would increase. Although he could hold onto the put position until its expiration, he might find it preferable to close out the option prior to maturity and recover some of the premium paid.<sup>26</sup>

#### 4.2.2. Profit and Loss at Expiration

Exhibit 12 shows the profit and loss diagram for the protective put.<sup>27</sup> The stock can rise to any level, and the position would benefit fully from the appreciation; the maximum gain is unlimited. On the downside, losses are “cut off” once the stock price falls to the exercise price. With a protective put, the maximum loss is the depreciation to the exercise price plus the premium paid, or  $S_0 - X + p_0$ . At the option expiration, the value of the protective put is the greater of the stock price or the exercise price. The reason is because the stock can rise to any level but has a floor value of the put exercise price. In symbols, the value of the combination of put and stock at expiration =  $\text{Max}(S_T, X)$ . The profit or loss at expiration is the ending value minus the beginning value. The initial value of the protective put is the starting stock price plus the put premium. In symbols, Profit of protective put at expiration =  $\text{Max}(S_T, X) - S_0 - p_0$ .

EXHIBIT 12 Protective Put Profit and Loss Diagram: With stock at 15.84, buy 15 put at 1.46



<sup>26</sup>A price shock to the underlying asset might increase the market’s expectations of future volatility, thereby likely increasing the put premium. By selling the option early, the investor would capture this increase. Also, once the adverse event occurred, there may not be a reason to continue to hold the insurance policy. If the investor no longer needs it, he should cancel it and get part of the purchase price back. In other words, he should sell the put and recapture some of its cost.

<sup>27</sup>Note that the profit and loss diagram for a protective put has a shape similar to a long call position, which is the result of put–call parity. Long the asset and long the put is equivalent to long a call plus long a risk-free bond. The bond has no impact on the shape of the profit and loss diagram.

In order to break even, the underlying asset must rise by enough to offset the price of the put that was purchased. The breakeven point is the initial stock price plus the option premium. In symbols, Breakeven point =  $S_0 + p_0$ .

In summary:

$$\text{Maximum profit} = S_T - S_0 - p_0 = \text{Unlimited}$$

$$\text{Maximum loss} = S_0 - X + p_0$$

$$\text{Breakeven point} = S_0 + p_0$$

$$\text{Expiration value} = \text{Max}(S_T, X)$$

$$\text{Profit at expiration} = \text{Max}(S_T, X) - S_0 - p_0$$

#### EXAMPLE 4 Characteristics of Protective Puts

$$S_0 = \text{Stock price when option position opened} = 25.00$$

$$X = \text{Option exercise price} = 20.00$$

$$S_T = \text{Stock price at option expiration} = 31.33$$

$$p_0 = \text{Put premium paid} = 1.15$$

1. Which of the following correctly calculates the profit with the protective put?
  - $S_T - S_0 - p_0 = 31.33 - 25.00 - 1.15 = 5.18$
  - $S_T - S_0 + p_0 = 31.33 - 25.00 + 1.15 = 7.48$
  - $S_T - X - p_0 = 31.33 - 20.00 - 1.15 = 10.18$
2. Which of the following correctly calculates the breakeven stock price with the protective put?
  - $S_0 - p_0 = 25.00 - 1.15 = 23.85$
  - $S_0 + p_0 = 25.00 + 1.15 = 26.15$
  - $S_T + p_0 = 31.33 - 1.15 = 30.18$
3. Which of the following correctly calculates the maximum loss with the protective put?
  - $S_0 - X + p_0 = 25.00 - 20.00 + 1.15 = 6.15$
  - $S_T - X - p_0 = 31.33 - 20.00 - 1.15 = 10.18$
  - $S_0 - p_0 = 25.00 - 1.15 = 23.85$

*Solution to 1:* A is correct. If the stock price is above the put exercise price at expiration, the put will expire worthless. The profit is the gain on the stock ( $S_T - S_0$ ) minus the cost of the put. Note that the maximum profit with a protective put is theoretically unlimited because the stock can rise to any level, and the entire profit is earned by the stockholder.

*Solution to 2:* B is correct. Because the option buyer pays the put premium, he does not begin to make money until the stock rises by enough to recover the premium paid.

*Solution to 3:* A is correct. Once the stock falls to the put exercise price, further losses are eliminated. The investor paid the option premium, so the total loss is the “deductible” plus the cost of the insurance.

### 4.3. Equivalence to Long Asset/Short Forward Position

All investors who consider option strategies should understand that some options are more sensitive to changes in the underlying asset than others. This relationship is measured by **delta**, an indispensable tool to a sophisticated options user. Delta measures how the option price changes as the underlying asset price changes.<sup>28</sup> Because a call increases in value and a put decreases in value as the underlying asset increases in price, call deltas range from 0 to 1 and put deltas range from 0 to -1. A long position in the underlying asset has a delta of 1.0, whereas a short position has a delta of -1.0. A rough rule of thumb is that an at-the-money option will have a delta that is approximately 0.5 (for a call) or -0.5 (for a put).

Suppose on the Tokyo Stock Exchange, Honda Motor Company stock sells for ¥3,500. A portfolio contains 100 shares, and the manager writes one exchange-traded covered call contract with a ¥3,500 strike. Because the call is at the money, meaning that the stock price and exercise price are equal, it will have a delta of about 0.5. The portfolio, however, is not long the call. The manager wrote it, and someone else owns it. From the portfolio's perspective, the delta is -0.5. A short call *loses* money as the underlying price rises. So, this covered call has an overall or **position delta** of 50: 100 points for the stock and -50 for the short call. Compare this call with a protective put in which someone buys 100 shares of stock and one contract of an at-the-money put. Its position delta would also be 50: 100 points for the stock and -50 points for the long put.

Finally, consider a long stock position of 100 shares and a short forward position of 50 shares. Because futures and forwards are essentially proxies for the stock, their deltas are also 1.0 for a long position and -1.0 for a short position. In this example, the short forward position "cancels" half the long stock position, so the position delta is also 50. These examples show three different positions: an at-the-money covered call, an at-the-money protective put, and a long stock/short forward position that all have the same delta. For small movements in the price of the underlying asset, these positions will show very similar gains and losses.

### 4.4. Writing Cash-Secured Puts

If someone writes a put option and simultaneously deposits an amount of money equal to the exercise price into a designated account, it is called writing a **cash-secured put**.<sup>29</sup> This strategy is appropriate for someone who is bullish on a stock or who wants to acquire shares at a particular price. The fact that the option exercise price is escrowed provides assurance that the put writer will be able to purchase the stock if the option holder chooses to exercise. Think of the cash in a cash-secured put as being similar to the stock part of a covered call. When an investor sells a call, she takes on the obligation to sell a stock, and this obligation is covered by ownership in the shares. When a put option is sold to create a new position, the obligation that accompanies this position is to purchase shares. In order to cover the obligation to purchase shares, the portfolio should have enough cash in the account to make good on this obligation. The short put position is covered or secured by cash in the account.

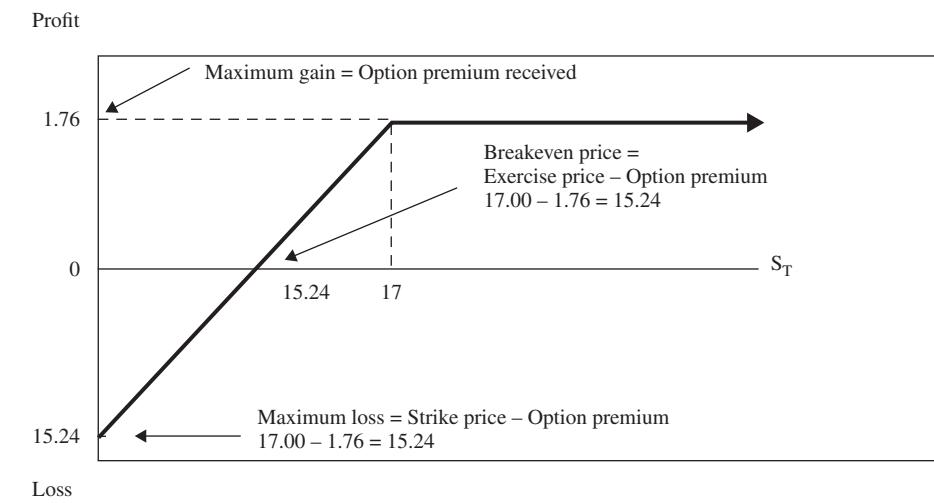
<sup>28</sup>Delta is the calculus first derivative of the option price with respect to the underlying asset price.

<sup>29</sup>This strategy is also called a *fiduciary put*. When someone writes a put but does not escrow the exercise price, it is sometimes called a *naked put*. Note that this is a slightly different use of the adjective "naked" than with a naked call. When writing a naked call, the call writer does not have the underlying *asset* to deliver if the call is exercised. When an investor writes a naked put, the investor has not set aside the *cash* necessary to buy the asset if the put is exercised.

Now consider two slightly different scenarios using the price data from Exhibit 7. One investor might be bullish on PBR and, with the stock at 15.84, believes it very likely that the stock will remain above 15 at the September option expiration. Suppose the investor writes the SEP 15 put for 0.65. As is always the case, the investor will keep the option premium regardless of what the stock price does. If, as expected, the stock is above 15 at expiration, the option holder will not want to sell at the lower price, so the put option will expire unexercised. If the stock is below 15 at expiration, the put would be exercised and the option writer would be obliged to purchase shares at the exercise price of 15. Netting out the option premium means that the effective purchase price would be  $15 - 0.65 = 14.35$ .

In another scenario, an institutional investor might be interested in purchasing PBR. Suppose the investor wrote the SEP 17 put for 1.76. If the stock is below 17 at expiration, the puts will be exercised and the investor would pay 17 for the shares, resulting in a net price of  $17 - 1.76 = 15.24$ . Anytime someone writes an option, the maximum gain is the option premium received, so in this case, the maximum gain is 1.76. The maximum loss when writing a put occurs when the stock falls to zero. The option writer pays the exercise price for worthless stock, but still keeps the premium. In this example, the maximum loss would be  $17 - 1.76 = 15.24$ . Exhibit 13 shows the corresponding profit and loss diagram.

EXHIBIT 13 Short Put Profit and Loss Diagram: Write SEP 17 put at 1.76



Note the similar shape of the covered call position in Exhibit 9 and the short put in Exhibit 13. Writing a covered call and writing a put are very similar with regard to their risk and reward characteristics.<sup>30</sup>

#### 4.5. The Risk of Covered Calls and Protective Puts

Some market observers believe that any derivative activity is inherently risky and inappropriate in accounts with conservative objectives. It is easy to show that this belief is not true.

<sup>30</sup>The two strategies are very similar because of put–call parity.

#### 4.5.1. Covered Calls

Consider the individual who owns 100 shares of a stock, with a position delta of 100. Suppose the investor now writes calls against this entire position, and suppose that each of these options has a delta of 0.4. This covered call position has a position delta of  $(100 \times 1.0) - (100 \times 0.4) = 60$ . A position delta of 60 is equivalent to owning 60 shares of the underlying asset. An investor can lose more money on a 100-share position than on a 60-share position. Even if the stock declines to nearly zero, the loss is less with the covered call because the option writer always gets to keep the option premium. Viewed this way, the covered call position is less risky than the underlying asset held alone.

There is, however, an important aspect of risk with covered calls. This is the risk that the underlying asset moves substantially above the option exercise price, in which case there can be a significant opportunity cost. Remember Exhibit 8 and the implication the short call has on the return distribution. The call writer sells the potential for “big gains” to the option buyer and is likely to experience regret if the stock price advances sharply. When the share price is 100, having to sell at 75 is not pleasant.

#### 4.5.2. Protective Puts

Similar logic applies to the use of protective puts. An investor who buys a put is essentially buying insurance on the stock. The put provides protection from the left tail of the return distribution. Someone buys insurance to protect against a risk and should not feel bad if the risk event does not materialize and the policyholder does not get to use the insurance. Stated another way, a homeowner should be happy if the fire insurance on their house goes unused. Still, we do not want to buy insurance we do not need, especially if it is expensive. Continually purchasing puts to protect against a possible stock price decline is an expensive strategy that would wipe out most of the long-term gain on an otherwise good investment. If risk is defined as something that could negatively affect the ability to achieve long-term investment goals, many people would say that the continuous purchase of protective puts is a risky strategy. The occasional purchase of a protective put to deal with a temporary situation, however, can be a sensible risk-reducing activity.

### 4.6. Collars

A **collar** is an option position in which the investor is long shares of stock and then buys a put with an exercise price below the current stock price and writes a call with an exercise price above the current stock price.<sup>31</sup> Collars allow a shareholder to acquire downside protection through a protective put but reduce the cash outlay by writing a covered call. By carefully selecting the options, an investor can often offset most of the put premium paid by the call premium received. In Exhibit 7, for instance, the NOV 15 put costs 1.46 and the NOV 17 call is 1.44, very nearly the same. A collar written in the over-the-counter market can be easily structured to provide a precise offset of the put premium with the call premium.<sup>32</sup>

<sup>31</sup>A collar is also called a *fence* or a *hedge wrapper*. In a foreign exchange transaction, it might be called a *risk reversal*.

<sup>32</sup>Most collars are structured so that the call and put premiums completely offset each other. If the investor starts with the put at a specific exercise price, he then sells a call that has the same premium. There is one specific call with the same premium, and it has a particular exercise price, which is above the exercise price of the put. An algorithm can be used to search for the exercise price on the call that has the same premium as that of the put, which is then the call that the investor should sell. Most collars are done in the over-the-counter market because the exercise price on the call must be a specific one. Exchange-traded options have standardized exercise prices, whereas the exercise prices of over-the-counter options can be set at whatever the investor wants.

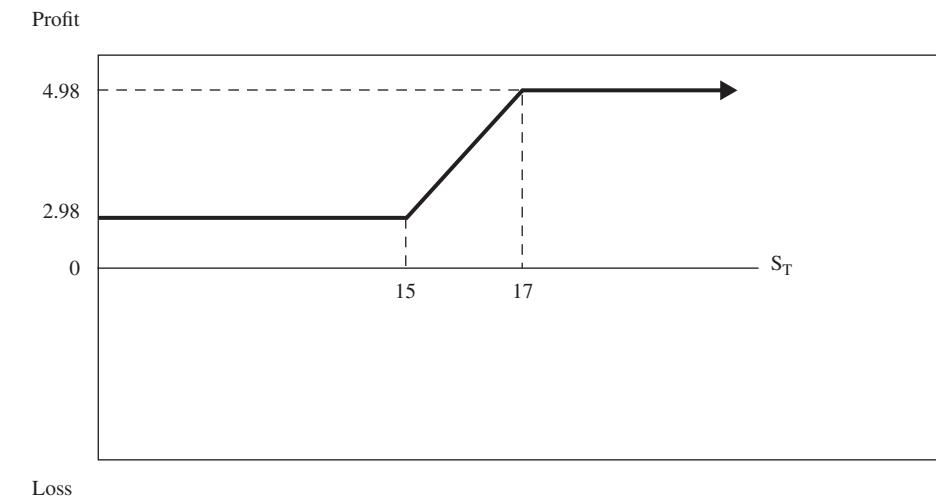
#### 4.6.1. Collars on an Existing Holding

Consider the risk–return trade-off for a shareholder who previously bought PBR stock at 12 and now buys the NOV 15 put for 1.46 and simultaneously writes the NOV 17 covered call for 1.44. Exhibit 14 shows a profit and loss worksheet for the three positions. Exhibit 15 shows the profit and loss diagram.

EXHIBIT 14 Collar Profit and Loss Worksheet: Stock purchased at 12, NOV 15 put purchased at 1.46, NOV 17 call written at 1.44

Stock price at expiration →	5	10	15	16	17	20
Profit/loss from long stock	−7.00	−2.00	3.00	4.00	5.00	8.00
Profit/loss from long 15 put	8.54	3.54	−1.46	−1.46	−1.46	−1.46
Profit/loss from short 17 call	1.44	1.44	1.44	1.44	1.44	−1.56
Total	2.98	2.98	2.98	3.98	4.98	4.98

EXHIBIT 15 Collar Profit and Loss Diagram: Stock purchased at 12, NOV 15 put purchased at 1.46, NOV 17 call written at 1.44



At or below the put exercise price of 15, the collar locks in a profit of 2.98. At or above the call exercise price of 17, the profit is constant at 4.98.

In this example, because the stock price had appreciated before establishing the collar, the position locks in a profit of at least 2.98. Investors typically establish a collar on a position that is already outstanding.

#### 4.6.2. Same-Strike Collar

What happens if an investor combines a same-strike collar with a long position in the underlying asset? Exhibit 16 shows that regardless of the stock price at option expiration, the combined position is worth the option exercise price. There is essentially no risk, and the position

is protected from a decline in market value. As previously shown, long a put and short a call is a synthetic short position. When a long position is combined with a synthetic short position, logically the risk is completely neutralized.

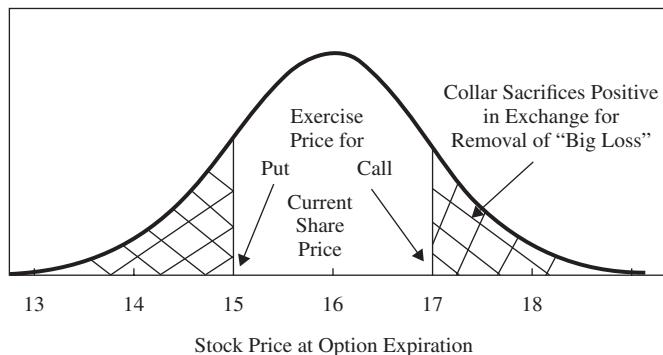
EXHIBIT 16 Long Position plus Same-Strike Collar

Stock price at expiration →	0	20	40	50	60	80	100
Long 50-strike put payoff	50	30	10	0	0	0	0
Short 50-strike call payoff	0	0	0	0	-10	-30	-50
Long stock	0	20	40	50	60	80	100
Total payoff	50	50	50	50	50	50	50

#### 4.6.3. The Risk of a Collar

We have already discussed the risks of covered calls and protective puts. The collar is essentially the simultaneous holding of both of these. See Exhibit 17 for the return distribution of a collar. A collar sacrifices the positive part of the return distribution in exchange for the removal of the adverse portion. With the short call option, the option writer sold the right side of the return distribution, which includes the most desirable outcomes. With the long put, the investor is protected against the left side of the distribution and the associated losses. The cost of the put is largely and often precisely offset by the income from writing the call. The collar dramatically narrows the distribution of possible investment outcomes, which is risk reducing. In exchange for the risk reduction, the return potential is limited.

EXHIBIT 17 Collars and the Return Distribution: With stock at 15.84, write 17 call and buy 15 put



## 5. SPREADS AND COMBINATIONS

Option spreads and combinations can be useful option strategies. In a typical **option spread**, the investor buys one call and writes another or buys one put and writes another.<sup>33</sup> Someone

<sup>33</sup>One important exception to the typical option spread is a *butterfly spread*, which is really two simultaneous spreads and can be done using only calls, only puts, or both puts and calls.

might, for instance, buy a NOV 16 call and simultaneously write a NOV 17 call, or one might buy a SEP 17 put and write a SEP 15 put. An **option combination** typically uses both puts and calls. The most important option combination is the straddle, which is the only combination we cover in this chapter. We will investigate spreads first.

## 5.1. Bull Spreads and Bear Spreads

Spreads are classified in two ways: by market sentiment and by the direction of the initial cash flows. A spread that becomes more valuable when the price of the underlying asset rises is a **bull spread**; a spread that becomes more valuable when the price declines is a **bear spread**. Because the investor buys one option and sells another, there is typically a net cash outflow or inflow. If establishing the spread requires a cash payment, it is referred to as a debit spread. Debit spreads are effectively long because the long option value exceeds the short option value. If the spread initially results in a cash inflow, it is referred to as a credit spread. Credit spreads are effectively short because the short option value exceeds the long option value. Any of these strategies can be created with puts or calls. The motivation for a spread is usually to place a directional bet, giving up part of the profit potential in exchange for a lower cost of the position. Some examples will help make this clear.

### 5.1.1. Bull Spread

Regardless of whether someone constructs a bull spread with puts or with calls, the strategy requires buying one option and writing another with a *higher* exercise price. Suppose, for instance, an investor thought it likely that by the September option expiration, PBR would rise to around 17 from its current level of 15.84. Based on the price data in Exhibit 7, what option strategy would capitalize on this anticipated price movement? If someone were to buy the SEP 15 call for 1.64 and the stock rose to 17, at expiration the call would be worth  $S_T - X = 17 - 15 = 2$ . If the price of the option was 1.64, the profit is 0.36. The maximum loss is the price paid for the option, or 1.64. If, instead, an investor bought the SEP 16 call for 0.97, at an expiration stock price of 17, the call would be worth 1.00 for a gain of 0.03.

A spread could make more sense with these option values. If someone believes the stock will not rise above 17 by September expiration, it may make sense to “sell off” the part of the return distribution above that price. He would receive 0.51 for each SEP 17 call sold. Consider two alternatives: 1) buy the SEP 15 call as the other “leg” of the spread, or 2) buy the SEP 16 call instead. Which is preferred? With Alternative 1, the SEP 15 call costs 1.64. Writing the SEP 17 call brings in 0.51, so the net cost is  $1.64 - 0.51 = 1.13$ . Traders would refer to this position as a PBR SEP 15/17 bull call spread. The maximum profit would occur at or above the exercise price of 17 because all gains above this level belong to the owner of the 17 call. At an underlying price of 17 or higher, from the trader’s perspective, the position is worth 2, which represents the appreciation from 15 to 17. Having paid 1.13 for the spread, the maximum profit is  $2.00 - 1.13 = 0.87$ . Another way to look at it is that at a price above 17, the trader exercises the long call, buying the stock at 15, and is forced to sell the stock at 17 to the holder of his short call.

With Alternative 2, the investor buys the SEP 16 call and pays 0.97 for it. Writing the SEP 17 call brings in 0.51, so the net cost would be  $0.97 - 0.51 = 0.46$ . At an underlying price of 17 or higher, the spread would be worth 1.00, so the maximum profit is  $1.00 - 0.46 = 0.54$ . Again, at 17 or higher, the trader exercises the call struck at 15, thereby buying the call, and has the other call exercised on him, thereby forcing him to sell the stock at 17. Exhibit 18 compares the profit and loss diagrams for these two alternatives.<sup>34</sup>

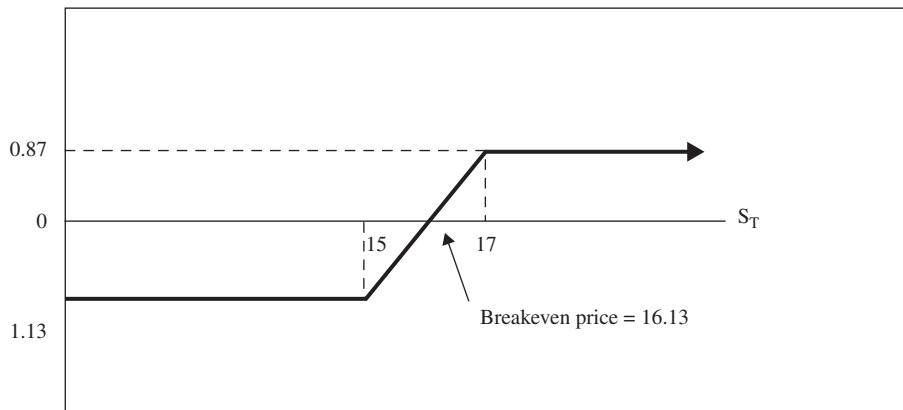
<sup>34</sup>Note that the shape of the profit and loss diagram is similar for a bull spread and for a collar.

To determine the breakeven price with a spread, find the underlying asset price that will cause the exercise value of the two options combined to equal the initial cost of the spread. A spread has two exercise prices, which we can denote as  $X_L$  for the lower exercise price and  $X_H$  for the higher exercise price. There are also two option premiums, which we can denote as  $c_L$  for the lower-strike call and  $c_H$  for the higher-strike call. Mathematically, the breakeven price for a call bull spread is  $X_L + (c_L - c_H)$ , which represents the lower exercise price plus the cost of the spread. In the examples here, Alternative 1 costs 1.13. If at option expiration the stock is 16.13, the 15-strike option would be worth 1.13 and the 17-strike call would be worthless. The breakeven price is  $15.00 + 1.13 = 16.13$  as Exhibit 18 shows.

**EXHIBIT 18 Bull Spreads: Current PBR stock price = 15.84**

**Alternative 1: Buy SEP 15 call at 1.64, write SEP 17 call at 0.51**

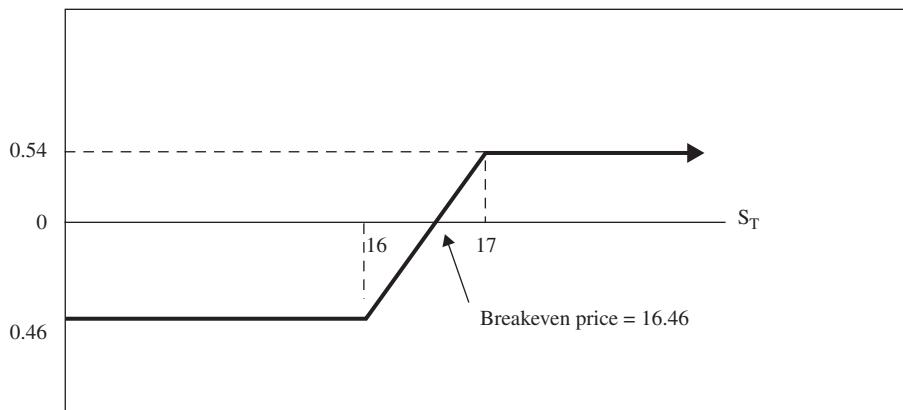
Profit



Loss

**Alternative 2: Buy SEP 16 call at 0.97, write SEP 17 call at 0.51**

Profit



Loss

Which of the alternatives is preferable? There is no clear-cut answer. The SEP 15/17 spread becomes profitable with a smaller price rise in PBR stock.<sup>35</sup> With Alternative 1, the breakeven point of 16.13 is less than 2% above the current level, whereas with Alternative 2, reaching the breakeven point requires almost a 4% rise in the stock price. By carefully selecting the expiration and exercise prices for the two legs of the spread, an investor can choose the risk–return mix that most closely matches his investment outlook.

### 5.1.2. Bear Spread

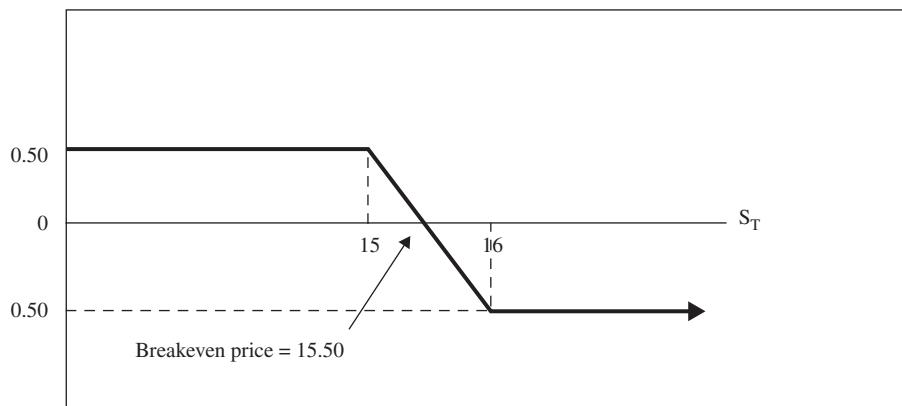
With a bull spread, the investor buys the lower exercise price and writes the higher exercise price. It is the opposite with a bear spread: buy the higher exercise price and sell the lower. If someone believed PBR stock would be below 15 by the November expiration, one strategy would be to buy the PBR NOV 16 put at 1.96 and write the NOV 15 put at 1.46. This spread has a net cost of 0.50; this amount is the maximum loss, and it occurs at a PBR stock price of 16 or higher. The maximum gain is also 0.50, which occurs at a stock price of 15 or lower. Finding the breakeven point uses the same logic as with a bull spread: find the underlying asset price at which the exercise value equals the initial cost. Let  $p_L$  represent the lower-strike put and  $p_H$  the higher-strike put.

Mathematically, the breakeven point is  $X_H - (p_H - p_L)$ . In this example,  $16 - (1.96 - 1.46) = 15.50$ . That is, at a stock price of 15.50, the 16-strike put would be worth 0.50 and the 15-strike put would be worthless. Exhibit 19 shows the profit and loss diagram for a NOV 15/16 bear spread.<sup>36</sup>

EXHIBIT 19 Bear Spread: Current PBR stock price = 15.84

**Buy NOV 16 put at 1.96, write Nov 15 put at 1.46**

Profit



Loss

<sup>35</sup>With a bull spread, this notation implies that the investor buys the 15 call and writes the 17 call.

<sup>36</sup>Bull spreads can also be done with puts, and bear spreads can also be done with calls. If this is the case, the result is a credit spread with an initial cash inflow. Recall that American exercise-style options may be exercised at any time prior to expiration. Bull spreads with American puts have an additional risk, which is that the short put gets exercised early, whereas the long put is not yet in the money. If the bull spread uses American calls and the short call is exercised, the long call is deeper in the money, which offsets that risk. A similar point can be applied to bear spreads using calls. Thus, with American options, bull spreads with calls and bear spreads with puts are generally preferred but of course, not required.

### 5.1.3. Refining Spreads

It is not necessary that both legs of a spread be established at the same time or maintained for the same period of time. Options are very versatile, and positions can be quickly adjusted as market conditions change. Here are a few examples of different tactical adjustments an option trader might consider.

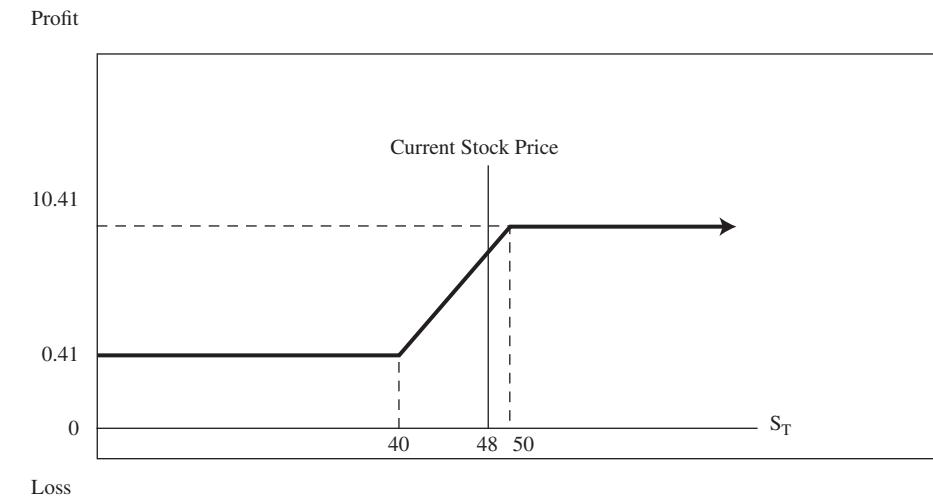
*5.1.3.1. Adding a Short Leg to a Long Position* Consider Carlos Aguila, a speculator who in September paid a premium of 1.50 for a NOV 40 call when the underlying stock was selling for 37. A month later, in October, the stock has risen to 48. He observes the following premiums for one-month call options.

Strike	Premium
40	8.30
45	4.42
50	1.91

This position has become very profitable. The call he bought is now worth 8.30. He paid 1.50, so his profit at this point is  $8.30 - 1.50 = 6.80$ . He thinks the stock is likely to stabilize around its new level and doubts that it will go much higher. Aguila is considering writing another call option with an exercise price of either 45 or 50, thereby converting his long call position into a bull spread. Looking first at the NOV 50 call, he notes that the 1.91 premium would more than cover the initial cost of the NOV 40 call. If he were to write this call, the new profit and loss diagram would look like Exhibit 20. To review, consider the following points:

- At stock prices of 50 or higher, the exercise value of the spread is 10.00. The reason is because both options would be in the money, and a call with an exercise price of 40 would always be worth 10 more than a call with an exercise price of 50. The initial cost of the call with an exercise price of 40 was 1.50, and there was a 1.91 cash inflow after writing the call with an exercise price of 50. The profit is  $10.00 - 1.50 + 1.91 = 10.41$ .
- At stock prices of 40 or lower, the exercise value of the spread is zero; both options would be out of the money. The initial cost of the call with an exercise price of 40 was 1.50, and there was a 1.91 cash inflow after writing the call with an exercise price of 50. The profit is  $0 - 1.50 + 1.91 = 0.41$ .
- Between the two striking prices (40 and 50), the exercise value of the spread rises steadily as the stock price increases. For every unit increase up to the higher striking price, the exercise value of this spread increases by 1.0. For instance, if the stock price remains unchanged at 48, the exercise value of the spread is 8.00. The reason is because the call with an exercise price of 40 would be worth 8.00 and the call with an exercise price of 50 would be worthless. The initial cost of the 40-strike call was 1.50, and there was a 1.91 cash inflow when the 50-strike call was written. The profit is  $8.00 - 1.50 + 1.91 = 8.41$ .

EXHIBIT 20 Spread Creation: Buy a call with an exercise price of 40 at 1.50; write a call later with an exercise price of 50 at 1.91



Now that he has written the NOV 50 call, Aguila needs to be careful how he views this new situation. No matter what happens to the stock price between now and expiration, the position is profitable, relative to his purchase price of the calls with an exercise price of 40. If the stock were to fall by any amount from its current level, however, he would have an opportunity loss: His profit would get progressively smaller if the price trended back to 40. Aguila would be correct in saying that the bull spread “locks in a profit,” but it does not completely hedge against a decline in the value of his new strategy.

**5.1.3.2. Multiple Strikes** Let us expand the prior example by assuming that Aguila owned 10 of the NOV 40 calls. He may choose to write higher-strike calls against just part of this position, which would leave additional upside potential if the stock were to continue to rise. Another choice would be writing a mix of two or more options, such as five of the Nov 50 options and five of the NOV 45 options. This approach would bring in premium income of

$$\begin{array}{rcl} (5 \times 1.91) & + (5 \times 4.42) & = 31.65 \\ \text{(50-strike call)} & \text{(45-strike call)} & \end{array}$$

If the stock remains at 48 at option expiration, the NOV 50 call will expire worthless and the 45 call would be exercised. Aguila would receive a call notice indicating that he had been “assigned to sell” shares at 45 because of the call he wrote. Aguila owns 10 of the NOV 40 calls, so he could exercise 5 of these options and buy the shares he needs to deliver. Thus, for these five options, Aguila effectively buys the stock at 40 and sells at 45, making 5.00 per share. Remember, though, that he initially paid 1.50 for the 40 calls, so his profit is 3.50 per share. He makes  $5 \times 3.50 = 17.50$ . He still owns five of the NOV 40 calls, and these would each be worth their exercise value of the stock price minus the strike price:  $48.00 - 40.00 = 8.00$ . He paid 1.50 for them, so his gain is  $5 \times (8.00 - 1.50) = 32.50$ . He could sell these five contracts and close out his position, or exercise and sell the stock. His aggregate profit would be 81.65:

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Option premium received:	31.65
Gain on five 40-strike calls used to deal with call exercise:	17.50
Gain on five 40-strike calls owned at option expiration:	<u>32.50</u>
Total:	81.65

If Aguila had kept the 10 NOV 40 contracts, their exercise value at expiration would have been  $48.00 - 40.00 = 8.00$ . Subtracting his purchase price of 1.50, his profit on 10 calls would have been  $10 \times 6.50 = 65.00$ . In this instance, the spread was a better performer than the long call by itself.

*5.1.3.3. Spreads as a Volatility Play* A spread strategy may make sense with a volatile stock in a trending market. Suppose the market has been rising, and Lars Clive, an options speculator, expects this trend to continue. Hypothetical company ZKQ currently sells for 44. Suppose Clive buys a NOV 45 call for 5.25. Three days later the stock price has risen to 49. With this new price, a NOV 50 call sells for 5.74. Clive establishes a 45/50 bull call spread by writing the NOV 50 call. Now suppose five days pass and the stock price falls to 45. The new option values would be 5.41 for the NOV 45 call and 3.55 for the NOV 50 call. Clive closes out the Nov 50 leg of his spread by buying it back. He sold the call for 5.74, and bought it back for 3.55, so he makes  $5.74 - 3.55 = 2.19$ , or 2.19 per contract. He still holds the long position in the NOV 45 call. Another four days pass, and ZKQ has risen to 48. The new price for the NOV 50 call would be 4.71; Clive then decides to write a call at this price. At this point, he has had two cash outflows totaling 8.80: the initial 5.25 plus the 3.55 to buy the NOV 50 call back. He has two inflows totaling 10.45: the premium income of 5.74 and then 4.71 from the two instances of writing the NOV 50 calls. Because the inflows of 10.45 exceed the outflows of 8.80, he has a resulting profit and loss diagram with a shape similar to the plot in Exhibit 20 that we saw in the previous example. The entire plot lies in profitable territory.

Time	Activity	Cash Out	Cash In
Day 1	Buy NOV 45 call	5.25	
Day 4	Sell NOV 50 call		5.74
Day 9	Buy NOV 50 call	3.55	
Day 13	Sell NOV 50 call		4.71
	Total	8.80	10.45
		Net Inflow	1.65

Spreads are primarily a directional play on the underlying spot price; still, spread traders can take advantage of changes in the level of volatility, and it is easy to create a hypothetical example like this one. There obviously is no guarantee that prices will continue to be volatile, or that any assumed price trend will continue. Still, the experienced option user knows to look for opportunistic plays that arise from price swings. Spreads are a relatively low-risk way to do so.

### EXAMPLE 5 Spreads

$S_0 = 44.50$   
 OCT 45 call = 2.55 OCT 45 put = 2.92  
 OCT 50 call = 1.45 OCT 50 put = 6.80

1. What is the maximum gain with an OCT 45/50 bull call spread?  
 A. 1.10  
 B. 3.05  
 C. 3.90
2. What is the maximum loss with an OCT 45/50 bear put spread?  
 A. 1.12  
 B. 3.88  
 C. 4.38
3. What is the breakeven point with an OCT 45/50 bull call spread?  
 A. 46.10  
 B. 47.50  
 C. 48.88

*Solution to 1:* C is correct. With a bull spread, the maximum gain occurs at the high exercise price. At an underlying price of 50 or higher, the spread is worth the difference in the strikes, or  $50 - 45 = 5$ . The cost of establishing the spread is the price of the lower-strike option minus the price of the higher-strike option:  $2.55 - 1.45 = 1.10$ . The maximum gain is  $5.00 - 1.10 = 3.90$ .

*Solution to 2:* B is correct. With a bear spread, you buy the higher exercise price and write the lower exercise price. When this strategy is done with puts, the higher exercise price option costs more than the lower exercise price option. Thus, you have a debit spread with an initial cash outlay, which is the most you can lose. The initial cash outlay is the cost of the OCT 50 put minus the premium received from writing the OCT 45 put:  $6.80 - 2.92 = 3.88$ .

*Solution to 3:* A is correct. You buy the OCT 45 call for 2.55 and sell the OCT 50 call for 1.45, for a net cost of 1.10. You breakeven when the position is worth the price you paid. The long call is worth 1.10 at a stock price of 46.10, and the OCT 50 call would expire out of the money and thus be worthless. The breakeven point is the lower exercise price of 45 plus the 1.10 cost of the spread, or 46.10.

#### 5.1.4. The Risk of Spreads

Note that the shape of the profit and loss diagram for the bull spread in Exhibit 18 is similar to that of the collar in Exhibit 15. The upside return potential is limited, but so is the maximum loss. Just like the risk-return trade-off with the collar, an option spread takes the tails of the distribution out of play and leaves only price uncertainty between the option exercise prices. Looking at this another way, if someone were to simply buy a long call, the maximum gain

would be unlimited and the maximum loss would be the option premium paid. If someone decides to convert this to a spread, it limits the maximum gain while simultaneously reducing the cost.

## 5.2. Calendar Spread

A strategy in which someone sells a near-dated call and buys a longer-dated one on the same underlying asset and with the same strike is commonly referred to as a **calendar spread**. When the investor buys the more distant option, it is a long calendar spread. The investor could also buy a near-term option and sell a longer-dated one, which would be a short calendar spread. Calendar spreads can also be done with puts; the investor would still buy one put and sell another put with a different expiration. As discussed previously, a portion of the option premium is time value. Time value decays over time and approaches zero as the option expiration date approaches. Taking advantage of this time decay is a primary motivation behind a calendar spread. Time decay is more pronounced for a short-term option than for one with a long time until expiration. A calendar spread trade seeks to exploit this characteristic by purchasing a longer-term option and writing a shorter-term option.

Here is an example of how someone might use such a spread. Suppose XYZ stock is trading at 45 a share in August. XYZ has a new product that is to be introduced to the public early the following year. A trader believes this new product introduction is going to have a positive impact on the shares. Until the excitement associated with this announcement starts to affect the stock price, the trader believes that the stock will languish around the current level. See the option prices in Exhibit 21. Based on the bullish outlook for the stock going into January, the trader purchases the XYZ JAN 45 call at 3.81. Noting that the near-term price forecast is neutral, the trader also decides to sell a XYZ SEP 45 call for 1.55.

### EXHIBIT 21 Calendar Spread Call Option Prices

*150 days until January option expiration  
Underlying stock price = 45*

Exercise Price	SEP	OCT	JAN
40	5.15	5.47	6.63
45	1.55	2.19	3.81
50	0.22	0.62	1.99

*Just before September option expiration  
Underlying stock price = 45*

Exercise Price	SEP	OCT	JAN
40	5.00	5.15	6.39
45	0.00	1.55	3.48
50	0.00	0.22	1.69

Now move forward to the September expiration and assume that XYZ is trading at 45. The September option will now expire with no value, which is a good outcome for the calendar spread trader. If the trader still believes that XYZ will stay around 45 into October before starting to move higher, the trader may continue to execute this strategy. An XYZ OCT 45 call might be sold for 1.55 with the hope that it also expires with no value.

In this example, the calendar spread trader has a directional opinion on the stock but does not believe that the price movement is imminent. Rather, the trader sees an opportunity to capture time value in one or more shorter-lived options that are expected to expire worthless.

### 5.3. Straddle

A long **straddle** is an option combination in which someone buys *both* puts and calls, with the same exercise price, on the same underlying asset.<sup>37</sup> If someone *writes* both options, it is a short straddle. Because a long call is bullish and a long put is bearish, this strategy may seem illogical. There are occasions, however, when a straddle might make sense. The classic example is in anticipation of some event in which the outcome is uncertain but likely to significantly affect the price of the underlying asset regardless of how the event gets resolved. From the shareholders' perspective for an option on a stock, if the news is bad, the stock price falls. If the news is good, it rises.

A straddle is an example of a directional play on the underlying volatility, expressing the view that volatility will either increase or decrease from its current level. A profitable outcome from a long straddle, however, usually requires a significant price movement in the underlying asset. The straddle buyer pays the premium for two options, so to make a profit, the underlying asset has to move either above or below the option exercise price by the total amount spent on the straddle. As an example, suppose in the next few days there is a verdict expected in a liability lawsuit against an automobile manufacturer. An investor expects the stock to move sharply one way or the other once the verdict is revealed. Because a straddle is neither a bullish nor a bearish strategy, the chosen options usually have an exercise price close to the current stock price. With any other exercise price, there is a directional bias because initially one of the options will be in the money and one will be out of the money.

Experienced option traders know that it is difficult to make money with a straddle. In the example, other people will also be watching the court proceedings. The collective wisdom will predict higher volatility once the verdict is announced, and option prices rise when volatility expectations rise. This increased volatility means that both the puts and the calls become expensive well before the verdict is revealed, and the long straddle requires the purchase of both options. To make money, the straddle buyer has to be correct in his view that the "true" underlying volatility is higher than the market consensus. Essentially, the bet is that the straddle buyer is right and the other market participants, on average, are wrong about the volatility.

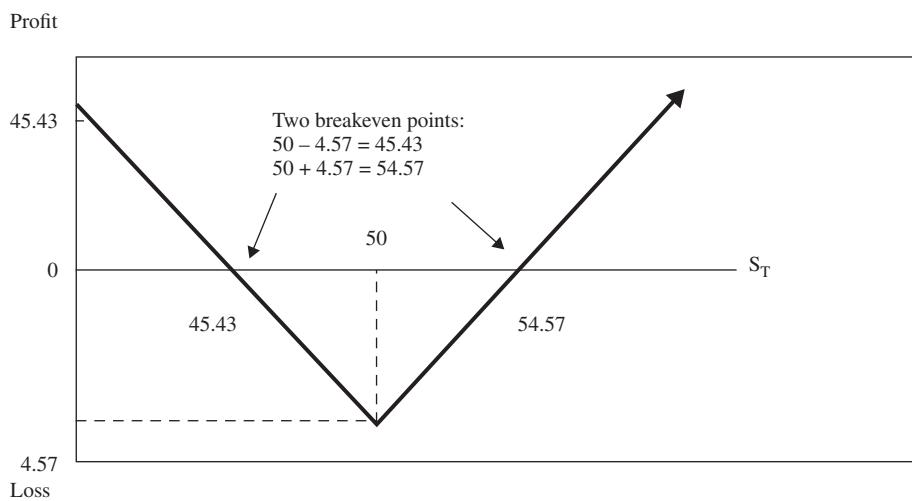
Suppose the underlying stock sells for 50, and an investor selects 30-day options with an exercise price of 50. The call sells for 2.29 and the put for 2.28, for a total investment of 4.57. To recover this cost, the underlying asset must either rise or fall by at least 4.57. At prices above 54.57 the call is in the money. At prices below 45.43 the put is in the money. See Exhibit 22. Theoretically, the stock can rise to any level, so the maximum profit with the long call is unlimited. If the stock declines, it can fall to no lower than zero. If that happens, the long put would be worth 50. Subtracting the 4.57 cost of the straddle gives a maximum profit of 45.43 from a stock drop.

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<sup>37</sup>If someone buys puts and calls with different exercise prices, the position is called a strangle.

For the straddle buyer, the worst outcome is if the stock closes exactly at 50, meaning both the put and the call would expire worthless. At any other price one of the options will have a positive exercise value. Note that at expiration, the straddle is not profitable if the stock price is in the range 45.43 to 54.57. It requires more than a 9% price move in one month to make money with this strategy. A speculator who believes such a move was unlikely might be inclined to *write* the straddle, in which case the profit and loss diagram in Exhibit 22 gets reversed, with a maximum gain of 4.57 and a theoretically unlimited loss if prices rise.

EXHIBIT 22 Long Straddle: Current stock price = 50; buy 50-strike call at 2.29, buy 50-strike put at 2.28



The risk of a long straddle is limited to the amount paid for the two option positions. At expiration, the only way both options expire worthless is if the stock price and exercise price are exactly equal; at any other final stock price, either the call or the put has a positive exercise value. Still, both options lose their time value, so it requires a significant price change in the underlying asset in order to move past one of the breakeven points.

#### 5.4. Consequences of Exercise

When someone writes an option, it is important to remember that it creates a contingent claim, but the writer is not the owner of the claim. The writer sells the claim and has been paid to take on an obligation.

The option writer has an obligation to perform if the option holder chooses to exercise and has no control over whether or not exercise occurs. The consequences of exercise can be significant. When someone owns an option, they decide when and if to exercise. Unexpected exercise can be quite inconvenient.

Suppose, for instance, an investor buys a NOV 95 call and writes a NOV 105 call to form a 95/105 bull spread; later, the stock price rises to 113. At this price the spread has reached its maximum value of 10. Two weeks before expiration the owner of the 105 call exercises. Unless the investor already own these shares, it will be necessary to purchase them in the open market

for 113. With a contract size of 100, it would cost  $100 \times 113 = 11,300$  to make the purchase, and this purchase might be very inconvenient. Alternatively, depending on the brokerage firm's policies, it might be able to exercise the 95 call to acquire the needed shares. Even this approach may be undesirable because it is rarely a good idea to exercise an option very far ahead of the expiration date. Doing so essentially amounts to throwing away the time value.

For instance, suppose that with the underlying stock at 113 the NOV 95 call the investor owns sells for 18.75. The option is in the money by 18, so that is its exercise value. If the investor exercised the option, he would buy shares at this discount. But the economic gain would be just the 18, whereas the option given up in exchange for the shares was worth 18.75. If the investor wanted the shares, he would be better off selling the option for 18.75 and then buying shares on the open market for 113. The net price would be  $113.00 - 18.75 = 94.25$ , or 0.75 less than the strike price. With capital constraints, though, the investor might not be able to do this. The key point in this discussion is that an investor should think through the consequences of exercise before implementing any strategy that requires writing options.

## 6. INVESTMENT OBJECTIVES AND STRATEGY SELECTION

The risk of a derivative product depends on what you do with it. It is not the fork's fault if someone sticks it in an electric outlet and gets shocked. People should think of derivatives as *neutral products* that can be combined with other assets to create a more preferred risk–return trade-off. If used wisely, they can help an investor or a portfolio manager quickly adapt to changing market conditions or client needs.

### 6.1. The Necessity of Setting an Objective

Every trade should begin with an opinion of the underlying market. Only then can an informed trading decision be made. With stocks and most assets, one thinks about the direction of the market: Is it going up or down? With options, it is not enough to think about only market *direction*; it is also necessary to think about the volatility. In other words, what matters to users of options is not just the direction the underlying is headed, but the volatility of the underlying. For example, in a simple call option purchase, just having the underlying go up is not sufficient to make money. The underlying must go up enough that the option reaches breakeven. The gain in the value of the stock must be sufficient that the call overcomes the loss of its time value. A long call must have some upside volatility, and a long put must have some downside volatility.

Suppose an investor is neutral on market direction. Again, he should consider the volatility. A straddle buyer is neither bullish nor bearish but expects a sharp increase in volatility. Someone who is neutral directionally but does not anticipate a sudden price change may want to write the straddle. Spreads tend to be middle-of-the-road strategies that fit best in neutral, flat markets. A spread trade is often appropriate when the outlook for an underlying market is either neutral or non-trending, meaning that it does not have a strong bullish or bearish outlook. Exhibit 23 shows one way of looking at the interplay of direction and volatility.

EXHIBIT 23 Direction and Volatility with Options

		Direction		
		Bearish	Neutral/No Bias	Bullish
Volatility	High	Buy puts	Buy straddle	Buy calls
	Average	Write calls and buy puts	Spreads	Buy calls and write puts
	Low	Write calls	Write straddle	Write puts

Some option traders refer to the concept of volatility as *speed*, with a high volatility market being a *fast market* and a low volatility market being a *slow market*. Market speed can, however, also refer to the rate at which a market moves, so investors should take care to be aware of the distinction in the uses of these characterizations.

## 6.2. Spectrum of Market Risk

Derivatives enable market participants to take a position that is extremely bearish, extremely bullish, or somewhere in between and to quickly and efficiently shift along this continuum as desired. Suppose a pension fund owns one million shares of HSBC Holdings. For some reason, the portfolio manager would like to temporarily reduce the position by 10%, meaning to reduce the market exposure and convert the equivalent funds to cash. As this chapter has already shown, there are a variety of ways in which this might be done. The portfolio manager could

1. sell 100,000 shares, which is 10% of the holding.
2. enter into a futures or forward contract to sell 100,000 shares.
3. write call contracts sufficient to generate minus 100,000 delta points.
4. buy put contracts sufficient to generate minus 100,000 delta points.
5. enter into a collar sufficient to generate minus 100,000 delta points.

Each of these alternatives has its own strengths and weaknesses. The first alternative (selling shares) has the advantage of clearly accomplishing the goal of a 10% reduction. For some investors, though, this solution could create a tax problem or result in inadvertently putting downward pressure on the stock price. Forward contracts are simple and effective, but they involve counterparty risk and are not easily canceled if later there is a desire to unwind the trade.<sup>38</sup> Writing calls brings in a cash premium, but leaves the writer subject to exercise risk. Buying puts requires a cash outlay but leaves the investor in control with respect to exercise risk. In short, all of the outcomes have advantages and disadvantages, but with derivatives, the investor has multiple choices instead of just one.

## 6.3. Analytics of the Breakeven Price

Investors often construct a profit and loss diagram for an option strategy that is under consideration. These diagrams are helpful in understanding the range of possible outcomes. It may

<sup>38</sup>It is possible to enter into an offsetting trade with the same counterparty, or a different counterparty, to net the position to zero. This approach is essentially the same as covering a futures contract position in which the counterparty is the clearinghouse. Offsetting a forward with a new forward is relatively easy to do.

also make sense to use option pricing theory to learn more about what the breakeven points on the profit and loss diagram really mean. The current option price is based on an outlook that takes into account an assumed future volatility of the prices of the underlying asset. Volatility is measured by standard deviation of percentage changes in the spot price of the underlying asset, and with an understanding of some basic statistical principles, this volatility can be used to estimate the likelihood of achieving a particular price target.

There are a variety of factors that come into play when determining the value of an option. The underlying market price, the exercise price of the option contract, the time left until option expiration, the current risk-free rate, and any dividends paid before expiration are all taken into account by the market when valuing an option. The exercise price is known, and the underlying market price and risk-free rate are easily accessible. The time remaining until expiration is consistently changing, but we always are aware of how much time is left until expiration. Dividends paid by companies are fairly stable as well.

There is another pricing factor that is taken into account, and that is the expected price movement or volatility of the underlying stock. The more volatility that is expected from the underlying over the life of an option, the higher the option premium. Consider two stocks, QRS and TUV, that are both trading at 50. QRS is a utility company that is expected to not have much volatility over the next month. TUV, however, is a biotech company that often experiences 5% price moves in a single day. All else being the same, options on QRS would have lower premiums than those on TUV. See the price data in Exhibit 24.

#### EXHIBIT 24 Volatility and Option Prices

*30 days until March option expiration  
QRS and TUV underlying stock price = 50*

QRS Mar 50 Call = 1.00  
QRS Mar 50 Put = 0.95  
TUV Mar 50 Call = 2.50  
TUV Mar 50 Put = 2.45

A trader buying a QRS Mar 50 call and QRS Mar 50 put would be purchasing a QRS Mar 50 straddle for 1.95. Buying a TUV Mar 50 straddle would involve purchasing the TUV Mar 50 call at 2.50 and TUV Mar 50 put at 2.45 for a net cost of 4.95. Breakeven at expiration for the QRS straddle occurs if the stock moves up or down 1.95 whereas breakeven for the TUV straddle would require a move of 4.95. In percentage terms, this means that break-even for the QRS straddle is 3.90% ( $1.95/50.00$ ) whereas TUV needs to rise or fall 9.90% ( $4.95/50.00$ ) for the straddle just to break even.

We say that TUV options have a higher **implied volatility** than comparable QRS option contracts. Implied volatility is the standard deviation that causes an option pricing model to give the current option price. Option users, in fact, use implied volatility as a form of option currency, meaning that prices are quoted as implied volatilities. For instance, knowing that an option sells for 2.00 reveals nothing about its relative price because that depends on the moneyness (i.e., the extent to which the option is in or out of the money) and the remaining life of the option. If, instead, someone says an option sells for an annualized implied volatility of 45%, that is a standalone statistic that can be directly compared with other options. TUV option premiums are higher than QRS option premiums because the market expects greater potential price moves out of TUV than from QRS.

There are a few different perspectives on how we should measure the time periods with option volatility. There are usually 365 calendar days in a year, but the markets are not open on Saturdays or Sundays or official holidays, which vary by country. Because the stock price does not have the opportunity to change when the market is closed, most experts believe that those days should not count. For this reason, some people use 252 or some other appropriate number for the number of trading days in a year. Dispersion is a function of both the *size* of the “jumps” in the variable and the *number* of those jumps. We convert an annual variance ( $\sigma^2$ ) to a daily variance by dividing by 252, and we convert an annual standard deviation ( $\sigma$ ) to a daily standard deviation by dividing by  $\sqrt{252}$ . With options, volatility is measured by the annual standard deviation.

Suppose the underlying stock in Exhibit 22 typically has an annual volatility of 30%. An investor can obtain some information on the likelihood of reaching the breakeven points before entering into the trade. In order for the straddle to be profitable at expiration, the stock must move up or down by 4.57 units from the current price of 50, which is a 9.14% movement. Expiration is in 30 days, but this includes four weekends and possibly a holiday. Suppose there are only 21 trading days until expiration. We convert a 9.14% movement in 21 days to an annual volatility by multiplying by the square root of the number of 21-day periods in a 252-day “year”:

$$\sigma_{\text{annual}} = 0.0914 \times \sqrt{\frac{252}{21}} = 32.6\%$$

The required price movement to the breakeven point represents an annual volatility that is only slightly greater than the historical level, so someone contemplating establishing the straddle might view this scenario favorably. If, instead, the straddle in Exhibit 22 costs 7 to establish, it would require a 14% move to reach a breakeven point. Using the formula just presented, this move is about 48.5% on an annual basis. You might not believe that such a price change could reasonably be expected in a 30-day period and thus elect not to enter into the strategy.<sup>39</sup>

### EXAMPLE 6 Straddle Analytics

XYZ stock = 100.00  
 100-strike call = 8.00  
 100-strike put = 7.50  
 Options are three months until expiration

1. If Smith buys a straddle on XYZ stock, he is *best* described as expecting a:
  - A. high volatility market.
  - B. low volatility market.
  - C. average volatility market.

<sup>39</sup>As described, volatility for equity markets is typically defined as a relative price change (lognormal volatility) whereas in the interest rate market, market practice changes more and more to an absolute definition of volatility, using absolute movement in basis points per annum.

2. This strategy will break even at expiration stock prices of:
  - A. 92.50 and 108.50.
  - B. 92.00 and 108.00.
  - C. 84.50 and 115.50.
3. Reaching a breakeven point implies an annualized rate of return *closest* to:
  - A. 16%.
  - B. 31%.
  - C. 62%.

*Solution to 1:* A is correct. A straddle is directionally neutral; it is neither bullish nor bearish. The straddle buyer wants volatility and wants it quickly, but does not care in which direction. The worst outcome is for the underlying asset to remain stable.

*Solution to 2:* C is correct. To break even, the stock price must move enough to recover the cost of both the put and the call. These premiums total \$15.50, so the stock must move up to \$115.50 or down to \$84.50.

*Solution to 3:* C is correct. The price change to a breakeven point is 15.50 points, or 15.5% on a 100 stock. This is for three months. This outcome is equivalent to an annualized rate of 62%, found by multiplying by 4 ( $15.5\% \times 4 = 62\%$ ).

## 6.4. Applications

This section illustrates by means of mini cases some of the ways in which different market participants use derivative products as a tool to solve a problem or to alter a risk exposure. Note that with the wide variety of derivatives available there are almost always multiple ways in which derivatives might logically be used in a particular situation. These mini cases cover only a few of them.

### 6.4.1. Writing Covered Calls

Carlos Rivera is a portfolio manager in a small asset management firm focusing on high-net-worth clients. In mid-April he is preparing for an upcoming meeting with Dr. Mary Parker, a client whose daughter is about to get married. Dr. Parker and her husband have just decided to pay for their daughter's honeymoon and need to raise \$30,000 relatively quickly. The client's portfolio is 100% invested in equities and, by policy, is aggressive. At the moment the Parkers are "asset rich and cash poor." They have largely depleted their cash reserves with the wedding expenses. The recently revised investment policy statement permits all option activity except the writing of naked calls. Over the next six months, Rivera's firm has a flat to slightly bearish market outlook. Dr. Parker's account contains 5,000 shares of Apple stock, a recent addition to the portfolio. Rivera is considering the following 30-day exchange-listed options, which expire in May.

Apple Options: Stock = \$99.72

Call	Exercise Price	Put
4.90	97	2.14
3.25	100	3.45
2.02	103	5.23

What strategy should Rivera recommend to Dr. Parker?

*Solution:* To generate income, Rivera will want to write options. The account permits the writing of covered, but not naked, calls and apparently also permits the writing of puts. Apple options trade on an organized exchange with a standard contract size of 100 options. With 5,000 shares in the account, 50 call contracts would be covered. If Rivera were to write either the MAY 100 or the MAY 103 calls, it would not generate the required income. Writing the MAY 97 calls would generate  $\$4.90 \times 100 \times 50 = \$24,500$ , but because this call is in the money, there is an increased risk of the option being in the money at expiration. Given the firm's market outlook, this risk might be acceptable. To make up the income shortfall of  $\$30,000 - \$24,500 = \$5,500$  Rivera could recommend writing a put.<sup>40</sup> To bring in the required premium, Rivera would have to write  $\$5,500/\$523 = 11$  contracts of the MAY 103 put, or  $\$5,500/\$345 = 16$  contracts of the MAY 100 put, or  $\$5,500/\$214 = 26$  contracts of the MAY 97 put.<sup>41</sup> The puts with exercise prices of 100 and 103 are already in the money, and given the market outlook of Rivera's firm, they are likely to be exercised, resulting in a new cash demand on the portfolio. If Rivera writes 26 contracts of the MAY 97 puts, it will bring in the needed income, and if Apple remains above 97, the puts will expire worthless. If Apple closes below \$97, the puts would be exercised and the account would require cash totaling \$252,200 to pay for the exercise of 26 put contracts for \$9,700/contract. The calls would have expired, so Rivera would be free to sell some of the Apple shares to raise cash if necessary. Rivera could recommend writing 50 contracts of the MAY 97 calls and 26 contracts of the MAY 97 puts. At the same time, it is important to note that writing puts is generally only appropriate for experienced, financially secure investors, and even for them, a financial adviser should review the adverse consequences of the puts being exercised.

**6.4.2.1. Portfolio Protection** Eliot Skaves manages a discretionary account with complete derivatives authority. The account holds 100,000 shares of Salar Limited, currently trading at HK\$42.00. Salar has an earnings announcement scheduled in one week. Although Skaves expects an earnings increase, he expects the company to narrowly miss the consensus earnings estimate. He would like to protect the client's position in the company until the report is released but wants to keep the cost of the protection to a minimum. There are no exchange-traded options on Salar. Skaves contacts a Hong Kong dealer and receives the following quotes for one-month options.

<sup>40</sup>If Rivera also writes puts, he has written a straddle. Note, though, that the short calls are not naked because Rivera also owns the underlying shares. This means that his maximum loss from an increase in the price of the underlying asset is known and limited. He does not face the potentially unlimited losses associated with writing a naked call.

<sup>41</sup>The number of contracts is rounded up in order to get the required income.

Salar Options: Stock = HK\$42.00

Call	Exercise Price	Put
3.05	40.00	1.04
1.69	42.50	2.19
0.84	45.00	3.83

What strategy should Skaves use?

*Solution:* Given the need to keep the cost of the protection to a minimum, Skaves might first investigate writing calls against the stock. The option premium received provides limited downside protection. For instance, if Skaves wrote the call with an exercise price of 40.00, the 3.05 premium would offset a price decline down to  $42.00 - 3.05 = 38.95$ . He would suffer the full loss of any decline below that. If he were to write the call with an exercise price of 42.50, he is protected down to  $42.50 - 1.69 = 40.81$ . The disadvantages of obtaining protection this way are twofold: First, the protection is limited, and second, if the underlying stock rises above the exercise price, there is an opportunity cost. In this example, if Skaves is wrong about his earnings estimate and the company beats expectations, the stock is likely to rise, in which case he foregoes any gains from the rise above the option exercise price.

An alternative to writing the calls would be to purchase a protective put, perhaps the put with an exercise price of 40.00, with the intent of selling it shortly after the earnings announcement. If the earnings are less than market expectations, the stock is likely to fall, thereby increasing the put value and partially offsetting the loss. If the earnings meet market expectations, the put value may be sold at a price near its purchase price. If the earnings are better than expected and the stock price rises, the put will decline in value. Skaves no longer would need the “insurance” and could sell the put, recovering part of the purchase price.

**6.4.2.2. Portfolio Protection: Adjustment** Assume that Salar’s earnings turned out to be surprisingly good; they beat the consensus estimate by 7 cents. Immediately after the announcement the stock rose 10% to HK\$46.20. Skaves believes this sharp jump in stock price is not justified by the new earnings level and expects the stock to give up about half this gain in the next few weeks. The new options prices are shown in the table. They reflect a jump in the anticipated volatility in the stock, which increases the value of both puts and calls.<sup>42</sup> Skaves notes that the put with an exercise price of 40.00 has declined from 1.04 to 0.83.

Salar Options Stock = HK\$46.20

Call	Exercise Price	Put
7.03	40.00	0.83
5.24	42.50	1.54
3.76	45.00	2.56
2.61	47.50	3.90

Now that the earnings announcement has been made, what should Skaves do?

<sup>42</sup>The original table is based on an annual volatility of 40%; the new volatility is 60%.

*Solution:* Skaves no longer needs the protection from the put option, so he should stick with his original strategy and sell the put. Because of the increase in volatility, the price of the put with an exercise price of 40.00 has fallen from 1.04 to 0.83. Skaves can recover 80% of the “insurance premium” he paid. Given that Skaves has a rather specific short-term belief about Salar stock, he might also consider writing a call with an exercise price of either 45.00 or 47.50, especially because the volatility has risen. When options are expensive, writing them can be attractive. He believes the stock is likely to retreat to the HK\$44.00 range soon. If he were to write the 47.50 call against all 100,000 shares, he would receive premium income of HK\$261,000. This option would expire worthless as long as Salar stock is below HK\$47.50 in 30 days. The call option with an exercise price of 45.00 would provide more income (HK\$3,760), but if the stock does not fall as much as Skaves expects, he might face the inconvenience of the option being assigned and shares called away.

#### 6.4.3. Collar/Equity Swap

Bernhard Steinbacher has a client with a 100,000 share holding in Targa, currently trading for €14. The client has a very low tax basis on this stock. Steinbacher wants to safeguard the value of the position but does not want to sell because of the substantial tax burden the sale would involve. He does not find exchange-traded options on the stock. He wants to present two different ways in which the client could protect an investment portfolio from a stock price decline.

*Solution 1:* In the over-the-counter market, he might buy a put and then write a call to offset the put premium. This strategy is a collar. The put provides downside protection below the put exercise price, and the call brings in income to help offset the cost of the put. Recalling Exhibit 17 and the underlying return distribution, this strategy effectively sells the right tail of the distribution, which represents large gains, in exchange for eliminating the left tail, which represents large losses.

*Solution 2:* Another possible solution is to enter into an equity swap trading the Targa return for Libor. The Targa shares are worth €1.4 million, so Steinbacher can agree to exchange the total return on the shares for the Libor return on this sum of money. He needs to decide the period of time for which the protection is needed and match the swap tenor to this. Perhaps he decides on six months, and six-month Libor is 0.34%, expressed as an annual rate.

*Scenario A:* Over the six months, Targa pays a €0.10 dividend and the share price rises 1%.

The total return on the stock is  $\frac{(14 \times 1.01) - 14 + 0.10}{14} = 1.71\%$ . For a six-month period, the Libor return would be half the annual rate, or 0.17%. The Targa stock return exceeds the Libor return, so Steinbacher would pay  $(1.71\% - 0.17\%) \times €1.4 \text{ million} = €21,560$ .

*Scenario B:* Over the six months Targa pays a €0.10 dividend and the share price falls 1%.

The total return on the stock is  $\frac{(14 \times 0.99) - 14 + 0.10}{14} = -0.29\%$ . The Targa stock return is less than the Libor return, so Steinbacher would “get paid” the negative return plus the Libor return and receive the difference  $(0.29\% + 0.17\%) \times €1.4 \text{ million} = €6,440$ .

#### 6.4.4. Writing Put Options

Oscar Quintera is the chief financial officer for Tres Jotas, a private investment firm in Puerto Rico. Quintera wants to enter a new equity position, but the shares are trading above the price Quintera wants to pay. The company wants to acquire 500,000 shares, so a few dollars difference in the purchase price makes a big dollar difference. The current share price is \$89.00, and Quintera is willing to buy the stock at a price of \$87.50 or less.

*Solution:* Quintera can write in-the-money puts to effectively “get paid” to buy the stock. Quintera sells puts and keeps the cash regardless of what happens in the future. If the stock is above the exercise price at expiration, the option will not be exercised. Otherwise, the option is exercised, Quintera buys, and as desired, becomes an owner of the stock. With the stock at \$89.00, a 30-day put option with an exercise price of 95.00 sells for 7.85. Quintera writes 5,000 contracts and receives premium income of  $100 \times 5,000 \times \$7.85 = \$3,925,000$ . The company keeps these funds regardless of future stock price movements. He is obligated to buy stock at \$95.00 if the put holder chooses to exercise.

*Scenario A:* The stock is \$92.00 per share on the option expiration day. With an exercise price of 95.00, the put is in the money and will be exercised. Quintera will be assigned to buy 500,000 shares at the exercise price of 95.00. The cost is  $500,000 \times \$95.00 = \$47,500,000$ . Quintera is satisfied with the outcome, though, because the firm keeps the premium income of \$3,925,000, so the net cost of purchase is  $\$47,500,000 - \$3,925,000 = \$43,575,000$ . On 500,000 shares, this means the effective purchase price is  $\$43,575,000/500,000 = \$87.15$ , which is below the \$87.50 price Quintera was willing to pay.

*Scenario B:* The stock price is \$97.00 on the option expiration day. With an exercise price of 95.00, the put is out of the money and would not be exercised. Quintera keeps the \$3,925,000 premium received from writing the option. This adds to the company’s profitability, but it did not acquire the shares and experienced an opportunity cost relative to an outright purchase of the stock at \$89.00.

#### 6.4.5. Long Straddle

Katrina Hamlet has been following McMillan Holdings for the past year. The company is involved in a potentially quite costly lawsuit, and she has been considering speculating with a straddle. The stock is currently trading for \$75.00, and she is focused on at-the-money calls and puts selling for 2.58 and 2.57, respectively. After the market closed today, she hears a news story indicating that a jury decision is expected later tomorrow. Hamlet expects that the stock will move at least 10% either way once the verdict is read, making the straddle strategy potentially appropriate. The following morning after the market opens, she goes to place her trade and finds that although the stock price remains at \$75.00, the option prices have adjusted to 6.00 for the call and 5.99 for the put. She wonders if these new option premiums have any implications for her intended strategy.

*Solution:* Hamlet is betting on a price movement in the underlying asset to make money with this trade. That price movement, up or down, must be large enough to recover the two premiums paid. In her early planning, that total was  $2.58 + 2.57 = 5.15$ . She expects at least a 10% price movement, which on a stock selling for \$75.00 would be an increase of \$7.50. This price movement would be enough to cover the 5.15 cost and make her strategy profitable.

The news report about the imminent verdict, however, increased the implied volatility in the options, raising their price and making it more difficult to achieve the breakeven points. The straddle now costs  $6.00 + 5.99 = 11.99$ . To reach the breakeven point, she now needs the stock to move by almost 16%. If she expects a 10% price movement, she is not likely to be happy with the outcome of this trade.

#### 6.4.6. Long Call

Olivier Akota believes a stock is going to move from £60 to £65 over the next 30 days. Akota checks the 30-day call options with an exercise price of 60 and believes they are overpriced at 4. He then looks at the 30-day call option with an exercise price of 65 and sees that he can sell it for 1.50.

*Solution:* Akota anticipates a sharp price increase in a short period of time, which is characteristic of a bullish, volatile market. The indicated strategy would be a long call position, but Akota believes the calls with an exercise price of 60 are overpriced. Also, at a premium of 4, if he is right and the stock advances to 65, his profit would be just 1 on an investment of 4. He might not like this risk–reward trade-off. If he were to buy the 60 call and also sell a call with an exercise price of 65 and form a call bull spread, his cost would be reduced by the 1.50 premium from writing the call, for a net cost of £2.50. Now, if the stock rises to £65 as he expects, he could earn a £2.50 profit on a £2.50 investment, doubling his money. In this case, a bull spread is reasonable, even though Akota believes the option he is buying is overpriced.

#### 6.4.7. Calendar Spread

Britta Olofsson thinks that XYZ stock, which is trading at SEK30, is going to remain in a narrow range for the next month until a new product is announced. She then thinks the stock will take off on a bullish run. She wants to have long exposure in case the stock moves early but does not want to pay the premium for a two-month call.

*Solution:* Olofsson could benefit from a calendar spread. She expects little price movement in the next 30 days. For example, by writing a 15-day, at-the-money call with a strike of 30 for 1, she is essentially selling time value that would reduce the cost of, perhaps, a 45-day, 30-strike call that might sell for 1.50. So, assume she sells the 15-day call and buys the 45-day call. Her net cost would be SEK0.50. If the stock does not begin to advance until after the call she wrote expires, she participates fully in the rising stock through her long call position. If the stock fails to rise, her maximum loss is the SEK0.50 she paid. If the stock rises before the short option expires, both options would increase in value, and the net wealth effect on Olofsson would be modest. She would be inconvenienced, though, because when the shorter-term option was exercised she would be assigned to deliver shares that she might not have. This situation would require a significant cash outlay.

#### 6.4.8. Currency Forward Contract

A Mexico-based firm anticipates receipt of a \$4 million payment in three months. The firm wants to reduce the associated foreign exchange risk.

*Solution:* The firm could enter into a three-month forward contract in which it agrees to a future delivery of \$4 million in exchange for pesos. Because the exchange rate is set today, the foreign exchange risk disappears.

#### 6.4.9. Interest Rate Swap

Les Poiriers is the chief financial officer for Bonshaw Bank, a Canadian commercial bank. Because of a very successful marketing program by one of the bank's competitors, her bank has found it necessary to react to the competitive pressures by issuing a substantial number of long-term, fixed-rate mortgage loans. Although the bank has always done some of these, the current demand has resulted in unacceptable interest rate risk and has caused frequent violations of the bank's asset/liability management policy. Bonshaw Bank currently has C\$400 million of these mortgages on its books, and Les Poiriers would like to reduce this amount by half.

*Solution:* Bonshaw needs to reduce its long-term, fixed-rate exposure to mortgage loans by C\$200 million. The mortgages the bank owns are a receive-fixed product and can be hedged with an offsetting pay-fixed swap. Such a swap would reduce the duration of the loans, and hence, reduce the interest rate risk. Specifically, Les Poiriers could enter into a C\$200 million notional value pay-fixed interest rate swap in which her bank pays the fixed rate to the counterparty and receives a floating rate. A floating-rate loan has a much lower duration than a long-term fixed-rate mortgage because the floating rate resets periodically as market interest rates change, bringing its market value back to par value.

## 7. SUMMARY

This chapter on derivatives strategies shows a number of ways in which market participants might use derivatives to enhance returns or to reduce risk to better meet portfolio objectives. The following are the key points.

- Interest rate, currency, and equity futures and swaps can be used to modify risk and return by altering the characteristics of the cash flows of an investment portfolio.
- Buying a call and writing a put with the same exercise price creates a synthetic long position.
- A long position plus a short futures position in the same underlying asset creates a synthetic risk-free asset earning the risk-free rate.
- A covered call, in which the holder of a stock writes a call giving someone the right to buy the shares, is one of the most common uses of options by individual investors.
- Covered calls can be used to generate income, to acquire shares at a lower-than-market price, or to exit a position when the shares hit a target price.
- A covered call position has a limited maximum return because of the transfer of the right tail of the return distribution to the option buyer.
- The maximum loss of a covered call position is less than the maximum loss of the underlying shares alone, but the covered call carries the potential for an opportunity loss if the underlying shares rise sharply.
- A protective put is the simultaneous holding of a long stock position and a long put on the same asset. The put provides protection or insurance against a price decline.
- Although the continuous purchase of protective puts is expensive and probably suboptimal, the occasional purchase of a protective put to deal with a bearish short-term outlook can be a reasonable risk-reducing activity.
- The maximum loss with a protective put is limited because the downside risk is transferred to the option writer in exchange for the payment of the option premium.
- With an option spread, an investor buys one option and writes another of the same type. This reduces the position cost but caps the maximum payoff.

- A bull spread is normally constructed by buying a call option and writing another call option with a higher exercise price.
- A bear spread is normally constructed by buying a put option and writing another put option with a lower exercise price.
- With either a bull spread or a bear spread, both the maximum gain and the maximum loss are known and limited.
- A collar is an option position in which the investor is long shares of stock and simultaneously writes a covered call and buys a protective put.
- A calendar spread involves buying a long-dated option and writing a shorter-dated option of the same type with the same exercise price, or vice versa. The primary motivation for such a spread is to take advantage of the faster time decay with the shorter-term option.
- A straddle is an option combination in which the investor buys puts and calls with the same exercise price. The straddle holder typically needs a substantial price movement in the underlying asset in order to make a profit.
- The risk of a derivative product depends on how it is used. Derivatives should always be used in connection with a well-defined investment objective.

## PROBLEMS

Aline Nuñes is a junior analyst in the derivatives research division of an international securities firm. Nuñes's supervisor, Cátia Pereira, asks her to conduct an analysis of various options trading strategies relating to shares of three companies: IZD, QWY, and XDF. On 1 February, Nuñes gathers selected option premium data on the companies, which is presented in Exhibit 1.

EXHIBIT 1 Share Price and Options Premiums as of 1 February (share prices and option premiums are in euros)

	Share Price	Call Premium	Option Date/Strike	Put Premium
IZD	93.93	9.45	April/87.50	1.67
		2.67	April/95.00	4.49
		1.68	April/97.50	5.78
QWY	28.49	4.77	April/24.00	0.35
		3.96	April/25.00	0.50
		0.32	April/31.00	3.00
XDF	74.98	0.23	February/80.00	5.52
		2.54	April/75.00	3.22
		2.47	December/80.00	9.73

Nuñes considers the following option strategies relating to IZD.

*Strategy 1:* Constructing a synthetic long put position in IZD

*Strategy 2:* Buying 100 shares of IZD and writing the April €95.00 strike call option on IZD

*Strategy 3:* Implementing a covered call position in IZD using the April €97.50 strike option

Nuñes next reviews the following option strategies relating to QWY.

*Strategy 4:* Implementing a protective put position in QWY using the April €25.00 strike option

*Strategy 5:* Buying 100 shares of QWY, buying the April €24.00 strike put option, and writing the April €31.00 strike call option

*Strategy 6:* Implementing a bear spread in QWY using the April €25.00 and April €31.00 strike options

Finally, Nuñes considers two option strategies relating to XDF.

*Strategy 7:* Writing both the April €75.00 strike call option and the April €75.00 strike put option on XDF

*Strategy 8:* Writing the February €80.00 strike call option and buying the December €80.00 strike call option on XDF

Over the past few months, Nuñes and Pereira have followed news reports on a proposed merger between XDF and one of its competitors. A government antitrust committee is currently reviewing the potential merger. Pereira expects the share price to move sharply up or down depending on whether the committee decides to approve or reject the merger next week.

Pereira asks Nuñes to recommend an option trade that might allow the firm to benefit from a significant move in the XDF share price regardless of the direction of the move.

1. Strategy 1 would require Nuñes to buy:
  - A. shares of IZD.
  - B. a put option on IZD.
  - C. a call option on IZD.
2. Based on Exhibit 1, Nuñes should expect Strategy 2 to be *least* profitable if the share price of IZD at option expiration is:
  - A. less than €91.26.
  - B. between €91.26 and €95.00.
  - C. more than €95.00.
3. Based on Exhibit 1, the breakeven share price of Strategy 3 is *closest* to:
  - A. €92.25.
  - B. €95.61.
  - C. €95.82.
4. Based on Exhibit 1, the maximum loss per share that would be incurred if Strategy 4 was implemented is:
  - A. €2.99.
  - B. €3.99.
  - C. unlimited.

5. Strategy 5 is *best* described as a:
  - A. collar.
  - B. straddle.
  - C. bear spread.
6. Based on Exhibit 1, Strategy 5 offers:
  - A. unlimited upside.
  - B. a maximum profit of €2.48 per share.
  - C. protection against losses if QWY's share price falls below €28.14.
7. Based on Exhibit 1, the breakeven share price for Strategy 6 is *closest* to:
  - A. €22.50.
  - B. €28.50.
  - C. €33.50.
8. Based on Exhibit 1, the maximum gain per share that could be earned if Strategy 7 is implemented is:
  - A. €5.74.
  - B. €5.76.
  - C. unlimited.
9. Based on Exhibit 1, the *best* explanation for Nuñes to implement Strategy 8 would be that, between the February and December expiration dates, she expects the share price of XDF to:
  - A. decrease.
  - B. remain unchanged.
  - C. increase.
10. The option trade that Nuñes should recommend relating to the government committee's decision is a:
  - A. collar.
  - B. bull spread.
  - C. long straddle.

# CHAPTER 6

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## RISK MANAGEMENT

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### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- discuss features of the risk management process, risk governance, risk reduction, and an enterprise risk management system;
- evaluate strengths and weaknesses of a company's risk management process;
- describe steps in an effective enterprise risk management system;
- evaluate a company's or a portfolio's exposures to financial and nonfinancial risk factors;
- calculate and interpret value at risk (VaR) and explain its role in measuring overall and individual position market risk;
- compare the analytical (variance–covariance), historical, and Monte Carlo methods for estimating VaR and discuss the advantages and disadvantages of each;
- discuss advantages and limitations of VaR and its extensions, including cash flow at risk, earnings at risk, and tail value at risk;
- compare alternative types of stress testing and discuss advantages and disadvantages of each;
- evaluate the credit risk of an investment position, including forward contract, swap, and option positions;
- demonstrate the use of risk budgeting, position limits, and other methods for managing market risk;

- demonstrate the use of exposure limits, marking to market, collateral, netting arrangements, credit standards, and credit derivatives to manage credit risk;
- discuss the Sharpe ratio, risk-adjusted return on capital, return over maximum drawdown, and the Sortino ratio as measures of risk-adjusted performance;
- demonstrate the use of VaR and stress testing in setting capital requirements.

## 1. INTRODUCTION

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Investment is an intrinsically risky activity. Indeed, risk taking is an innate characteristic of human activity and as old as humankind itself. Without risk, we have little possibility of reward. We thus need to treat risk management as a critical component of the investment process. Specifically, with regard to both individual investments and entire portfolios, we should examine and compare the full spectrum of risks and expected returns to ensure that to the greatest extent possible the exposures we assume are at all times justified by the rewards we can reasonably expect to reap. Proper identification, measurement, and control of risk are key to the process of investing, and we put our investment objectives at risk unless we commit appropriate resources to these tasks.

A portfolio manager must be familiar with risk management not only as it relates to portfolio management but also as it relates to managing an enterprise, because a portfolio manager is a responsible executive in an enterprise (his investment firm). He must also understand the risks and risk management processes of companies in which he invests. The risk management framework presented in this reading is an inclusive one, applicable to the management of both enterprise and portfolio risk.

Although portfolio managers and enterprises may occasionally hedge their risks or engage in other risk-reducing transactions, they should not, and indeed cannot, restrict their activities to those that are risk free, as discussed in more detail later. The fact that these entities engage in risky activities raises a number of important questions:

- What is an effective process for identifying, measuring, and managing risk?
- Which risks are worth taking on a regular basis, which are worth taking on occasion, and which should be avoided altogether?
- How can our success or lack of success in risk taking be evaluated?
- What information should be reported to investors and other stakeholders concerning the risk of an enterprise or a portfolio?

The answers to these questions and many others collectively define the process of *risk management*. Over the course of this reading, we endeavor to explain this process and some of its most important concepts. Consistent with the book's focus on portfolio management, this reading concentrates on managing risks arising from transactions that are affected by interest rates, stock prices, commodity prices, and exchange rates. We also survey the other risks that most enterprises face and illustrate the discussion from a variety of perspectives. The reading is organized as follows. Section 2 defines and explains a risk management framework. Section 3 discusses what constitutes good risk management. Sections 4, 5, and 6 discuss the individual steps in the risk management process, and we conclude with a summary.

## 2. RISK MANAGEMENT AS A PROCESS

We can formally define risk management as follows:

Risk management is a process involving the identification of exposures to risk, the establishment of appropriate ranges for exposures (given a clear understanding of an entity's objectives and constraints), the continuous measurement of these exposures (either present or contemplated), and the execution of appropriate adjustments whenever exposure levels fall outside of target ranges. The process is continuous and may require alterations in any of these activities to reflect new policies, preferences, and information.

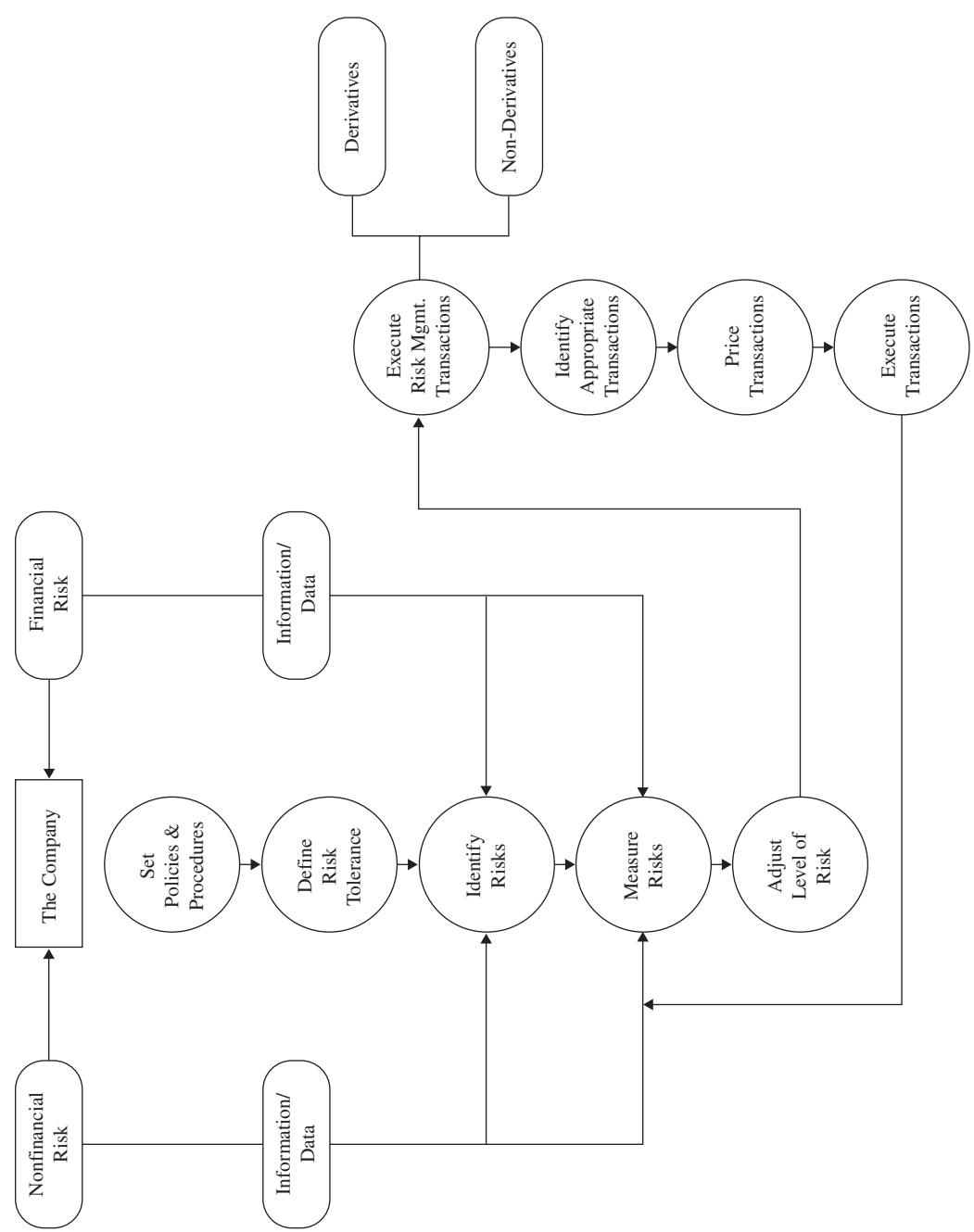
This definition highlights that risk management should be a *process*, not just an activity. A process is continuous and subject to evaluation and revision. Effective risk management requires the constant and consistent monitoring of exposures, with an eye toward making adjustments, whenever and wherever the situation calls for them.<sup>1</sup> Risk management in its totality is all at once a proactive, anticipative, and reactive process that continuously monitors and controls risk.

Exhibit 1 illustrates the *practical application of the process* of risk management as it applies to a hypothetical business enterprise. We see at the top that the company faces a range of financial and nonfinancial risks; moving down the exhibit, we find that the company has responded to this challenge by establishing a series of risk management policies and procedures. First, it defines its risk tolerance, which is the level of risk it is willing and able to bear.<sup>2</sup> It then identifies the risks, drawing on all sources of information, and attempts to measure these risks using information or data related to all of its identified exposures. The process of risk measurement can be as simple as Exhibit 1 illustrates, but more often than not it involves expertise in the practice of modeling and sometimes requires complex analysis. Once the enterprise has built effective risk identification and measurement mechanisms, it is in a position to adjust its risk exposures, wherever and whenever exposures diverge from previously identified target ranges. These adjustments take the form of risk-modifying transactions (broadly understood to include the possible complete transfer of risk). The execution of risk management transactions is itself a distinct process; for portfolios, this step consists of trade identification, pricing, and execution. The process then loops around to the measurement of risk and continues in that manner, and to the constant monitoring and adjustment of the risk, to bring it into or maintain it within the desired range.

<sup>1</sup>For brevity, we often refer to an exposure to risk or **risk exposure** (the state of being exposed to or vulnerable to a risk) as simply an *exposure*.

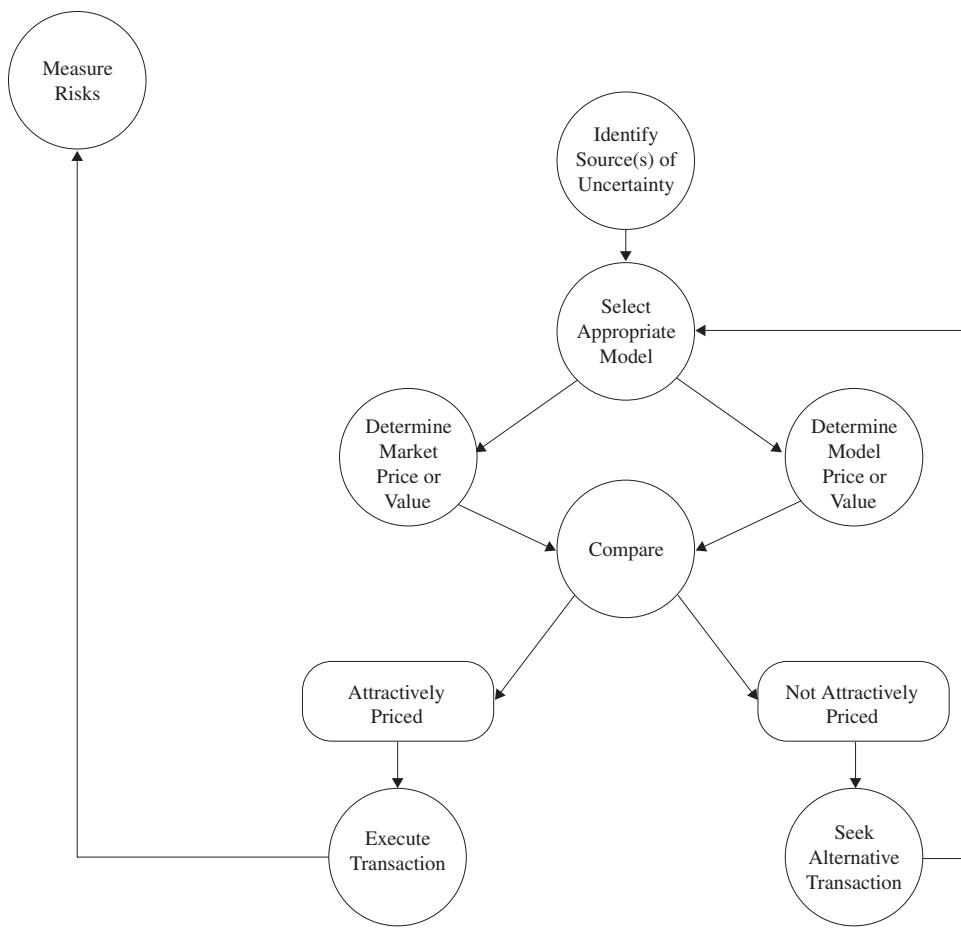
<sup>2</sup>An enterprise may have different risk tolerances for different types of risk in a manner that does not readily permit averaging, so we should view risk tolerance in this context as potentially multidimensional.

**EXHIBIT 1 Risk Management Process: The Practice of Risk Management**



In applying the risk management process to portfolio management, managers must devote a considerable amount of attention to measuring and pricing the risks of financial transactions or positions, particularly those involving derivatives. Exhibit 2 illustrates this process of pricing and measuring risk, expanding on the detail given in Exhibit 1. In Exhibit 2, we see at the top that in pricing the transaction, we first identify the source(s) of uncertainty. Then we select the appropriate pricing model and enter our desired inputs to derive our most accurate estimate of the instrument's model value (which we hope reflects its true economic value). Next, we look to the marketplace for an indication of where we can actually execute the transaction. If the execution price is "attractive" (i.e., the market will buy the instrument from us at a price at or above, or sell it to us at a price at or below, the value indicated by our model), it fits our criteria for acceptance; if not, we should seek an alternative transaction. After executing the transaction, we would then return to the process of measuring risk.

EXHIBIT 2 Risk Management Process: Pricing and Measuring Risk



Our discussion of Exhibit 1 highlighted that risk management involves adjusting levels of risk to appropriate levels, not necessarily eliminating risk altogether. It is nearly impossible to operate a successful business or investment program without taking risks. Indeed, a company that accepted no risk would not be an operating business. Corporations take risks for the purpose of generating returns that increase their owners' wealth. Corporation owners, the shareholders, risk their capital with the same objective in mind. *Companies that succeed in doing the activities they should be able to do well, however, cannot afford to fail overall because of activities in which they have no expertise.* Accordingly, many companies hedge risks that arise from areas in which they have no expertise or comparative advantage. In areas in which they do have an edge (i.e., their primary line of business), they tend to hedge only tactically. They hedge when they think they have sufficient information to suggest that a lower risk position is appropriate. They manage risk, increasing it when they perceive a competitive advantage and decreasing it when they perceive a competitive disadvantage. In essence, they attempt to efficiently allocate risk. Similarly, portfolio managers attempt to efficiently use risk to achieve their return objectives.

We have illustrated that risk management involves far more than risk reduction or hedging (one particular risk-reduction method). Risk management is a general practice that involves risk modification (e.g., risk reduction or risk expansion) as deemed necessary and appropriate by the custodians of capital and its beneficial owners.

For the risk management process to work, managers need to specify thoughtfully the business processes they use to put risk management into practice. We refer to these processes collectively as risk governance, the subject of the next section.

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### 3. RISK GOVERNANCE

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Senior management is ultimately responsible for *every* activity within an organization. Their involvement is thus essential for risk management to succeed. The process of setting overall policies and standards in risk management is called risk governance. Risk governance involves choices of governance structure, infrastructure, reporting, and methodology. The quality of risk governance can be judged by its transparency, accountability, effectiveness (achieving objectives), and efficiency (economy in the use of resources to achieve objectives).

Risk governance begins with choices concerning governance structure. Organizations must determine whether they wish their risk management efforts to be centralized or decentralized. Under a centralized risk management system, a company has a single risk management group that monitors and ultimately controls all of the organization's risk-taking activities. By contrast, a decentralized system places risk management responsibility on individual business unit managers. In a decentralized approach, each unit calculates and reports its exposures independently. Decentralization has the advantage of allowing the people closer to the actual risk taking to more directly manage it. Centralization permits economies of scale and allows a company to recognize the offsetting nature of distinct exposures that an enterprise might assume in its day-to-day operations. For example, suppose one subsidiary of a company buys from Japan and another subsidiary sells to Japan, with both engaged in yen-denominated transactions. Each subsidiary would perceive some foreign exchange exposure. From a centralized viewpoint, however, these risks have offsetting effects, thereby reducing the overall need to hedge.

Moreover, even when exposures to a single risk factor do not directly offset one another, enterprise-level risk estimates may be lower than those derived from individual units because of the risk-mitigating benefits of diversification. For example, one corporate division may borrow US dollars at five-year maturities, and another division may fund its operation by issuing 90-day commercial paper. In theory, the corporation's overall sensitivity to rising interest rates may be less than the sum of that reported by each division, because the five-year and 90-day rate patterns are less than perfectly correlated.

In addition, centralized risk management puts the responsibility on a level closer to senior management, where we have argued it belongs. It gives an overall picture of the company's risk position, and ultimately, the overall picture is what counts. This centralized type of risk management is now called **enterprise risk management** (ERM) or sometimes firmwide risk management because its distinguishing feature is a firmwide or across-enterprise perspective.<sup>3</sup> In ERM, an organization must consider each risk factor to which it is exposed—both in isolation and in terms of any interplay among them.

Risk governance is an element of **corporate governance** (the system of internal controls and procedures used to manage individual companies). As risk management's role in corporate governance has become better appreciated, the importance of ERM has risen proportionately. Indeed, for risk-taking entities (this means nearly the entire economic universe), it is contradictory to suggest that an organization has sound corporate governance without maintaining a clear and continuously updated understanding of its exposures at the enterprise level. Senior managers who have an adequate understanding of these factors are in a superior governance position to those who do not, and over time this advantage is almost certain to accrue to the bottom line. Therefore, the risk management system of a company that chooses a decentralized risk management approach requires a mechanism by which senior managers can inform themselves about the enterprise's overall risk exposures.

At the enterprise level, companies should control not only the sensitivity of their earnings to fluctuations in the stock market, interest rates, foreign exchange rates, and commodity prices, but also their exposures to credit spreads and default risk, to gaps in the timing match of their assets and liabilities, and to operational/systems failures, financial fraud, and other factors that can affect corporate profitability and even survival.

### EXAMPLE 1 Some Risk Governance Concerns of Investment Firms

Regardless of the risk governance approach chosen, effective risk governance for investment firms demands that the trading function be separated from the risk management function. An individual or group that is independent of the trading function must monitor the positions taken by the traders or risk takers and price them independently. The risk manager has the responsibility for monitoring risk levels for all portfolio positions

<sup>3</sup>The Committee of Sponsoring Organizations of the Treadway Commission defines ERM as follows: "Enterprise risk management is a process, effected by an entity's board of directors, management, and other personnel, applied in strategy setting and across the enterprise, designed to identify potential events that may affect the entity, and manage risk to be within its risk appetite, to provide reasonable assurance regarding the achievement of entity objectives" (2004, p. 2).

(as well as for portfolios as a whole) and executing any strategies necessary to control the level of risk. To do this, the risk manager must have timely and accurate information, authority, and independence from the trading function. That is not to say that the trading function will not need its own risk management expertise in order to allocate capital in an optimal fashion and maximize risk-adjusted profit. Ideally, the risk manager will work with the trading desks in the development of risk management specifications, such that everyone in the organization is working from a common point of reference in terms of measuring and controlling exposures.

Effective risk governance for an investment firm also requires that the back office be fully independent from the front office, so as to provide a check on the accuracy of information and to forestall collusion. (The **back office** is concerned with transaction processing, record keeping, regulatory compliance, and other administrative functions; the **front office** is concerned with trading and sales.) Besides being independent, the back office of an investment firm must have a high level of competence, training, and knowledge because failed trades, errors, and over-sights can lead to significant losses that may be amplified by leverage. The back office must effectively coordinate with external service suppliers, such as the firm's **global custodian**. The global custodian effects **trade settlement** (completion of a trade wherein purchased financial instruments are transferred to the buyer and the buyer transfers money to the seller), safekeeping of assets, and the allocation of trades to individual custody accounts. Increasingly, financial institutions are seeking risk reduction with cost efficiencies through **straight-through processing** (STP) systems that obviate manual and/or duplicative intervention in the process from trade placement to settlement.

An effective ERM system typically incorporates the following steps:

1. Identify each *risk factor* to which the company is exposed.
2. Quantify each exposure's size in money terms.
3. Map these inputs into a risk estimation calculation.<sup>4</sup>
4. Identify overall risk exposures as well as the contribution to overall risk deriving from each risk factor.
5. Set up a process to report on these risks periodically to senior management, who will set up a committee of division heads and executives to determine capital allocations, risk limits, and risk management policies.
6. Monitor compliance with policies and risk limits.

<sup>4</sup>For example, using value at risk or another of the concepts that we will discuss later.

Steps 5 and 6 help enormously in allowing an organization to quantify the magnitude and distribution of its exposures and in enabling it to use the ERM system's output to more actively align its risk profile with its opportunities and constraints on a routine, periodic basis.

As a final note, effective ERM systems always feature centralized data warehouses, where a company stores all pertinent risk information, including position and market data, in a technologically efficient manner. Depending on the organization's size and complexity, developing and maintaining a high-quality data warehouse can require a significant and continuing investment. In particular, the process of identifying and correcting errors in a technologically efficient manner can be enormously resource intensive—especially when the effort requires storing historical information on complex financial instruments. It is equally clear, however, that the return on such an investment can be significant.

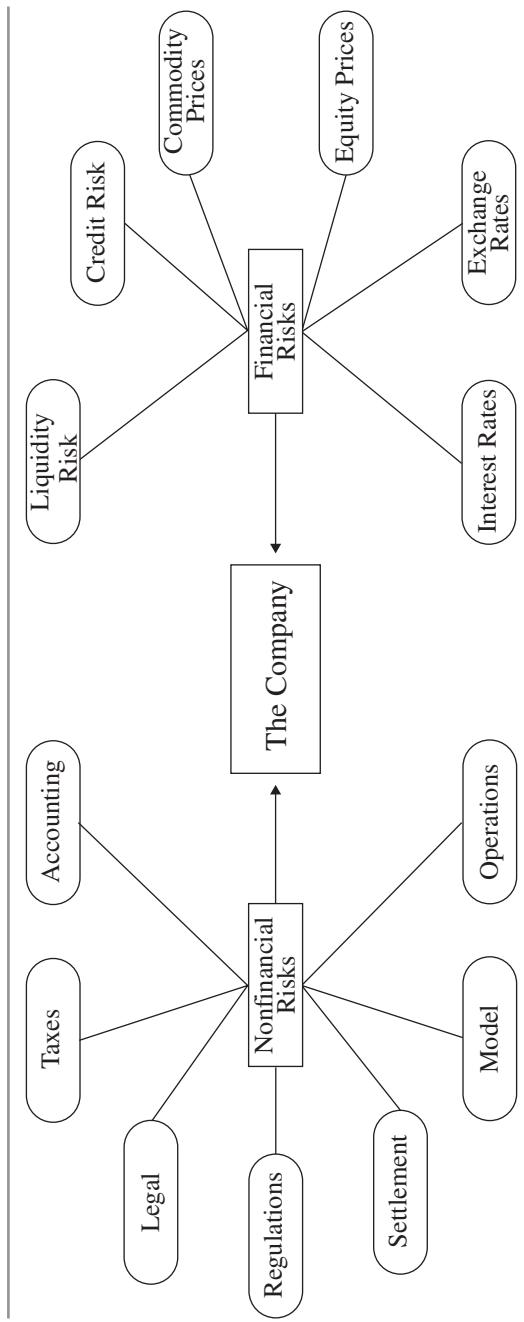
#### 4. IDENTIFYING RISKS

As indicated above, economic agents of all types assume different types of exposures on a near-continuous basis. Moreover, these risk exposures take very different forms, each of which, to varying extents, may call for customized treatment. Effective risk management demands the separation of risk exposures into specific categories that reflect their distinguishing characteristics. Once a classification framework is in place, we can move on to the next steps in the risk management process: identification, classification, and measurement.

Although the list is far from exhaustive, many company (or portfolio) exposures fall into one of the following categories: market risk (including interest rate risk, exchange rate risk, equity price risk, commodity price risk); credit risk; liquidity risk; operational risk; model risk; settlement risk; regulatory risk; legal/contract risk; tax risk; accounting risk; and sovereign/political risk. These risks may be grouped into financial risks and nonfinancial risks as shown in Exhibit 3.<sup>5</sup> **Financial risk** refers to all risks derived from events in the external financial markets; **nonfinancial risk** refers to all other forms of risk.

<sup>5</sup>A notable risk that could be included in a comprehensive listing (particularly as pertains to commercial enterprises) is **business risk**, defined by Ross, Westerfield, and Jordan (1993, p. 527) as “the equity risk that comes from the nature of the firm's operating activities.” For example, the risk for a hotel business that arises from variability in room occupancy rates would be classified as business risk. In a later section on other risks, we also discuss two types of risks related to netting.

EXHIBIT 3 The Sources of Risk



Example 2 illustrates a simple analysis of risk exposures. In the example, we have detailed the subtypes of market risk; each one may pose unique issues of measurement and management.

### EXAMPLE 2 An Analysis of Risk Exposures

Liam McNulty is the risk manager for a large multinational agricultural concern, Agripure. The company grows its own corn, wheat, and soybeans but pays large sums to third parties for pesticides, fertilizer, and other supplies. For this, it must borrow heavily to finance its purchases. Customers typically purchase Agripure's goods on credit. Moreover, Agripure buys and sells its products and raw materials worldwide, often transacting in the domestic currency of its customers and suppliers. Finally, to finance its own expansion, Agripure intends to issue stock.

Recommend and justify the risk exposures that McNulty should report as part of an enterprise risk management system for Agripure.

*Solution:* McNulty should report on the following risk exposures:

- Market risk, including these subtypes:
  - Commodity price risk, because Agripure has exposures in raw materials and finished products.
  - Foreign exchange risk, because it buys and sells products world-wide, often transacting in the home currency of the entity on the other side of the transaction.
  - Equity market risk, because Agripure's expansion financing is affected by the price it receives for its share issuance.
  - Interest rate risk, because Agripure has exposures in financing its raw material purchases and because its customers typically purchase their goods on credit.
- Credit risk, because Agripure's customers typically purchase their goods on credit.
- Operational risk, because as an agricultural producer Agripure is subject to weather-related risk (an external event).

In the following sections, we discuss each of these risks in detail.

#### 4.1. Market Risk

**Market risk** is the risk associated with interest rates, exchange rates, stock prices, and commodity prices. It is linked to supply and demand in various marketplaces. Although we may distinguish among interest rate risk, currency risk, equity market risk, and commodity risk when discussing measurement and management issues, for example, these subtypes all have exposure to supply and demand. Much of the evolution that has taken place in the field of risk management has emanated from a desire to understand and control market risks, and we will have a good deal to say about this topic throughout the balance of this reading.

One set of market risk takers with special requirements for market risk are defined-benefit (DB) pension funds, which manage retirement assets generally under strict regulatory regimes. Pension fund risk management necessarily concerns itself with funding the stream of promised payments to pension plan participants. Therefore, a DB plan must measure its market exposures not purely on the basis of its assets but also in terms of the risks of pension assets in relation to liabilities. Other investors as well can have strong asset/liability management concerns.<sup>6</sup> This has important implications for exposure measurement, risk control, capital allocation and risk budgeting, which we will address later.

#### 4.2. Credit Risk

Apart from market risk, credit risk is the primary type of financial risk that economic agents face. **Credit risk** is the risk of loss caused by a counterparty or debtor's failure to make a promised payment. This definition reflects a traditional binary concept of credit risk, by and large embodied by default risk (i.e., the risk of loss associated with the nonperformance of a debtor or counterparty). For the last several years, however, credit markets have taken on more and more of the characteristics typically associated with full-scale trading markets. As this pattern has developed, the lines between credit risk and market risk have blurred as markets for credit derivatives have developed.<sup>7</sup> For example, the holder of a traded credit instrument could suffer a loss as a result of a short-term supply–demand imbalance without the underlying probability of default changing. Some subset of market participants often suffers losses whether credit is improving or deteriorating because it is now quite easy to take long and short positions in credit markets. Finally, note that pricing conventions for credit typically take the form of spreads against market benchmarks, for example, government bond yields or swap rates.<sup>8</sup> Thus when a given credit instrument is said to be priced at 150 over, it typically means that the instrument can be purchased to yield 150 basis points over the rate on the market benchmark (e.g., the government bond with the same maturity).

Until the era of over-the-counter derivatives, credit risk was more or less exclusively a concern in the bond and loan markets. Exchange-traded derivatives are guaranteed against credit loss. OTC derivatives, however, contain no explicit credit guaranty and, therefore, subject participants to the threat of loss if their counterparty fails to pay.

Before OTC derivatives became widely used, bond portfolio managers and bank loan officers were the primary credit risk managers. They assessed credit risk in a number of ways,<sup>9</sup> including the qualitative evaluation of corporate fundamentals through the review of financial statements, the calculation of credit scores, and by relying on consensus information that was and still is widely available for virtually every borrower. The synthesis of this “credit consensus” resides with rating agencies and credit bureaus, which were historically, and to some extent still are, the primary sources of information on credit quality. The proliferation and complexity of

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<sup>6</sup>See the readings on managing institutional investor portfolios and asset allocation in particular.

<sup>7</sup>A **credit derivative** is a contract in which one party has the right to claim a payment from another party in the event that a specific credit event occurs over the life of the contract.

<sup>8</sup>A **swap rate** is the interest rate applicable to the pay-fixed-rate side of an interest rate swap. See Chance (2003) to review the basics of swaps.

<sup>9</sup>Credit risk in the more general context of fixed-income securities is discussed in more detail in Fabozzi (2004a), Chapter 15. Many of the principles of credit risk analysis for fixed-income securities also apply to derivatives.

financial instruments with credit elements in the OTC derivatives market, however, has placed new demands on the understanding of credit risk. Indeed, the need to better understand credit risk has led to significant progress in developing tools to measure and manage this risk.

### 4.3. Liquidity Risk

**Liquidity risk** is the risk that a financial instrument cannot be purchased or sold without a significant concession in price because of the market's potential inability to efficiently accommodate the desired trading size.<sup>10</sup> In some cases, the market for a financial instrument can dry up completely, resulting in a total inability to trade an asset. This risk is present in both initiating and liquidating transactions, for both long and short positions, but can be particularly acute for liquidating transactions—especially when such liquidation is motivated by the need to reduce exposures in the wake of large losses. Those wishing to sell securities under these circumstances can find the market bereft of buyers at prices acceptable to the seller, particularly in periods of unusually high market stress. Perhaps less frequently, short sellers in need of covering losing positions are at risk to short squeezes. This situation is often exacerbated by the fact that for most cash instruments, short sellers establish positions by borrowing the securities in question from brokerage firms and other entities that typically can require the securities to be returned with little or no advance warning. Although derivatives can be used to effectively sell an asset or liquidate a short position, they often will not help in managing liquidity risk. If the underlying is illiquid, there is a good possibility that the universe of associated derivative instruments may also be illiquid.

For traded securities, the size of the **bid–ask spread** (the spread between the bid and ask prices), stated as a proportion of security price, is frequently used as an indicator of liquidity.<sup>11</sup> When markets are illiquid, dealers expect to sell at relatively high prices and buy at relatively low prices to justify their assumption of exposure to liquidity risk. However, bid–ask quotations apply only to specified, usually small size, trades, and are thus an imprecise measure of liquidity risk. Other, more complex measures of liquidity have been developed to address the issue of trading volume. For example, Amihud's (2002) illiquidity ratio measures the price impact per \$1 million traded in a day, expressed in percentage terms. Note, however, that no explicit transaction volume is available for many OTC instruments. Less formally, one of the best ways to measure liquidity is through the monitoring of transaction volumes, with the obvious rule of thumb being that the greater the average transaction volume, the more liquid the instrument in question is likely to be. Historical volume patterns, however, may not repeat themselves at times when the liquidity they imply is most needed.

Liquidity risk is a serious problem and often is difficult to observe and quantify. It is not always apparent that certain securities are illiquid: Some that are liquid when purchased (or sold short) can be illiquid by the time they are sold (or repurchased to cover short positions). Valuation models rarely encompass this liquidity risk in estimating fair value. Those models that do attempt to incorporate transaction costs do so in a nonformulaic manner. Of course, these problems typically reach their apex when the markets themselves are under stress and the

<sup>10</sup>Liquidity has been used in various senses. For example, **funding risk** (the risk that liabilities funding long asset positions cannot be rolled over at reasonable cost) has sometimes been referred to as a type of liquidity risk; liquidity in this sense relates to the availability of cash. One would still distinguish between market liquidity risk (discussed in the reading) and funding liquidity risk.

<sup>11</sup>For example, see Amihud and Mendelson (1986). We must state the bid–ask spread as a proportion of stock price to control for differences in securities' prices.

need for liquidity is most acute. Liquidity assessments that fail to consider the problems that might arise during periods of market stress are incomplete from a risk management perspective. For all of these reasons, liquidity risk is one of the more complex aspects of risk management.

We now turn our attention to nonfinancial risks, starting with operational risk.

#### 4.4 Operational Risk

**Operational risk**, sometimes called operations risk, is the risk of loss from failures in a company's systems and procedures or from external events. These risks can arise from computer breakdowns (including bugs, viruses, and hardware problems), human error, and events completely outside of companies' control, including "acts of God" and terrorist actions.

Computer failures are quite common, but the development of backup systems and recovery procedures has reduced their impact in recent years. Technology bugs and viruses are potentially quite risky but have become more manageable with the proper personnel, software, and systems. Even the smallest business has learned to back up files and take them off the premises. Larger businesses have much more extensive computer risk management practices.

Human failures include the typically manageable unintentional errors that occur in every business, along with more critical and potentially disastrous incidences of willful misconduct.

#### EXAMPLE 3 An Operational Risk for Financial Services Companies: The Rogue Trader

Among the more prominent examples of operational risk for financial service companies is that of the so-called rogue trader: an individual who has either assumed an irresponsibly high level of risk, engaged in unauthorized transactions, or some combination of the two. The risks associated with this type of activity increase the longer it goes undetected, and often the very lack of controls that creates the opportunity for a rogue trader in the first place renders it difficult to quickly determine that a problem exists. In some extreme cases, such as an incident that occurred in the Singapore office of Barings Bank, a rogue trader can cause an entire organization to fold. The incidence of high-profile rogue trading episodes has multiplied since the early 1990s, but in nearly all of these episodes, the problem's major source was a lack of rudimentary corporate controls and oversight.<sup>12</sup>

Our definition of operational risk includes losses from external events. Insurance typically covers damage from fires, floods and other types of natural disasters, but insurance provides only cash compensation for losses. If a flood destroys the trading room of a bank, the monies recovered likely will not come close to paying for the loss of customers who may take their

<sup>12</sup>For more on the subject of operational risk in financial services companies, see Marshall (2001).

trading business elsewhere. Hence, most companies have backup facilities they can activate in such cases. The 1993 World Trade Center bombing in New York City led many companies to establish backup systems in the event of another terrorist attack, which sadly took place on a greater scale eight years later. The speed with which trading enterprises, including the New York Stock Exchange, domiciled inside or near the World Trade Center reestablished full-scale operations after such a devastating attack is but one indication of the increased importance placed on operational risk management by these enterprises.

In some cases, companies manage operational risk by using insurance contracts, which involves a transfer of risk. A few types of derivative contracts even pay off for operational losses, but the market for these has not fully developed. These instruments are essentially insurance contracts. Most companies manage operational risk, however, by monitoring their systems, taking preventive actions, and having a plan in place to respond if such events occur.

#### 4.5. Model Risk

**Model risk** is the risk that a model is incorrect or misapplied; in investments, it often refers to valuation models. Model risk exists to some extent in any model that attempts to identify the fair value of financial instruments, but it is most prevalent in models used in derivatives markets.

Since the development of the seminal Black–Scholes–Merton option pricing model, both derivatives and derivative pricing models have proliferated.<sup>13</sup> The development of so many models has brought model risk to prominence. If an investor chooses an inappropriate model, misinterprets the results, or uses incorrect inputs, the chance of loss increases at the same time that control over risk is impaired. Therefore, investors must scrutinize and objectively validate all models they use.

#### 4.6. Settlement (Herstatt) Risk

The payments associated with the purchase and sale of cash securities such as equities and bonds, along with cash transfers executed for swaps, forwards, options, and other types of derivatives, are referred to collectively as settlements. The process of settling a contract involves one or both parties making payments and/or transferring assets to the other. We define **settlement risk** as the risk that one party could be in the process of paying the counterparty while the counterparty is declaring bankruptcy.<sup>14</sup>

Most regulated futures and options exchanges are organized in such a way that they themselves (or a closely affiliated entity) act as the central counterparty to all transactions. This facility usually takes the form of a clearing house, which is backed by large and credible financial guarantees. All transactions on the exchange take place between an exchange member and the central counterparty, which removes settlement risk from the transaction. The possibility always exists, however, that the exchange member is acting in an agency capacity and/or that its end client fails to settle. Clearly in these circumstances, the responsibility falls to the exchange member to make good and bear any loss on the trade.

OTC markets, including those for bonds and derivatives, do not rely on a clearing house. Instead, they effect settlement through the execution of agreements between the actu-

<sup>13</sup>See Chance (2003).

<sup>14</sup>Note that settlement can also fail because of operational problems even when the counterparty is creditworthy; the risk in that case would be an operational risk.

al counterparties to the transaction. With swaps and forward contracts, settlements take the form of two-way payments. Two-way payments create the problem that one party could be in the process of paying its counterparty while that counterparty is declaring bankruptcy and failing to make its payment. Netting arrangements, used in interest rate swaps and certain other derivatives, can reduce settlement risk. In such arrangements, the financial instrument is periodically marked to market (under an agreed-upon methodology) and the “loser” pays the “winner” the difference for the period. This mechanism reduces the magnitude of any settlement failures to the net payment owed plus the cost of replacing the defaulted contract. Transactions with a foreign exchange component, however (e.g., currency forwards and currency swaps, but also spot trades), do not lend themselves to netting. Furthermore, such contracts often involve two parties in different countries, increasing the risk that one party will be unaware that the other party is declaring bankruptcy. The risk has been called Herstatt risk because of a famous incident in 1974 when Bank Herstatt failed at a time when counterparties were sending money to it.

Fortunately, bankruptcy does not occur often. Furthermore, through continuously linked settlement (CLS) in which payments on foreign exchange contracts are executed simultaneously, this risk has been even further mitigated.<sup>15</sup>

#### 4.7. Regulatory Risk

**Regulatory risk** is the risk associated with the uncertainty of how a transaction will be regulated or with the potential for regulations to change. Equities (common and preferred stock), bonds, futures, and exchange-traded derivatives markets usually are regulated at the federal level, whereas OTC derivative markets and transactions in alternative investments (e.g., hedge funds and private equity partnerships) are much more loosely regulated. Federal authorities in most countries take the position that these latter transactions are private agreements between sophisticated parties, and as such should not be regulated in the same manner as publicly traded markets. Indeed, in some circumstances, unsophisticated investors are excluded altogether from participating in such investments.

With regard to derivatives, companies that are regulated in other ways may have their derivatives business indirectly regulated. For example, in the United States, banks are heavily regulated by federal and state banking authorities, which results in indirect regulation of their derivatives business. Beyond these *de facto* restrictions, however, in most countries, the government does not regulate the OTC derivatives business.<sup>16</sup>

Regulation is a source of uncertainty. Regulated markets are always subject to the risk that the existing regulatory regime will become more onerous, more restrictive, or more costly. Unregulated markets face the risk of becoming regulated, thereby imposing costs and restrictions where none existed previously. Regulatory risk is difficult to estimate because laws are written by politicians and regulations are written by civil servants; laws, regulations, and enforcement activities may change with changes in political parties and regulatory personnel. Both the regulations and their enforcement often reflect attitudes and philosophies that may change over time. Regulatory risk and the degree of regulation also vary widely from country to country.

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<sup>15</sup>The execution takes place in a five-hour window (three hours in Asia Pacific), representing the overlapping business hours of different settlement systems. For more information, see [www.cls-group.com](http://www.cls-group.com).

<sup>16</sup>Of course, contract law always applies to any such transaction.

Regulatory risk often arises from the arbitrage nature of derivatives and structured transactions. For example, a long position in stock accompanied by borrowing can replicate a forward contract or a futures contract. Stocks are regulated by securities regulators, and loans are typically regulated by banking oversight entities. Forward contracts are essentially unregulated. Futures contracts are regulated at the federal level in most countries, but not always by the same agency that regulates the stock market. Equivalent combinations of cash securities and derivatives thus are not always regulated in the same way or by the same regulator. Another example of inconsistent or ambiguous regulatory treatment might arise from a position spanning different geographic regions, such as the ownership of a NASDAQ-listed European-domiciled technology company in a European stock portfolio.

#### 4.8. Legal/Contract Risk

Nearly every financial transaction is subject to some form of contract law. Any contract has two parties, each obligated to do something for the other. If one party fails to perform or believes that the other has engaged in a fraudulent practice, the contract can be abrogated. A dispute would then likely arise, which could involve litigation, especially if large losses occur. In some cases, the losing party will claim that the counterparty acted fraudulently or that the contract was illegal in the first place and, therefore, should be declared null and void. The possibility of such a claim being upheld in court creates a form of **legal/contract risk**: the possibility of loss arising from the legal system's failure to enforce a contract in which an enterprise has a financial stake.

Derivative transactions often are arranged by a dealer acting as a principal. The legal system has upheld many claims against dealers, which is not to say that the dealer has always been in the wrong but simply that dealers have sometimes put themselves into precarious situations. Dealers are indeed often advisors to their counterparties, giving the impression that if the dealer and counterparty enter into a contract, the counterparty expects the contract to result in a positive outcome. To avoid that misunderstanding, dealers may go to great lengths to make clear that they are the opposite party, not an advisor. Dealers also write contracts more carefully to cover the various contingencies that have been used against them in litigation. But a government or regulator might still take the legal view that a dealer has a higher duty of care for a less experienced counterparty. Contract law is in most circumstances federally or nationally governed. As such, the added possibility exists in arbitrage transactions that different laws might apply to each side of the transaction, thus adding more risk.

#### 4.9. Tax Risk

**Tax risk** arises because of the uncertainty associated with tax laws. Tax law covering the ownership and transaction of financial instruments can be extremely complex, and the taxation of derivatives transactions is an area of even more confusion and uncertainty. Tax rulings clarify these matters on occasion, but on other occasions, they confuse them further. In addition, tax policy often fails to keep pace with innovations in financial instruments. When this happens, investors are left to guess what type and level of taxation will ultimately apply, creating the risk that they have guessed wrongly and could later be subject to back taxes. In some cases, transactions that appear upfront to be exempt from taxation could later be found to be taxable, thereby creating a future expense that was unanticipated (and perhaps impossible to anticipate) at the time that the transaction was executed. We noted, in discussing regulatory risk, that equivalent combinations of financial instruments are not always regulated the same way.

Likewise, equivalent combinations of financial instruments are not always subject to identical tax treatment. This fact creates a tremendous burden of inconsistency and confusion, but on occasion the opportunity arises for arbitrage gains, although the tax authorities often quickly close such opportunities.

Like regulatory risk, tax risk is affected by the priorities of politicians and regulators. Many companies invest considerable resources in lobbying as well as hiring tax experts and consultants to control tax risk.

#### 4.10. Accounting Risk

**Accounting risk** arises from uncertainty about how a transaction should be recorded and the potential for accounting rules and regulations to change. Accounting statements are a key, if not primary, source of information on publicly traded companies. In the United States, accounting standards are established primarily by the Financial Accounting Standards Board (FASB). Legal requirements in the area of accounting are enforced for publicly traded companies by federal securities regulators and by the primary stock exchange associated with the security. Non-US domiciled companies that raise capital in the United States are also subject to these standards and laws. The law demands accurate accounting statements, and inaccurate financial reporting can subject corporations and their principals to civil and criminal litigation for fraud. In addition, the market punishes companies that do not provide accurate accounting statements, as happened for Enron and its auditor Arthur Andersen.

The International Accounting Standards Board (IASB) sets global standards for accounting. The FASB and the IASB have been working together toward convergence of accounting standards worldwide with 2005 targeted for harmonization. Historically, accounting standards have varied from country to country, with some countries requiring a higher level of disclosure than others.

#### EXAMPLE 4 Accounting Risk: The Case of Derivative Contracts

Accounting for derivative contracts has raised considerable confusion. When confusion occurs, companies run the risk that the accounting treatment for transactions could require adjustment, which could possibly lead to a need to restate earnings. Earnings restatements are almost always embarrassing for a company, because they suggest either a desire to hide information, the company's failure to fully understand material elements of its business, or some combination of the two. Restatements are very detrimental to corporate valuations because they cause investors to lose confidence in the accuracy of corporate financial disclosures. Beyond that, if negligence or intent to mislead was involved, the company could face civil and criminal liabilities as well.

Confusion over the proper accounting for derivatives gives rise to accounting as a source of risk. As with regulatory and tax risk, sometimes equivalent combinations of derivatives are not accounted for uniformly. The accounting profession typically moves to close such loopholes, but it does not move quickly and certainly does not keep up with the pace of innovation in financial engineering, so problems nearly always remain.

The IASB in IAS 39 (International Accounting Standard No. 39) requires the inclusion of derivatives and their associated gains and losses on financial statements, as does the FASB in SFAS 133 (Statement of Financial Accounting Standard No. 133). These rulings contain some areas of confusion and inconsistency, however, affording considerable room for interpretation.<sup>17</sup>

Most companies deal with accounting risk by hiring personnel with the latest accounting knowledge. In addition, companies lobby and communicate actively with accounting regulatory bodies and federal regulators in efforts to modify accounting rules in a desired direction and to make them clearer. Companies have tended to fight rules requiring more disclosure, arguing that disclosure *per se* is not always beneficial and can involve additional costs. A trade-off exists between the rights of corporations to protect proprietary information from competitors and the need to adequately inform investors and the public. This controversy is unlikely to go away, suggesting that accounting risk will always remain.

#### 4.11. Sovereign and Political Risks

Although they are covered indirectly above in areas such as regulatory, accounting, and tax risk, we can also isolate, and to a certain extent evaluate, the risks associated with changing political conditions in countries where portfolio managers may choose to assume exposure. Although this topic merits more discussion than can reasonably be devoted in this space, we can broadly define two types of exposures.

**Sovereign risk** is a form of credit risk in which the borrower is the government of a sovereign nation. Like other forms of credit risk, it has a current and a potential component, and like other forms, its magnitude has two components: the likelihood of default and the estimated recovery rate. Of course, the task of evaluating sovereign risk is in some ways more complex than that of evaluating other types of credit exposure because of the additional political component involved. Like other types of borrowers, debtor nations have an asset/liability/cash flow profile that competent analysts can evaluate. In addition to this profile, however, lenders to sovereigns (including bondholders) must consider everything from the country's willingness to meet its credit obligations (particularly in unstable political environments) to its alternative means of financing (seeking help from outside entities such as the International Monetary Fund, imposing capital controls, etc.) and other measures it might take, such as currency devaluation, to stabilize its situation.

The presence of sovereign risk is real and meaningful, and perhaps the most salient example of its deleterious effects can be found in Russia's 1998 default. This episode represented the first time in many decades that a nation of such size and stature failed to meet its obligations to its lenders. Moreover, although the country was experiencing considerable trauma at that time—in part as the result of a contagion in emerging markets—it is abundantly clear that Russia was *unwilling* rather than *unable* to meet these obligations. The end result was a global financial crisis, in which investors lost billions of dollars and the country's robust development arc was slowed down for the better part of a decade.

<sup>17</sup>Gastineau, Smith, and Todd (2001) provides excellent information on accounting for derivatives in the United States.

**Political risk** is associated with changes in the political environment. Political risk can take many forms, both overt (e.g., the replacement of a pro-capitalist regime with one less so) and subtle (e.g., the potential impact of a change in party control in a developed nation), and it exists in every jurisdiction where financial instruments trade.

#### 4.12. Other Risks

Companies face nonfinancial and financial risks other than those already mentioned. **ESG risk** is the risk to a company's market valuation resulting from environmental, social, and governance factors. Environmental risk is created by the operational decisions made by the company managers, including decisions concerning the products and services to offer and the processes to use in producing those products and services. Environmental damage may lead to a variety of negative financial and other consequences. Social risk derives from the company's various policies and practices regarding human resources, contractual arrangements, and the workplace. Liability from discriminatory workplace policies and the disruption of business resulting from labor strikes are examples of this type of risk. Flaws in corporate governance policies and procedures increase governance risk, with direct and material effects on a company's value in the marketplace.

One little-discussed but very large type of risk that some investment companies face is that of performance netting risk, often referred to simply as netting risk. **Performance netting risk**, which applies to entities that fund more than one strategy, is the potential for loss resulting from the failure of fees based on net performance to fully cover contractual payout obligations to individual portfolio managers that have positive performance when other portfolio managers have losses and when there are asymmetric incentive fee arrangements with the portfolio managers. The problem is best explained through an example.

Consider a hedge fund that charges a 20% incentive fee of any positive returns and funds two strategies equally, each managed by independent portfolio managers (call them Portfolio Managers A and B). The hedge fund pays Portfolio Managers A and B 10% of any gains they achieve. Now assume that in a given year, Portfolio Manager A makes \$10 million and Portfolio Manager B loses the same amount. The net incentive fee to the hedge fund is zero because it has generated zero returns. Unless otherwise negotiated, however (and such clauses are rare), the hedge fund remains obligated to pay Portfolio Manager A \$1 million. As a result, the hedge fund company has incurred a loss, despite breaking even overall in terms of returns.<sup>18</sup> Note that the asymmetric nature of incentive fee contracts (i.e., losses are not penalized as gains are rewarded) plays a critical role in creating the problem the hedge fund faces. Because such arrangements are effectively a call option on a percentage of profits, in some circumstances they may provide an incentive to take excessive risk (the value of a call option is positively related to the underlying's volatility). Nevertheless, such arrangements are widespread.

Performance netting risk occurs only in multistrategy, multimanager environments and only manifests itself when individual portfolio managers within a jointly managed product generate actual losses over the course of a fee-generating cycle—typically one year. Moreover, an investment entity need not be flat or down on the year to experience netting-associated

<sup>18</sup>The asymmetric nature of the incentive fee contract (currently typical for hedge funds) plays a critical role in this example; were the arrangement symmetric, with negative returns penalized as positive returns are rewarded, the issue discussed would disappear.

losses. For any given level of net returns, its portion of fees will be higher if all portfolio managers generate no worse than zero performance over the period than they would if some portfolio managers generate losses. As mentioned earlier, an asymmetric incentive fee contract must exist for this problem to arise.

Performance netting risk applies not just to hedge funds but also to banks' and broker/dealers' trading desks, commodity trading advisors, and indeed, to any environment in which individuals have asymmetric incentive fee arrangements but the entity or unit responsible for paying the fees is compensated on the basis of net results. Typically this risk is managed through a process that establishes absolute negative performance thresholds for individual accounts and aggressively cuts risk for individual portfolio managers at performance levels at, near, or below zero for the period in question.<sup>19</sup>

Distinct from performance netting risk, **settlement netting risk** (or again, simply netting risk) refers to the risk that a liquidator of a counterparty in default could challenge a netting arrangement so that profitable transactions are realized for the benefit of creditors.<sup>20</sup> Such risk is mitigated by netting agreements that can survive legal challenge.

## 5. MEASURING RISK

Having spent some time identifying some of the major sources of risk, both financial and nonfinancial, we now turn our attention toward the measurement of those risks. In particular, we look at some techniques for measuring market risk and credit risk. Subsequently, we briefly survey some of the issues for measuring nonfinancial risk, a very difficult area but also a very topical one—particularly after the advent of the Basel II standards on risk management for international banks, which we will discuss.

### 5.1. Measuring Market Risk

Market risk refers to the exposure associated with actively traded financial instruments, typically those whose prices are exposed to the changes in interest rates, exchange rates, equity prices, commodity prices, or some combination thereof.<sup>21</sup>

Over the years, financial theorists have created a simple and finite set of statistical tools to describe market risk. The most widely used and arguably the most important of these is the standard deviation of price outcomes associated with an underlying asset. We usually refer to this measure as the asset's **volatility**, typically represented by the Greek letter sigma ( $\sigma$ ). Volatility is often an adequate description of portfolio risk, particularly for those portfolios composed of instruments with linear payoffs.<sup>22</sup> In some applications, such as indexing, volatility relative to a benchmark is paramount. In those cases, our focus should be on the volatility of the deviation of a portfolio's returns in excess of a stated benchmark portfolio's returns, known as **active risk, tracking risk**, tracking error volatility, or by some simply as tracking error.

<sup>19</sup>For more information on this topic, see Grant (2004).

<sup>20</sup>See [www.foa.co.uk/documentation/netting/index.jsp](http://www.foa.co.uk/documentation/netting/index.jsp).

<sup>21</sup>The definition of market risk given here is the one used in the practice of risk management. The term market risk, however, is often used elsewhere to refer to the risk of the market as a whole, which is usually known as systematic risk. In this reading, we define market risk as risk management professionals do.

<sup>22</sup>The contrast is with instruments such as options that have nonlinear or piecewise linear payoffs. See Chance (2003) for more on the payoff functions of options.

As we will see shortly, the volatility associated with individual positions, in addition to being a very useful risk management metric in its own right, can be combined with other simple statistics, such as correlations, to form the building blocks for the portfolio-based risk management systems that have become the industry standard in recent years. We cover these systems in the next section of this reading.

A portfolio's exposure to losses because of market risk typically takes one of two forms: sensitivity to adverse movements in the value of a key variable in valuation (primary or first-order measures of risk) and risk measures associated with *changes in* sensitivities (secondary or second-order measures of risk). Primary measures of risk often reflect linear elements in valuation relationships; secondary measures often take account of curvature in valuation relationships. Each asset class (e.g., bonds, foreign exchange, equities) has specific first- and second-order measures.

Let us consider measures of primary sources of risk first. For a stock or stock portfolio, **beta** measures sensitivity to market movements and is a linear risk measure. For bonds, **duration** measures the sensitivity of a bond or bond portfolio to a small parallel shift in the yield curve and is a linear measure, as is **delta** for options, which measures an option's sensitivity to a small change in the value of its underlying. These measures all reflect the expected change in price of a financial instrument for a unit change in the value of another instrument.

Second-order measures of risk deal with the change in the price sensitivity of a financial instrument and include convexity for fixed-income portfolios and gamma for options. **Convexity** measures how interest rate sensitivity changes with changes in interest rates.<sup>23</sup> **Gamma** measures the delta's sensitivity to a change in the underlying's value. Delta and gamma together capture first- and second-order effects of a change in the underlying.

For options, two other major factors determine price: volatility and time to expiration, both first-order or primary effects. Sensitivity to volatility is reflected in **vega**, the change in the price of an option for a change in the underlying's volatility. Most early option-pricing models (e.g., the Black–Scholes–Merton model) assume that volatility does not change over the life of an option, but in fact, volatility does generally change. Volatility changes are sometimes easy to observe in markets: Some days are far more volatile than others. Moreover, new information affecting the value of an underlying instrument, such as pending product announcements, will discernibly affect volatility. Because of their nonlinear payoff structure, options are typically very responsive to a change in volatility. Swaps, futures, and forwards with linear payoff functions are much less sensitive to changes in volatility. Option prices are also sensitive to changes in time to expiration, as measured by **theta**, the change in price of an option associated with a one-day reduction in its time to expiration.<sup>24</sup> Theta, like vega, is a risk that is associated exclusively with options. Correlation is a source of risk for certain types of options—for example, options on more than one underlying (when the correlations between the underlyings' returns constitute a risk variable).<sup>25</sup>

Having briefly reviewed traditional notions of market risk measurement, we introduce a new topic, one that took the industry by storm: value at risk.

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<sup>23</sup>Convexity is covered in some detail in Fabozzi (2004a), Chapter 7.

<sup>24</sup>For more information on theta, see Chance (2003).

<sup>25</sup>For more information, see Chance (2003).

## 5.2. Value at Risk

During the 1990s, value at risk—or VaR, as it is commonly known—emerged as the financial service industry's premier risk management technique.<sup>26</sup> JPMorgan (now JPMorgan Chase) developed the original concept for internal use but later published the tools it had developed for managing risk (as well as related information).<sup>27</sup> Probably no other risk management topic has generated as much attention and controversy as has value at risk. In this section, we take an introductory look at VaR, examine an application, and look at VaR's strengths and limitations.

VaR is a probability-based measure of loss potential for a company, a fund, a portfolio, a transaction, or a strategy. It is usually expressed either as a percentage or in units of currency. Any position that exposes one to loss is potentially a candidate for VaR measurement. VaR is most widely and easily used to measure the loss from market risk, but it can also be used—subject to much greater complexity—to measure the loss from credit risk and other types of exposures.

We have noted that VaR is a probability-based measure of loss potential. This definition is very general, however, and we need something more specific. More formally: **Value at risk (VaR)** is an estimate of the loss (in money terms) that we expect to be exceeded with a given level of probability over a specified time period.<sup>28</sup>

Readers are encouraged to think very carefully about the implications of this definition, which has a couple of important elements. First, we see that VaR is an estimate of the loss that we expect to be exceeded. Hence, it measures a minimum loss. The actual loss may be much worse without necessarily impugning the VaR model's accuracy. Second, we see that VaR is associated with a given probability. Say the VaR is €10,000,000 at a probability of 5% for a given time period. All else equal, if we lower the probability from 5% to 1%, the VaR will be larger in magnitude because we now are referring to a loss that we expect to be exceeded with only a 1% probability. Third, we see that VaR has a time element and that as such, VaRs cannot be compared directly unless they share the same time interval. There is a big difference among potential losses that are incurred daily, weekly, monthly, quarterly, or annually. Potential losses over longer periods should be larger than those over shorter periods, but in most instances, longer time periods will not increase exposure in a linear fashion.

Consider the following example of VaR for an investment portfolio: *The VaR for a portfolio is \$1.5 million for one day with a probability of 0.05.* Recall what this statement says: *There is a 5% chance that the portfolio will lose at least \$1.5 million in a single day.* The emphasis here should be on the fact that *the \$1.5 million loss is a minimum.* With due care, it is also possible to describe VaR as a maximum: The probability is 95% that the portfolio will lose no more than \$1.5 million in a single day. We see this equivalent perspective in the common practice of stating VaR using a confidence level: For the example just given, we would say that *with 95% confidence (or for a 95% confidence level), the VaR for a portfolio is \$1.5 million*

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<sup>26</sup>The terminology “Value-at-Risk” is expressed in different ways. For example, sometimes hyphens are used and sometimes it is just written as “Value at Risk.” Sometimes it is abbreviated as VAR and sometimes as VaR. Those who have studied econometrics should be alert to the fact that the letters VAR also refer to an estimation technique called Vector Autoregression, which has nothing to do with value at risk. We shall use the abbreviation “VaR.”

<sup>27</sup>RiskMetrics Group has now spun off from JPMorgan and is an independent company. See [www.riskmetrics.com](http://www.riskmetrics.com).

<sup>28</sup>In the terminology of statistics, VaR with an  $x$  percent probability for a given time interval represents the  $x$ th percentile of the distribution of outcomes (ranked from worst to best) over that time period.

for one day.<sup>29</sup> We prefer to express VaR in the form of a minimum loss with a given probability. This approach is a bit more conservative, because it reminds us that the loss could be worse.<sup>30</sup>

### 5.2.1. Elements of Measuring Value at Risk

Although VaR has become an industry standard, it may be implemented in several forms, and establishing an appropriate VaR measure requires the user to make a number of decisions about the calculation's structure. Three important ones are picking a probability level, selecting the time period over which to measure VaR, and choosing the specific approach to modeling the loss distribution.<sup>31</sup>

The probability chosen is typically either 0.05 or 0.01 (corresponding to a 95% or 99% confidence level, respectively). The use of 0.01 leads to a more conservative VaR estimate, because it sets the figure at the level where there should be only a 1% chance that a given loss will be worse than the calculated VaR. The trade-off, however, is that the VaR risk estimate will be much larger with a 0.01 probability than it will be for a 0.05 probability. In the above example, we might have to state that the VaR is \$2.1 million for one day at a probability of 0.01. The risk manager selects 0.01 or 0.05; no definitive rule exists for preferring one probability to the other. For portfolios with largely linear risk characteristics, the two probability levels will provide essentially identical information. However, the tails of the loss distribution may contain a wealth of information for portfolios that have a good deal of optionality or nonlinear risks, and in these cases risk managers may need to select the more conservative probability threshold.

The second important decision for VaR users is choosing the time period. VaR is often measured over a day, but other, longer time periods are common. Banking regulators prefer two-week period intervals. Many companies report quarterly and annual VaRs to match their performance reporting cycles. Investment banks, hedge funds, and dealers seem to prefer daily VaR, perhaps because of the high turnover in their positions. Regardless of the time interval selected, the longer the period, the greater the VaR number will be because the magnitude of potential losses varies directly with the time span over which they are measured. The individual or individuals responsible for risk management will choose the time period.

Once these primary parameters are set, one can proceed to actually obtain the VaR estimate. This procedure involves another decision: the choice of technique. The basic idea behind estimating VaR is to identify the probability distribution characteristics of portfolio returns. Consider the information in Exhibit 4, which is a simple probability distribution for the return on a portfolio over a specified time period. Suppose we were interested in the VaR at a probability of 0.05. We would add up the probabilities for the class intervals until we reached a cumulative probability of 0.05. Observe that the probability is 0.01 that the portfolio will lose at least 40%, 0.01 that the portfolio will lose between 30% and 40%, and 0.03 that the portfolio will lose between 20% and 30%. Thus, the probability is 0.05 that the portfolio will lose at least 20%. Because we want to express our risk measure in units of money, we would then multiply 20% by the portfolio's initial market value to obtain VaR. The VaR for a probability of 0.01 would be 40% multiplied by the market value. From a confidence-level perspective,

<sup>29</sup>This would be referred to as 95% one-day VaR.

<sup>30</sup>For a long position, the maximum possible loss is the entire value of the portfolio. For a short position, or a portfolio with both long and short positions, it is impossible to state the maximum possible loss because at least in theory, a short faces the possibility of unlimited losses.

<sup>31</sup>As we will learn in this section, users can select from three basic VaR methodologies, each of which uses a slightly different algorithm to estimate exposure.

we estimate with 99% confidence that our portfolio will lose no more than 40% of its value over the specified time period.

EXHIBIT 4 Sample Probability Distribution of Returns on a Portfolio

Return on Portfolio	Probability
Less than -40%	0.010
-40% to -30%	0.010
-30% to -20%	0.030
-20% to -10%	0.050
-10% to -5%	0.100
-5% to -2.5%	0.125
-2.5% to 0%	0.175
0% to 2.5%	0.175
2.5% to 5%	0.125
5% to 10%	0.100
10% to 20%	0.050
20% to 30%	0.030
30% to 40%	0.010
Greater than 40%	0.010
	1.000

Exhibit 4 offers a simplified representation of the information necessary to estimate VaR. This method for calculating VaR is rather cumbersome, and the information is not always easy to obtain. As such, the industry has developed a set of three standardized methods for estimating VaR: the analytical or variance–covariance method, the historical method, and the Monte Carlo simulation method. We will describe and illustrate each of these in turn.

### 5.2.2. The Analytical or Variance–Covariance Method

The analytical or variance–covariance method begins with the assumption that portfolio returns are normally distributed. Recall from your study of portfolio management that a normal distribution can be completely described by its expected value and standard deviation.

Consider the standard normal distribution, a special case of the normal distribution centered on an expected value of zero and having a standard deviation of 1.0. We can convert any outcome drawn from a nonstandard normal distribution to a standard normal value by taking the outcome of interest, subtracting its mean, and dividing the result by its standard deviation. The resulting value then conforms to the standard normal distribution.<sup>32</sup> With the standard normal distribution, 5% of possible outcomes are likely to be smaller than -1.65.<sup>33</sup> Therefore,

<sup>32</sup>For example, suppose you were interested in knowing the probability of obtaining a return of -15% or less when the expected return is 12% and the standard deviation is 20%. You would calculate the standard normal value, called a “z”, as  $(-0.15 - 0.12)/0.20 = -1.35$ . Then you would look up this value in a table or use a spreadsheet function, such as Microsoft Excel’s “=normsdist()” function. In this case, the probability is 0.0885.

<sup>33</sup>See DeFusco, McLeavey, Pinto, and Runkle (2004), pp. 255–56.

to calculate a 5% VaR for a portfolio (i.e., VaR at a probability of 0.05), we would estimate its expected return and subtract 1.65 times its estimated standard deviation of returns. So, the key to using the analytical or variance–covariance method is to estimate the portfolio's expected return and standard deviation of returns. An example follows.<sup>34</sup>

Suppose the portfolio contains two asset classes, with 75% of the money invested in an asset class represented by the S&P 500 Index and 25% invested in an asset class represented by the NASDAQ Composite Index.<sup>35</sup> Recall that a portfolio's expected return is a weighted average of the expected returns of its component stocks or asset classes. A portfolio's variance can be derived using a simple quadratic formula that combines the variances and covariances of the component stocks or asset classes. For example, assume that  $\mu_S$  and  $\mu_N$  are the expected returns of the S&P 500 and NASDAQ, respectively;  $\sigma_S$  and  $\sigma_N$  are their standard deviations; and  $\rho$  is the correlation between the two asset classes. The expected return,  $\mu_p$ , and variance,  $\sigma_p^2$ , of the combined positions are given as

$$\mu_p = w_S\mu_S + w_N\mu_N$$

$$\sigma_p^2 = w_S^2\sigma_S^2 + w_N^2\sigma_N^2 + 2\rho w_S w_N \sigma_S \sigma_N$$

where  $w$  indicates the percentage allocated to the respective classes. The portfolio's standard deviation is just the square root of its variance. Exhibit 5 provides estimates of the portfolio's expected value and standard deviation using actual numbers, where we obtain  $\mu_p$  of 0.135 and  $\sigma_p$  of 0.244.

**EXHIBIT 5** Estimating the Expected Return and Standard Deviation of a Portfolio Combining Two Asset Classes

	S&P 500	NASDAQ	Combined Portfolio
Percentage invested ( $w$ )	0.75	0.25	1.00
Expected annual return ( $\mu$ )	0.12	0.18	0.135 <sup>a</sup>
Standard deviation ( $\sigma$ )	0.20	0.40	0.244 <sup>b</sup>
Correlation ( $\rho$ )	0.90		

<sup>a</sup>Expected return of portfolio:  $\mu_p = w_S\mu_S + w_N\mu_N = 0.75(0.12) + 0.25(0.18) = 0.135$

<sup>b</sup>Standard deviation of portfolio:

$$\sigma_p^2 = w_S^2\sigma_S^2 + w_N^2\sigma_N^2 + 2\rho w_S w_N \sigma_S \sigma_N$$

$$= (0.75)^2(0.20)^2 + (0.25)^2(0.40)^2 + 2(0.90)(0.75)(0.25)(0.20)(0.40) = 0.0595$$

$$\sigma_p = (\sigma_p^2)^{1/2} = (0.0595)^{1/2} = 0.244$$

Note that the example provided above is quite simplistic, involving only two assets, and thus only two variances and one covariance. As such, the calculation of portfolio variance is relatively manageable. As the number of instruments in the portfolio increases, however, the

<sup>34</sup>For more detailed information, see DeFusco et al. (2004), Chapter 11.

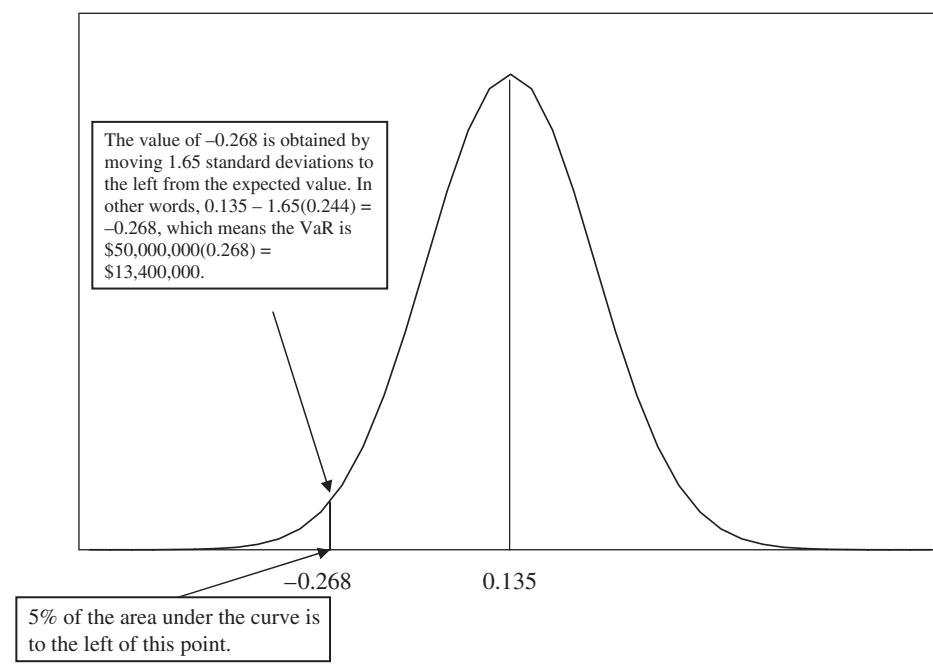
<sup>35</sup>The extension to three or more classes is relatively straightforward once one knows how to calculate the variance of a portfolio of more than two assets. We shall focus here on the two-asset-class case.

calculation components expand dramatically and the equation quickly becomes unwieldy. The important thing to remember is that in order to derive the variance for a portfolio of multiple financial instruments, all we require are the associated variances and covariances, along with the ability to calculate their quadratic relationship.

If we are comfortable with the assumption of a normal distribution and the accuracy of our estimates of the expected returns, variances, and correlations, we can confidently use the analytical-method estimate of VaR. Exhibit 6 illustrates the calculation of this estimate. VaR is first expressed in terms of the return on the portfolio. With an expected return of 0.135, we move 1.65 standard deviations along the  $x$ -axis in the direction of lower returns. Each standard deviation is 0.244. Thus we would obtain  $0.135 - 1.65(0.244) = -0.268$ .<sup>36</sup> At this point, VaR could be expressed as a loss of 26.8%. We could say that there is a 5% chance that the portfolio will lose at least 26.8% in a year. It is also customary to express VaR in terms of the portfolio's currency unit. Therefore, if the portfolio is worth \$50 million, we can express VaR as  $\$50,000,000(-0.268) = \$13.4$  million.

This figure is an annual VaR. If we prefer a daily VaR, we can adjust the expected return to its daily average of approximately  $0.135/250 = 0.00054$  and the standard deviation to its daily value of  $0.244/\sqrt{250} = 0.01543$ , which are based on the assumption of 250 trading days in a year and statistical independence between days. Then the daily VaR is  $0.00054 - 1.65(0.01543) = -0.0249$ . On a dollar basis, the daily VaR is  $\$50,000,000(0.0249) = \$1.245$  million.

EXHIBIT 6 Annual VaR for a Portfolio with Expected Return of 0.135 and Standard Deviation of 0.244



<sup>36</sup>The reader can confirm that 1.65 and 2.33 standard deviations give the correct VaR at the 5% and 1 % probability levels, respectively, using the Microsoft Excel function “=normsdist()”.

For a 1% VaR, we would move 2.33 standard deviations in the direction of lower returns. Thus the annual VaR would be  $0.135 - 2.33(0.244) = -0.434$  or  $\$50,000,000(0.434) = \$21.7$  million. The daily VaR would be  $0.00054 - 2.33(0.01543) = -0.035$  or  $\$50,000,000(0.035) = \$1.75$  million.

Some approaches to estimating VaR using the analytical method assume an expected return of zero. This assumption is generally thought to be acceptable for daily VaR calculations because expected daily return will indeed tend to be close to zero. Because expected returns are typically positive for longer time horizons, shifting the distribution by assuming a zero expected return will result in a larger projected loss, so the VaR estimate will be greater. Therefore, this small adjustment offers a slightly more conservative result and avoids the problem of having to estimate the expected return, a task typically much harder than that of estimating associated volatility. Another advantage of this adjustment is that it makes it easier to adjust the VaR for a different time period. For example, if the daily VaR is estimated at \$100,000, the annual VaR will be  $\$100,000\sqrt{250} = \$1,581,139$ . This simple conversion of a shorter-term VaR to a longer-term VaR (or vice versa) does not work, however, if the average return is not zero. In these cases, one would have to convert the average return and standard deviation to the different time period and compute the VaR from the adjusted average and standard deviation.

### EXAMPLE 5 VaR with Different Probability Levels and Time Horizons

Consider a portfolio consisting of stocks as one asset class and bonds as another. The expected return on the portfolio's stock portion is 12%, and the standard deviation is 22%. The expected return on the bond portion is 5%, and the standard deviation is 7%. All of these figures are annual. The correlation between the two asset classes is 0.15. The portfolio's market value is \$150 million and is allocated 65% to stocks and 35% to bonds. Determine the VaR using the analytical method for the following cases:

1. a 5% yearly VaR.
2. a 1% yearly VaR.
3. a 5% weekly VaR.
4. a 1% weekly VaR.

*Solutions:* First, we must calculate the annual portfolio expected return and standard deviation. Using  $S$  to indicate stocks and  $B$  to indicate bonds, we have

$$\begin{aligned}\mu_P &= w_S\mu_S + w_B\mu_B = 0.65(0.12) + 0.35(0.05) = 0.0955 \\ \sigma_P^2 &= w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2\rho w_S w_B \sigma_S \sigma_B \\ &= (0.65)^2(0.22)^2 + (0.35)^2(0.07)^2 + 2(0.15)(0.65)(0.35)(0.22)(0.07) \\ &= 0.0221 \\ \sigma_P &= \sqrt{0.0221} = 0.1487\end{aligned}$$

*Solution to 1:* For a 5% yearly VaR, we have  $\mu_p - 1.65\sigma_p = 0.0955 - 1.65(0.1487) = -0.1499$ . Then the VaR is  $\$150,000,000(0.1499) = \$22.485$  million.

*Solution to 2:* For a 1% yearly VaR, we have  $\mu_p - 2.33\sigma_p = 0.0955 - 2.33(0.1487) = -0.251$ . Then the VaR is  $\$150,000,000(0.251) = \$37.65$  million.

*Solution to 3:* For weekly VaR, we adjust the expected return to  $0.0955/52 = 0.00184$  and the standard deviation to  $0.1487/\sqrt{52} = 0.02062$ .

The 5% weekly VaR is then  $\mu_p - 1.65\sigma = 0.00184 - 1.65(0.02062) = -0.03218$ . Then the VaR is  $\$150,000,000(0.03218) = \$4.827$  million.

*Solution to 4:* The 1% weekly VaR is  $\mu_p - 2.33\sigma_p = 0.00184 - 2.33(0.02062) = -0.0462$ . Then the VaR is  $\$150,000,000(0.0462) = \$6.93$  million.

The analytical or variance–covariance method’s primary advantage is its simplicity. Its primary disadvantage is its reliance on several simplifying assumptions, including the normality of return distributions. In principle, there is no reason why the calculation demands a normal distribution, but if we move away from the normality assumption, we cannot rely on variance as a complete measure of risk. Distributions can deviate from normality because of skewness and kurtosis. Skewness is a measure of a distribution’s deviation from the perfect symmetry (the normal distribution has a skewness of zero). A positively skewed distribution is characterized by relatively many small losses and a few extreme gains and has a long tail on its right side. A negatively skewed distribution is characterized by relatively many small gains and a few extreme losses and has a long tail on its left side. When a distribution is positively or negatively skewed, the variance–covariance method of estimating VaR will be inaccurate.

In addition, many observed distributions of returns have an abnormally large number of extreme events. This quality is referred to in statistical parlance as leptokurtosis but is more commonly called the property of fat tails.<sup>37</sup> Equity markets, for example, tend to have more frequent large market declines than a normal distribution would predict. Therefore, using a normality assumption to estimate VaR for a portfolio that features fat tails could underestimate the actual magnitude and frequency of large losses. VaR would then fail at precisely what it is supposed to do: measure the risk associated with large losses.

A related problem that surfaces with the analytical or variance–covariance method is that the normal distribution assumption is inappropriate for portfolios that contain options. The return distributions of options portfolios are often far from normal. Remember that a normal distribution has an unlimited upside and an unlimited downside. Call options have unlimited upside potential, as in a normal distribution, but their downside is a fixed value (the call’s

<sup>37</sup>See DeFusco, McLeavey, Pinto, and Runkle (2004), Chapter 5.

premium) and the distribution of call returns is highly skewed. Put options have a large but limited upside and a fixed downside (the put's premium), and the distribution of put returns is also highly skewed. In the same vein, covered calls and protective puts have return distributions that are sharply skewed in one direction or the other.

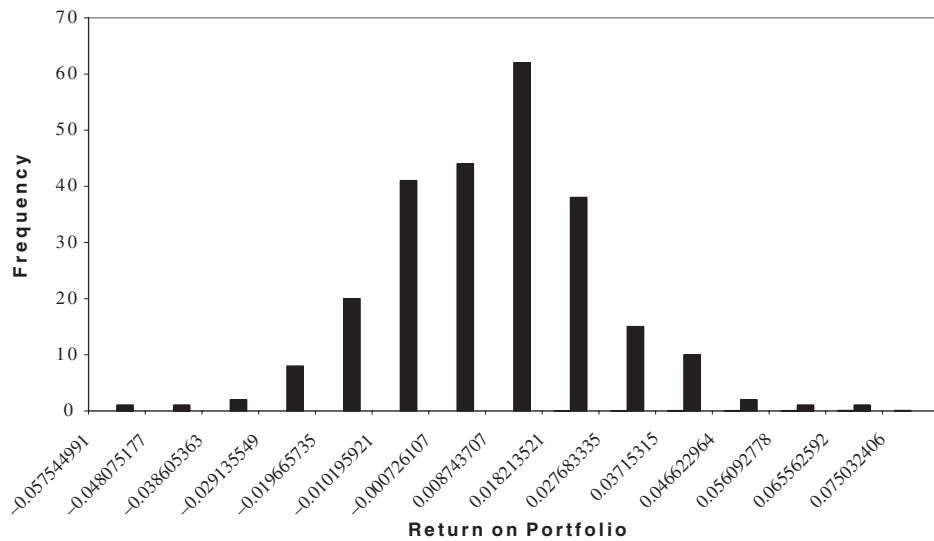
Therefore, when portfolios contain options, the assumption of a normal distribution to estimate VaR presents a significant problem. One common solution is to estimate the option's price sensitivity using its delta. Recall that delta expresses a linear relationship between an option's price and the underlying's price (i.e.,  $\text{Delta} = \text{Change in option price}/\text{Change in underlying}$ ). A linear relationship lends itself more easily to treatment with a normal distribution. That is, a normally distributed random variable remains normally distributed when multiplied by a constant. In this case, the constant is the delta. The change in the option price is assumed to equal the change in the underlying price multiplied by the delta. This trick converts the normal distribution for the return on the underlying into a normal distribution for the option return. As such, the use of delta to estimate the option's price sensitivity for VaR purposes has led some to call the analytical method (or variance-covariance method) the **delta-normal method**. The use of delta is appropriate only for small changes in the underlying, however. As an alternative, some users of the delta-normal method add the second-order effect, captured by gamma. Unfortunately, as these higher-order effects are added, the relationship between the option price and the underlying's price begins to approximate the true nonlinear relationship. At that point, using a normal distribution becomes completely inappropriate. Therefore, using the analytical method could cause problems if a portfolio has options or other financial instruments that do not follow the normal distribution. Moreover, it is often difficult, if not impossible, to come up with a single second-order estimate that both is accurate and fits seamlessly into a variance/covariance VaR model.

### 5.2.3. The Historical Method

Another widely used VaR methodology is the historical method. Using historical VaR, we calculate returns for a given portfolio using actual daily prices from a user-specified period in the recent past, graphing these returns into a histogram. From there, it becomes easy to identify the loss that is exceeded with a probability of 0.05 (or 0.01%, if preferred).

Consider the portfolio we have been examining, consisting of 75% invested in the S&P 500 and 25% invested in the NASDAQ Composite Index. Exhibit 7, a histogram, shows the daily returns on this portfolio for a recent calendar year. First, we note that the distribution is similar, but by no means identical, to that of a normal distribution. This portfolio has a few more returns slightly lower than the midpoint of the return sample than it would if its distribution were perfectly normal. With the historical method, however, we are not constrained to using the normal distribution. We simply collect the historical data and identify the return below which 5 (or 1)% of returns fall. Although we could attempt to read this number from the histogram, it is much easier to simply rank-order the returns and determine the VaR figure from the sorted returns and the portfolio's dollar value.

EXHIBIT 7 Historical Daily Returns on a Portfolio Invested 75% in S&P 500 and 25% in NASDAQ



The year examined here contains 248 returns. Having 5% of the returns in the distribution's lower tail would mean that about 12 return observations should be less than the VaR estimate. Thus the approximate VaR figure would be indicated by the 12th-worst return. A rank ordering of the data reveals that the 12th-worst return is  $-0.0294$ . For a \$50,000,000 portfolio, the one-day VaR would thus be  $0.0294(\$50,000,000) = \$1.47$  million.<sup>38</sup>

The historical method is also sometimes called the **historical simulation method**. This term is somewhat misleading because the approach involves not a *simulation* of the past returns but rather what *actually happened* in the past. In this context, note that a portfolio that an investor might have held in the past might not be the same as the one that an investor will have in the future. When using the historical method, one must always keep in mind that the purpose of the exercise is to apply *historical* price changes to the *current* portfolio.<sup>39</sup> In addition, instruments such as bonds and most derivatives behave differently at different times in their lives, and any accurate historical VaR calculation must take this into account by adjusting current bond/derivative pricing parameters to simulate their current characteristics across the period of analysis. For example, a historical VaR calculation that goes back one year for a portfolio that contains bonds that mature in the year 2027 should actually use otherwise identical bonds maturing in 2026 as proxies; these bonds are the most accurate representations of the current risk profile because they would have presented themselves one year ago in time. When a company uses

<sup>38</sup>Technically, the VaR would fall between the 12th- and 13th-worst returns. Using the 13th-worst return gives a more conservative VaR. Alternatively, we might average the 12th- and 13th-worst returns.

<sup>39</sup>For example, in the two-asset portfolio we illustrated here, the weights were 75% S&P 500 and 25% NASDAQ. If the company were going forward with a different set of weights, it would obviously need to use the weights it planned to use in the future when calculating the VaR by the historical method.

a different portfolio composition to calculate its historical VaR than the one it actually had in the past, it may be more appropriate to call the method a historical simulation.

The historical method has the advantage of being **nonparametric** (i.e., involving minimal probability-distribution assumptions), enabling the user to avoid any assumptions about the type of probability distribution that generates returns. The disadvantage, however, is that this method relies completely on events of the past, and whatever distribution prevailed in the past might not hold in the future. In particular, periods of unusually large negative returns, such as the 23% one-day decline in the Dow Jones Industrial Average on 19 October 1987, might be questionable as an assumption for the future. This problem applies to the other types of VaR methodologies as well, however, including the analytical method and Monte Carlo simulation, both of which derive their inputs, more often than not, entirely from the historical prices associated with the securities contained in the portfolio.

### EXAMPLE 6 Calculating VaR Using the Historical Method

For simplicity, we use a one-stock portfolio. Exhibit 8 shows the 40 worst monthly returns on IBM stock during the last 20 years, in descending order, as of 2011 (minus signs omitted):

EXHIBIT 8 IBM Stock: Worst Monthly Returns

0.26190	0.11692	0.09077	0.07537
0.22645	0.11553	0.08926	0.07298
0.20511	0.10838	0.08585	0.07260
0.19462	0.10805	0.08481	0.07247
0.18802	0.10687	0.08422	0.07075
0.17183	0.10503	0.08356	0.06894
0.16415	0.09873	0.08234	0.06782
0.14834	0.09550	0.08197	0.06746
0.14773	0.09276	0.08143	0.06501
0.12444	0.09091	0.07547	0.06437

For both calculations below, assume the portfolio value is \$100,000.

1. Calculate a 5% monthly VaR using the historical method.
2. Calculate a 1% monthly VaR using the historical method.

*Solutions:* First, we note that during the last 20 years, there were 240 monthly returns. We see here only the worst 40 returns. Therefore, although we lack the entire distribution of returns, we do have enough to calculate the VaR.

*Solution to 1:* Out of 240 returns, the 5% worst are the 12 worst returns. Therefore, the historical VaR would be about the 12th-worst return. From the exhibit, we see that this return is  $-0.11553$ . So, the one-month VaR is  $0.11553(\$100,000) = \$11,553$ .

*Solution to 2:* The 1% worst returns include 2.4 returns. We would probably use the second-worst return, which is  $-0.22645$ . The VaR is  $0.22645(\$100,000) = \$22,645$ . Alternatively, we might average the second- and third-worst returns to obtain  $(-0.22645 + -0.20511)/2 = -0.21578$ . Then the one-month VaR would be  $0.21578(\$100,000) = \$21,578$ .

The excerpt from The Goldman Sachs Group, Inc. Form 10-K that follows in Example 7 shows how this firm reports its VaR. We see that Goldman Sachs reports average daily VaR (at a 95% confidence level) for its last three fiscal years. In addition, the firm reports the high and low daily VaR values for the last fiscal year along with the year end values for the last two fiscal years. Goldman Sachs reports VaRs for these four risk categories (interest rate, equity prices, currency rates, and commodity prices) as well as its firm-wide risk exposure (total VaR). Total VaR is less than the sum of the individual VaRs because Goldman Sachs' exposures to the various risk categories are less than perfectly correlated. The **diversification effect** reported in the Average Daily VaR table in Example 7 equals the difference between the total VaR and the sum of the individual VaRs. For example, for 2010, the diversification effect is  $\$134 - (\$93 + \$68 + \$32 + \$33) = -\$92$ .

### EXAMPLE 7 Value at Risk and the Management of Market Risk at Goldman Sachs

*The following excerpt is from the 2010 Form 10-K of Goldman Sachs:*

#### *Value at Risk*

VaR is the potential loss in value of inventory positions due to adverse market movements over a defined time horizon with a specified confidence level. We typically employ a one-day time horizon with a 95% confidence level. Thus, we would expect to see reductions in the fair value of inventory positions at least as large as the reported VaR once per month. The VaR model captures risks including interest rates, equity prices, currency rates and commodity prices. As such, VaR facilitates comparison across portfolios of different risk characteristics. VaR also captures the diversification of aggregated risk at the firmwide level.

Inherent limitations to VaR include:

- VaR does not estimate potential losses over longer time horizons where moves may be extreme.
- VaR does not take account of the relative liquidity of different risk positions.
- Previous moves in market risk factors may not produce accurate predictions of all future market moves.

The historical data used in our VaR calculation is weighted to give greater importance to more recent observations and reflect current asset volatilities. This improves the accuracy of our estimates of potential loss. As a result, even if our inventory positions were unchanged, our VaR would increase with increasing market volatility and vice versa.

Given its reliance on historical data, VaR is most effective in estimating risk exposures in markets in which there are no sudden fundamental changes or shifts in market conditions.

We evaluate the accuracy of our VaR model through daily backtesting (i.e., comparing daily trading net revenues to the VaR measure calculated as of the prior business day) at the firmwide level and for each of our businesses and major regulated subsidiaries.

VaR does not include:

- positions that are best measured and monitored using sensitivity measures; and
- the impact of changes in counterparty and our own credit spreads on derivatives as well as changes in our own credit spreads on unsecured borrowings for which the fair value option was elected.

#### *Stress Testing*

We use stress testing to examine risks of specific portfolios as well as the potential impact of significant risk exposures across the firm. We use a variety of scenarios to calculate the potential loss from a wide range of market moves on the firm's portfolios. These scenarios include the default of single corporate or sovereign entities, the impact of a move in a single risk factor across all positions (e.g., equity prices or credit spreads) or a combination of two or more risk factors.

Unlike VaR measures, which have an implied probability because they are calculated at a specified confidence level, there is generally no implied probability that our stress test scenarios will occur. Instead, stress tests are used to model both moderate and more extreme moves in underlying market factors. When estimating potential loss, we generally assume that our positions cannot be reduced or hedged (although experience demonstrates that we are generally able to do so).

Stress test scenarios are conducted on a regular basis as part of the firm's routine risk management process and on an ad hoc basis in response to market events or concerns. Stress testing is an important part of the firm's risk management process because it allows us to highlight potential loss concentrations, undertake risk/reward analysis, and assess and mitigate our risk positions.

#### *Limits*

We use risk limits at various levels in the firm (including firmwide, product and business) to govern risk appetite by controlling the size of our exposures to market risk. Limits are reviewed frequently and amended on a permanent or temporary basis to reflect changing market conditions, business conditions or tolerance for risk.

The Firmwide Risk Committee sets market risk limits at firmwide and product levels and our Securities Division Risk Committee sets sub-limits for market-making and investing activities at a business level. The purpose of the firmwide limits is to assist

senior management in controlling the firm's overall risk profile. Sub-limits set the desired maximum amount of exposure that may be managed by any particular business on a day-to-day basis without additional levels of senior management approval, effectively leaving day-to-day trading decisions to individual desk managers and traders. Accordingly, sub-limits are a management tool designed to ensure appropriate escalation rather than to establish maximum risk tolerance. Sub-limits also distribute risk among various businesses in a manner that is consistent with their level of activity and client demand, taking into account the relative performance of each area.

Our market risk limits are monitored daily by Market Risk Management, which is responsible for identifying and escalating, on a timely basis, instances where limits have been exceeded. The business-level limits that are set by the Securities Division Risk Committee are subject to the same scrutiny and limit escalation policy as the firmwide limits.

When a risk limit has been exceeded (e.g., due to changes in market conditions, such as increased volatilities or changes in correlations), it is reported to the appropriate risk committee and a discussion takes place with the relevant desk managers, after which either the risk position is reduced or the risk limit is temporarily or permanently increased.

#### *Metrics*

We analyze VaR at the firmwide level and a variety of more detailed levels, including by risk category, business, and region. The tables below present average daily VaR and year-end VaR by risk category.

Average Daily VaR (in millions)

Risk Categories	Year Ended		
	December 2010	December 2009	November 2008
Interest rates	\$ 93	\$176	\$ 142
Equity prices	68	66	72
Currency rates	32	36	30
Commodity prices	33	36	44
Diversification effect <sup>a</sup>	(92)	(96)	(108)
<b>Total</b>	<b>\$134</b>	<b>\$218</b>	<b>\$ 180</b>

<sup>a</sup>Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.

Our average daily VaR decreased to \$134 million in 2010 from \$218 million in 2009, principally due to a decrease in the interest rates category which was primarily due to reduced exposures, lower levels of volatility and tighter spreads.

Our average daily VaR increased to \$218 million in 2009 from \$180 million in 2008, principally due to an increase in the interest rates category and a reduction in the diversification benefit across risk categories, partially offset by a decrease in the commodity prices category. The increase in the interest rates category was primarily due to wider spreads. The decrease in the commodity prices category was primarily due to lower energy prices.

## Year-End VaR and High and Low VaR (in millions)

Risk Categories	As of December		Year Ended December 2010	
	2010	2009	High	Low
Interest rates	\$ 78	\$ 122	\$123	\$ 76
Equity prices	51	99	186	39
Currency rates	27	21	62	14
Commodity prices	25	33	62	18
Diversification effect <sup>a</sup>	(70)	(122)		
<b>Total</b>	<b>\$111</b>	<b>\$ 153</b>	<b>\$223</b>	<b>\$105</b>

<sup>a</sup>Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.

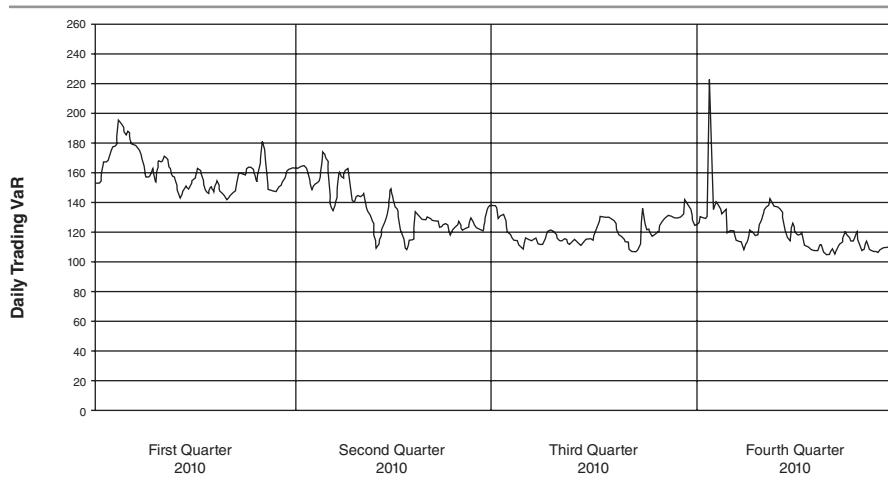
Our daily VaR decreased to \$111 million as of December 2010 from \$153 million as of December 2009, principally due to a decrease in the equity prices and interest rates categories, partially offset by a decrease in the diversification benefit across risk categories. The decreases in the equity prices and interest rates categories were primarily due to reduced exposures and lower levels of volatility.

During the year ended December 2010, the firmwide VaR risk limit was exceeded on one occasion in order to facilitate a client transaction and was resolved by a reduction in the risk position on the following day. Separately, during the year ended December 2010, the firmwide VaR risk limit was reduced on one occasion reflecting lower risk utilization.

During the year ended December 2009, the firmwide VaR risk limit was exceeded on two successive days. It was resolved by a reduction in the risk position without a permanent or temporary VaR limit increase. Separately, during the year ended December 2009, the firmwide VaR risk limit was raised on one occasion and reduced on two occasions as a result of changes in the risk utilization and the market environment.

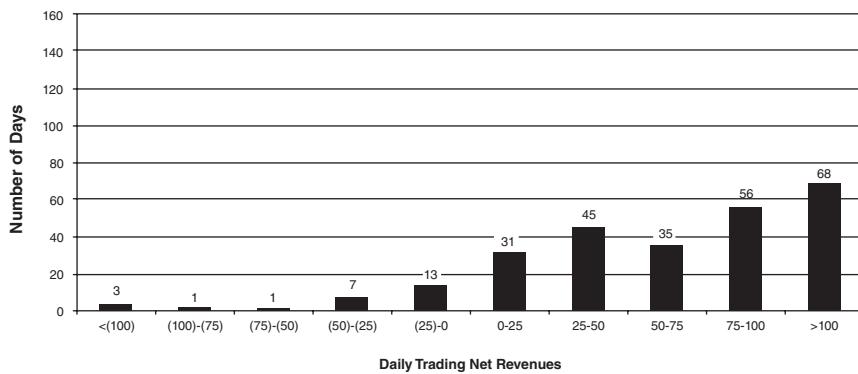
The chart below reflects the VaR over the last four quarters.

## Daily VaR (\$ in millions)



The chart below presents the frequency distribution of our daily trading net revenues for substantially all inventory positions included in VaR for the year ended December 2010.

Daily Trading Net Revenues (\$ in millions)



As noted above, daily trading net revenues are compared with VaR calculated as of the end of the prior business day. Trading losses incurred on a single day exceeded our 95% one-day VaR on two occasions during 2010. Trading losses incurred on a single day did not exceed our 95% one-day VaR during 2009.

*Source:* Goldman Sachs 2010 Form 10-K, pp. 85-87. The Goldman Sachs Group, Inc. All rights reserved.

The next section addresses the third method of estimating VaR, Monte Carlo simulation.

#### 5.2.4. The Monte Carlo Simulation Method

The third approach to estimating VaR is Monte Carlo simulation. In general, Monte Carlo simulation produces random outcomes so we can examine what might happen given a particular set of risks. It is used widely in the sciences as well as in business to study a variety of problems. In the financial world in recent years, it has become an extremely important technique for measuring risk. Monte Carlo simulation generates random outcomes according to an assumed probability distribution and a set of input parameters. We can then analyze these outcomes to gauge the risk associated with the events in question. When estimating VaR, we use Monte Carlo simulation to produce random portfolio returns. We then assemble these returns into a summary distribution from which we can determine at which level the lower 5% (or 1%, if preferred) of return outcomes occur. We then apply this figure to the portfolio value to obtain VaR.

Monte Carlo simulation uses a probability distribution for each variable of interest and a mechanism to randomly generate outcomes according to each distribution. Our goal here is to gain a basic understanding of the technique and how to use it. Therefore, we illustrate it without explaining the full details of how to generate the random values.

Suppose we return to the example of our \$50 million portfolio invested 75% in the S&P 500 and 25% in the NASDAQ Composite Index. We assume, as previously, that this portfolio

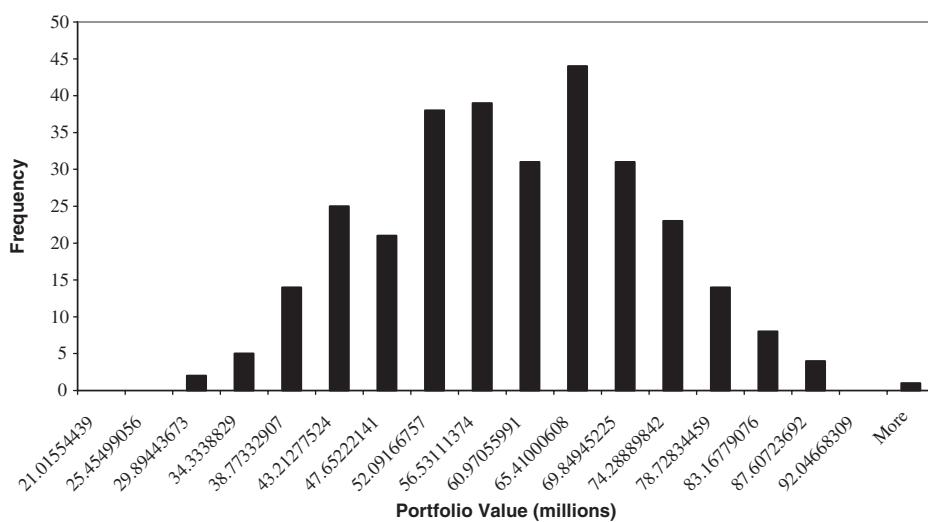
should have an annual expected return of 13.5% and a standard deviation of 24.4%. We shall now conduct a Monte Carlo simulation using the normal distribution with these parameters. Keep in mind that in practice, one advantage of Monte Carlo simulation is that it does not require a normal distribution, but the normal distribution is often used and we shall stay with it for illustrative purposes.

We use a random number generator to produce a series of random values, which we then convert into a normally distributed stream of outcomes representing a rate of return for this portfolio over a period of one year. Suppose the first value it produces is a return of  $-21.87\%$ . This rate corresponds to an end-of-year portfolio value of \$39.07 million. The second random return it produces is  $-4.79\%$ , which takes the portfolio value to \$47.61 million.<sup>40</sup> The third random return it produces is 31.38%, which makes the portfolio value \$65.69 million. We continue this process a large number of times, perhaps several thousand or even several million. To keep the simulation to a manageable size for illustrative purposes, we generate only 300 outcomes.

Exhibit 9 shows the histogram of portfolio outcomes. Notice that even though we used a normal distribution to generate the outcomes, the resulting distribution does not look entirely normal. Of course, we should be surprised if it did because we used only 300 random outcomes, a relatively small sample.

To obtain the point in the lower tail that 5% of the outcomes exceed, we rank order the data and find the 15th-lowest outcome, which is a portfolio value of \$34.25 million, corresponding to a loss of \$15.75 million. This value is higher than the annual VaR estimated using the analytical method (\$13.4 million). These two values would be identical (or nearly so) if we had employed a sufficiently large sample size in the Monte Carlo simulation so that the sample VaR would converge to the true population VaR.

EXHIBIT 9 Simulated Values after One Year for a Portfolio Invested 75% in S&P 500 and 25% in NASDAQ



<sup>40</sup>The random outcomes are independent, not sequential. Each outcome thus represents a return relative to the full initial portfolio value of \$50 million.

In Monte Carlo simulation, we can make any distributional assumption that we believe is appropriate. In many practical applications, it is inappropriate to assume a normal return distribution. In particular, for many derivatives dealers, the problems in managing the risk of these instruments are compounded by the fact that an extremely large number of random variables may affect the value of their overall position. These variables are often not normally distributed, and furthermore, they often interact with each other in complex ways. Monte Carlo simulation is often the only practical means of generating the information necessary to manage the risk.

#### 5.2.5. “Surplus at Risk”: VaR as It Applies to Pension Fund Portfolios

You will recall from earlier points in our discussion that pension funds face a slightly different set of challenges in the measurement of market exposures, primarily because of the fact that the assets must fund pension obligations whose present value is itself subject to interest rate risk and other risks.<sup>41</sup> The difference between the value of the pension fund’s assets and liabilities is referred to as the **surplus**, and it is this value that pension fund managers seek to enhance and protect. If this surplus falls into negative territory, the plan sponsor must contribute funds to make up the deficit over a period of time that is specified as part of the fund’s plan.

In order to reflect this set of realities in their risk estimations, pension fund managers typically apply VaR methodologies not to their portfolio of assets but to the surplus. To do so, they simply express their liability portfolio as a set of short securities and calculate VaR on the net position. VaR handles this process quite elegantly, and once this adjustment is made, all three VaR methodologies can be applied to the task.

### 5.3. The Advantages and Limitations of VaR

Although value at risk has become the industry standard for risk assessment, it also has widely documented imperfections. VaR can be difficult to estimate, and different estimation methods can give quite different values. VaR can also lull one into a false sense of security by giving the impression that the risk is properly measured and under control. VaR often underestimates the magnitude and frequency of the worst returns, although this problem often derives from erroneous assumptions and models. As we discuss later, VaR for individual positions does not generally aggregate in a simple way to portfolio VaR. Also, VaR fails to incorporate positive results into its risk profile, and as such, it arguably provides an incomplete picture of overall exposures.

Users of VaR should routinely test their system to determine whether their VaR estimates prove accurate in predicting the results experienced over time. For example, if daily VaR at 0.05 is estimated at \$1 million, then over a reasonable period of time, such as a year, a loss of at least \$1 million should be exceeded approximately  $250(0.05) = 12.5$  days. If the frequency of losses equal to or greater than this amount is markedly different, then the model is not accomplishing its objectives. This process of comparing the number of violations of VaR thresholds with the figure implied by the user-selected probability level is part of a process known as **backtesting**. It is extremely important to go through this exercise, ideally across multiple time intervals, to ensure that the VaR estimation method adopted is reasonably accurate. For example, if the

<sup>41</sup>An example of a defined-benefit pension plan’s obligation is the promise to pay, for each year of service, a certain percentage of a vested participant’s average salary in their final five years of service; this promise may include cost-of-living adjustments.

VaR estimate is based on daily observations and targets a 0.05 probability, then in addition to ensuring that approximately a dozen threshold violations occur during a given year, it is also useful to check other, shorter time intervals, including the most recent quarter (for which, given 60-odd trading days, we would expect approximately three VaR exceptions—i.e., losses greater than the calculated VaR), and the most recent month (20 observations, implying a single VaR exception). Note that the results should not be expected to precisely match the probability level predictions but should at a minimum be of similar magnitude. If the results vary much from those that the model predicts, then users must examine the reasons and make appropriate adjustments.

An accurate VaR estimate can also be extremely difficult to obtain for complex organizations. In the simple example we used previously, VaR was driven solely by the large- and small-cap US stocks. For a large international bank, however, the exposures might be to a variety of domestic and international interest rate markets, numerous exchange rates, perhaps some equity markets, and even some commodity markets. A bank could have exposure to literally thousands of risks. Consolidating the effects of these exposures into a single risk measure can be extremely difficult. Nonetheless, most large banks manage to do so.

VaR has the attraction of quantifying the potential loss in simple terms and can be easily understood by senior management. Regulatory bodies have taken note of VaR as a risk measure, and some require that institutions provide it in their reports. In the United States, the Securities and Exchange Commission now requires publicly traded companies to report how they are managing financial risk. VaR is one acceptable method of reporting that information.

Another advantage of VaR is its versatility. Many companies use VaR as a measure of their capital at risk. They will estimate the VaR associated with a particular activity, such as a line of business, an individual asset manager, a subsidiary, or a division. Then, they evaluate performance, taking into account the VaR associated with this risky activity. In some cases, companies allocate capital based on VaR. For example, a pension fund might determine its overall acceptable VaR and then inform each asset class manager that it can operate subject to its VaR not exceeding a certain amount. The manager's goal is to earn the highest return possible given its VaR allocation. This activity is known as risk budgeting; we cover it in more detail in a later section.

In summary, VaR has notable advantages and disadvantages. Controversy and criticism have surrounded it.<sup>42</sup> Nevertheless, if a risk manager uses VaR with full awareness of its limitations, he should definitely gain useful information about risk. Even if VaR gives an incorrect measure of the loss potential, the risk manager can take this risk measurement error into account when making the key overall decisions—provided, of course, that the magnitude of the error can be measured and adjusted for with some level of precision, e.g., through backtesting a VaR method against historical data. The controversy remains, but VaR as a risk measure is unlikely to ever be completely rejected. It should not, however, be used in isolation. VaR is often paired with stress testing, discussed in a subsequent section. Remember too that no risk

<sup>42</sup>A well-known critic of VaR has likened its use to flying an aircraft with a potentially flawed altimeter. With an altimeter, a pilot may think he knows the correct altitude. Without an altimeter, the pilot will look out the window. Of course, this argument presumes that there are no clouds below. The probability of hitting trees or a mountain is the joint probability that the aircraft is too low and that the altimeter gives a false signal, which is less than the simple probability that the aircraft is too low. Aware of the potential for the altimeter to be flawed, the pilot will also seek information from other sources, which themselves are less than 100% accurate. So will the risk manager when using VaR. Both will gauge the risk against their tolerance for risk and take appropriate action. We look at some of these other sources of risk information in the next section.

measure can precisely predict future losses. It is important to ensure that the inputs to the VaR calculation are as reliable as possible and relevant to the current investment mix.

#### 5.4. Extensions and Supplements to VaR

Risk managers have developed several useful extensions and supplements to VaR. In this section, we review several of the more noteworthy.

A key concern to risk managers is the evaluation of the portfolio effect of a given risk. The ability to isolate the effect of a risk, particularly in complex portfolios with high correlation effects, is very important. We can use incremental VaR (IVaR) to investigate the effect. **Incremental VaR** measures the incremental effect of an asset on the VaR of a portfolio by measuring the difference between the portfolio's VaR while including a specified asset and the portfolio's VaR with that asset eliminated.<sup>43</sup> We can also use IVaR to assess the incremental effect of a subdivision on an enterprise's overall VaR. Although IVaR gives an extremely limited picture of the asset's or portfolio's contribution to risk, it nonetheless provides useful information about how adding the asset will affect the portfolio's overall risk as reflected in its VaR.

Some variations of VaR are **cash flow at risk** (CFAR) and **earnings at risk** (EAR). CFAR and EAR measure the risk to a company's cash flow or earnings, respectively, instead of its market value as in the case of VaR. CFAR is the minimum cash flow loss that we expect to be exceeded with a given probability over a specified time period. EAR is defined analogously to CFAR but measures risk to accounting earnings. CFAR and EAR can be used when a company (or portfolio of assets) generates cash flows or profits but cannot be readily valued in a publicly traded market, or when the analyst's focus is on the risk to cash flow and earnings, for example, in a valuation. CFAR and EAR can complement VaR's perspective on risk.

Another useful tool to supplement VaR is the **tail value at risk** (TVaR), also known as the conditional tail expectation. TVaR is defined as the VaR plus the expected loss in excess of VaR, when such excess loss occurs. For example, given a 5% daily VaR, TVaR might be calculated as the average of the worst 5% of outcomes in a simulation.

VaR developed initially as a measure for market risk, which is the risk associated with the primary market forces of interest rates, exchange rates, stock prices, and commodity prices. With some difficulty, VaR can be extended to handle credit risk, the risk that a counterparty will not pay what it owes. More recent extensions of VaR have tended to focus on modeling assets with nonnormal underlying distributions. The use of conditional normal distribution based on different regimes is a very intriguing concept, but the mathematics used in this area can be daunting.<sup>44</sup>

#### 5.5. Stress Testing

Managers often use stress testing (a term borrowed from engineering) to supplement VaR as a risk measure. The main purpose of VaR analysis is to quantify potential losses under normal market conditions. Stress testing, by comparison, seeks to identify unusual circumstances that could lead to losses in excess of those typically expected. Clearly, different scenarios will have attached probabilities of occurring that vary from the highly likely to the almost totally

<sup>43</sup>For more details, see Crouhy, Galai, and Mark (2001), Chapter 6.

<sup>44</sup>For an extremely entertaining tour of some of the pitfalls of traditional risk analysis and some solutions, see Osband (2002).

improbable. It is, therefore, the natural complement to VaR analysis. Two broad approaches exist in stress testing: scenario analysis and stressing models.

### 5.5.1. Scenario Analysis

Scenario analysis is the process of evaluating a portfolio under different states of the world. Quite often it involves designing scenarios with deliberately large movements in the key variables that affect the values of a portfolio's assets and derivatives.

One type of scenario analysis, that of **stylized scenarios**, involves simulating a movement in at least one interest rate, exchange rate, stock price, or commodity price relevant to the portfolio. These movements might range from fairly modest changes to quite extreme shifts. Many practitioners use standard sets of stylized scenarios to highlight potentially risky outcomes for the portfolio. Some organizations have formalized this process; for example, the Derivatives Policy Group recommends its members look at the following seven scenarios:

- parallel yield curve shifting by  $\pm 100$  basis points (1 percentage point);
- yield curve twisting by  $\pm 25$  basis points;<sup>45</sup>
- each of the four combinations of the above shifts and twists;
- implied volatilities changing by  $\pm 20\%$  from current levels;
- equity index levels changing by  $\pm 10\%$ ;
- major currencies moving by  $\pm 6\%$  and other currencies by  $\pm 20\%$ ;
- swap spread changing by  $\pm 20$  basis points.

In 1988, the Chicago Mercantile Exchange introduced a system call SPAN to calculate collateral requirements based on their members' total portfolios of futures and options. The objective of this system was to stress portfolios under a variety of scenarios. SPAN has become a very popular system among futures and options exchanges worldwide to set margin requirements. It offers a very useful, generalized form of scenario analysis that combines elements of VaR with some specified overlay based on real-world observation of the relationship among financial instruments.

Scenario analysis is a very useful enhancement to VaR, enabling those interested in risk analysis to identify and analyze specific exposures that might affect a portfolio. The results, of course, are only as good as implied by the accuracy of the scenarios devised. One problem with the stylized scenario approach is that the shocks tend to be applied to variables in a sequential fashion. In reality, these shocks often happen at the same time, have much different correlations than normal, or have some causal relationship connecting them.

Another approach to scenario analysis involves using **actual extreme events** that have occurred in the past. Here, we might want to put our portfolio through price movements that simulate the stock market crash of October 1987; the collapse of Long-Term Capital Management in 1998; the technology stock bubble of the late 1990s; the abrupt bursting of said bubble, beginning in the spring of 2000; or the market reaction to the terrorist attacks of 11 September 2001. This type of scenario analysis might be particularly useful if we think that the occurrence of extreme market breaks has a higher probability than that given by the probability model or historical time period being used in developing the VaR estimate. Stress testing of actual extreme events forces one to direct attention to these outcomes.

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<sup>45</sup>A **twist** is a nonparallel movement in the yield curve. An example of a twist is a 25-bps increase in short rates and no change in long rates, which would result in a flattening of the yield curve.

We might also create scenarios based on **hypothetical events**—events that have never happened in the markets or market outcomes to which we attach a small probability. These types of scenarios are very difficult to analyze and may generate confusing outcomes, so it is important to carefully craft hypothetical analyses if they are to generate information that adds value to the risk management processes.

Having devised a series of appropriate scenarios, the next step in the process is to apply them to the portfolio. The key task here is to understand the instruments' sensitivities to the underlying risk factors being stressed. This process is often a complex one that demands an understanding of the portfolio's risk parameters such that we can make appropriate approximations from standardized risk characteristics such as betas, deltas, gammas, duration, and convexity. Market liquidity is often a consideration also, especially when the underlying valuation models for assets assume arbitrage-free pricing, which assumes the ability to transact in any quantity. In addition, liquidity often dries up completely in a market crisis.

#### 5.5.2. Stressing Models

Given the difficulty in estimating the sensitivities of a portfolio's instruments to the scenarios we might design, another approach might be to use an existing model and apply shocks and perturbations to the model inputs in some mechanical way. This approach might be considered more scientific because it emphasizes a range of possibilities rather than a single set of scenarios, but it will be more computationally demanding. It is also possible to glean some idea of the likelihood of different scenarios occurring.

The simplest form of stressing model is referred to as **factor push**, the basic idea of which is to push the prices and risk factors of an underlying model in the most disadvantageous way and to work out the combined effect on the portfolio's value. This exercise might be appropriate for a wide range of models, including option-pricing models such as Black–Scholes–Merton, multifactor equity risk models, and term structure factor models. But factor push also has its limitations and difficulties—principally the enormous model risk that occurs in assuming the underlying model will function in an extreme risk climate.

Other approaches include **maximum loss optimization**—in which we would try to optimize mathematically the risk variable that will produce the maximum loss—and **worst-case scenario analysis**—in which we can examine the worst case that we actually expect to occur.

Overall stress testing is a valuable complement to VaR analysis and can highlight weaknesses in risk management procedures.

### 5.6. Measuring Credit Risk

Credit risk is present when there is a positive probability that one party owing money to another will renege on the obligation (i.e., the counterparty could default). If the defaulting party has insufficient resources to cover the loss or the creditor cannot impose a claim on any assets the debtor has that are unrelated to the line of business in which the credit was extended, the creditor can suffer a loss.<sup>46</sup> A creditor might be able to recover some of the loss, perhaps by having the debtor sell assets and pay the creditors a portion of their claim.

<sup>46</sup>The personal assets of a corporation's owners are shielded from creditors by the principle of limited liability, which can also apply to certain partnerships. The law supporting limited liability is a fundamental one in most societies and supports the notion that default is a right. Indeed, option-pricing theory has been used to value this right as the option that it actually is.

Credit losses have two dimensions: the likelihood of loss and the associated amount of loss (reflecting, of course, the amount of credit outstanding and the associated recovery rate). The likelihood of loss is a probabilistic concept: In every credit-based transaction, a given probability exists that the debtor will default. When a default does occur, however, creditors are often able to recover at least a portion of their investment, and as such, it is necessary and appropriate to assess the magnitude of this recovery (i.e., the recovery rate) in order to fully understand the risk profile of the credit dynamic. In relation to data on market risk, the amount of information available on credit losses is much smaller. Credit losses occur infrequently, and as such, the empirical data set from which to draw exposure inferences is quite limited. Although some statistical data are available, historical recovery rates can be unreliable. It can be hard to predict what an asset could be sold for in bankruptcy proceedings, and claims are not always paid in the order specified by bankruptcy law.

In the risk management business, exposure must often be viewed from two different time perspectives. We must assess first the risk associated with immediate credit events and second the risk associated with events that may happen later. With respect to credit, the risk of events happening in the immediate future is called **current credit risk** (or, alternatively, jump-to-default risk); it relates to the risk that amounts due at the present time will not be paid. For example, some risk exists that the counterparty could default on an interest or swap payment due immediately. Assuming, however, that the counterparty is solvent and that it will make the current payment with certainty, the risk remains that the entity will default at a later date. This risk is called potential credit risk, and it can differ quite significantly from current credit risk; the relationship between the two is a complex one. A company experiencing financial difficulties at present could, with sufficient time, work out its problems and be in better financial condition at a later date. Regardless of which risk is greater, however, a creditor must assess credit risk at different points in time. In doing so, the creditor must understand how different financial instruments have different patterns of credit risk, both across instruments and across time within a given instrument. This point will be discussed later in this section.

Another element of credit risk, which blends current and potential credit risk, is the possibility that a counterparty will default on a current payment to a different creditor. Most direct lending or derivative-based credit contracts stipulate that if a borrower defaults on any outstanding credit obligations, the borrower is in default on them all (this is known as a **cross-default provision**). Creditors stipulate this condition as one means of controlling credit exposure; in particular, it allows them to act quickly to mitigate losses to counterparties unable to meet any of their obligations. For example, suppose Party A owes Party B, but no payments are due for some time. Party A, however, currently owes a payment to Party C and is unable to pay. A is, therefore, in default to Party C. Depending on what actions C takes, A may be forced into bankruptcy. If so, then B's claim simply goes into the pool of other claims on A. In that case, A has technically defaulted to B without actually having a payment due.

In a previous section, we discussed how VaR is used to measure market risk. VaR is also used, albeit with greater difficulty, to measure credit risk. This measure is sometimes called **credit VaR**, default VaR, or credit at risk. Like ordinary VaR, it reflects the minimum loss with a given probability during a period of time. A company might, for example, quote a credit VaR of €10 million for one year at a probability of 0.05 (or a confidence level of 95%). In other words, the company has a 5% chance of incurring default-related losses of at least €10 million in one year. Note that credit VaR cannot be separated from market VaR because credit risk arises from gains on market positions held. Therefore, to accurately measure credit VaR, a risk manager must focus on the upper tail of the distribution of market returns, where the

return to the position is positive, in contrast to market risk VaR, which focuses on the lower tail. Suppose the 5% upper tail of the market risk distribution is €5 million. The credit VaR can be roughly thought of as €5 million, but this thinking assumes that the probability of loss is 100% and the net amount recovered in the event of a loss is zero. Further refinements incorporating more-accurate measures of the default probability and recovery rate should lead to a lower and more accurate credit VaR. In addition, the explosion of volume and liquidity in the credit derivatives market has vastly increased the amount of information available to risk managers with respect to the problem of understanding how the marketplace values credit risk on a real-time basis. Nevertheless, estimating credit VaR is more complicated than estimating market VaR because credit events are rare and recovery rates are hard to estimate. Credit risk is less easily aggregated than market risk; the correlations between the credit risks of counterparties must be considered.

In the next sections, we present the perspective of option pricing theory on credit risk and the measurement of credit risk exposures for certain derivative contracts.

### 5.6.1. Option-Pricing Theory and Credit Risk

Option theory enables us to better understand the nature of credit risk. In this section, we will see that the stock of a company with leverage can be viewed as a call option on its assets. This approach will lead to the result that a bond with credit risk can be viewed as a default-free bond plus an implicit short put option written by the bondholders for the stockholders.

Consider a company with assets with a market value of  $A_0$  and debt with a face value of  $F$ . The debt is in the form of a single zero-coupon bond due at time  $T$ . The bond's market value is  $B_0$ . Thus the stock's market value is

$$S_0 = A_0 - B_0$$

At time  $T$ , the assets will be worth  $A_T$  and the company will owe  $F$ . If  $A_T \geq F$ , the company will pay off its debt, leaving the amount  $A_T - F$  for the stockholders. Thus  $S_T$  will be worth  $A_T - F$ . If the assets' value is insufficient to pay off the debt ( $A_T < F$ ), the stockholders will discharge their obligation by turning over the assets to the bondholders. Thus the bondholders will receive  $A_T$ , which is less than their claim of  $F$ , and the stockholders will receive nothing. The company is, therefore, bankrupt. Exhibit 10 illustrates these results by showing the payoffs to the two suppliers of capital.

EXHIBIT 10 Payoffs to the Suppliers of Capital to the Company

Source of Capital	Market Value at Time 0	Payoffs at Time $T$	
		$A_T < F$	$A_T \geq F$
Bondholders	$B_0$	$A_T$	$F$
Stockholders	$S_0$	0	$A_T - F$
Total	$B_0 + S_0 = A_0$	$A_T$	$A_T$

Notice that the payoffs to the stockholders resemble those of a call option in which the underlying is the assets, the exercise price is  $F$ , and the option expires at time  $T$ , the bond maturity date. Indeed, the stock of a company with a single zero-coupon bond issue is a call option on the assets.

To better understand the nature of stock as a call option, let us recall the concept of put–call parity,<sup>47</sup> where  $p_0 + S_0 = c_0 + X/(1+r)^T$ . The put price plus the underlying price equals the call price plus the present value of the exercise price. So, working this through for our own problem, we find the correspondences shown in Exhibit 11.

EXHIBIT 11 Equity as a Call Option on the Value of a Company

Variable	Traditional Framework	Current Framework
Underlying	$S_0$ (stock)	$A_0$ (value of assets)
Exercise price	$X$	$F$ (face value of bond)
Time to expiration	$T$	$T$ (maturity of bond)
Risk-free rate	$r$	$r$
Call price	$c_0$	$S_0$ (value of stock)
Put price	$p_0$	$p_0$

Note the last line. We see that in the traditional framework, there is a put option, which we know is an option to sell the underlying at a fixed price. In fact, we know from put–call parity that  $p_0 = c_0 - S_0 + X/(1+r)^T$ . The put is equivalent to a long call, a short position in the underlying stock, and a long position in a risk-free bond with face value equal to the exercise price. In the current framework, the standard expression of put–call parity is  $p_0 + A_0$  (put plus underlying) =  $S_0 + F/(1+r)^T$  (stock plus present value of bond principal). Turning this expression around and reversing the order of the put and bond, we obtain

$$A_0 = S_0 + F/(1+r)^T - p_0$$

Noting, however, that by definition the asset value,  $A_0$ , equals the stock's market value,  $S_0$ , plus the bond's market value,  $B_0$ ,

$$A_0 = S_0 + B_0$$

we see that the bond's market value must be equivalent to

$$B_0 = F/(1+r)^T - p_0$$

The first term on the right-hand side is equivalent to a default-free zero-coupon bond paying  $F$  at maturity. The second term is a short put. The bondholders' claim, which is subject to default, can thus be viewed as a default-free bond and a short put on the assets. In other words, the bondholders have implicitly written the stockholders a put on the assets. From the stockholders' perspective, this put is their right to fully discharge their liability by turning over the assets to the bondholders, even though those assets could be worth less than the bondholders' claim. In legal terminology, this put option is called the stockholders' right of limited liability.

The existence of this implicit put option is the difference between a default-free bond and a bond subject to default. This approach to understanding credit risk forms the basis for models that use option-pricing theory to explain credit risk premiums, probabilities of default, and the valuation of companies that use leverage. In practice, the capital structures of most

<sup>47</sup>See Chance (2003), Chapter 4.

companies are more complex than the one used here, but practical applications of model variants appear in the financial industry.

### 5.6.2. The Credit Risk of Forward Contracts

Recall that forward contracts involve commitments on the part of each party. No cash is due at the start, and no cash is paid until expiration, at which time one party owes the greater amount to the other. The party that owes the larger amount could default, leaving the other with a claim of the defaulted amount. Each party assumes the other's credit risk. Prior to expiration, no current credit risk exists, because no current payments are owed, but there is potential credit risk in connection with the payments to be made at expiration. Current credit risk arises when the contract is at its expiration. Below we will examine how potential credit risk changes during the life of the contract as the value of the underlying changes.

From the perspective of a given party, a forward contract's market value can be easily calculated as the present value of the amount owed to the party minus the present value of the amount it owes. So, the market value at a given time reflects the potential credit risk. This is another reason why the calculation of market value is important: It indicates the amount of a claim that would be subject to loss in the event of a default.

For example, look at a forward contract that expires in one year. The underlying asset price is \$100 and the risk-free interest rate is 5%. We can determine that the forward price is  $\$100(1.05) = \$105$ . We could then assume that three months later, the asset price is \$102. We can determine that the long forward contract's value at that time is  $\$102 - \$105/(1.05)^{0.75} = \$0.7728$ . This is the value to the long because the contract is a claim on the asset, which is currently worth \$102, and an obligation to pay \$105 for it in nine months. To the holder of the long position, this contract is worth \$0.7728, and to the holder of the short position, it is worth  $-\$0.7728$ .

Which party bears the potential credit risk? The long's claim is positive; the short's claim is negative. Therefore, the long currently bears the credit risk. As it stands right now, the value of the long's claim is \$0.7728. No payment is currently due, and hence no current credit risk exists, but the payments that are due later have a present value of \$0.7728. Actual default may or may not occur at expiration. Moreover, at expiration, the amount owed is unlikely to be this same amount. In fact, if the spot price falls enough, the situation will have turned around and the long could owe the short the greater amount. Nonetheless, in assessing the credit risk three months into the contract, the long's claim is \$0.7728. This claim has a probability of not being paid and also has the potential for recovery of a portion of the loss in the event of default. If the counterparty declares bankruptcy before the contract expires, the claim of the non-defaulting counterparty is the forward contract's market value at the time of the bankruptcy, assuming this value is positive. So, if the short declares bankruptcy at this time, the long has a claim worth \$0.7728. If the long declares bankruptcy, the long holds an asset worth \$0.7728.

### 5.6.3. The Credit Risk of Swaps

A swap is similar to a series of forward contracts. The periodic payments associated with a swap imply, however, that credit risk will be present at a series of points during the contract's life. As with forward contracts, the swap's market value can be calculated at any time and reflects the present value of the amount at risk for a credit loss (i.e., the potential credit risk).

Consider, for example, the case of a plain vanilla interest rate swap with a one-year life and quarterly payments at Libor. Using the term structure, we can determine that the swap has a fixed rate of 3.68%, leading to quarterly fixed payments of \$0.0092 per \$1 notional principal.

We then can move forward 60 days into the life of the swap and, with a new term structure, we can determine that the swap's market value is \$0.0047 per \$1 notional principal. To the party that is long (i.e., paying fixed and receiving floating), the swap has a positive market value. To the counterparty, which pays floating and receives fixed, the claim has a market value of  $-\$0.0047$ .

As with a forward contract, the market value indicates the present value of the payments owed to the party minus the present value of the payments the party owes. Only 60 days into the life of a swap with quarterly payments, no payment is due for 30 more days. Thus there is no current credit risk. There is, however, potential credit risk. The market value of \$0.0047 represents the amount that is at risk of loss for default. Of course, if default occurs, it will be at a later date when the amount will probably be different. Moreover, the market value could reverse its sign. At this time, the amount owed by the short to the long is greater, but at a later date, the amount owed by the long to the short could be greater. As with forward contracts, if the party to which the value is negative defaults, the counterparty has a claim of that value. If the party to which the value is positive defaults, the defaulting party holds an asset with the positive market value. Also, the counterparty could default to someone else, thereby being forced to declare bankruptcy before a payment on this swap is due. In that case, the swap's market value at that time is either the claim of the creditor or the asset held by the bankrupt party in bankruptcy proceedings.

The credit risk of swaps can vary greatly across product types within this asset class and over a given swap's lifetime. For interest rate and equity swaps, the potential credit risk is largest during the middle period of the swap's life. During the beginning of a swap's life, typically we would assume that the credit risk is small because, presumably, the involved counterparties have performed sufficient current credit analysis on one another to be comfortable with the arrangement or otherwise they would not engage in the transaction. At the end of the life of the swap, the credit risk is diminished because most of the underlying risk has been amortized through the periodic payment process. There are fewer payments at the end of a swap than at any other time during its life; hence, the amount a party can lose because of a default is smaller. This leaves the greatest exposure during the middle period, a point at which 1) the credit profile of the counterparties may have changed for the worse and 2) the magnitude and frequency of expected payments between counterparties remain material. One exception to this pattern involves currency swaps, which often provide for the payment of the notional principal at the beginning and at the end of the life of the transaction. Because the notional principal tends to be a large amount relative to the payments, the potential for loss caused by the counterparty defaulting on the final notional principal payment is great. Thus, whereas interest rate swaps have their greatest credit risk midway during the life of the swap, currency swaps have their greatest credit risk between the midpoint and the end of the life of the swap.

#### 5.6.4. The Credit Risk of Options

Forward contracts and swaps have bilateral default risk. Although only one party will end up making a given payment, each party could potentially be the party owing the net amount. Options, on the other hand, have unilateral credit risk. The buyer of an option pays a cash premium at the start and owes nothing more unless, under the buyer's sole discretion, he decides to exercise the option. Once the premium is paid, the seller assumes no credit risk from the buyer. Instead, credit risk accrues entirely to the buyer and can be quite significant. If the buyer exercises the option, the seller must meet certain terms embedded in the contract. If the option is a call, the seller must deliver the underlying or pay an equivalent cash settlement.

If the option is a put, the seller must accept delivery of the underlying and pay for it or meet these obligations in the form of cash payments. If the seller fails to fulfill her end of the obligation, she is in default. Like forward contracts, European options have no payments due until expiration. Hence, they have no current credit risk until expiration, although significant potential credit risk exists.

Consider a European call option for which the underlying security has a price of 52.75 and a standard deviation of 0.35. The exercise price is 50, the risk-free rate is 4.88% continuously compounded, and the option expires in nine months. Using the Black–Scholes–Merton model, we find that the value of the option is 8.5580. The holder thus has potential credit risk represented by a present claim of 8.5580. This amount can be thought of as the amount that is at risk, even though at expiration the option will probably be worth a different amount. In fact, the option might even expire out of the money, in which case it would not matter if the short were bankrupt. If the short declares bankruptcy before expiration, the long has a claim on the value of the option under bankruptcy law.

If the option were American, the value could be greater. Moreover, with American options, current credit risk could arise if the option holder decides to exercise the option early. This alternative creates the possibility of the short defaulting before expiration.

### EXAMPLE 8 Calculating Credit Risk Exposures

Calculate the amount at risk of a credit loss in the following situations:

1. A US party goes long a forward contract on €1 denominated in dollars in which the underlying is the euro. The original term of the contract was two years, and the forward rate was \$0.90. The contract now has 18 months or 1.5 years to maturity. The spot or current exchange rate is \$0.862. The US interest rate is 6%, and the euro interest rate is 5%. The interest rates are based on discrete compounding/discounting. At the point when the contract has 1.5 years remaining, the value of the contract to the long per \$1 notional principal equals the spot exchange rate, \$0.862, discounted at the international interest rate for 1.5 years, minus the forward rate, \$0.90, discounted at the domestic interest rate for 1.5 years:<sup>48</sup>

$$\frac{\$0.862}{(1.05)^{1.5}} - \frac{\$0.90}{(1.06)^{1.5}} = -\$0.0235$$

Evaluate the credit risk characteristics of this situation.

2. Consider a plain vanilla interest rate swap with two months to go before the next payment. Six months after that, the swap will have its final payment. The swap fixed rate is 7%, and the upcoming floating payment is 6.9%. All payments are based on 30 days in a month and 360 days in a year. Two-month Libor is 7.250%, and

<sup>48</sup>See Chance (2003), pp. 58–59.

eight-month Libor is 7.375%. The present value factors for two and eight months can be calculated as follows:

$$\frac{1}{1+0.0725(60/360)} = 0.9881$$

$$\frac{1}{1+0.07375(240/360)} = 0.9531$$

The next floating payment will be  $0.069(180/360) = 0.0345$ . The present value of the floating payments (plus hypothetical notional principal) is  $1.0345(0.9881) = 1.0222$ . Given an annual rate of 7%, the fixed payments will be  $0.07(180/360) = 0.035$ .

The present value of the fixed payments (plus hypothetical notional principal) is, therefore,  $0.035(0.9881) + 1.035(0.9531) = 1.0210$ . Determine the amount at risk of a credit loss and state which party currently bears the risk. Assume a \$1 notional principal.

3. A dealer has sold a call option on a stock for \$35 to an investor. The option is currently worth \$46, as quoted in the market. Determine the amount at risk of a credit loss and state which party currently bears the risk.

*Solution to 1:* The position has a negative value to the long, so the credit risk is currently borne by the short. From the short's point of view, the contract has a value of \$0.0235 per \$1 notional principal. No payments are due for 18 months, but the short's claim on the long is worth \$0.0235 more than the long's claim on the short. Therefore, this amount is the current value of the amount at risk for a credit loss. Of course, the amount could, and probably will, change over the life of the contract. The credit risk exposure might even shift to the other party.

*Solution to 2:* The market value of the swap to the party paying fixed and receiving floating is  $1.0222 - 1.0210 = 0.0012$ . This value is positive to the party paying fixed and receiving floating; thus this party currently assumes the credit risk. Of course, the value will change over the life of the swap and may turn negative, meaning that the credit risk is then assumed by the party paying floating and receiving fixed.

*Solution to 3:* All of the credit risk is borne by the investor (the owner of the call), because he will look to the dealer (the seller) for the payoff if the owner exercises the option. The current value of the amount at risk is the market price of \$46.

Derivatives' credit risk can be quite substantial, but this risk is considerably less than that faced by most lenders. When a lender makes a loan, the interest and principal are at risk. The loan principal corresponds closely to the notional principal of most derivative contracts. With the exception of currency swaps, the notional principal is never exchanged in a swap. Even with currency swaps, however, the risk is much smaller than on a loan. If a counterparty defaults on a currency swap, the amount owed to the defaulting counterparty serves as a type of collateral because the creditor is not required to pay it to the defaulting party. Therefore, the credit risk on derivative transactions tends to be quite small relative to that on loans. On forward

and swap transactions, the netting of payments makes the risk extremely small relative to the notional principal and to the credit risk on a bond or loan of equivalent principal.

### 5.7. Liquidity Risk

One of the implicit assumptions in risk management with VaR is that positions can be liquidated when they approach or move outside pre-agreed risk limits. In practice, some assets are far more liquid than others and practitioners will often liquidity-adjust VaR estimates accordingly. Wide bid–ask spreads in proportion to price are an obvious measure of the cost of trading an illiquid instrument or underlying security. But some instruments simply trade very infrequently at any price—a far more complex problem, because infrequently quoted prices often give the statistical illusion of low or lower volatility. This dynamic is counterintuitive, because we would expect instruments that are illiquid to have a higher bid–ask spread and higher volatilities.

A famous case of underestimating liquidity risk is the failure of the hedge fund Long-Term Capital Management (LTCM) in 1998. LTCM was set up by a group of bond traders and academics and was engaged in arbitrage or relative value trading on world fixed-income markets through the use of the swap market. The total equity in the fund peaked at around \$5 billion, but this amount was leveraged around 25 times (perhaps substantially more when the full impact of derivatives is considered). The BIS estimated that the notional value of the swaps entered into by LTCM was around 2.4% of the entire world swap market. LTCM failed to appreciate the market moves that would occur when it attempted to liquidate positions, particularly those in illiquid, emerging, fixed-income markets. The New York Federal Reserve was forced to act for fear of a global financial crisis and organized a consortium of 14 international banks to manage the assets of the fund. In the end, and after substantial financial help, LTCM's investors lost more than 90% of their equity.

### 5.8. Measuring Nonfinancial Risks

Nonfinancial risks are intrinsically very difficult to measure. Indeed, some of the nonfinancial exposures we have discussed, such as regulatory risk, tax risk, legal risk, and accounting risk, could easily be thought of as not measurable in any precise mathematical way. They are unlike market risk and the VaR concept because we usually lack an observable distribution of losses related to these factors.

Some of these risks could be thought of as more suitable for insurance than measurement and hedging. Like a flood that occurs every 50 years, they might well affect a large number of instruments or contracts. Here, it is possible to learn from best practice in the insurance industry. Insurance companies usually have sufficient assets and are capitalized to withstand these uncertain events. Where it is possible to model a source of risk, actuaries often use techniques like extreme value theory, but even these techniques are only as good as the historical data on which they are based.

#### 5.8.1. Operational Risk

Until a few years ago, the subject of operational risk received little attention, and ideas about actually measuring operational risk were practically unheard of. But a number of well-publicized losses at financial institutions, ranging from a breakdown of internal systems to rogue employees and in some cases employee theft, have put operational risk justifiably into the forefront.

Furthermore, the explicit mention of operational risk requirements in the Basel II banking regulations has created real advantages for banks that can credibly measure their operational risks. This, in turn, has led to an explosion in the academic literature relating to the measurement of operational risk and its role in enterprise risk systems.

### EXAMPLE 9 Basel II—A Brief Overview

The Basel banking regulations apply only to large international banks, but national governments use them as a guideline in formulating their own financial laws and regulations, so the regulations have much more widespread importance. In January 2001, the Basel Committee on Banking Supervision issued a proposal for a New Basel Capital Accord that would replace the 1988 Capital Accord. This first accord, “Basel I,” was widely criticized for being too inflexible in applying an across-the-board 8% capital adequacy ratio that made no discrimination between a well risk-managed bank and one that was not.<sup>49</sup>

The Basel II proposal incorporates three mutually reinforcing pillars that allow banks and supervisors to evaluate properly the various risks that banks face:

- Pillar 1: Capital Requirements;
- Pillar 2: Supervisory Review;
- Pillar 3: Market Discipline.

The first pillar of Basel II moves away from a blanket, one-size-fits-all approach and allows banks to develop their own mathematically based financial models. Once these internally developed techniques have been successfully demonstrated to the regulators, banks are able to progress to higher levels of risk management that within the accord are offset by reduced regulatory capital charges. Key to these higher levels of risk management are advanced systems for managing credit risk and operational risk.

The second pillar, supervisory review, requires banks to meet Basel-recommended operational risk requirements that have been tailored by their host country. “Risky” banks, whose risk management systems score lowly in the areas of market risk and operational risk, face penalties. Better-risk-managed banks will have major competitive advantages over rivals, in that, all else equal, they are likely to be subject to reduced capital requirements per unit of risk.

The third pillar says that banks must fulfill the Basel requirements for transparency and disclosing company data. A key point here is that banks must reveal more detail about their profits and losses, which may lead to a supervisory authority reviewing risk systems and changing the capital allocation under the first pillar.

<sup>49</sup> A **capital adequacy ratio** is a measure of the adequacy of capital in relation to assets. The purpose of capital is to absorb unanticipated losses with sufficient margin to permit the entity to continue as a going concern. Basel I specified a capital adequacy ratio as a percent of the credit-risk-weighted assets on the bank’s balance sheet, where bank assets were divided into four broad categories. For more details, see Saunders and Cornett (2003).

## 6. MANAGING RISK

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Having established methods for the identification and measurement of risk, we turn our attention to a critical stage of any solid risk management program: that of *managing* risk. The key components, which by now should be somewhat intuitive to you, are as follows:

- An effective risk governance model, which places overall responsibility at the senior management level, allocates resources effectively and features the appropriate separation of tasks between revenue generators and those on the control side of the business.
- Appropriate systems and technology to combine information analysis in such a way as to provide timely and accurate risk information to decision makers.
- Sufficient and suitably trained personnel to evaluate risk information and articulate it to those who need this information for the purposes of decision making.

A recent advertisement for the RiskMetrics Group ([www.riskmetrics.com](http://www.riskmetrics.com)) identified the following nine principles of effective risk management:

- There is no return without risk. Rewards go to those who take risks.
- Be transparent. Risk should be fully understood.
- Seek experience. Risk is measured and managed by people, not mathematical models.
- Know what you don't know. Question the assumptions you make.
- Communicate. Risk should be discussed openly.
- Diversify. Multiple risks will produce more consistent rewards.
- Show discipline. A consistent and rigorous approach will beat a constantly changing strategy.
- Use common sense. It is better to be approximately right than to be precisely wrong.
- Return is only half the equation. Decisions should be made only by considering the risk and return of the possibilities.

Risk management is in so many ways just good common business sense. It is quite remarkable, however, that commonsense rules are violated so easily and so often. But that problem is not unique to risk management.

Currently, two professional organizations are devoted to risk management. The Global Association of Risk Professionals (GARP) and the Professional Risk Managers' International Association (PRMIA) are actively involved in promoting knowledge in the field of risk management. You may wish to visit their websites at [www.garp.com](http://www.garp.com) and [www.prmia.org](http://www.prmia.org).

With these principles in mind, in the following section, we will discuss the various components of a well-adapted risk-control program.

### 6.1. Managing Market Risk

Let us assume we have correctly identified the sources of market risk that affect our business. Further assume that we have decided on an appropriate way to measure market risk and successfully deployed the systems we need to monitor our positions and measure our risk in a timely way. The result is an appropriate firmwide VaR estimate and associated breakdown by business area. Now we must ask ourselves the following questions: How do we know how much risk is acceptable for us to take? What is the overall exposure assumption capacity for the enterprise, and how close to full capacity should we run? We already know that VaR is not a

measure of the maximum possible loss but only a probabilistic guide to the minimum loss we might expect with a certain frequency over a certain time frame.

Our **enterprise risk management** system will be incomplete without a well-thought-out approach to setting appropriate **risk tolerance** levels and identifying the proper corrective behavior to take if our actual risks turn out to be significantly higher or lower than is consistent with our risk tolerance. Note here that in many circumstances, it could cause as many problems to take too little risk as to take too much risk. As we noted at the beginning of this reading, companies are in business to take risk and taking too little risk will more than likely reduce the possible rewards; it could even make the company vulnerable to takeover. In a more extreme scenario, insufficient risk-taking may lead to situations in which the expected return stands little chance of covering variable (let alone fixed) costs.

Corrective behavior in the case of excessive market risk will almost always result in the need for additional hedging or the scaling back of tradable positions. Quite often, however, liquidity and other factors will prevent perfect hedging, perhaps exacerbating risk concerns rather than mitigating them.

#### 6.1.1. Risk Budgeting

In recent years, companies and portfolio managers have begun to implement a new approach to risk management called **risk budgeting**. It focuses on questions such as, “Where do we want to take risk?” and “What is the efficient allocation of risk across various units of an organization or investment opportunities?” Risk budgeting is relevant in both an organizational and a portfolio management context.

To take an organizational perspective first, risk budgeting involves establishing objectives for individuals, groups, or divisions of an organization that take into account the allocation of an acceptable level of risk. As an example, the foreign exchange (FX) trading desk of a bank could be allocated capital of €100 million and permitted a daily VaR of €5 million. In other words, the desk is granted a budget, expressed in terms of allocated capital and an acceptable level of risk, expressed in euro amounts of VaR. In variations on this theme, instead of using VaR units an organization might allocate risk based on individual transaction size, the amount of working capital needed to support the portfolio, or the amount of losses acceptable for any given time period (e.g., one month). In any case, the innovation here is that the enterprise allocates risk capital before the fact in order to provide guidance on the acceptable amount of risky activities that a given unit can undertake.

A well-run risk-taking enterprise manages these limits carefully and constantly monitors their implementation. Any excesses are reported to management immediately for corrective action. Under this type of regime, management can compare the profits generated by each unit with the amount of capital and risk employed. So, to continue our example from above, say the FX trading desk made a quarterly profit of €20 million from its allocation. The bank’s fixed-income trading desk was allocated capital of €200 million and permitted a daily VaR of €5 million; the fixed-income trading desk made €25 million in quarterly trading profits. We note that the allocated daily VaRs for the two business areas are the same, so each area has the same risk budget, and that the fixed-income desk generated better returns on the VaR allocation, but worse on the allocation of actual capital, than did the FX desk. (The FX desk shows a  $\text{€20/€100} = 20\%$  return on capital versus  $\text{€25/€200} = 12.5\%$  for the fixed-income desk.) This type of scenario is quite common and highlights the complexities of the interaction between risk management and capital allocation. Risk and capital are finite resources that must be allocated carefully.

The sum of risk budgets for individual units will typically exceed the risk budget for the organization as a whole because of the impacts of diversification. Returning to our example, let us assume that for the enterprise in question, its FX and bond trading desks engage in activities that are only weakly correlated. In this case, our present allocation of capital and risk might make perfect sense. For example, the daily VaR of the two business areas combined might be €7 million (i.e., 70% of the combined risk allocation for the two desks), for which we again generate a total quarterly profit of  $\text{€}20,000,000 + \text{€}25,000,000 = \text{€}45$  million.

Alternatively, say the two business areas are very highly correlated (their correlation coefficient equals 1) and their combined daily VaR is  $\text{€}5,000,000 + \text{€}5,000,000 = \text{€}10$  million (i.e., 100% of the aggregate VaR allocation across desks). The combined profit is still  $\text{€}20,000,000 + \text{€}25,000,000 = \text{€}45$  million. Under these circumstances—and particularly if the bank's management believes that the correlations will remain strong—management might consider closing down the fixed-income desk to generate  $0.20(\text{€}100,000,000 + \text{€}200,000,000) = \text{€}60$  million of returns on the  $3(\text{€}5,000,000) = \text{€}15$  million of VaR. Contrast this strategy with that of closing down the foreign exchange trading desk and allocating all of the capital and risk to the bond trading desk, which would produce  $0.125(\text{€}200,000,000 + \text{€}100,000,000) = \text{€}37.5$  million in profit for the €7.5 million in VaR, representing a lower return on capital and a higher return on VaR.

A risk-budgeting perspective has also been applied to allocating funds to portfolio managers. Consider an active investor who wants to allocate funds optimally to several domestic and nondomestic equity and fixed-income investment managers. Such an investor might focus on tracking risk as the primary risk measure and decide on an overall maximum acceptable level for it, such as 200 basis points. The expected information ratio (IR) for each manager is one possible measure of each manager's ability to add value, considering the managers in isolation.<sup>50</sup> In this application, however, it is appropriate for the investor to adjust each manager's IR to eliminate the effect of asset class correlations; such correlation-adjusted IRs will capture each manager's incremental ability to add value in a portfolio context. Using such correlation-adjusted IRs, we can determine the optimal tracking risk allocation for each investment manager (which, intuitively, is positively related to his correlation-adjusted IR).<sup>51</sup>

Through these two examples, we edge toward some understanding of risk-adjusted performance measures, which we will discuss in greater detail later in the reading. The point about risk budgeting is that it is a comprehensive methodology that empowers management to allocate capital and risk in an optimal way to the most profitable areas of a business, taking account of the correlation of returns in those business areas.

It once again bears mention that for many portfolio managers, risk budget allocations should be measured in relation to risk to the surplus—that is, the difference between the values of assets and liabilities.

<sup>50</sup>The information ratio is active return divided by active risk; it measures active return per unit of active risk.

<sup>51</sup>See Waring, Whitney, Pirone, and Castile (2000) and references therein for further reading.

### EXAMPLE 10 A Fund Management Company and Risk Budgeting

We can readily illustrate the methodology and underlying economics of risk budgeting with the example of a fund management company. We choose, for this example, a multistrategy hedge fund, because although mutual funds and other types of institutional money managers certainly face similar risk management issues, they are often bound by strict guidelines that tie their risk budgeting to factors such as the performance of a benchmark index and other mandated fund management protocols. For example, the Vanguard family of mutual funds offers a wide range of indexed mutual funds. These funds' associated risk budgets are very narrowly defined, as the managers are called on at all times to track the underlying index very closely in terms of securities held, associated portfolio weightings, and so forth. As investor funds flow in and out of these securities, portfolio managers execute trades that do little more than reestablish this replication balance. Of course, many institutional fund products allow for much broader deviations from market benchmarks; in most cases, however, risk budgeting will be constrained by certain principles associated with benchmarking.

Hedge funds with multiple portfolio managers (as well as, in some cases, the proprietary trading divisions of banks and broker/dealers) have many fewer risk constraints than indexed mutual funds; they have more freedom, therefore, in establishing risk budgets. Because of the absolute return (as opposed to benchmark-driven) nature of their performance, and because of issues such as performance netting risk covered earlier in this reading, it is very much in their interest to ensure that each portfolio in the enterprise operates within a well-conceived risk budget framework. Included among the critical components of such a program might be the following:

- **Performance Stopouts** A performance stopout is the maximum amount that a given portfolio is allowed to lose in a period (e.g., a month or a year).
- **Working Capital Allocations** Most funds will allocate a specific amount of working capital to each portfolio manager, both as a means of enforcing risk disciplines and also to ensure the ability to fund all operations.
- **VaR Limits** Discussed above.
- **Scenario Analysis Limits** The risk manager of the fund company may establish risk limits based on the scenario analysis discussed in the preceding section. Under such an approach, the portfolio manager would be compelled to construct a portfolio such that under specified scenarios, it did not produce losses greater than certain predetermined amounts.
- **Risk Factor Limits** Portfolio managers may be subject to limits on individual risk factors, as generated by a VaR analysis (e.g., VaR exposure to a certain risk cannot exceed, say, \$X or X%) or driven by linear (e.g., duration, beta) or nonlinear (e.g., convexity, gamma) risk estimation methodologies.
- **Position Concentration Limits** Many risk managers seek to enforce diversification by mandating a specific maximum amount for individual positions.
- **Leverage Limits** A maximum amount of leverage in the portfolio may be specified.
- **Liquidity Limits** To help manage liquidity exposure, large funds will often also set position limits as a specified maximum percentage of daily volume, float, or open interest.

Of course, other types of limits are imposed on portfolio managers in a multistrategy environment, and by the same token, the risk-budgeting strategy of a given enterprise may include only a subset of the examples provided immediately above. Nevertheless, some subset of these limit structures is present in nearly every multistrategy fund vehicle, and it is difficult to imagine an effective risk control system that does not set limits.

## 6.2. Managing Credit Risk

It is important that creditors do a good job of measuring and controlling credit risk. Estimating default probabilities is difficult because of the infrequency of losses for many situations where credit risk exists. Moreover, credit losses differ considerably from losses resulting from market moves. Credit is a one-sided risk. If Party B owes Party A the amount of £1,000, B will end up paying A either £1,000 or some amount ranging from zero to £1,000. A's rate of return is certainly not normally distributed and not even symmetric. All of the risk is downside. Thus credit risk is not easily analyzed or controlled using such measures as standard deviation and VaR. Creditors need to regularly monitor the financial condition of borrowers and counterparties. In addition, they can use the risk management techniques for credit discussed below.

### 6.2.1. Reducing Credit Risk by Limiting Exposure

Limiting the amount of exposure to a given party is the primary means of managing credit risk. Just as a bank will not lend too much money to one entity, neither will a party engage in too many derivatives transactions with one counterparty. Exactly how much exposure to a given counterparty is "too much" is still not easy to quantify. Experienced risk managers often have a good sense of when and where to limit their exposure, and they make extensive use of quantitative credit exposure measures to guide them in this process. Banks have regulatory constraints on the amount of credit risk they can assume, which are specified in terms of formulas.

### 6.2.2. Reducing Credit Risk by Marking to Market

One device that the futures market uses to control credit risk is marking tradable positions to market. The OTC derivatives market also uses **marking to market** to deal with credit risk: Some OTC contracts are marked to market periodically during their lives. Recall that a forward contract or swap has a market value that is positive to one party and negative to another. When a contract calls for marking to market, the party for which the value is negative pays the market value to the party for which the value is positive. Then the fixed rate on the contract is recalculated, taking into account the new spot price, interest rate, and time to expiration.

Recall that we examined a one-year forward contract with an initial forward price of \$105. Three months later, when the asset price was \$102, its value was \$0.7728 to the long. If the contract were marked to market at that time, the short would pay the long \$0.7728. Then, the two parties would enter into a new contract expiring in nine months with a new forward price, which would be  $\$102(1.05)^{0.75} = \$105.80$ .

### EXAMPLE 11 Repricing a Forward Contract

Consider a one-year forward contract established at a rate of \$105. The contract is four months into its life. The spot price is \$108, the risk-free rate is 4.25%, and the underlying makes no cash payments. The two parties decided at the start that they will mark the contract to market every four months. The market value of the contract is  $\$108 - \$105 / (1.0425)^{8/12} = \$5.873$ . Determine how the cash flows and resets would work under these circumstances.

*Solution:* The contract is positive to the long, so the short pays the long \$5.873. The parties then reprice the contract. The new price is  $\$108(1.0425)^{8/12} = \$111.04$ . At this point, the forward price is reset to \$111.04. The parties will then mark to market again at the eight-month point and reset the forward price. This price will then stay in force until expiration.

OTC options usually are not marked to market because their value is always positive to one side of the transaction. Of course, one party of the option certainly bears credit risk, but marking to market is usually done only with contracts with two-way credit risk. Option credit risk is normally handled by collateral.

#### 6.2.3. Reducing Credit Risk with Collateral

The posting of collateral is a widely accepted credit exposure mitigant in both lending and derivatives transactions. One very prominent example of its use comes from futures markets, which require that all market participants post margin collateral. Beyond this, many OTC derivative markets have collateral posting provisions, with the collateral usually taking the form of cash or highly liquid, low-risk securities. A typical arrangement involves the routine, periodic posting of values sufficient to cover mark-to-market deficiencies. To illustrate, if a given derivatives contract has a positive value to Party A and a negative value to Party B, then Party B owes more than Party A, and Party B must put collateral into an account designated for this purpose. As the contract's market value changes, the amount of collateral that must be maintained will vary, increasing as the market value increases and vice versa. At some point, if the market value of the transaction changes sign (i.e., goes from positive to negative for one of the participants), the collateral position will typically reverse itself, with the entity previously posting collateral seeing a release of these assets and the other participant in the transaction experiencing a collateral obligation. In addition to market values, collateral requirements are sometimes also based on factors such as participants' credit ratings.

#### 6.2.4. Reducing Credit Risk with Netting

One of the most common features used in two-way contracts with a credit risk component, such as forwards and swaps, is netting. This process, which we have already briefly discussed, involves the reduction of all obligations owed between counterparties into a single cash transaction that eliminates these liabilities. For example, if a payment is due and Party A owes more to Party B than B owes to A, the difference between the amounts owed is calculated and Party

A pays the net amount owed. This procedure, called **payment netting**, reduces the credit risk by reducing the amount of money that must be paid. If Party A owes €100,000 to Party B, which owes €40,000 to A, then the net amount owed is €60,000, which A owes to B. Without netting, B would need to send €40,000 to A, which would send €100,000 to B. Suppose B was in the process of sending its €40,000 to A but was unaware that A was in default and unable to send the €100,000 to B. If the €40,000 is received by A, B might be unable to get it back until the bankruptcy court decides what to do, which could take years. Using netting, only the €60,000 owed by A to B is at risk.

In the examples we have seen so far, netting is applied on the payment date. The concept of netting can be extended to the events and conditions surrounding a bankruptcy. Suppose A and B are counterparties to a number of derivative contracts. On some of the contracts, the market value to A is positive, while on others, the market value to B is positive. If A declares bankruptcy, the parties can use netting to solve a number of problems. If A and B agree to do so before the bankruptcy, they can net the market values of *all* of their derivative contracts to determine one overall value owed by one party to another. It could well be the case that even though A is bankrupt, B might owe more to A than A owes to B. Then, rather than B being a creditor to A, A's claim on B becomes one of A's remaining assets. This process is referred to as **closeout netting**.

During this bankruptcy process, netting plays an important role in reducing a practice known in the financial services industry as cherry picking, which in this case would involve a bankrupt company attempting to enforce contracts that are favorable to it while walking away from those that are unprofitable. In our example, without netting, A could default on the contracts in which it owes more to B than B owes to A, but B could be forced to pay up on those contracts in which it owes more to A than A owes to B.

To be supported through the bankruptcy process, however, netting must be recognized by the legal system and works best when each party's rights and obligations are specified at the time before or contemporaneous to the executions of transactions. Most, but not all, legal jurisdictions recognize netting.

#### 6.2.5. Reducing Credit Risk with Minimum Credit Standards and Enhanced Derivative Product Companies

As noted above, the first line of defense against credit risk is limiting the amount of business one party engages in with another. An important and related concept is to ensure that all credit-based business is undertaken with entities that have adequate levels of credit quality. The historical standard measures for such credit quality come from rating agencies such as Moody's Investors Service and Standard & Poor's. Some companies will not do business with an enterprise unless its rating from these agencies meets a prescribed level of credit quality. This practice can pose a problem for some derivatives dealers, most of which engage in other lines of business that expose them to a variety of other risks; for example, banks are the most common derivatives dealers. To an end user considering engaging in a derivative contract with a dealer, the potential for the dealer's other business to cause the dealer to default is a serious concern. Banks, in particular, are involved in consumer and commercial lending, which can be quite risky. In the United States, for example, we have seen banking crises involving bad loans to the real estate industry and underdeveloped countries.

The possibility that bad loans will cause a bank to default on its derivatives transactions is quite real, and credit ratings often reflect this possibility. In turn, ratings are a major determinant in business flows for banks that act as dealers. Hence, many derivatives dealers have taken

action to control their exposure to rating downgrades. One such action is the formation of a type of subsidiary that is separate from the dealer's other activities. These subsidiaries are referred to as **enhanced derivatives products companies** (EDPCs), sometimes known as special purpose vehicles (SPVs). These companies are usually completely separate from the parent organization and are not liable for the parent's debts. They tend to be very heavily capitalized and are committed to hedging all of their derivatives positions. As a result of these features, these subsidiaries almost always receive the highest credit quality rating by the rating agencies. In the event that the parent goes bankrupt, the EDPC is not liable for the parent company's debts; if the EDPC goes under, however, the parent is liable for an amount up to its equity investment and may find it necessary to provide even more protection. Hence, an EDPC would typically have a higher credit rating than its parent. In fact, it is precisely for the purpose of obtaining the highest credit rating, and thus the most favorable financing terms with counterparties, that banks and broker dealers go through the expense of putting together EDPCs.

#### 6.2.6. Transferring Credit Risk with Credit Derivatives

Another mechanism for managing credit risk is to transfer it to another party. Credit derivatives provide mechanisms for such transfers. Credit derivatives include such contracts as credit default swaps, total return swaps, credit spread options, and credit spread forwards. These transactions are typically customized, although the wording of contract provisions is often standardized. In a **credit default swap**, the protection buyer pays the protection seller in return for the right to receive a payment from the seller in the event of a specified credit event. In a **total return swap**, the protection buyer pays the total return on a reference obligation (or basket of reference obligations) in return for floating-rate payments. If the reference obligation has a credit event, the total return on the reference obligation should fall; the total return should also fall in the event of an increase in interest rates, so the protection seller (total return receiver) in this contract is actually exposed to both credit risk and interest rate risk. A **credit spread option** is an option on the yield spread of a reference obligation and over a referenced benchmark (such as the yield on a specific default-free security of the same maturity); by contrast, a **credit spread forward** is a forward contract on a yield spread. Credit derivatives may be used not only to eliminate credit risk but also to assume credit risk. For example, an investor may be well positioned to assume a credit risk because it is uncorrelated with other credit risks in her portfolio.<sup>52</sup>

### 6.3. Performance Evaluation

In order to maximize risk-adjusted return through the capital allocation process, we must measure performance against risks assumed and budgeted at both the business unit or substrategy level and enterprise or overall portfolio level. All business activities should be evaluated against the risk taken, and a considerable body of knowledge has developed concerning the evaluation of investment performance from a risk-adjusted perspective. Traditional approaches, which take into account return against a risk penalty, are now used in other areas of business activity besides portfolio management. Some banks and service providers have developed sophisticated performance evaluation systems that account for risk, and they have marketed these systems successfully to clients. Risk-adjusted performance, as measured against sensible benchmarks, is

<sup>52</sup>For more information on credit derivatives, see Fabozzi (2004b), Chapter 9, and Chance (2003), Chapter 9.

a critically important capital allocation tool because it allows for the comparison of results in terms of homogeneous units of exposure assumption. Absent these measurement tools, market participants with high risk profiles are likely to be given higher marks for positive performance than they arguably deserve because they derive more from increased exposure assumption than they do from superior portfolio management methodologies. Furthermore, most investment professionals are compensated on the basis of the performance of their portfolios, trading positions, or investment ideas, and it is appropriate to judge performance in risk-adjusted terms.

Following is a list of standard methodologies for expressing return in units of exposure assumption:

- **Sharpe Ratio** The seminal measure for risk-adjusted return, the Sharpe ratio has become the industry standard. The traditional definition of this measure is as follows:<sup>53</sup>

$$\text{Sharpe ratio} = \frac{\text{Mean portfolio return} - \text{Risk-free rate}}{\text{Standard deviation of portfolio return}}$$

The basic idea, therefore, is entirely intuitive: The Sharpe ratio is the mean return earned in excess of the risk-free rate per unit of volatility or total risk. By subtracting a risk-free rate from the mean return, we can isolate the performance associated with risk-taking activities. One elegant outcome of the calculation is that a portfolio engaging in “zero risk” investment, such as the purchase of Treasury bills for which the expected return is precisely the risk-free rate, earns a Sharpe ratio of exactly zero.

The Sharpe ratio calculation is the most widely used method for calculating risk-adjusted return. Nevertheless, it can be inaccurate when applied to portfolios with significant nonlinear risks, such as options positions. In part for these reasons, alternative risk-adjusted return methodologies have emerged over the years, including the following.

- **Risk-Adjusted Return on Capital (RAROC)** This concept divides the expected return on an investment by a measure of capital at risk, a measure of the investment’s risk that can take a number of different forms and can be calculated in a variety of ways that may have proprietary features. The company may require that an investment’s expected RAROC exceed a RAROC benchmark level for capital to be allocated to it.<sup>54</sup>
- **Return over Maximum Drawdown (RoMAD)** Drawdown, in the field of hedge fund management, is defined as the difference between a portfolio’s maximum point of return (known in industry parlance as its “high-water” mark), and any subsequent low point of performance. *Maximum* drawdown is the largest difference between a high-water and a subsequent low. Maximum drawdown is a preferred way of expressing the risk of a given portfolio—particularly as associated track records become longer—for investors who believe that observed loss patterns over longer periods of time are the best available proxy for actual exposure.

<sup>53</sup>This traditional definition of the Sharpe ratio can be directly linked to the capital market line and related capital market theory concepts (see Elton, Gruber, Brown, and Goetzmann, 2003). Sharpe (1994), however, defines the Sharpe ratio as a general construct using the mean excess return in relation to a benchmark in the numerator and the standard deviation of returns in excess of the benchmark in the denominator (see the discussion of the information ratio in the reading on evaluating portfolio performance for an illustration of this usage). Using the risk-free rate as the benchmark, the numerator would be as given in the text but the denominator would be the standard deviation of returns in excess of the risk-free rate (which, in practice, would infrequently result in significant discrepancies).

<sup>54</sup>For more information on RAROC, see Saunders and Cornett (2003).

Return over maximum drawdown is simply the average return in a given year that a portfolio generates, expressed as a percentage of this drawdown figure. It enables investors to ask the following question: Am I willing to accept an occasional drawdown of  $X\%$  in order to generate an average return of  $Y\%$ ? An investment with  $X = 10\%$  and  $Y = 15\%$  ( $\text{RoMAD} = 1.5$ ) would be more attractive than an investment with  $X = 40\%$  and  $Y = 10\%$  ( $\text{RoMAD} = 0.25$ ).

- **Sortino Ratio** One school of thought concerning the measurement of risk-adjusted returns argues, with some justification, that portfolio managers should not be penalized for volatility deriving from outsized positive performance. The Sortino ratio adopts this perspective. The numerator of the Sortino ratio is the return in excess of the investor's minimum acceptable return (MAR). The denominator is the downside deviation using the MAR as the target return.<sup>55</sup> **Downside deviation** computes volatility using only rate of return data points below the MAR. Thus the expression for the Sortino ratio is

$$\text{Sortino ratio} = (\text{Mean portfolio return} - \text{MAR}) / \text{Downside deviation}$$

If the MAR is set at the risk-free rate, the Sortino ratio is identical to the Sharpe ratio, save for the fact that it uses downside deviation instead of the standard deviation in the denominator. A side-by-side comparison of rankings of portfolios according to the Sharpe and Sortino ratios can provide a sense of whether outperformance may be affecting assessments of risk-adjusted performance. Taken together, the two ratios can tell a more detailed story of risk-adjusted return than either will in isolation, but the Sharpe ratio is better grounded in financial theory and analytically more tractable. Furthermore, departures from normality of returns can raise issues for the Sortino ratio as much as for the Sharpe ratio.

These approaches are only a subset of the methodologies available to investors wishing to calculate risk-adjusted returns. Each approach has both its merits and its drawbacks. Perhaps the most important lesson to bear in mind with respect to this mosaic is the critical need to understand the inputs to any method, so as to be able to interpret the results knowledgeably, with an understanding of their possible limitations.

#### 6.4. Capital Allocation

In addition to its unquestionable value in the task of capital preservation, risk management has become a vital, if not central, component in the process of allocating capital across units of a risk-taking enterprise. The use of inputs, such as volatility/correlation analysis, risk-adjusted return calculations, scenario analysis, etc., provides the allocators of risk capital with a much more informed means of arriving at the appropriate conclusions on how best to distribute this scarce resource. The risk management inputs to the process can be used in formal, mathematical, “optimization” routines, under which enterprises input performance data into statistical programs that will then offer appropriate capital allocation combinations to make efficient use of risk. Quantitative output may simply serve as background data for qualitative decision-making processes. One way or another, however, risk management has become a vital input into the capital allocation process, and it is fair to describe this development as positive from a systemic perspective.

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<sup>55</sup> Downside deviation, the term usually used in presenting the Sortino ratio, could also be called a target semideviation (using MAR as the target).

As part of the task of allocating capital across business units, organizations must determine how to measure such capital. Here there are multiple methodologies, and we will discuss five of them in further detail:

1. **Nominal, Notional, or Monetary Position Limits** Under this approach, the enterprise simply defines the amount of capital that the individual portfolio or business unit can use in a specified activity, based on the actual amount of money exposed in the markets. It has the advantage of being easy to understand, and, in addition, it lends itself very nicely to the critical task of calculating a percentage-based return on capital allocated. Such limits, however, may not capture effectively the effects of correlation and offsetting risks. Furthermore, an individual may be able to work around a nominal position using other assets that can replicate a given position. For these reasons, although it is often useful to establish notional position limits, it is seldom a *sufficient* capital allocation method from a risk control perspective.
2. **VaR-Based Position Limits** As an alternative or supplement to notional limits, enterprises often assign a VaR limit as a proxy for allocated capital. This approach has a number of distinct advantages, most notably the fact that it allocates capital in units of estimated exposure and thus acts in greater harmony with the risk control process. This approach has potential problems as well, however. Most notably, the limit regime will be only as effective as the VaR calculation itself; when VaR is cumbersome, less than completely accurate, not well understood by traders, or some combination of the above, it is difficult to imagine it providing rational results from a capital allocation perspective. In addition, the relation between overall VaR and the VaRs of individual positions is complex and can be counterintuitive.<sup>56</sup> Nevertheless, VaR limits probably have an important place in any effective capital allocation scheme.
3. **Maximum Loss Limits** Irrespective of other types of limit regimes that it might have in place, it is crucial for any risk-taking enterprise to establish a maximum loss limit for each of its risk-taking units. In order to be effective, this figure must be large enough to enable the unit to achieve performance objectives but small enough to be consistent with the preservation of capital. This limit must represent a firm constraint on risk-taking activity. Nevertheless, even when risk-taking activity is generally in line with policy, management should recognize that extreme market discontinuities can cause such limits to be breached.
4. **Internal Capital Requirements** Internal capital requirements specify the level of capital that management believes to be appropriate for the firm. Some regulated financial institutions, such as banks and securities firms, typically also have regulatory capital requirements that, if they are higher, overrule internal requirements. Traditionally, internal capital requirements have been specified heuristically in terms of the capital ratio (the ratio of capital to assets). Modern tools permit a more rigorous approach. If the value of assets declines by an amount that exceeds the value of capital, the firm will be insolvent. Say a 0.01 probability of insolvency over a one-year horizon is acceptable. By requiring capital to equal at least one-year aggregate VaR at the 1% probability level, the capital should be adequate in terms of the firm's risk tolerance. If the company can assume a normal return distribution, the required amount of capital can be stated in standard deviation units (e.g., 1.96 standard deviations would reflect a 0.025 probability of insolvency). A capital

<sup>56</sup>For example, one cannot add the VaR of individual positions to obtain a conservative measure (i.e., maximum) of overall VaR because it is possible for the sum of the VaRs to be greater than the VaR of the combined positions.

requirement based on aggregate VaR has an advantage over regulatory capital requirements in that it takes account of correlations. Furthermore, to account for extraordinary shocks, we can stress test the VaR-based recommendation.

5. **Regulatory Capital Requirements** In addition, many institutions (e.g., securities firms and banks) must calculate and meet regulatory capital requirements. Wherever and whenever this is the case, it of course makes sense to allocate this responsibility to business units. Meeting regulatory capital requirements can be a difficult process, among other reasons because such requirements are sometimes inconsistent with rational capital allocation schemes that have capital preservation as a primary objective. Nevertheless, when regulations demand it, firms must include regulatory capital as part of their overall allocation process.

Depending on such factors as the type of enterprise, its corporate culture, fiduciary obligations, etc., the most effective approach to capital allocation probably involves a combination of most, if not all, of the above methodologies. The trick, of course, is to combine the appropriate ones in a rational and consistent manner that creates the proper incentives for balance between the dual objectives of profit maximization and capital preservation.

### 6.5. Psychological and Behavioral Considerations

Over the past several years, a body of research has emerged that seeks to model the behavioral aspects of portfolio management. This concept has important implications for risk management for two reasons. First, risk takers may behave differently at different points in the portfolio management cycle, depending on such factors as their recent performance, the risk characteristics of their portfolios, and market conditions. Second, and on a related note, risk management would improve if these dynamics could be modeled.

Although the topic merits more discussion than we can possibly include in this context, the main factor to consider from a risk management perspective is the importance of establishing a risk governance framework that anticipates the points in a cycle when the incentives of risk takers diverge from those of risk capital allocators. One prominent example (although by no means the only one) occurs when portfolio managers who are paid a percentage of their profits in a given year fall into a negative performance situation. The trader's situation does not deteriorate from a compensation perspective with incremental losses at this point (i.e., the trader is paid zero, no matter how much he loses), but of course the organization as a whole suffers from the trader's loss. Moreover, the risks at the enterprise level can be nonlinear under these circumstances because of concepts of netting risk covered earlier in this reading. These and other behavioral issues can be handled best by risk control and governance processes that contemplate them. One such example is limit setting, which can, with some thought, easily incorporate many of these issues.<sup>57</sup>

## 7. SUMMARY

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Financial markets reward competence and knowledge in risk management and punish mistakes. Portfolio managers must therefore study and understand the discipline of successful

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<sup>57</sup>Those interested in studying these topics further may wish to refer to Grant (2004) and Kiev (2002).

risk management. In this reading, we have introduced basic concepts and techniques of risk management and made the following points:

- Risk management is a process involving the identification of the exposures to risk, the establishment of appropriate ranges for exposures, the continuous measurement of these exposures, and the execution of appropriate adjustments to bring the actual level and desired level of risk into alignment. The process involves continuous monitoring of exposures and new policies, preferences, and information.
- Typically, risks should be minimized wherever and whenever companies lack comparative advantages in the associated markets, activities, or lines of business.
- Risk governance refers to the process of setting risk management policies and standards for an organization. Senior management, which is ultimately responsible for all organizational activities, must oversee the process.
- Enterprise risk management is a centralized risk management system whose distinguishing feature is a firm-wide or across-enterprise perspective on risk.
- Financial risk refers to all risks derived from events in the external financial markets. Non-financial risk refers to all other forms of risk. Financial risk includes market risk (risk related to interest rates, exchange rates, stock prices, and commodity prices), credit risk, and liquidity risk. The primary sources of nonfinancial risk are operations risk, model risk, settlement risk, regulatory risk, legal risk, tax risk, and accounting risk.
- Traditional measures of market risk include linear approximations such as beta for stocks, duration for fixed income, and delta for options, as well as second-order estimation techniques such as convexity and gamma. For products with option-like characteristics, techniques exist to measure the impact of changes in volatility (vega) and the passage of time (theta). Sensitivity to movements in the correlation among assets is also relevant for certain types of instruments.
- Value at risk (VaR) estimates the minimum loss that a party would expect to experience with a given probability over a specified period of time. Using the complementary probability (i.e., 100% minus the given probability stated as a percent), VaR can be expressed as a maximum loss at a given confidence level. VaR users must make decisions regarding appropriate time periods, confidence intervals, and specific VaR methodologies.
- The analytical or variance–covariance method can be used to determine VaR under the assumption that returns are normally distributed by subtracting a multiple of the standard deviation from the expected return, where the multiple is determined by the desired probability level. The advantage of the method is its simplicity. Its disadvantages are that returns are not normally distributed in any reliable sense and that the method does not work well when portfolios contain options and other derivatives.
- The historical method estimates VaR from data on a portfolio's performance during a historical period. The returns are ranked, and VaR is obtained by determining the return that is exceeded in a negative sense 5% or 1% (depending on the user's choice) of the time. The historical method has the advantage of being simple and not requiring the assumption of a normal distribution. Its disadvantage is that accurate historical time-series information is not always easily available, particularly for instruments such as bonds and options, which behave differently at different points in their life spans.
- Monte Carlo simulation estimates VaR by generating random returns and determining the 5% or 1% (depending on the user's choice) worst outcomes. It has the advantages that it does not require a normal distribution and can handle complex relationships among risks.

- VaR can be difficult to estimate, can give a wide range of values, and can lead to a false sense of security that risk is accurately measured and under control. VaR for individual positions do not generally aggregate in a simple way to portfolio VaR. VaR also puts all emphasis on the negative outcomes, ignoring the positive outcomes. It can be difficult to calculate VaR for a large complex organization with many exposures. On the other hand, VaR is a simple and easy-to-understand risk measure that is widely accepted. It is also adaptable to a variety of uses, such as allocating capital.
- Incremental VaR measures the incremental effect of an asset on the VaR of a portfolio. Cash flow at risk and earnings at risk measure the risk to a company's cash flow or earnings instead of market value, as in the case of VaR. Tail value at risk is VaR plus the expected loss in excess of VaR, when such excess loss occurs. Stress testing is another important supplement to VaR.
- Credit risk has two dimensions, the probability of default and the associated recovery rate.
- Credit risk in a forward contract is assumed by the party to which the market value is positive. The market value represents the current value of the claim that one party has on the other. The actual payoff at expiration could differ, but the market value reflects the current value of that future claim.
- Credit risk in swaps is similar to credit risk in forward contracts. The market value represents the current value of the claim on the future payments. The party holding the positive market value assumes the credit risk at that time. For interest rate and equity swaps, credit risk is greatest near the middle of the life of the swap. For currency swaps with payment of notional principal, credit risk is greatest near the end of the life of the swap.
- Credit risk in options is one-sided. Because the seller is paid immediately and in full, she faces no credit risk. By contrast, the buyer faces the risk that the seller will not meet her obligations in the event of exercise. The market value of the option is the current value of the future claim the buyer has on the seller.
- VaR can be used to measure credit risk. The interpretation is the same as with standard VaR, but a credit-based VaR is more complex because it must interact with VaR based on market risk. Credit risk arises only when market risk results in gains to trading. Credit VaR must take into account the complex interaction of market movements, the possibility of default, and recovery rates. Credit VaR is also difficult to aggregate across markets and counterparties.
- Risk budgeting is the process of establishing policies to allocate the finite resource of risk capacity to business units that must assume exposure in order to generate return. Risk budgeting has also been applied to allocation of funds to investment managers.
- The various methods of controlling credit risk include setting exposure limits for individual counterparties, exchanging cash values that reflect mark-to-market levels, posting collateral, netting, setting minimum credit, using special-purpose vehicles that have higher credit ratings than the companies that own them, and using credit derivatives.
- Among the measures of risk-adjusted performance that have been used in a portfolio context are the Sharpe ratio, risk-adjusted return on capital, return over maximum drawdown, and the Sortino ratio. The Sharpe ratio uses standard deviation, measuring total risk as the risk measure. Risk-adjusted return on capital accounts for risk using capital at risk. The Sortino ratio measures risk using downside deviation, which computes volatility using only rate-of-return data points below a minimum acceptable return. Return over maximum drawdown uses maximum drawdown as a risk measure.
- Methods for allocating capital include nominal position limits, VaR-based position limits, maximum loss limits, internal capital requirements, and regulatory capital requirements.

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## PROBLEMS

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1. Discuss the difference between centralized and decentralized risk management systems, including the advantages and disadvantages of each.
2. Stewart Gilchrist follows the automotive industry, including Ford Motor Company. Based on Ford's 2003 annual report, Gilchrist writes the following summary:

Ford Motor Company has businesses in several countries around the world. Ford frequently has expenditures and receipts denominated in non-US currencies, including purchases and sales of finished vehicles and production parts, subsidiary dividends, investments in non-US operations, etc. Ford uses a variety of commodities in the production of motor vehicles, such as non-ferrous metals, precious metals, ferrous alloys, energy, and plastics/resins. Ford typically purchases these commodities from outside suppliers. To finance its operations, Ford uses a variety of funding sources, such as commercial paper, term debt, and lines of credit from major commercial banks. The company invests any surplus cash in securities of various types and maturities, the value of which are subject to fluctuations in interest rates. Ford has a credit division, which provides financing to customers wanting to purchase Ford's vehicles on credit. Overall, Ford faces several risks. To manage some of its risks, Ford invests in fixed-income instruments and derivative contracts. Some of these investments do not rely on a clearing house and instead effect settlement through the execution of bilateral agreements.

Based on the above discussion, recommend and justify the risk exposures that should be reported as part of an Enterprise Risk Management System for Ford Motor Company.

3. NatWest Markets (NWM) was the investment banking arm of National Westminster Bank, one of the largest banks in the United Kingdom. On 28 February 1997, NWM revealed that a substantial loss had been uncovered in its trading books. During the 1990s, NatWest was engaged in trading interest rate options and swaptions on several underlying currencies. This trading required setting appropriate prices of the options by the traders at NatWest. A key parameter in setting the price of an interest rate option is the implied volatility of the underlying asset—that is, the interest rate on a currency. In contrast to other option parameters that affect the option prices, such as duration to maturity and exercise price, implied volatility is not directly observable and must be estimated. Many option pricing models imply that the implied volatility should be the same for all options on the same underlying, irrespective of their exercise price or maturity. In practice, however, implied volatility is often observed to have a curvilinear relationship with the option's moneyness (i.e., whether the option is out of the money, at the money, or in the money), a relationship sometimes called the *volatility smile*. Implied volatility tended to be higher for out-of-the-money options than for at-the-money options on the same underlying.

NWM prices on certain contracts tended to consistently undercut market prices, as if the out-of-the-money options were being quoted at implied volatilities that were too low. When trading losses mounted in an interest rate option contract, a trader undertook a series of off-market-price transactions between the options portfolio and a swaptions

portfolio to transfer the losses to a type of contract where losses were easier to conceal. A subsequent investigation revealed that the back office did not independently value the trading positions in question and that lapses in trade reconciliation had occurred.

What type or types of risk were inadequately managed in the above case?

4. Sue Ellicott supervises the trading function at an asset management firm. In conducting an in-house risk management training session for traders, Ellicott elicits the following statements from traders:

Trader 1 "Liquidity risk is not a major concern for buyers of a security as opposed to sellers."

Trader 2 "In general, derivatives can be used to substantially reduce the liquidity risk of a security."

Ellicott and the traders then discuss two recent cases of a similar risk exposure in an identical situation that one trader (Trader A) hedged and another trader (Trader B) assumed as a speculation. A participant in the discussion makes the following statement concerning the contrasting treatment:

Trader 3 "Our traders have considerable experience and expertise in analyzing the risk, and this risk is related to our business. Trader B was justified in speculating on the risk within the limits of his risk allocation."

State and justify whether each trader's statement is correct or incorrect.

5. A large trader on the government bond desk of a major bank loses €20 million in a year, in the process reducing the desk's overall profit to €10 million. Senior management, on looking into the problem, determines that the trader repeatedly violated his position limits during the year. They also determine that the bulk of the loss took place in the last two weeks of the year, when the trader increased his position dramatically and experienced 80% of his negative performance. The bank dismisses both the trader and his desk manager. The bank has an asymmetric incentive compensation contract arrangement with its traders.
  - A. Discuss the performance netting risk implications of this scenario.
  - B. Are there any reasons why the timing of the loss is particularly significant?
  - C. What mistakes did senior management make? Explain how these errors can be corrected.
6. Ford Credit is the branch of Ford Motor Company that provides financing to Ford's customers. For this purpose, it obtains funding from various sources. As a result of its interest rate risk management process, including derivatives, Ford Credit's debt reprices faster than its assets. This situation means that when interest rates are rising, the interest rates paid on Ford Credit's debt will increase more rapidly than the interest rates earned on assets, thereby initially reducing Ford Credit's pretax net interest income. The reverse will be true when interest rates decline.

Ford's annual report provides a quantitative measure of the sensitivity of Ford Credit's pretax net interest income to changes in interest rates. For this purpose, it uses interest rate scenarios that assume a hypothetical, instantaneous increase or decrease in interest rates of 1 percentage point across all maturities. These scenarios are compared with a base case that assumes that interest rates remain constant at existing levels. The differences between the scenarios and the base case over a 12-month period represent an estimate of the sensitivity of Ford Credit's pretax net interest income. This sensitivity as of year-end 2003 and 2002 is as follows:

	Pretax Net Interest Income Impact Given a One Percentage Point Instantaneous <i>Increase</i> in Interest Rates (in Millions)	Pretax Net Interest Income Impact Given a One Percentage Point Instantaneous <i>Decrease</i> in Interest Rates (in Millions)
December 31, 2003	(\$179)	\$179
December 31, 2002	(\$153)	\$156

Source: Annual Report of Ford Motor Company, 2003.

Describe the strengths and weaknesses of the interest rate risk analysis presented in the foregoing table.

7. A. An organization's risk management function has computed that a portfolio held in one business unit has a 1% weekly value at risk (VaR) of £4.25 million. Describe what is meant in terms of a minimum loss.  
B. The portfolio of another business unit has a 99% weekly VaR of £4.25 million (stated using a confidence limit approach). Describe what is meant in terms of a maximum loss.
8. Each of the following statements about VaR is true *except*:  
A. VaR is the loss that would be exceeded with a given probability over a specific time period.  
B. Establishing a VaR involves several decisions, such as the probability and time period over which the VaR will be measured and the technique to be used.  
C. VaR will be larger when it is measured at 5% probability than when it is measured at 1% probability.  
D. VaR will be larger when it is measured over a month than when it is measured over a day.
9. Suppose you are given the following sample probability distribution of annual returns on a portfolio with a market value of \$10 million.

Return on Portfolio	Probability
Less than -50%	0.005
-50% to -40%	0.005
-40% to -30%	0.010
-30% to -20%	0.015
-20% to -10%	0.015
-10% to -5%	0.165
-5% to 0%	0.250
0% to 5%	0.250
5% to 10%	0.145
10% to 20%	0.075
20% to 30%	0.025
30% to 40%	0.020
40% to 50%	0.015
Greater than 50%	0.005
	1.000

Based on this probability distribution, determine the following:

- A. 1% yearly VaR.
  - B. 5% yearly VaR.
10. An analyst would like to know the VaR for a portfolio consisting of two asset classes: long-term government bonds issued in the United States and long-term government bonds issued in the United Kingdom. The expected monthly return on US bonds is 0.85%, and the standard deviation is 3.20%. The expected monthly return on UK bonds, in US dollars, is 0.95%, and the standard deviation is 5.26%. The correlation between the US dollar returns of UK and US bonds is 0.35. The portfolio market value is \$100 million and is equally weighted between the two asset classes. Using the analytical or variance-covariance method, compute the following:
- A. 5% monthly VaR.
  - B. 1% monthly VaR.
  - C. 5% weekly VaR.
  - D. 1% weekly VaR.
11. You invested \$25,000 in the stock of Dell Computer Corporation in early 2011. You have compiled the monthly returns on Dell's stock during the period 2006–2010, as given below.

2006	2007	2008	2009	2010
-0.0214	-0.0347	-0.1824	-0.0723	-0.1017
-0.0106	-0.0566	-0.0070	-0.1021	0.0264
0.0262	0.0158	0.0010	0.1114	0.1344
-0.1196	0.0862	-0.0648	0.2257	0.0786
-0.0313	0.0675	0.2378	-0.0043	-0.1772
-0.0362	0.0609	-0.0512	0.1867	-0.0953
-0.1137	-0.0203	0.1229	-0.0255	0.0978
0.0401	0.0100	-0.1156	0.1831	-0.1110
0.0129	-0.0230	-0.2416	-0.0360	0.1020
0.0652	0.1087	-0.2597	-0.0531	0.1099
0.1196	-0.1980	-0.0844	-0.0228	-0.0816
-0.0789	-0.0012	-0.0833	0.0170	0.0250

Using the historical method, compute the following:

- A. 5% monthly VaR.
  - B. 1% monthly VaR.
12. Consider a \$10 million portfolio of stocks. You perform a Monte Carlo simulation to estimate the VaR for this portfolio. You choose to perform this simulation using a normal distribution of returns for the portfolio, with an expected annual return of 14.8% and a

standard deviation of 20.5%. You generate 700 random outcomes of annual return for this portfolio, of which the worst 40 outcomes are given below.

-0.400	-0.320	-0.295	-0.247
-0.398	-0.316	-0.282	-0.233
-0.397	-0.314	-0.277	-0.229
-0.390	-0.310	-0.273	-0.226
-0.355	-0.303	-0.273	-0.223
-0.350	-0.301	-0.261	-0.222
-0.347	-0.301	-0.259	-0.218
-0.344	-0.300	-0.253	-0.216
-0.343	-0.298	-0.251	-0.215
-0.333	-0.296	-0.248	-0.211

Using the above information, compute the following:

- A. 5% annual VaR.
  - B. 1% annual VaR.
13. A firm runs an investment portfolio consisting of stocks as well as options on stocks. Management would like to determine the VaR for this portfolio and is thinking about which technique to use. Discuss a problem with using the analytical or variance-covariance method for determining the VaR of this portfolio.
- B. Describe a situation in which an organization might logically select each of the three VaR methodologies.
14. An organization's 5% daily VaR shows a number fairly consistently around €3 million. A backtest of the calculation reveals that, as expected under the calculation, daily portfolio losses in excess of €3 million tend to occur about once a month. When such losses do occur, however, they typically are more than double the VaR estimate. The portfolio contains a very large short options position.
- A. Is the VaR calculation accurate?
  - B. How can the VaR figure best be interpreted?
  - C. What additional measures might the organization take to increase the accuracy of its overall exposure assessments?
15. Indicate which of the following statements about credit risk is (are) false, and explain why.
- A. Because credit losses occur often, it is easy to assess the probability of a credit loss.
  - B. One element of credit risk is the possibility that the counterparty to a contract will default on an obligation to another (i.e., third) party.
  - C. Like the buyer of a European-style option, the buyer of an American-style option faces no current credit risk until the expiration of the option.
16. Ricardo Colón, an analyst in the investment management division of a financial services firm, is developing an earnings forecast for a local oil services company. The company's

income is closely linked to the price of oil. Furthermore, the company derives the majority of its income from sales to the United States. The economy of the company's home country depends significantly on export oil sales to the United States. As a result, movements in world oil prices in US dollar terms and the US dollar value of the home country's currency are strongly positively correlated. A decline in oil prices would reduce the company's sales in US dollar terms, all else being equal. On the other hand, the appreciation of the home country's currency relative to the US dollar would reduce the company's sales in terms of the home currency.

According to Colón's research, Raúl Rodriguez, the company's chief risk officer, has made the following statement:

"The company has rejected hedging the market risk of a decline in oil prices by selling oil futures and hedging the currency risk of a depreciation of the US dollar relative to our home currency by buying home currency futures in US markets. We have decided that a more effective risk management strategy for our company is to not hedge either market risk or currency risk."

- A. State whether the company's decision to not hedge market risk was correct. Justify your answer with one reason.
  - B. State whether the company's decision to not hedge currency risk was correct. Justify your answer with one reason.
  - C. Critique the risk management strategy adopted.
17. Tony Smith believes that the price of a particular underlying, currently selling at \$96, will increase substantially in the next six months, so he purchases a European call option expiring in six months on this underlying. The call option has an exercise price of \$101 and sells for \$6.
- A. How much is the current credit risk, if any?
  - B. How much is the current value of the potential credit risk, if any?
  - C. Which party bears the credit risk(s), Tony Smith or the seller?
18. Following are four methods for calculating risk-adjusted performance: the Sharpe ratio, risk-adjusted return on capital (RAROC), return over maximum drawdown (Ro-MAD), and the Sortino ratio. Compare and contrast the measure of risk that each method uses.

### The following information relates to Questions 19–24

Monika Kreuzer chairs the risk management committee for DGI Investors, a European money management firm. The agenda for the 1 June committee meeting includes three issues concerning client portfolios:

1. Estimating a new value at risk (VaR) for the Stimson Industries portfolio.
2. Answering questions from Kalton Corporation managers.
3. Revising the VaR for Muth Company given new capital market expectations.

**1. VaR for Stimson Industries.** DGI currently provides a 5% yearly VaR on the equity portfolio that it manages for Stimson. The €50 million portfolio has an expected annual return of 9.6% and an annual standard deviation of 18.0%. With a standard normal distribution, 5% of the possible outcomes are 1.65 standard deviations or more below the mean. Using the analytical (variance–covariance) method for calculating VaR, DGI estimates the 5% yearly VaR to

be €10.05 million. Assuming that monthly returns are independent, committee member Eric Stulz wants to estimate a 5% *monthly* VaR for Stimson's portfolio.

Stulz asks his fellow committee members for feedback on the following statements about VaR in a report he is preparing for Stimson Industries:

- Statement #1: "Establishing a VaR involves several decisions, such as the probability and time period over which the VaR will be measured and the technique to be used."
- Statement #2: "A portfolio's VaR will be larger when it is measured at a 5% probability than when it is measured at a 1% probability."
- Statement #3: "A portfolio's VaR will be larger when it is measured over a month than when it is measured over a day."

**2. Questions from Kalton Corporation Managers.** Kalton Corporation has two large derivatives positions with a London securities house. The first position is a long forward currency contract to buy pounds at €1.4500. The current exchange rate is €1.4000 per pound. The second position is a long put option on the DJ Euro STOXX Index with a strike price of 305.00. The current closing price of the index is 295.00. A Kalton manager has written, "I am concerned about the risks of these two large positions. Who is bearing the credit risks, Kalton Corporation or the counterparty (the London securities house)?" Kreuzer suggests that DGI reply: "Kalton Corporation is bearing the credit risk of the currency forward contract, but the London securities house is bearing the credit risk of the put option on the DJ Euro STOXX Index."

Because they believe that the credit risk in corporate bonds is going to decline, Kalton Corporation managers have decided to increase Kalton's credit risk exposure in corporate bonds. They have asked Kreuzer and the risk management committee to recommend derivatives positions to accomplish this change.

**3. Revising the VaR for Muth Company.** Kreuzer provides a variety of statistics to Muth, for whom DGI manages a portfolio composed of 50% in Asia-Pacific equities and 50% in European equities. One of the statistics that Kreuzer supplies Muth is a 5% monthly VaR estimate based on the analytical (variance-covariance) method. Kreuzer is concerned that changes in the market outlook will affect Muth's risk. DGI is updating its capital market expectations, which will include 1) an increase in the expected return on Asia-Pacific equities and 2) an increase in the correlation between Asia-Pacific equities and European equities. Kreuzer comments: "Considered independently, and assuming that other variables are held constant, each of these changes in capital market expectations will increase the monthly VaR estimate for the Muth portfolio."

Kreuzer also discusses the limitations and strengths of applying VaR to the Muth portfolio. She states that: "One of the advantages of VaR is that the VaR of individual positions can be simply aggregated to find the portfolio VaR." Kreuzer also describes how VaR can be supplemented with performance evaluation measures, such as the Sharpe ratio. She states: "The Sharpe ratio is widely used for calculating a risk-adjusted return, although it can be an inaccurate measure when applied to portfolios with significant options positions."

19. The monthly VaR that Stulz wants to estimate for the Stimson portfolio is *closest* to:

- A. €0.8 million.
- B. €2.9 million.
- C. €3.9 million.

20. Regarding the three statements in the report that Stulz is preparing for Stimson Industries, the statement that is *incorrect* is:

- A. Statement #1.
- B. Statement #2.
- C. Statement #3.

21. Regarding Kalton's two derivatives positions, is Kreuzer correct about which party is bearing the credit risk of the currency forward contract and the put option on the DJ Euro STOXX Index, respectively?

	Currency Forward Contract	Put Option on the DJ Euro STOXX Index
A.	No	No
B.	No	Yes
C.	Yes	Yes

22. To make the desired change in Kalton's credit risk exposure in corporate bonds, Kreuzer could recommend that Kalton take a position as a:

- A. seller in a credit default swap.
- B. buyer in a credit default swap.
- C. buyer of a put option on a corporate bond.

23. Is Kreuzer correct in predicting the independent effects of the increase in the expected return and the increase in the correlation, respectively, on the calculated VaR of the Muth portfolio?

	Effect of Increase in the Expected Return	Effect of Increase in the Correlation
A.	No	No
B.	No	Yes
C.	Yes	No

24. Are Kreuzer's statements about an advantage of VaR and about the Sharpe ratio, respectively, correct?

	Statement about an Advantage of VaR	Statement about the Sharpe Ratio
A.	No	No
B.	No	Yes
C.	Yes	Yes

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# CHAPTER 7

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## RISK MANAGEMENT APPLICATIONS OF FORWARD AND FUTURES STRATEGIES

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### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- demonstrate the use of equity futures contracts to achieve a target beta for a stock portfolio and calculate and interpret the number of futures contracts required;
- construct a synthetic stock index fund using cash and stock index futures (equitizing cash);
- explain the use of stock index futures to convert a long stock position into synthetic cash;
- demonstrate the use of equity and bond futures to adjust the allocation of a portfolio between equity and debt;
- demonstrate the use of futures to adjust the allocation of a portfolio across equity sectors and to gain exposure to an asset class in advance of actually committing funds to the asset class;
- explain exchange rate risk and demonstrate the use of forward contracts to reduce the risk associated with a future receipt or payment in a foreign currency;
- explain the limitations to hedging the exchange rate risk of a foreign market portfolio and discuss feasible strategies for managing such risk.

## 1. INTRODUCTION

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In preceding readings, we examined the characteristics and pricing of forwards, futures, options, and swaps. On occasion, we made reference to possible ways in which these instruments could be used. In the readings that follow, we examine more specifically the strategies and applications that are commonly used with these instruments. Here we focus on forward and futures contracts. These instruments are quite similar. Both involve commitments for one party to buy and the other to sell an underlying instrument at a future date at a price agreed on at the start of the contract. The underlying instrument might be an interest payment, a bond, a stock, or a currency. Forward contracts are customized agreements between two parties: The terms are agreed on by both parties in a formal legal contract that exists in an environment outside of regulatory constraints. Each party is subject to potential default on the part of the other. Futures contracts, on the other hand, are standardized instruments created on a futures exchange, protected against credit losses by the clearinghouse, and subject to federal regulatory oversight.

In this reading, we examine a number of scenarios in which parties facing risk management problems use forward and futures contracts to alter the risk of their positions. In some situations we use forwards and in others we use futures. For cases in which either would suffice, we pick the instrument that is most commonly used in that type of situation. Although we shall not devote a great deal of space up front to justifying why we picked the instrument we did, we shall provide some discussion of this point in Section 6.

After completing this reading, you may be surprised to observe that we do not cover an important class of derivative strategies, those that are called *arbitrage*. This omission is not because they are not important enough to cover or that they are not risk management strategies; in fact, we have *already* covered them. When we covered the pricing of forwards, futures, options, and swaps, we explained how these instruments are priced by combining the underlying and risk-free bonds to replicate the derivative or by combining a long position in the underlying and a short position in the derivative to replicate a risk-free position. From there we obtained a formula that gives us the correct price of the derivative. An arbitrage profit is possible if the derivative is not priced according to the formula. We have already looked at how those strategies are executed. We should not expect to encounter arbitrage opportunities very often in practice. They are quickly captured by derivatives trading firms, which themselves cannot expect to be able to *consistently* claim such opportunities before they disappear.<sup>1</sup>

Businesses make products and provide services as they attempt to increase shareholder wealth. In doing so, they face a variety of risks. Managing risk lies at the heart of what companies do. All companies specialize in managing the risk of whatever market their primary business is in: Airlines deal with the risk associated with the demand for air travel, software companies deal with the risk associated with the demand for new computer programs, movie companies deal with the risk associated with the demand for their films. But these companies also deal with other risks, such as the risk of interest rates and exchange rates. Usually these

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<sup>1</sup>Suppose market participants assume that arbitrage opportunities are so infrequent and difficult to capture before they are gone that no one monitors market prices looking for arbitrage opportunities. Then these arbitrage opportunities will begin to occur more frequently. A market in which arbitrage opportunities are rare, and therefore prices are fair and accurate, is ironically a market in which participants believe they can indeed uncover and exploit arbitrage opportunities. Thus, an arbitrage-free market requires disbelievers.

companies take calculated risks in their primary lines of business and avoid risks they do not feel qualified to take, such as interest rate risk and exchange rate risk. Naturally this approach involves a practice called **hedging**.

Hedging involves taking a market position to protect against an undesirable outcome. Suppose a company has a strong belief that interest rates will increase. It engages in a forward rate agreement (FRA) transaction to lock in the rate on a loan it will take out at a later date. This position protects the company from the undesirable outcome of an increase in interest rates, but it also prevents the company from enjoying any decline in rates. In that sense, the position is as much a speculative position as if a speculator had made the following prediction: *We believe that interest rates will rise to an unacceptable level, and we intend to trade on that basis to make money.* By engaging in the FRA to hedge this outcome, the company trades to make a profit from its FRA that will help offset any increase in the interest rate on its future loan. But by locking in a rate, it forgoes the possibility of benefiting from a decline in interest rates. The company has made a bet that rates will rise. If they fall, the company has lost the bet and lost money on its FRA that offsets the benefit of the lower interest rate on this loan planned for a later date.

In this reading we shall not overindulge in the use of the term hedging. We shall say that companies do more than hedge: *They manage risk.* They carefully consider scenarios and elect to adjust the risk they face to a level they feel is acceptable. In many cases, this adjustment will involve the reduction of risk; in some cases, however, the scenario will justify increasing the company's risk. In all cases, the company is just altering the risk from its current level to the level the company desires. And that is what managing risk is all about.

This reading is divided into five main parts. Sections 2 and 3 focus on the management of interest rate and equity market risk, respectively. Section 4 combines interest rate and equity risk management applications by looking at how investors can manage an asset portfolio using futures. Section 5 looks at the management of foreign currency risk. In Section 6 we examine the general question of whether to use forwards or futures to manage risk, and in Section 7 we look at a few final issues.

## 2. STRATEGIES AND APPLICATIONS FOR MANAGING INTEREST RATE RISK

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Almost every business borrows money from time to time. A company borrowing at a fixed rate may think it is immune to interest rate risk, but that is not the case. Risk arises from the possibility that interest rates can increase from the time the company decides to take the loan to the time it actually takes the loan. Most companies make plans to borrow based on their cash needs at specific future dates. The rates they pay on these loans are important determinants of their future cash needs, as reflected in their planned interest payments. Exposure to interest rate risk is, therefore, a major concern. Failing to manage interest rate risk can hinder the planning process, as well as result in unexpected demands on cash necessitated by unexpectedly higher interest payments.

### 2.1. Managing the Interest Rate Risk of a Loan Using an FRA

There are several situations in which a company might want to manage the interest rate risk of a loan. The two we look at here involve a company planning to take out a loan at a later

date. In one situation, the loan has a single interest rate and a single interest payment. In another situation, a company takes out a floating-rate loan in which the interest rate is reset periodically.

### 2.1.1. Single-Payment Loan

Exhibit 1 presents the case of Global BioTechnology (GBT), which determines that it will need to borrow money at a later date at a rate of Libor plus 200 basis points. Fearing an increase in interest rates between now and the day it takes out the loan, it enters into a long position in an FRA. The FRA has a fixed rate, called the FRA rate. If the underlying rate at expiration is above the FRA rate, GBT as the holder of the long position will receive a lump sum of cash based on the difference between the FRA rate and the market rate at that time. This payment will help offset the higher rate GBT would be paying on its loan. If the rate in the market falls below the FRA rate, however, GBT will end up paying the counterparty, which will offset the lower rate GBT will be paying on its loan. The end result is that GBT will pay approximately a fixed rate, the FRA rate.

## EXHIBIT 1 Using an FRA to Lock in the Rate on a Loan

### Scenario (15 April)

Global BioTechnology (GBT) is a US corporation that occasionally undertakes short-term borrowings in US dollars with the rate tied to Libor. To facilitate its cash flow planning, it uses an FRA to lock in the rate on such loans as soon as it determines that it will need the money.

On 15 April, GBT determines that it will borrow \$40 million on 20 August. The loan will be repaid 180 days later on 16 February, and the rate will be at Libor plus 200 basis points. Because GBT believes that interest rates will increase, it decides to manage this risk by going long an FRA. An FRA will enable it to receive the difference between Libor on 20 August and the FRA rate quoted by the dealer on 15 April. The quoted rate from the dealer is 5.25%. GBT wants to lock in a 7.25% rate: 5.25% plus 200 basis points.

### Action

GBT confirms that it will borrow \$40 million at Libor plus 200 basis points on 20 August. GBT goes long an FRA at a rate of 5.25% to expire on 20 August with the underlying being 180-day Libor.

### Scenario (20 August)

At contract expiration, 180-day Libor is 6%.

### Outcome and Analysis

The FRA payoff is given by the general formula:

$$\begin{aligned}
 & \text{Notional principal} \times \\
 & \left[ \frac{\left( \frac{\text{Underlying rate at expiration} - \text{Forward contract rate}}{360} \right) \left( \frac{\text{Days in underlying rate}}{360} \right)}{1 + \text{Underlying rate} \left( \frac{\text{Days in underlying rate}}{360} \right)} \right] \\
 & \text{or } \$40,000,000 \times \left[ \frac{(0.06 - 0.0525)(180/360)}{1 + 0.06(180/360)} \right] = \$145,631
 \end{aligned}$$

## EXHIBIT 1 (Continued)

GBT receives this amount in cash. Therefore, to obtain \$40 million in cash, it has to borrow \$40,000,000 – \$145,631 = \$39,854,369 at Libor plus 200 basis points,  $0.06 + 0.02 = 0.08$ , or 8%.

On 16 February GBT pays back \$39,854,369[1 + 0.08(180/360)] = \$41,448,544. So, it effectively pays a rate of

$$\left( \frac{\$41,448,544}{\$40,000,000} - 1 \right) \left( \frac{360}{180} \right) = 0.0724$$

The net effect is that GBT receives \$40 million on 20 August and pays back \$41,448,544 on 16 February, a rate of 7.24%. This rate was effectively locked in on 15 April at the FRA rate of 5.25% plus the 200 basis points GBT pays over Libor.

Shown below are the results for possible Libors on 20 August of 2%, 4%, ..., 10%.

Libor on 20 August	FRA Payoff	Amount Borrowed	Libor + 200 bps Loan Rate	Amount Repaid on 16 February	Effective Loan Rate
0.02	-\$643,564	\$40,643,564	0.04	\$41,456,435	0.0728
0.04	-245,098	40,245,098	0.06	41,452,451	0.0726
0.06	145,631	39,854,369	0.08	41,448,544	0.0724
0.08	528,846	39,471,154	0.10	41,444,712	0.0722
0.10	904,762	39,095,238	0.12	41,440,952	0.0720

In this problem, the FRA rate is 5.25%. In the exhibit, we described an outcome in which the underlying rate, 180-day Libor, is 6%. GBT ends up paying  $6\% + 2\% = 8\%$  on the loan, but the FRA pays off an amount sufficient to reduce the effective rate to 7.24%. Note the table at the end of the exhibit showing other possible outcomes. In all cases, the rate GBT pays is approximately the FRA rate of 5.25% plus 200 basis points. This rate is not precisely 7.25%, however, because of the way in which the FRA is constructed to pay off at expiration. When Libor on 20 August is above 5.25%, the FRA payoff on that day reduces the amount that has to be borrowed at Libor plus 200 basis points. This reduction works to the advantage of GBT. Conversely, when rates are below 5.25%, the amount that must be borrowed increases but that amount is borrowed at a lower rate. Thus, there is a slight asymmetric effect of a few basis points that prevents the effective loan rate from precisely equaling 7.25%.

In a similar manner, a lender could lock in a rate on a loan it plans to make by going short an FRA. Lenders are less inclined to do such transactions, however, because they cannot anticipate the exact future borrowing needs of their customers. In some cases, banks that offer credit lines at floating rates might wish to lock in lending rates using FRAs. But because the choice of whether to borrow is the borrower's and not the lender's, a lender that uses an FRA is taking considerable risk that the loan will not even be made. In that case, the lender would do better to use an option so that, in the worst case, it loses only the option premium.

### EXAMPLE 1

ABTech plans to borrow \$10 million in 30 days at 90-day Libor plus 100 basis points. To lock in a borrowing rate of 7%, it purchases an FRA at a rate of 6%. This contract would be referred to as a  $1 \times 4$  FRA because it expires in one month (30 days) and the underlying Eurodollar matures four months (120 days) from now. Thirty days later, Libor is 7.5%. Demonstrate that ABTech's effective borrowing rate is 7% if Libor in 30 days is 7.5%.

*Solution:* If Libor is 7.5% at the expiration of the FRA in 30 days, the payoff of the FRA is

$$\text{Notional principal} \times \left[ \frac{\left( \frac{\text{Underlying rate at expiration} - \text{Forward contract rate}}{360} \right) \left( \frac{\text{Days in underlying rate}}{360} \right)}{1 + \text{Underlying rate} \left( \frac{\text{Days in underlying rate}}{360} \right)} \right]$$

which is

$$\$10,000,000 \times \left[ \frac{(0.075 - 0.06)(90/360)}{1 + 0.075(90/360)} \right] = \$36,810$$

Because this amount is a cash inflow, ABTech will not need to borrow a full \$10,000,000. Instead, it will borrow \$10,000,000 - \$36,810 = \$9,963,190.

The amount it will pay back in 90 days is

$$\$9,963,190[1 + (0.075 + 0.01)(90/360)] = \$10,174,908$$

The effective rate is, therefore,

$$\left( \frac{\$10,174,908}{\$10,000,000} - 1 \right) \left( \frac{360}{90} \right) \approx 0.07$$

ABTech borrows at Libor plus 100 basis points. Therefore, using an FRA, it should be able to lock in the FRA rate (6%) plus 100 basis points, which it does.

#### 2.1.2. Floating-Rate Loan

In the example above, the loan involved only a single payment and, therefore, we had only one setting of an interest rate to worry about. Many loans are floating-rate loans, meaning that their rates are reset several times during the life of the loan. This resetting of the rate poses a series of risks for the borrower.

Suppose a corporation is taking out a two-year loan. The rate for the initial six months is set today. The rate will be reset in 6, 12, and 18 months. Because the current rate is already in place, there is nothing the corporation can do to mitigate that risk.<sup>2</sup> It faces, however, the risk of rising interest rates over the remaining life of the loan, which would result in higher interest payments.

One way to control this risk is to enter into a series of FRA transactions with each component FRA tailored to expire on a date on which the rate will be reset. This strategy will not lock in the *same* fixed rate for each semiannual period, but different rates for each period will be locked in. Another alternative would be to use futures. For example, for a Libor-based loan, the Eurodollar futures contract would be appropriate. Nonetheless, the use of futures to manage this risk poses significant problems. One problem is that the Eurodollar futures contract has expirations only on specific days during the year. The Chicago Mercantile Exchange offers contract expirations on the current month, the next month, and a sequence of months following the pattern of March, June, September, and December. Thus, it is quite likely that no contracts would exist with expirations that align with the later payment reset dates. The Eurodollar futures contract expires on the second London business day before the third Wednesday of the month. This date might not be the exact day of the month on which the rate is reset. In addition, the Eurodollar futures contract is based only on the 90-day Eurodollar rate, whereas the loan rate is pegged to the 180-day rate. Although many dealer firms use the Eurodollar futures contract to manage the risk associated with their over-the-counter derivatives, they do so using sophisticated techniques that measure and balance the volatility of the futures contract to the volatility of their market positions. Moreover, they adjust their positions rapidly in response to market movements. Without that capability, borrowers who simply need to align their interest rate reset dates with the dates on which their derivatives expire can do so more easily with swaps. We cover how this is done in the reading on risk management. Nevertheless, an understanding of how FRAs are used will help with an understanding of this application of swaps.

## 2.2. Strategies and Applications for Managing Bond Portfolio Risk

In Section 2.1, we dealt with the risk associated with short-term borrowing interest rates, which obviously affects short-term borrowers and lenders. The risk associated with longer-term loans primarily takes the form of bond market risk. Here we shall take a look at a firm managing a government bond portfolio, that is, a lending position. The firm can manage the risk associated with interest rates by using futures on government bonds. In the next three sections, we explore how to measure the risk of a bond portfolio, measure the risk of bond futures, and balance those risks.

### 2.2.1. Measuring the Risk of a Bond Portfolio

The sensitivity of a bond to a general change in interest rates is usually captured by assuming that the bond price changes in response to a change in its yield, which is driven by the general

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<sup>2</sup>If a corporation were planning to take out a floating-rate loan at a later date, it would also be concerned about the first interest rate and might attempt to lock in that rate. In the example used here, we placed the company in a situation in which it already knows its initial rate and, therefore, is worried only about the remaining rate resets.

level of rates. The responsiveness of a bond price to a yield change is captured in two ways: duration and basis point value.<sup>3</sup>

**Duration** is a measure of the size and timing of the cash flows paid by a bond. It quantifies these factors by summarizing them in the form of a single number, which is interpreted as an average maturity of the bond. To speak in terms of an average maturity of a bond of a given specific maturity sounds somewhat strange, but remember that a coupon bond is really just a combination of zero-coupon bonds.<sup>4</sup> The average maturity of these component zero-coupon bonds is the duration. The average is not an ordinary average but a weighted average, with the weights based on the present values of the respective cash payments on the bonds. Hence, the weights are not equal, and the large principal repayment places the greatest emphasis on the final payment.

Suppose the bond price is  $B$ , the yield is  $y_B$ , and Macaulay duration is  $DUR_B$ . Then the relationship between the change in the bond price and its yield is given as

$$\Delta B \approx -DUR_B B \frac{\Delta y_B}{1+y_B}$$

where the Greek symbol  $\Delta$  indicates “change in” and where the overall relationship is shown as an approximation ( $\approx$ ). For this relationship to be exact requires that the yield change be very small.<sup>5</sup> The left-hand side,  $\Delta B$ , is the change in the bond price. The negative sign on the right-hand side is consistent with the inverse relationship between the bond price and its yield.<sup>6</sup>

A somewhat simplified version of the above equation is

$$\Delta B \approx -MDUR_B B \Delta y_B$$

where  $MDUR_B = DUR_B / (1 + y_B)$ .  $MDUR_B$  is called the **modified duration** and is just an adjustment of the duration for the level of the yield. We shall use the relationship as captured by the modified duration.<sup>7</sup>

As an example, suppose the bond price is \$922.50, modified duration is 5.47 years, and the yield increases by 15 basis points. Then the price change should be approximately

$$\Delta B \approx -5.47(\$922.50)(0.0015) = -\$7.57$$

<sup>3</sup>Readers may first wish to review some fixed-income securities material. See especially Chapter 7 of *Fixed Income Analysis for the Chartered Financial Analyst Program*, Frank J. Fabozzi (Frank J. Fabozzi Associates, 2000).

<sup>4</sup>This analogy comes about because the coupons and final principal on a bond can be viewed as zero-coupon bonds, each maturing on the date on which a coupon or the final principal is paid. The value of a coupon or the final principal is analogous to the face value of a zero-coupon bond. In the US Treasury bond market, companies buy coupon bonds and sell claims on the individual coupons and principal, which are referred to as Treasury strips.

<sup>5</sup>If the yield change is not sufficiently small, it may be necessary to incorporate second-order effects, which are captured by a bond's convexity.

<sup>6</sup>The above relationship is based on annual coupons. If the coupons are paid semiannually, then  $1 + y_B$  should be  $1 + y_B/2$ . In this case, the duration will be stated as the number of semiannual, rather than annual, periods.

<sup>7</sup>The duration before dividing by  $1 + y_B$  is sometimes called the **Macaulay duration**, to distinguish it from the modified duration. It is named for Frederick Macaulay, one of the economists who first derived it.

In response to a 15 basis point increase in yield, the bond price should decrease by approximately \$7.57. So the new bond price would be predicted to be  $\$922.50 - \$7.57 = \$914.93$ .

The relationship between the bond price and its yield is sometimes stated another way. We often speak in terms of the change in the bond price for a 1 basis point change in yield. This value is sometimes referred to as **basis point value (BPV)**, **present value of a basis point (PVBP)**, or **price value of a basis point** (again PVBP). We refer to this concept as PVBP, defined as

$$PVBP_B \approx MDUR_B(0.0001)$$

The multiplication by 0.0001 enables PVBP to capture how much the bond price changes for a 1 basis point change. In the example above, the PVBP for our bond is

$$PVBP_B \approx (5.47)(\$922.50)(0.0001) = \$0.5046$$

So for a 1 basis point change, the bond price would change by approximately \$0.5046. Accordingly, a 15 basis point change produces a price change of  $15(\$0.5046) = \$7.57$ . Both duration and PVBP measure the same thing, however, and we shall use only duration.

Duration and PVBP are usually thought of with respect to individual bonds, but in practice, they are typically used at the portfolio level. Hence, we should care more about the duration of a bond portfolio than about the duration of an individual bond. With respect to yield, we do not usually speak in terms of the yield of a bond portfolio, but in this case we must. A given bond portfolio can be thought of as a series of cash flows that can be captured in terms of a representative bond. Thus, we might describe this bond as a bond portfolio with a market value of \$922.5 million, a modified duration of 5.47 years, and a portfolio yield that is a complex weighted average of the yields on the component bonds of the portfolio. The portfolio yield can change by a certain number of basis points. That yield change is a weighted average of the yield changes on the component bonds. Given such a yield change, the bond portfolio value will change in an approximate manner according to the duration formula shown above.

The way a bond price changes according to a yield change indicates its responsiveness to interest rates. Given a bond futures contract, we can also measure its sensitivity to interest rate changes.

### 2.2.2. Measuring the Risk of Bond Futures

Having measured the responsiveness of a bond portfolio to an interest rate change, we now need to measure the responsiveness of a futures contract to an interest rate change. Most bond futures contracts are based on a hypothetical benchmark bond. The Chicago Board of Trade's US Treasury bond futures contract is based on a 6% bond with at least 15 years from the futures expiration to maturity or the first call date. Even though the benchmark bond has a 6% coupon, any bond meeting the maturity requirement can be delivered. At any time, a single bond exists that the holder of the short position would find optimal to deliver if current conditions continued. That bond is called the **cheapest-to-deliver** and can be thought of as the bond on which the futures contract is based. In other words, the cheapest-to-deliver bond is the underlying. The responsiveness of the futures contract to an interest rate change is equivalent to the responsiveness of that bond on the futures expiration day to an interest rate change.

We can think of this concept as the responsiveness of the underlying bond in a forward context. This responsiveness can be measured as that bond's modified duration on the futures expiration and, as such, we can use the price sensitivity formula to capture the sensitivity of the futures contract to a yield change. Accordingly, we shall, somewhat loosely, refer to this as the implied

duration of the futures contract, keeping in mind that what we mean is the duration of the underlying bond calculated as of the futures expiration. Moreover, we also mean that the underlying bond has been identified as the cheapest bond to deliver and that if another bond takes its place, the duration of that bond must be used. We use the term *implied* to emphasize that a futures contract does not itself have a duration but that its duration is implied by the underlying bond. In addition to the duration, we also require an **implied yield** on the futures, which reflects the yield on the underlying bond implied by pricing it as though it were delivered at the futures contract expiration.

Hence, we can express the sensitivity of the futures price to a yield change as

$$\Delta f \approx -MDUR_f f \Delta y_f \quad (1)$$

where  $MDUR_f$  is the implied modified duration of the futures,  $f$  is the futures price, and  $\Delta y_f$  is the basis point change in the implied yield on the futures.

Now that we have a measure of the responsiveness of a bond portfolio and the responsiveness of a bond futures contract to interest rate changes, we should be able to find a way to balance the two to offset the risk.

### 2.2.3. Balancing the Risk of a Bond Portfolio against the Risk of Bond Futures

We now make the simple assumption that a single interest rate exists that drives all interest rates in the market. We assume that a 1 basis point change in this interest rate will cause a 1 basis point change in the yield on the bond portfolio and a 1 basis point change in the implied yield on the futures. We will relax that assumption later. For now, consider a money manager who holds a bond portfolio of a particular market value and will not be adding to it or removing some of it to balance the risk. In other words, the manager will not make any transactions in the actual bonds themselves. The manager can, however, trade any number of futures contracts to adjust the risk. Let  $N_f$  be the number of futures contracts traded. To balance the risk, suppose we combine the change in the value of the bond portfolio and the change in the value of  $N_f$  futures and set these equal to zero:  $\Delta B + N_f \Delta f = 0$ . Solving for  $N_f$  produces  $N_f = -\Delta B / \Delta f$ . Substituting our formulas for  $\Delta B$  and  $\Delta f$ , we obtain

$$N_f = -\left( \frac{MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right) \left( \frac{\Delta y_B}{\Delta y_f} \right) = -\left( \frac{MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right)$$

where we assume that  $\Delta y_B / \Delta y_f = 1$ ; or in other words, the bond portfolio yield changes one-for-one with the implied yield on the futures.<sup>8</sup>

Now let us go back to the major simplifying assumption we made. We assumed that an interest rate change occurs in the market and drives the yield on the bond and the implied yield on the futures one-for-one. In reality, this assumption is unlikely to hold true. Suppose, for example, the rate driving all rates in the United States is the overnight Fed funds rate.<sup>9</sup> If this rate changes by 1 basis point, not all rates along the term structure are likely to change by 1 basis point. What actually matters, however, is not that all rates change by the same amount but that the yield on

<sup>8</sup>Technically, this equation is the ratio of two approximate formulas, but we remove the approximation symbol from this point onward.

<sup>9</sup>The overnight Fed funds rate is the rate that banks charge each other to borrow and lend excess reserves for one night.

the bond portfolio and the implied yield on the futures change by the same amount for a 1 basis point change in this rate. If that is not the case, we need to make an adjustment.

Suppose the yield on the bond portfolio changes by a multiple of the implied yield on the futures in the following manner:

$$\Delta y_B = \beta_y \Delta y_f \quad (2)$$

We refer to the symbol  $\beta_y$  as the **yield beta**. It can be more or less than 1, depending on whether the bond yield is more sensitive or less sensitive than the implied futures yield. If we take the formula we previously obtained for  $\Delta B$ , substitute  $\beta_y \Delta y_f$  where we previously had  $\Delta y_B$ , and use this new variation of the formula in the formula  $N_f = -\Delta B / \Delta f$ , we obtain

$$N_f = -(MDUR_B / MDUR_f) (B/f) \beta_y \quad (3)$$

This is the more general formula, because  $\beta_y = 1.0$  is just the special case we assumed at the start.

We can modify Equation 3 so that it gives us the number of futures contracts needed to change our portfolio's modified duration to meet a target. What we have done so far *completely* balances the risk of the futures position against the risk of the bond portfolio, eliminating the risk. In the practice of risk management, however, we might not always want to eliminate the risk; we might want to adjust it only a little. At some times we might even want to increase it.

The risk of the overall bond portfolio reflects the duration of the bonds and the duration of the futures. Suppose we consider a target overall modified duration of the portfolio,  $MDUR_T$ . This amount is our desired overall modified duration. Because the portfolio consists of bonds worth  $B$  and futures, which have zero value, the overall portfolio value is  $B$ .<sup>10</sup> Now we introduce the notion of a dollar duration, which is the duration times the market value. The target dollar duration of our portfolio is set equal to the dollar duration of the bonds we hold and the dollar duration of the futures contracts:

$$B(MDUR_T) = B(MDUR_B) + f(MDUR_f)N_f$$

Solving for  $N_f$ , we obtain

$$N_f = \left( \frac{MDUR_T - MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right)$$

Observe that if we wish to increase the modified duration from  $MDUR_B$  to something higher, then  $MDUR_T$  is greater than  $MDUR_B$  and the overall sign of  $N_f$  will be positive, so we buy futures. This relationship should make sense: Buying futures would add volatility and increase duration. If we wish to reduce the modified duration from  $MDUR_B$  to something lower, then  $MDUR_T$  will be less than  $MDUR_B$  and the sign of  $N_f$  will be negative, meaning that we need to sell futures. Selling futures would reduce duration and volatility. In the extreme case in which we want to eliminate risk completely, we want  $MDUR_T$  to equal zero. In that case, the above formula reduces to the original one we obtained earlier in this section for the case of

<sup>10</sup>Recall that futures contracts have value through the accumulation of price changes during a trading day. At the end of the day, all gains and losses are paid out through the marking-to-market process and the value then goes back to zero. We assume we are at one of those points at which the value is zero.

completely eliminating risk. In a similar manner, if the bond and futures yields do not change one-for-one, we simply alter the above formula to

$$N_f = \left( \frac{MDUR_T - MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right) \beta_y \quad (4)$$

to incorporate the yield beta.

Now we explore how to use what we have learned in this section.

#### 2.2.4. Managing the Risk of a Government Bond Portfolio

A money manager can use Equation 4 to determine the number of futures contracts to buy or sell to adjust the duration of a portfolio. Such a transaction might be done in anticipation of a strong or weak market in bonds over a temporary period of time. In Exhibit 2, we illustrate the case of a pension fund that wants to increase the portfolio duration. We see that the futures transaction was successful in increasing the duration but not as precisely as planned. In fact, even without doing the futures transaction, the portfolio duration was not exactly as the company had believed. Duration is not an exact measure, nor does the bond price change occur precisely according to the duration formula.<sup>11</sup>

#### EXHIBIT 2 Using Bond Futures to Manage the Risk of a Bond Portfolio

##### Scenario (7 July)

A portion of the pension fund of United Energy Services (UES) is a portfolio of US government bonds. On 7 July, UES obtained a forecast from its economist that over the next month, interest rates are likely to make a significant unexpected decline. Its portfolio manager would like to take a portion of the bond portfolio and increase the duration to take advantage of this forecasted market movement.

Specifically, UES would like to raise the duration on \$75 million of bonds from its current level of 6.22 to 7.50. Both of these durations and all durations used in this problem are modified durations. UES has identified an appropriate Treasury bond futures contract that is currently priced at \$82,500 and has an implied modified duration of 8.12. UES has estimated that the yield on the bond portfolio is about 5% more volatile than the implied yield on the futures. Thus, the yield beta is 1.05.

##### Action

To increase the duration, UES will need to buy futures contracts. Specifically, the number of futures contracts UES should use is

$$\begin{aligned} N_f &= \left( \frac{MDUR_T - MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right) \beta_y \\ &= \left( \frac{7.50 - 6.22}{8.12} \right) \left( \frac{\$75,000,000}{\$82,500} \right) 1.05 = 150.47 \end{aligned}$$

Because fractional contracts cannot be traded, UES will buy 150 contracts.

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<sup>11</sup>For this reason, we stated that the bond price change, given the duration and yield change, is *approximately* given by the formula in the text.

## EXHIBIT 2 (Continued)

**Scenario (6 August)**

The implied yield on the futures has decreased by 35 basis points, and the futures price has now moved to \$85,000.<sup>12</sup> The yield on the bond portfolio has decreased by 40 basis points, and the portfolio has increased in value by \$1,933,500.

**Outcome and Analysis**

The profit on the futures transaction is found by multiplying the number of futures contracts by the difference between the new price and the old price:

$$\text{Profit on futures contract} = N_f(\text{New futures price} - \text{Old futures price})$$

In this case, the profit on the futures contract is  $150(\$85,000 - \$82,500) = \$375,000$ . Thus, the overall gain is  $\$1,933,500 + \$375,000 = \$2,308,500$ .

How effective was the transaction? To answer this question, we compare the *ex post* duration to the planned duration. The purpose was to increase the duration from 6.22 to a planned 7.50. The return on the portfolio was

$$\frac{\$1,933,500}{\$75,000,000} = 0.0258$$

or 2.58% without the futures transaction, and

$$\frac{\$2,308,500}{\$75,000,000} = 0.0308$$

or 3.08% with the futures transaction. What does this set of calculations imply about the portfolio's *ex post* duration? Recall that duration is a measure of the percentage change in portfolio value with respect to a basis point change in yield. The *ex post* duration<sup>13</sup> of the portfolio can be measured by dividing the percentage change in portfolio value by the 40 basis point change in the portfolio yield:

$$\frac{0.0258}{0.0040} = 6.45$$

without the futures transaction and

$$\frac{0.0308}{0.0040} = 7.70$$

with the futures transaction. UES came fairly close to achieving its desired increase in duration using futures.

<sup>12</sup>In the examples in this reading, bond futures prices move to a new level in the course of the scenario. These new futures prices come from the cost-of-carry model (assuming there is no mispricing in the market).

<sup>13</sup>Of course, the *ex post* duration without the futures transaction is not exactly 6.22 because duration is an inexact measure, and the actual bond price change may not be precisely what is given by the modified duration formula.

In the example here, the fund increased its modified duration during a time when interest rates fell and the bond portfolio value increased. It leveraged itself to take advantage of a favorable outlook. Not all such decisions work out so well. Suppose in this example the economist had a different forecast, and as a result, UES wanted to eliminate all interest rate risk. So let us rework the problem under the assumption that the fund put on a full hedge, thereby reducing the modified duration to zero.

With  $MDUR_T = 0$ , the number of futures contracts would be

$$N_f = \left( \frac{0 - 6.22}{8.12} \right) \left( \frac{\$75,000,000}{\$82,500} \right) 1.05 = -731.19$$

Thus, the fund would sell 731 contracts. The profit from the futures transaction<sup>14</sup> would be  $-731(\$85,000 - \$82,500) = -\$1,827,500$ . The overall transaction earned a profit of  $\$1,933,500 - \$1,827,500 = \$106,000$ , a gain of

$$\frac{\$106,000}{\$75,000,000} = 0.0014$$

or 0.14%. Thus, shorting the futures contracts virtually wiped out all of the gain from the decrease in interest rates. Our *ex ante* objective was to reduce the modified duration to zero. The *ex post* modified duration, however, turned out to be

$$\frac{0.0014}{0.0040} = 0.35$$

Thus, the modified duration was reduced almost to zero.

## EXAMPLE 2

Debt Management Associates (DMA) offers fixed-income portfolio management services to institutional investors. It would like to execute a duration-changing strategy for a €100 million bond portfolio of a particular client. This portfolio has a modified duration of 7.2. DMA plans to change the modified duration to 5.00 by using a futures contract priced at €120,000, which has an implied modified duration of 6.25. The yield beta is 1.15.

- A. Determine how many futures contracts DMA should use and whether it should buy or sell futures.
- B. Suppose that the yield on the bond has decreased by 20 basis points at the horizon date. The bond portfolio increases in value by 1.5%. The futures price increases to €121,200. Determine the overall gain on the portfolio and the *ex post* modified duration as a result of the futures transaction.

<sup>14</sup>Notice that in calculating the profit from a futures transaction, we multiply the number of futures contracts by the futures price at the close of the strategy minus the original futures price. It is important to maintain the correct sign for the number of futures contracts. This formulation always results in a positive number for  $N_f$  times the futures selling price and a negative number for  $N_f$  times the futures buying price, which should make sense. Of course, as previously noted, we also ignore the marking-to-market feature of futures contracts.

*Solution to A:* The appropriate number of futures contracts is

$$N_f = \left( \frac{5 - 7.2}{6.25} \right) \left( \frac{100,000,000}{120,000} \right) 1.15 = -337.33$$

So DMA should sell 337 contracts.

*Solution to B:* The value of the bond portfolio will be  $\$100,000,000(1.015) = \$101,500,000$ . The profit on the futures transaction is  $-337(\$121,200 - 120,000) = -\$404,400$ ; a loss of  $\$404,400$ . Thus, the overall value of the position is  $\$101,500,000 - \$404,400 = \$101,095,600$ , a return of approximately 1.1%. The bond yield decreases by 20 basis points and the portfolio gains 1.1%. The *ex post* modified duration would be  $0.0110/0.0020 = 5.50$ .

Changing the duration—whether increasing it, reducing it partially, or reducing it all the way to zero—is an inexact process. More importantly, however, risk management by adjusting duration is only a means of implementing a strategy in response to an outlook. No one can guarantee that the outlook will not be wrong.

### 2.2.5. Some Variations and Problems in Managing Bond Portfolio Risk

In the examples used here, the bond portfolio consisted of government bonds. Of course, corporate and municipal bonds are widely held in bond portfolios. Unfortunately, there is no corporate bond futures contract.<sup>15</sup> A municipal bond futures contract exists in the United States, based on an index of municipal bonds, but its volume is relatively light and the contract may not be sufficiently liquid for a large-size transaction.<sup>16</sup> Government bond futures contracts tend to be relatively liquid. In fact, in the United States, different contracts exist for government securities of different maturity ranges, and most of these contracts are relatively liquid.

If one uses a government bond futures to manage the risk of a corporate or municipal bond portfolio, there are some additional risks to deal with. For instance, the relationship between the yield change that drives the futures contract and the yield change that drives the bond portfolio is not as reliable. The yield on a corporate or municipal bond is driven not only by interest rates but also by the perceived default risk of the bond. We might believe that the yield beta is 1.20, meaning that the yield on a corporate bond portfolio is about 20% more volatile than the implied yield that drives the futures contract. But this relationship is usually estimated from a regression of corporate bond yield changes on government bond yield changes. This relationship is less stable than if we were running a regression of government

<sup>15</sup>There have been attempts to create futures contracts on corporate bonds, but these contracts have not been successful in generating enough trading volume to survive.

<sup>16</sup>In the Commodity Futures Trading Commission's fiscal year 2001, the Chicago Board of Trade's municipal bond futures contract traded about 1,400 contracts a day. Each contract is worth about \$100,000 of municipal bonds. Thus, the average daily volume amounts to about \$140 million of municipal bonds—not a very large amount relative to the size of the municipal bond market.

bond yield changes on yield changes of a different government bond, the one underlying the futures.

In addition, corporate and municipal bonds often have call features that can greatly distort the relationship between duration and yield change and also make the measurement of duration more complicated. For example, when a bond's yield decreases, its price should increase. The duration is meant to show approximately how much the bond's price should increase. But when the bond is callable and the yield enters into the region in which a call becomes more likely, its price will increase by far less than predicted by the duration. Moreover, the call feature complicates the measurement of duration itself. Duration is no longer a weighted-average maturity of the bond.

Finally, we should note that corporate and municipal bonds are subject to default risk that is not present in government bonds. As the risk of default changes, the yield spread on the defaultable bond relative to the default-free government bond increases. This effect further destabilizes the relationship between the bond portfolio value and the futures price so that duration-based formulas for the number of futures contracts tend to be unreliable.

It is tempting to think that if one wants to increase (decrease) duration and buys (sells) futures contracts, that at least the transaction was the right type even if the number of futures contracts is not exactly correct. The problem, however, is that changes in the bond portfolio value that are driven by changes in default risk or the effects of call provisions will not be matched by movements in the futures contract. The outcome will not always be what is expected.

Another problem associated with the modified duration approach to measuring and managing bond portfolio risk is that the relationship between duration and yield change used here is an instantaneous one. It captures *approximately* how a bond price changes in response to an immediate and very small yield change. As soon as the yield changes or an instant of time passes, the duration changes. Then the number of futures contracts required would change. Thus, the positions described here would need to be revised. Most bond portfolio managers do not perform these kinds of frequent adjustments, however, and simply accept that the transaction will not work precisely as planned.

We should also consider the alternative that the fund could adjust the duration by making transactions in the bonds themselves. It could sell relatively low-duration bonds and buy relatively high-duration bonds to raise the duration to the desired level. There is still no guarantee, however, that the actual duration will be exactly as desired. Likewise, to reduce the duration to zero, the fund could sell out the entire bond portfolio and place the proceeds in cash securities that have low duration. Reducing the duration to essentially zero would be easier to do than increasing it, because it would not be hard to buy bonds with essentially zero duration. Liquidating the entire portfolio, however, would be quite a drastic thing to do, especially given that the fund would likely remain in that position for only a temporary period.

Raising the duration by purchasing higher-duration bonds would be a great deal of effort to expend if the position is being altered only temporarily. Moreover, the transaction costs of buying and selling actual securities are much greater than those of buying and selling futures.

In this reading, we shall consider these adjustments as advanced refinements that one should understand before putting these types of transactions into practice. Although we need to be aware of these technical complications, we shall ignore them in the examples here.

Now let us take a look at managing risk in the equity market.

### 3. STRATEGIES AND APPLICATIONS FOR MANAGING EQUITY MARKET RISK

Even though interest rates are volatile, the stock market is even more volatile. Hence, the risk associated with stock market volatility is greater than that of bond market volatility. Fortunately, the stock market is generally more liquid than the bond market, at least compared with long-term and corporate and municipal bonds. The risk associated with stock market volatility can be managed relatively well with futures contracts. As we have previously noted, these contracts are based on stock market indices and not individual stocks. Although futures on individual stocks are available, most diversified investors manage risk at the portfolio level, thereby preferring futures on broad-based indices. Accordingly, this will be our focus in this reading. We look more specifically at the risk of managing individual stocks in the reading on risk management applications of option strategies.

#### 3.1. Measuring and Managing the Risk of Equities

Futures provide the best way to manage the risk of diversified equity portfolios. Although the standard deviation, or volatility, is a common measure of stock market risk, we prefer a measure that more accurately reflects the risk of a diversified stock portfolio. One reason for this preference is that we shall use futures that are based on broadly diversified portfolios. The most common risk measure of this type is the **beta**,<sup>17</sup> often denoted with the Greek symbol  $\beta$ . Beta is an important factor in capital market and asset pricing theory and, as we see here, it plays a major role in risk management. Although you may have encountered beta elsewhere, we shall take a quick review of it here.

Beta measures the relationship between a stock portfolio and the market portfolio, which is an abstract hypothetical measure of the portfolio containing *all* risky assets, not just stocks. The market portfolio is the most broadly diversified portfolio of all. We know, however, that it is impossible to identify the composition of the true market portfolio. We tend to use proxies, such as the S&P 500 Index, which do not really capture the true market portfolio. Fortunately, for the purposes of risk management, precision in the market portfolio does not matter all that much. Obviously there are no futures contracts on the true market portfolio; there can be futures contracts only on proxies such as the S&P 500. That being the case, it is appropriate to measure the beta of a portfolio relative to the index on which the futures is based.

Beta is a relative risk measure. The beta of the index we use as a benchmark is 1.0. Ignoring any asset-specific risk, an asset with a beta of 1.10 is 10% more volatile than the index. A beta of 0.80 is 20% less volatile than the index. Beta is formally measured as

$$\beta = \frac{\text{cov}_{SI}}{\sigma_I^2}$$

where  $\text{cov}_{SI}$  is the covariance between the stock portfolio and the index and  $\sigma_I^2$  is the variance of the index. Covariance is a measure of the extent to which two assets, here the portfolio and

<sup>17</sup>At this point, we must distinguish this beta from the yield beta. When we use the term “yield beta,” we mean the relationship of the yield on the instrument being hedged to the implied yield on the futures. When we use the term “beta” without a modifier, we mean the relationship of a stock or portfolio to the market.

the index, move together.<sup>18</sup> If the covariance is positive (negative), the portfolio and the index tend to move in the same (opposite) direction. By itself, the magnitude of the covariance is difficult to interpret, but the covariance divided by the product of the standard deviations of the stock and the index produces the familiar measure called the correlation coefficient. For beta, however, we divide the covariance by the variance of the index and obtain a measure of the volatility of the portfolio relative to the market.

It is important to emphasize that beta measures only the portfolio volatility relative to the index. Thus, it is a measure only of the risk that cannot be eliminated by diversifying a portfolio. This risk is called the systematic, nondiversifiable, or market risk. A portfolio that is not well diversified could contain additional risk, which is called the nonsystematic, diversifiable, or asset-specific risk.<sup>19</sup> Systematic risk is the risk associated with broad market movements; nonsystematic risk is the risk unique to a company. An example of the former might be a change in interest rates by the Federal Reserve; an example of the latter might be a labor strike on a particular company. Because it captures only systematic risk, beta may seem to be a limited measure of risk, but the best way to manage nonsystematic risk, other than diversification, is to use options, as we do in the reading on risk management applications of option strategies. At this point, we focus on managing systematic or market risk.

As a risk measure, beta is similar to duration. Recall that we captured the dollar risk by multiplying the modified duration by the dollar value of the portfolio. For the bond futures contract, we multiplied its implied modified duration by the futures price. We called this the dollar-implied modified duration. In a similar manner, we shall specify a dollar beta by multiplying the beta by the dollar value of the portfolio. For the futures, we shall multiply its beta by the futures price,  $f$ . For the futures contract, beta is often assumed to be 1.0, but that is not exactly the case, so we will specify it as  $\beta_f$ . The dollar beta of the futures contract is  $\beta_f f$ . The dollar beta of the stock portfolio is written as  $\beta_S S$ , where  $\beta_S$  is the beta of the stock portfolio and  $S$  is the market value of the stock portfolio.

If we wish to change the beta, we specify the desired beta as a target beta of  $\beta_T$ . Because the value of the futures starts off each day as zero, the dollar beta of the combination of stock and futures if the target beta is achieved is  $\beta_T S$ .<sup>20</sup> The number of futures we shall use is  $N_f$ , which is the unknown that we are attempting to determine. We set the target dollar beta to the dollar beta of the stock portfolio and the dollar beta of  $N_f$  futures:

$$\beta_T S = \beta_S S + N_f \beta_f f$$

We then solve for  $N_f$  and obtain

$$N_f = \left( \frac{\beta_T - \beta_S}{\beta_f} \right) \left( \frac{S}{f} \right) \quad (5)$$

<sup>18</sup>More specifically, the covariance measures the extent to which the *returns* on the stock and the index move together.

<sup>19</sup>We also sometimes use the term “idiosyncratic risk.”

<sup>20</sup>Recall that the market value of the portfolio will still be the same as the market value of the stock, because the value of the futures is zero. The futures value becomes zero whenever it is marked to market, which takes place at the end of each day. In other words, the target beta does not appear to be applied to the value of the futures in the above analysis because the value of the futures is zero.

Observe that if we want to increase the beta,  $\beta_T$  will exceed  $\beta_S$  and the sign of  $N_f$  will be positive, which means that we must buy futures. If we want to decrease the beta,  $\beta_T$  will be less than  $\beta_S$ , the sign of  $N_f$  will be negative, and we must sell futures. This relationship should make sense: Selling futures will offset some of the risk of holding the stock. Alternatively, buying futures will add risk as  $\beta_T > \beta_S$  and  $N_f > 0$ .

In the special case in which we want to completely eliminate the risk,  $\beta_T$  would be zero and the formula would reduce to

$$N_f = -\left(\frac{\beta_S}{\beta_f}\right)\left(\frac{S}{f}\right)$$

In this case, the sign of  $N_f$  will always be negative, which makes sense. To hedge away all of the risk, we definitely need to sell futures.

In the practical implementation of a stock index futures trade, we need to remember that stock index futures prices are quoted on an order of magnitude the same as that of the stock index. The actual futures price is the quoted futures price times a designated multiplier. For example, if the S&P 500 futures price is quoted at 1225, the multiplier of \$250 makes the actual futures price  $1225(\$250) = \$306,250$ . This amount would be the value of  $f$  in the above formulas. In some situations, the futures price will simply be stated, as for example \$306,250. In that case, we can assume the price is quoted as  $f = \$306,250$  and the multiplier is 1.

We also need to remember that the futures contract will hedge only the risk associated with the relationship between the portfolio and the index on which the futures contract is based. Thus, for example, a portfolio consisting mostly of small-cap stocks should not be paired with a futures contract on a large-cap index such as the S&P 500. Such a transaction would manage only the risk that large-cap stocks move with small-cap stocks. If any divergence occurs in the relationship between these two sectors, such as large-cap stocks going up and small-cap stocks going down, a transaction designed to increase (decrease) risk could end up decreasing (increasing) risk.

Recall also that dividends can interfere with how this transaction performs. Index futures typically are based only on price indices; they do not reflect the payment and reinvestment of dividends. Therefore, dividends will accrue on the stocks but are not reflected in the index. This is not a major problem, however, because dividends in the short-term period covered by most contracts are not particularly risky.

### 3.2. Managing the Risk of an Equity Portfolio

To adjust the beta of an equity portfolio, an investment manager could use Equation 5 to calculate the number of futures contracts needed. She can use the formula to either increase or decrease the portfolio's systematic risk. The manager might increase the beta if she expects the market to move up, or decrease the beta if she expects the market to move down. Also, the betas of equity portfolios change constantly by virtue of the market value of the portfolio changing.<sup>21</sup> Therefore, futures can be used to adjust the beta from its actual level to the desired level.

<sup>21</sup>Consider, for example, a portfolio in which \$3 million is invested in stock with a beta of 1.0 and \$1 million is invested in cash with a beta of 0.0 and a rate of 5%. The equity market weight is, therefore, 0.75, and the overall beta is  $1.0(0.75) + 0.0(0.25) = 0.75$ . Now suppose the following year, the stock increases by 20%. Then the stock value will be \$3.6 million and the cash balance will be \$1.05 million. The overall portfolio value will be \$4.65 million, so the equity market weight will be  $3.6/4.65 = 0.77$ . Thus, 77% of the portfolio will now have a beta of  $1.0(0.77)$ , and the overall beta will have drifted upward to 0.77.

Exhibit 3 illustrates the case of a pension fund that wants to increase its equity portfolio beta during a period in which it expects the market to be strong. It increases its beta from 0.90 to 1.10 by purchasing 29 futures contracts. Betas, however, are notoriously difficult to measure. We see after the fact that the beta actually was increased to 1.15. As long as we buy (sell) futures contracts, however, we will increase (decrease) the beta.

### EXHIBIT 3 Using Stock Index Futures to Manage the Risk of a Stock Portfolio

#### Scenario (2 September)

BB Holdings (BBH) is a US conglomerate. Its pension fund generates market forecasts internally and receives forecasts from an independent consultant. As a result of these forecasts, BBH expects the market for large-cap stocks to be stronger than it believes everyone else is expecting over the next two months.

#### Action

BBH decides to adjust the beta on \$38,500,000 of large-cap stocks from its current level of 0.90 to 1.10 for the period of the next two months. It has selected a futures contract deemed to have sufficient liquidity; the futures price is currently \$275,000 and the contract has a beta of 0.95. The appropriate number of futures contracts to adjust the beta would be

$$N_f = \left( \frac{\beta_T - \beta_S}{\beta_f} \right) \left( \frac{S}{f} \right) = \left( \frac{1.10 - 0.90}{0.95} \right) \left( \frac{\$38,500,000}{\$275,000} \right) = 29.47$$

So it buys 29 contracts.

#### Scenario (3 December)

The market as a whole increases by 4.4%. The stock portfolio increases to \$40,103,000. The stock index futures contract rises to \$286,687.50,<sup>22</sup> an increase of 4.25%.

#### Outcome and Analysis

The profit on the futures contract is  $29(\$286,687.50 - \$275,000.00) = \$338,937.50$ . The rate of return for the stock portfolio is

$$\frac{\$40,103,000}{\$38,500,000} - 1 = 0.0416$$

or 4.16%. Adding the profit from the futures gives a total market value of  $\$40,103,000.00 + \$338,937.50 = \$40,441,937.50$ . The rate of return for the stock portfolio is

$$\frac{\$40,441,937.50}{\$38,500,000.00} - 1 = 0.0504$$

<sup>22</sup>In the examples in this reading, stock futures prices move to a new level in the course of the scenario. These new futures prices come from the cost-of-carry model (assuming there is no mispricing in the market).

## EXHIBIT 3 (Continued)

or 5.04%. Because the market went up by 4.4% and the overall gain was 5.04%, the effective beta of the portfolio was

$$\frac{0.0504}{0.044} = 1.15$$

Thus, the effective beta is quite close to the target beta of 1.10.

Of course, be aware that increasing the beta increases the risk. Therefore, if the beta is increased and the market falls, the loss on the portfolio will be greater than if beta had not been increased. Decreasing the beta decreases the risk, so if the market rises, the portfolio value will rise less. As an example, consider the outcome described in Exhibit 3. Suppose that instead of being optimistic, the fund manager was very pessimistic and wanted to decrease the beta to zero. Therefore, the target beta,  $\beta_T$ , is 0.0. Then the number of futures contracts would be

$$N_f = \left( \frac{0.0 - 0.90}{0.95} \right) \frac{\$38,500,000}{\$275,000.00} = -132.63$$

So the fund sells 133 futures. Given the same outcome as in Exhibit 3, the profit on the futures contracts would be

$$-133(\$286,687.50 - \$275,000.00) = -\$1,554,437.50$$

There would be a loss of more than \$1.5 million on the futures contracts. The market value of the stock after it moved up was \$40,103,000, but with the futures loss, the market value is effectively reduced to  $\$40,103,000.00 - \$1,554,437.50 = \$38,548,562.50$ . This is a return of

$$\frac{\$38,548,562.50}{\$38,500,000.00} - 1 = 0.0013$$

Thus, the effective beta is

$$\frac{0.0013}{0.044} = 0.030$$

The beta has been reduced almost to zero. This reduction costs the company virtually all of the upward movement, but such a cost is to be expected if the beta were changed to zero.

## EXAMPLE 3

Equity Analysts Inc. (EQA) is an equity portfolio management firm. One of its clients has decided to be more aggressive for a short period of time. It would like EQA to move the beta on its \$65 million portfolio from 0.85 to 1.05. EQA can use a futures contract priced at \$188,500, which has a beta of 0.92, to implement this change in risk.

- A. Determine the number of futures contracts EQA should use and whether it should buy or sell futures.

- B. At the horizon date, the equity market is down 2%. The stock portfolio falls 1.65%, and the futures price falls to \$185,000. Determine the overall value of the position and the effective beta.

*Solution to A:* The number of futures contracts EQA should use is

$$N_f = \left( \frac{1.05 - 0.85}{0.92} \right) \left( \frac{\$65,000,000}{\$188,500} \right) = 74.96$$

So EQA should buy 75 contracts.

*Solution to B:* The value of the stock portfolio will be  $\$65,000,000(1 - 0.0165) = \$63,927,500$ . The profit on the futures transaction is  $75(\$185,000 - \$188,500) = -\$262,500$ . The overall value of the position is  $\$63,927,500 - \$262,500 = \$63,665,000$ .

Thus, the overall return is  $\frac{\$63,665,000}{\$65,000,000} - 1 = -0.0205$

Because the market went down by 2%, the effective beta is  $0.0205/0.02 = 1.025$ .

### 3.3. Creating Equity out of Cash

Stock index futures are an excellent tool for creating synthetic positions in equity, which can result in significant transaction cost savings and preserve liquidity. In this section, we explore how to create a synthetic index fund and how to turn cash into synthetic equity.

The relationship between a futures or forward contract and the underlying asset is determined by a formula that relates the risk-free interest rate to the dividends on the underlying asset. Entering into a hypothetical arbitrage transaction in which we buy stock and sell futures turns an equity position into a risk-free portfolio. In simple terms, we say that

$$\text{Long stock} + \text{Short futures} = \text{Long risk-free bond}$$

We can turn this equation around to obtain<sup>23</sup>

$$\text{Long stock} = \text{Long risk-free bond} + \text{Long futures}$$

If we buy the risk-free bonds and buy the futures, we replicate a position in which we would be buying the stock. This synthetic replication of the underlying asset can be a very useful transaction when we wish to construct a synthetic stock index fund, or when we wish to convert into equity a cash position that we are required to maintain for liquidity purposes. Both of these situations involve holding cash and obtaining equity market exposure through the use of futures.

<sup>23</sup>We turn the equation around by noting that to remove a short futures position from the left-hand side, we should buy futures. If we add a long futures position to the left-hand side, we have to add it to the right-hand side.

### 3.3.1. Creating a Synthetic Index Fund

A synthetic index fund is an index fund position created by combining risk-free bonds and futures on the desired index. Suppose a US money manager would like to offer a new product, a fund on an index of UK stock as represented by the Financial Times Stock Exchange (FTSE) 100 Index. The manager will initiate the fund with an investment of £100 million. In other words, the US money manager would offer clients an opportunity to invest in a position in British stock with the investment made in British pounds.<sup>24</sup> The manager believes the fund is easier to create synthetically using futures contracts.

To create this synthetic index fund, we need to know several more pieces of information. The dividend yield on the UK stocks is 2.5%, and the FTSE 100 Index futures contract that we shall use expires in three months, has a quoted price of £4,000, and has a multiplier of £10.<sup>25</sup> The UK risk-free interest rate is 5%.<sup>26</sup> When the futures contract expires, it will be rolled over into a new contract.

To create this synthetic index fund, we must buy a certain number of futures. Let the following be the appropriate values of the inputs:

$V$  = amount of money to be invested, £100 million

$f$  = futures price, £4,000

$T$  = time to expiration of futures, 0.25

$\delta$  = dividend yield on the index, 0.025

$r$  = risk-free rate, 0.05

$q$  = multiplier, £10

We would like to replicate owning the stock and reinvesting the dividends. How many futures contracts would we need to buy and add to a long bond position? We designate  $N_f$  as the required number of futures contracts and  $N_f^*$  as its rounded-off value.

Now observe that the payoff of  $N_f^*$  futures contracts will be  $N_f^*q(S_T - f)$ . This equation is based on the fact that we have  $N_f^*$  futures contracts, each of which has a multiplier of  $q$ . The futures contracts are established at a price of  $f$ . When it expires, the futures price will be the spot price,  $S_T$ , reflecting the convergence of the futures price at expiration to the spot price.

The futures payoff can be rewritten as  $N_f^*qS_T - N_f^*qf$ . The minus sign on the second term means that we shall have to pay  $N_f^*qf$ . The (implied) plus sign on the first term means that we shall receive  $N_f^*qS_T$ . Knowing that we buy  $N_f^*$  futures contracts, we also want to know how

<sup>24</sup>If you are wondering why US investors would like to invest in a position denominated in British pounds rather than dollars, remember that the currency risk can be a source of diversification. Adding a position in the UK equity market provides one tier of diversification, while adding the risk of the dollar/pound exchange rate adds another tier of diversification, especially because the exchange rate is likely to have a low correlation with the US stock market.

<sup>25</sup>Recall that the multiplier is a number multiplied by the quoted futures price to obtain the actual futures price. In this section, accurately pricing the futures contract is important to the success of these strategies. For example, assume the S&P 500 is at 1,000 and the multiplier is \$250, so the full price is  $(1,000)(\$250) = \$250,000$ . We wish to trade a futures contract priced at  $f$ , where  $f$  is based on the index value of 1,000 grossed up by the risk-free rate and reduced by the dividends. It is far easier to think of  $f$  in terms of its relationship to  $S$  without the multiplier. In one case, however, we shall let the multiplier be 1, so you should be able to handle either situation.

<sup>26</sup>It might be confusing as to why we care about the UK interest rate and not the US interest rate. This transaction is completely denominated in pounds, and the futures contract is priced in pounds based on the UK dividend yield and interest rate. Hence, the UK interest rate plays a role here, and the US interest rate does not.

much to invest in bonds. We shall call this  $V^*$  and calculate it based on  $N_f^*$ . Below we shall show how to calculate  $N_f^*$  and  $V^*$ . If we invest enough money in bonds to accumulate a value of  $N_f^*qf$ , this investment will cover the amount we agree to pay for the FTSE:  $N_f^* \times q \times f$ . The present value of this amount is  $N_f^*qf/(1 + r)^T$ .

Because the amount of money we start with is  $V$ , we should have  $V$  equal to  $N_f^*qf/(1 + r)^T$ . From here we can solve for  $N_f^*$  to obtain

$$N_f^* = \frac{V(1+r)^T}{qf} \quad (\text{rounded to an integer}) \quad (6)$$

But once we round off the number of futures, we do not truly have  $V$  dollars invested. The amount we actually have invested is

$$V^* = \frac{N_f^*qf}{(1+r)^T} \quad (7)$$

We can show that investing  $V^*$  in bonds and buying  $N_f^*$  futures contracts at a price of  $f$  is equivalent to buying  $N_f^*q/(1 + \delta)^T$  units of stock.

As noted above, if we have bonds maturing to the value  $N_f^*qf$ , we have enough cash on hand to pay the obligation of  $N_f^*qf$  on our futures contract. The futures contract will pay us the amount  $N_f^*qS_T$ . If we had actually purchased units of stock, the reinvestment of dividends into new units means that we would end up with the equivalent of  $N_f^*q$  units, and means that we implicitly started off with  $N_f^*q/(1 + \delta)^T$  units.

In short, this transaction implies that we synthetically start off with  $N_f^*q/(1 + \delta)^T$  units of stock, collect and reinvest dividends, and end up with  $N_f^*q$  units. We emphasize that all of these transactions are synthetic. We do not actually own the stock or collect and reinvest the dividends. We are attempting only to replicate what would happen if we actually owned the stock and collected and reinvested the dividends.

Exhibit 4 illustrates this transaction. The interest plus principal on the bonds is a sufficient amount to buy the stock in settlement of the futures contract, so the fund ends up holding the stock, as it originally wanted.

#### EXHIBIT 4 Constructing a Synthetic Index Fund

##### Scenario (15 December)

On 15 December, a US money manager for a firm called Strategic Money Management (SMM) wants to construct a synthetic index fund consisting of a position of £100 million invested in UK stock. The index will be the FTSE 100, which has a dividend yield of 2.5%. A futures contract on the FTSE 100 is priced at £4,000 and has a multiplier of £10. The position will be held until the futures expires in three months, at which time it will be renewed with a new three-month futures. The UK risk-free rate is 5%. Both the risk-free rate and the dividend yield are stated as annually compounded figures.

##### Action

The number of futures contracts will be

$$N_f = \frac{V(1+r)^T}{qf} = \frac{\text{£}100,000,000(1.05)^{0.25}}{\text{£}10(4,000)} = 2,530.68$$

## EXHIBIT 4 (Continued)

Because we cannot buy fractions of futures contracts, we round  $N_f$  to  $N_f^* = 2,531$ . With this rounding, we are actually synthetically investing

$$\frac{2,531(\£10)\£4,000}{(1.05)^{0.25}} = \£100,012,622$$

in stock. So we put this much money in risk-free bonds, which will grow to  $\£100,012,622(1.05)^{0.25} = \£101,240,000$ . The number of units of stock that we have effectively purchased at the start is

$$\frac{N_f^* q}{(1+\delta)^T} = \frac{2,531(10)}{(1.025)^{0.25}} = 25,154.24$$

If the stock had actually been purchased, dividends would be received and reinvested into additional shares. Thus, the number of shares would grow to  $25,154.24(1.025)^{0.25} = 25,310$ .

### Scenario (15 March)

The index is at  $S_T$  when the futures expires.

### Outcome and Analysis

The futures contracts will pay off the amount

$$\text{Futures payoff} = 2,531(\£10)(S_T - \£4,000) = \£25,310S_T - \£101,240,000$$

This means that the fund will pay  $\£101,240,000$  to settle the futures contract and obtain the market value of 25,310 units of the FTSE 100, each worth  $S_T$ . Therefore, the fund will need to come up with  $\£101,240,000$ , but as noted above, the money invested in risk-free bonds grows to a value of  $\£101,240,000$ .

SMM, therefore, pays this amount to settle the futures contracts and effectively ends up with 25,310 units of the index, the position it wanted in the market.

There are a few other considerations to note. One is that we rounded according to the usual rules of rounding, going up if the fraction is 0.5 or greater. By rounding up, we shall have to invest more than  $V$  in bonds. If we rounded down, we shall invest less than  $V$ . It does not really matter whether we always round up on 0.5 or greater, but that is the rule we shall use here. It should also be noted that this transaction does not capture the dividends that would be earned if one held the underlying stocks directly. The yield of 2.5% is important in the computations here, but the fund does not earn these dividends. All this transaction does is capture the performance of the index. Because the index is a price index only and does not include dividends, this synthetic replication strategy can capture only the index performance without the dividends.<sup>27</sup> Another concern that could be encountered in practice is that the futures contract could expire later than the desired date. If so, the strategy will still be successful if the futures contract is correctly priced when the strategy is completed. Consistent with that point, we should note that any strategy using futures will be effective only to the extent that the futures contract is correctly priced when the position is opened and also when

<sup>27</sup>The values of some stock indices, called total return indices, include reinvested dividends. If a futures contract on the total return index is used, then the strategy would capture the dividends. Doing so would, however, require a few changes to the formulas given here.

it is closed. This point underscores the importance of understanding the pricing of futures contracts.

### 3.3.2. Equitizing Cash

The strategy of combining risk-free bonds and futures is used not only to replicate an index; it is also used to take a given amount of cash and turn it into an equity position while maintaining the liquidity provided by the cash. This type of transaction is sometimes called equitizing cash. Consider an investment fund that has a large cash balance. It would like to invest in equity but either is not allowed to do so or cannot afford to take the risk that it might need to liquidate a large amount of stock in a short period of time, which could be difficult to do or might result in significant losses. Nonetheless, the fund is willing to take the risk of equity market exposure provided it can maintain the liquidity. The above transaction can be altered just slightly to show how this is done.

Suppose the fund in Exhibit 4 is actually a UK insurance company that has about £100 million of cash invested at the risk-free rate. It would like to gain equity market exposure by investing in the FTSE 100 index. By policy, it is allowed to do so, provided that it maintains sufficient liquidity. If it engages in the synthetic index strategy described above, it maintains about £100 million invested in cash in the form of risk-free bonds and yet gains the exposure to about £100 million of UK stock. In the event that it must liquidate its position, perhaps to pay out insurance claims, it need only liquidate the UK risk-free bonds and close out the futures contracts. Given the liquidity of the futures market and the obvious liquidity of the risk-free bond market, doing so would be relatively easy.

There is one important aspect of this problem, however, over which the fund has no control: the pricing of the futures. Because the fund will take a long position in futures, the futures contract must be correctly priced. If the futures contract is overpriced, the fund will pay too much for the futures. In that case, the risk-free bonds will not be enough to offset the excessively high price effectively paid for the stock. If, however, the futures contract is underpriced, the fund will get a bargain and will come out much better.

Finally, we should note that these strategies can be illustrated with bond futures to gain bond market exposure, but they are more commonly implemented using stock index futures to gain equity market exposure.

### EXAMPLE 4

Index Advantage (INDEXA) is a money management firm that specializes in turning the idle cash of clients into equity index positions at very low cost. INDEXA has a new client with about \$500 million of cash that it would like to invest in the small-cap equity sector. INDEXA will construct the position using a futures contract on a small-cap index. The futures price is 1,500, the multiplier is \$100, and the contract expires in six months. The underlying small-cap index has a dividend yield of 1%. The risk-free rate is 3% per year.

- A. Determine exactly how the cash can be equitized using futures contracts.
- B. When the futures contract expires, the index is at  $S_T$ . Demonstrate how the position produces the same outcome as an actual investment in the index.

*Solution to A:* INDEXA should purchase

$$N_f = \frac{\$500,000,000(1.03)^{0.5}}{\$100(1,500)} = 3,382.96$$

futures contracts. Round this amount to  $N_f^* = 3,383$ . Then invest

$$\frac{3,383(\$100)(1,500)}{(1.03)^{0.5}} = \$500,005,342$$

in risk-free bonds paying 3% interest. Note that this is not exactly an initial investment of \$500 million, because one cannot purchase fractions of futures contracts. The bonds will grow to a value of  $\$500,005,342(1.03)^{0.5} = \$507,450,000$ . The number of units of stock effectively purchased through the use of futures is

$$\frac{N_f^* q}{(1 + \delta)^T} = \frac{3,383(100)}{(1.01)^{0.5}} = 336,621.08$$

If 336,621.08 shares were actually purchased, the accumulation and reinvestment of dividends would result in there being  $336,621.08 (1.01)^{0.5} = 338,300$  shares at the futures expiration.

*Solution to B:* At expiration, the payoff on the futures is

$$3,383(100)(S_T - 1500) = 338,300S_T - \$507,450,000$$

In other words, to settle the futures, INDEXA will owe \$507,450,000 and receive the equivalent of 338,300 units of stock worth  $S_T$ .

### 3.4. Creating Cash out of Equity

Because we have the relation Long stock + Short futures = Long risk-free bonds, we should be able to construct a synthetic position in cash by selling futures against a long stock position. Indeed we have already done a similar transaction when we sold futures to reduce the stock portfolio beta to zero. Therefore, if we wish to sell stock, we can do so by converting it to synthetic cash. This move can save transaction costs and avoid the sale of large amounts of stock at a single point in time.

Suppose the market value of our investment in stock is  $V$ , and we would like to create synthetic cash roughly equivalent to that amount. We shall sell futures, with the objective that at the horizon date, we shall have  $V(1 + r)^T$ . Money in the amount of  $V$  will have grown in value at the risk-free rate. Each unit of the index is priced at  $S$ . The number of units of the index we shall effectively convert to cash would appear to be  $(V/S)$ , but because of reinvested dividends, we actually end up with  $(1 + \delta)^T$  units of stock for every unit we start with. Hence, the number of units we are effectively converting to cash is  $(V/S)(1 + \delta)^T$ .

As in the example of the synthetic index fund, we shall again have a problem in that the number of futures contracts must be rounded off to an integer. Keeping that in mind, the

payoff of the futures contracts will be  $qN_f^*(S_T - f) = qN_f^*S_T - qN_f^*f$ . If the number of units of stock is  $(V/S)(1 + \delta)^T$ , then the value of the overall position (long stock plus short futures) will be  $(V/S)(1 + \delta)^T S_T + qN_f^*S_T - qN_f^*f$ . Because we are trying to convert to risk-free bonds (cash), we need to find a way to eliminate the  $S_T$  term. We just solve for the value of  $N_f^*$  that will cause the first two terms to offset.<sup>28</sup> We obtain a previous equation, Equation 6

$$N_f^* = -\frac{V(1+r)^T}{qf} \quad (\text{rounded to an integer})$$

As usual, the minus sign means that  $N_f^*$  is less than zero, which means we are selling futures. Because of rounding, the amount of stock we are actually converting is

$$V^* = \frac{-N_f^*qf}{(1+r)^T} \quad (8)$$

Therefore, if we use  $N_f^*$  futures contracts, we have effectively converted stock worth  $V^*$  to cash. This will not be the exact amount of stock we own, but it will be close. As in the case of the synthetic index fund, reinvestment of dividends means that the number of units of stock will be  $-N_f^*q/(1 + \delta)^T$  at the start and  $-N_f^*q$  when the futures expires. In Exhibit 5, we illustrate the application of this strategy for a pension fund that would like to convert \$50 million of stock to synthetic cash.

## EXHIBIT 5 Creating Synthetic Cash

### Scenario (2 June)

The pension fund of Interactive Industrial Systems (IIS) holds a \$50 million portion of its portfolio in an indexed position of the NASDAQ 100, which has a dividend yield of 0.75%. It would like to convert that position to cash for a two-month period. It can do this using a futures contract on the NASDAQ 100, which is priced at 1484.72, has a multiplier of \$100, and expires in two months. The risk-free rate is 4.65%.

### Action

The fund needs to use

$$N_f = \frac{-V(1+r)^T}{qf} = -\frac{\$50,000,000(1.0465)^{2/12}}{\$100(1484.72)} = -339.32$$

futures contracts. This amount should be rounded to  $N_f^* = -339$ . Because of rounding, the amount of stock synthetically converted to cash is really

$$\frac{-N_f^*qf}{(1+r)^T} = \frac{339(\$100)(1484.72)}{(1.0465)^{2/12}} = \$49,952,173$$

<sup>28</sup>In order to get this solution, we must take the result that  $f = S(1 + r)^T/(1 + \delta)^T$  and turn it around so that  $S = f(1 + \delta)^T/(1 + r)^T$  to find the value of  $S$ .

## EXHIBIT 5 (Continued)

This amount should grow to  $\$49,952,173(1.0465)^{2/12} = \$50,332,008$ . The number of units of stock is

$$\frac{-N_f * q}{(1 + \delta)^T} = \frac{339(\$100)}{(1.0075)^{2/12}} = 33,857.81$$

at the start, which grows to  $33,857.81(1.0075)^{2/12} = 33,900$  units when the futures expires.

**Scenario (4 August)**

The stock index is at  $S_T$  when the futures expires.

**Outcome and Analysis**

The payoff of the futures contract is

$$-339(\$100)(S_T - 1484.72) = -\$33,900S_T + \$50,332,008$$

As noted, dividends are reinvested and the number of units of the index grows to 33,900 shares. The overall position of the fund is

Stock worth 33,900S

Futures payoff of  $-33,900S_T + \$50,332,008$

or an overall total of  $\$50,332,008$ . This is exactly the amount we said the fund would have if it invested  $\$49,952,173$  at the risk-free rate of 4.65% for two months. Thus, the fund has effectively converted a stock position to cash.

**EXAMPLE 5**

Synthetics Inc. (SYNINC) executes a variety of synthetic strategies for pension funds. One such strategy is to enable the client to maintain a liquid balance in cash while retaining exposure to equity market movements. A similar strategy is to enable the client to maintain its position in the market but temporarily convert it to cash. A client with a \$100 million equity position wants to convert it to cash for three months. An equity market futures contract is priced at \$325,000, expires in three months, and is based on an underlying index with a dividend yield of 2%. The risk-free rate is 3.5%.

- A. Determine the number of futures contracts SYNINC should trade and the effective amount of money it has invested in risk-free bonds to achieve this objective.
- B. When the futures contracts expire, the equity index is at  $S_T$ . Show how this transaction results in the appropriate outcome.

*Solution to A:* First note that no multiplier is quoted here. The futures price of \$325,000 is equivalent to a quoted price of \$325,000 and a multiplier of 1.0. The number of futures contracts is

$$N_f = -\frac{\$100,000,000(1.035)^{0.25}}{\$325,000} = -310.35$$

Rounding off, SYNINC should sell 310 contracts. This is equivalent to selling futures contracts on stock worth

$$\frac{310(\$325,000)}{(1.035)^{0.25}} = \$99,887,229$$

and is the equivalent of investing \$99,887,229 in risk-free bonds, which will grow to a value of  $\$99,887,229(1.035)^{0.25} = \$100,750,000$ . The number of units of stock being effectively converted to cash is (ignoring the minus sign)

$$\frac{N_f * q}{(1 + \delta)^T} = \frac{310(1)}{(1.02)^{0.25}} = 308.47$$

The accumulation and reinvestment of dividends would make this figure grow to  $308.47(1.02)^{0.25} = 310$  units when the futures expires.

*Solution to B:* At expiration, the profit on the futures is  $-310(S_T - \$325,000) = -310S_T + \$100,750,000$ . That means SYNINC will have to pay  $310S_T$  and will receive \$100,750,000 to settle the futures contract. Due to reinvestment of dividends, it will end up with the equivalent of 310 units of stock, which can be sold to cover the amount  $-310S_T$ . This will leave \$100,750,000, the equivalent of having invested in risk-free bonds.

You might be wondering about the relationship between the number of futures contracts given here and the number of futures contracts required to adjust the portfolio beta to zero. Here we are selling a given number of futures contracts against stock to effectively convert the stock to a risk-free asset. Does that not mean that the portfolio would then have a beta of zero? In Section 3.2, we gave a different formula to reduce the portfolio beta to zero. These formulas do not appear to be the same. Would they give the same value of  $N_f$ ? In the example here, we sell the precise number of futures to completely hedge the stock portfolio. The stock portfolio, however, has to be identical to the index. It cannot have a different beta. The other formula, which reduces the beta to zero, is more general and can be used to eliminate the systematic risk on any portfolio. Note, however, that only systematic risk is eliminated. If the portfolio is not fully diversified, some risk will remain, but that risk is diversifiable, and the expected return on that portfolio would still be the risk-free rate. If we apply that formula to a portfolio that is identical to the index on which the futures is based, the two formulas are the same and the number of futures contracts to sell is the same in both cases.<sup>29</sup>

<sup>29</sup>A key element in this statement is that the futures beta is the beta of the underlying index, multiplied by the present value interest factor using the risk-free rate. This is a complex and subtle point, however, that we simply state without going into the mathematical proof.

Finally, we should note that we could have changed the beta of the portfolio by making transactions in individual securities. To raise (lower) the beta we could sell (buy) low-beta stocks and buy (sell) high-beta stocks. Alternatively, we could do transactions in the portfolio itself and the risk-free asset. To reduce the beta to zero, for example, we could sell the entire portfolio and invest the money in the risk-free asset. To increase the beta, we could reduce any position we hold in the risk-free asset, even to the point of borrowing by issuing the risk-free asset.<sup>30</sup> In this reading, we illustrate how these transactions can be better executed using derivatives, which have lower transaction costs and generally greater liquidity. There is no guarantee that either approach will result in the portfolio having the exact beta the investor desired. Betas are notoriously difficult to measure. But executing the transactions in derivatives provides an attractive alternative to having to make a large number of transactions in individual securities. In light of the fact that many of these adjustments are intended to be only temporary, it makes far more sense to do the transactions in derivatives than to make the transactions in the underlying securities, provided that one is willing to keep re-entering positions upon contract expirations.

## 4. ASSET ALLOCATION WITH FUTURES

It has been widely noted that the most important factor in the performance of an asset portfolio is the allocation of the portfolio among asset classes. In this reading, we do not develop techniques for determining the best allocation among asset classes any more than we attempt to determine what beta to set as a target for our stock portfolio or what duration to set as a target for our bond portfolio. We focus instead on how derivative strategies can be used to implement a plan based on a market outlook. As we saw previously in this reading, we can adjust the beta or duration effectively with lower cost and greater liquidity by using stock index or bond futures. In this section, we look at how to allocate a portfolio among asset classes using futures.

### 4.1. Adjusting the Allocation among Asset Classes

Consider the case of a \$300 million portfolio that is allocated 80% (\$240 million) to stock and 20% (\$60 million) to bonds. The manager wants to change the allocation to 50% (\$150 million) stock and 50% (\$150 million) bonds. Therefore, the manager wants to reduce the allocation to stock by \$90 million and increase the allocation to bonds by \$90 million. The trick, however, is to use the correct number of futures contracts to set the beta and duration to the desired level. To do this, the manager should sell stock index futures contracts to reduce the beta on the \$90 million of stock from its current level to zero. This transaction will effectively convert the stock to cash. She should then buy bond futures contracts to increase the duration on the cash from its current level to the desired level.

Exhibit 6 presents this example. The manager sells 516 stock index futures contracts and buys 772 bond futures contracts. Two months later, the position is worth \$297,964,852. As we

<sup>30</sup>Students of capital market theory will recognize that the transactions we describe in this paragraph are those involving movements up and down the capital market line, which leads to investors finding their optimal portfolios. This kind of trading activity in turn leads to the well-known capital asset pricing model.

show, had the transactions been done by selling stocks and buying bonds, the portfolio would be worth \$297,375,000, a difference of only about 0.2% relative to the original market value. Of course, the futures transactions can be executed in a more liquid market and with lower transaction costs.

#### EXHIBIT 6 Adjusting the Allocation between Stocks and Bonds

##### Scenario (15 November)

Global Asset Advisory Group (GAAG) is a pension fund management firm. One of its funds consists of \$300 million allocated 80% to stock and 20% to bonds. The stock portion has a beta of 1.10 and the bond portion has a duration of 6.5. GAAG would like to temporarily adjust the asset allocation to 50% stock and 50% bonds. It will use stock index futures and bond futures to achieve this objective. The stock index futures contract has a price of \$200,000 (after accounting for the multiplier) and a beta of 0.96. The bond futures contract has an implied modified duration of 7.2 and a price of \$105,250. The yield beta is 1. The transaction will be put in place on 15 November, and the horizon date for termination is 10 January.

##### Action

The market value of the stock is  $0.80(\$300,000,000) = \$240,000,000$ . The market value of the bonds is  $0.20(\$300,000,000) = \$60,000,000$ . Because it wants the portfolio to be temporarily reallocated to half stock and half bonds, GAAG needs to change the allocation to \$150 million of each.

Thus, GAAG effectively needs to sell \$90 million of stock by converting it to cash using stock index futures and buy \$90 million of bonds by using bond futures. This would effectively convert the stock into cash and then convert that cash into bonds. Of course, this entire series of transactions will be synthetic; the actual stock and bonds in the portfolio will stay in place.

Using Equation 5, the number of stock index futures, denoted as  $N_{sf}$ , will be

$$N_{sf} = \left( \frac{\beta_T - \beta_S}{\beta_f} \right) \frac{S}{f_s}$$

where  $\beta_T$  is the target beta of zero,  $\beta_S$  is the stock beta of 1.10,  $\beta_f$  is the futures beta of 0.96,  $S$  is the market value of the stock involved in the transaction of \$90 million, and  $f_s$  is the price of the stock index futures, \$200,000. We obtain

$$N_{sf} = \left( \frac{0.00 - 1.10}{0.96} \right) \frac{\$90,000,000}{\$200,000} = -515.63$$

Rounding off, GAAG sells 516 contracts.

Using Equation 4, the number of bond futures, denoted as  $N_{bf}$ , will be

$$N_{bf} = \left( \frac{MDUR_T - MDUR_B}{MDUR_f} \right) \frac{B}{f_b}$$

where  $MDUR_T$  is the target modified duration of 6.5,  $MDUR_B$  is the modified duration of the existing bonds,  $MDUR_f$  is the implied modified duration of the futures (here 7.2),  $B$  is the market value of the bonds of \$90 million, and  $f_b$  is the bond futures price of \$105,250. The modified duration of the existing bonds is the modified duration of a cash position. The sale of stock index futures provides \$90 million of synthetic cash that is now converted into bonds using bond futures. Because no movement of actual

## EXHIBIT 6 (Continued)

cash is involved in these futures market transactions, the modified duration of cash is effectively equal to zero. We obtain

$$N_{bf} = \left( \frac{6.5 - 0.0}{7.2} \right) \left( \frac{\$90,000,000}{\$105,250} \right) = 771.97$$

So GAAG buys 772 contracts.

**Scenario (10 January)**

During this period, the stock portion of the portfolio returns  $-3\%$  and the bond portion returns  $1.25\%$ . The stock index futures price goes from  $\$200,000$  to  $\$193,600$ , and the bond futures price increases from  $\$105,250$  to  $\$106,691$ .

**Outcome and Analysis**

The profit on the stock index futures transaction is  $-516(\$193,600 - \$200,000) = \$3,302,400$ . The profit on the bond futures transaction is  $772(\$106,691 - \$105,250) = \$1,112,452$ . The total profit from the futures transaction is, therefore,  $\$3,302,400 + \$1,112,452 = \$4,414,852$ . The market value of the stocks and bonds will now be

Stocks: $\$240,000,000(1 - 0.03)$	=	$\$232,800,000$
Bonds: $\$60,000,000(1.0125)$	=	$\$ 60,750,000$
Total:		$\$293,550,000$

Thus, the total portfolio value, including the futures gains, is  $\$293,550,000 + \$4,414,852 = \$297,964,852$ . Had GAAG sold stocks and then converted the proceeds to bonds, the value would have been

Stocks: $\$150,000,000(1 - 0.03)$	=	$\$145,500,000$
Bonds: $\$150,000,000(1.0125)$	=	$\$151,875,000$
Total:		$\$297,375,000$

This total is a slight difference of about  $0.2\%$  relative to the market value of the portfolio using derivatives.

Exhibit 7 shows a variation of this problem in which a portfolio management firm wants to convert a portion of a bond portfolio to cash to meet a liquidity requirement and another portion to a higher duration. On the portion it wants to convert to cash, it sells 104 futures contracts. This is the correct amount to change the duration to 0.25, the approximate duration of a short-term money market instrument. It then buys 33 futures contracts to raise the duration on the other part of the portfolio. The net is that it executes only one transaction of 71 contracts, and the end result is a portfolio worth  $\$3,030,250$  at the end of the period. Had the transactions been done by selling and buying securities, the portfolio would have been worth  $\$3,048,000$ , or about the same amount. Another question we shall examine is whether this strategy actually meets the liquidity requirement.

We note in Exhibit 7 that the manager wants to convert a portion of the portfolio to cash to increase liquidity. By selling the futures contracts, the manager maintains the securities in long-term bonds but reduces the volatility of those bonds to the equivalent

of that of a short-term instrument. We might, however, question whether liquidity has actually been improved. If cash is needed, the fund would have to sell the long-term bonds and buy back the futures. The latter would not present a liquidity problem, but the sale of the long-term bonds could be a problem. Reducing the duration to replicate a short-term instrument does not remove the problem that long-term instruments, which are still held, may have to be liquidated. What it does is convert the volatility of the instrument to that of a short-term instrument. This conversion in no way handles the liquidity problem. It simply means that given an interest rate change, the position will have the sensitivity of a short-term instrument.

#### EXHIBIT 7 Adjusting the Allocation between One Bond Class and Another

##### Scenario (15 October)

Fixed Income Money Advisors (FIMA) manages bond portfolios for wealthy individual investors. It uses various tactical strategies to alter its mix between long-and short-term bonds to adjust its portfolio to a composition appropriate for its outlook for interest rates. Currently, it would like to alter a \$30 million segment of its portfolio that has a modified duration of 6.5. To increase liquidity, it would like to move \$10 million into cash but adjust the duration on the remaining \$20 million to 7.5. These changes will take place on 15 October and will likely be reversed on 12 December.

##### Action

The bond futures contract that FIMA will use is priced at \$87,500 and has an implied modified duration of 6.85. To convert \$10 million of bonds at a duration of 6.5 into cash requires adjusting the duration to that of a cash equivalent. A cash equivalent is a short-term instrument with a duration of less than 1.0. The equivalent instruments that FIMA would use if it did the transactions in cash would be six-month instruments. The average duration of a six-month instrument is three months or 0.25. The interest rate that drives the long-term bond market is assumed to have a yield beta of 1.0 with respect to the interest rate that drives the futures market.

FIMA could solve this problem in either of two ways. It could lower the duration on \$10 million of bonds from 6.5 to 0.25. Then it could raise the duration on \$20 million from 6.5 to 7.5. If FIMA converts \$10 million to a duration of 0.25 and \$20 million to a duration of 7.5, the overall duration would be  $(10/30)0.25 + (20/30)7.50 = 5.08$ . As an alternative, FIMA could just aim for lowering the overall duration to 5.08, but we shall illustrate the approach of adjusting the duration in two steps.

Thus, FIMA needs to lower the duration on \$10 million from 6.5 to 0.25. Accordingly, the appropriate number of futures contracts is

$$N_f = \left( \frac{MDUR_T - MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right) = \left( \frac{0.25 - 6.50}{6.85} \right) \left( \frac{\$10,000,000}{\$87,500} \right) = -104.28$$

So, FIMA should sell 104 contracts.

To increase the duration on \$20 million from 6.5 to 7.5, the appropriate number of futures contracts is

$$N_f = \left( \frac{MDUR_T - MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right) = \left( \frac{7.5 - 6.5}{6.85} \right) \left( \frac{\$20,000,000}{\$87,500} \right) = 33.37$$

Thus, FIMA should buy 33 futures contracts.

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EXHIBIT 7 (Continued)

Because these transactions involve the same futures contract, the net effect is that FIMA should sell 71 contracts. Therefore, FIMA does just one transaction to sell 71 contracts.

**Scenario (12 December)**

During this period, interest rates rose by 2% and the bonds decreased in value by 13% (6.5 duration times 2%). The futures price fell to \$75,250. Thus, the \$30 million bond portfolio fell by  $\$30,000,000(0.13) = \$3,900,000$ .

**Outcome and Analysis**

The profit on the futures contracts is  $-71(\$75,250 - \$87,500) = \$869,750$ . So the overall loss is  $\$3,900,000 - \$869,750 = \$3,030,250$ . The change in the portfolio value of 13% was based on an assumed yield change of 2% (6.5 duration times 0.02 = 0.13). A portfolio with a modified duration of 5.08 would, therefore, change by approximately  $5.08(0.02) = 0.1016$ , or 10.16%. The portfolio thus would decrease by  $\$30,000,000(0.1016) = \$3,048,000$ .

The difference in this result and what was actually obtained is \$17,750, or about 0.06% of the initial \$30 million value of the portfolio. Some of this difference is due to rounding and some is due to the fact that bonds do not respond in the precise manner predicted by duration.

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In Exhibit 8, we illustrate a similar situation involving a pension fund that would like to shift the allocation of its portfolio from large-cap stock to mid-cap stock. With futures contracts available on indices of both the large-cap and mid-cap sectors, the fund can do this by selling futures on the large-cap index and buying futures on the mid-cap index. The results come very close to replicating what would happen if it undertook transactions in the actual stocks. The futures transactions, however, take place in a market with much greater liquidity and lower transaction costs.

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EXHIBIT 8 Adjusting the Allocation between One Equity Class and Another**Scenario (30 April)**

The pension fund of US Integrated Technology (USIT) holds \$50 million of large-cap domestic equity. It would like to move \$20 million from large-cap stocks to mid-cap stocks. The large-cap stocks have an average beta of 1.03. The desired beta of mid-cap stocks is 1.20. A futures contract on large-cap stocks has a price of \$263,750 and a beta of 0.98. A futures contract on mid-cap stocks has a price of \$216,500 and a beta of 1.14. The transaction will be initiated on 30 April and terminated on 29 May.

To distinguish the futures contracts, we use  $N_{LF}$  and  $N_{MF}$  as the number of large-cap and mid-cap futures contracts,  $f_L$  and  $f_M$  as the prices of large-cap and mid-cap futures contracts (\$263,750 and \$216,500, respectively),  $\beta_L$  and  $\beta_M$  as the betas of large-cap and mid-cap stocks (1.03 and 1.20, respectively), and  $\beta_{LF}$  and  $\beta_{MF}$  as the betas of the large-cap and mid-cap futures (0.98 and 1.14, respectively).

**Action**

USIT first wants to convert \$20 million of stock to cash and then convert \$20 million of cash into mid-cap stock. It can use large-cap futures to convert the beta from 1.03 to zero and then use mid-cap futures to convert the beta from 0 to 1.20.

(continued)

## EXHIBIT 8 (Continued)

To convert the large-cap stock to cash will require

$$N_{Lf} = \left( \frac{\beta_T - \beta_L}{\beta_{Lf}} \right) \left( \frac{S}{f_L} \right) = \left( \frac{0.0 - 1.03}{0.98} \right) \left( \frac{\$20,000,000}{\$263,750} \right) = -79.70$$

So USIT sells 80 large-cap futures contracts. At this point, it has changed the beta to zero. Now it uses mid-cap futures to convert the beta from 0.0 to 1.20:

$$N_{Mf} = \left( \frac{\beta_M - \beta_T}{\beta_{Mf}} \right) \left( \frac{B}{f_M} \right) = \left( \frac{1.20 - 0.0}{1.14} \right) \left( \frac{\$20,000,000}{\$216,500} \right) = 97.24$$

So USIT buys 97 mid-cap futures contracts.

#### Scenario (29 May)

Large-cap stocks increase by 2.47%, and the large-cap futures price increases to \$269,948. Mid-cap stocks increase by 2.88%, and the mid-cap futures price increases to \$222,432. The \$50 million large-cap portfolio is now worth  $\$50,000,000(1.0247) = \$51,235,000$ .

#### Outcome and Analysis

The profit on the large-cap futures contracts is  $-80(\$269,948 - \$263,750) = -\$495,840$ . The profit on the mid-cap futures contracts is  $97(\$222,432 - \$216,500) = \$575,404$ . The total value of the fund is, therefore,  $\$51,235,000 - \$495,840 + \$575,404 = \$51,314,564$ .

Had the transactions been executed by selling \$20 million of large-cap stock and buying \$20 million of mid-cap stock, the value of the large-cap stock would be  $\$30,000,000(1.0247) = \$30,741,000$ , and the value of the mid-cap stock would be  $\$20,000,000(1.0288) = \$20,576,000$ , for a total value of  $\$30,741,000 + \$20,576,000 = \$51,317,000$ .

This amount produces a difference of \$2,436 compared with making the allocation synthetically, an insignificant percentage of the original portfolio value. The difference comes from the fact that stocks do not always respond in the exact manner predicted by their betas and also that the number of futures contracts is rounded off.

#### EXAMPLE 6

Q-Tech Advisors manages a portfolio consisting of \$100 million, allocated 70% to stock at a beta of 1.05 and 30% to bonds at a modified duration of 5.5. As a tactical strategy, it would like to temporarily adjust the allocation to 60% stock and 40% bonds. Also, it would like to change the beta on the stock position from 1.05 to 1.00 and the modified duration from 5.5 to 5.0. It will use a stock index futures contract, which is priced at \$280,000 and has a beta of 0.98, and a bond futures contract, which is priced at \$125,000 and has an implied modified duration of 6.50.

- Determine how many stock index and bond futures contracts it should use and whether to go long or short.

- B. At the horizon date, the stock portfolio has fallen by 3% and the bonds have risen by 1%. The stock index futures price is \$272,160, and the bond futures price is \$126,500. Determine the market value of the portfolio assuming the transactions specified in Part A are done, and compare it to the market value of the portfolio had the transactions been done in the securities themselves.

*Solution to A:* To reduce the allocation from 70% stock (\$70 million) and 30% bonds (\$30 million) to 60% stock (\$60 million) and 40% bonds (\$40 million), Q-Tech must synthetically sell \$10 million of stock and buy \$10 million of bonds. First, assume that Q-Tech will sell \$10 million of stock and leave the proceeds in cash. Doing so will require

$$N_{sf} = \left( \frac{0 - 1.05}{0.98} \right) \left( \frac{\$10,000,000}{\$280,000} \right) = -38.27$$

futures contracts. It should sell 38 contracts, which creates synthetic cash of \$10 million. To buy \$10 million of bonds, Q-Tech should buy

$$N_{bf} = \left( \frac{5.50 - 0.0}{6.50} \right) \left( \frac{\$10,000,000}{\$125,000} \right) = 67.69$$

futures contracts, which rounds to 68. This transaction allows Q-Tech to synthetically borrow \$10 million (selling a stock futures contract is equivalent to borrowing cash) and buy \$10 million of bonds. Because we have created synthetic cash and a synthetic loan, these amounts offset. Thus, at this point, having sold 38 stock index futures and bought 68 bond futures, Q-Tech has effectively sold \$10 million of stock and bought \$10 million of bonds. It has produced a synthetically re-allocated portfolio of \$60 million of stock and \$40 million of bonds.

Now it needs to adjust the beta on the \$60 million of stock to its target of 1.00. The number of futures contracts would, therefore, be

$$N_{sf} = \left( \frac{1.00 - 1.05}{0.98} \right) \left( \frac{\$60,000,000}{\$280,000} \right) = -10.93$$

So it should sell an additional 11 contracts. In total, it should sell  $38 + 11 = 49$  contracts.

To adjust the modified duration from 5.50 to its target of 5.00 on the \$40 million of bonds, the number of futures contracts is

$$N_{bf} = \left( \frac{5 - 5.50}{6.50} \right) \left( \frac{\$40,000,000}{\$125,000} \right) = -24.62$$

So it should sell 25 contracts. In total, therefore, it should buy  $68 - 25 = 43$  contracts.

*Solution to B:* The value of the stock will be  $\$70,000,000(1 - 0.03) = \$67,900,000$ .

The profit on the stock index futures will be  $-49(\$272,160 - \$280,000) = \$384,160$ .

The total value of the stock position is therefore  $\$67,900,000 + \$384,160 = \$68,284,160$ .

The value of the bonds will be  $\$30,000,000(1.01) = \$30,300,000$ .  
 The profit on the bond futures will be  $43(\$126,500 - \$125,000) = \$64,500$ .  
 The total value of the bond position is, therefore,  $\$30,300,000 + \$64,500 = \$30,364,500$ .  
 Therefore, the overall position is worth  $\$68,284,160 + \$30,364,500 = \$98,648,660$ .  
 Had the transactions been done in the securities themselves, the stock would be worth  $\$60,000,000(1 - 0.03) = \$58,200,000$ . The bonds would be worth  $\$40,000,000(1.01) = \$40,400,000$ . The overall value of the portfolio would be  $\$58,200,000 + \$40,400,000 = \$98,600,000$ , which is a difference of only  $\$48,660$  or 0.05% of the original value of the portfolio.

So far, we have looked only at allocating funds among different asset classes. In the next section, we place ourselves in the position that funds are not available to invest in any asset classes, but market opportunities are attractive. Futures contracts enable an investor to place itself in the market without yet having the actual cash in place.

#### 4.2. Pre-Investing in an Asset Class

In all the examples so far, the investor is already in the market and wants to either alter the position to a different asset allocation or get out of the market altogether. Now consider that the investor might not be in the market but wants to get into the market. The investor might not have the cash to invest at a time when opportunities are attractive. Futures contracts do not require a cash outlay but can be used to add exposure. We call this approach pre-investing.

An advisor to a mutual fund would like to pre-invest \$10 million in cash that it will receive in three months. It would like to allocate this money to a position of 60% stock and 40% bonds. It can do this by taking long positions in stock index futures and bond futures. The trick is to establish the position at the appropriate beta and duration. This strategy is illustrated in Exhibit 9. We see that the result using futures is very close to what it would have been if the fund had actually had the money and invested it in stocks and bonds.

#### EXHIBIT 9 Pre-Investing in Asset Classes

##### Scenario (28 February)

Quantitative Mutual Funds Advisors (QMFA) uses modern analytical techniques to manage money for a number of mutual funds. QMFA is not necessarily an aggressive investor, but it does not like to be out of the market. QMFA has learned that it will receive an additional \$10 million to invest. Although QMFA would like to receive the money now, the money is not available for three months. If it had the money now, QMFA would invest \$6 million in stocks at an average beta of 1.08 and \$4 million in bonds at a modified duration of 5.25. It believes the market outlook over the next three months is highly attractive. Therefore, QMFA would like to invest now, which it can do by trading stock and bond futures. An appropriate stock index futures contract is selling at \$210,500 and has a beta of 0.97. An appropriate bond futures contract is selling for \$115,750 and has an implied modified duration of 6.05. The current date is 28 February, and the money will be available on 31 May. The number of stock index futures contracts will be denoted as  $N_{sf}$ , and the number of bond futures contracts will be denoted as  $N_{bf}$ .

## EXHIBIT 9 (Continued)

**Action**

QMFA wants to take a position in \$6 million of stock index futures at a beta of 1.08. It currently has no position; hence, its beta is zero. The required number of stock index futures contracts to obtain this position is

$$N_{sf} = \left( \frac{\beta_T - \beta_S}{\beta_f} \right) \left( \frac{S}{f} \right) = \left( \frac{1.08 - 0.0}{0.97} \right) \left( \frac{\$6,000,000}{\$210,500} \right) = 31.74$$

So QMFA buys 32 stock index futures contracts.

To gain exposure at a duration of 5.25 on \$4 million of bonds, the number of bond futures contracts is

$$N_{bf} = \left( \frac{MDUR_T - MDUR_B}{MDUR_f} \right) \left( \frac{B}{f} \right) = \left( \frac{5.25 - 0.0}{6.05} \right) \left( \frac{\$4,000,000}{\$115,750} \right) = 29.99$$

Thus, QMFA buys 30 bond futures contracts.

**Scenario (31 May)**

During this period, the stock increased by 2.2% and the bonds increased by 0.75%. The stock index futures price increased to \$214,500, and the bond futures price increased to \$116,734.

**Outcome and Analysis**

The profit on the stock index futures contracts is  $32(\$214,500 - \$210,500) = \$128,000$ . The profit on the bond futures contracts is  $30(\$116,734 - \$115,750) = \$29,520$ . The total profit is, therefore,  $\$128,000 + \$29,520 = \$157,520$ .

Had QMFA actually invested the money, the stock would have increased in value by  $\$6,000,000(0.022) = \$132,000$ , and the bonds would have increased in value by  $\$4,000,000(0.0075) = \$30,000$ , for a total increase in value of  $\$132,000 + \$30,000 = \$162,000$ , which is relatively close to the futures gain of \$157,520. The difference of \$4,480 between this approach and the synthetic one is about 0.04% of the \$10 million invested. This difference is due to the fact that stocks and bonds do not always respond in the manner predicted by their betas and durations and also that the number of futures contracts is rounded off.

In a transaction like the one just described, the fund is effectively borrowing against the cash it will receive in the future by pre-investing. Recall that

$$\text{Long underlying} + \text{Short futures} = \text{Long risk-free bond}$$

which means that

$$\text{Long underlying} = \text{Long risk-free bond} + \text{Long futures}$$

In this example, however, the investor does not have the long position in the risk-free bond. That would require cash. We can remove the long risk-free bond in the equation above

by off-setting it with a loan in which we borrow the cash. Hence, adding a loan to both sides gives<sup>31</sup>

$$\text{Long underlying} + \text{Loan} = \text{Long futures}$$

An outright long position in futures is like a fully leveraged position in the underlying. So in this example, we have effectively borrowed against the cash we will receive in the future and invested in the underlying.

### EXAMPLE 7

Total Asset Strategies (TAST) specializes in a variety of risk management strategies, one of which is to enable investors to take positions in markets in anticipation of future transactions in securities. One of its popular strategies is to have the client invest when it does not have the money but will be receiving it later. One client interested in this strategy will receive \$6 million at a later date but wants to proceed and take a position of \$3 million in stock and \$3 million in bonds. The desired stock beta is 1.0, and the desired bond duration is 6.2. A stock index futures contract is priced at \$195,000 and has a beta of 0.97. A bond futures contract is priced at \$110,000 and has an implied modified duration of 6.0.

- A. Find the number of stock and bond futures contracts TAST should trade and whether it should go long or short.
- B. At expiration, the stock has gone down by 5%, and the stock index futures price is down to \$185,737.50. The bonds are up 2%, and the bond futures price is up to \$112,090. Determine the value of the portfolio and compare it with what it would have been had the transactions been made in the actual securities.

*Solution to A:* The approximate number of stock index futures is

$$\left( \frac{1.00 - 0.0}{0.97} \right) \left( \frac{\$3,000,000}{\$195,000} \right) = 15.86$$

So TAST should buy 16 contracts. The number of bond futures is

$$\left( \frac{6.2 - 0.0}{6.0} \right) \left( \frac{\$3,000,000}{\$110,000} \right) = 28.18$$

So it should buy 28 contracts.

*Solution to B:* The profit on the stock index futures is  $16(\$185,737.50 - \$195,000) = -\$148,200$ .

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<sup>31</sup>The right-hand side is a long risk-free bond and a loan of the same amount, which offset each other.

The profit on the bond futures is  $28(\$112,090 - \$110,000) = \$58,520$ . The total profit is  $-\$148,200 + \$58,520 = -\$89,680$ , a loss of  $\$89,680$ . Suppose TAST had invested directly. The stock would have been worth  $\$3,000,000(1 - 0.05) = \$2,850,000$ , and the bonds would have been worth  $\$3,000,000(1.02) = \$3,060,000$ , for a total value of  $\$2,850,000 + \$3,060,000 = \$5,910,000$ , or a loss of  $\$90,000$ , which is about the same as the loss using only the futures.

When the cash is eventually received, the investor will close out the futures position and invest the cash. This transaction is equivalent to paying off this implicit loan. The investor will then be long the underlying.

We should remember that this position is certainly a speculative one. By taking a leveraged long position in the market, the investor is speculating that the market will perform well enough to cover the cost of borrowing. If this does not happen, the losses could be significant. But such is the nature of leveraged speculation with a specific horizon.

So far, all of the strategies we have examined have involved domestic transactions. We now take a look at how foreign currency derivatives can be used to handle common transactions faced in global commerce.

## 5. STRATEGIES AND APPLICATIONS FOR MANAGING FOREIGN CURRENCY RISK

The risk associated with changes in exchange rates between currencies directly affects many companies. Any company that engages in business with companies or customers in other countries is exposed to this risk. The volatility of exchange rates results in considerable uncertainty for companies that sell products in other countries as well as for those companies that buy products in other countries. Companies are affected not only by the exchange rate uncertainty itself but also by its effects on their ability to plan for the future. For example, consider a company with a foreign subsidiary. This subsidiary generates sales in the foreign currency that will eventually be converted back into its domestic currency. To implement a business plan that enables the company to establish a realistic target income, the company must not only predict its foreign sales, but it must also predict the exchange rate at which it will convert its foreign cash flows into domestic cash flows. The company may be an expert on whatever product it makes or service it provides and thus be in a good position to make reasonable forecasts of sales. But predicting foreign exchange rates with much confidence is extremely difficult, even for experts in the foreign exchange business. A company engaged in some other line of work can hardly expect to be able to predict foreign exchange rates very well. Hence, many such businesses choose to manage this kind of risk by locking in the exchange rate on future cash flows with the use of derivatives. This type of exchange rate risk is called **transaction exposure**.

In addition to the risk associated with foreign cash flows, exchange rate volatility also affects a company's accounting statements. When a company combines the balance sheets of foreign subsidiaries into a consolidated balance sheet for the entire company, the numbers

from the balance sheets of foreign subsidiaries must be converted into its domestic currency at an appropriate exchange rate. Hence, exchange rate risk manifests itself in this arena as well. This type of exchange rate risk is called **translation exposure**.

Finally, we should note that exchange rate uncertainty can also affect a company by making its products or services either more or less competitive with those of comparable foreign companies. This type of risk can affect any type of company, even if it does not sell its goods or services in foreign markets. For example, suppose the US dollar is exceptionally strong. This condition makes US products and services more expensive to non-US residents and will lead to a reduction in travel to the United States. Hence, the owner of a hotel in the Disney World area, even though her cash flow is entirely denominated in dollars, will suffer a loss of sales when the dollar is strong because fewer non-US residents will travel to the United States, visit Disney World, and stay in her hotel. Likewise, foreign travel will be cheaper for US citizens, and more of them will visit foreign countries instead of Disney World.<sup>32</sup> This type of risk is called **economic exposure**.

In this reading, we shall focus on managing the risk of transaction exposure. The management of translation exposure requires a greater focus on accounting than we can provide here. Managing economic exposure requires the forecasting of demand in light of competitive products and exchange rates, and we shall not address this risk.

The management of a single cash flow that will have to be converted from one currency to another is generally done using forward contracts. Futures contracts tend to be too standardized to meet the needs of most companies. Futures are primarily used by dealers to manage their foreign exchange portfolios.<sup>33</sup> Therefore, in the two examples here, we use forward contracts to manage the risk of a single foreign cash flow.

### 5.1. Managing the Risk of a Foreign Currency Receipt

When due to receive cash flows denominated in a foreign currency, companies can be viewed as being long the currency. They will convert the currency to their domestic currency and, hence, will be selling the foreign currency to obtain the domestic currency. If the domestic currency increases in value while the company is waiting to receive the cash flow, the domestic currency will be more expensive, and the company will receive fewer units of the domestic currency for the given amount of foreign currency. Thus, being long the foreign currency, the company should consider selling the currency in the forward market by going short a currency forward contract.

Exhibit 10 illustrates the case of a company that anticipates the receipt of a future cash flow denominated in euros. By selling a forward contract on the amount of euros it expects to receive, the company locks in the exchange rate at which it will convert the euros. We assume the contract calls for actual delivery of the euros, as opposed to a cash settlement, so the company simply transfers the euros to the dealer, which sends the domestic currency to the

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<sup>32</sup>Even US citizens who would never travel abroad would not increase their trips to Disney World because of the more favorable exchange rate.

<sup>33</sup>In some cases, single cash flows are managed using currency options, which we cover in the reading on risk management applications of option strategies. A series of foreign cash flows is usually managed using currency swaps, which we cover in the reading on risk management applications of swap strategies.

company. If the transaction were structured to be settled in cash, the company would sell the euros on the market for the exchange rate at that time,  $S_T$ , and the forward contract would be cash settled for a payment of  $-(S_T - F)$ , where  $F$  is the rate agreed on at the start of the forward contract—in other words, the forward exchange rate. The net effect is that the company receives  $F$ , the forward rate for the euros.

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#### EXHIBIT 10 Managing the Risk of a Foreign Currency Receipt

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##### **Scenario (15 August)**

H-Tech Hardware, a US company, sells its products in many countries. It recently received an order for some computer hardware from a major European government. The sale is denominated in euros and is in the amount of €50 million. H-Tech will be paid in euros; hence, it bears exchange rate risk. The current date is 15 August, and the euros will be received on 3 December.

##### **Action**

On 15 August, H-Tech decides to lock in the 3 December exchange rate by entering into a forward contract that obligates it to deliver €50 million and receive a rate of \$0.877. H-Tech is effectively long the euro in its computer hardware sale, so a short position in the forward market is appropriate.

##### **Scenario (3 December)**

The exchange rate on this day is  $S_T$ , but as we shall see, this value is irrelevant for H-Tech because it is hedged.

##### **Outcome and Analysis**

The company receives its €50 million, delivers it to the dealer, and is paid \$0.877 per euro for a total payment of €50,000,000(\$0.877) = \$43,850,000. H-Tech thus pays the €50 million and receives \$43.85 million, based on the rate locked in on 15 August.

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## 5.2. Managing the Risk of a Foreign Currency Payment

In Exhibit 11, we see the opposite type of problem. A US company is obligated to purchase a foreign currency at a later date. Because an increase in the exchange rate will hurt it, the US company is effectively short the currency. Hence, to lock in the rate now, it needs to go long the forward contract. Regardless of the exchange rate at expiration, the company purchases the designated amount of currency at the forward rate agreed to in the contract now.

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#### EXHIBIT 11 Managing the Risk of a Foreign Currency Payment

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##### **Scenario (2 March)**

American Manufacturing Catalyst (AMC) is a US company that occasionally makes steel and copper purchases from non-US companies to meet unexpected demand that cannot be filled through its domestic suppliers. On 2 March, AMC determines that it will need to buy a large quantity of steel from a Japanese company on 1 April. It has entered into a contract with the Japanese company to pay ¥900 million for the steel. At a current exchange rate of \$0.0083 per yen, the purchase will currently cost ¥900,000,000(\$0.0083) = \$7,470,000. AMC faces the risk of the yen strengthening.

*(continued)*

## EXHIBIT 11 (Continued)

**Action**

In its future steel purchase, AMC is effectively short yen, because it will need to purchase yen at a later date. Thus, a long forward contract is appropriate. AMC decides to lock in the exchange rate for 1 April by entering into a long forward contract on ¥900 million with a dealer. The forward rate is \$0.008309. AMC will be obligated to purchase ¥900 million on 1 April and pay a rate of \$0.008309.

**Scenario (1 April)**

The exchange rate for yen is  $S_T$ . As we shall see, this value is irrelevant for AMC, because it is hedged.

**Outcome and Analysis**

The company purchases ¥900 million from the dealer and pays \$0.008309, for a total payment of  $\$900,000,000(\$0.008309) = \$7,478,100$ . This amount was known on 2 March. AMC gets the yen it needs and uses it to purchase the steel.

In Exhibit 10, a company agreed to accept a fixed amount of the foreign currency for the sale of its computer hardware. In Exhibit 11, a company agreed to pay a fixed amount of the foreign currency to acquire the steel. You may be wondering why in both cases the transaction was denominated in the foreign currency. In some cases, a company might be able to lock in the amount of currency in domestic units. It all depends on the relative bargaining power of the buyer and the seller and on how badly each wants to make the sale. Companies with the expertise to manage foreign exchange risk can use that expertise to offer contracts denominated in either currency to their counterparts on the other side of the transaction. For example, in the second case, suppose the Japanese company was willing to lock in the exchange rate using a forward contract with one of its derivatives dealers. Then the Japanese company could offer the US company the contract in US dollars. The ability to manage exchange rate risk and offer customers a price in either currency can be an attractive feature for a seller.

**EXAMPLE 8**

Royal Tech Ltd. is a UK technology company that has recently acquired a US subsidiary. The subsidiary has an underfunded pension fund, and Royal Tech has absorbed the subsidiary's employees into its own pension fund, bringing the US subsidiary's defined-benefit plan up to an adequate level of funding. Soon Royal Tech will be making its first payments to retired employees in the United States. Royal Tech is obligated to pay about \$1.5 million to these retirees. It can easily set aside in risk-free bonds the amount of pounds it will need to make the payment, but it is concerned about the foreign currency risk in making the US dollar payment. To manage this risk, Royal Tech is considering using a forward contract that has a contract rate of £0.60 per dollar.

- A. Determine how Royal Tech would eliminate this risk by identifying an appropriate forward transaction. Be sure to specify the notional principal and state whether to go long or short. What domestic transaction should it undertake?

- B. At expiration of the forward contract, the spot exchange rate is  $S_T$ . Explain what happens.

*Solution to A:* Royal Tech will need to come up with \$1,500,000 and is obligated to buy dollars at a later date. It is thus short dollars. To have \$1,500,000 secured at the forward contract expiration, Royal Tech would need to go long a forward contract on the dollar. With the forward rate equal to £0.60, the contract will need a notional principal of £900,000. So Royal Tech must set aside funds so that it will have £900,000 available when the forward contract expires. When it delivers the £900,000, it will receive  $\text{£900,000}(1/\text{£0.60}) = \$1,500,000$ , where  $1/\text{£0.60} \approx \$1.67$  is the dollar-per-pound forward rate.

*Solution to B:* At expiration, it will not matter what the spot exchange rate is. Royal Tech will deliver £900,000 and receive \$1,500,000.

### 5.3. Managing the Risk of a Foreign-Market Asset Portfolio

One of the dominant themes in the world of investments in the last 20 years has been the importance of diversifying internationally. The increasing globalization of commerce has created a greater willingness on the part of investors to think beyond domestic borders and add foreign securities to a portfolio.<sup>34</sup> Thus, more asset managers are holding or considering holding foreign stocks and bonds. An important consideration in making such a decision is the foreign currency risk. Should a manager accept this risk, hedge the foreign market risk and the foreign currency risk, or hedge only the foreign currency risk?

It is tempting to believe that the manager should accept the foreign market risk, using it to further diversify the portfolio, and hedge the foreign currency risk. In fact, many asset managers claim to do so. A closer look, however, reveals that it is virtually impossible to actually do this.

Consider a US asset management firm that owns a portfolio currently invested in euro-denominated stock worth  $S_0$ , where  $S_0$  is the current stock price in euros. The exchange rate is  $FX_0$  dollars per euro. Therefore, the portfolio is currently worth  $S_0(FX_0)$  in dollars. At a future time,  $t$ , the portfolio is worth  $S_t$  in euros and the exchange rate is  $FX_t$ . So the portfolio would then be worth  $S_t(FX_t)$ . The firm is long both the stock and the euro.

A forward contract on the euro would require the firm to deliver a certain number of euros and receive the forward rate,  $F$ . The number of euros to be delivered, however, would need to be specified in the contract. In this situation, the firm would end up delivering  $S_t$  euros. This amount is unknown at the time the forward contract is initiated. Thus, it would not be possible to know how many euros the firm would need to deliver.

Some companies manage this problem by estimating an expected future value of the portfolio. They enter into a hedge based on that expectation and adjust the hedge to accommodate

<sup>34</sup>Ironically, the increasing globalization of commerce has increased the correlation among the securities markets of various countries. With this higher correlation, the benefits of international diversification are much smaller.

any changes in expectations. Other companies hedge a minimum portfolio value. They estimate that it is unlikely the portfolio value will fall below a certain level and then sell a forward contract for a size based on this minimum value.<sup>35</sup> This approach leaves the companies hedged for a minimum value, but any increase in the value of the portfolio beyond the minimum would not be hedged. Therefore, any such gains could be wiped out by losses in the value of the currency.

So, with the exception of one special and complex case we discuss below, it is not possible to leave the local equity market return exposed and hedge the currency risk.<sup>36</sup> If the local market return is hedged, then it would be possible to hedge the currency risk. The hedge of the local market return would lock in the amount of the foreign currency that would be converted at the hedge termination date. Of course, the company can hedge the local market return and leave the currency risk unhedged. Or it can hedge neither.<sup>37</sup>

In Exhibit 12, we examine the two possibilities that can be executed: hedging the local market risk and hedging both the local market risk *and* the foreign currency risk. We first use futures on the foreign equity portfolio as though no currency risk existed. This transaction attempts to lock in the future value of the portfolio. This locked-in return should be close to the foreign risk-free rate. If we also choose to hedge the currency risk, we then know that the future value of the portfolio will tell us the number of units of the foreign currency that we shall have available to convert to domestic currency at the hedge termination date. Then we would know the amount of notional principal to use in a forward contract to hedge the exchange rate risk.

#### EXHIBIT 12 Managing the Risk of a Foreign-Currency-Denominated Asset Portfolio

##### Scenario (31 December)

AZ Asset Management is a US firm that invests money for wealthy individual investors. Concerned that it does not know how to manage foreign currency risk, so far AZ has invested only in US markets. Recently, it began learning about managing currency risk and would like to begin investing in foreign markets with a small position worth €10 million. The proposed portfolio has a beta of 1.10. AZ is considering either hedging the European equity market return and leaving the currency risk unhedged, or hedging the currency risk as well as the European equity market return. If it purchases the €10 million portfolio, it will put this hedge in place on 31 December and plans to leave the position open until 31 December of the following year.

For hedging the European equity market risk, it will use a stock index futures contract on a euro-denominated stock index. This contract is priced at €120,000 and has a beta of 0.95. If it hedges the currency risk, it will use a dollar-denominated forward contract on the euro. That contract has a price of \$0.815 and can have any notional principal that the parties agree on at the start. The current spot exchange rate is \$0.80. The foreign risk-free rate is 4%, which is stated as an annually compounded rate. The domestic risk-free rate is 6%.

<sup>35</sup>One way to assure a minimum value would be to use a put option. We shall take up this strategy in the reading on risk management applications of option strategies.

<sup>36</sup>The foreign equity market return is often referred to as the local market return, a term we shall use henceforth.

<sup>37</sup>In fact, some compelling arguments exist for hedging neither. The currency risk can be unrelated to the domestic market risk, thereby offering some further diversifying risk-reduction possibilities.

## EXHIBIT 12 (Continued)

**Action**

*Hedging the equity market risk only:* To eliminate the risk on the portfolio of stock that has a beta of 1.10 would require

$$N_f = \left( \frac{0 - 1.10}{0.95} \right) \left( \frac{10,000,000}{120,000} \right) = -96.49$$

contracts. This amount would be rounded to 96, so AZ would sell 96 contracts.

*Hedging the equity market risk and the currency risk:* Again, AZ would sell 96 stock index futures contracts. It would enter into a forward contract to lock in the exchange rate on a certain amount of euros on 31 December. The question is, how many euros will it have? If the futures contract hedges the stock portfolio, it should earn the foreign risk-free rate. Thus, the portfolio should be worth  $\text{€}10,000,000(1.04) = \text{€}10,400,000$ . So, AZ expects to have  $\text{€}10,400,000$  on the following 31 December and will convert this amount back to dollars. So the notional principal on the forward contract should be  $\text{€}10.4$  million. Note that the starting portfolio value in dollars is  $\text{€}10,000,000(\$/0.80) = \$8,000,000$ .

**Scenario (31 December of the Following Year)**

During the year, the European stock market went down 4.55%. Given the portfolio beta of 1.10, it declines by  $4.55(1.10) = 5\%$ . The portfolio is now worth  $\text{€}10,000,000(1 - 0.05) = \text{€}9,500,000$ . The exchange rate fell to \$0.785, and the futures price fell to €110,600.

**Outcome and Analysis**

*If nothing is hedged:* The portfolio is converted to dollars at \$0.785 and is worth  $\text{€}9,500,000(0.785) = \$7,457,500$ . This amount represents a loss of 6.8% over the initial value of \$8,000,000.

*If only the European stock market is hedged:* The profit on the futures would be  $-96(\text{€}110,600 - \text{€}120,000) = \text{€}902,400$ . Adding this amount to the value of the portfolio gives a value of  $\text{€}9,500,000 + \text{€}902,400 = \text{€}10,402,400$ , which is an increase in value of 4.02%, or approximately the foreign risk-free rate, as it should be. This amount is converted to dollars to obtain  $\text{€}10,402,400(\$/0.785) = \$8,165,884$ , a gain of 2.07%.

*If the European stock market and the currency risk are both hedged:* AZ sold €10.4 million of euros in the forward market at \$0.815. The contract will settle in cash and show a profit of  $\text{€}10,400,000(\$/0.815 - \$/0.785) = \$312,000$ . This leaves the overall portfolio value at  $\text{€}8,165,884 + \$312,000 = \$8,477,884$ , a gain of 5.97%, or approximately the domestic risk-free rate.

In this case, the foreign stock market went down and the foreign currency went down. Without the hedge, the loss was almost 7%. With the foreign stock market hedge, the loss turns into a gain of 2%. With the currency hedge added, the loss becomes a gain of almost 6%. Of course, different outcomes could occur. Gains from a stronger foreign stock market and a stronger currency would be lost if the company had made these same hedges.

Note, however, that once AZ hedges the foreign market return, it can expect to earn only the foreign risk-free rate. If it hedges the foreign market return and the exchange rate, it can expect to earn only its domestic risk-free rate. Therefore, neither strategy makes much sense for the long run. In the short run, however, this strategy can be a good tactic for investors who are already in foreign markets and who wish to temporarily take a more defensive position without liquidating the portfolio and converting it to cash.

### EXAMPLE 9

FCA Managers (FCAM) is a US asset management firm. Among its asset classes is a portfolio of Swiss stocks worth SF10 million, which has a beta of 1.00. The spot exchange rate is \$0.75, the Swiss interest rate is 5%, and the US interest rate is 6%. Both of these interest rates are compounded in the Libor manner: Rate  $\times$  (Days/360). These rates are consistent with a six-month forward rate of \$0.7537. FCAM is considering hedging the local market return on the portfolio and possibly hedging the exchange rate risk for a six-month period. A futures contract on the Swiss market is priced at SF300,000 and has a beta of 0.90.

- What futures position should FCAM take to hedge the Swiss market return? What return could it expect?
- Assuming that it hedges the Swiss market return, how could it hedge the exchange rate risk as well, and what return could it expect?

*Solution to A:* To hedge the Swiss local market return, the number of futures contracts is

$$N_f = \left( \frac{0 - 1.00}{0.90} \right) \left( \frac{\text{SF}10,000,000}{\text{SF}300,000} \right) = -37.04$$

So FCAM should sell 37 contracts. Because the portfolio is perfectly hedged, its return should be the Swiss risk-free rate of 5%.

*Solution to B:* If hedged, the Swiss portfolio should grow to a value of  $\text{SF}10,000,000[1 + 0.05(180/360)] = \text{SF}10,250,000$ .

FCAM could hedge this amount with a forward contract with this much notional principal. If the portfolio is hedged, it will convert to a value of  $\text{SF}10,250,000(\$0.7537) = \$7,725,425$ .

In dollars, the portfolio was originally worth  $\text{SF}10,000,000(\$0.75) = 7,500,000$ . Thus, the return is  $\frac{\$7,725,425}{\$7,500,000} - 1 \approx 0.03$ , which is the US risk-free rate for six months.

We see that if only the foreign stock market return is hedged, the portfolio return is the foreign risk-free rate before converting to the domestic currency. If both the foreign stock market and the exchange rate risk are hedged, the return equals the domestic risk-free rate.

As a temporary and tactical strategy, hedging one or both risks can make sense. There are certainly periods when one might be particularly concerned about these risks and might wish to eliminate them. Executing this sort of strategy can be much easier than selling all of the foreign stocks and possibly converting the proceeds into domestic currency. But in the long run, a strategy of investing in foreign markets, hedging that risk, and hedging the exchange rate risk hardly makes much sense.

## 6. FUTURES OR FORWARDS?

As we have seen, numerous opportunities and strategies exist for managing risk using futures and forwards. We have largely ignored the issue of which instrument—futures or forwards—is better. Some types of hedges are almost always executed using futures, and some are almost always executed using forwards. Why the preference for one over the other? First, let us recall the primary differences between the two:

- Futures contracts are standardized, with all terms except for the price set by the futures exchange. Forward contracts are customized. The two parties set the terms according to their needs.
- Futures contracts are guaranteed by the clearinghouse against default. Forward contracts subject each party to the possibility of default by the other party.
- Futures contracts require margin deposits and the daily settlement of gains and losses. Forward contracts pay off the full value of the contract at expiration. Some participants in forward contracts agree prior to expiration to use margin deposits and occasional settlements to reduce the default risk.
- Futures contracts are regulated by federal authorities. Forward contracts are essentially unregulated.
- Futures contracts are conducted in a public arena, the futures exchange, and are reported to the exchanges and the regulatory authority. Forward contracts are conducted privately, and individual transactions are not generally reported to the public or regulators.

Risks that are associated with very specific dates, such as when interest rates are reset on a loan, usually require forward contracts. Thus, we used an FRA to lock in the rate on a loan. That rate is set on a specific day. A futures contract has specific expirations that may not correspond to the day on which the rate is reset. Although it is possible to use sophisticated models and software to compensate for this problem, typical borrowers do not usually possess the expertise to do so. It is much easier for them to use an FRA.

Oddly enough, however, the risk of most bond portfolios is managed using Treasury bond futures. Those portfolios have horizon dates for which the company is attempting to lock in values. But usually rates are not being reset on that date, and the hedge does not need to be perfect. Often there is flexibility with respect to the horizon date. Treasury bond futures work reasonably well for these investors. Likewise, the risk of equity portfolios tends to be managed with stock index futures. Even though the offsetting of risks is not precise, that is not a necessity. Equity and debt portfolio managers usually need only satisfactory protection against market declines. Nonetheless, in some cases equity and debt portfolio managers use over-the-counter instruments such as forward contracts. In fact, sometimes a portfolio manager will ask a derivatives dealer to write a forward contract on a specific portfolio. This approach is more costly than using futures and provides a better hedge, but, as noted, a perfect hedge is usually not needed. In practice, portfolio managers have traded off the costs of customized hedges with the costs of using standardized futures contracts and have found the latter to be preferable.<sup>38</sup>

Forward contracts are the preferred vehicle for the risk management of foreign currency. This preference partly reflects the deep liquidity in the forward market, which has been around

<sup>38</sup>Portfolio managers do use swaps on occasion, as we cover in the reading on risk management applications of swap strategies.

longer than the futures market. Moreover, much of this trading is undertaken by corporations managing the risk of either the issuance of a bond or the inflows and outcomes of specific currency transactions, in which case the precision provided by customized transactions is preferred.

Nevertheless, one might wonder why certain contracts do not die out. Recall that most corporations do not use the Eurodollar futures market to hedge their floating-rate loans. Yet the Eurodollar futures contract is one of the most active of all futures contracts in the world. Where does this volume come from? It is from the dealers in swaps, options, and FRAs. When they enter into transactions with end users in which the underlying rate is Libor, they must manage the risk they have assumed, and they must do so very quickly. They cannot afford to leave their positions exposed for long. It is rarely possible for them to simply pick up the phone and find another customer to take the opposite side of the transaction. A corporate client with the exact opposite needs at the exact same time as some other end user would be rare. Therefore, dealers need to execute offsetting transactions very quickly. There is no better place to do this than in the Eurodollar futures markets, with its extremely deep liquidity. These dealer companies have sophisticated analysts and software and are able to manage the risk caused by such problems as the futures contract expiring on one day and the payoffs on the FRAs being set on another day. Thus, these risks can be more effectively measured and managed by dealers than by end users.

As we have emphasized, futures contracts require margin deposits and the daily settling of gains and losses. This process causes some administrative problems because money must be deposited into a futures account and cash flows must be managed on a daily basis. When futures brokers call for more money to cover losses, companies using futures contracts must send cash or very liquid securities. Although the brokers may be generating value on the other side of a hedge transaction, that value may not produce actual cash.<sup>39</sup> On the other hand, forward transactions, while not necessarily requiring margin deposits, generate concerns over whether the counterparty will be able to pay at expiration. Of course, those concerns lead some counterparties to require margins and periodic settlements.

Although forward contracts are essentially unregulated while futures contracts are heavily regulated, this factor is not usually a major consideration in deciding which type of contract to use. In some cases, however, regulation prevents use of a specific contract. For example, a country might prohibit foreign futures exchanges from offering their products in its markets. There would probably be no such prohibition on forward contracts.<sup>40</sup> In some cases, regulation prevents or delays usage of certain futures products, making it possible for innovative companies that can create forward products to offer them ahead of the comparable products of futures exchanges. The futures exchanges claim this is unfair by making it more difficult for them to compete with forward markets in providing risk management products.

We also noted that futures contracts are public transactions, whereas forward contracts are private transactions. This privacy characteristic can cause a company to prefer a forward transaction if it does not want others, such as traders on the futures exchange, to know its views.

<sup>39</sup> Consider, for example, a company that sells futures contracts to hedge the value of a bond portfolio. Suppose interest rates fall, the bond portfolio rises, and the futures price also rises. Losses will be incurred on the futures contracts and additional margin deposits will be required. The bond portfolio has increased in value, but it may not be practical to liquidate the portfolio to generate the necessary cash.

<sup>40</sup> In some less developed but highly regulated countries, private financial transactions such as forward contracts can be prohibited.

## 7. FINAL COMMENTS

A few points are worth repeating. Because they can be somewhat unstable, betas and durations are difficult to measure, even under the best of circumstances. Even when no derivatives transactions are undertaken, the values believed to be the betas and durations may not truly turn out to reflect the sensitivities of stocks and bonds to the underlying sources of risk. Therefore, if derivatives transactions do not work out to provide the exact hedging results expected, users should not necessarily blame derivatives. If, however, speculative long (short) positions are added to an otherwise long position, risk should increase (decrease), although the exact amount of the increase (decrease) cannot be known for sure in advance.<sup>41</sup> Derivatives should not be maligned for their speculative use when there are valuable hedging uses.

We have mentioned that transaction costs are a major consideration in the use of derivatives, and this is clearly the case with futures and forwards. By some reports, transaction costs for stock index derivatives are approximately 95% lower than for stock indices.<sup>42</sup> Indeed, one of the major reasons that derivatives exist is that they provide a means of trading at lower transaction costs. To survive as risk management products, derivatives need to be much less expensive than the value of the underlying instruments. There are almost no situations in which transacting in the underlying securities would be preferable to using derivatives on a transaction-cost basis, when taking a position for a specified short horizon.

Transacting in futures and forwards also has a major advantage of being less disruptive to the portfolio and its managers. For example, the asset classes of many portfolios are managed by different persons or firms. If the manager of the overall portfolio wants to change the risk of certain asset classes or alter the allocations between asset classes, he can do so using derivatives. Instead of telling one manager that she must sell securities and another manager that he must buy securities, the portfolio manager can use derivatives to reduce the allocation to one class and increase the allocation to the other. The asset class managers need not even know that the overall asset allocation has been changed. They can concentrate on doing the best they can within their respective areas of responsibility.

In the matter of liquidity, however, futures and forwards do not always offer the advantages often attributed to them. They require less capital to trade than the underlying securities, but they are not immune to liquidity problems. Nowhere is this concern more evident than in using a futures contract that expires a long time from the present. The greatest liquidity in the futures markets is in the shortest expirations. Although there may be futures contracts available with long-term expirations, their liquidity is much lower. Many forward markets are very liquid, but others may not be. High liquidity should not automatically be assumed of all derivatives, although in general, derivatives are more liquid than the underlying securities.

Many organizations are not permitted to use futures or forwards. Futures and forwards are fully leveraged positions, because they essentially require no equity. Some companies might

<sup>41</sup>We use the expression “should increase (decrease)” to reflect the fact that some other factors could cause perverse results. We previously mentioned call features and credit risk of such assets as corporate and municipal bonds that could result in a bond price not moving in the same direction as a move in the general level of bond prices. A poorly diversified stock portfolio could move opposite to the market, thereby suggesting a negative beta that is really only diversifiable risk. We assume that these situations are rare or that their likelihood and consequences have been properly assessed by risk managers.

<sup>42</sup>These statements are based on trading all individual stocks that make up an index. Trading through exchange-traded funds would reduce some of these stock trading costs.

have a policy against fully leveraged positions but might permit options, which are not fully leveraged. Loss potential is much greater on purchased or sold futures or forwards, whereas losses are capped on purchased options. Some organizations, however, permit futures and forwards but prohibit options. Other organizations might prohibit credit-risky instruments, such as forwards and over-the-counter options, but permit credit-risk-free instruments, such as futures and exchange-listed options. These restrictions, although sometimes misguided, are realistic constraints that must be considered when deciding how to manage risk.

In conclusion, futures and forward contracts are both alike and different. On some occasions, one is preferred over the other. Both types of contracts have their niches. The most important point they have in common is that they have zero value at the start and offer linear payoffs, meaning that no one “invests” any money in either type of contract at the start, but the cost is paid for by the willingness to give up gains and incur losses resulting from movements in the underlying. Options, which require a cash investment at the start, allow a party to capture favorable movements in the underlying while avoiding unfavorable movements. This type of payoff is nonlinear. In some cases, options will be preferred to other types of derivatives.

## 8. SUMMARY

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- A borrower can lock in the rate that will be set at a future date on a single-payment loan by entering into a long position in an FRA. The FRA obligates the borrower to make a fixed interest payment and receive a floating interest payment, thereby protecting the borrower if the loan rate is higher than the fixed rate in the FRA but also eliminating gains if the loan rate is lower than the fixed rate in the FRA.
- The duration of a bond futures contract is determined as the duration of the bond underlying the futures contract as of the futures expiration, based on the yield of the bond underlying the futures contract. The modified duration is obtained by dividing the duration by 1 plus the yield. The duration of a futures contract is implied by these factors and is called the implied (modified) duration.
- The implied yield of a futures contract is the yield implied by the futures price on the bond underlying the futures contract as of the futures expiration.
- The yield beta is the sensitivity of the yield on a bond portfolio relative to the implied yield on the futures contract.
- The number of bond futures contracts required to change the duration of a bond portfolio is based on the ratio of the market value of the bonds to the futures price multiplied by the difference between the target or desired modified duration and the actual modified duration, divided by the implied modified duration of the futures.
- The actual adjusted duration of a bond portfolio may not equal the desired duration for a number of reasons, including that the yield beta may be inaccurate or unstable or the bonds could contain call features or default risk. In addition, duration is a measure of instantaneous risk and may not accurately capture the risk over a long horizon without frequent portfolio adjustments.
- The number of equity futures contracts required to change the beta of an equity portfolio is based on the ratio of the market value of the stock to the futures price times the difference between the target or desired beta and the actual beta, divided by the beta of the futures.
- A long position in stock is equivalent to a long position in futures and a long position in a risk-free bond; therefore, it is possible to synthetically create a long position in stock by

buying futures on stock and a risk-free bond. This process is called equitizing cash and can be used to create a synthetic stock index fund.

- A long position in cash is equivalent to a long position in stock and a short position in stock futures. Therefore, it is possible to synthetically create a long position in cash by buying stock and selling futures.
- The allocation of a portfolio between equity and debt can be adjusted using stock index and bond futures. Buy futures to increase the allocation to an asset class, and sell futures to decrease the allocation to an asset class.
- The allocation of a bond portfolio between cash and high-duration bonds can be adjusted by using bond futures. Sell futures to increase the allocation to cash, and buy futures to increase the allocation to long-term bonds.
- The allocation of an equity portfolio among different equity sectors can be adjusted by using stock index futures. Sell futures on an index representing one sector to decrease the allocation to that sector, and buy futures on an index representing another sector to increase the allocation to that sector.
- A portfolio manager can buy bond or stock index futures to take a position in an asset class without having cash to actually invest in the asset class. This type of strategy is sometimes used in anticipation of the receipt of a sum of cash at a later date, which will then be invested in the asset class and the futures position will be closed.
- Transaction exposure is the risk associated with a foreign exchange rate on a specific business transaction such as a purchase or sale. Translation exposure is the risk associated with the conversion of foreign financial statements into domestic currency. Economic exposure is the risk associated with changes in the relative attractiveness of products and services offered for sale, arising out of the competitive effects of changes in exchange rates.
- The risk of a future foreign currency receipt can be eliminated by selling a forward contract on the currency. This transaction locks in the rate at which the foreign currency will be converted to the domestic currency.
- The risk of a future foreign currency payment can be eliminated by buying a forward contract on the currency. This transaction locks in the rate at which the domestic currency will be converted to the foreign currency.
- It is not possible to invest in a foreign equity market and precisely hedge the currency risk only. To hedge the currency risk, one must know the exact amount of foreign currency that will be available at a future date. Without locking in the equity return, it is not possible to know how much foreign currency will be available.
- It is possible to hedge the foreign equity market return and accept the exchange rate risk or hedge the foreign equity market return *and* hedge the exchange rate risk. By hedging the equity market return, one would know the proper amount of currency that would be available at a later date and could use a futures or forward contract to hedge the currency risk. The equity return, however, would equal the risk-free rate.
- Forward contracts are usually preferred over futures contracts when the risk is related to an event on a specific date, such as an interest rate reset. Forward contracts on foreign currency are usually preferred over futures contracts, primarily because of the liquidity of the market. Futures contracts require margins and daily settlements but are guaranteed against credit losses and may be preferred when credit concerns are an issue. Either contract may be preferred or required if there are restrictions on the use of the other. Dealers use both instruments in managing their risk, occasionally preferring one instrument and sometimes preferring the other. Forward contracts are preferred if privacy is important.

- Futures and forwards, as well as virtually all derivatives, have an advantage over transactions in the actual instruments by virtue of their significantly lower transaction costs. They also allow a portfolio manager to make changes in the risk of certain asset classes or the allocation among asset classes without disturbing the asset class or classes themselves. This feature allows the asset class managers to concentrate on their respective asset classes without being concerned about buying and selling to execute risk-altering changes or asset allocation changes.
- Although futures and forwards tend to be more liquid than their underlying assets, they are not always highly liquid. Therefore, it cannot always be assumed that futures and forwards can solve liquidity problems.

## PROBLEMS

1. An investment management firm wishes to increase the beta for one of its portfolios under management from 0.95 to 1.20 for a three-month period. The portfolio has a market value of \$175,000,000. The investment firm plans to use a futures contract priced at \$105,790 in order to adjust the portfolio beta. The futures contract has a beta of 0.98.
  - A. Calculate the number of futures contracts that should be bought or sold to achieve an increase in the portfolio beta.
  - B. At the end of three months, the overall equity market is up 5.5%. The stock portfolio under management is up 5.1%. The futures contract is priced at \$111,500. Calculate the value of the overall position and the effective beta of the portfolio.
2. Consider an asset manager who wishes to create a fund with exposure to the Russell 2000 stock index. The initial amount to be invested is \$300,000,000. The fund will be constructed using the Russell 2000 Index futures contract, priced at 498.30 with a \$500 multiplier. The contract expires in three months. The underlying index has a dividend yield of 0.75%, and the risk-free rate is 2.35% per year.
  - A. Indicate how the money manager would go about constructing this synthetic index using futures.
  - B. Assume that at expiration, the Russell 2000 is at 594.65. Show how the synthetic position produces the same result as investment in the actual stock index.
3. An investment management firm has a client who would like to temporarily reduce his exposure to equities by converting a \$25 million equity position to cash for a period of four months. The client would like this reduction to take place without liquidating his equity position. The investment management firm plans to create a synthetic cash position using an equity futures contract. This futures contract is priced at 1170.10, has a multiplier of \$250, and expires in four months. The dividend yield on the underlying index is 1.25%, and the risk-free rate is 2.75%.
  - A. Calculate the number of futures contracts required to create synthetic cash.
  - B. Determine the effective amount of money committed to this risk-free transaction and the effective number of units of the stock index that are converted to cash.
  - C. Assume that the stock index is at 1031 when the futures contract expires. Show how this strategy is equivalent to investing the risk-free asset, cash.

4. Consider a portfolio with a 65% allocation to stocks and 35% to bonds. The portfolio has a market value of \$200 million. The beta of the stock position is 1.15, and the modified duration of the bond position is 6.75. The portfolio manager wishes to increase the stock allocation to 85% and reduce the bond allocation to 15% for a period of six months. In addition to altering asset allocations, the manager would also like to increase the beta on the stock position to 1.20 and increase the modified duration of the bonds to 8.25. A stock index futures contract that expires in six months is priced at \$157,500 and has a beta of 0.95. A bond futures contract that expires in six months is priced at \$109,000 and has an implied modified duration of 5.25. The stock futures contract has a multiplier of one.
  - A. Show how the portfolio manager can achieve his goals by using stock index and bond futures. Indicate the number of contracts and whether the manager should go long or short.
  - B. After six months, the stock portfolio is up 5% and bonds are up 1.35%. The stock futures price is \$164,005 and the bond futures price is \$110,145. Compare the market value of the portfolio in which the allocation is adjusted using futures to the market value of the portfolio in which the allocation is adjusted by directly trading stocks and bonds.
5. A pension fund manager expects to receive a cash inflow of \$50,000,000 in three months and wants to use futures contracts to take a \$17,500,000 synthetic position in stocks and \$32,500,000 in bonds today. The stock would have a beta of 1.15 and the bonds a modified duration of 7.65. A stock index futures contract with a beta of 0.93 is priced at \$175,210. A bond futures contract with a modified duration of 5.65 is priced at \$95,750.
  - A. Calculate the number of stock and bond futures contracts the fund manager would have to trade in order to synthetically take the desired position in stocks and bonds today. Indicate whether the futures positions are long or short.
  - B. When the futures contracts expire in three months, stocks have declined by 5.4% and bonds have declined by 3.06%. Stock index futures are priced at \$167,559, and bond futures are priced at \$93,586. Show that profits on the futures positions are essentially the same as the change in the value of stocks and bonds during the three-month period.
6. A. Consider a US company, GateCorp, that exports products to the United Kingdom. GateCorp has just closed a sale worth £200,000,000. The amount will be received in two months. Because it will be paid in pounds, the US company bears the exchange risk. In order to hedge this risk, GateCorp intends to use a forward contract that is priced at \$1.4272 per pound. Indicate how the company would go about constructing the hedge. Explain what happens when the forward contract expires in two months.  
B. ABCorp is a US-based company that frequently imports raw materials from Australia. It has just entered into a contract to purchase A\$175,000,000 worth of raw wool, to be paid in one month. ABCorp fears that the Australian dollar will strengthen, thereby raising the US dollar cost. A forward contract is available and is priced at \$0.5249 per Australian dollar. Indicate how ABCorp would go about constructing a hedge. Explain what happens when the forward contract expires in one month.



# CHAPTER 8

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## RISK MANAGEMENT APPLICATIONS OF OPTION STRATEGIES

Don M. Chance, PhD, CFA

### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- compare the use of covered calls and protective puts to manage risk exposure to individual securities;
- calculate and interpret the value at expiration, profit, maximum profit, maximum loss, breakeven underlying price at expiration, and general shape of the graph for the following option strategies: bull spread, bear spread, butterfly spread, collar, straddle, box spread;
- calculate the effective annual rate for a given interest rate outcome when a borrower (lender) manages the risk of an anticipated loan using an interest rate call (put) option;
- calculate the payoffs for a series of interest rate outcomes when a floating rate loan is combined with 1) an interest rate cap, 2) an interest rate floor, or 3) an interest rate collar;
- explain why and how a dealer delta hedges an option position, why delta changes, and how the dealer adjusts to maintain the delta hedge;
- interpret the gamma of a delta-hedged portfolio and explain how gamma changes as in-the-money and out-of-the-money options move toward expiration.

## 1. INTRODUCTION

In the chapter on risk management applications of forward and futures strategies, we examined strategies that employ forward and futures contracts. Recall that forward and futures contracts have linear payoffs and do not require an initial outlay. Options, on the other hand, have non-linear payoffs and require the payment of cash up front. By having nonlinear payoffs, options permit their users to benefit from movements in the underlying in one direction and to not be harmed by movements in the other direction. In many respects, they offer the best of all worlds, a chance to profit if expectations are realized with minimal harm if expectations turn out to be wrong. The price for this opportunity is the cash outlay required to establish the position. From the standpoint of the holder of the short position, options can lead to extremely large losses. Hence, sellers of options must be well compensated in the form of an adequate up-front premium and must skillfully manage the risk they assume.

In this chapter we examine the most widely used option strategies. The chapter is divided into three parts. In the first part, we look at option strategies that are typically used in equity investing, which include standard strategies involving single options and strategies that combine options with the underlying. In the second part, we look at the specific strategies that are commonly used in managing interest rate risk. In the third part, we examine option strategies that are used primarily by dealers and sophisticated traders to manage the risk of option positions.

Let us begin by reviewing the necessary notation. First recall that time 0 is the time at which the strategy is initiated and time T is the time the option expires, stated as a fraction of a year. Accordingly, the amount of time until expiration is simply  $T - 0 = T$ , which is  $(\text{Days to expiration})/365$ . The other symbols are

- $c_0, c_T$  = price of the call option at time 0 and time T
- $p_0, p_T$  = price of the put option at time 0 and time T<sup>1</sup>
- X = exercise price
- $S_0, S_T$  = price of the underlying at time 0 and time T
- $V_0, V_T$  = value of the position at time 0 and time T
- $\Pi$  = profit from the transaction:  $V_T - V_0$
- r = risk-free rate

Some additional notation will be introduced when necessary.

Note that we are going to measure the profit from an option transaction, which is simply the final value of the transaction minus the initial value of the transaction. Profit does not take into account the time value of money or the risk. Although a focus on profit is not completely satisfactory from a theoretical point of view, it is nonetheless instructive, simple, and a common approach to examining options. Our primary objective here is to obtain a general picture of the manner in which option strategies perform. With that in mind, discussing profit offers probably the best trade-off in terms of gaining the necessary knowledge with a minimum of complexity.

In this chapter, we assume that the option user has a view regarding potential movements of the underlying. In most cases that view is a prediction of the direction of the underlying, but in some cases it is a prediction of the volatility of the underlying. In all cases, we assume this view is specified over a horizon that corresponds to the option's life or that the option expiration can

<sup>1</sup>Lower case indicates European options, and upper case indicates American options. In this chapter, all options are European.

be tailored to the horizon date. Hence, for the most part, these options should be considered customized, over-the-counter options.<sup>2</sup> Every interest rate option is a customized option.

Because the option expiration corresponds to the horizon date for which a particular view is held, there is no reason to use American options. Accordingly, all options in this chapter are European options. Moreover, we shall not consider terminating the strategy early. Putting an option in place and closing the position prior to expiration is certainly a legitimate strategy. It could reflect the arrival of new information over the holding period, but it requires an understanding of more complex issues, such as valuation of the option and the rate at which the option loses its time value. Thus, we shall examine the outcome of a particular strategy over a range of possible values of the underlying only on the expiration day.

Section 2 of this chapter focuses on option strategies that relate to equity investments. Section 3 concentrates on strategies using interest rate options. In Section 4, we focus on managing an option portfolio.

## 2. OPTION STRATEGIES FOR EQUITY PORTFOLIOS

Many typical illustrations of option strategies use individual stocks, but we shall use options on a stock index, the NASDAQ 100, referred to simply as the NASDAQ. We shall assume that in addition to buying and selling options on the NASDAQ, we can also buy the index, either through construction of the portfolio itself, through an index mutual fund, or an exchange-traded fund.<sup>3</sup> We shall simply refer to this instrument as a stock. We are given the following numerical data:

$$S_0 = 2000, \text{ value of the NASDAQ 100 when the strategy is initiated}$$

$$T = 0.0833, \text{ the time to expiration (one month = 1/12)}$$

The options available will be the following:<sup>4</sup>

Exercise Price	Call Price	Put Price
1950	108.43	56.01
2000	81.75	79.25
2050	59.98	107.39

Let us start by examining an initial strategy that is the simplest of all: to buy or sell short the underlying. Panel A of Exhibit 1 illustrates the profit from the transaction of buying a share of stock. We see the obvious result that if you buy the stock and it goes up, you make a profit; if it goes down, you incur a loss. Panel B shows the case of selling short the stock. Recall that this strategy involves borrowing the shares from a broker, selling them at the current price, and then buying them back at a later date. In this case, if you sell short the stock and it goes down, you make a profit. Conversely, if it goes up, you incur a loss. Now we shall move on to strategies involving options, but we shall use the stock strategies again when we combine options with stock.

<sup>2</sup>If the options discussed were exchange-listed options, it would not significantly alter the material in this chapter.

<sup>3</sup>Exchange-traded shares on the NASDAQ 100 are called NASDAQ 100 Trust Shares and QQQs, for their ticker symbol. They are commonly referred to as Qubes, trade on the AMEX, and are the most active exchange-traded fund and often the most actively traded of all securities. Options on the NASDAQ 100 are among the most actively traded as well.

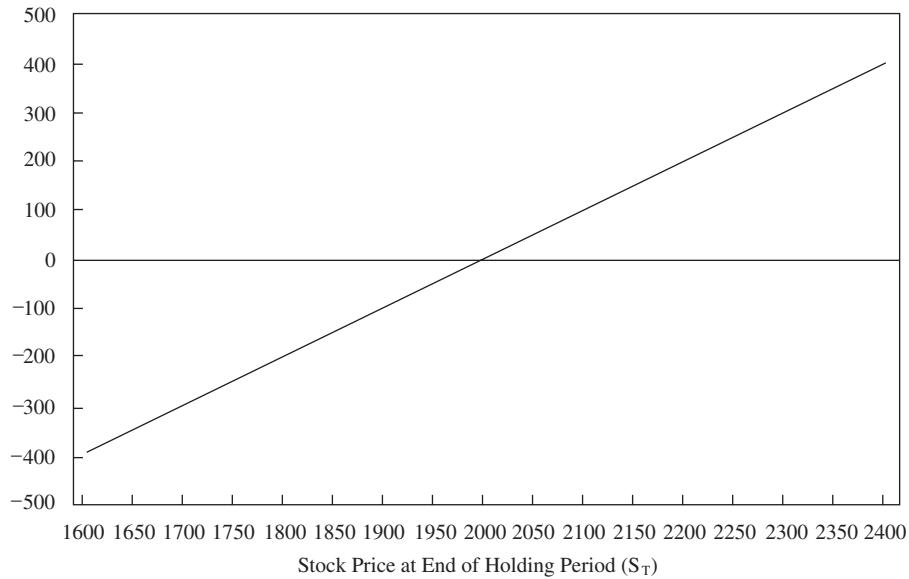
<sup>4</sup>These values were obtained using the Black–Scholes–Merton model. By using this model, we know we are working with reasonable values that do not permit arbitrage opportunities.

EXHIBIT 1 Simple Stock Strategies

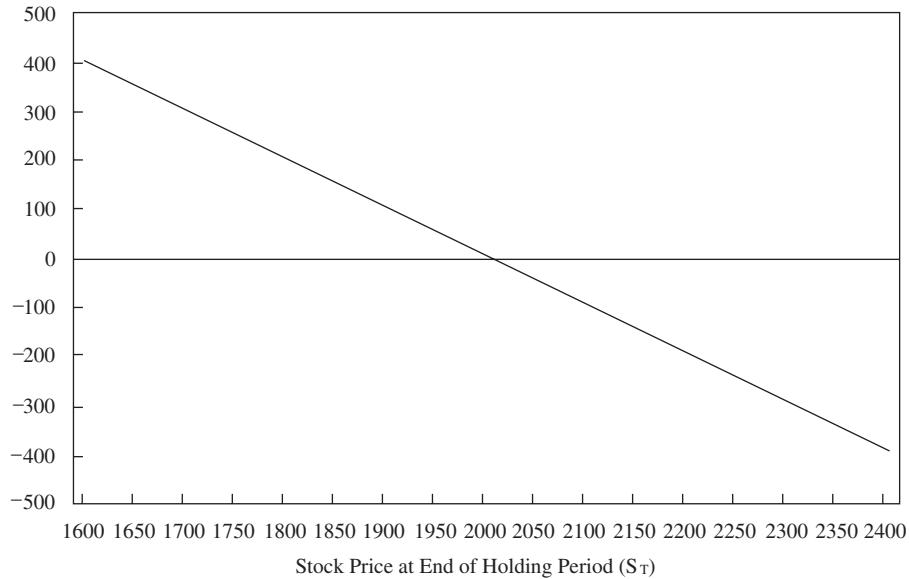
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**A. Buy Stock**

Profit from Transaction (\$)

**B. Sell Short Stock**

Profit from Transaction (\$)



In this section we examine option strategies in the context of their use in equity portfolios. Although these strategies are perfectly applicable for fixed-income portfolios, corporate borrowing scenarios, or even commodity risk management situations, they are generally more easily explained and understood in the context of investing in equities or equity indices.

To analyze an equity option strategy, we first assume that we establish the position at the current price. We then determine the value of the option at expiration for a specific value of the index at expiration. We calculate the profit as the value at expiration minus the current price. We then generate a graph to illustrate the value at expiration and profit for a range of index values at expiration. Although the underlying is a stock index, we shall just refer to it as the underlying to keep things as general as possible. We begin by examining the most fundamental option transactions, long and short positions in calls and puts.

## 2.1. Standard Long and Short Positions

### 2.1.1. Calls

Consider the purchase of a call option at the price  $c_0$ . The value at expiration,  $c_T$ , is  $c_T = \max(0, S_T - X)$ . Broken down into parts,

$$\begin{aligned} c_T &= 0 && \text{if } S_T \leq X \\ c_T &= S_T - X && \text{if } S_T > X \end{aligned}$$

The profit is obtained by subtracting the option premium, which is paid to purchase the option, from the option value at expiration,  $\Pi = c_T - c_0$ . Broken down into parts,

$$\begin{aligned} \Pi &= -c_0 && \text{if } S_T \leq X \\ \Pi &= S_T - X - c_0 && \text{if } S_T > X \end{aligned}$$

Now consider this example. We buy the call with the exercise price of 2000 for 81.75. Consider values of the index at expiration of 1900 and 2100.

For  $S_T = 1900$ ,

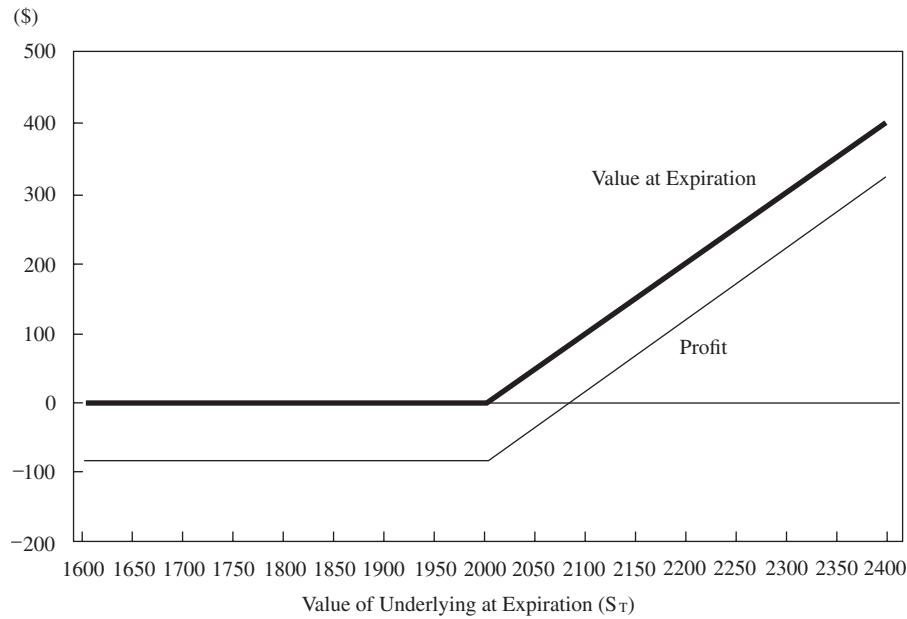
$$\begin{aligned} c_T &= \max(0, 1900 - 2000) = 0 \\ \Pi &= 0 - 81.75 = -81.75 \end{aligned}$$

For  $S_T = 2100$ ,

$$\begin{aligned} c_T &= \max(0, 2100 - 2000) = 100 \\ \Pi &= 100 - 81.75 = 18.25 \end{aligned}$$

Exhibit 2 illustrates the value at expiration and profit when  $S_T$ , the underlying price at expiration, ranges from 1600 to 2400. We see that buying a call results in a limited loss of the premium, 81.75. For an index value at expiration greater than the exercise price of 2000, the value and profit move up one-for-one with the index value, and there is no upper limit.

## EXHIBIT 2 Buy Call



It is important to identify the breakeven index value at expiration. Recall that the formula for the profit is  $\Pi = \max(0, S_T - X) - c_0$ . We would like to know the value of  $S_T$  for which  $\Pi = 0$ . We shall call that value  $S_T^*$ . It would be nice to be able to solve  $\Pi = \max(0, S_T^* - X) - c_0 = 0$  for  $S_T^*$ , but that is not directly possible. Instead, we observe that there are two ranges of outcomes, one in which  $\Pi = S_T^* - X - c_0$  for  $S_T^* > X$ , the case of the option expiring in-the-money, and the other in which  $\Pi = -c_0$  for  $S_T \leq X$ , the case of the option expiring out-of-the-money. It is obvious from the equation and by observing Exhibit 2 that in the latter case, there is no possibility of breaking even. In the former case, we see that we can solve for  $S_T^*$ . Setting  $\Pi = S_T^* - X - c_0 = 0$ , we obtain  $S_T^* = X + c_0$ .

Thus, the breakeven is the exercise price plus the option premium. This result should be intuitive: The value of the underlying at expiration must exceed the exercise price by the amount of the premium to recover the cost of the premium. In this problem, the breakeven is  $S_T^* = 2000 + 81.75 = 2081.75$ . Observe in Exhibit 2 that the profit line crosses the axis at this value.

In summarizing the strategy, we have the following results for the option buyer:

$$\begin{aligned}
 c_T &= \max(0, S_T - X) \\
 \text{Value at expiration} &= c_T \\
 \text{Profit: } \Pi &= c_T - c_0 \\
 \text{Maximum profit} &= \infty \\
 \text{Maximum loss} &= c_0 \\
 \text{Breakeven: } S_T^* &= X + c_0
 \end{aligned}$$

Call options entice naive speculators, but it is important to consider the *likely* gains and losses more than the *potential* gains and losses. For example, in this case, the underlying

must go up by about 4.1% in one month to cover the cost of the call. This increase equates to an annual rate of almost 50% and is an unreasonable expectation by almost any standard. If the underlying does not move at all, the loss is 100% of the premium.

For the seller of the call, the results are just the opposite. The sum of the positions of the seller and buyer is zero. Hence, we can take the value and profit results for the buyer and change the signs. The results for the maximum profit and maximum loss are changed accordingly, and the breakeven is the same. Hence, for the option seller,

$$c_T = \max(0, S_T - X)$$

$$\text{Value at expiration} = -c_T$$

$$\text{Profit: } \Pi = -c_T + c_0$$

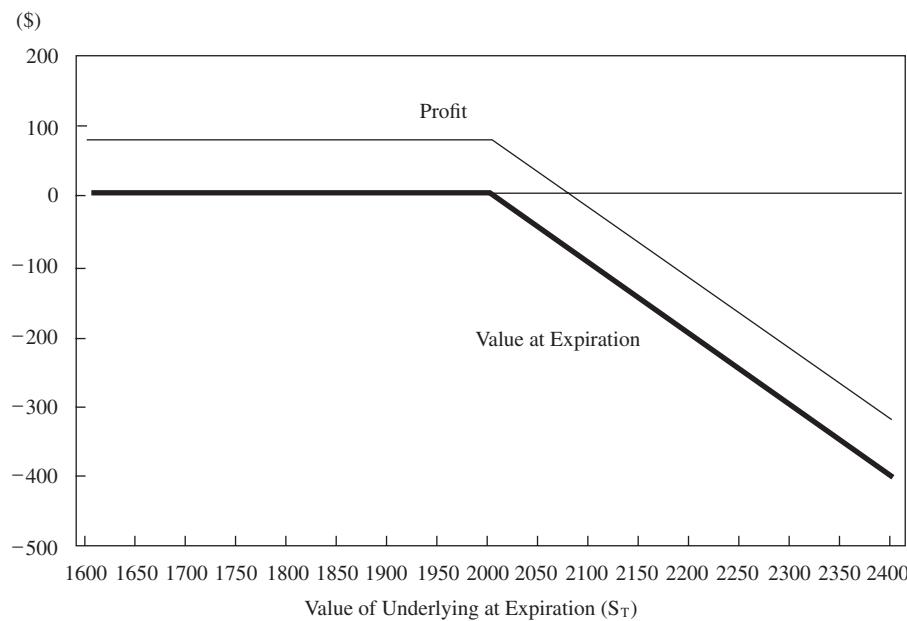
$$\text{Maximum profit} = c_0$$

$$\text{Maximum loss} = \infty$$

$$\text{Breakeven: } S_T^* = X + c_0$$

Exhibit 3 shows the results for the seller of the call. Note that the value and profit have a fixed maximum. The worst case is an infinite loss. Just as there is no upper limit to the buyer's potential gain, there is no upper limit to how much the seller can lose.

### EXHIBIT 3 Sell Call



Call options are purchased by investors who are bullish. We now turn to put options, which are purchased by investors who are bearish.

### EXAMPLE 1

Consider a call option selling for \$7 in which the exercise price is \$100 and the price of the underlying is \$98.

- A. Determine the value at expiration and the profit for a buyer under the following outcomes:
  - i. The price of the underlying at expiration is \$102.
  - ii. The price of the underlying at expiration is \$94.
- B. Determine the value at expiration and the profit for a seller under the following outcomes:
  - i. The price of the underlying at expiration is \$91.
  - ii. The price of the underlying at expiration is \$101.
- C. Determine the following:
  - i. the maximum profit to the buyer (maximum loss to the seller).
  - ii. the maximum loss to the buyer (maximum profit to the seller).
- D. Determine the breakeven price of the underlying at expiration.

*Solutions:*

A. Call buyer

$$\begin{aligned}
 \text{i. Value at expiration} &= c_T = \max(0, S_T - X) \\
 &= \max(0, 102 - 100) = 2 \\
 \Pi &= c_T - c_0 = 2 - 7 = -5
 \end{aligned}$$

$$\begin{aligned}
 \text{ii. Value at expiration} &= c_T = \max(0, S_T - X) \\
 &= \max(0, 94 - 100) = 0 \\
 \Pi &= c_T - c_0 = 0 - 7 = -7
 \end{aligned}$$

B. Call seller

$$\begin{aligned}
 \text{i. Value at expiration} &= -c_T = -\max(0, S_T - X) \\
 &= -\max(0, 91 - 100) = 0 \\
 \Pi &= -c_T + c_0 = -0 + 7 = 7
 \end{aligned}$$

$$\begin{aligned}
 \text{ii. Value at expiration} &= -c_T = -\max(0, S_T - X) \\
 &= -\max(0, 101 - 100) = -1 \\
 \Pi &= -c_T + c_0 = -1 + 7 = 6
 \end{aligned}$$

C. Maximum and minimum

- i. Maximum profit to buyer (loss to seller) =  $\infty$
- ii. Maximum loss to buyer (profit to seller) =  $c_0 = 7$

$$\text{D. } S_T^* = X + c_0 = 100 + 7 = 107$$

### 2.1.2. Puts

The value of a put at expiration is  $p_T = \max(0, X - S_T)$ . Broken down into parts,

$$\begin{aligned} p_T &= X - S_T && \text{if } S_T < X \\ p_T &= 0 && \text{if } S_T \geq X \end{aligned}$$

The profit is obtained by subtracting the premium on the put from the value at expiration:

$$\Pi = p_T - p_0$$

Broken down into parts,

$$\begin{aligned} \Pi &= X - S_T - p_0 && \text{if } S_T < X \\ \Pi &= -p_0 && \text{if } S_T \geq X \end{aligned}$$

For our example and outcomes of  $S_T = 1900$  and  $2100$ , the results are as follows:

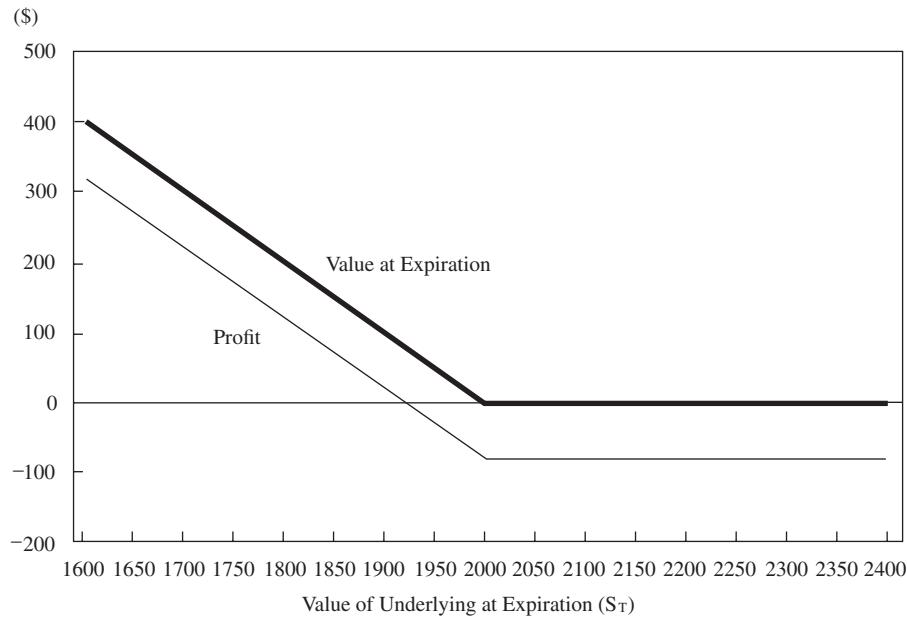
$$\begin{aligned} S_T &= 1900: \\ P_T &= \max(0, 2000 - 1900) = 100 \\ \Pi &= 100 - 79.25 = 20.75 \\ S_T &= 2100: \\ P_T &= \max(0, 2000 - 2100) = 0 \\ \Pi &= 0 - 79.25 = -79.25 \end{aligned}$$

These results are shown in Exhibit 4. We see that the put has a maximum value and profit and a limited loss, the latter of which is the premium. The maximum value is obtained when the underlying goes to zero.<sup>5</sup> In that case,  $p_T = X$ . So the maximum profit is  $X - p_0$ . Here that will be  $2000 - 79.25 = 1920.75$ .

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<sup>5</sup>The maximum value and profit are not visible on the graph because we do not show  $S_T$  all the way down to zero.

## EXHIBIT 4 Buy Put



The breakeven is found by breaking up the profit equation into its parts,  $\Pi = X - S_T - p_0$  for  $S_T < X$  and  $\Pi = -p_0$  for  $S_T \geq X$ . In the latter case, there is no possibility of breaking even. It refers to the range over which the entire premium is lost. In the former case, we denote the breakeven index value as  $S_T^*$ , set the equation to zero, and solve for  $S_T^*$  to obtain  $S_T^* = X - p_0$ . In our example, the breakeven is  $S_T^* = 2000 - 79.25 = 1920.75$ .

In summary, for the strategy of buying a put we have

$$\begin{aligned}
 p_T &= \max(0, X - S_T) \\
 \text{Value at expiration} &= p_T \\
 \text{Profit: } \Pi &= p_T - p_0 \\
 \text{Maximum profit} &= X - p_0 \\
 \text{Maximum loss} &= p_0 \\
 \text{Breakeven: } S_T^* &= X - p_0
 \end{aligned}$$

Now consider the *likely* outcomes for the holder of the put. In this case, the underlying must move down by almost 4% in one month to cover the premium. One would hardly ever expect the underlying to move down at an annual rate of almost 50%. Moreover, if the underlying does not move downward at all (a likely outcome given the positive expected return on most assets), the loss is 100% of the premium.

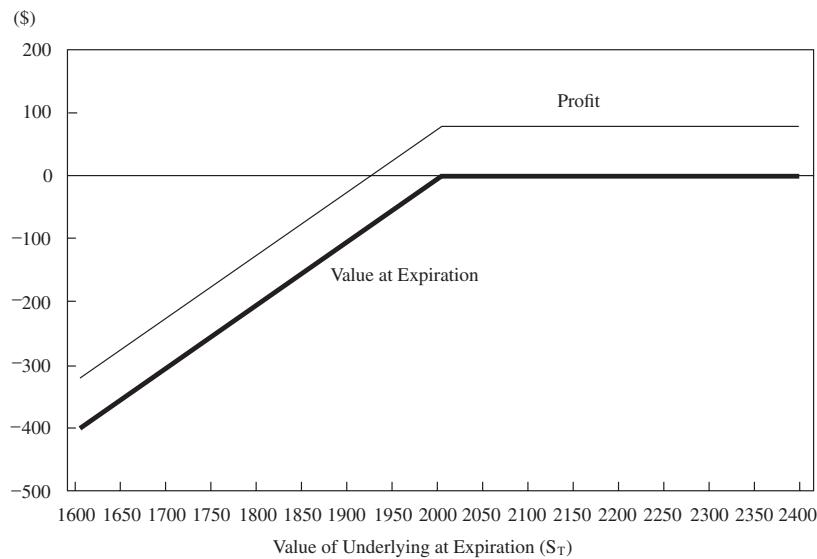
For the sale of a put, we simply change the sign on the value at expiration and profit. The maximum profit for the buyer becomes the maximum loss for the seller and the maximum loss

for the buyer becomes the maximum profit for the seller. The breakeven for the seller is the same as for the buyer. So, for the seller,

$$\begin{aligned}
 p_T &= \max(0, X - S_T) \\
 \text{Value at expiration} &= -p_T \\
 \text{Profit: } \Pi &= -p_T + p_0 \\
 \text{Maximum profit} &= p_0 \\
 \text{Maximum loss} &= X - p_0 \\
 \text{Breakeven: } S_T^* &= X - p_0
 \end{aligned}$$

Exhibit 5 graphs the value at expiration and the profit for this transaction.

#### EXHIBIT 5 Sell Put



#### EXAMPLE 2

Consider a put option selling for \$4 in which the exercise price is \$60 and the price of the underlying is \$62.

- Determine the value at expiration and the profit for a buyer under the following outcomes:
  - The price of the underlying at expiration is \$62.
  - The price of the underlying at expiration is \$55.

- B. Determine the value at expiration and the profit for a seller under the following outcomes:
- The price of the underlying at expiration is \$51.
  - The price of the underlying at expiration is \$68.
- C. Determine the following:
- the maximum profit to the buyer (maximum loss to the seller).
  - the maximum loss to the buyer (maximum profit to the seller).
- D. Determine the breakeven price of the underlying at expiration.

*Solutions:*

A. Put buyer

$$\begin{aligned}\text{i. Value at expiration} &= p_T = \max(0, X - S_T) \\ &= \max(0, 60 - 62) = 0 \\ \Pi &= p_T - p_0 = 0 - 4 = -4\end{aligned}$$

$$\begin{aligned}\text{ii. Value at expiration} &= p_T = \max(0, X - S_T) \\ &= \max(0, 60 - 55) = 5 \\ \Pi &= p_T - p_0 = 5 - 4 = 1\end{aligned}$$

B. Put seller

$$\begin{aligned}\text{i. Value at expiration} &= -p_T = -\max(0, X - S_T) \\ &= -\max(0, 60 - 51) = -9 \\ \Pi &= -p_T + p_0 = -9 + 4 = -5\end{aligned}$$

$$\begin{aligned}\text{ii. Value at expiration} &= -p_T = -\max(0, X - S_T) \\ &= -\max(0, 60 - 68) = 0 \\ \Pi &= -p_T + p_0 = 0 + 4 = 4\end{aligned}$$

C. Maximum and minimum

- Maximum profit to buyer (loss to seller) =  $X - p_0 = 60 - 4 = 56$
- Maximum loss to buyer (profit to seller) =  $p_0 = 4$

D.  $S_T^* = X - p_0 = 60 - 4 = 56$

It may be surprising to find that we have now covered all of the information we need to examine all of the other option strategies. We need to learn only a few basic facts. We must know the formula for the value at expiration of a call and a put. Then we need to know how to calculate the profit for the purchase of a call and a put, but that calculation is simple: the value at expiration minus the initial value. If we know these results, we can calculate the value at expiration of the option and the profit for any value of the underlying at expiration. If we can do that, we can graph the results for a range of possible values of the underlying at expiration. Because graphing can take a long time, however, it is probably helpful to learn the basic shapes of the value and profit graphs for calls and puts. Knowing the profit equation and the shapes of the graphs, it is easy to determine the maximum profit and maximum loss. The breakeven can be determined by setting the profit equation to zero for the case in which the profit equation contains  $S_T$ . Once we have these results for the long call and put, it is an easy matter to turn

them around and obtain the results for the short call and put. Therefore, little if any memorization is required. From there, we can go on to strategies that combine an option with another option and combine options with the underlying.

## 2.2. Risk Management Strategies with Options and the Underlying

In this section, we examine two of the most widely used option strategies, particularly for holders of the underlying. One way to reduce exposure without selling the underlying is to sell a call on the underlying; the other way is to buy a put.

### 2.2.1. Covered Calls

A **covered call** is a relatively conservative strategy, but it is also one of the most misunderstood strategies. A covered call is a position in which you own the underlying and sell a call. The value of the position at expiration is easily found as the value of the underlying plus the value of the short call:

$$V_T = S_T - \max(0, S_T - X)$$

Therefore,

$$\begin{aligned} V_T &= S_T && \text{if } S_T \leq X \\ V_T &= S_T - (S_T - X) = X && \text{if } S_T > X \end{aligned}$$

We obtain the profit for the covered call by computing the change in the value of the position,  $V_T - V_0$ . First recognize that  $V_0$ , the value of the position at the start of the contract, is the initial value of the underlying minus the call premium. We are long the underlying and short the call, so we must subtract the call premium that was received from the sale of the call. The initial investment in the position is what we pay for the underlying less what we receive for the call. Hence,  $V_0 = S_0 - c_0$ . The profit is thus

$$\begin{aligned} \Pi &= S_T - \max(0, S_T - X) - (S_0 - c_0) \\ &= S_T - S_0 - \max(0, S_T - X) + c_0 \end{aligned}$$

With the equation written in this manner, we see that the profit for the covered call is simply the profit from buying the underlying,  $S_T - S_0$ , plus the profit from selling the call,  $-\max(0, S_T - X) + c_0$ . Breaking it down into ranges,

$$\begin{aligned} \Pi &= S_T - S_0 + c_0 && \text{if } S_T \leq X \\ \Pi &= S_T - S_0 - (S_T - X) + c_0 = X - S_0 + c_0 && \text{if } S_T > X \end{aligned}$$

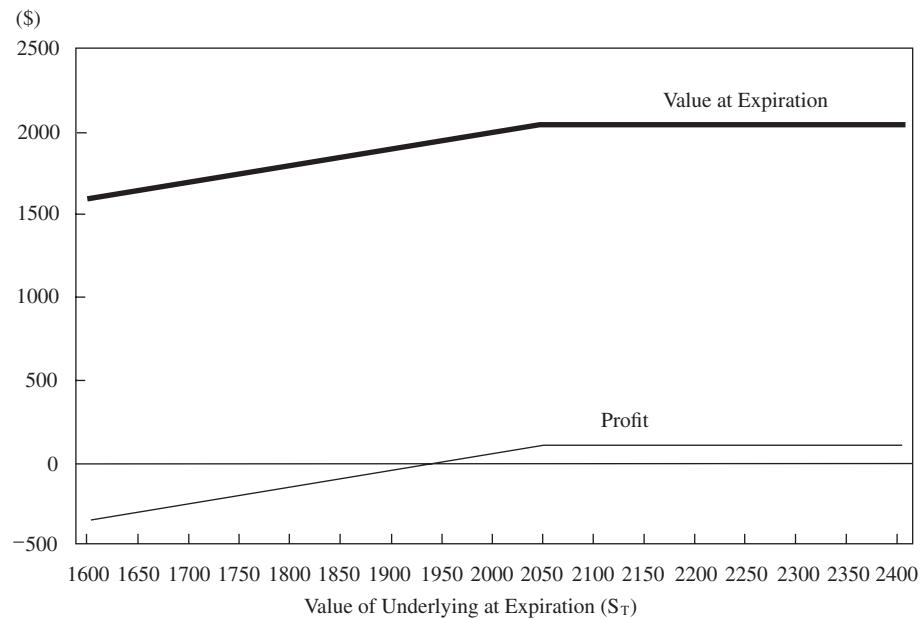
In our example,  $S_0 = 2000$ . In this section we shall use a call option with the exercise price of 2050. Thus  $X = 2050$ , and the premium,  $c_0$ , is 59.98. Let us now examine two outcomes:  $S_T = 2100$  and  $S_T = 1900$ . The value at expiration when  $S_T = 2100$  is  $V_T = 2100 - (2100 - 2050) = 2050$ , and when  $S_T = 1900$ , the value of the position is  $V_T = 1900$ .

In the first case, we hold the underlying worth 2100 but are short a call worth 50. Thus, the net value is 2050. In the second case, we hold the underlying worth 1900 and the option expires out-of-the-money.

In the first case,  $S_T = 2100$ , the profit is  $\Pi = 2050 - 2000 + 59.98 = 109.98$ . In the second case,  $S_T = 1900$ , the profit is  $\Pi = 1900 - 2000 + 59.98 = -40.02$ . These results are graphed for a range of values of  $S_T$  in Exhibit 6. Note that for all values of  $S_T$  greater than 2050, the value and profit are maximized. Thus, 2050 is the maximum value and 109.98 is the maximum profit.<sup>6</sup>

As evident in Exhibit 6 and the profit equations, the maximum loss would occur when  $S_T$  is zero. Hence, the profit would be  $S_T - S_0 + c_0$ . The profit is  $-S_0 + c_0$  when  $S_T = 0$ . This means that the maximum loss is  $S_0 - c_0$ . In this example,  $-S_0 + c_0$  is  $-2000 + 59.98 = -1940.02$ . Intuitively, this would mean that you purchased the underlying for 2000 and sold the call for 59.98. The underlying value went to zero, resulting in a loss of 2000, but the call expired with no value, so the gain from the option is the option premium. The total loss is 1940.02.

EXHIBIT 6 Covered Call (Buy Underlying, Sell Call)



The breakeven underlying price is found by examining the profit equations and focusing on the equation that contains  $S_T$ . In equation form,  $\Pi = S_T - S_0 + c_0$  when  $S_T \leq X$ . We let  $S_T^*$  denote the breakeven value of  $S_T$ , set the equation to zero, and solve for  $S_T^*$  to obtain  $S_T^* = S_0 - c_0$ . The breakeven and the maximum loss are identical. In this example, the breakeven is  $S_T^* = 2000 - 59.98 = 1940.02$ , which is seen in Exhibit 6.

<sup>6</sup>Note in Exhibit 6 that there is large gap between the value at expiration and profit, especially compared with the graphs of buying and selling calls and puts. This difference occurs because a covered call is mostly a position in the underlying asset. The initial value of the asset,  $S_0$ , accounts for most of the difference in the two lines. Note also that because of the put–call parity relationship, a covered call looks very similar to a short put.

To summarize the covered call, we have the following:

Value at expiration:  $V_T = S_T - \max(0, S_T - X)$

Profit:  $\Pi = V_T - S_0 + c_0$

Maximum profit =  $X - S_0 + c_0$

Maximum loss =  $S_0 - c_0$

Breakeven:  $S_T^* = S_0 - c_0$

Because of the importance and widespread use of covered calls, it is worthwhile to discuss this strategy briefly to dispel some misunderstandings. First of all, some investors who do not believe in using options fail to see that selling a call on a position in the underlying reduces the risk of that position. Options do not automatically increase risk. The option part of this strategy alone, viewed in isolation, seems an extremely risky strategy. We noted in Section 2.1.1 that selling a call without owning the stock exposes the investor to unlimited loss potential. But selling a covered call—adding a short call to a long position in a stock—reduces the overall risk. Thus, any investor who holds a stock cannot say he is too conservative to use options.

Following on that theme, however, one should also view selling a covered call as a strategy that reduces not only the risk but also the expected return compared with simply holding the underlying. Hence, one should not expect to make a lot of money writing calls on the underlying. It should be apparent that in fact the covered call writer could miss out on significant gains in a strong bull market. The compensation for this willingness to give up potential upside gains, however, is that in a bear market the losses on the underlying will be cushioned by the option premium.

It may be disconcerting to some investors to look at the profit profile of a covered call. The immediate response is to think that no one in their right mind would invest in a strategy that has significant downside risk but a limited upside. Just owning the underlying has significant downside risk, but at least there is an upside. But it is important to note that the visual depiction of the strategy, as in Exhibit 6, does not tell the whole story. It says nothing about the likelihood of certain outcomes occurring.

For example, consider the covered call example we looked at here. The underlying starts off at 2000. The maximum profit occurs when the option expires with the underlying at 2050 or above, an increase of 2.5% over the life of the option. We noted that this option has a one-month life. Thus, the underlying would have to increase at an approximate annual rate of at least  $2.5\%(12) = 30\%$  for the covered call writer to forgo all of the upside gain. There are not many stocks, indices, or other assets in which an investor would expect the equivalent of an annual move of at least 30%. Such movements obviously do occur from time to time, but they are not common. Thus, covered call writers do not often give up large gains.

But suppose the underlying did move to 2050 or higher. As we previously showed, the value of the position would be 2050. Because the initial value of the position is  $2000 - 59.98 = 1940.02$ , the rate of return would be 5.7% for one month. Hence, the maximum return is still outstanding by almost anyone's standards.<sup>7</sup>

Many investors believe that the initial value of a covered call should not include the value of the underlying if the underlying had been previously purchased. Suppose, for example, that

<sup>7</sup>Of course, we are not saying that the performance reflects a positive alpha. We are saying only that the upside performance given up reflects improbably high returns, and therefore the limits on the upside potential are not too restrictive.

this asset, currently worth 2000, had been bought several months ago at 1900. It is tempting to ignore the current value of the underlying; there is no current outlay. This view, however, misses the notion of opportunity cost. If an investor currently holding an asset chooses to write a call on it, she has made a conscious decision not to sell the asset. Hence, the current value of the asset should be viewed as an opportunity cost that is just as real as the cost to an investor buying the underlying at this time.

Sellers of covered calls must make a decision about the chosen exercise price. For example, one could sell the call with an exercise price of 1950 for 108.43, or sell the call with exercise price of 2000 for 81.75, or sell the call with exercise price of 2050 for 59.98. The higher the exercise price, the less one receives for the call but the more room for gain on the upside. There is no clear-cut solution to deciding which call is best; the choice depends on the risk preferences of the investor.

Finally, we should note that anecdotal evidence suggests that writers of call options make small amounts of money, but make it often. The reason for this phenomenon is generally thought to be that buyers of calls tend to be overly optimistic, but that argument is fallacious. The real reason is that the expected profits come from rare but large payoffs. For example, consider the call with exercise price of 2000 and a premium of 81.75. As we learned in Section 2.1, the breakeven underlying price is 2081.75—a gain of about 4.1% in a one-month period, which would be an exceptional return for almost any asset. These prices were obtained using the Black–Scholes–Merton model, so they are fair prices. Yet the required underlying price movement to profit on the call is exceptional. Obviously someone buys calls, and naturally, someone must be on the other side of the transaction. Sellers of calls tend to be holders of the underlying or other calls, which reduces the enormous risk they would assume if they sold calls without any other position.<sup>8</sup> Hence, it is reasonable to expect that sellers of calls would make money often, because large underlying price movements occur only rarely. Following this line of reasoning, however, it would appear that sellers of calls can consistently take advantage of buyers of calls. That cannot possibly be the case. What happens is that buyers of calls make money less often than sellers, but when they do make money, the leverage inherent in call options amplifies their returns. Therefore, when call writers lose money, they tend to lose big, but most call writers own the underlying or are long other calls to offset the risk.

### EXAMPLE 3

Consider a bond selling for \$98 per \$100 face value. A call option selling for \$8 has an exercise price of \$105. Answer the following questions about a covered call.

- A. Determine the value of the position at expiration and the profit under the following outcomes:
  - i. The price of the bond at expiration is \$110.
  - ii. The price of the bond at expiration is \$88.

<sup>8</sup>Sellers of calls who hold other calls are engaged in transactions called spreads. We discuss several types of spreads in Section 2.3.

- B. Determine the following:
- The maximum profit.
  - The maximum loss.
- C. Determine the breakeven bond price at expiration.

*Solutions:*

- A. i.  $V_T = S_T - \max(0, S_T - X) = 110 - \max(0, 110 - 105)$   
 $= 110 - 110 + 105 = 105$   
 $\Pi = V_T - V_0 = 105 - (S_0 - c_0) = 105 - (98 - 8) = 15$
- ii.  $V_T = S_T - \max(0, S_T - X) = 88 - \max(0, 88 - 105)$   
 $= 88 - 0 = 88$   
 $\Pi = V_T - V_0 = 88 - (S_0 - c_0) = 88 - (98 - 8) = -2$
- B. i. Maximum profit  $= X - S_0 + c_0 = 105 - 98 + 8 = 15$   
ii. Maximum loss  $= S_0 - c_0 = 98 - 8 = 90$
- C.  $S_T^* = S_0 - c_0 = 98 - 8 = 90$

Covered calls represent one widely used way to protect a position in the underlying. Another popular means of providing protection is to buy a put.

### 2.2.2. Protective Puts

Because selling a call provides some protection to the holder of the underlying against a fall in the price of the underlying, buying a put should also provide protection. A put, after all, is designed to pay off when the price of the underlying moves down. In some ways, buying a put to add to a long stock position is much better than selling a call. As we shall see here, it provides downside protection while retaining the upside potential, but it does so at the expense of requiring the payment of cash up front. In contrast, a covered call generates cash up front but removes some of the upside potential.

Holding an asset and a put on the asset is a strategy known as a **protective put**. The value at expiration and the profit of this strategy are found by combining the value and profit of the two strategies of buying the asset and buying the put. The value is  $V_T = S_T + \max(0, X - S_T)$ . Thus, the results can be expressed as

$$\begin{aligned} V_T &= S_T + (X - S_T) = X && \text{if } S_T \leq X \\ V_T &= S_T && \text{if } S_T > X \end{aligned}$$

When the underlying price at expiration exceeds the exercise price, the put expires with no value. The position is then worth only the value of the underlying. When the underlying price at expiration is less than the exercise price, the put expires in-the-money and is worth  $X - S_T$ , while the underlying is worth  $S_T$ . The combined value of the two instruments is  $X$ . When the underlying is worth less than the exercise price at expiration, the put can be used to sell the underlying for the exercise price.

The initial value of the position is the initial price of the underlying,  $S_0$ , plus the premium on the put,  $p_0$ . Hence, the profit is  $\Pi = S_T + \max(0, X - S_T) - (S_0 + p_0)$ . The profit can be broken down as follows:

$$\begin{aligned}\Pi &= X - (S_0 + p_0) && \text{if } S_T \leq X \\ \Pi &= S_T - (S_0 + p_0) && \text{if } S_T > X\end{aligned}$$

In this example, we are going to use the put with an exercise price of 1950. Its premium is 56.01. Recalling that the initial price of the underlying is 2000, the value at expiration and profit for the case of  $S_T = 2100$  are

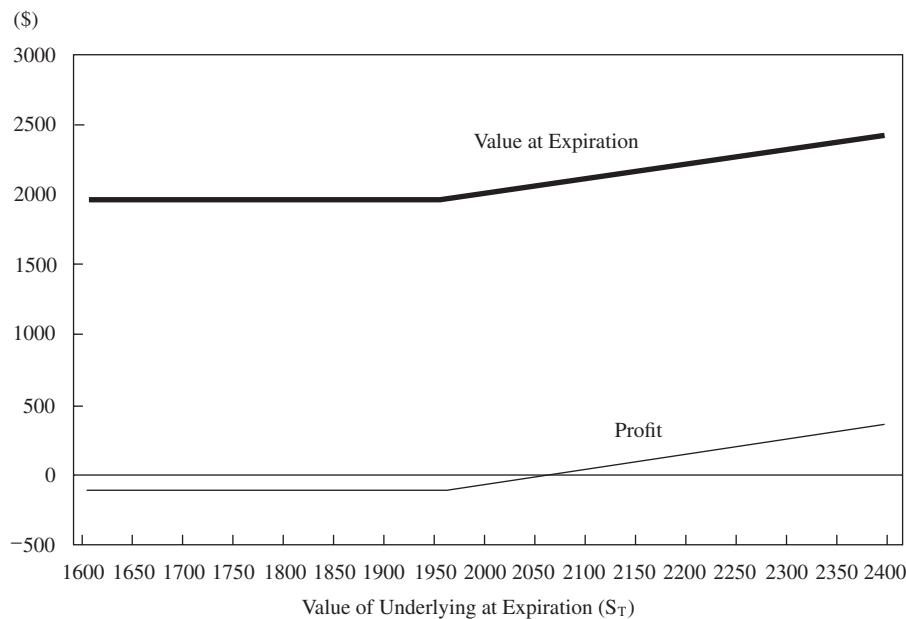
$$\begin{aligned}V_T &= 2100 \\ \Pi &= 2100 - (2000 + 56.01) = 43.99\end{aligned}$$

For the case of  $S_T = 1900$ , the value at expiration and profit are

$$\begin{aligned}V_T &= 1950 \\ \Pi &= 1950 - (2000 + 56.01) = -106.01\end{aligned}$$

The results for a range of outcomes are shown in Exhibit 7. Note how the protective put provides a limit on the downside with no limit on the upside.<sup>9</sup> Therefore, we can say that the upper limit is infinite. The lower limit is a loss of 106.01. In the worst possible case, we can sell the underlying for the exercise price, but the up-front cost of the underlying and put are 2056.01, for a maximum loss of 106.01.

EXHIBIT 7 Protective Put (Buy Underlying, Buy Put)



<sup>9</sup>Note that the graph for a protective put looks like the graph for a call. This result is due to put–call parity.

Now let us find the breakeven price of the underlying at expiration. Note that the two profit equations are  $\Pi = S_T - (S_0 + p_0)$  if  $S_T > X$  and  $\Pi = X - (S_0 + p_0)$  if  $S_T \leq X$ . In the latter case, there is no value of the underlying that will allow us to break even. In the former case,  $S_T > X$ , we change the notation on  $S_T$  to  $S_T^*$  to denote the breakeven value, set this expression equal to zero, and solve for  $S_T^*$ :

$$S_T^* = S_0 + p_0$$

To break even, the underlying must be at least as high as the amount expended up front to establish the position. In this problem, this amount is  $2000 + 56.01 = 2056.01$ .

To summarize the protective put, we have the following:

Value at expiration:  $V_T = S_T + \max(0, X - S_T)$

Profit:  $\Pi = V_T - S_0 - p_0$

Maximum profit =  $\infty$

Maximum loss =  $S_0 + p_0 - X$

Breakeven:  $S_T^* = S_0 + p_0$

A protective put can appear to be a great transaction with no drawbacks. It provides downside protection with upside potential, but let us take a closer look. First recall that this is a one-month transaction and keep in mind that the option has been priced by the Black–Scholes–Merton model and is, therefore, a fair price. The maximum loss of 106.01 is a loss of  $106.01/2056.01 = 5.2\%$ . The breakeven of 2056.01 requires an upward move of 2.8%, which is an annual rate of about 34%. From this angle, the protective put strategy does not look quite as good, but in fact, these figures simply confirm that protection against downside loss is expensive. When the protective put is fairly priced, the protection buyer must give up considerable upside potential that may not be particularly evident from just looking at a graph.

The purchase of a protective put also presents the buyer with some choices. In this example, the buyer bought the put with exercise price of 1950 for 56.01. Had he bought the put with exercise price of 2000, he would have paid 79.25. The put with exercise price of 2050 would have cost 107.39. The higher the price for which the investor wants to be able to sell the underlying, the more expensive the put will be.

The protective put is often viewed as a classic example of insurance. The investor holds a risky asset and wants protection against a loss in value. He then buys insurance in the form of the put, paying a premium to the seller of the insurance, the put writer. The exercise price of the put is like the insurance deductible because the magnitude of the exercise price reflects the risk assumed by the party holding the underlying. The higher the exercise price, the less risk assumed by the holder of the underlying and the more risk assumed by the put seller. The lower the exercise price, the more risk assumed by the holder of the underlying and the less risk assumed by the put seller. In insurance, the higher the deductible, the more risk assumed by the insured party and the less risk assumed by the insurer. Thus, a higher exercise price is analogous to a lower insurance deductible.

Like traditional insurance, this form of insurance provides coverage for a period of time. At the end of the period of time, the insurance expires and either pays off or not. The buyer of the insurance may or may not choose to renew the insurance by buying another put.

### EXAMPLE 4

Consider a currency selling for \$0.875. A put option selling for \$0.075 has an exercise price of \$0.90. Answer the following questions about a protective put.

- A. Determine the value at expiration and the profit under the following outcomes:
  - i. The price of the currency at expiration is \$0.96.
  - ii. The price of the currency at expiration is \$0.75.
- B. Determine the following:
  - i. the maximum profit.
  - ii. the maximum loss.
- C. Determine the breakeven price of the currency at expiration.

*Solutions:*

- A. i.  $V_T = S_T + \max(0, X_T - S_T) = 0.96 + \max(0, 0.90 - 0.96) = 0.96$   
 $\Pi = V_T - V_0 = 0.96 - (S_0 + p_0) = 0.96 - (0.875 + 0.075)$   
 $= 0.01$
- ii.  $V_T = S_T + \max(0, X - S_T) = 0.75 + \max(0, 0.90 - 0.75) = 0.90$   
 $\Pi = V_T - V_0 = 0.90 - (S_0 + p_0) = 0.90 - (0.875 + 0.075)$   
 $= -0.05$
- B. i. Maximum profit =  $\infty$   
 ii. Maximum loss =  $S_0 + p_0 - X = 0.875 + 0.075 - 0.90 = 0.05$
- C.  $S_T^* = S_0 + p_0 = 0.875 + 0.075 = 0.95$

Finally, we note that a protective put can be modified in a number of ways. One in particular is to sell a call to generate premium income to pay for the purchase of the put. This strategy is known as a collar. We shall cover collars in detail in Section 2.4.1 when we look at combining puts and calls. For now, however, let us proceed with strategies that combine calls with calls and puts with puts. These strategies are called spreads.

### 2.3. Money Spreads

A spread is a strategy in which you buy one option and sell another option that is identical to the first in all respects except either exercise price or time to expiration. If the options differ by time to expiration, the spread is called a time spread. Time spreads are strategies designed to exploit differences in perceptions of volatility of the underlying. They are among the more specialized strategies, and we do not cover them here. Our focus is on money spreads, which are spreads in which the two options differ only by exercise price. The investor buys an option with a given expiration and exercise price and sells an option with the same expiration but a different exercise price. Of course, the options are on the same underlying asset. The term *spread* is used here because the payoff is based on the difference, or spread, between option exercise prices.

### 2.3.1. Bull Spreads

A **bull spread** is designed to make money when the market goes up. In this strategy we combine a long position in a call with one exercise price and a short position in a call with a higher exercise price. Let us use  $X_1$  as the lower of the two exercise prices and  $X_2$  as the higher. The European call prices would normally be denoted as  $c(X_1)$  and  $c(X_2)$ , but we shall simplify this notation somewhat in this chapter by using the symbols  $c_1$  and  $c_2$ , respectively. The value of a call at expiration is  $c_T = \max(0, S_T - X)$ . So, the value of the spread at expiration is

$$V_T = \max(0, S_T - X_1) - \max(0, S_T - X_2)$$

Therefore,

$$\begin{aligned} V_T &= 0 - 0 = 0 && \text{if } S_T \leq X_1 \\ V_T &= S_T - X_1 - 0 = S_T - X_1 && \text{if } X_1 < S_T < X_2 \\ V_T &= S_T - X_1 - (S_T - X_2) = X_2 - X_1 && \text{if } S_T \geq X_2 \end{aligned}$$

The profit is obtained by subtracting the initial outlay for the spread from the above value of the spread at expiration. To determine the initial outlay, recall that a call option with a lower exercise price will be more expensive than a call option with a higher exercise price. Because we are buying the call with the lower exercise price and selling the call with the higher exercise price, the call we buy will cost more than the call we sell. Hence, the spread will require a net outlay of funds. This net outlay is the initial value of the position of  $V_0 = c_1 - c_2$ , which we call the net premium. The profit is  $V_T - V_0$ . Therefore,

$$\Pi = \max(0, S_T - X_1) - \max(0, S_T - X_2) - (c_1 - c_2)$$

In this manner, we see that the profit is the profit from the long call,  $\max(0, S_T - X_1) - c_1$ , plus the profit from the short call,  $-\max(0, S_T - X_2) + c_2$ . Broken down into ranges, the profit is

$$\begin{aligned} \Pi &= -c_1 + c_2 && \text{if } S_T \leq X_1 \\ \Pi &= S_T - X_1 - c_1 + c_2 && \text{if } X_1 < S_T < X_2 \\ \Pi &= X_2 - X_1 - c_1 + c_2 && \text{if } S_T \geq X_2 \end{aligned}$$

If  $S_T$  is below  $X_1$ , the strategy will lose a limited amount of money. The profit on the upside, if  $S_T$  is at least  $X_2$ , is also limited. When both options expire out-of-the-money, the investor loses the net premium,  $c_1 - c_2$ .

In this example, we use exercise prices of 1950 and 2050. Thus  $X_1 = 1950$ ,  $c_1 = 108.43$ ,  $X_2 = 2050$ , and  $c_2 = 59.98$ . Let us examine the outcomes in which the asset price at expiration is 2100, 2000, and 1900. In one outcome, the underlying is above the upper exercise price at expiration, and in one, the underlying is below the lower exercise price at expiration. Let us also examine one case between the exercise prices with  $S_T$  equal to 2000.

When  $S_T = 2100$ , the value at expiration is  $V_T = 2050 - 1950 = 100$

When  $S_T = 2000$ , the value at expiration is  $V_T = 2000 - 1950 = 50$

When  $S_T = 1900$ , the value at expiration is  $V_T = 0$

To calculate the profit, we simply subtract the initial value for the call with exercise price  $X_1$  and add the initial value for the call with exercise price  $X_2$ .

When  $S_T = 2100$ , the profit is  $\Pi = 100 - 108.43 + 59.98 = 51.55$

When  $S_T = 2000$ , the profit is  $\Pi = 50 - 108.43 + 59.98 = 1.55$

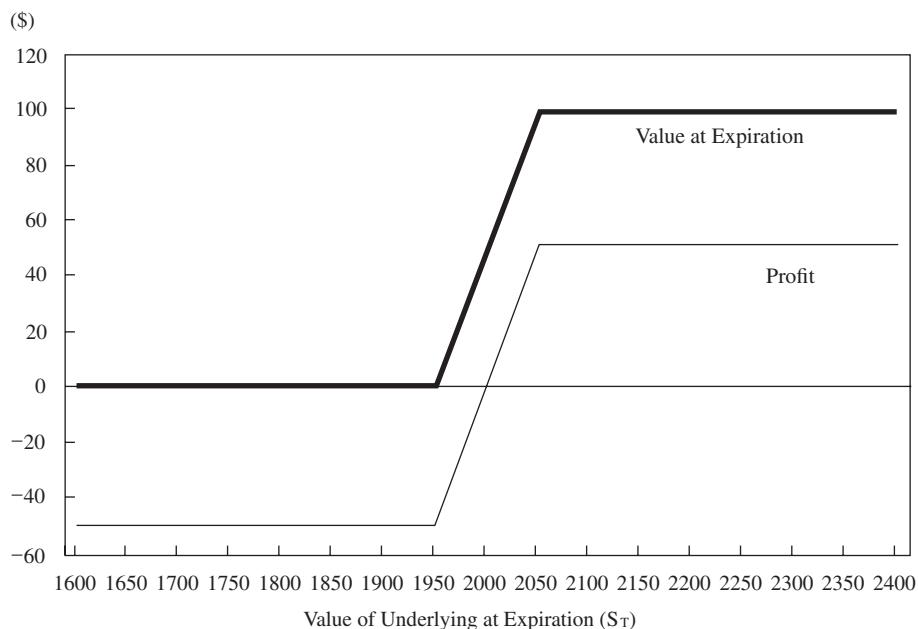
When  $S_T = 1900$ , the profit is  $\Pi = -108.43 + 59.98 = -48.45$

When  $S_T$  is greater than 2100, we would obtain the same outcome as when  $S_T$  equals 2100.

When  $S_T$  is less than 1900, we would obtain the same outcome as when  $S_T$  equals 1900.

Exhibit 8 depicts these results graphically. Note how the bull spread provides a limited gain as well as a limited loss. Of course, just purchasing a call provides a limited loss. But when selling the call in addition to buying the call, the investor gives up the upside in order to reduce the downside. In the bull spread, the investor sells gains from the call beyond the higher exercise price. Thus, a bull spread has some similarities to the covered call. With a covered call, the long position in the underlying "covers" the short position in the call. In a bull spread, the long position in the call with the lower exercise price "covers" the short position in the call with the higher exercise price. For both strategies, the short call can be viewed as giving up the gains beyond its exercise price. The upside gain can also be viewed as paying a premium of  $c_1 - c_2$  to buy the underlying for  $X_1$  and sell it for  $X_2$ . Accordingly, the maximum gain is  $X_2 - X_1 - c_1 + c_2 = 2050 - 1950 - 108.43 + 59.98 = 51.55$ , as computed above. This amount represents a maximum return of about 106%.<sup>10</sup> The maximum loss is the net premium, 48.45, which is a 100% loss.

EXHIBIT 8 Bull Spread (Buy Call with Exercise Price  $X_1$ , Sell Call with Exercise Price  $X_2$ )



<sup>10</sup>This calculation is based on the fact that the initial value of the position is  $108.43 - 59.98 = 48.45$  and the maximum value is 100, which is a gain of 106.4%.

As can be seen from the graph and the profit equations, there is a breakeven asset price at expiration that falls between the two exercise prices. We let  $S_T^*$  be the breakeven asset price at expiration and set the profit for the case of  $X_1 < S_T < X_2$  to zero:

$$S_T^* = X_1 + c_1 - c_2$$

To achieve a profit of zero or more, the asset price at expiration must exceed the lower exercise price by at least the net premium paid for the options. The long option must expire in-the-money by enough to cover the net premium. In our example,

$$S_T^* = 1950 + 108.43 - 59.98 = 1,998.45$$

What this result means is that the underlying must not move down by more than 0.08%.

To summarize the bull spread, we have

Value at expiration:  $V_T = \max(0, S_T - X_1) - \max(0, S_T - X_2)$

Profit:  $\Pi = V_T - c_1 + c_2$

Maximum profit =  $X_2 - X_1 - c_1 + c_2$

Maximum loss =  $c_1 - c_2$

Breakeven:  $S_T^* = X_1 + c_1 - c_2$

### EXAMPLE 5

Consider two call options on a stock selling for \$72. One call has an exercise price of \$65 and is selling for \$9. The other call has an exercise price of \$75 and is selling for \$4. Both calls expire at the same time. Answer the following questions about a bull spread:

- A. Determine the value at expiration and the profit under the following outcomes:
  - i. The price of the stock at expiration is \$78.
  - ii. The price of the stock at expiration is \$69.
  - iii. The price of the stock at expiration is \$62.
- B. Determine the following:
  - i. the maximum profit.
  - ii. the maximum loss.
- C. Determine the breakeven stock price at expiration.

*Solutions:*

$$\begin{aligned} A. \quad i. \quad V_T &= \max(0, S_T - X_1) - \max(0, S_T - X_2) \\ &= \max(0, 78 - 65) - \max(0, 78 - 75) = 13 - 3 = 10 \\ \Pi &= V_T - V_0 = V_T - (c_1 - c_2) = 10 - (9 - 4) = 5 \end{aligned}$$

$$\begin{aligned} ii. \quad V_T &= \max(0, S_T - X_1) - \max(0, S_T - X_2) \\ &= \max(0, 69 - 65) - \max(0, 69 - 75) = 4 - 0 = 4 \\ \Pi &= V_T - V_0 = V_T - (c_1 - c_2) = 4 - (9 - 4) = -1 \end{aligned}$$

$$\begin{aligned}
 \text{iii. } V_T &= \max(0, S_T - X_1) - \max(0, S_T - X_2) \\
 &= \max(0, 62 - 65) - \max(0, 62 - 75) = 0 - 0 = 0 \\
 \Pi &= V_T - V_0 = 0 - (c_1 - c_2) = 0 - (9 - 4) = -5
 \end{aligned}$$

B. i. Maximum profit =  $X_2 - X_1 - (c_1 - c_2) = 75 - 65 - (9 - 4) = 5$

ii. Maximum loss =  $c_1 - c_2 = 9 - 4 = 5$

C.  $S_T^* = X_1 + c_1 - c_2 = 65 + 9 - 4 = 70$

Bull spreads are used by investors who think the underlying price is going up. There are also bear spreads, which are used by investors who think the underlying price is going down.

### 2.3.2. Bear Spreads

If one uses the opposite strategy, selling a call with the lower exercise price and buying a call with the higher exercise price, the opposite results occur. The graph is completely reversed: The gain is on the downside and the loss is on the upside. This strategy is called a **bear spread**. The more intuitive way of executing a bear spread, however, is to use puts. Specifically, we would buy the put with the higher exercise price and sell the put with the lower exercise price.

The value of this position at expiration would be  $V_T = \max(0, X_2 - S_T) - \max(0, X_1 - S_T)$ . Broken down into ranges, we have the following relations:

$$\begin{aligned}
 V_T &= X_2 - S_T - (X_1 - S_T) = X_2 - X_1 && \text{if } S_T \leq X_1 \\
 V_T &= X_2 - S_T - 0 = X_2 - S_T && \text{if } X_1 < S_T < X_2 \\
 V_T &= 0 - 0 = 0 && \text{if } S_T \geq X_2
 \end{aligned}$$

To obtain the profit, we subtract the initial outlay. Because we are buying the put with the higher exercise price and selling the put with the lower exercise price, the put we are buying is more expensive than the put we are selling. The initial value of the bear spread is  $V_0 = p_2 - p_1$ . The profit is, therefore,  $V_T - V_0$ , which is

$$\Pi = \max(0, X_2 - S_T) - \max(0, X_1 - S_T) - p_2 + p_1$$

We see that the profit is the profit from the long put,  $\max(0, X_2 - S_T) - p_2$ , plus the profit from the short put,  $-\max(0, X_1 - S_T) + p_1$ . Broken down into ranges, the profit is

$$\begin{aligned}
 \Pi &= X_2 - X_1 - p_2 + p_1 && \text{if } S_T \leq X_1 \\
 \Pi &= X_2 - S_T - p_2 + p_1 && \text{if } X_1 < S_T < X_2 \\
 \Pi &= -p_2 + p_1 && \text{if } S_T \geq X_2
 \end{aligned}$$

In contrast to the profit in a bull spread, the bear spread profit occurs on the downside; the maximum profit occurs when  $S_T \leq X_1$ . This profit reflects the purchase of the underlying at  $X_1$ , which occurs when the short put is exercised, and the sale of the underlying at  $X_2$ , which occurs when the long put is exercised. The worst outcome occurs when  $S_T > X_2$ , in which case both puts expire out-of-the-money and the net premium is lost.

In the example, we again use options with exercise prices of 1950 and 2050. Their premiums are  $p_1 = 56.01$  and  $p_2 = 107.39$ . We examine the three outcomes we did with the bull spread:  $S_T$  is 1900, 2000, or 2100.

When  $S_T = 1900$ , the value at expiration is  $V_T = 2050 - 1950 = 100$

When  $S_T = 2000$ , the value at expiration is  $V_T = 2050 - 2000 = 50$

When  $S_T = 2100$ , the value at expiration is  $V_T = 0$

The profit is obtained by taking the value at expiration, subtracting the premium of the put with the higher exercise price, and adding the premium of the put with the lower exercise price:

When  $S_T = 1900$ , the profit is  $\Pi = 100 - 107.39 + 56.01 = 48.62$

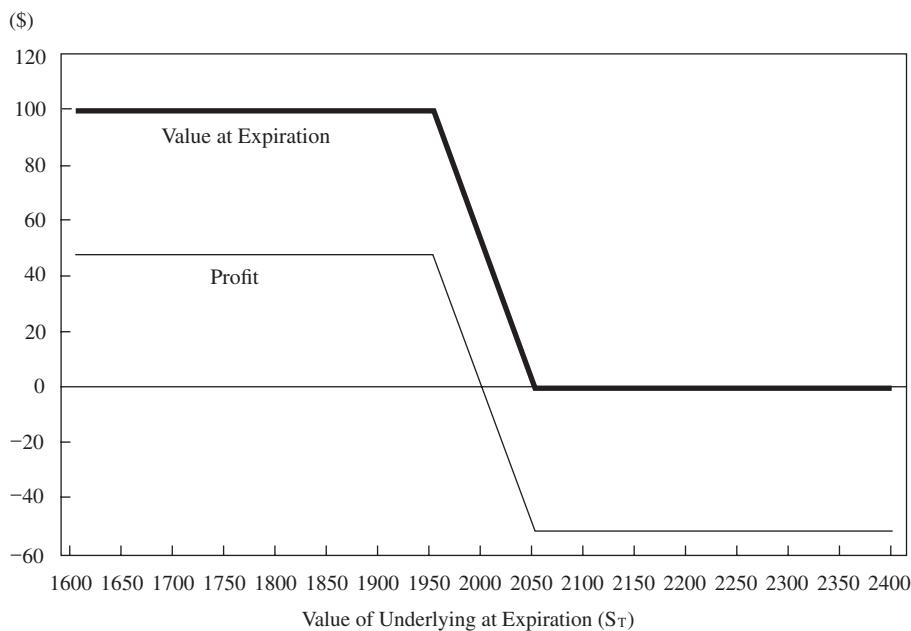
When  $S_T = 2000$ , the profit is  $\Pi = 50 - 107.39 + 56.01 = -1.38$

When  $S_T = 2100$ , the profit is  $\Pi = -107.39 + 56.01 = -51.38$

When  $S_T$  is less than 1900, the outcome is the same as when  $S_T$  equals 1900. When  $S_T$  is greater than 2100, the outcome is the same as when  $S_T$  equals 2100.

The results are graphed in Exhibit 9. Note how this strategy is similar to a bull spread but with opposite outcomes. The gains are on the downside underlying moves and the losses are on the upside underlying. The maximum profit occurs when both puts expire in-the-money. You end up using the short put to buy the asset and the long put to sell the asset. The maximum profit is  $X_2 - X_1 - p_2 + p_1$ , which in this example is  $100 - 107.39 + 56.01 = 48.62$ , a return of 94%.<sup>11</sup> The maximum loss of  $p_2 - p_1$  occurs when both puts expire out-of-the-money, and in this case is  $107.39 - 56.01 = 51.38$ , a loss of 100%.

EXHIBIT 9 Bear Spread (Buy Put with Exercise Price  $X_2$ , Sell Put with Exercise Price  $X_1$ )



<sup>11</sup>The net premium is  $107.39 - 56.01 = 51.38$ , so the maximum value of 100 is a return of about 94%.

The breakeven asset price occurs between the two exercise prices. Let  $S_T^*$  be the breakeven asset price at expiration, set the profit equation for the middle case to zero, and solve for  $S_T^*$  to obtain  $S_T^* = X_2 - p_2 + p_1$ . In this case, the breakeven is  $S_T^* = 2050 - 107.39 + 56.01 = 1,998.62$ . The underlying need move down only as little as 0.07% to make a profit.

To summarize the bear spread, we have

Value at expiration:  $V_T = \max(0, X_2 - S_T) - \max(0, X_1 - S_T)$

Profit:  $\Pi = V_T - p_2 + p_1$

Maximum profit =  $X_2 - X_1 - p_2 + p_1$

Maximum loss =  $p_2 - p_1$

Breakeven:  $S_T^* = X_2 - p_2 + p_1$

### EXAMPLE 6

Consider two put options on a bond selling for \$92 per \$100 par. One put has an exercise price of \$85 and is selling for \$3. The other put has an exercise price of \$95 and is selling for \$11. Both puts expire at the same time. Answer the following questions about a bear spread:

- Determine the value at expiration and the profit under the following outcomes:
  - The price of the bond at expiration is \$98.
  - The price of the bond at expiration is \$91.
  - The price of the bond at expiration is \$82.
- Determine the following:
  - the maximum profit.
  - the maximum loss.
- Determine the breakeven bond price at expiration.

*Solutions:*

$$\begin{aligned} A. \quad i. \quad V_T &= \max(0, X_2 - S_T) - \max(0, X_1 - S_T) \\ &= \max(0, 95 - 98) - \max(0, 85 - 98) = 0 - 0 = 0 \\ \Pi &= V_T - V_0 = V_T - (p_2 - p_1) = 0 - (11 - 3) = -8 \end{aligned}$$

$$\begin{aligned} ii. \quad V_T &= \max(0, X_2 - S_T) - \max(0, X_1 - S_T) \\ &= \max(0, 95 - 91) - \max(0, 85 - 91) = 4 - 0 = 4 \\ \Pi &= V_T - V_0 = V_T - (p_2 - p_1) = 4 - (11 - 3) = -4 \end{aligned}$$

$$\begin{aligned} iii. \quad V_T &= \max(0, X_2 - S_T) - \max(0, X_1 - S_T) \\ &= \max(0, 95 - 82) - \max(0, 85 - 82) = 13 - 3 = 10 \\ \Pi &= V_T - V_0 = 10 - (p_2 - p_1) = 10 - (11 - 3) = 2 \end{aligned}$$

- i. Maximum profit =  $X_2 - X_1 - (p_2 - p_1) = 95 - 85 - (11 - 3) = 2$   
ii. Maximum loss =  $p_2 - p_1 = 11 - 3 = 8$
- $S_T^* = X_2 - p_2 + p_1 = 95 - 11 + 3 = 87$

The bear spread with calls involves selling the call with the lower exercise price and buying the one with the higher exercise price. Because the call with the lower exercise price will be more expensive, there will be a cash inflow at initiation of the position and hence a profit if the calls expire worthless.

Bull and bear spreads are but two types of spread strategies. We now take a look at another strategy, which combines bull and bear spreads.

### 2.3.3. Butterfly Spreads

In both the bull and bear spread, we used options with two different exercise prices. There is no limit to how many different options one can use in a strategy. As an example, the **butterfly spread** combines a bull and bear spread. Consider three different exercise prices,  $X_1$ ,  $X_2$ , and  $X_3$ . Suppose we first construct a bull spread, buying the call with exercise price of  $X_1$  and selling the call with exercise price of  $X_2$ . Recall that we could construct a bear spread using calls instead of puts. In that case, we would buy the call with the higher exercise price and sell the call with the lower exercise price. This bear spread is identical to the sale of a bull spread.

Suppose we sell a bull spread by buying the call with exercise price  $X_3$  and selling the call with exercise price  $X_2$ . We have now combined a long bull spread and a short bull spread (or a bear spread). We own the calls with exercise price  $X_1$  and  $X_3$  and have sold two calls with exercise price  $X_2$ . Combining these results, we obtain a value at expiration of

$$V_T = \max(0, S_T - X_1) - 2\max(0, S_T - X_2) + \max(0, S_T - X_3)$$

This can be broken down into ranges of

$$\begin{aligned} V_T &= 0 - 2(0) + 0 = 0 && \text{if } S_T \leq X_1 \\ V_T &= S_T - X_1 - 2(0) + 0 = S_T - X_1 && \text{if } X_1 < S_T < X_2 \\ V_T &= S_T - X_1 - 2(S_T - X_2) + 0 = -S_T + 2X_2 - X_1 && \text{if } X_2 \leq S_T < X_3 \\ V_T &= S_T - X_1 - 2(S_T - X_2) + S_T - X_3 = 2X_2 - X_1 - X_3 && \text{if } S_T \geq X_3 \end{aligned}$$

If the exercise prices are equally spaced,  $2X_2 - X_1 - X_3$  would equal zero.<sup>12</sup> In virtually all cases in practice, the exercise prices are indeed equally spaced, and we shall make that assumption. Therefore,

$$V_T = 2X_2 - X_1 - X_3 = 0 \quad \text{if } S_T \geq X_3$$

To obtain the profit, we must subtract the initial value of the position, which is  $V_0 = c_1 - 2c_2 + c_3$ . Is this value positive or negative? It turns out that it will always be positive. The bull spread we buy is more expensive than the bull spread we sell, because the lower exercise price on the bull spread we buy ( $X_1$ ) is lower than the lower exercise price on the bull spread we sell ( $X_2$ ). Because the underlying is more likely to move higher than  $X_1$  than to move higher than  $X_2$ , the bull spread we buy is more expensive than the bull spread we sell.

The profit is thus  $V_T - V_0$ , which is

$$\Pi = \max(0, S_T - X_1) - 2\max(0, S_T - X_2) + \max(0, S_T - X_3) - c_1 + 2c_2 - c_3$$

<sup>12</sup>For example, suppose the exercise prices are equally spaced with  $X_1 = 30$ ,  $X_2 = 40$ , and  $X_3 = 50$ . Then  $2X_2 - X_3 - X_1 = 2(40) - 50 - 30 = 0$ .

Broken down into ranges,

$$\begin{aligned}
 \Pi &= -c_1 + 2c_2 - c_3 && \text{if } S_T \leq X_1 \\
 \Pi &= S_T - X_1 - c_1 + 2c_2 - c_3 && \text{if } X_1 < S_T < X_2 \\
 \Pi &= -S_T + 2X_2 - X_1 - c_1 + 2c_2 - c_3 && \text{if } X_2 \leq S_T < X_3 \\
 \Pi &= -c_1 + 2c_2 - c_3 && \text{if } S_T \geq X_3
 \end{aligned}$$

Note that in the lowest and highest ranges, the profit is negative; a loss. It is not immediately obvious what happens in the middle two ranges. Let us look at our example. In this example, we buy the calls with exercise prices of 1950 and 2050 and sell two calls with exercise price of 2000. So,  $X_1 = 1950$ ,  $X_2 = 2000$ , and  $X_3 = 2050$ . Their premiums are  $c_1 = 108.43$ ,  $c_2 = 81.75$ , and  $c_3 = 59.98$ . Let us examine the outcomes in which  $S_T = 1900$ , 1975, 2025, and 2100. These outcomes fit into each of the four relevant ranges.

$$\begin{aligned}
 \text{When } S_T = 1900, \text{ the value at expiration is } V_T &= 0 - 2(0) + 0 = 0 \\
 \text{When } S_T = 1975, \text{ the value at expiration is } V_T &= 1975 - 1950 = 25 \\
 \text{When } S_T = 2025, \text{ the value at expiration is } V_T &= -2025 + 2(2000) - 1950 = 25 \\
 \text{When } S_T = 2100, \text{ the value at expiration is } V_T &= 0
 \end{aligned}$$

Now, turning to the profit,

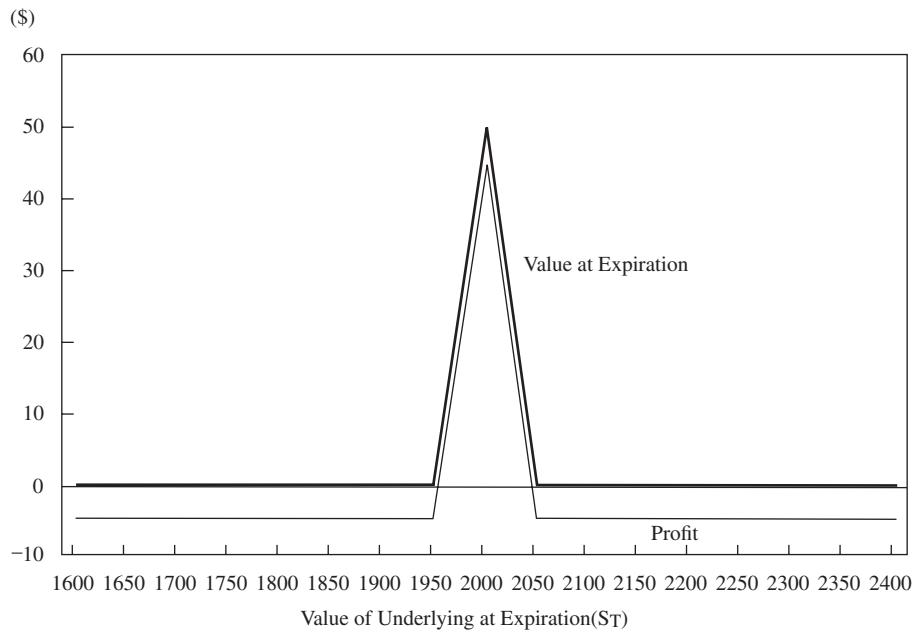
$$\begin{aligned}
 \text{When } S_T = 1900, \text{ the profit will be } \Pi &= 0 - 108.43 + 2(81.75) - 59.98 = -4.91 \\
 \text{When } S_T = 1975, \text{ the profit will be } \Pi &= 25 - 108.43 + 2(81.75) - 59.98 = 20.09 \\
 \text{When } S_T = 2025, \text{ the profit will be } \Pi &= 25 - 108.43 + 2(81.75) - 59.98 = 20.09 \\
 \text{When } S_T = 2100, \text{ the profit will be } \Pi &= 0 - 108.43 + 2(81.75) - 59.98 = -4.91
 \end{aligned}$$

Exhibit 10 depicts these results graphically. Note that the strategy is based on the expectation that the volatility of the underlying will be relatively low. The expectation must be that the underlying will trade near the middle exercise price. The maximum loss of 4.91 occurs if the underlying ends up below the lower strike, 1950, or above the upper strike, 2050. The maximum profit occurs if the underlying ends up precisely at the middle exercise price. This maximum profit is found by examining either of the middle two ranges with  $S_T$  set equal to  $X_2$ :

$$\begin{aligned}
 \Pi(\text{maximum}) &= S_T - X_1 - c_1 + 2c_2 - c_3 \\
 &= X_2 - X_1 - c_1 + 2c_2 - c_3 && \text{if } S_T = X_2 \\
 \Pi(\text{maximum}) &= -S_T + 2X_2 - X_1 - c_1 + 2c_2 - c_3 \\
 &= X_2 - X_1 - c_1 + 2c_2 - c_3 && \text{if } S_T = X_2
 \end{aligned}$$

In this case, the maximum profit is  $\Pi(\text{maximum}) = 2000 - 1950 - 108.43 + 2(81.75) - 59.98 = 45.09$ , which is a return of 918%.<sup>13</sup>

<sup>13</sup>This return is based on a maximum value of  $2000 - 1950 = 50$  versus the initial value of 4.91, a return of 918%.

EXHIBIT 10 Butterfly Spread (Buy Calls with Exercise Price  $X_1$  and  $X_3$ , Sell Two Calls with Exercise Price  $X_2$ )

There are two breakeven prices, and they lie within the two middle profit ranges. We find them as follows:

For  $X_1 < S_T < X_2$ :

$$\begin{aligned}\Pi &= S_T^* - X_1 - c_1 + 2c_2 - c_3 = 0 \\ S_T^* &= X_1 + c_1 - 2c_2 + c_3\end{aligned}$$

For  $X_2 < S_T < X_3$ :

$$\begin{aligned}\Pi &= -S_T^* + 2X_2 - X_1 - c_1 + 2c_2 - c_3 = 0 \\ S_T^* &= 2X_2 - X_1 - c_1 + 2c_2 - c_3\end{aligned}$$

In this example, therefore, the breakeven prices are

$$\begin{aligned}S_T^* &= X_1 + c_1 - 2c_2 + c_3 \\ &= 1950 + 108.43 - 2(81.75) + 59.98 = 1954.91 \\ S_T^* &= 2X_2 - X_1 - c_1 + 2c_2 - c_3 \\ &= 2(2000) - 1950 - 108.43 + 2(81.75) - 59.98 = 2045.09\end{aligned}$$

These movements represent a range of roughly  $\pm 2.3\%$  from the starting value of 2000. Therefore, if the underlying stays within this range, the strategy will be profitable.

In summary, for the butterfly spread

Value at expiration:  $V_T = \max(0, S_T - X_1) - 2\max(0, S_T - X_2) + \max(0, S_T - X_3)$

Profit:  $\Pi = V_T - c_1 + 2c_2 - c_3$

Maximum profit =  $X_2 - X_1 - c_1 + 2c_2 - c_3$

Maximum loss =  $c_1 - 2c_2 + c_3$

Breakeven:  $S_T^* = X_1 + c_1 - 2c_2 + c_3$  and  $S_T^* = 2X_2 - X_1 - c_1 + 2c_2 - c_3$

As we noted, a butterfly spread is a strategy based on the expectation of low volatility in the underlying. Of course, for a butterfly spread to be an appropriate strategy, the user must believe that the underlying will be less volatile than the market expects. If the investor buys into the strategy and the market is more volatile than expected, the strategy is likely to result in a loss. If the investor expects the market to be more volatile than he believes the market expects, the appropriate strategy could be to sell the butterfly spread. Doing so would involve selling the calls with exercise prices of  $X_1$  and  $X_3$  and buying two calls with exercise prices of  $X_2$ .<sup>14</sup>

Alternatively, a butterfly spread can be executed using puts. Note that the initial value of the spread using calls is  $V_0 = c_1 - 2c_2 + c_3$ . Recall that from put-call parity,  $c = p + S - X/(1 + r)^T$ . If we use the appropriate subscripts and substitute  $p_i + S - X_i/(1 + r)^T$  for  $c_i$  where  $i = 1, 2$ , and  $3$ , we obtain  $V_0 = p_1 - 2p_2 + p_3$ . The positive signs on  $p_1$  and  $p_3$  and the negative sign on  $2p_2$  mean that we could buy the puts with exercise prices  $X_1$  and  $X_3$  and sell two puts with exercise price of  $X_2$  to obtain the same result. We would, in effect, be buying a bear spread with puts consisting of buying the put with exercise price of  $X_3$  and selling the put with exercise price of  $X_2$ , and also selling a bear spread by selling the put with exercise price of  $X_2$  and buying the put with exercise price of  $X_1$ . If the options are priced correctly, it does not really matter whether we use puts or calls.<sup>15</sup>

### EXAMPLE 7

Consider three put options on a currency that is currently selling for \$1.45. The exercise prices are \$1.30, \$1.40, and \$1.50. The put prices are \$0.08, \$0.125, and \$0.18, respectively. The puts all expire at the same time. Answer the following questions about a butterfly spread.

- A. Determine the value at expiration and the profit under the following outcomes:
  - i. The price of the currency at expiration is \$1.26.
  - ii. The price of the currency at expiration is \$1.35.
  - iii. The price of the currency at expiration is \$1.47.
  - iv. The price of the currency at expiration is \$1.59.

<sup>14</sup>A short butterfly spread is sometimes called a **sandwich spread**.

<sup>15</sup>If puts were underpriced, it would be better to buy the butterfly spread using puts. If calls were underpriced, it would be better to buy the butterfly spread using calls. Of course, other strategies could also be used to take advantage of any mispricing.

- B. Determine the following:
- the maximum profit.
  - the maximum loss.
- C. Determine the breakeven currency price at expiration.

*Solutions:*

A. i.  $V_T = \max(0, X_1 - S_T) - 2 \max(0, X_2 - S_T) + \max(0, X_3 - S_T)$   
 $= \max(0, 1.30 - 1.26) - 2 \max(0, 1.40 - 1.26) + \max(0, 1.50 - 1.26)$   
 $= 0.04 - 2(0.14) + 0.24 = 0.0$   
 $\Pi = V_T - V_0 = V_T - (p_1 - 2p_2 + p_3) = 0.0 - [0.08 - 2(0.125) + 0.18]$   
 $= -0.01$

ii.  $V_T = \max(0, X_1 - S_T) - 2 \max(0, X_2 - S_T) + \max(0, X_3 - S_T)$   
 $= \max(0, 1.30 - 1.35) - 2 \max(0, 1.40 - 1.35) + \max(0, 1.50 - 1.35)$   
 $= 0.0 - 2(0.05) + 0.15 = 0.05$   
 $\Pi = V_T - V_0 = V_T - (p_1 - 2p_2 + p_3)$   
 $= 0.05 - [0.08 - 2(0.125) + 0.18] = 0.04$

iii.  $V_T = \max(0, X_1 - S_T) - 2 \max(0, X_2 - S_T) + \max(0, X_3 - S_T)$   
 $= \max(0, 1.30 - 1.47) - 2 \max(0, 1.40 - 1.47) + \max(0, 1.50 - 1.47)$   
 $= 0.0 - 2(0) + 0.03 = 0.03$   
 $\Pi = V_T - V_0 = V_T - (p_1 - 2p_2 + p_3)$   
 $= 0.03 - [0.08 - 2(0.125) + 0.18] = 0.02$

iv.  $V_T = \max(0, X_1 - S_T) - 2 \max(0, X_2 - S_T) + \max(0, X_3 - S_T)$   
 $= \max(0, 1.30 - 1.59) - 2 \max(0, 1.40 - 1.59) + \max(0, 1.50 - 1.59)$   
 $= 0.0 - 2(0) + 0.0 = 0.0$   
 $\Pi = V_T - V_0 = V_T - (p_1 - 2p_2 + p_3)$   
 $= 0.0 - [0.08 - 2(0.125) + 0.18] = -0.01$

- B. i. Maximum profit  $= X_3 - X_2 - (p_1 - 2p_2 + p_3)$   
 $= 1.50 - 1.40 - [0.08 - 2(0.125) + 0.18] = 0.09$
- ii. Maximum loss  $= p_1 - 2p_2 + p_3$   
 $= 0.08 - 2(0.125) + 0.18 = 0.01$
- C.  $S_T^* = X_1 + p_1 - 2p_2 + p_3 = 1.30 + 0.08 - 2(0.125) + 0.18 = 1.31$   
 $S_T^* = 2X_2 - X_1 - p_1 + 2p_2 - p_3 = 2(1.40) - 1.30 - 0.08$   
 $+ 2(0.125) - 0.18 = 1.49$

So far, we have restricted ourselves to the use of either calls or puts, but not both. We now look at strategies that involve positions in calls *and* puts.

## 2.4. Combinations of Calls and Puts

### 2.4.1. Collars

Recall that in Section 2.2 we examined the protective put. In that strategy, the holder of the underlying asset buys a put to provide protection against downside loss. Purchasing the put requires the payment of the put premium. One way to get around paying the put premium is to sell another option with a premium equal to the put premium, which can be done by selling a call with an exercise price above the current price of the underlying.

Although it is not necessary that the call premium offset the put premium, and the call premium can even be more than the put premium, the typical collar has the call and put premiums offset. When this offsetting occurs, no net premium is required up front. In effect, the holder of the asset gains protection below a certain level, the exercise price of the put, and pays for it by giving up gains above a certain level, the exercise price of the call. This strategy is called a **collar**. When the premiums offset, it is sometimes called a **zero-cost collar**. This term is a little misleading, however, as it suggests that there is no “cost” to this transaction. The cost takes the form of forgoing upside gains. The term “zero-cost” refers only to the fact that no cash is paid up front.

A collar is a modified version of a protective put and a covered call and requires different exercise prices for each. Let the put exercise price be  $X_1$  and the call exercise price be  $X_2$ . With  $X_1$  given, it is important to see that  $X_2$  is not arbitrary. If we want the call premium to offset the put premium, the exercise price on the call must be set such that the price of the call equals the price of the put. We thus can select any exercise price of the put. Then the call exercise price is selected by determining which exercise price will produce a call premium equal to the put premium. Although the put can have any exercise price, typically the put exercise price is lower than the current value of the underlying. The call exercise price then must be above the current value of the underlying.<sup>16</sup>

So let  $X_1$  be set. The put with this exercise price has a premium of  $p_1$ . We now need to set  $X_2$  such that the premium on the call,  $c_2$ , equals the premium on the put,  $p_1$ . To do so, we need to use an option valuation model, such as Black–Scholes–Merton, to find the exercise price of the call that will make  $c_2 = p_1$ . Recall that the Black–Scholes–Merton formula is

$$c = S_0 N(d_1) - X e^{-r^c T} N(d_2)$$

where

$$d_1 = \frac{\ln(S_0/X) + (r^c + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and where  $r^c$  is the continuously compounded risk-free rate and  $N(d_1)$  and  $N(d_2)$  are normal probabilities associated with the values  $d_1$  and  $d_2$ . Ideally we would turn the equation around and solve for  $X$  in terms of  $c$ , but the equation is too complex to be able to isolate  $X$  on one

<sup>16</sup>It can be proven in general that the call exercise price would have to be above the current value of the underlying. Intuitively, it can be shown through put–call parity that if the call and put exercise prices were equal to the current value of the underlying, the call would be worth more than the put. If we lower the put exercise price below the price of the underlying, the put price would decrease. Then the gap between the call and put prices would widen further. We would then need to raise the call exercise price above the current price of the underlying to make its premium come down.

side. So, we must solve for  $X$  by trial and error. We substitute in values of  $X$  until the option price equals  $c$ , where  $c$  is the call premium that we want to equal the put premium.

Consider the Nasdaq example. Suppose we use the put with exercise price of 1950. Its premium is 56.01. So now we need a call with a premium of 56.01. The call with exercise price of 2000 is worth 81.75. So to get a lower call premium, we need a call with an exercise price higher than 2000. By trial and error, we insert higher and higher exercise prices until the call premium falls to 56.01, which occurs at an exercise price of about 2060.<sup>17</sup> So now we have it. We buy the put with an exercise price of 1950 for 56.01 and sell the call with exercise price of 2060 for 56.01. This transaction requires no cash up front.

The value of the position at expiration is the sum of the value of the underlying asset, the value of the put, and the value of the short call:

$$V_T = S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2)$$

Broken down into ranges, we have

$$\begin{aligned} V_T &= S_T + X_1 - S_T - 0 = X_1 && \text{if } S_T \leq X_1 \\ V_T &= S_T + 0 - 0 = S_T && \text{if } X_1 < S_T < X_2 \\ V_T &= S_T + 0 - (S_T - X_2) = X_2 && \text{if } S_T \geq X_2 \end{aligned}$$

The initial value of the position is simply the value of the underlying asset,  $S_0$ . The profit is  $V_T - V_0$ :

$$\Pi = S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2) - S_0$$

Broken down into ranges, we have

$$\begin{aligned} \Pi &= X_1 - S_0 && \text{if } S_T \leq X_1 \\ \Pi &= S_T - S_0 && \text{if } X_1 < S_T < X_2 \\ \Pi &= X_2 - S_0 && \text{if } S_T \geq X_2 \end{aligned}$$

Using our example where  $X_1 = 1950$ ,  $p_1 = 56.01$ ,  $X_2 = 2060$ ,  $c_2 = 56.01$ , and  $S_0 = 2000$ , we obtain the following values at expiration:

If  $S_T = 1900$ ,  $V_T = 1950$

If  $S_T = 2000$ ,  $V_T = 2000$

If  $S_T = 2100$ ,  $V_T = 2060$

The profit for  $S_T = 1900$  is  $\Pi = 1950 - 2000 = -50$ .

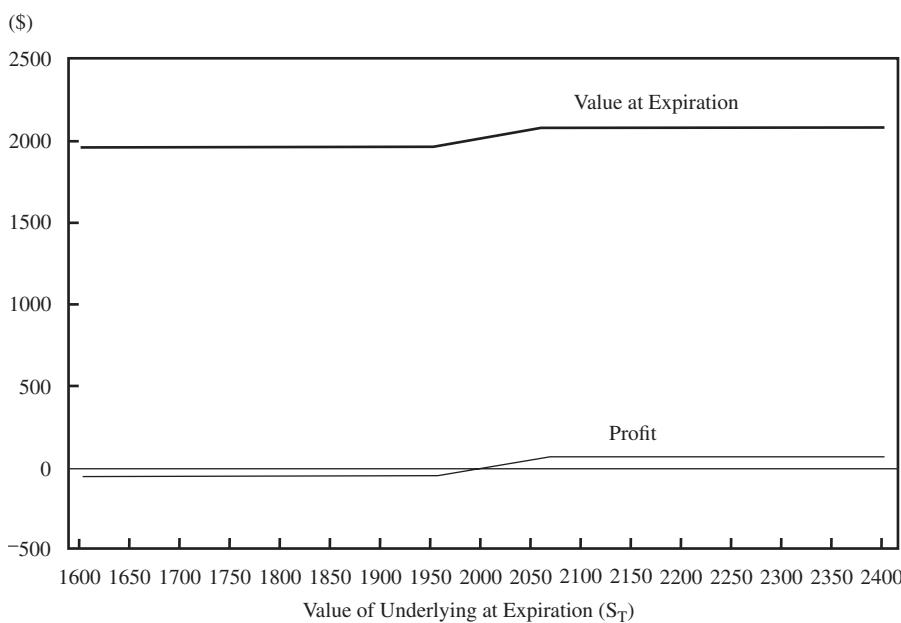
If  $S_T = 2000$ ,  $\Pi = 2000 - 2000 = 0$

If  $S_T = 2100$ ,  $\Pi = 2060 - 2000 = 60$

<sup>17</sup>The other necessary information to obtain the exercise price of the call is that the volatility is 0.35, the risk-free rate is 0.02, and the dividend yield is 0.005. The actual call price at a stock price of 2060 is 56.18. At 2061, the call price is 55.82. Thus, the correct exercise price lies between 2060 and 2061; we simply round to 2060.

A graph of this strategy is shown in Exhibit 11. Note that the lines are flat over the range of  $S_T$  up to the put exercise price of 1950 and in the range beyond the call exercise price of 2060. Below 1950, the put provides protection against loss. Above 2060, the short call forces a relinquishment of the gains, which are earned by the buyer of the call. In between these ranges, neither the put nor the call has value. The profit is strictly determined by the underlying and moves directly with the value of the underlying. The maximum profit is  $X_2 - S_0$ , which here is  $2060 - 2000 = 60$ , a return of 3%. The maximum loss is  $S_0 - X_1$ , which here is  $2000 - 1950 = 50$ , a loss of 2.5%. Keep in mind that these options have lives of one month, so those numbers represent one-month returns. The breakeven is simply the original underlying price of 2000.

EXHIBIT 11 Zero-Cost Collar (Buy Put with Exercise Price  $X_1$ , Sell Call with Exercise Price  $X_2$ , Put and Call Premiums Offset)



In summary, for the collar

$$\text{Value at expiration: } V_T = S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2)$$

$$\text{Profit: } \Pi = V_T - S_0$$

$$\text{Maximum profit} = X_2 - S_0$$

$$\text{Maximum loss} = S_0 - X_1$$

$$\text{Breakeven: } S_T^* = S_0$$

Collars are also known as range forwards and risk reversals.<sup>18</sup> Asset managers often use them to guard against losses without having to pay cash up front for the protection. Clearly,

<sup>18</sup> It is not clear why a collar is sometimes called a risk reversal. It is clear, however, why a collar is sometimes called a range forward. Like a forward contract, it requires no initial outlay other than for the underlying. Unlike a forward contract, which has a strictly linear payoff profile, the collar payoff breaks at the two exercise prices, thus creating a range.

however, they are virtually the same as bull spreads. The latter has a cap on the gain and a floor on the loss but does not involve actually holding the underlying. In Section 3 we shall encounter this strategy again in the form of an interest rate collar, which protects floating-rate borrowers against high interest rates.

### EXAMPLE 8

The holder of a stock worth \$42 is considering placing a collar on it. A put with an exercise price of \$40 costs \$5.32. A call with the same premium would require an exercise price of \$50.59.

- A. Determine the value at expiration and the profit under the following outcomes:
  - i. The price of the stock at expiration is \$55.
  - ii. The price of the stock at expiration is \$48.
  - iii. The price of the stock at expiration is \$35.
- B. Determine the following:
  - i. the maximum profit.
  - ii. the maximum loss.
- C. Determine the breakeven stock price at expiration.

*Solutions:*

$$\begin{aligned}
 \text{A. i. } V_T &= S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2) \\
 &= 55 + \max(0, 40 - 55) - \max(0, 55 - 50.59) \\
 &= 55 + 0 - (55 - 50.59) = 50.59 \\
 \Pi &= V_T - S_0 = 50.59 - 42 = 8.59
 \end{aligned}$$

$$\begin{aligned}
 \text{ii. } V_T &= S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2) \\
 &= 48 + \max(0, 40 - 48) - \max(0, 48 - 50.59) \\
 &= 48 + 0 - 0 = 48 \\
 \Pi &= V_T - S_0 = 48 - 42 = 6
 \end{aligned}$$

$$\begin{aligned}
 \text{iii. } V_T &= S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2) \\
 &= 35 + \max(0, 40 - 35) - \max(0, 35 - 50.59) \\
 &= 35 + 5 - 0 = 40 \\
 \Pi &= V_T - S_0 = 40 - 42 = -2
 \end{aligned}$$

- B. i. Maximum profit =  $X_2 - S_0 = 50.59 - 42 = 8.59$
- ii. Maximum loss =  $S_0 - X_1 = 42 - 40 = 2$
- C.  $S_T^* = S_0 = 42$

Collars are one of the many directional strategies, meaning that they perform based on the direction of the movement in the underlying. Of course, butterfly spreads perform based on the volatility of the underlying. Another strategy in which performance is based on the volatility of the underlying is the straddle.

#### 2.4.2. Straddle

To justify the purchase of a call, an investor must be bullish. To justify the purchase of a put, an investor must be bearish. What should an investor do if he believes the market will be volatile but does not feel particularly strongly about the direction? We discussed earlier that a short butterfly spread is one strategy. It benefits from extreme movements, but its gains are limited. There are other, more complex strategies, such as time spreads, that can benefit from high volatility; however, one simple strategy, the **straddle**, also benefits from high volatility.

Suppose the investor buys both a call and a put with the same exercise price on the same underlying with the same expiration. This strategy enables the investor to profit from upside or downside moves. Its cost, however, can be quite heavy. In fact, a straddle is a wager on a large movement in the underlying.

The value of a straddle at expiration is the value of the call and the value of the put:  $V_T = \max(0, S_T - X) + \max(0, X - S_T)$ . Broken down into ranges,

$$\begin{aligned} V_T &= X - S_T && \text{if } S_T < X \\ V_T &= S_T - X && \text{if } S_T \geq X \end{aligned}$$

The initial value of the straddle is simply  $V_0 = c_0 + p_0$ . The profit is  $V_T - V_0$  or  $\Pi = \max(0, S_T - X) + \max(0, X - S_T) - c_0 - p_0$ . Broken down into ranges,

$$\begin{aligned} \Pi &= X - S_T - c_0 - p_0 && \text{if } S_T < X \\ \Pi &= S_T - X - c_0 - p_0 && \text{if } S_T \geq X \end{aligned}$$

In our example, let  $X = 2000$ . Then  $c_0 = 81.75$  and  $p_0 = 79.25$ .

If  $S_T = 2100$ , the value of the position at expiration is

$$V_T = 2100 - 2000 = 100$$

If  $S_T = 1900$ , the value of the position at expiration is

$$V_T = 2000 - 1900 = 100$$

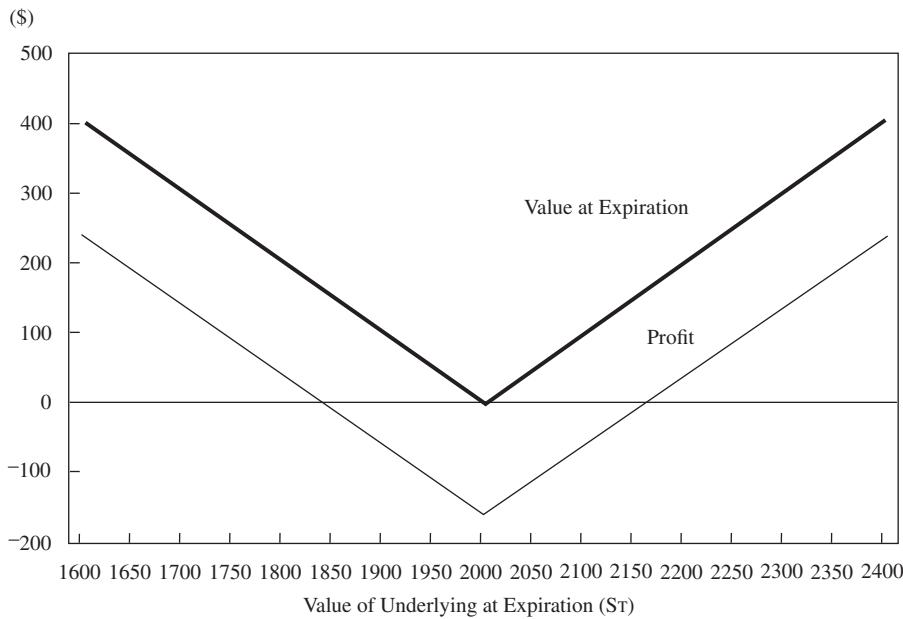
If  $S_T = 2100$ , the profit is  $\Pi = 100 - 81.75 - 79.25 = -61$

If  $S_T = 1900$ , the profit is  $\Pi = 100 - 81.75 - 79.25 = -61$

Note the symmetry, whereby a move of 100 in either direction results in a change in value of 61. The put and call payoffs are obviously symmetric. It is also apparent that these outcomes are below breakeven.

Observe the results in Exhibit 12. Note that the value and profit are V-shaped, thereby benefiting from large moves in the underlying in either direction. Like the call option the straddle contains, the gain on the upside is unlimited. Like the put, the downside gain is not unlimited, but it is quite large. The underlying can go down no further than zero. Hence, on the downside the maximum profit is  $X - c_0 - p_0$ , which in this case is  $2000 - 81.75 - 79.25 = 1839$ . The maximum loss occurs if the underlying ends up precisely at the exercise price. In that case, neither the call nor the put expires with value and the premiums are lost on both. Therefore, the maximum loss is  $c_0 + p_0$ , which is  $81.75 + 79.25 = 161$ .

## EXHIBIT 12 Straddle (Buy Call and Put with Exercise Price X)



There are two breakevens. Using  $S_T^*$  to denote the breakevens, we set each profit equation to zero and solve for  $S_T^*$ :

If  $S_T \geq X$ ,

$$\begin{aligned}\Pi &= S_T^* - X - c_0 - p_0 = 0 \\ S_T^* &= X + c_0 + p_0\end{aligned}$$

If  $S_T < X$ ,

$$\begin{aligned}\Pi &= X - S_T^* - c_0 - p_0 = 0 \\ S_T^* &= X - c_0 - p_0\end{aligned}$$

The breakevens thus equal the exercise price plus or minus the premiums. So in this case, the breakevens are  $2000 \pm 161 = 2161$  and  $1839$ . A move of 161 is a percentage move of 8.1% over a one-month period. Hence, in this example, the purchase of a straddle is a bet that the underlying will move at nearly a 100% annual rate over a one-month period, quite a risky bet. An investor would make such a bet only when he felt that the underlying would be exceptionally volatile. An obvious time to use a straddle would be around major events such as earnings announcements. But because earnings announcements are known and anticipated events, the greater uncertainty surrounding them should already be reflected in the options' prices. Recall that the greater the volatility, the higher the prices of both puts and calls. Therefore, using a straddle in anticipation of an event that everyone knows is coming is not necessarily a good idea. Only when the investor believes the market will be more volatile than everyone else believes would a straddle be advised.

In summary, for a straddle

Value at expiration:  $V_T = \max(0, S_T - X) + \max(0, X - S_T)$

Profit:  $\Pi = V_T - (c_0 + p_0)$

Maximum profit =  $\infty$

Maximum loss =  $c_0 + p_0$

Breakeven:  $S_T^* = X \pm (c_0 + p_0)$

As we have noted, a straddle would tend to be used by an investor who is expecting the market to be volatile but does not have strong feelings one way or the other on the direction. An investor who leans one way or the other might consider adding a call or a put to the straddle. Adding a call to a straddle is a strategy called a **strap**, and adding a put to a straddle is called a **strip**. It is even more difficult to make a gain from these strategies than it is for a straddle, but if the hoped-for move does occur, the gains are leveraged. Another variation of the straddle is a **strangle**, in which the put and call have different exercise prices. This strategy creates a graph similar to a straddle but with a flat section instead of a point on the bottom.

### EXAMPLE 9

Consider a stock worth \$49. A call with an exercise price of \$50 costs \$6.25 and a put with an exercise price of \$50 costs \$5.875. An investor buys a straddle.

- Determine the value at expiration and the profit under the following outcomes:
  - The price of the stock at expiration is \$61.
  - The price of the stock at expiration is \$37.
- Determine the following:
  - the maximum profit.
  - the maximum loss.
- Determine the breakeven stock price at expiration.

*Solutions:*

- $$\begin{aligned} \text{i. } V_T &= \max(0, S_T - X) + \max(0, X - S_T) \\ &= \max(0, 61 - 50) + \max(0, 50 - 61) = 11 - 0 = 11 \\ \Pi &= V_T - (c_0 + p_0) = 11 - (6.25 + 5.875) = -1.125 \end{aligned}$$

- $$\begin{aligned} \text{i. } V_T &= \max(0, S_T - X) + \max(0, X - S_T) \\ &= \max(0, 37 - 50) + \max(0, 50 - 37) = 0 + 13 = 13 \\ \Pi &= V_T - S_0 = 13 - (6.25 + 5.875) = 0.875 \end{aligned}$$

- Maximum profit =  $\infty$
  - Maximum loss =  $c_0 + p_0 = 6.25 + 5.875 = 12.125$
- $S_T^* = X \pm (c_0 + p_0) = 50 \pm (6.25 + 5.875) = 62.125, 37.875$

Now we turn to a strategy that combines more than one call and more than one put. It should not be surprising that we shall recognize this strategy as just a combination of something we have already learned.

### 2.4.3. Box Spreads

We can exploit an arbitrage opportunity with a neutral position many alternative ways: using put–call parity, using the binomial model, or using the Black–Scholes–Merton model. Exploiting put–call parity requires a position in the underlying. Using the binomial or Black–Scholes–Merton model requires that the model holds in the market. In addition, both models require a position in the underlying and an estimate of the volatility.

A **box spread** can also be used to exploit an arbitrage opportunity but it requires that neither the binomial nor Black–Scholes–Merton model holds, it needs no estimate of the volatility, and all of the transactions can be executed within the options market, making implementation of the strategy simpler, faster, and with lower transaction costs.

In basic terms, a box spread is a combination of a bull spread and a bear spread. Suppose we buy the call with exercise price  $X_1$  and sell the call with exercise price  $X_2$ . This set of transactions is a bull spread. Then we buy the put with exercise price  $X_2$  and sell the put with exercise price  $X_1$ . This is a bear spread. Intuitively, it should sound like a combination of a bull spread and a bear spread would leave the investor with a fairly neutral position, and indeed, that is the case.

The value of the box spread at expiration is

$$V_T = \max(0, S_T - X_1) - \max(0, S_T - X_2) + \max(0, X_2 - S_T) - \max(0, X_1 - S_T)$$

Broken down into ranges, we have

$$\begin{aligned} V_T &= 0 - 0 + X_2 - S_T - (X_1 - S_T) = X_2 - X_1 && \text{if } S_T \leq X_1 \\ V_T &= S_T - X_1 - 0 + X_2 - S_T - 0 = X_2 - X_1 && \text{if } X_1 < S_T < X_2 \\ V_T &= S_T - X_1 - (S_T - X_2) + 0 - 0 = X_2 - X_1 && \text{if } S_T \geq X_2 \end{aligned}$$

These outcomes are all the same. In each case, two of the four options expire in-the-money, and the other two expire out-of-the-money. In each case, the holder of the box spread ends up buying the underlying with one option, using either the long call at  $X_1$  or the short put at  $X_1$ , and selling the underlying with another option, using either the long put at  $X_2$  or the short call at  $X_2$ . The box spread thus results in buying the underlying at  $X_1$  and selling it at  $X_2$ . This outcome is known at the start.

The initial value of the transaction is the value of the long call, short call, long put, and short put,  $V_0 = c_1 - c_2 + p_2 - p_1$ . The profit is, therefore,  $\Pi = X_2 - X_1 - c_1 + c_2 - p_2 + p_1$ .

In contrast to all of the other strategies, the outcome is simple. In all cases, we end up with the same result. Using the options with exercise prices of 1950 and 2050, which have premiums of  $c_1 = 108.43$ ,  $c_2 = 59.98$ ,  $p_1 = 56.01$ , and  $p_2 = 107.39$ , the value at expiration is always  $2050 - 1950 = 100$  and the profit is always  $\Pi = 100 - 108.43 + 59.98 - 107.39 + 56.01 = 0.17$ . This value may seem remarkably low. We shall see why momentarily.

The initial value of the box spread is  $c_1 - c_2 + p_2 - p_1$ . The payoff at expiration is  $X_2 - X_1$ . Because the transaction is risk free, the present value of the payoff, discounted using the risk-free rate, should equal the initial outlay. Hence, we should have

$$(X_2 - X_1)/(1+r)^T = c_1 - c_2 + p_2 - p_1$$

If the present value of the payoff exceeds the initial value, the box spread is underpriced and should be purchased.

In this example, the initial outlay is  $V_0 = 108.43 - 59.98 + 107.39 - 56.01 = 99.83$ . To obtain the present value of the payoff, we need an interest rate and time to expiration. The prices of these options were obtained using a time to expiration of one month and a risk-free rate of 2.02%. The present value of the payoff is

$$(X_2 - X_1)/(1+r) = (2050 - 1950)/(1.0202)^{1/12} = 99.83$$

In other words, this box spread is correctly priced. This result should not be surprising, because we noted that we used the Black–Scholes–Merton model to price these options. The model should not allow arbitrage opportunities of any form.

Recall that the profit from this transaction is 0.17, a very low value. This profit reflects the fact that the box spread is purchased at 99.83 and matures to a value of 100, a profit of 0.17, which is a return of the risk-free rate for one month.<sup>19</sup> The reason the profit seems so low is that it is just the risk-free rate.

Let us assume that one of the long options, say the put with exercise price of 2050, is underpriced. Let its premium be 105 instead of 107.39. Then the net premium would be  $108.43 - 59.98 + 105 - 56.01 = 97.44$ . Again, the present value of the payoff is 99.83. Hence, the box spread would generate a gain in value clearly in excess of the risk-free rate. If some combination of the options was such that the net premium is more than the present value of the payoff, then the box spread would be overpriced. Then we should sell the  $X_1$  call and  $X_2$  put and buy the  $X_2$  call and  $X_1$  put. Doing so would generate an outlay at expiration with a present value less than the initial value.

So to summarize the box spread, we say that

Value at expiration:  $V_T = X_2 - X_1$

Profit:  $\Pi = X_2 - X_1 - (c_1 - c_2 + p_2 - p_1)$

Maximum profit = (same as profit)

Maximum loss = (no loss is possible, given fair option prices)

Breakeven: no breakeven; the transaction always earns the risk-free rate, given fair option prices

### EXAMPLE 10

Consider a box spread consisting of options with exercise prices of 75 and 85. The call prices are 16.02 and 12.28 for exercise prices of 75 and 85, respectively. The put prices are 9.72 and 15.18 for exercise prices of 75 and 85, respectively. The options expire in six months and the discrete risk-free rate is 5.13%.

- A. Determine the value of the box spread and the profit for any value of the underlying at expiration.
- B. Show that this box spread is priced such that an attractive opportunity is available.

<sup>19</sup>That is,  $99.83(1.0202)^{1/12} \approx 100$ . Hence, the profit of 0.17 is about 2.02%, for one month.

*Solutions:*

- A. The box spread always has a value at expiration of  $X_2 - X_1 = 85 - 75 = 10$

$$\begin{aligned}\Pi &= V_T - (c_1 - c_2 + p_2 - p_1) \\ &= 10 - (16.02 - 12.28 + 15.18 - 9.72) = 0.80\end{aligned}$$

- B. The box spread should be worth  $(X_2 - X_1)/(1 + r)^T$ , or

$$(85 - 75)/(1.0513)^{0.5} = 9.75$$

The cost of the box spread is  $16.02 - 12.28 + 15.18 - 9.72 = 9.20$ . The box spread is thus underpriced. At least one of the long options is priced too low or at least one of the short options is priced too high; we cannot tell which. Nonetheless, we can execute this box spread, buying the call with exercise price  $X_1 = 75$  and put with exercise price  $X_2 = 85$  and selling the call with exercise price  $X_2 = 85$  and put with exercise price  $X_1 = 75$ . This would cost 9.20. The present value of the payoff is 9.75. Therefore, the box spread would generate an immediate increase in value of 0.55.

We have now completed our discussion of equity option strategies. Although the strategies are applicable, with minor changes, to fixed-income securities, we shall not explore that area here. We shall, however, look at interest rate option strategies, which require some significant differences in presentation and understanding compared with equity option strategies.

### 3. INTEREST RATE OPTION STRATEGIES

Consider a group of options in which the underlying is an interest rate and the exercise price is expressed in terms of a rate. Recall that this group of options consists of calls, which pay off if the option expires with the underlying interest rate above the exercise rate, and puts, which pay off if the option expires with the underlying interest rate below the exercise rate. Interest rate call and put options are usually purchased to protect against changes in interest rates. For dollar-based interest rate derivatives, the underlying is usually Libor but is always a specific rate, such as the rate on a 90- or 180-day underlying instrument. An interest rate option is based on a specific notional principal, which determines the payoff when the option is exercised. Traditionally, the payoff does not occur immediately upon exercise but is delayed by a period corresponding to the life of the underlying instrument from which the interest rate is taken, an issue we review below.

The payoff of an interest rate call option is

$$\begin{aligned}&(\text{Notional principal}) \max (0, \text{Underlying rate at expiration} \\ &\quad - \text{Exercise rate}) \left( \frac{\text{Days in underlying rate}}{360} \right)\end{aligned}$$

where “days in underlying” refers to the maturity of the instrument from which the underlying rate is taken. In some cases, “days in underlying” may be the exact day count during a period. For example, if an interest rate option is used to hedge the interest paid over an  $m$ -day

period, then “days in underlying” would be  $m$ . Even though Libor of 30, 60, 90, 180 days, etc., whichever is closest to  $m$ , might be used as the underlying rate, the actual day count would be  $m$ , the exact number of days. In such cases, the payment date is usually set at 30, 60, 90, 180, etc. days after the option expiration date. So, for example, 180-day Libor might be used as the underlying rate, and “days in underlying” could be 180 or perhaps 182, 183, etc. The most important point, however, is that the rate is determined on one day, the option expiration, and payment is made  $m$  days later. This practice is standard in floating-rate loans and thus is used with interest rate options, which are designed to manage the risk of floating-rate loans.

Likewise, the payoff of an interest rate put is

$$\begin{aligned} & (\text{Notional principal}) \max (0, \text{Exercise rate} \\ & - \text{Underlying rate at expiration}) \left( \frac{\text{Days in underlying rate}}{360} \right) \end{aligned}$$

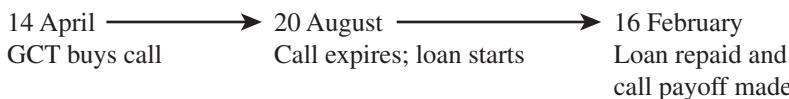
Now let us take a look at some applications of interest rate options.

### 3.1. Using Interest Rate Calls with Borrowing

Let us examine an application of an interest rate call to establish a maximum interest rate for a loan to be taken out in the future. In brief, a company can buy an interest rate call that pays off from increases in the underlying interest rate beyond a chosen level. The call payoff then compensates for the higher interest rate the company has to pay on the loan.

Consider the case of a company called Global Computer Technology (GCT), which occasionally takes out short-term loans in US dollars with the rate tied to Libor. Anticipating that it will take out a loan at a later date, GCT recognizes the potential for an interest rate increase by that time. In this example, today is 14 April, and GCT expects to borrow \$40 million on 20 August at Libor plus 200 basis points. The loan will involve the receipt of the money on 20 August with full repayment of principal and interest 180 days later on 16 February. GCT would like protection against higher interest rates, so it purchases an interest rate call on 180-day Libor to expire on 20 August. GCT chooses an exercise rate of 5%. This option gives it the right to receive an interest payment of the difference between the 20 August Libor and 5%. If GCT exercises the option on 20 August, the payment will occur 180 days later on 16 February when the loan is paid off. The cost of the call is \$100,000, which is paid on 14 April. Libor on 14 April is 5.5%.

The transaction is designed such that if Libor is above 5% on 20 August, GCT will benefit and be protected against increases in interest rates. To determine how the transaction works, we need to know the effective rate on the loan. Note that the sequence of events is as follows:



So cash is paid for the call on 14 April. Cash proceeds from the loan are received on 20 August. On 16 February, the loan is repaid and the call payoff (if any) is made.

To evaluate the effectiveness of the overall transaction, we need to determine how the call affects the loan. Therefore, we need to incorporate the payment of the call premium on 14 April into the cash flow on the loan. So, it would be appropriate to compound the call

premium from 14 April to 20 August. In effect, we need to know what the call, purchased on 14 April, effectively costs on 20 August. We compound its premium for the 128 days from 14 April to 20 August at the rate at which GCT would have to borrow on 14 April. This rate would be Libor on 14 April plus 200 basis points, or 7.5%. The call premium thus effectively costs

$$\$100,000 \left[ 1 + 0.075 \left( \frac{128}{360} \right) \right] = \$102,667$$

on 20 August.<sup>20</sup> On that date, GCT takes out the loan, thereby receiving \$40 million. We should, however, reduce this amount by \$102,667, because GCT effectively receives less money because it must buy the call. So, the loan proceeds are effectively  $\$40,000,000 - \$102,667 = \$39,897,333$ .

Next we must calculate the amount of interest paid on the loan and the amount of any call payoff. Let us assume that Libor on 20 August is 8%. In that case, the loan rate will be 10%. The interest on the loan will be

$$\$40,000,000 (0.10) \left( \frac{180}{360} \right) = \$2,000,000$$

This amount, plus \$40 million principal, is repaid on 16 February. With Libor assumed to be 8% on 20 August, the option payoff is

$$\$40,000,000 \max (0, 0.08 - 0.05) \left( \frac{180}{360} \right) = \$40,000,000 (0.03) \left( \frac{180}{360} \right) = \$600,000$$

This amount is paid on 16 February. The effective interest paid on 16 February is thus  $\$2,000,000 - \$600,000 = \$1,400,000$ . So, GCT effectively receives \$39,897,333 on 20 August and pays back \$40,000,000 plus \$1,400,000 or \$41,400,000 on 16 February. The effective annual rate is

$$\left( \frac{\$41,400,000}{\$39,897,333} \right)^{365/180} - 1 = 0.0779$$

Exhibit 13 presents a complete description of the transaction and the results for a range of possible Libors on 20 August. Exhibit 14 illustrates the effective loan rate compared with Libor on 20 August. We see that the strategy places an effective ceiling on the rate on the loan of about 7.79% while enabling GCT to benefit from decreases in Libor. Of course, a part of this maximum rate is the 200 basis point spread over Libor that GCT must pay.<sup>21</sup> In effect, the company's maximum rate without the spread is 5.79%. This reflects the exercise rate of 5% plus the effect of the option premium.

<sup>20</sup>The interpretation of this calculation is that GCT's cost of funds is 7.5%, making the option premium effectively \$102,667 by the time the loan is taken out.

<sup>21</sup>It should be noted that the effective annual rate is actually more than 200 basis points. For example, if someone borrows \$100 at 2% for 180 days, the amount repaid would be  $\$100[1 + 0.02(180/360)] = \$101$ . The effective annual rate would be  $(\$101/\$100)^{365/180} - 1 = 0.0204$ .

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**EXHIBIT 13 Outcomes for an Anticipated Loan Protected with an Interest Rate Call**


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**Scenario (14 April)**

Global Computer Technology (GCT) is a US corporation that occasionally undertakes short-term borrowings in US dollars with the rate tied to Libor. To facilitate its cash flow planning, it buys an interest rate call to put a ceiling on the rate it pays while enabling it to benefit if rates fall. A call gives GCT the right to receive the difference between Libor on the expiration date and the exercise rate it chooses when it purchases the option. The payoff of the call is determined on the expiration date, but the payment is not received until a certain number of days later, corresponding to the maturity of the underlying Libor. This feature matches the timing of the interest payment on the loan.

**Action**

GCT determines that it will borrow \$40 million at Libor plus 200 basis points on 20 August. The loan will be repaid with a single payment of principal and interest 180 days later on 16 February.

To protect against increases in Libor between 14 April and 20 August, GCT buys a call option on Libor with an exercise rate of 5% to expire on 20 August with the underlying being 180-day Libor. The call premium is \$100,000. We summarize the information as follows:

Loan amount	\$40,000,000
Underlying	180-day Libor
Spread	200 basis points over Libor
Current Libor	5.5%
Expiration	20 August (128 days later)
Exercise rate	5%
Call premium	\$100,000

**Scenario (20 August)**

Libor on 20 August is 8%.

**Outcome and Analysis**

For any Libor, the call payoff at expiration is given below and will be received 180 days later:

$$\$40,000,000 \max (0, \text{Libor} - 0.05) \left( \frac{180}{360} \right)$$

For Libor of 8%, the payoff is

$$\$40,000,000 \max (0, 0.08 - 0.05) \left( \frac{180}{360} \right) = \$600,000$$

The premium compounded from 14 April to 20 August at the original Libor of 5.5% plus 200 basis points is

$$\$100,000 \left[ 1 + (0.055 + 0.02) \left( \frac{128}{360} \right) \right] = \$102,667$$

So the call costs \$100,000 on 14 April, which is equivalent to \$102,667 on 20 August. The effective loan proceeds are  $\$40,000,000 - \$102,667 = \$39,897,333$ . The loan interest is

$$\$40,000,000 (\text{Libor on 20 August} + 200 \text{ Basis points}) \left( \frac{180}{360} \right)$$

For Libor of 8%, the loan interest is

$$\$40,000,000 (0.08 + 0.02) \left( \frac{180}{360} \right) = \$2,000,000$$

The call payoff was given above. The loan interest minus the call payoff is the effective interest. The effective rate on the loan is

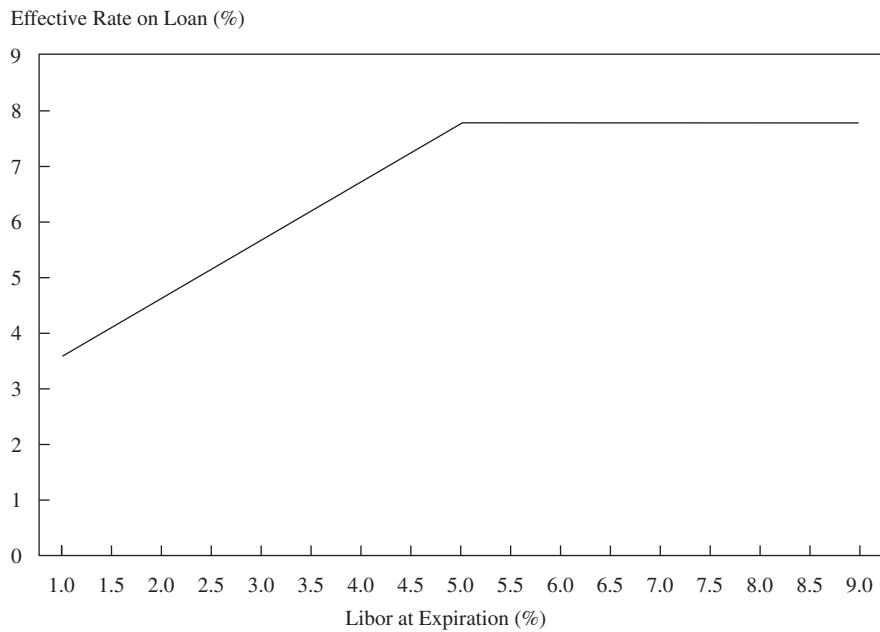
$$\begin{aligned} & \left( \frac{\$40,000,000 \text{ plus Effective interest}}{\$39,897,333} \right)^{365/180} \\ & -1 = \left( \frac{\$40,000,000 + \$2,000,000 - \$600,000}{\$39,897,333} \right)^{365/180} - 1 = 0.0779 \end{aligned}$$

or 7.79%.

The results are shown below for a range of Libors on 20 August.

Libor on 20 August	Loan Rate	Loan Interest Paid on 16 February	Call Payoff	Effective Interest	Effective Loan Rate
0.010	0.030	\$600,000	\$0	\$600,000	0.0360
0.015	0.035	700,000	0	700,000	0.0412
0.020	0.040	800,000	0	800,000	0.0464
0.025	0.045	900,000	0	900,000	0.0516
0.030	0.050	1,000,000	0	1,000,000	0.0568
0.035	0.055	1,100,000	0	1,100,000	0.0621
0.040	0.060	1,200,000	0	1,200,000	0.0673
0.045	0.065	1,300,000	0	1,300,000	0.0726
0.050	0.070	1,400,000	0	1,400,000	0.0779
0.055	0.075	1,500,000	100,000	1,400,000	0.0779
0.060	0.080	1,600,000	200,000	1,400,000	0.0779
0.065	0.085	1,700,000	300,000	1,400,000	0.0779
0.070	0.090	1,800,000	400,000	1,400,000	0.0779
0.075	0.095	1,900,000	500,000	1,400,000	0.0779
0.080	0.100	2,000,000	600,000	1,400,000	0.0779
0.085	0.105	2,100,000	700,000	1,400,000	0.0779
0.090	0.110	2,200,000	800,000	1,400,000	0.0779

**EXHIBIT 14** The Effective Rate on an Anticipated Future Loan Protected with an Interest Rate Call Option



**EXAMPLE 11**

On 10 January, ResTex Ltd. determines that it will need to borrow \$5 million on 15 February at 90-day Libor plus 300 basis points. The loan will be an add-on interest loan in which ResTex will receive \$5 million and pay it back plus interest on 16 May. To manage the risk associated with the interest rate on 15 February, ResTex buys an interest rate call that expires on 15 February and pays off on 16 May. The exercise rate is 5%, and the option premium is \$10,000. The current 90-day Libor is 5.25%. Assume that this rate, plus 300 basis points, is the rate it would borrow at for any period of up to 90 days if the loan were taken out today. Interest is computed on the exact number of days divided by 360.

Determine the effective annual rate on the loan for each of the following outcomes:

1. 90-day Libor on 15 February is 6%.
2. 90-day Libor on 15 February is 4%.

*Solutions:* First we need to compound the premium from 10 January to 15 February, which is 36 days. This calculation tells us the effective cost of the call as of the time the loan is taken out:

$$\$10,000 \left[ 1 + (0.0525 + 0.03) \left( \frac{36}{360} \right) \right] = \$10,083$$

The loan proceeds will therefore be  $\$5,000,000 - \$10,083 = \$4,989,917$ .

*Solution to 1:* Libor is 6%. The loan rate will be 9%.

The interest on the loan will be  $\$5,000,000(0.06 + 0.03) (90/360) = \$112,500$ .

The option payoff will be  $\$5,000,000 \max(0, 0.06 - 0.05) (90/360) = \$12,500$ .

Therefore, the effective interest will be  $\$112,500 - \$12,500 = \$100,000$ .

The effective rate on the loan will be

$$\left( \frac{\$5,000,000 + \$100,000}{\$4,989,917} \right)^{365/90} - 1 = 0.0925$$

Of course, a little more than 300 basis points of this amount is the spread.

*Solution to 2:* Libor is 4%. The loan rate will be 7%.

The interest on the loan will be  $\$5,000,000(0.04 + 0.03) (90/360) = \$87,500$ .

The option payoff will be  $\$5,000,000 \max(0, 0.04 - 0.05) (90/360) = \$0.00$ .

The effective interest will, therefore, be  $\$87,500$ .

The effective rate on the loan will be

$$\left( \frac{\$5,000,000 + \$87,500}{\$4,989,917} \right)^{365/90} - 1 = 0.0817$$

Of course, a little more than 300 basis points of this amount is the spread.

Whereas interest rate call options are appropriate for borrowers, lenders also face the risk of interest rates changing. As you may have guessed, they make use of interest rate puts.

### 3.2. Using Interest Rate Puts with Lending

Now consider an application of an interest rate put to establish a minimum interest rate for a commitment to give a loan in the future. A lender can buy a put that pays off if the interest rate falls below a chosen level. The put payoff then compensates the bank for the lower interest rate on the loan.

For example, consider Arbitrage Bank Inc. (ABInc) which makes loan commitments to corporations. It stands ready to make a loan at Libor at a future date. To protect itself against decreases in interest rates between the time of the commitment and the time the loan is taken out, it buys interest rate puts. These options pay off if Libor is below the exercise rate at expiration. If Libor is above the exercise rate at expiration, the option expires unexercised and the lender benefits from the higher rate on the loan.

In this example, ABInc makes a commitment on 15 March to lend \$50 million at 90-day Libor plus 2.5% on 1 May, which is 47 days later. Current Libor is 7.25%. It buys a put with an exercise rate of 7% for \$62,500. Assume that the opportunity cost of lending in the

Libor market is Libor plus a spread of 2.5%. Therefore, the effective cost of the premium compounded to the option's expiration is<sup>22</sup>

$$\$62,500 \left[ 1 + (0.0725 + 0.025) \left( \frac{47}{360} \right) \right] = \$63,296$$

When it lends \$50 million on 1 May, it effectively has an outlay of  $\$50,000,000 + \$63,296 = \$50,063,296$ . The loan rate is set on 1 May and the interest, paid 90 days later on 30 July, is

$$\$50,000,000 \left[ \text{Libor on 1 May plus 250 Basis points} \left( \frac{90}{360} \right) \right]$$

The put payoff is

$$\$50,000,000 \max (0, 0.07 - \text{Libor on 1 May}) \left( \frac{90}{360} \right)$$

The loan interest plus the put payoff make up the effective interest. The effective rate on the loan is

$$\left( \frac{\text{Principal plus Effective interest}}{\$50,063,296} \right)^{365/90} - 1$$

Suppose Libor on 1 May is 6%. In that case, the loan rate will be 8.5%, and the interest on the loan will be

$$\$50,000,000 \left[ (0.06 + 0.025) \left( \frac{90}{360} \right) \right] = \$1,062,500$$

The put payoff is

$$\$50,000,000 \max (0, 0.07 - 0.06) \left( \frac{90}{360} \right) = \$125,000$$

This amount is paid on 30 July. The put cost of \$62,500 on 15 March is equivalent to paying \$63,296 on 1 May. Thus, on 1 May the bank effectively commits  $\$50,000,000 + \$63,296 = \$50,063,296$ . The effective interest it receives is the loan interest of \$1,062,500 plus the put payoff of \$125,000, or \$1,187,500. The effective annual rate is

$$\left( \frac{\$50,000,000 + \$1,187,500}{\$50,063,296} \right)^{365/90} - 1 = 0.0942$$

Exhibit 15 presents the results for a range of possible Libors at expiration, and Exhibit 16 graphs the effective loan rate against Libor on 1 May. Note how there is a minimum effective

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<sup>22</sup>The interpretation of this calculation is that the bank could have otherwise made a loan of \$62,500, which would have paid back \$63,296 on 1 May.

loan rate of 9.42%. Of this rate, 250 basis points is automatically built in as the loan spread.<sup>23</sup> The remaining amount reflects the exercise rate on the put of 7% minus the cost of the put premium.

#### EXHIBIT 15 Outcomes for an Anticipated Loan Protected with an Interest Rate Put

##### Scenario (15 March)

Arbitrage Bank Inc. (ABInc) is a US bank that makes loan commitments to corporations. When ABInc makes these commitments, it recognizes the risk that Libor will fall by the date the loan is taken out. ABInc protects itself against interest rate decreases by purchasing interest rate puts, which give it the right to receive the difference between the exercise rate it chooses and Libor at expiration. Libor is currently 7.25%.

##### Action

ABInc commits to lending \$50 million to a company at 90-day Libor plus 250 basis points. The loan will be a single-payment loan, meaning that it will be made on 1 May and the principal and interest will be repaid 90 days later on 30 July.

To protect against decreases in Libor between 15 March and 1 May, ABInc buys a put option with an exercise rate of 7% to expire on 1 May with the underlying being 90-day Libor. The put premium is \$62,500. We summarize the information as follows:

Loan amount	\$50,000,000
Underlying	90-day Libor
Spread	250 basis points over Libor
Current Libor	7.25%
Expiration	1 May
Exercise rate	7%
Put premium	\$62,500

##### Scenario (1 May)

Libor is now 6%.

##### Outcome and Analysis

For any Libor, the payoff at expiration is given below and will be received 90 days later:

$$\$50,000,000 \max (0, 0.07 - \text{Libor}) \left( \frac{90}{360} \right)$$

*(continued)*

<sup>23</sup>As in the case of the borrower, the spread is effectively more than 250 basis points when the effective annual rate is determined. For this 90-day loan, this effectively amounts to 256 basis points.

## EXHIBIT 15 (Continued)

For Libor of 6%, the payoff is

$$\$50,000,000 \max (0, 0.07 - 0.060) \left( \frac{90}{360} \right) = \$125,000$$

The premium compounded from 15 March to 1 May at current Libor plus 250 basis points is

$$\$62,500 \left[ 1 + (0.0725 + 0.025) \left( \frac{47}{360} \right) \right] = \$63,296$$

So the put costs \$62,500 on 15 March, which is equivalent to \$63,296 on 1 May. The effective amount loaned is \$50,000,000 + \$63,296 = \$50,063,296. For any Libor, the loan interest is

$$\$50,000,000 \left[ \text{Libor on 1 May plus 250 Basis points} \left( \frac{90}{360} \right) \right]$$

With Libor at 6%, the interest is

$$\$50,000,000 \left[ (0.06 + 0.025) \left( \frac{90}{360} \right) \right] = \$1,062,500$$

The loan interest plus the put payoff is the effective interest on the loan. The effective rate on the loan is

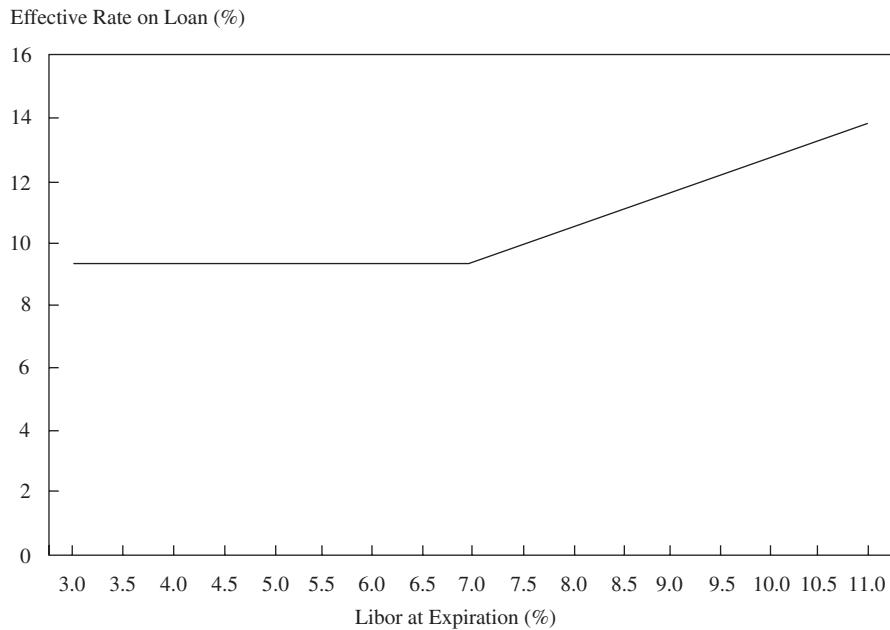
$$\begin{aligned} & \left( \frac{\text{Principal plus Effective interest}}{\$50,063,296} \right)^{365/90} - 1 \\ & = \left( \frac{\$50,000,000 + \$1,062,500 + \$125,000}{\$50,063,296} \right)^{365/90} - 1 = 0.0942 \end{aligned}$$

or 9.42%. The results that follow are for a range of Libors on 1 May.

Libor on 1 May	Loan Rate	Loan Interest Paid on 30 July	Put Payoff	Effective Interest	Effective Loan Rate
0.030	0.055	\$687,500	\$500,000	\$1,187,500	0.0942
0.035	0.060	750,000	437,500	1,187,500	0.0942
0.040	0.065	812,500	375,000	1,187,500	0.0942
0.045	0.070	875,000	312,500	1,187,500	0.0942
0.050	0.075	937,500	250,000	1,187,500	0.0942
0.055	0.080	1,000,000	187,500	1,187,500	0.0942
0.060	0.085	1,062,500	125,000	1,187,500	0.0942
0.065	0.090	1,125,000	62,500	1,187,500	0.0942

Libor on 1 May	Loan Rate	Loan Interest Paid on 30 July	Put Payoff	Effective Interest	Effective Loan Rate
0.070	0.095	1,187,500	0	1,187,500	0.0942
0.075	0.100	1,250,000	0	1,250,000	0.0997
0.080	0.105	1,312,500	0	1,312,500	0.1051
0.085	0.110	1,375,000	0	1,375,000	0.1106
0.090	0.115	1,437,500	0	1,437,500	0.1161
0.095	0.120	1,500,000	0	1,500,000	0.1216
0.100	0.125	1,562,500	0	1,562,500	0.1271
0.105	0.130	1,625,000	0	1,625,000	0.1327
0.110	0.135	1,687,500	0	1,687,500	0.1382

EXHIBIT 16 The Effective Rate on an Anticipated Loan with an Interest Rate Put Option



### EXAMPLE 12

State Bank and Trust (SBT) is a lender in the floating-rate instrument market, but it has been hurt by recent interest rate decreases. SBT often makes loan commitments for its customers and then accepts the rate in effect on the day the loan is taken out. SBT has avoided floating-rate financing in the past. It takes out a certain amount of fixed-rate

financing in advance to cover its loan commitments. One particularly large upcoming loan has it worried. This is a \$100 million loan to be made in 65 days at 180-day Libor plus 100 basis points. The loan will be paid back 182 days after being taken out, and interest will be based on an exact day count and 360 days in a year. Current Libor is 7.125%, which is the rate it could borrow at now for any period less than 180 days. SBT considers the purchase of an interest rate put to protect it against an interest rate decrease over the next 65 days. The put will have an exercise price of 7% and a premium of \$475,000.

Determine the effective annual rate on the loan for the following outcomes:

1. 180-day Libor at the option expiration is 9%.
2. 180-day Libor at the option expiration is 5%.

*Solutions:* First we need to compound the premium for 65 days. This calculation tells us the effective cost of the put as of the time the loan is made:

$$\$475,000 \left[ 1 + (0.07125 + 0.01) \left( \frac{65}{360} \right) \right] = \$481,968$$

The outlay will effectively be  $\$100,000,000 + \$481,968 = \$100,481,968$ .

*Solution to 1:* Libor is 9%. The loan rate will be 10%.

The interest on the loan will be  $\$100,000,000 (0.09 + 0.01)(182/360) = \$5,055,556$ .

The option payoff will be  $\$100,000,000 \max(0, 0.07 - 0.09)(182/360) = \$0.0$ .

Because there is no option payoff, the effective interest will be \$5,055,556. The effective rate on the loan will be

$$\left( \frac{\$100,000,000 + \$5,055,556}{\$100,481,968} \right)^{365/182} - 1 = 0.0934$$

Of course, a little more than 100 basis points of this amount is the spread.

*Solution to 2:* Libor is 5%. The loan will be 6%. The interest on the loan will be  $\$100,000,000 (0.05 + 0.01)(182/360) = \$3,033,333$ .

The option payoff will be  $\$100,000,000 \max(0, 0.07 - 0.05)(182/360) = \$1,011,111$ .

The effective interest will, therefore, be  $\$3,033,333 + \$1,011,111 = \$4,044,444$ .

The effective rate on the loan will be

$$\left( \frac{\$100,000,000 + \$4,044,444}{\$100,481,968} \right)^{365/182} - 1 = 0.0724$$

Of course, a little more than 100 basis points of this amount is the spread.

Interest rate calls and puts can be combined into packages of multiple options, which are widely used to manage the risk of floating-rate loans.

### 3.3. Using an Interest Rate Cap with a Floating-Rate Loan

Many corporate loans are floating-rate loans. They require periodic interest payments in which the rate is reset on a regularly scheduled basis. Because there is more than one interest payment, there is effectively more than one distinct risk. If a borrower wanted to use an interest rate call to place a ceiling on the effective borrowing rate, it would require more than one call. In effect, it would require a distinct call option expiring on each interest rate reset date. A combination of interest rate call options designed to align with the rates on a loan is called a **cap**. The component options are called **caplets**. Each caplet is distinct in having its own expiration date, but typically the exercise rate on each caplet is the same.

To illustrate the use of a cap, consider a company called Measure Technology (MesTech), which borrows in the floating-rate loan market. It usually takes out a loan for several years at a spread over Libor, paying the interest semiannually and the full principal at the end. On 15 April, MesTech takes out a \$10 million three-year loan at 100 basis points over 180-day Libor from a bank called SenBank. Current 180-day Libor is 9%, which sets the rate for the first six-month period at 10%. Interest payments will be on the 15th of October and April for three years. This means that the day counts for the six payments will be 183, 182, 183, 182, 183, and 182.

To protect against increases in interest rates, MesTech purchases an interest rate cap with an exercise rate of 8%. The component caplets expire on 15 October, the following 15 April, and so forth until the last caplet expires on a subsequent 15 October. The loan has six interest payments, but because the first rate is already set, there are only five risky payments so the cap will contain five caplets. The payoff of each caplet will be determined on its expiration date, but the caplet payoff, if any, will actually be made on the next payment date. This enables the caplet payoff to line up with the date on which the loan interest is paid. The cap premium, paid up front on 15 April, is \$75,000.

In the example of a single interest rate call, we looked at a range of outcomes several hundred basis points around the exercise rate. In a cap, however, many more outcomes are possible. Ideally we would examine a range of outcomes for each caplet. In the example of a single cap, we looked at the exercise rate and 8 rates above and below for a total of 17 rates. For five distinct rate resets, this same procedure would require  $5^{17}$  or more than 762 billion different possibilities. So, we shall just look at one possible combination of rates.

We shall examine a set of outcomes in which Libor is

- 8.50% on 15 October
- 7.25% on 15 April the following year
- 7.00% on the following 15 October
- 6.90% on the following 15 April
- 8.75% on the following 15 October

The loan interest is computed as

$$\begin{aligned} & \$10,000,000 (\text{Libor on previous reset date} + 100 \text{ Basis points}) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

Thus, the first interest payment is

$$\$10,000,000 (0.10) \left( \frac{183}{360} \right) = \$508,333$$

which is based on 183 days between 15 April and 15 October. This amount is certain, because the first interest rate has already been set. The remaining interest payments are based on the assumption we made above about the course of Libor over the life of the loan.

The results for these assumed rates are shown in the table at the end of Exhibit 17. Note several things about the effective interest, displayed in the last column. First, the initial interest payment is much higher than the other interest payments because the initial rate is somewhat higher than the remaining rates that prevailed over the life of the loan. Also, recall that the initial rate is already set, and it would make no sense to add a caplet to cover the initial rate, because the caplet would have to expire immediately in order to pay off on the first 15 October. If the caplet expired immediately, the amount MesTech would have to pay for it would be the amount of the caplet payoff, discounted for the deferral of the payoff. In other words, it would make no sense to have an option, or any derivative for that matter, that is purchased and expires immediately. Note also the variation in the effective interest payments, which occurs for two reasons. One is that, in contrast to previous examples, interest is computed over the exact number of days in the period. Thus, even if the rate were the same, the interest could vary by the effect of one or two days of interest. The other reason is that in some cases the caplets do expire with value, thereby reducing the effective interest paid.

## EXHIBIT 17 Interest Rate Cap

### Scenario (15 April)

Measure Technology (MesTech) is a corporation that borrows in the floating-rate instrument market. It typically takes out a loan for several years at a spread over Libor. MesTech pays the interest semiannually and the full principal at the end.

To protect against rising interest rates over the life of the loan, MesTech usually buys an interest rate cap in which the component caplets expire on the dates on which the loan rate is reset. The cap seller is a derivatives dealer.

### Action

MesTech takes out a \$10 million three-year loan at 100 basis points over Libor. The payments will be made semiannually. The lender is SenBank. Current Libor is 9%, which means that the first rate will be at 10%. Interest will be based on 1/360 of the exact number of days in the six-month period. MesTech selects an exercise rate of 8%. The caplets will expire on 15 October, 15 April of the following year, and so on for three years, but the caplet payoffs will occur on the next payment date to correspond with the interest payment

based on Libor that determines the cap payoff. The cap premium is \$75,000. We thus have the following information:

Loan amount	\$10,000,000
Underlying	180-day Libor
Spread	100 basis points over Libor
Current Libor	9%
Interest based on	actual days/360
Component caplets	five caplets expiring 15 October, 15 April, etc.
Exercise rate	8%
Cap premium	\$75,000

### Scenario (Various Dates throughout the Loan)

Shown below is one particular set of outcomes for Libor:

- 8.50% on 15 October
- 7.25% on 15 April the following year
- 7.00% on the following 15 October
- 6.90% on the following 15 April
- 8.75% on the following 15 October

### Outcome and Analysis

The loan interest due is computed as

$$\begin{aligned} & \$10,000,000 (\text{Libor on previous reset date} + 100 \text{ Basis points}) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

The caplet payoff is

$$\begin{aligned} & \$10,000,000 \max (0, \text{Libor on previous reset date} - 0.08) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

The previous reset date is the expiration date of the caplet. The effective interest is the interest due minus the caplet payoff.

The first caplet expires on the first 15 October and pays off the following April, because Libor on 15 October was 8.5%. The payoff is computed as

$$\begin{aligned} & \$10,000,000 \max (0, 0.085 - 0.08) \left( \frac{182}{360} \right) \\ & = \$10,000,000 (0.005) \left( \frac{182}{360} \right) = \$25,278 \end{aligned}$$

(continued)

## EXHIBIT 17 (Continued)

which is based on 182 days between 15 October and 15 April. The following table shows the payments on the loan and cap:

Date	Libor	Loan Rate	Days in Period	Interest Due	Caplet Payoffs	Effective Interest
15 April	0.0900	0.1000				
15 October	0.0850	0.0950	183	\$508,333		\$508,333
15 April	0.0725	0.0825	182	480,278	\$25,278	455,000
15 October	0.0700	0.0800	183	419,375	0	419,375
15 April	0.0690	0.0790	182	404,444	0	404,444
15 October	0.0875	0.0975	183	401,583	0	401,583
15 April			182	492,917	37,917	455,000

Note that on the following three dates, the caplets are out-of-the-money, because the Libors are all lower than 8%. On the final 15 October, however, Libor is 8.75%, which leads to a final caplet payoff of \$37,917 on the following 15 April, at which time the loan principal is repaid.

We do not show the effective rate on the loan. Because the loan has multiple payments, the effective rate would be analogous to the internal rate of return on a capital investment project or the yield-to-maturity on a bond. This rate would have to be found with a financial calculator or spreadsheet, and we would have to account for the principal received up front and paid back at maturity, as well as the cap premium. It is sufficient for us to see that the cap protects the borrower any time the rate rises above the exercise rate and allows the borrower to benefit from rates lower than the exercise rate.

Finally, there is one circumstance under which this cap might contain a sixth caplet, one expiring on the date on which the loan is taken out. If the borrower purchased the cap in advance of taking out the loan, the first loan rate would not be set until the day the loan is actually taken out. The borrower would thus have an incentive to include a caplet that would protect the first rate setting.

## EXAMPLE 13

Healthy Biosystems (HBIO) is a typical floating-rate borrower, taking out loans at Libor plus a spread. On 15 January 2002, it takes out a loan of \$25 million for one year with quarterly payments on 12 April, 14 July, 16 October, and the following 14 January. The underlying rate is 90-day Libor, and HBIO will pay a spread of 250 basis points. Interest is based on the exact number of days in the period. Current 90-day Libor is 6.5%. HBIO purchases an interest rate cap for \$20,000 that has an exercise rate of 7% and has caplets expiring on the rate reset dates.

Determine the effective interest payments if Libor on the following dates is as given:

12 April	7.250%
14 July	6.875%
16 October	7.125%

*Solution:* The interest due for each period is computed as  $\$25,000,000(\text{Libor on previous reset date} + 0.0250)(\text{Days in period}/360)$ . For example, the first interest payment is calculated as  $\$25,000,000(0.065 + 0.025)(87/360) = \$543,750$ , based on the fact that there are 87 days between 15 January and 12 April. Each caplet payoff is computed as  $\$25,000,000 \max(0, \text{Libor on previous reset date} - 0.07)(\text{Days in period}/360)$ , where the “previous reset date” is the caplet expiration. Payment is deferred until the date on which the interest is paid at the given Libor. For example, the caplet expiring on 12 April is worth  $\$25,000,000 \max(0, 0.0725 - 0.07)(93/360) = \$16,145$ , which is paid on 14 July and is based on the fact that there are 93 days between 12 April and 14 July.

The effective interest is the actual interest minus the caplet payoff. The payments are shown in the table below:

Date	Libor	Loan Rate	Days in Period	Interest Due	Caplet Payoff	Effective Interest
15 January	0.065	0.09				
12 April	0.0725	0.0975	87	\$543,750		\$543,750
14 July	0.06875	0.09375	93	629,688	\$16,146	613,542
16 October	0.07125	0.09625	94	611,979	0	611,979
14 January			90	601,563	7,813	593,750

Lenders who use floating-rate loans face the same risk as borrowers. As such they can make use of combinations of interest rate puts.

### 3.4. Using an Interest Rate Floor with a Floating-Rate Loan

Let us now consider the same problem from the point of view of the lender, which is SenBank in this example. It would be concerned about falling interest rates. It could, therefore, buy a combination of interest rate put options that expire on the various interest rate reset dates. This combination of puts is called a **floor**, and the component options are called **floorlets**. Specifically, let SenBank buy a floor with floorlets expiring on the interest rate reset dates and with an exercise rate of 8%. The premium is \$72,500.<sup>24</sup> Exhibit 18 illustrates the results using the same

<sup>24</sup>Note that the premiums for the cap and floor are not the same. This difference occurs because the premiums for a call and a put with the same exercise price are not the same, as can be seen by examining put-call parity.

outcomes we looked at when examining the interest rate cap. Note that the floorlet expires in-the-money on three dates when Libor is less than 8%, and out-of-the-money on two dates when Libor is greater than 8%. In those cases in which the floorlet expires in-the-money, the actual payoff does not occur until the next settlement period. This structure aligns the floorlet payoffs with the interest payments they are designed to protect. We see that the floor protects the lender against falling interest rates. Any time the rate is below 8%, the floor compensates the bank for any difference between the rate and 8%. When the rate is above 8%, the floorlets simply expire unused.

#### EXHIBIT 18 Interest Rate Floor

##### Scenario (15 April)

SenBank lends in the floating-rate instrument market. Often it uses floating-rate financing, thereby protecting itself against decreases in the floating rates on its loans. Sometimes, however, it finds it can get a better rate with fixed-rate financing, but it then leaves itself exposed to interest rate decreases on its floating-rate loans. Its loans are typically for several years at a spread over Libor with interest paid semi-annually and the full principal paid at the end.

To protect against falling interest rates over the life of the loan, SenBank buys an interest rate floor in which the component floorlets expire on the dates on which the loan rate is reset. The floor seller is a derivatives dealer.

##### Action

SenBank makes a \$10 million three-year loan at 100 basis points over Libor to MesTech (see cap example). The payments will be made semiannually. Current Libor is 9%, which means that the first interest payment will be at 10%. Interest will be based on the exact number of days in the six-month period divided by 360. SenBank selects an exercise rate of 8%. The floorlets will expire on 15 October, 15 April of the following year, and so on for three years, but the floorlet payoffs will occur on the next payment date so as to correspond with the interest payment based on Libor that determines the floorlet payoff. The floor premium is \$72,500. We thus have the following information:

Loan amount	\$10,000,000
Underlying	180-day Libor
Spread	100 basis points over Libor
Current Libor	9%
Interest based on	actual days/360
Component floorlets	five floorlets expiring 15 October, 15 April, etc.
Exercise rate	8%
Floor premium	\$72,500

##### Outcomes (Various Dates throughout the Loan)

Shown below is one particular set of outcomes for Libor:

- 8.50% on 15 October
- 7.25% on 15 April the following year
- 7.00% on the following 15 October
- 6.90% on the following 15 April
- 8.75% on the following 15 October

### Outcome and Analysis

The loan interest is computed as

$$\begin{aligned} & \$10,000,000 (\text{Libor on previous reset date} + 100 \text{ Basis points}) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

The floorlet payoff is

$$\begin{aligned} & \$10,000,000 \max (0, 0.08 - \text{Libor on previous reset date}) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

The effective interest is the interest due plus the floorlet payoff. The following table shows the payments on the loan and floor:

Date	Libor	Loan Rate	Days in Period	Interest Due	Floorlet Payoffs	Effective Interest
15 April	0.0900	0.1000				
15 October	0.0850	0.0950	183	\$508,333		\$508,333
15 April	0.0725	0.0825	182	480,278	\$0	480,278
15 October	0.0700	0.0800	183	419,375	38,125	457,500
15 April	0.0690	0.0790	182	404,444	50,556	455,000
15 October	0.0875	0.0975	183	401,583	55,917	457,500
			182	492,917	0	492,917

### EXAMPLE 14

Capitalized Bank (CAPBANK) is a lender in the floating-rate loan market. It uses fixed-rate financing on its floating-rate loans and buys floors to hedge the rate. On 1 May 2002, it makes a loan of \$40 million at 180-day Libor plus 150 basis points. Interest will be paid on 1 November, the following 5 May, the following 1 November, and the following 2 May, at which time the principal will be repaid. The exercise rate is 4.5%, the floorlets expire on the rate reset dates, and the premium will be \$120,000. Interest will be calculated based on the actual number of days in the period over 360. The current 180-day Libor is 5%.

Determine the effective interest payments CAPBANK will receive if Libor on the following dates is as given:

1 November	4.875%
5 May	4.25%
1 November	5.125%

*Solution:* The interest due for each period is computed as  $\$40,000,000(\text{Libor on previous reset date} + 0.0150)(\text{Days in period}/360)$ . For example, the first interest payment is  $\$40,000,000(0.05 + 0.0150)(184/360) = \$1,328,889$ , based on the fact that there are 184 days between 1 May and 1 November. Each floorlet payoff is computed as  $\$40,000,000 \max(0, 0.045 - \text{Libor on previous reset date})(\text{Days in period}/360)$ , where the “previous reset date” is the floorlet expiration. Payment is deferred until the date on which the interest is paid at the given Libor. For example, the floorlet expiring on 5 May is worth  $\$40,000,000 \max(0, 0.045 - 0.0425)(180/360) = \$50,000$ , which is paid on 1 November and is based on the fact that there are 180 days between 5 May and 1 November.

The effective interest is the actual interest plus the floorlet payoff. The payments are shown in the table below:

Date	Libor	Loan Rate	Days in Period	Interest Due	Floorlet Payoff	Effective Interest
1 May	0.05	0.065				
1 November	0.04875	0.06375	184	\$1,328,889		\$1,328,889
5 May	0.0425	0.0575	185	1,310,417	\$0	1,310,417
1 November	0.05125	0.06625	180	1,150,000	50,000	1,200,000
2 May			182	1,339,722	0	1,339,722

When studying equity option strategies, we combined puts and calls into a single transaction called a collar. In a similar manner, we now combine caps and floors into a single transaction, also called a collar.

### 3.5. Using an Interest Rate Collar with a Floating-Rate Loan

As we showed above, borrowers are attracted to caps because they protect against rising interest rates. They do so, however, at the cost of having to pay a premium in cash up front. A collar combines a long position in a cap with a short position in a floor. The sale of the floor generates a premium that can be used to offset the premium on the cap. Although it is not necessary that the floor premium completely offset the cap premium, this arrangement is common.<sup>25</sup>

<sup>25</sup>It is even possible for the floor premium to be greater than the cap premium, thereby *generating cash* up front.

The exercise rate on the floor is selected such that the floor premium is precisely the cap premium. As with equity options, this type of strategy is called a zero-cost collar. Recall, however, that this term is a bit misleading because it suggests that this transaction has no true "cost." The cost is simply not up front in cash. The sale of the floor results in the borrower giving up any gains from interest rates below the exercise rate on the floor. Therefore, the borrower pays for the cap by giving away some of the gains from the possibility of falling rates.

Recall that for equity investors, the collar typically entails ownership of the underlying asset and the purchase of a put, which is financed with the sale of a call. In contrast, an interest rate collar is more commonly seen from the borrower's point of view: a position as a borrower and the purchase of a cap, which is financed by the sale of a floor. It is quite possible, however, that a lender would want a collar. The lender is holding an asset, the loan, and wants protection against falling interest rates, which can be obtained by buying a floor, which itself can be financed by selling a cap. Most interest rate collars, however, are initiated by borrowers.

In the example we used previously, MesTech borrows \$10 million at Libor plus 100 basis points. The cap exercise rate is 8%, and the premium is \$75,000. We now change the numbers a little and let MesTech set the exercise rate at 8.625%. To sell a floor that will generate the same premium as the cap, the exercise rate is set at 7.5%. It is not necessary for us to know the amounts of the cap and floor premiums; it is sufficient to know that they offset.

Exhibit 19 shows the collar results for the same set of interest rate outcomes we have been previously using. Note that on the first 15 October, Libor is between the cap and floor exercise rates, so neither the caplet nor the floorlet expires in-the-money. On the following 15 April, 15 October, and the next 15 April, the rate is below the floor exercise rate, so MesTech has to pay up on the expiring floorlets. On the final 15 October, Libor is above the cap exercise rate, so MesTech gets paid on its cap.

#### EXHIBIT 19 Interest Rate Collar

##### **Scenario (15 April)**

Consider the Measure Technology (MesTech) scenario described in the cap and floor example in Exhibits 17 and 18. MesTech is a corporation that borrows in the floating-rate instrument market. It typically takes out a loan for several years at a spread over Libor. MesTech pays the interest semiannually and the full principal at the end.

To protect against rising interest rates over the life of the loan, MesTech usually buys an interest rate cap in which the component caplets expire on the dates on which the loan rate is reset. To pay for the cost of the interest rate cap, MesTech can sell a floor at an exercise rate lower than the cap exercise rate.

##### **Action**

Consider the \$10 million three-year loan at 100 basis points over Libor. The payments are made semiannually. Current Libor is 9%, which means that the first rate will be at 10%. Interest is based on the exact number of days in the six-month period divided by 360. MesTech selects an exercise rate of 8.625% for the cap. Generating a floor premium sufficient to offset the cap premium requires a floor exercise rate of 7.5%. The caplets and floorlets will expire on 15 October, 15 April of the following year, and so on for three years, but the payoffs will occur on the following payment date to correspond with the interest

*(continued)*

## EXHIBIT 19 (Continued)

payment based on Libor that determines the caplet and floorlet payoffs. Thus, we have the following information:

Loan amount	\$10,000,000
Underlying	180-day Libor
Spread	100 basis points over Libor
Current Libor	9%
Interest based on	actual days/360
Component options	five caplets and floorlets expiring 15 October, 15 April, etc.
Exercise rate	8.625% on cap, 7.5% on floor
Premium	no net premium

**Scenario (Various Dates throughout the Loan)**

Shown below is one particular set of outcomes for Libor:

8.50% on 15 October  
 7.25% on 15 April the following year  
 7.00% on the following 15 October  
 6.90% on the following 15 April  
 8.75% on the following 15 October

**Outcome and Analysis**

The loan interest is computed as

$$\begin{aligned} & \$10,000,000 (\text{Libor on previous reset date} + 100 \text{ Basis points}) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

The caplet payoff is

$$\begin{aligned} & \$10,000,000 \max (0, \text{Libor on previous reset date} - 0.08625) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

The floorlet payoff is

$$\begin{aligned} & (\$10,000,000 \max (0, 0.075 - \text{Libor on previous reset date})) \\ & \times \left( \frac{\text{Days in settlement period}}{360} \right) \end{aligned}$$

The effective interest is the interest due minus the caplet payoff minus the floorlet payoff. Note that because the floorlet was sold, the floorlet payoff is either negative (so we would subtract a negative number, thereby adding an amount to obtain the total interest due) or zero.

The following table shows the payments on the loan and collar:

Date	Libor	Loan Rate	Days in Period	Interest Due	Caplet Payoffs	Floorlet Payoffs	Effective Interest
15 April	0.0900	0.1000					
15 October	0.0850	0.0950	183	\$508,333			\$508,333
15 April	0.0725	0.0825	182	480,278	\$0	\$0	480,278
15 October	0.0700	0.0800	183	419,375	0	-12,708	432,083
15 April	0.0690	0.0790	182	404,444	0	-25,278	429,722
15 October	0.0875	0.0975	183	401,583	0	-30,500	432,083
			182	492,917	6,319	0	486,598

A collar establishes a range, the cap exercise rate minus the floor exercise rate, within which there is interest rate risk. The borrower will benefit from falling rates and be hurt by rising rates within that range. Any rate increases above the cap exercise rate will have no net effect, and any rate decreases below the floor exercise rate will have no net effect. The net cost of this position is zero, provided that the floor exercise rate is set such that the floor premium offsets the cap premium.<sup>26</sup> It is probably easy to see that collars are popular among borrowers.

### EXAMPLE 15

Exegesis Systems (EXSYS) is a floating-rate borrower that manages its interest rate risk with collars, purchasing a cap and selling a floor in which the cost of the cap and floor are equivalent. EXSYS takes out a \$35 million one-year loan at 90-day Libor plus 200 basis points. It establishes a collar with a cap exercise rate of 7% and a floor exercise rate of 6%. Current 90-day Libor is 6.5%. The interest payments will be based on the exact day count over 360. The caplets and floorlets expire on the rate reset dates. The rates will be set on the current date (5 March), 4 June, 5 September, and 3 December, and the loan will be paid off on the following 3 March.

Determine the effective interest payments if Libor on the following dates is as given:

4 June	7.25%
5 September	6.5%
3 December	5.875%

*Solution:* The interest due for each period is computed as  $\$35,000,000(\text{Libor on previous reset date} + 0.02)(\text{Days in period}/360)$ . For example, the first interest payment is

<sup>26</sup>It is certainly possible that the floor exercise rate would be set first, and the cap exercise rate would then be set to have the cap premium offset the floor premium. This would likely be the case if a lender were doing the collar. We assume, however, the case of a borrower who wants protection above a certain level and then decides to give up gains below a particular level necessary to offset the cost of the protection.

$\$35,000,000(0.065 + 0.02)(91/360) = \$752,014$ , based on the fact that there are 91 days between 5 March and 4 June. Each caplet payoff is computed as  $\$35,000,000 \max(0, \text{Libor on previous reset date} - 0.07)(\text{Days in period}/360)$ , where the “previous reset date” is the caplet expiration. Payment is deferred until the date on which the interest is paid at the given Libor. For example, the caplet expiring on 4 June is worth  $\$35,000,000 \max(0, 0.0725 - 0.07)(93/360) = \$22,604$ , which is paid on 5 September and is based on the fact that there are 93 days between 4 June and 5 September. Each floorlet payoff is computed as  $\$35,000,000 \max(0, 0.06 - \text{Libor on previous reset date})(\text{Days in period}/360)$ . For example, the floorlet expiring on 3 December is worth  $\$35,000,000 \max(0, 0.06 - 0.05875)(90/360) = \$10,938$ , based on the fact that there are 90 days between 3 December and 3 March. The effective interest is the actual interest minus the caplet payoff minus the floorlet payoff. The payments are shown in the table below:

Date	Libor	Loan Rate	Days in Period	Interest Due	Caplet Payoff	Floorlet Payoff	Effective Interest
5 March	0.065	0.085					
4 June	0.0725	0.0925	91	\$752,014			\$752,014
5 September	0.065	0.085	93	836,354	\$22,604	\$0	813,750
3 December	0.05875	0.07875	89	735,486	0	0	735,486
3 March			90	689,063	0	-10,938	700,001

Of course, caps, floors, and collars are not the only forms of protection against interest rate risk. We have previously covered FRAs and interest rate futures. The most widely used protection, however, is the interest rate swap. We cover swap strategies in the chapter on risk management applications of swap strategies.

In the final section of this chapter, we examine the strategies used to manage the risk of an option portfolio.

#### 4. OPTION PORTFOLIO RISK MANAGEMENT STRATEGIES

So far we have looked at examples of how companies and investors use options. As we have described previously, many options are traded by dealers who make markets in these options, providing liquidity by first taking on risk and then hedging their positions in order to earn the bid–ask spread without taking the risk. In this section, we shall take a look at the strategies dealers use to hedge their positions.<sup>27</sup>

Let us assume that a customer contacts a dealer with an interest in purchasing a call option. The dealer, ready to take either side of the transaction, quotes an acceptable ask price and the customer buys the option. Recall from earlier in this chapter that a short position in a call option is a

<sup>27</sup>For over-the-counter options, these dealers are usually the financial institutions that make markets in these options. For exchange-traded options, these dealers are the traders at the options exchanges, who may trade for their own accounts or could represent firms.

very dangerous strategy, because the potential loss on an upside underlying move is open ended. The dealer would not want to hold a short call position for long. The ideal way to lay off the risk is to find someone else who would take the exact opposite position, but in most cases, the dealer will not be so lucky.<sup>28</sup> Another ideal possibility is for the dealer to lay off the risk using put–call parity. Recall that put–call parity says that  $c = p + S - X/(1 + r)^T$ . The dealer that has sold a call needs to buy a call to hedge the position. The put–call parity equation means that a long call is equivalent to a long put, a long position in the asset, and issuing a zero-coupon bond with a face value equal to the option exercise price and maturing on the option expiration date. Therefore, if the dealer could buy a put with the same exercise price and expiration, buy the asset, and sell a bond or take out a loan with face value equal to the exercise price and maturity equal to that of the option’s expiration, it would have the position hedged. Other than buying an identical call, as described above, this hedge would be the best because it is static: No change to the position is required as time passes.

Unfortunately, neither of these transactions can be commonly employed. The necessary options may not be available or may not be favorably priced. As the next best alternative, dealers **delta hedge** their positions using an available and attractively priced instrument. The dealer is short the call and will need an offsetting position in another instrument. An obvious offsetting instrument would be a long position of a certain number of units of the underlying. The size of that long position will be related to the option’s delta. Let us briefly review delta here. By definition,

$$\text{Delta} = \frac{\text{Change in option price}}{\text{Change in underlying price}}$$

Delta expresses how the option price changes relative to the price of the underlying. Technically, we should use an approximation sign ( $\approx$ ) in the above equation, but for now we shall assume the approximation is exact. Let  $\Delta S$  be the change in the underlying price and  $\Delta c$  be the change in the option price. Then  $\text{Delta} = \Delta c / \Delta S$ . The delta usually lies between 0.0 and 1.0.<sup>29</sup> Delta will be 1.0 only at expiration and only if the option expires in-the-money. Delta will be 0.0 only at expiration and only if the option expires out-of-the-money. So most of the time, the delta will be between 0.0 and 1.0. Hence, 0.5 is often given as an “average” delta, but one must be careful because even before expiration the delta will tend to be higher than 0.5 if the option is in-the-money.

Now, let us assume that we construct a portfolio consisting of  $N_S$  units of the underlying and  $N_c$  call options. The value of the portfolio is, therefore,

$$V = N_S S + N_c c$$

The change in the value of the portfolio is

$$\Delta V = N_S \Delta S + N_c \Delta c$$

<sup>28</sup>Even luckier would be the dealer’s original customer who might stumble across a party who wanted to sell the call option. The two parties could then bypass the dealer and negotiate a transaction directly between each other, which would save each party half of the bid–ask spread.

<sup>29</sup>In the following text, we always make reference to the delta lying between 0.0 and 1.0, which is true for calls. For puts, the delta is between -1.0 and 0.0. It is common, however, to refer to a put delta of -1.0 as just 1.0, in effect using its absolute value and ignoring the negative. In all discussions in this chapter, we shall refer to delta as ranging between 1.0 and 0.0, recalling that a put delta would range from -1.0 to 0.0.

If we want to hedge the portfolio, then we want the change in  $V$ , given a change in  $S$ , to be zero. Dividing by  $\Delta S$ , we obtain

$$\begin{aligned}\frac{\Delta V}{\Delta S} &= N_S \frac{\Delta S}{\Delta S} + N_c \frac{\Delta c}{\Delta S} \\ &= N_S + N_c \frac{\Delta c}{\Delta S}\end{aligned}$$

Setting this result equal to zero and solving for  $N_c/N_S$ , we obtain

$$\frac{N_c}{N_S} = -\frac{1}{\Delta c/\Delta S}$$

The ratio of calls to shares has to be the negative of 1 over the delta. Thus, if the dealer sells a given number of calls, say 100, it will need to own 100(Delta) shares.

How does delta hedging work? Let us say that we sell call options on 200 shares (this quantity is 2 standardized call contracts on an options exchange) and the delta is 0.5. We would, therefore, need to hold  $200(0.5) = 100$  shares. Say the underlying falls by \$1. Then we lose \$100 on our position in the underlying. If the delta is accurate, the option should decline by \$0.50. By having 200 options, the loss in value of the options collectively is \$100. Because we are short the options, the loss in value of the options is actually a gain. Hence, the loss on the underlying is offset by the gain on the options. If the dealer were long the option, it would need to sell short the shares.

This illustration may make delta hedging sound simple: Buy (sell) delta shares for each option short (long). But there are three complicating issues. One is that delta is only an approximation of the change in the call price for a change in the underlying. A second issue is that the delta changes if anything else changes. Two factors that change are the price of the underlying and time. When the price of the underlying changes, delta changes, which affects the number of options required to hedge the underlying. Delta also changes as time changes; because time changes continuously, delta also changes continuously. Although a dealer can establish a delta-hedged position, as soon as anything happens—the underlying price changes or time elapses—the position is no longer delta hedged. In some cases, the position may not be terribly out of line with a delta hedge, but the more the underlying changes, the further the position moves away from being delta hedged. The third issue is that the number of units of the underlying per option must be rounded off, which leads to a small amount of imprecision in the balancing of the two opposing positions.

In the following section, we examine how a dealer delta hedges an option position, carrying the analysis through several days with the additional feature that excess cash will be invested in bonds and any additional cash needed will be borrowed.

#### 4.1. Delta Hedging an Option over Time

In the previous section, we showed how to set up a delta hedge. As we noted, a delta-hedged position will not remain delta hedged over time. The delta will change as the underlying changes and as time elapses. The dealer must account for these effects.

Let us first examine how actual option prices are sensitive to the underlying and what the delta tells us about that sensitivity. Consider a call option in which the underlying is worth

1210, the exercise price is 1200, the continuously compounded risk-free rate is 2.75%, the volatility of the underlying is 20%, and the expiration is 120 days. There are no dividends or cash flows on the underlying. Substituting these inputs into the Black–Scholes–Merton model, the option is worth 65.88. Recall from our study of the Black–Scholes–Merton model that delta is the term “ $N(d_1)$ ” in the formula and represents a normal probability associated with the value  $d_1$ , which is provided as part of the Black–Scholes–Merton formula. In this example, the delta is 0.5826.<sup>30</sup>

Suppose that the underlying price instantaneously changes to 1200, a decline of 10. Using the delta, we would estimate that the option price would be

$$65.88 + (1200 - 1210)(0.5826) = 60.05$$

If, however, we plugged into the Black–Scholes–Merton model the same parameters but with a price of the underlying of 1200, we would obtain a new option price of 60.19—not much different from the previous result. But observe in Exhibit 20 what we obtain for various other values of the underlying. Two patterns become apparent: 1) The further away we move from the current price, the worse the delta-based approximation, and 2) the effects are asymmetric. A given move in one direction does not have the same effect on the option as the same move in the other direction. Specifically, for calls, the delta underestimates the effects of increases in the underlying and overestimates the effects of decreases in the underlying.<sup>31</sup> Because of this characteristic, the delta hedge will not be perfect. The larger the move in the underlying, the worse the hedge. Moreover, whenever the underlying price changes, the delta changes, which requires a rehedging or adjustment to the position. Observe in the last column of the table in Exhibit 20 we have recomputed the delta using the new price of the underlying. A dealer must adjust the position according to this new delta.

#### EXHIBIT 20 Delta and Option Price Sensitivity

$S = 1210$   
 $X = 1200$   
 $r^c = 0.0275$  (continuously compounded)  
 $\sigma = 0.20$   
 $T = 0.328767$  (based on 120 days/365)  
 No dividends  
 $c = 65.88$  (from the Black–Scholes–Merton model)

New Price of Underlying	Delta-Estimated Call Price <sup>a</sup>	Actual Call Price <sup>b</sup>	Difference (Actual – Estimated)	New Delta
1180	48.40	49.69	1.29	0.4959
1190	54.22	54.79	0.57	0.5252
1200	60.05	60.19	0.14	0.5542
1210	65.88	65.88	0.00	0.5826
1220	71.70	71.84	0.14	0.6104

(continued)

<sup>30</sup>All calculations were done on a computer for best precision.

<sup>31</sup>For puts, delta underestimates the effects of price decreases and overestimates the effects of price increases.

## EXHIBIT 20 (Continued)

New Price of Underlying	Delta-Estimated Call Price <sup>a</sup>	Actual Call Price <sup>b</sup>	Difference (Actual - Estimated)	New Delta
1230	77.53	78.08	0.55	0.6374
1240	83.35	84.59	1.24	0.6635

<sup>a</sup> Delta-estimated call price = Original call price + (New price of underlying – Original price of underlying)Delta.

<sup>b</sup> Actual call price obtained from Black–Scholes–Merton model using new price of underlying; all other inputs are the same.

Now let us consider the effect of time on the delta. Exhibit 21 shows the delta and the number of units of underlying required to hedge 1,000 short options when the option has 120 days, 119, etc. on down to 108. A critical assumption is that we are holding the underlying price constant. Of course, this constancy would not occur in practice, but to focus on understanding the effect of time on the delta, we must hold the underlying price constant. Observe that the delta changes slowly and the number of units of the underlying required changes gradually over this 12-day period. Another not-so-obvious effect is also present: When we round up, we have more units of the underlying than needed, which has a negative effect that hurts when the underlying goes down. When we round down, we have fewer units of the underlying than needed, which hurts when the underlying goes up.

## EXHIBIT 21 The Effect of Time on the Delta

$$S = 1210$$

$$X = 1200$$

$$r^c = 0.0275 \text{ (continuously compounded)}$$

$$\sigma = 0.20$$

$$T = 0.328767 \text{ (based on 120 days/365)}$$

No dividends

$$c = 65.88 \text{ (from the Black–Scholes–Merton model)}$$

$$\text{Delta} = 0.5826$$

Delta hedge 1,000 short options by holding  $1,000(0.5826) = 582.6$  units of the underlying.

Time to Expiration (Days)	Delta	Number of Units of Underlying Required
120	0.5826	582.6
119	0.5825	582.5
118	0.5824	582.4
117	0.5823	582.3
116	0.5822	582.2
115	0.5821	582.1
114	0.5820	582.0
113	0.5819	581.9
112	0.5818	581.8

Time to Expiration (Days)	Delta	Number of Units of Underlying Required
111	0.5817	581.7
110	0.5816	581.6
109	0.5815	581.5
108	0.5814	581.4

The combined effects of the underlying price changing and the time to expiration changing interact to present great challenges for delta hedgers. Let us set up a delta hedge and work through a few days of it. Recall that for the option we have been working with, the underlying price is \$1,210, the option price is \$65.88, and the delta is 0.5826. Suppose a customer comes to us and asks to buy calls on 1,000 shares. We need to buy a sufficient number of shares to offset the sale of the 1,000 calls. Because we are short 1,000 calls, and this number is fixed, we need 0.5826 shares per call or about 583 shares. So we buy 583 shares to balance the 1,000 short calls. The value of this portfolio is

$$583(\$1,210) - 1,000(\$65.88) = \$639,550$$

So, to initiate this delta hedge, we would need to invest \$639,550. To determine if this hedge is effective, we should see this value grow at the risk-free rate. Because the Black–Scholes–Merton model uses continuously compounded interest, the formula for compounding a value at the risk-free rate for one day is  $\exp(r^c/365)$ , where  $r^c$  is the continuously compounded risk-free rate. One day later, this value should be  $\$639,550 \exp(0.0275/365) = \$639,598$ . This value becomes our benchmark.

Now, let us move forward one day and have the underlying go to \$1,215. We need a new value of the call option, which now has one less day until expiration and is based on an underlying with a price of \$1,215. The market would tell us the option price, but we do not have the luxury here of asking the market for the price. Instead, we have to appeal to a model that would tell us an appropriate price. Naturally, we turn to the Black–Scholes–Merton model. We recalculate the value of the call option using Black–Scholes–Merton, with the price of the underlying at \$1,215 and the time to expiration at  $119/365 = 0.3260$ . The option value is \$68.55, and the new delta is 0.5966. The portfolio is now worth

$$583(\$1,215) - 1,000(\$68.55) = \$639,795$$

This value differs from the benchmark by a small amount:  $\$639,795 - \$639,598 = \$197$ . Although the hedge is not perfect, it is off by only about 0.03%.

Now, to move forward and still be delta hedged, we need to revise the position. The new delta is 0.5966. So now we need  $1,000(0.5966) = 597$  units of the underlying and must buy 14 units of the underlying. This purchase will cost  $14(\$1,215) = \$17,010$ . We obtain this money by borrowing it at the risk-free rate. So we issue bonds in the amount of \$17,010. Now our

position is 597 units of the underlying, 1,000 short calls, and a loan of \$17,010. The value of this position is still

$$597(\$1,215) - 1,000(\$68.55) - \$17,010 = \$639,795$$

Of course, this is the same value we had before adjusting the position. We could not expect to generate or lose money just by rearranging our position. As we move forward to the next day, we should see this value grow by one day's interest to  $\$639,795 \exp(0.0275/365) = \$639,843$ . This amount is the benchmark for the next day.

Suppose the next day the underlying goes to \$1,198, the option goes to 58.54, and its delta goes to 0.5479. Our loan of \$17,010 will grow to  $\$17,010 \exp(0.0275/365) = \$17,011$ . The new value of the portfolio is

$$597(\$1,198) - 1,000(\$58.54) - \$17,011 = \$639,655$$

This amount differs from the benchmark by  $\$639,655 - \$639,843 = -\$188$ , an error of about 0.03%.

With the new delta at 0.5479, we now need 548 shares. Because we have 597 shares, we now must sell  $597 - 548 = 49$  shares. Doing so would generate  $49(\$1,198) = \$58,702$ . Because the value of our debt was \$17,011 and we now have \$58,702 in cash, we can pay back the loan, leaving  $\$58,702 - \$17,011 = \$41,691$  to be invested at the risk-free rate. So now we have 548 units of the underlying, 1,000 short calls, and bonds of \$41,691. The value of this position is

$$548(\$1,198) - 1,000(\$58.54) + \$41,691 = \$639,655$$

Of course, this is the same value we had before buying the underlying. Indeed, we cannot create or destroy any wealth by just rearranging the position.

Exhibit 22 illustrates the delta hedge, carrying it through one more day. After the third day, the value of the position should be  $\$639,655 \exp(0.0275/365) = \$639,703$ . The actual value is \$639,870, a difference of  $\$639,870 - \$639,703 = \$167$ .

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#### EXHIBIT 22 Delta Hedge of a Short Options Position

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$S = \$1,210$   
 $X = \$1,200$   
 $r^c = 0.0275$  (continuously compounded)  
 $\sigma = 0.20$   
 $T = 0.328767$  (based on 120 days/365)  
 No dividends  
 $c = \$65.88$  (from the Black–Scholes–Merton model)  
 $\text{Delta} = 0.5826$   
 Units of option constant at 1,000  
 Units of underlying required =  $1000 \times \text{Delta}$   
 Units of underlying purchased = (Units of underlying required one day) – (Units of underlying required previous day)  
 Bonds purchased =  $-S(\text{Units of underlying purchased})$   
 Bond balance = (Previous balance)  $\exp(r^c/365) + \text{Bonds purchased}$   
 Value of portfolio = (Units of underlying) $S + (\text{Units of options})c + \text{Bond balance}$

Day	S	c	Delta	Options Sold	Units of Underlying Required	Units of Underlying Purchased	Value of Bonds Purchased	Bond Balance	Value of Portfolio
0	\$1,210	\$65.88	0.5826	1,000	583	583	\$0	\$0	\$639,550
1	1,215	68.55	0.5965	1,000	597	14	-17,010	-17,010	639,795
2	1,198	58.54	0.5479	1,000	548	-49	58,702	41,691	639,655
3	1,192	55.04	0.5300	1,000	530	-18	21,456	63,150	639,870

As we can see, the delta hedge is not perfect, but it is pretty good. After three days, we are off by \$167, only about 0.03% of the benchmark.

In our example and the discussions here, we have noted that the dealer would typically hold a position in the underlying to delta-hedge a position in the option. Trading in the underlying would not, however, always be the preferred hedge vehicle. In fact, we have stated quite strongly that trading in derivatives is often easier and more cost-effective than trading in the underlying. As noted previously, ideally a short position in a particular option would be hedged by holding a long position in that same option, but such a hedge requires that the dealer find another customer or dealer who wants to sell that same option. It is possible, however, that the dealer might be able to more easily buy a different option on the same underlying and use that option as the hedging instrument.

For example, suppose one option has a delta of  $\Delta_1$  and the other has a delta of  $\Delta_2$ . These two options are on the same underlying but are not identical. They differ by exercise price, expiration, or both. Using  $c_1$  and  $c_2$  to represent their prices and  $N_1$  and  $N_2$  to represent the quantity of each option in a portfolio that hedges the value of one of the options, the value of the position is

$$V = N_1 c_1 + N_2 c_2$$

Dividing by  $\Delta S$ , we obtain

$$\frac{\Delta V}{\Delta S} = N_1 \frac{\Delta c_1}{\Delta S} + N_2 \frac{\Delta c_2}{\Delta S}$$

To delta hedge, we set this amount to zero and solve for  $N_1/N_2$  to obtain

$$\frac{N_1}{N_2} = -\frac{\Delta c_2}{\Delta c_1}$$

The negative sign simply means that a long position in one option will require a short position in the other. The desired quantity of Option 1 relative to the quantity of Option 2 is the ratio of the delta of Option 2 to the delta of Option 1. As in the standard delta-hedge example, however, these deltas will change and will require monitoring and modification of the position.<sup>32</sup>

<sup>32</sup>Because the position is long one option and short another, whenever the options differ by exercise price, expiration, or both, the position has the characteristics of a spread. In fact, it is commonly called a **ratio spread**.

### EXAMPLE 16

DynaTrade is an options trading company that makes markets in a variety of derivative instruments. DynaTrade has just sold 500 call options on a stock currently priced at \$125.75. Suppose the trade date is 18 November. The call has an exercise price of \$125, 60 days until expiration, a price of \$10.89, and a delta of 0.5649. DynaTrade will delta-hedge this transaction by purchasing an appropriate number of shares. Any additional transactions required to adjust the delta hedge will be executed by borrowing or lending at the continuously compounded risk-free rate of 4%.

DynaTrade has begun delta hedging the option. Two days later, 20 November, the following information applies:

Stock price	\$122.75
Option price	\$9.09
Delta	0.5176
Number of options	500
Number of shares	328
Bond balance	-\$6,072
Market value	\$29,645

- At the end of 19 November, the delta was 0.6564. Based on this number, show how 328 shares of stock is used to delta hedge 500 call options.
- Show the allocation of the \$29,645 market value of DynaTrade's total position among stock, options, and bonds on 20 November.
- Show what transactions must be done to adjust the portfolio to be delta hedged for the following day (21 November).
- On 21 November, the stock is worth \$120.50 and the call is worth \$7.88. Calculate the market value of the delta-hedged portfolio and compare it with a benchmark, based on the market value on 20 November.

*Solution to A:* If the stock moves up (down) \$1, the 328 shares should change by \$328. The 500 calls should change by  $500(0.6564) = \$328.20$ , rounded off to \$328. The calls are short, so any change in the value of the stock position is an opposite change in the value of the options.

*Solution to B:*

$$\begin{aligned} \text{Stock worth } 328(\$122.75) &= \$40,262 \\ \text{Options worth } -500(\$9.09) &= -\$4,545 \\ \text{Bonds worth } -\$6,072 & \\ \text{Total of } \$29,645 & \end{aligned}$$

*Solution to C:* The new required number of shares is  $500(0.5176) = 258.80$ . Round this number to 259. So we need to have 259 shares instead of 328 shares and must sell

69 shares, generating  $69(\$122.75) = \$8,470$ . We invest this amount in risk-free bonds. We had a bond balance of  $-\$6,072$ , so the proceeds from the sale will pay off all of this debt, leaving a balance of  $\$8,470 - \$6,072 = \$2,398$  going into the next day. The composition of the portfolio would then be as follows:

Shares worth  $259(\$122.75) = \$31,792$   
Options worth  $-500(\$9.09) = -\$4,545$   
Bonds worth  $\$2,398$   
Total of  $\$29,645$

*Solution to D:* The benchmark is  $\$29,645 \exp(0.04/365) = \$29,648$ . Also, the value of the bond one day later will be  $\$2,398 \exp(0.04/365) = \$2,398$ . (This is less than a half-dollar's interest, so it essentially leaves the balance unchanged.) Now we have

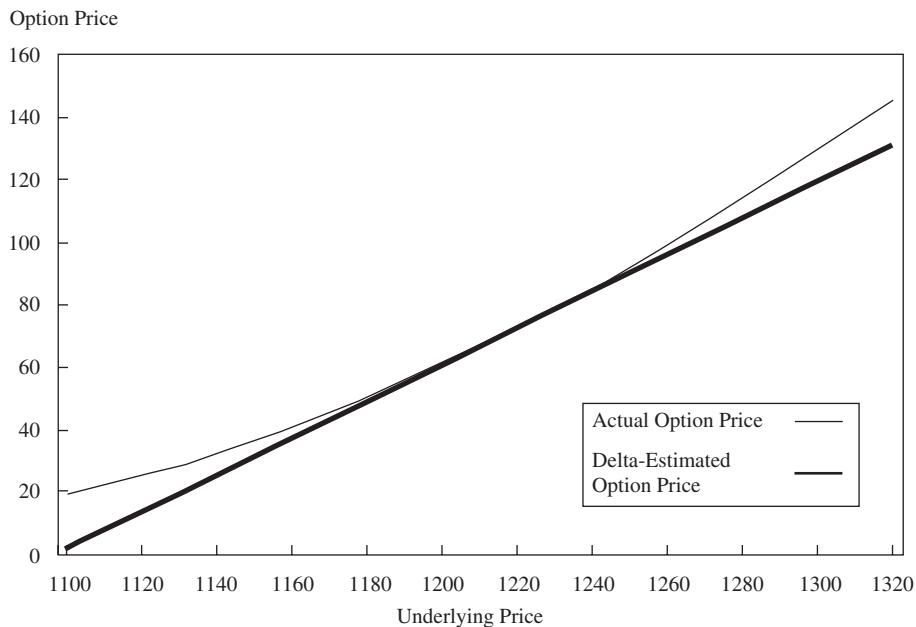
Shares worth  $259(\$120.50) = \$31,210$   
Options worth  $-500(\$7.88) = -\$3,940$   
Bonds worth  $\$2,398$   
Total of  $\$29,668$

This is about \$20 more than the benchmark.

As previously noted, the delta is a fairly good approximation of the change in the option price for a very small and rapid change in the price of the underlying. But the underlying does not always change in such a convenient manner, and this possibility introduces a risk into the process of delta hedging.

Note Exhibit 23, a graph of the actual option price and the delta-estimated option price from the perspective of day 0 in Exhibit 20. At the underlying price of \$1,210, the option price is \$65.88. The curved line shows the exact option price, calculated with the Black–Scholes–Merton model, for a range of underlying prices. The heavy line shows the option price estimated using the delta as we did in Exhibit 20. In that exhibit, we did not stray too far from the current underlying price. In Exhibit 23, we let the underlying move a little further. Note that the further we move from the current price of the underlying of \$1,210, the further the heavy line deviates from the solid line. As noted earlier, the actual call price moves up more than the delta approximation and moves down less than the delta approximation. This effect occurs because the option price is convex with respect to the underlying price. This convexity, which is quite similar to the convexity of a bond price with respect to its yield, means that a first-order price sensitivity measure like delta, or its duration analog for bonds, is accurate only if the underlying moves by a small amount. With duration, a second-order measure called convexity reflects the extent of the deviation of the actual pricing curve from the approximation curve. With options, the second-order measure is called **gamma**.

EXHIBIT 23 Actual Option Price and Delta-Estimated Option Price



#### 4.2. Gamma and the Risk of Delta

A gamma is a measure of several effects. It reflects the deviation of the exact option price change from the price change as approximated by the delta. It also measures the sensitivity of delta to a change in the underlying. In effect, it is the delta of the delta. Specifically,

$$\text{Gamma} = \frac{\text{Change in delta}}{\text{Change in underlying price}}$$

Like delta, gamma is actually an approximation, but we shall treat it as exact. Although a formula exists for gamma, we need to understand only the concept.

If a delta-hedged position were risk free, its gamma would be zero. The larger the gamma, the more the delta-hedged position deviates from being risk free. Because gamma reflects movements in the delta, let us first think about how delta moves. Focusing on call options, recall that the delta is between 0.0 and 1.0. At expiration, the delta is 1.0 if the option expires in-the-money and 0.0 if it expires out-of-the-money. During its life, the delta will tend to be above 0.5 if the option is in-the-money and below 0.5 if the option is out-of-the-money. As expiration approaches, the deltas of in-the-money options will move toward 1.0 and the deltas of out-of-the-money options will move toward 0.0.<sup>33</sup> They will, however, move slowly in their respective directions. The largest moves occur near expiration, when the deltas of at-the-money options move quickly toward 1.0 or 0.0. These rapid movements are the ones that cause the

<sup>33</sup>The deltas of options that are very slightly in-the-money will temporarily move down as expiration approaches. Exhibit 21 illustrates this effect. But they will eventually move up toward 1.0.

most problems for delta hedgers. Options that are deep in-the-money or deep out-of-the-money tend to have their deltas move closer to 1.0 or 0.0 well before expiration. Their movements are slow and pose fewer problems for delta hedgers. Thus, it is the rapid movements in delta that concern delta hedgers. These rapid movements are more likely to occur on options that are at-the-money and/or near expiration. Under these conditions, the gammas tend to be largest and delta hedges are hardest to maintain.

When gammas are large, some delta hedgers choose to also gamma hedge. This somewhat advanced strategy requires adding a position in another option, combining the underlying and the two options in such a manner that the delta is zero and the gamma is zero. Because it is a somewhat advanced and specialized topic, we do not cover the details of how this is done.

The delta is not the only important factor that changes in the course of managing an option position. The volatility of the underlying can also change.

### 4.3. Vega and Volatility Risk

The sensitivity of the option price to the volatility is called the vega and is defined as

$$\text{Vega} = \frac{\text{Change in option price}}{\text{Change in volatility}}$$

As with delta and gamma, the relationship above is an approximation, but we shall treat it as exact. An option price is very sensitive to the volatility of the underlying. Moreover, the volatility is the only unobservable variable required to value an option. Hence, volatility is the most critical variable. When we examined option-pricing models, we studied the Black–Scholes–Merton and binomial models. In neither of these models is the volatility allowed to change. Yet no one believes that volatility is constant; on some days the stock market is clearly more volatile than on other days. This risk of changing volatility can greatly affect a dealer's position in options. A delta-hedged position with a zero or insignificant gamma can greatly change in value if the volatility changes. If, for example, the dealer holds the underlying and sells options to delta hedge, an increase in volatility will raise the value of the options, generating a potentially large loss for the dealer.

Measuring the sensitivity of the option price to the volatility is difficult. The vega from the Black–Scholes–Merton or binomial models is a somewhat artificial construction. It represents how much the model price changes if one changes the volatility by a small amount. But in fact, the model itself is based on the assumption that volatility does not change. Forcing the volatility to change in a model that does not acknowledge that volatility can change has unclear implications.<sup>34</sup> It is clear, however, that an option price is more sensitive to the volatility when it is at-the-money.

<sup>34</sup>If this point seems confusing, consider this analogy. In the famous Einstein equation  $E = mc^2$ ,  $E$  is energy,  $m$  is mass, and  $c$  is the constant representing the speed of light. For a given mass, we could change  $c$ , which would change  $E$ . The equation allows this change, but in fact the speed of light is constant at 186,000 miles per second. So far as scientists know, it is a universal constant and can never change. In the case of option valuation, the model assumes that volatility of a given stock is like a universal constant. We can change it, however, and the equation would give us a new option price. But are we allowed to do so? Unlike the speed of light, volatility does indeed change, even though our model says that it does not. What happens when we change volatility in our model? We do not know.

Dealers try to measure the vega, monitor it, and in some cases hedge it by taking on a position in another option, using that option's vega to offset the vega on the original option. Managing vega risk, however, cannot be done independently of managing delta and gamma risk. Thus, the dealer is required to jointly monitor and manage the risk associated with the delta, gamma, and vega. We should be aware of the concepts behind managing these risks.

## 5. FINAL COMMENTS

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In the chapter on risk management applications of forward and futures strategies, we examined forward and futures strategies. These types of contracts provide gains from movements of the underlying in one direction but result in losses from movements of the underlying in the other direction. The advantage of a willingness to incur losses is that no cash is paid at the start. Options offer the advantage of having one-directional effects: The buyer of an option gains from a movement in one direction and loses only the premium from movements in the other direction. The cost of this advantage is that options require the payment of cash at the start. Some market participants choose forwards and futures because they do not have to pay cash at the start. They can justify taking positions without having to come up with the cash to do so. Others, however, prefer the flexibility to benefit when their predictions are right and suffer only a limited loss when wrong. The trade-off between the willingness to pay cash at the start versus incurring losses, given one's risk preferences, is the deciding factor in whether to use options or forwards/futures.

All option strategies are essentially rooted in the transactions of buying a call or a put. Understanding a short position in either type of option means understanding the corresponding long position in the option. All remaining strategies are just combinations of options, the underlying, and risk-free bonds. We looked at a number of option strategies associated with equities, which can apply about equally to index options or options on individual stocks. The applicability of these strategies to bonds is also fairly straightforward. The options must expire before the bonds mature, but the general concepts associated with equity option strategies apply similarly to bond option strategies.

Likewise, strategies that apply to equity options apply in nearly the same manner to interest rate options. Nonetheless, significant differences exist between interest rate options and equity or bond options. If nothing else, the notion of bullishness is quite opposite. Bullish (bearish) equity investors buy calls (puts). In interest rate markets, bullish (bearish) investors buy puts (calls) on interest rates, because being bullish (bearish) on interest rates means that one thinks rates are going down (up). Interest rate options pay off as though they were interest payments. Equity or bond options pay off as though the holder were selling or buying stocks or bonds. Finally, interest rate options are very often combined into portfolios in the form of caps and floors for the purpose of hedging floating-rate loans. Standard option strategies such as straddles and spreads are just as applicable to interest rate options.

Despite some subtle differences between the option strategies examined in this chapter and comparable strategies using options on futures, the differences are relatively minor and do not warrant separate coverage here. If you have a good grasp of the basics of the option strategies presented in this chapter, you can easily adapt those strategies to ones in which the underlying is a futures contract.

In the chapter on risk management applications of swap strategies, we take up strategies using swaps. As we have so often mentioned, interest rate swaps are the most widely used financial derivative. They are less widely used with currencies and equities than are

forwards, futures, and options. Nonetheless, there are many applications of swaps to currencies and equities, and we shall certainly look at them. To examine swaps, however, we must return to the types of instruments with two-directional payoffs and no cash payments at the start. Indeed, swaps are a lot like forward contracts, which themselves are a lot like futures.

## 6. SUMMARY

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- The profit from buying a call is the value at expiration,  $\max(0, S_T - X)$ , minus  $c_0$ , the option premium. The maximum profit is infinite, and the maximum loss is the option premium. The breakeven underlying price at expiration is the exercise price plus the option premium. When one sells a call, these results are reversed.
- The profit from buying a put is the value at expiration,  $\max(0, X - S_T)$ , minus  $p_0$ , the option premium. The maximum profit is the exercise price minus the option premium, and the maximum loss is the option premium. The breakeven underlying price at expiration is the exercise price minus the option premium. When one sells a put, these results are reversed.
- The profit from a covered call—the purchase of the underlying and sale of a call—is the value at expiration,  $S_T - \max(0, S_T - X)$ , minus  $(S_0 - c_0)$ , the cost of the underlying minus the option premium. The maximum profit is the exercise price minus the original underlying price plus the option premium, and the maximum loss is the cost of the underlying less the option premium. The breakeven underlying price at expiration is the original price of the underlying minus the option premium.
- The profit from a protective put—the purchase of the underlying and a put—is the value at expiration,  $S_T + \max(0, X - S_T)$ , minus the cost of the underlying plus the option premium,  $(S_0 + p_0)$ . The maximum profit is infinite, and the maximum loss is the cost of the underlying plus the option premium minus the exercise price. The breakeven underlying price at expiration is the original price of the underlying plus the option premium.
- The profit from a bull spread—the purchase of a call at one exercise price and the sale of a call with the same expiration but a higher exercise price—is the value at expiration,  $\max(0, S_T - X_1) - \max(0, S_T - X_2)$ , minus the net premium,  $c_1 - c_2$ , which is the premium of the long option minus the premium of the short option. The maximum profit is  $X_2 - X_1$  minus the net premium, and the maximum loss is the net premium. The breakeven underlying price at expiration is the lower exercise price plus the net premium.
- The profit from a bear spread—the purchase of a put at one exercise price and the sale of a put with the same expiration but a lower exercise price—is the value at expiration,  $\max(0, X_2 - S_T) - \max(0, X_1 - S_T)$ , minus the net premium,  $p_2 - p_1$ , which is the premium of the long option minus the premium of the short option. The maximum profit is  $X_2 - X_1$  minus the net premium, and the maximum loss is the net premium. The breakeven underlying price at expiration is the higher exercise price minus the net premium.
- The profit from a butterfly spread—the purchase of a call at one exercise price,  $X_1$ , sale of two calls at a higher exercise price,  $X_2$ , and the purchase of a call at a higher exercise price,  $X_3$ —is the value at expiration,  $\max(0, S_T - X_1) - 2\max(0, S_T - X_2) + \max(0, S_T - X_3)$ , minus the net premium,  $c_1 - 2c_2 + c_3$ . The maximum profit is  $X_2 - X_1$  minus the net premium, and the maximum loss is the net premium. The breakeven underlying prices at expiration are  $2X_2 - X_1$  minus the net premium and  $X_1$  plus the net premium. A butterfly spread can also be constructed by trading the corresponding put options.

- The profit from a collar—the holding of the underlying, the purchase of a put at one exercise price,  $X_1$ , and the sale of a call with the same expiration and a higher exercise price,  $X_2$ , and in which the premium on the put equals the premium on the call—is the value at expiration,  $S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2)$ , minus  $S_0$ , the original price of the underlying. The maximum profit is  $X_2 - S_0$ , and the maximum loss is  $S_0 - X_1$ . The breakeven underlying price at expiration is the initial price of the underlying.
- The profit from a straddle—a long position in a call and a put with the same exercise price and expiration—is the value at expiration,  $\max(0, S_T - X) + \max(0, X - S_T)$ , minus the premiums on the call and put,  $c_0 + p_0$ . The maximum profit is infinite, and the maximum loss is the sum of the premiums on the call and put,  $c_0 + p_0$ . The breakeven prices at expiration are the exercise price plus and minus the premiums on the call and put.
- A box spread is a combination of a bull spread using calls and a bear spread using puts, with one call and put at an exercise price of  $X_1$  and another call and put at an exercise price of  $X_2$ . The profit is the value at expiration,  $X_2 - X_1$ , minus the net premiums,  $c_1 - c_2 + p_2 - p_1$ . The transaction is risk free, and the net premium paid should be the present value of this risk-free payoff.
- A long position in an interest rate call can be used to place a ceiling on the rate on an anticipated loan from the perspective of the borrower. The call provides a payoff if the interest rate at expiration exceeds the exercise rate, thereby compensating the borrower when the rate is higher than the exercise rate. The effective interest paid on the loan is the actual interest paid minus the call payoff. The call premium must be taken into account by compounding it to the date on which the loan is taken out and deducting it from the initial proceeds received from the loan.
- A long position in an interest rate put can be used to lock in the rate on an anticipated loan from the perspective of the lender. The put provides a payoff if the interest rate at expiration is less than the exercise rate, thereby compensating the lender when the rate is lower than the exercise rate. The effective interest paid on the loan is the actual interest received plus the put payoff. The put premium must be taken into account by compounding it to the date on which the loan is taken out and adding it to initial proceeds paid out on the loan.
- An interest rate cap can be used to place an upper limit on the interest paid on a floating-rate loan from the perspective of the borrower. A cap is a series of interest rate calls, each of which is referred to as a caplet. Each caplet provides a payoff if the interest rate on the loan reset date exceeds the exercise rate, thereby compensating the borrower when the rate is higher than the exercise rate. The effective interest paid is the actual interest paid minus the caplet payoff. The premium is paid at the start and is the sum of the premiums on the component caplets.
- An interest rate floor can be used to place a lower limit on the interest received on a floating-rate loan from the perspective of the lender. A floor is a series of interest rate puts, each of which is called a floorlet. Each floorlet provides a payoff if the interest rate at the loan reset date is less than the exercise rate, thereby compensating the lender when the rate is lower than the exercise rate. The effective interest received is the actual interest plus the floorlet payoff. The premium is paid at the start and is the sum of the premiums on the component floorlets.
- An interest rate collar, which consists of a long interest rate cap at one exercise rate and a short interest rate floor at a lower exercise rate, can be used to place an upper limit on the interest paid on a floating-rate loan. The floor, however, places a lower limit on the interest paid on the floating-rate loan. Typically the floor exercise rate is set such that the premium on the floor equals the premium on the cap, so that no cash outlay is required to initiate the

transaction. The effective interest is the actual interest paid minus any payoff from the long caplet plus any payoff from the short floorlet.

- Dealers offer to take positions in options and typically hedge their positions by establishing delta-neutral combinations of options and the underlying or other options. These positions require that the sensitivity of the option position with respect to the underlying be offset by a quantity of the underlying or another option. The delta will change, moving toward 1.0 for in-the-money calls (−1.0 for puts) and 0.0 for out-of-the-money options as expiration approaches. Any change in the underlying price will also change the delta. These changes in the delta necessitate buying and selling options or the underlying to maintain the delta-hedged position. Any additional funds required to buy the underlying or other options are obtained by issuing risk-free bonds. Any additional funds released from selling the underlying or other options are invested in risk-free bonds.
- The delta of an option changes as the underlying changes and as time elapses. The delta will change more rapidly with large movements in the underlying and when the option is approximately at-the-money and near expiration. These large changes in the delta will prevent a delta-hedged position from being truly risk free. Dealers usually monitor their gammas and in some cases hedge their gammas by adding other options to their positions such that the gammas offset.
- The sensitivity of an option to volatility is called the vega. An option's volatility can change, resulting in a potentially large change in the value of the option. Dealers monitor and sometimes hedge their vegas so that this risk does not impact a delta-hedged portfolio.

## PROBLEMS

1. You are bullish about an underlying that is currently trading at a price of \$80. You choose to go long one call option on the underlying with an exercise price of \$75 and selling at \$10, and go short one call option on the underlying with an exercise price of \$85 and selling at \$2. Both the calls expire in three months.
  - A. What is the term commonly used for the position that you have taken?
  - B. Determine the value at expiration and the profit for your strategy under the following outcomes:
    - i. The price of the underlying at expiration is \$89.
    - ii. The price of the underlying at expiration is \$78.
    - iii. The price of the underlying at expiration is \$70.
  - C. Determine the following:
    - i. the maximum profit.
    - ii. the maximum loss.
  - D. Determine the breakeven underlying price at expiration of the call options.
  - E. Verify that your answer to Part D above is correct.

2. You expect a currency to depreciate with respect to the US dollar. The currency is currently trading at a price of \$0.75. You decide to go long one put option on the currency with an exercise price of \$0.85 and selling at \$0.15, and go short one put option on the currency with an exercise price of \$0.70 and selling at \$0.03. Both the puts expire in three months.
  - A. What is the term commonly used for the position that you have taken?
  - B. Determine the value at expiration and the profit for your strategy under the following outcomes:
    - i. The price of the currency at expiration is \$0.87.
    - ii. The price of the currency at expiration is \$0.78.
    - iii. The price of the currency at expiration is \$0.68.
  - C. Determine the following:
    - i. the maximum profit.
    - ii. the maximum loss.
  - D. Determine the breakeven underlying price at the expiration of the put options.
  - E. Verify that your answer to Part D above is correct.
3. A stock is currently trading at a price of \$114. You construct a butterfly spread using calls of three different strike prices on this stock, with the calls expiring at the same time. You go long one call with an exercise price of \$110 and selling at \$8, go short two calls with an exercise price of \$115 and selling at \$5, and go long one call with an exercise price of \$120 and selling at \$3.
  - A. Determine the value at expiration and the profit for your strategy under the following outcomes:
    - i. The price of the stock at the expiration of the calls is \$106.
    - ii. The price of the stock at the expiration of the calls is \$110.
    - iii. The price of the stock at the expiration of the calls is \$115.
    - iv. The price of the stock at the expiration of the calls is \$120.
    - v. The price of the stock at the expiration of the calls is \$123.
  - B. Determine the following:
    - i. the maximum profit.
    - ii. the maximum loss.
    - iii. the stock price at which you would realize the maximum profit.
    - iv. the stock price at which you would incur the maximum loss.
  - C. Determine the breakeven underlying price at expiration of the call options.
4. A stock is currently trading at a price of \$114. You construct a butterfly spread using puts of three different strike prices on this stock, with the puts expiring at the same time. You go long one put with an exercise price of \$110 and selling at \$3.50, go short two puts with an exercise price of \$115 and selling at \$6, and go long one put with an exercise price of \$120 and selling at \$9.
  - A. Determine the value at expiration and the profit for your strategy under the following outcomes:
    - i. The price of the stock at the expiration of the puts is \$106.
    - ii. The price of the stock at the expiration of the puts is \$110.
    - iii. The price of the stock at the expiration of the puts is \$115.
    - iv. The price of the stock at the expiration of the puts is \$120.
    - v. The price of the stock at the expiration of the puts is \$123.

- B. Determine the following:
- the maximum profit.
  - the maximum loss.
  - the stock price at which you would realize the maximum profit.
  - the stock price at which you would incur the maximum loss.
- C. Determine the breakeven underlying price at expiration of the put options.
- D. Verify that your answer to Part C above is correct.
5. A stock is currently trading at a price of \$80. You decide to place a collar on this stock. You purchase a put option on the stock, with an exercise price of \$75 and a premium of \$3.50. You simultaneously sell a call option on the stock with the same maturity and the same premium as the put option. This call option has an exercise price of \$90.
- A. Determine the value at expiration and the profit for your strategy under the following outcomes:
- The price of the stock at expiration of the options is \$92.
  - The price of the stock at expiration of the options is \$90.
  - The price of the stock at expiration of the options is \$82.
  - The price of the stock at expiration of the options is \$75.
  - The price of the stock at expiration of the options is \$70.
- B. Determine the following:
- the maximum profit.
  - the maximum loss.
  - the stock price at which you would realize the maximum profit.
  - the stock price at which you would incur the maximum loss.
- C. Determine the breakeven underlying price at expiration of the put options.
6. You believe that the market will be volatile in the near future, but you do not feel particularly strongly about the direction of the movement. With this expectation, you decide to buy both a call and a put with the same exercise price and the same expiration on the same underlying stock trading at \$28. You buy one call option and one put option on this stock, both with an exercise price of \$25. The premium on the call is \$4 and the premium on the put is \$1.
- A. What is the term commonly used for the position that you have taken?
- B. Determine the value at expiration and the profit for your strategy under the following outcomes:
- The price of the stock at expiration is \$35.
  - The price of the stock at expiration is \$29.
  - The price of the stock at expiration is \$25.
  - The price of the stock at expiration is \$20.
  - The price of the stock at expiration is \$15.
- C. Determine the following:
- the maximum profit.
  - the maximum loss.
- D. Determine the breakeven stock price at expiration of the options.

**The following information relates to Questions 7–12**

Stanley Singh, CFA, is the risk manager at SS Asset Management. Singh works with individual clients to manage their investment portfolios. One client, Sherman Hopewell, is worried about how short-term market fluctuations over the next three months might impact his equity

position in Walnut Corporation. While Hopewell is concerned about short-term downside price movements, he wants to remain invested in Walnut shares as he remains positive about its long-term performance. Hopewell has asked Singh to recommend an option strategy that will keep him invested in Walnut shares while protecting against a short-term price decline. Singh gathers the information in Exhibit 1 to explore various strategies to address Hopewell's concerns.

**EXHIBIT 1** Walnut Corporation Current Stock Price: \$67.79 Walnut Corporation European Options

Exercise Price	Market Call Price	Call Delta	Market Put Price	Put Delta
\$ 55.00	\$ 12.83	4.7	\$ 0.24	-16.7
\$ 65.00	\$ 3.65	12.0	\$ 1.34	-16.9
\$ 67.50	\$ 1.99	16.5	\$ 2.26	-15.3
\$ 70.00	\$ 0.91	22.2	\$ 3.70	-12.9
\$ 80.00	\$ 0.03	35.8	\$ 12.95	-5.0

*Note:* Each option has 106 days remaining until expiration.

Another client, Nigel French, is a trader who does not currently own shares of Walnut Corporation. French has told Singh that he believes that Walnut shares will experience a large move in price after the upcoming quarterly earnings release in two weeks. However, French tells Singh he is unsure which direction the stock will move. French asks Singh to recommend an option strategy that would allow him to profit should the share price move in either direction.

A third client, Wanda Tills, does not currently own Walnut shares and has asked Singh to explain the profit potential of three strategies using options in Walnut: a bull call spread, a straddle, and a butterfly spread. In addition, Tills asks Singh to explain the gamma of a call option. In response, Singh prepares a memo to be shared with Tills that provides a discussion of gamma and presents his analysis on three option strategies:

- Strategy 1:** A straddle position at the \$67.50 strike option
- Strategy 2:** A bull call spread using the \$65 and \$70 strike options
- Strategy 3:** A butterfly spread using the \$65, \$67.50, and \$70 strike call options

7. The option strategy Singh is *most likely* to recommend to Hopewell is a:
  - A. collar.
  - B. covered call.
  - C. protective put.
8. The option strategy that Singh is *most likely* to recommend to French is a:
  - A. straddle.
  - B. butterfly.
  - C. box spread.
9. Based upon Exhibit 1, Strategy 1 is profitable when the share price at expiration is *closest* to:
  - A. \$63.24.
  - B. \$65.24.
  - C. \$69.49.

10. Based upon Exhibit 1, the maximum profit, on a per share basis, from investing in Strategy 2, is *closest* to:
  - A. \$2.26.
  - B. \$2.74.
  - C. \$5.00.
11. Based upon Exhibit 1, and assuming the market price of Walnut's shares at expiration is \$66, the profit or loss, on a per share basis, from investing in Strategy 3, is *closest* to:
  - A. -\$1.57.
  - B. \$0.42.
  - C. \$1.00.
12. Based on the data in Exhibit 1, Singh would advise Tills that the call option with the *largest* gamma would have a strike price *closest* to:
  - A. \$ 55.
  - B. \$ 67.50.
  - C. \$ 80.



# CHAPTER 9

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## RISK MANAGEMENT APPLICATIONS OF SWAP STRATEGIES

Don M. Chance, PhD, CFA

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### LEARNING OUTCOMES

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*After completing this chapter, you will be able to do the following:*

- demonstrate how an interest rate swap can be used to convert a floating-rate (fixed-rate) loan to a fixed-rate (floating-rate) loan;
- calculate and interpret the duration of an interest rate swap;
- explain the effect of an interest rate swap on an entity's cash flow risk;
- determine the notional principal value needed on an interest rate swap to achieve a desired level of duration in a fixed-income portfolio;
- explain how a company can generate savings by issuing a loan or bond in its own currency and using a currency swap to convert the obligation into another currency;
- demonstrate how a firm can use a currency swap to convert a series of foreign cash receipts into domestic cash receipts;
- explain how equity swaps can be used to diversify a concentrated equity portfolio, provide international diversification to a domestic portfolio, and alter portfolio allocations to stocks and bonds;
- demonstrate the use of an interest rate swaption (1) to change the payment pattern of an anticipated future loan and (2) to terminate a swap.

## 1. INTRODUCTION

This reading is the final in a series of three in which we examine strategies and applications of various derivative instruments. We now turn to swaps. Recall that a swap is a transaction in which two parties agree to exchange a series of cash flows over a specific period of time. At least one set of cash flows must be variable—that is, not known at the beginning of the transaction and determined over the life of the swap by the course of an underlying source of uncertainty. The other set of cash flows can be fixed or variable. Typically, no net exchange of money occurs between the two parties at the start of the contract.<sup>1</sup>

Because at least one set of swap payments is random, it must be driven by an underlying source of uncertainty. This observation provides a means for classifying swaps. The four types of swaps are interest rate, currency, equity, and commodity swaps. Interest rate swaps typically involve one side paying at a floating interest rate and the other paying at a fixed interest rate. In some cases both sides pay at a floating rate, but the floating rates are different. Currency swaps are essentially interest rate swaps in which one set of payments is in one currency and the other is in another currency. The payments are in the form of interest payments; either set of payments can be fixed or floating, or both can be fixed or floating. With currency swaps, a source of uncertainty is the exchange rate so the payments can be fixed and still have uncertain value. In equity swaps, at least one set of payments is determined by the course of a stock price or stock index. In commodity swaps at least one set of payments is determined by the course of a commodity price, such as the price of oil or gold. In this reading we focus exclusively on financial derivatives and, hence, do not cover commodity swaps.

Swaps can be viewed as combinations of forward contracts. A forward contract is an agreement between two parties in which one party agrees to buy from another an underlying asset at a future date at a price agreed on at the start. This agreed-upon price is a fixed payment, but the value received for the asset at the future date is a variable payment because it is subject to risk. A swap extends this notion of an exchange of variable and fixed payments to more than one payment. Hence, a swap is like a series of forward contracts.<sup>2</sup> We also saw that a swap is like a combination of options. We showed that pricing a swap involves determining the terms that the two parties agree to at the start, which usually involves the amount of any fixed payment. Because no net flow of money changes hands at the start, a swap is a transaction that starts off with zero market value. Pricing the swap is done by finding the terms that result in equivalence of the present values of the two streams of payments.

After a swap begins, market conditions change and the present values of the two streams of payments are no longer equivalent. The swap then has a nonzero market value. To one party, the swap has a positive market value; to the other, its market value is negative. The process of

<sup>1</sup>Currency swaps can be structured to have an exchange of the notional principals in the two currencies at the start, but because these amounts are equivalent after adjusting for the exchange rate, no *net* exchange of money takes place. At expiration of the swap, the two parties reverse the original exchange, which does result in a net flow of money if the exchange rate has changed, as will probably be the case. A few swaps, called *off-market swaps*, involve an exchange of money at the start, but they are the exception, not the rule.

<sup>2</sup>There are some technical distinctions between a series of forward contracts and a swap, but the essential elements of equivalence are there.

valuation involves determining this market value. For the most part, valuation and pricing is a process that requires only the determination of present values using current interest rates and, as necessary, stock prices or exchange rates.

We also examined the swaption, an instrument that combines swaps and options. Specifically, a swaption is an option to enter into a swap. There are two kinds of swaptions: those to make a fixed payment, called payer swaptions, and those to receive a fixed payment, called receiver swaptions. Like options, swaptions require the payment of a premium at the start and grant the right, but not the obligation, to enter into a swap.<sup>3</sup>

In this reading, we shall examine ways in which swaps can be used to achieve risk management objectives. We already examined certain risk management strategies when we discussed swaps in the reading on risk management applications of option strategies. Here, we go into more detail on these strategies and, of course, introduce quite a few more. We shall also discuss how swaptions are used to achieve risk management objectives.

## 2. STRATEGIES AND APPLICATIONS FOR MANAGING INTEREST RATE RISK

In previous readings, we examined the use of forwards, futures, and options to manage interest rate risk. The interest rate swap, however, is unquestionably the most widely used instrument to manage interest rate risk.<sup>4</sup> In the readings on risk management applications of forward, futures, and options strategies, we examined two primary forms of interest rate risk. One is the risk associated with borrowing and lending in short-term markets. This risk itself has two dimensions: the risk of rates changing from the time a loan is anticipated until it is actually taken out, and the risk associated with changes in interest rates once the loan is taken out. Swaps are not normally used to manage the risk of an anticipated loan; rather, they are designed to manage the risk on a series of cash flows on loans already taken out or in the process of being taken out.<sup>5</sup>

The other form of interest rate risk that concerns us is the risk associated with managing a portfolio of bonds. As we saw in the reading on risk management applications of forward and futures strategies, managing this risk generally involves controlling the portfolio duration. Although futures are commonly used to make duration changes, swaps can also be used, and we shall see how in this reading.

In this section, we look at one more situation in which swaps can be used to manage interest rate risk. This situation involves the use of a relatively new financial instrument called a **structured note**, which is a variation of a floating-rate note that has some type of unusual characteristic. We cover structured notes in Section 2.3.

<sup>3</sup>Forward swaps, on the other hand, are obligations to enter into a swap.

<sup>4</sup>The Bank for International Settlements, in its June 2002 survey of derivative positions of global banks published on 8 November 2002, indicates that swaps make up more than 75% of the total notional principal of all interest rate derivative contracts (see [www.bis.org](http://www.bis.org)).

<sup>5</sup>It is technically possible to use a swap to manage the risk faced in anticipation of taking out a loan, but it would not be easy and would require a great deal of analytical skill to match the volatility of the swap to the volatility of the gain or loss in value associated with changes in interest rates prior to the date on which a loan is taken out. Other instruments are better suited for managing this type of risk.

## 2.1. Using Interest Rate Swaps to Convert a Floating-Rate Loan to a Fixed-Rate Loan (and Vice Versa)

Because much of the funding banks receive is at a floating rate, most banks prefer to make floating-rate loans. By lending at a floating rate, banks pass on the interest rate risk to borrowers. Borrowers can use forwards, futures, and options to manage their exposure to rising interest rates, but swaps are the preferred instrument for managing this risk.<sup>6</sup> A typical situation involves a corporation agreeing to borrow at a floating rate even though it would prefer to borrow at a fixed rate. The corporation will use a swap to convert its floating-rate loan to a fixed-rate loan.

Internet Book Publishers (IBP) is a corporation that typically borrows at a floating rate from a lender called Prime Lending Bank (PLB). In this case, it takes out a one-year \$25 million loan at 90-day Libor plus 300 basis points. The payments will be made at specific dates about 91 days apart. The rate is initially set today, the day the loan is taken out, and is reset on each payment date: On the first payment date, the rate is reset for the second interest period. With four loan payments, the first rate is already set, but IBP is exposed to risk on the other three reset dates. Interest is calculated based on the actual day count since the last payment date, divided by 360. The loan begins on 2 March and the interest payment dates are 2 June, 2 September, 1 December, and the following 1 March.

IBP manages this interest rate risk by using a swap. It contacts a swap dealer, Swaps Provider Inc. (SPI), which is the derivatives subsidiary of a major investment banking firm. Under the terms of the swap, SPI will make payment to IBP at a rate of Libor, and IBP will pay SPI a fixed rate of 6.27%, with payments to be made on the dates on which the loan interest payments are made.

The dealer prices the fixed rate on a swap into the swap such that the present values of the two payment streams are equal. The floating rates on the swap will be set today and on the first, second, and third loan interest payment dates, thereby corresponding to the dates on which the loan interest rate is reset. The notional principal on the swap is \$25 million, the face value of the loan. The swap interest payments are structured so that the actual day count is used, as is done on the loan.

So, IBP borrows \$25 million at a floating rate and arranges for the swap, which involves no cash flows at the origination date. The flow of money on each loan/swap payment date is illustrated in Exhibit 1. We see that IBP makes its loan payments at Libor plus 0.03.<sup>7</sup> The actual calculation of the loan interest is as follows:

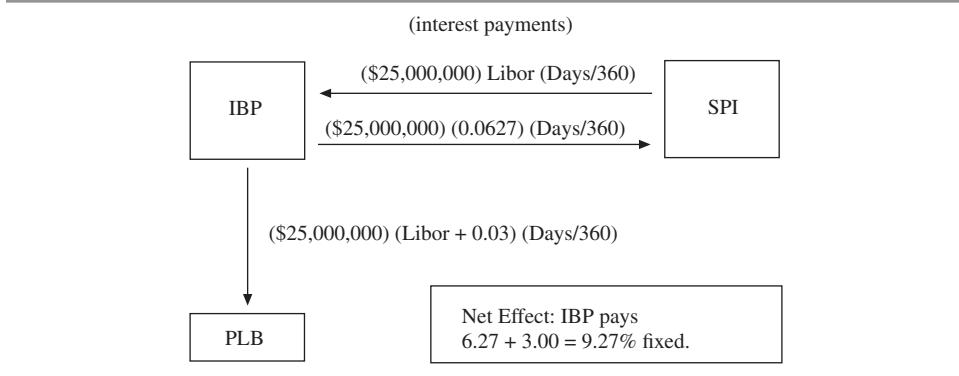
$$(\$25 \text{ million})(\text{Libor} + 0.03)(\text{Days}/360)$$

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<sup>6</sup>It is not clear why swaps are preferred over other instruments to manage the exposure to rising interest rates, but one possible reason is that when swaps were first invented, they were marketed as equivalent to a pair of loans. By being long one loan and short another, a corporation could alter its exposure without having to respond to claims that it was using such instruments as futures or options, which might be against corporate policy. In other words, while swaps are derivatives, their equivalence to a pair of loans meant that no policy existed to prevent their use. Moreover, because of the netting of payments and no exchange of notional principal, interest rate swaps were loans with considerably less credit risk than ordinary loans. Hence, the corporate world easily and widely embraced them.

<sup>7</sup>Remember that when we refer to the payment at a rate of Libor, that rate was established at the previous settlement date or at the beginning of the swap if this is the first settlement period.

## EXHIBIT 1 Converting a Floating-Rate Loan to a Fixed-Rate Loan Using an Interest Rate Swap



The swap payments are calculated in the same way but are based on either Libor or the fixed rate of 6.27%. The interest owed on the loan based on Libor is thus offset by the interest received on the swap payment based on Libor.<sup>8</sup> Consequently, IBP does not *appear to be exposed* to the uncertainty of changing Libor, but we shall see that it is indeed exposed. The net effect is that IBP pays interest at the swap fixed rate of 6.27% plus the 3% spread on the loan for a total of 9.27%.

IBP's swap transaction appears to remove its exposure to Libor. Indeed, having done this transaction, most corporations would consider themselves hedged against rising interest rates, which is usually the justification corporations give for doing swap transactions. It is important to note, however, that IBP is also speculating on rising interest rates. If rates fall, IBP will not be able to take advantage, as it is locked in to a synthetic fixed-rate loan at 9.27%. There can be a substantial opportunity cost to taking this position and being wrong. To understand this point, let us reintroduce the concept of duration.

We need to measure the sensitivity of the market value of the overall position compared to what it would have been had the loan been left in place as a floating-rate loan. For that we turn to duration, a measure of sensitivity to interest rates. If a default-free bond is a floating-rate bond, its duration is nearly zero because interest sensitivity reflects how much the market value of an asset changes for a given change in interest rates. A floating-rate bond is designed with the idea that its market value will not drift far from par. Because the coupon will catch up with the market rate periodically, only during the period between interest payment dates can the market value stray from par value. Moreover, during this period, it would take a substantial interest rate change to have much effect on the market value of the floating-rate bond. Without showing the details, we shall simply state the result that a floating-rate bond's duration is approximately the amount of time remaining until the next coupon payment. For a bond with quarterly payments, the maximum duration is 0.25 years and the minimum duration is zero. Consequently, the average duration is about 0.125 years. From the perspective of the *issuer* rather than the holder, the duration of the position is -0.125.

The duration of IBP's floating-rate loan position in this example is an average of -0.125, which is fairly low compared with most financial instruments. Therefore, the market value of the loan is not very interest-rate sensitive. If interest rates fall, the loan rate will fall in three

<sup>8</sup>Of course in practice, the swap payments are netted and only a single payment flows from one party to the other. Netting reduces the credit risk but does not prevent the Libor component of the net swap payment from offsetting the floating loan interest payment, which is the objective of the swap.

months, and IBP will not have much of a loss from the market value of the loan. If interest rates rise, IBP will not have much of a gain from the market value of the loan.

Now let us discuss the duration of a swap. Remember that entering a pay-fixed, receive-floating swap is similar to issuing a fixed-rate bond and using the proceeds to buy a floating-rate bond. The duration of a swap is thus equivalent to the duration of a long position in a floating-rate bond and a short position in a fixed-rate bond. The duration of the long position in the floating-rate bond would, again, be about 0.125. What would be the duration of the short position in the fixed-rate bond? A one-year fixed-rate bond with quarterly payments would probably have a duration of between 0.6 and 1.0. Let us assume this duration is about 0.75 (nine months) or 75% of the maturity, an assumption we shall make from here out. So the duration of the swap would be roughly  $0.125 - 0.75 = -0.625$ .

Combining the swap with the loan means that the duration of IBP's overall position will be  $-0.125 - 0.625 = -0.75$ . The swap was designed to convert the floating-rate loan to a fixed-rate loan. Hence, the position should be equivalent to that of taking out a fixed-rate loan. As we assumed for a one-year fixed-rate bond with quarterly payments, the duration would be 0.75. The duration of a borrower's position in a fixed-rate loan would be  $-0.75$ , the same as the duration of borrowing with the floating-rate loan and engaging in the swap. The negative duration means that a fixed-rate borrower will be helped by rising rates and a falling market value.<sup>9</sup>

Although the duration of the one-year fixed-rate loan is not large, at least relative to that of bonds and longer-term loans, it is nonetheless six times that of the floating-rate loan. Consequently, the sensitivity of the market value of the overall position is six times what it would have been had the loan been left in place as a floating-rate loan. From this angle, it is hard to see how such a transaction could be called a hedge because declining rates and increasing market values will hurt the fixed-rate borrower. The actual risk increases sixfold with this transaction!<sup>10</sup>

So, can this transaction be viewed as a hedge? If not, why is it so widely used? From a cash flow perspective, the transaction does indeed function as a hedge. IBP knows that its interest payments will all be  $\$25,000,000(0.0927)(\text{Days}/360)$ . Except for the slight variation in days per quarter, this amount is fixed and can be easily built into plans and budgets. So from a planning and accounting perspective, the transaction serves well as a hedge. From a market value perspective, however, it is tremendously speculative. But does market value matter? Indeed it does. From the perspective of finance theory, maximizing the market value of shareholders' equity is the objective of a corporation. Moreover, under recently enacted accounting rules, companies must mark derivative and asset positions to market values, which has improved transparency.

So, in summary, using a swap to convert a floating-rate loan to a fixed-rate loan is a common transaction, one ostensibly structured as a hedge. Such a transaction, despite stabilizing a company's cash outflows, however, increases the risk of the company's market value. Whether this issue is of concern to most companies is not clear. This situation remains one of the most widely encountered scenarios and the one for which interest rate swaps are most commonly employed.

<sup>9</sup>Remember from the reading on risk management applications of forward and futures strategies that the percentage change in the market value of an asset or portfolio is  $-1$  times the duration times the change in yield over 1 plus the yield. So, if the duration is negative, the double minus results in the position benefiting from rising interest rates.

<sup>10</sup>In the example here, the company is a corporation. A bank might have assets that would be interest sensitive and could be used to balance the duration. A corporation's primary assets have varying, inconsistent, and difficult-to-measure degrees of interest sensitivity.

### EXAMPLE 1

Consider a bank that holds a \$5 million loan at a fixed rate of 6% for three years, with quarterly payments. The bank had originally funded this loan at a fixed rate, but because of changing interest rate expectations, it has now decided to fund it at a floating rate. Although it cannot change the terms of the loan to the borrower, it can effectively convert the loan to a floating-rate loan by using a swap. The fixed rate on three-year swaps with quarterly payments at Libor is 7%. We assume the number of days in each quarter to be 90 and the number of days in a year to be 360.

- A. Explain how the bank could convert the fixed-rate loan to a floating-rate loan using a swap.
- B. Explain why the effective floating rate on the loan will be less than Libor.

*Solution to A:* The interest payments it will receive on the loan are  $\$5,000,000(0.06)(90/360) = \$75,000$ . The bank could do a swap to pay a fixed rate of 7% and receive a floating rate of Libor. Its fixed payment would be  $\$5,000,000(0.07)(90/360) = \$87,500$ . The floating payment it would receive is  $\$5,000,000L(90/360)$ , where L is Libor established at the previous reset date. The overall cash flow is thus  $\$5,000,000(L - 0.01)(90/360)$ , Libor minus 100 basis points.

*Solution to B:* The bank will effectively receive less than Libor because when the loan was initiated, the rate was 6%. Then when the swap was executed, the rate was 7%. This increase in interest rates hurts the fixed-rate lender. The bank cannot implicitly change the loan from fixed rate to floating rate without paying the price of this increase in interest rates. It pays this price by accepting a lower rate than Libor when the loan is effectively converted to floating. Another factor that could contribute to this rate being lower than Libor is that the borrower's credit risk at the time the loan was established is different from the bank's credit risk as reflected in the swap fixed rate, established in the Libor market when the swap is initiated.

Equipped with our introductory treatment of the duration of a swap, we are now in a position to move on to understanding how to use swaps to manage the risk of a longer-term position that is also exposed to interest rate risk.

## 2.2. Using Swaps to Adjust the Duration of a Fixed-Income Portfolio

We saw in the previous section that the duration of a swap is the net of the durations of the equivalent positions in fixed- and floating-rate bonds. Thus, the position of the pay-fixed party in a pay-fixed, receive-floating swap has the duration of a floating-rate bond minus the duration of a fixed-rate bond, where the floating- and fixed-rate bonds have cash flows equivalent to the corresponding cash flows of the swap.<sup>11</sup> The pay-fixed, receive-floating swap has

<sup>11</sup>Recall, however, that an interest rate swap does not involve a notional principal payment up front or at expiration. But because a swap is equivalent to being long a fixed- (or floating-) rate bond and short a floating- (or fixed-) rate bond, the principals on the bonds offset, leaving their cash flows identical to that of a swap.

a negative duration, because the duration of a fixed-rate bond is positive and larger than the duration of a floating-rate bond, which is near zero. Moreover, the negative duration of this position makes sense in that the position would be expected to benefit from rising interest rates.

Consider the following transaction. Quality Asset Management (QAM) controls a \$500 million fixed-income portfolio that has a duration of 6.75. It is considering reducing the portfolio duration to 3.50 by using interest rate swaps. QAM has determined that the interest sensitivity of the bond portfolio is adequately captured by its relationship with Libor; hence, a swap using Libor as the underlying rate would be appropriate. But first there are several questions to ask:

- Should the swap involve paying fixed, receiving floating or paying floating, receiving fixed?
- What should be the terms of the swap (maturity, payment frequency)?
- What should be the notional principal?

As for whether the swap should involve paying fixed or receiving fixed, the value of the bond portfolio is inversely related to interest rates. To reduce the duration, it would be necessary to hold a position that moves directly with interest rates. To do this we must add a negative-duration position. Hence, the swap should be a pay-fixed swap to receive floating.

The terms of the swap will affect the need to renew it as well as its duration and the notional principal required. It would probably be best for the swap to have a maturity at least as long as the period during which the duration adjustment applies. Otherwise, the swap would expire before the bond matures, and QAM would have to initiate another swap. The maturity and payment frequency of the swap affect the duration. Continuing with the assumption (for convenience) that the duration of the fixed-rate bond is approximated as 75% of its maturity, we find, for example, that a one-year swap with semi-annual payments would have a duration of  $0.25 - 0.75 = -0.50$ . A one-year swap with quarterly payments would have a duration of  $0.125 - 0.75 = -0.625$ . A two-year swap with semiannual payments would have a duration of  $0.25 - 1.50 = -1.25$ . A two-year swap with quarterly payments would have a duration of  $0.125 - 1.50 = -1.375$ .

These different durations affect the notional principal required, which leads us to the third question. Prior to the duration adjustment, the portfolio consists of \$500 million at a duration of 6.75. QAM then adds a position in a swap with a notional principal of NP and a modified duration of  $MDUR_S$ . The swap will have zero market value.<sup>12</sup> The bonds and the swap will then combine to make up a portfolio with a market value of \$500 million and a duration of 3.50. This relationship can be expressed as follows:

$$\$500,000,000(6.75) + NP(MDUR_S) = \$500,000,000(3.50)$$

The solution for NP is

$$NP = \$500,000,000 \left( \frac{3.50 - 6.75}{MDUR_S} \right)$$

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<sup>12</sup>Recall that the market value of a swap is zero at the start. This market value can obviously vary over time from zero, and such deviations should be taken into account, but to start, the market value will be zero.

The duration of the swap is determined once QAM decides which swap to use. Suppose it uses a one-year swap with semiannual payments. Then, as shown above, the duration would be  $-0.50$ . The amount of notional principal required would, therefore, be

$$NP = \$500,000,000 \left( \frac{3.50 - 6.75}{-0.50} \right) = \$3,250,000,000$$

In other words, this portfolio adjustment would require a swap with a notional principal of more than \$3 billion! This would be a very large swap, probably too large to execute. Consider the use of a five-year swap with semiannual payments. Its duration would be  $0.25 - 3.75 = -3.50$ . Then the notional principal would be

$$NP = \$500,000,000 \left( \frac{3.50 - 6.75}{-3.50} \right) = \$464,290,000$$

With this longer duration, the notional principal would be about \$464 million, a much more reasonable amount, although still a fairly large swap.

So, in general, the notional principal of a swap necessary to change the duration of a bond portfolio worth  $B$  from  $MDUR_B$  to a target duration,  $MDUR_T$ , is

$$NP = B \left( \frac{MDUR_T - MDUR_B}{MDUR_S} \right)$$

## EXAMPLE 2

A \$250 million bond portfolio has a duration of 5.50. The portfolio manager wants to reduce the duration to 4.50 by using a swap. Consider the possibility of using a one-year swap with monthly payments or a two-year swap with semiannual payments.

- A. Determine the durations of the two swaps under the assumption of paying fixed and receiving floating. Assume that the duration of a fixed-rate bond is 75% of its maturity.
- B. Choose the swap with the longer absolute duration and determine the notional principal of the swap necessary to change the duration as desired. Explain your results.

*Solution to A:* The duration of a one-year pay-fixed, receive-floating swap with monthly payments is the duration of a one-year floating-rate bond with monthly payments minus the duration of a one-year fixed-rate bond with monthly payments. The duration of the former is about one-half of the length of the payment interval. That is  $1/24$  of a year, or 0.042. Because the duration of the one-year fixed-rate bond is 0.75 (75% of one year), the duration of the swap is  $0.042 - 0.75 = -0.708$ .

The duration of a two-year swap with semiannual payments is the duration of a two-year floating-rate bond with semiannual payments minus the duration of a two-year fixed-rate bond. The duration of the former is about one-quarter of a year, or 0.25. The duration of the latter is 1.50 (75% of two years). The duration of the swap is thus  $0.25 - 1.50 = -1.25$ .

*Solution to B:* The longer (more negative) duration swap is the two-year swap with semiannual payments. The current duration of the \$250 million portfolio is 5.50 and the target duration is 4.50. Thus, the required notional principal is

$$\begin{aligned} NP &= B \left( \frac{MDUR_T - MDUR_B}{MDUR_S} \right) \\ &= \$250,000,000 \left( \frac{4.50 - 5.50}{-1.25} \right) = \$200,000,000 \end{aligned}$$

So, to lower the duration requires the addition of an instrument with a duration lower than that of the portfolio. The duration of a receive-floating, pay-fixed swap is negative and, therefore, lower than that of the existing portfolio.

### 2.3. Using Swaps to Create and Manage the Risk of Structured Notes

Structured notes are short- or intermediate-term floating-rate securities that have some type of unusual feature that distinguishes them from ordinary floating-rate notes. This unusual feature can be in the form of leverage, which results in the interest rate on the note moving at a multiple of market rates, or can be an inverse feature, meaning that the interest rate on the note moves opposite to market rates. Structured notes are designed to be sold to specific investors, who are often motivated by constraints that restrict their ability to hold derivatives or use leverage. For example, many insurance companies and pension funds are attracted to structured notes, because the instruments qualify as fixed-income securities but have features that are similar to options, swaps, and margin transactions. Issuers typically create the notes, sell them to these investors, and then manage the risk, earning a profit by replicating the opposite position at a cost lower than what they could sell the notes for.

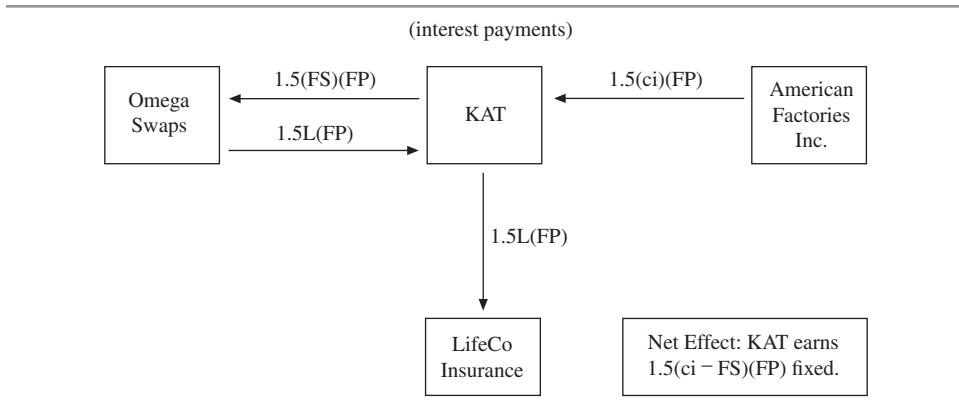
In this section, we shall use the notation FP as the principal/face value of the note,  $c_i$  as the fixed interest rate on a bond, and FS as the fixed interest rate on the swap.

#### 2.3.1. Using Swaps to Create and Manage the Risk of Leveraged Floating-Rate Notes

Kappa Alpha Traders (KAT) engages in a variety of arbitrage-related transactions designed to make small risk-free or low-risk profits. One such transaction involves the issuance of

structured notes, which it sells to insurance companies. KAT plans to issue a leveraged structured note with a principal of FP that pays an interest rate of 1.5 times Libor. This type of instrument is usually called a **leveraged floating-rate note**, or leveraged floater. The reference to *leverage* is to the fact that the coupon is a multiple of a specific market rate of interest such as Libor. The note will be purchased by an insurance company called LifeCo. KAT will use the proceeds to buy a fixed-rate bond that pays an interest rate of  $ci$ . It will then combine the position with a plain vanilla swap with dealer Omega Swaps. Exhibit 2 illustrates how this works.

EXHIBIT 2 Proceeds from a Leveraged Floater Used to Buy a Fixed-Rate Bond, with Risk Managed with a Plain Vanilla Swap



KAT issues the leveraged floater, selling it to LifeCo Insurance with the intent of financing it with a fixed-rate bond and swapping the fixed rate for a floating rate to match the leveraged floater. The periodic interest payment on the leveraged floater will be  $1.5L$ , where  $L$  is Libor, times  $FP$ .<sup>13</sup> It then takes the proceeds and buys a fixed-rate bond issued by a company called American Factories Inc. This bond will have face value of  $1.5(FP)$  and pay a coupon of  $ci$ . KAT is then in a position of receiving a fixed coupon of  $ci$  on principal of  $1.5(FP)$  and paying a floating coupon of  $1.5L$  on a principal of  $FP$ . It then enters into a swap with dealer Omega Swaps on notional principal of  $1.5FP$ . KAT will pay a fixed rate of  $FS$  and receive a floating rate of Libor ( $L$ ). Note the net effect: KAT's obligation on the leveraged floater of  $1.5L(FP)$  is matched by its receipt on the swap. KAT receives  $1.5(ci)(FP)$  on the fixed-rate bond and pays out  $1.5(FS)(FP)$  on the swap, netting  $1.5(FP)(ci - FS)$ . Is this amount an inflow or outflow? It depends. If the interest rate on American Factories' debt reflects greater credit risk than that implied by the fixed rate on the swap, then KAT receives a net payment. Generally that would be the case. Thus, KAT identifies an attractively priced fixed-rate note and captures its return over the swap rate, offsetting the floating rate on the swap with the structured note. Of course, KAT is assuming some credit risk, the risk of default by American Factories, as well as the risk of default by Omega Swaps. On the other hand, KAT put up no capital to engage in this transaction. The cost of the American Factories bond was financed by issuing the structured note.

<sup>13</sup>These payments could be made semiannually, in which case they would be half of  $1.5L(FP)$ .

### EXAMPLE 3

A company issues a floating-rate note that pays a rate of twice Libor on notional principal FP. It uses the proceeds to buy a bond paying a rate of  $ci$ . It also enters into a swap with a fixed rate of FS to manage the risk of the Libor payment on the leveraged floater.

- Demonstrate how the company can engage in these transactions, leaving it with a net cash flow of  $2(FP)(ci - FS)$ .
- Explain under what condition the amount  $(ci - FS)$  is positive.

*Solution to A:* The company has issued a leveraged floater at a rate of  $2L$  on notional principal FP. Then it should purchase a bond with face value of  $2(FP)$  and coupon  $ci$ . It enters into a swap to pay a fixed rate of FS and receive a floating rate of  $L$  on notional principal  $2(FP)$ . The net cash flows are as follows:

From leveraged floater	$-2L(FP)$
From bond	$+(ci)2(FP)$
Floating side of swap	$+(L)2(FP)$
Fixed side of swap	$-(FS)2(FP)$
Total	$2FP(ci - FS)$

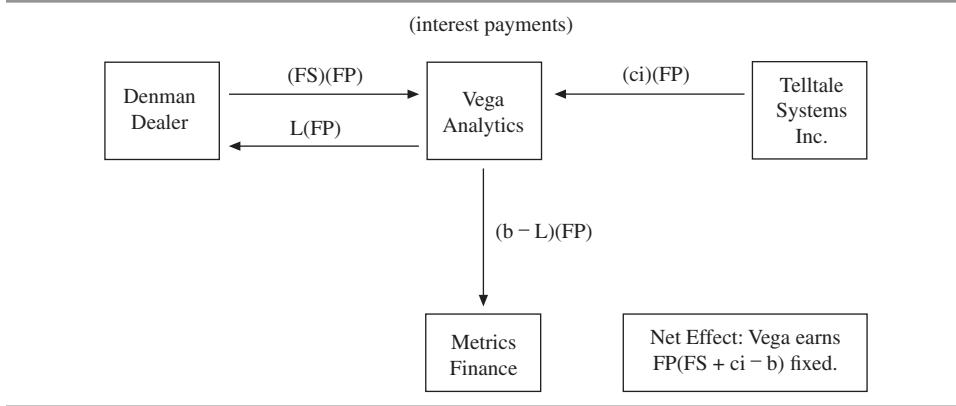
*Solution to B:* The difference between the bond coupon rate,  $ci$ , and the swap fixed rate, FS, will be positive if the bond has greater credit risk than is implied by the fixed rate in the swap, which is based on the Libor term structure and reflects the borrowing rate of London banks. Thus, the gain of  $2(ci - FS)(FP)$  is likely to reflect a credit risk premium.

#### 2.3.2. Using Swaps to Create and Manage the Risk of Inverse Floaters

Another type of structured note is the **inverse floater**. Consider a company called Vega Analytics that, like KAT, engages in a variety of arbitrage trades using structured notes. Vega wants to issue an inverse floater paying a rate of  $b$  minus Libor,  $b - L$ , on notional principal FP. Vega sets the value of  $b$  in negotiation with the buyer of the note, taking into account a number of factors. The rate on the note moves inversely with Libor, but if Libor is at the level  $b$ , the rate on the note goes to zero. If Libor rises above  $b$ , the rate on the note is negative! We shall address this point later in this section.

The pattern will be the same as the pattern used for the leveraged floater: Finance the structured note by a fixed-rate note and then swap the fixed rate for a floating rate to match the structured note. Exhibit 3 shows how Vega issues the note to a company called Metrics Finance and uses the proceeds to purchase a fixed-rate note issued by a company called Telltale Systems, Inc., which pays a rate of  $(ci)(FP)$ . Vega then enters into an interest rate swap with notional principal FP with a counterparty called Denman Dealer Holdings. In this swap, Vega receives a fixed rate of FS and pays  $L$ . Observe that the net effect is that Vega's overall cash flow is  $FP[-(b - L) + ci + FS - L] = FP(FS + ci - b)$ .

**EXHIBIT 3** Proceeds from an Inverse Floater Used to Buy a Fixed-Rate Bond, with Risk Managed with a Plain Vanilla Swap



Clearly if  $b$  is set below  $FS + ci$ , then the overall cash flow is positive. Vega can potentially do this because of the credit risk it assumes. Vega sets  $b$  but cannot set  $FS$ , and  $ci$  is based on both the level of market interest rates and the credit risk of Telltale. The lower Vega sets  $b$ , the larger its cash flow from the overall transactions. But one major consideration forces Vega to limit how low it sets  $b$ : The lower it sets  $b$ , the less attractive the note will be to potential investors.

Remember that the inverse floater pays  $b - L$ . When  $L$  reaches the level of  $b$ , the interest rate on the inverse floater is zero. If  $L$  rises above  $b$ , then the interest rate on the inverse floater becomes negative. A negative interest rate would imply that the lender (Metrics) pays interest to the borrower (Vega). Most lenders would find this result unacceptable, but the lower  $b$  is set, the more likely this outcome will occur. Thus, Vega will want to set  $b$  at a reasonably high level but below  $FS + ci$ .

Regardless of where Vega sets  $b$ , the possibility remains that  $L$  will exceed  $b$ . Metrics may have Vega guarantee that the interest rate on the floater will go no lower than 0%. To manage the risk associated with this guarantee, Vega will buy an interest rate cap. Let us see how all of this works with a numerical example.

Suppose the swap fixed rate,  $FS$ , is 6%, and  $ci$ , the rate on Telltale's note, is 7%. Vega sets  $b$  at 12% and guarantees to Metrics that the interest rate will go no lower than zero. Then the inverse floater pays  $12\% - L$ . As long as Libor is below 12%, Vega's cash flow is  $6 + 7 - 12 = 1\%$ . Suppose  $L$  is 14%. Then Vega's cash flows are

+7% from the Telltale note  
 0% to Metrics  
 +6% from Denman  
 14% to Denman  
 Net: outflow of 1%

Vega's net cash flow is negative. To avoid this problem, Vega would buy an interest rate cap in which the underlying is Libor and the exercise rate is  $b$ . The cap would have a notional principal of  $FP$  and consist of individual caplets expiring on the dates on which the inverse floater rates are set. Thus, on a payment date, when  $L$  exceeds  $b$ , the inverse floater does not pay interest, but the caplet expires in-the-money and pays  $L - b$ . Then the cash flows would be

+7% from the Telltale note  
 0% to Metrics  
 +6% from Denman  
 14% to Denman  
 $(14\% - 12\%) = 2\%$  from the caplet  
 Net: inflow of 1%

Thus, Vega has restored its guaranteed cash inflow of 1%.

Of course, the premium on the cap would be an additional cost that Vega would pass on in the form of a lower rate paid to Metrics on the inverse floater. In other words, for Metrics to not have to worry about ever having a negative interest rate, it would have to accept a lower overall rate. Thus,  $b$  would be set a little lower.

#### EXAMPLE 4

A company issues an inverse floating-rate note with a face value of \$30 million and a coupon rate of 14% minus Libor. It uses the proceeds to buy a bond with a coupon rate of 8%.

- Explain how the company would manage the risk of this position using a swap with a fixed rate of 7%, and calculate the overall cash flow given that Libor is less than 14%.
- Explain what would happen if Libor exceeds 14%. What could the company do to offset this problem?

*Solution to A:* The company would enter into a swap in which it pays Libor and receives a fixed rate of 7% on notional principal of \$30 million. The overall cash flows are as follows:

From the inverse floater	$-(0.14 - L)\$30,000,000$
From the bond it buys	$+(0.08)\$30,000,000$
From the swap	
Fixed payment	$+(0.07)\$30,000,000$
Floating payment	$-(L)\$30,000,000$
Overall total	$+(0.01)\$30,000,000$

*Solution to B:* If Libor is more than 14%, then the inverse floater payment of  $(0.14 - L)$  would be negative. The lender would then have to pay interest to the borrower (the company). For this reason, in most cases, an inverse floater has a floor at zero. In such a case, the total cash flow to this company would be  $(0 + 0.08 + 0.07 - L)\$30,000,000$ . There would be zero total cash flow at  $L = 15\%$ . But at an  $L$  higher than 15%, the otherwise positive cash flow to the company becomes negative.

To offset this effect, the company would typically buy an interest rate cap with an exercise rate of 14%. The cap would have caplets that expire on the interest rate reset dates of the swap/loan and have a notional principal of \$30 million. Then when  $L > 0.14$ , the caplet would pay off  $L - 0.14$  times the \$30 million. This payoff would make up the difference. The price paid for the cap would be an additional cost.

Interest rate swaps are special cases of currency swaps—cases in which the payments are made in different currencies. We now take a look at ways in which currency swaps are used.

### 3. STRATEGIES AND APPLICATIONS FOR MANAGING EXCHANGE RATE RISK

Currency swaps are designed for the purpose of managing exchange rate risk. They also play a role in managing interest rate risk, but only in cases in which exchange rate risk is present. In this section, we look at three situations in which exchange rate risk can be managed using currency swaps.

#### 3.1. Converting a Loan in One Currency into a Loan in Another Currency

Royal Technology Ltd. (ROTECH) is a British high-tech company that is currently planning an expansion of about £30 million into Europe. To implement this expansion, it requires funding in euros. The current exchange rate is €1.62/£, so the expansion will cost €48.6 million. ROTECH could issue a euro-denominated bond, but it is not as well known in the euro market as it is in the United Kingdom where, although not a top credit, its debt is rated investment grade. As an alternative, ROTECH could issue a pound-denominated bond and convert it to a euro-denominated bond using a currency swap. Exhibit 4 illustrates how it could do this.

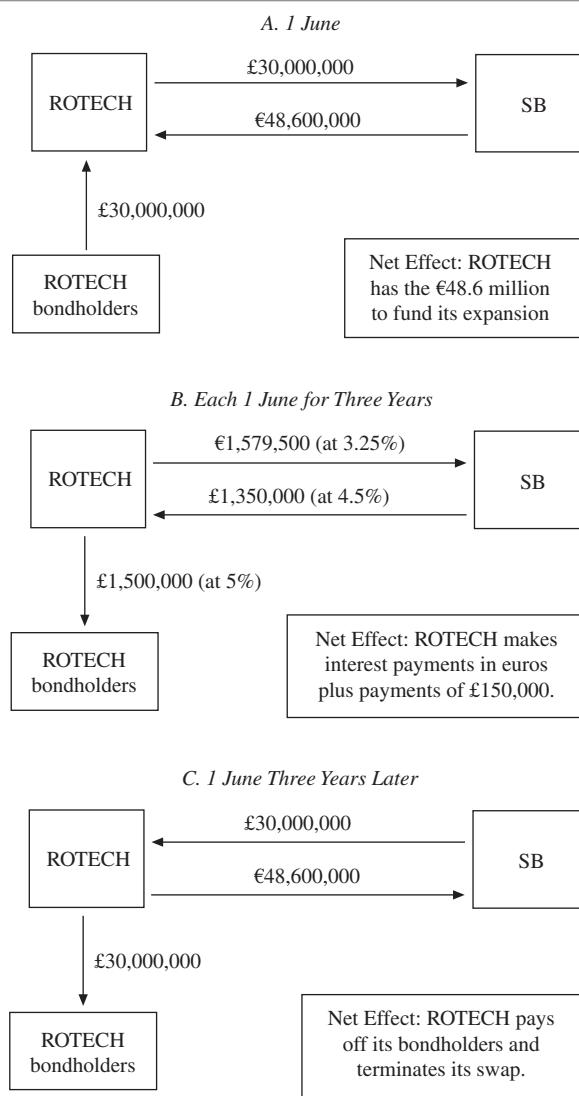
The transaction begins on 1 June. ROTECH will borrow for three years. It issues a bond for £30 million, receiving the proceeds from its bondholders. The bond carries an interest rate of 5% and will require annual interest payments each 1 June. ROTECH then enters into a currency swap with a dealer called Starling Bank (SB). It pays SB £30 million and receives from SB €48.6 million. The terms of the swap call for ROTECH to pay interest to SB at a rate of 3.25% in euros and receive interest from SB at a rate of 4.5% in pounds. With the exchange of principals up front, ROTECH then has the euros it needs to proceed with its expansion. Panel A of Exhibit 4 illustrates the flow of funds at the start of the transaction.

The interest payments and swap payments, illustrated in Panel B, occur each year on 1 June. The interest payments on the pound-denominated bond will be £30,000,000(0.05) = £1,500,000. The interest due to ROTECH from SB is £30,000,000(0.045) = £1,350,000. The interest ROTECH owes SB is €48,600,000(0.0325) = €1,579,500. The net effect is that ROTECH pays interest in euros. The interest received from the dealer, however, does not completely offset the interest it owes on its bond. ROTECH cannot borrow in pounds at the swap market fixed rate, because its credit rating is not as good as the rating implied in the Libor

market term structure.<sup>14</sup> The net effect is that ROTECH will pay additional interest of  $(0.05 - 0.045)\text{£}30,000,000 = \text{£}150,000$ .

Panel C of Exhibit 4 shows the cash flows that occur at the end of the life of the swap and the maturity date of the bond. ROTECH receives the principal of £30 million from SB and pays it to its bondholders, discharging its liability. It then pays €48.6 million to SB to make the final principal payment on the swap.

**EXHIBIT 4** Issuing a Pound-Denominated Bond and Using a Currency Swap to Convert to a Euro-Denominated Bond



<sup>14</sup>Remember that swap fixed rates are determined in the Libor market. This market consists of high-quality London banks, which borrow at an excellent rate. Hence, it is unlikely that ROTECH can borrow at as favorable a rate as these London banks.

This type of transaction is an extremely common use of currency swaps. The advantage of borrowing this way rather than directly in another currency lies in the fact that the borrower can issue a bond or loan in the currency in which it is better known as a creditor. Then, by engaging in a swap with a bank with which it is familiar and probably already doing business, it can borrow in the foreign currency indirectly. For example, in this case, SB is probably a large multinational bank and is well known in foreign markets. But SB also has a longstanding banking relationship with ROTech. Consequently, SB can operate in foreign exchange markets, using its advantage, and pass that advantage on to ROTech.<sup>15</sup>

Another reason this transaction is attractive for borrowers like ROTech is that the company can lower its borrowing cost by assuming some credit risk. If ROTech had issued debt in euros directly, it would face no credit risk.<sup>16</sup> By engaging in the swap, however, ROTech assumes the credit risk that SB will default on its swap payments. If SB defaults, ROTech would still have to make its interest and principal payments to its bondholders. In exchange for accepting this risk, it is likely that ROTech would get a better overall deal. Of course, the desired result would not be achieved if SB defaults. But ROTech would not engage in the transaction if it thought there was much chance of default. Therefore, ROTech acknowledges and accepts some credit risk in return for expecting a better overall rate than if it issues euro-denominated debt.

Because it cannot borrow at the same rate as the fixed rate on the swap, ROTech must pay £150,000 more in interest annually. Recall that the fixed rate on the swap is the rate that would be paid if a London bank issued a par bond. ROTech, like most companies, would not be able to borrow at a rate that attractive. The £150,000 in interest that ROTech pays can be viewed as a credit risk premium, which it would have to pay regardless of whether it borrowed directly in the euro market or indirectly through a swap.

In this transaction, the interest payments were made at a fixed rate. As we previously learned, a currency swap can be structured to have both sides pay fixed, both pay floating, or one pay fixed and the other floating. If ROTech wanted to issue debt in euros at a floating rate, it could issue the bond at a fixed rate and structure the swap so that the dealer pays it pounds at a fixed rate and it pays the dealer euros at a floating rate. Alternatively, it could issue the pound-denominated bond at a floating rate and structure the swap so that the dealer pays it pounds at a floating rate and ROTech pays euros at a floating rate.<sup>17</sup> A currency swap party's choice to pay a fixed or floating rate depends on its views about the direction of interest rate movements. Companies typically choose floating rates when they think interest rates are likely to fall. They choose fixed rates when they think interest rates are likely to rise.

It should also be noted that companies often choose a particular type of financing (fixed or floating) and then change their minds later by executing another swap. For example, suppose ROTech proceeds with this transaction as we illustrated it: paying a fixed rate on its pound-denominated bonds, receiving a fixed rate on the pound payments on its swap, and paying a fixed rate on its euro payments on the swap. Suppose that part of the way through the life of the swap, ROTech thinks that euro interest rates are going down. If it wants to take action based on this view, it could enter into a plain vanilla interest rate swap in euros with SB

<sup>15</sup>SB accepts the foreign exchange in the swap from ROTech and almost surely passes on that risk by hedging its position with some other type of foreign exchange transaction.

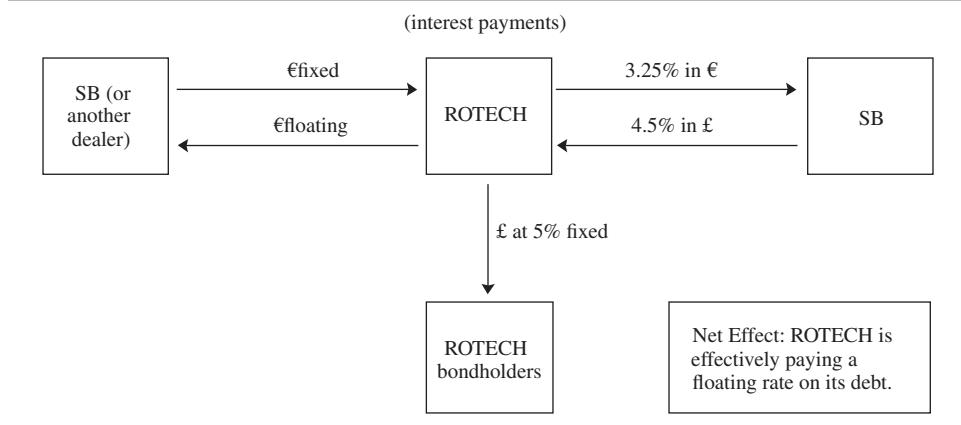
<sup>16</sup>Of course, ROTech's bondholders would face the credit risk that ROTech could default.

<sup>17</sup>It would not matter how ROTech structured the payments on the pound-denominated bond. Either type of payment would be passed through with the currency swap, which would be structured to match that type of payment.

or some other dealer. It would promise to pay the counterparty interest in euros at a floating rate and receive interest in euros at a fixed rate. This transaction would shift the euro interest obligation to floating.

Exhibit 5 illustrates this example. Of course, this transaction is speculative, based as it is on a perception of likely interest rate movements. Moreover, the fixed payments would not offset due to different interest rates.

**EXHIBIT 5** Reversing a Prior Swap to Change from a Fixed-Rate to an Overall Floating-Rate Status



One important way in which currency swaps differ from interest rate swaps is that currency swaps involve the payment of notional principal. However, not all currency swaps involve the payment of notional principal. In transactions such as the ROTECH swap with SB described here, the payment of notional principal is important. The notional principal payment is required, because it offsets the principal on the bond that ROTECH issued in pounds. In the next section, we look at a currency swap in which the notional principal is not paid.

**EXAMPLE 5**

A Japanese company issues a bond with face value of ¥1.2 billion and a coupon rate of 5.25%. It decides to use a swap to convert this bond into a euro-denominated bond. The current exchange rate is ¥120/€. The fixed rate on euro-denominated swaps is 6%, and the fixed rate on yen-denominated swaps is 5%. All payments will be made annually, so there is no adjustment such as Days/360.

- Describe the terms of the swap and identify the cash flows at the start.
- Identify all interest cash flows at each interest payment date.
- Identify all principal cash flows at the maturity of the bond.

*Solution to A:* The company will enter into a swap with notional principal of  $¥1,200,000,000/(\text{¥}120/\text{€}1) = \text{€}10,000,000$ . The swap will involve an exchange of

notional principals at the beginning and end. The annual cash flows will involve paying euros and receiving yen. The following cash flows occur at the start:

From issuance of yen bond	+ ¥1,200,000,000
From swap	- ¥1,200,000,000
	+ €10,000,000
Net	+ €10,000,000

*Solution to B:* The following cash flows occur at the annual interest payment dates:

Interest payments on bond	(¥1,200,000,000)(0.0525) = - ¥63,000,000
Swap payments	
Yen	+ (¥1,200,000,000)(0.05) = + ¥60,000,000
Euro	- (€10,000,000)(0.06) = - €600,000
Net	- ¥3,000,000 - €600,000

*Solution to C:* The following cash flows occur at the end of the life of the swap:

Principal repayment on bond	- ¥1,200,000,000
Swap principal payments	
Yen	+ ¥1,200,000,000
Euro	- €10,000,000
Net	- €10,000,000

### 3.2. Converting Foreign Cash Receipts into Domestic Currency

Companies with foreign subsidiaries regularly generate cash in foreign currencies. Some companies repatriate that cash back into their domestic currency on a regular basis. If these cash flows are predictable in quantity, the rate at which they are converted can be locked in using a currency swap.

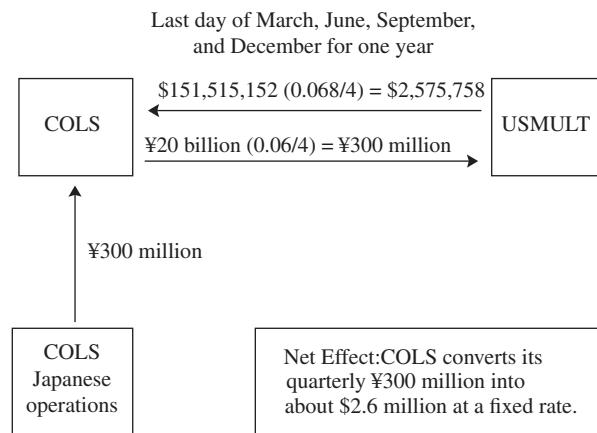
Colorama Software (COLS) is a US company that writes software for digital imaging. So far it has expanded internationally only into the Japanese market, where it generates a net cash flow of about ¥1.2 billion a year. It converts this cash flow into US dollars four times a year, with conversions taking place on the last day of March, June, September, and December. The amounts converted are equal to ¥300 million at each conversion.

COLS would like to lock in its conversion rate for several years, but it does not feel confident in predicting the amount it will convert beyond one year. Thus, it feels it can commit to only a one-year transaction to lock in the conversion rate. It engages in a currency swap with a dealer bank called US Multinational Bank (USMULT) in which COLS will make fixed payments in Japanese yen and receive fixed payments in US dollars. The current spot exchange rate is ¥132/\$, which is \$0.00757576/¥, or \$0.757576 per 100 yen.

The fixed rate on plain vanilla swaps in Japan is 6%, and the fixed rate on plain vanilla swaps in the United States is 6.8%. To create a swap that will involve the exchange of ¥300 million per quarter into US dollars would require a Japanese notional principal of  $\text{¥}300,000,000/(0.06/4) = \text{¥}20,000,000,000$ , which is equivalent to a US dollar notional principal of  $\text{¥}20,000,000,000/\text{¥}132 = \$151,515,152$ .<sup>18</sup>

Thus, COLS engages in a swap for ¥20 billion in which it will pay 6% on a quarterly basis, or 1.5% per quarter in Japanese yen, and receive 6.8% on a quarterly basis, or 1.7% on \$151,515,152. There is no initial or final exchange of notional principals. The cash flows in the swap are illustrated in Exhibit 6.

**EXHIBIT 6** Converting a Series of Foreign Cash Flows into Domestic Currency Using a Currency Swap



COLS pays USMULT ¥20,000,000,000(0.06/4) = ¥300,000,000 quarterly on the swap. This amount corresponds to the cash flow it generates on its Japanese operations. It then receives 6.8% on a dollar notional principal of \$151,515,152 for a total of \$151,515,152(0.068/4) = \$2,575,758. So the swap effectively locks in the conversion of its quarterly yen cash flows for one year.

COLS does face some risk in this transaction. Besides the credit risk of the swap counterparty defaulting, COLS faces the risk that its operations will not generate at least ¥300 million. Of course, COLS' operations could generate more than ¥300 million, but that would mean only that some of its cash flows would not convert at a locked-in rate. If its operations do not generate at least ¥300 million, COLS still must pay ¥300 million to the swap counterparty.

Currency swaps can be used for purposes other than managing conversion risks. These swaps are also used by dealers to create synthetic strategies that allow them to offer new instruments or hedge existing instruments. In the next section, we look at how currency swaps can be used to synthesize an instrument called a dual-currency bond.

<sup>18</sup>A currency swap at 6% with quarterly payments and a notional principal of ¥20 billion would require payments of  $(\text{¥}20,000,000,000)(0.06/4) = \text{¥}300,000,000$  per quarter.

### EXAMPLE 6

A Canadian corporation with a French subsidiary generates cash flows of €10 million a year. It wants to use a currency swap to lock in the rate at which it converts to Canadian dollars. The current exchange rate is C\$0.825/€. The fixed rate on a currency swap in euros is 4%, and the fixed rate on a currency swap in Canadian dollars is 5%.

- A. Determine the notional principals in euros and Canadian dollars for a swap with annual payments that will achieve the corporation's objective.
- B. Determine the overall periodic cash flow from the subsidiary operations and the swap.

*Solution to A:* With the euro fixed rate at 4%, the euro notional principal should be

$$\frac{\text{€}10,000,000}{0.04} = \text{€}250,000,000$$

The equivalent Canadian dollar notional principal would be  $\text{€}250,000,000 \times \text{C\$}0.825 = \text{C\$}206,250,000$ .

*Solution to B:* The cash flows will be as follows:

From subsidiary operations	€10,000,000
Swap euro payment	$-0.04(\text{€}250,000,000) = -\text{€}10,000,000$
Swap Canadian dollar payment	$0.05(\text{C\$}206,250,000) = \text{C\$}10,312,500$

The net effect is that the €10 million converts to C\$10,312,500.

### 3.3. Using Currency Swaps to Create and Manage the Risk of a Dual-Currency Bond

A financial innovation in recent years is the dual-currency bond, on which the interest is paid in one currency and the principal is paid in another. Such a bond can be useful to a multi-national company that might generate sufficient cash in a foreign currency to pay interest but not enough to pay the principal, which it thus might want to pay in its home currency. Dual-currency bonds can be shown to be equivalent to issuing an ordinary bond in one currency and combining it with a currency swap that has no principal payments. Consider the following transactions:

- Issue a bond in dollars.
- Engage in a currency swap with no principal payments. The swap will require the company to pay interest in the foreign currency and receive interest in dollars.

Because the company issued the bond in dollars, it will make interest payments in dollars. The currency swap, however, will result in the company receiving interest in dollars to offset

the interest paid on the dollar-denominated bond and making interest payments on the currency swap in the foreign currency.<sup>19</sup> Effectively, the company will make interest payments in the foreign currency. At the maturity date of the bond and swap, the company will pay off the dollar-denominated bond, and there will be no payments on the swap.

Of course, this example illustrates the synthetic creation of a dual-currency bond. Alternatively, a company can create the dual-currency bond directly by issuing a bond in which it promises to pay the principal in one currency and the interest in another. Then, it might consider offsetting the dual-currency bond by synthetically creating the opposite position. The company is short a dual-currency bond. A synthetic dual-currency bond can be created through the purchase of a domestic bond and a currency swap. If the synthetic dual-currency bond is cheaper than the actual dual-currency bond, the company can profit by offsetting the short position in the actual bond by a long position in the synthetic bond. Let us see how this strategy can be implemented using a trading firm that finds an opportunity to earn an arbitrage profit doing so.

Trans Mutual Arbitrage (TMARB) is such a firm. It has a major client, Omega Construction (OGCONS), that would like to purchase a five-year dual-currency bond. The bond will have a face value of \$10 million and an equivalent face value in euros of €12.5 million.<sup>20</sup> The bond will pay interest in euros at a rate of 4.5%. TMARB sees an opportunity to issue the bond, take the proceeds, and buy a 5.25% (coupon rate) US dollar-denominated bond issued by an insurance company called Kappa Insurance Co. (KINSCO). TMARB will also engage in a currency swap with dealer American Trading Bank (ATB) in which TMARB will receive interest payments in euros at a rate of 4.5% on notional principal of €12.5 million and pay interest at a rate of 5.0% on notional principal of \$10 million.<sup>21</sup> The swap does not involve the payment of notional principals. The swap and bond begin on 15 May and involve annual payments every 15 May for five years.

Exhibit 7 illustrates the structure of this swap. In Panel A, we see the initial cash flows. TMARB receives \$10 million from OGCNOS for the issuance of the dual-currency bond. It then takes the \$10 million and buys a \$10 million dollar-denominated bond issued by KINSCO. There are no initial cash flows on the currency swap.

Panel B shows the annual cash flows, which occur on 15 May for five years. TMARB pays interest of  $\$10,000,000(0.05) = \$500,000$  to OGCNOS on the dual-currency bond. It receives interest of an equivalent amount from ATB on the currency swap. It pays interest of  $\$10,000,000(0.045) = \$450,000$  on the currency swap and receives interest of  $\$10,000,000 \times 0.0525 = \$525,000$  from KINSCO on the dollar-denominated bond. TMARB's euro interest payments are fully covered, and it nets a gain from its dollar interest payments. This opportunity resulted because TMARB found a synthetic way to issue a bond at 5.00% and buy one paying 5.25%. Of course, TMARB will be accepting some credit risk, from both the swap dealer and KINSCO, and its gain may reflect only this credit risk.

Panel C provides the final payments. TMARB pays off OGCNOS its \$10 million obligation on the dual-currency bond and receives \$10 million from KINSCO on the dollar-denominated bond. There are no payments on the swap.

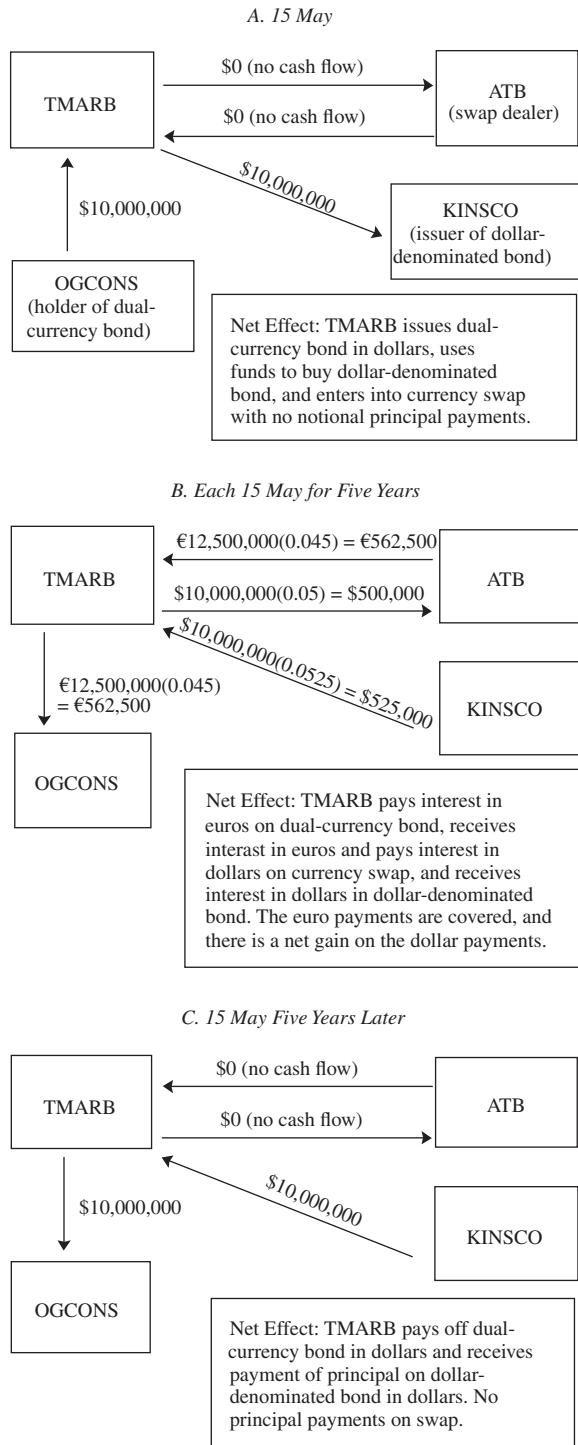
The end result is that TMARB issued a dual-currency bond and offset it with an ordinary dollar-denominated bond and a currency swap with no principal payments. TMARB earned a profit, which may be compensation for the credit risk taken.

<sup>19</sup>It does not matter if the dollar bond has fixed- or floating-rate interest. The currency swap would be structured to have the same type of interest to offset.

<sup>20</sup>The current exchange rate must, therefore, be \$0.80/€.

<sup>21</sup>We have made the fixed rate on the bond the same as the euro fixed rate on the swap for convenience. In practice, there probably would be a spread between the two rates, but the size of the spread would be fixed.

## EXHIBIT 7 Issuing a Dual-Currency Bond and Managing the Risk with an Ordinary Bond and a Currency Swap



### EXAMPLE 7

From the perspective of the issuer, construct a synthetic dual-currency bond in which the principal is paid in US dollars and the interest is paid in Swiss francs. The face value will be \$20 million, and the interest rate will be 5% in Swiss francs. The exchange rate is \$0.80/SF. Assume that the appropriate interest rate for a \$20 million bond in dollars is 5.5%. The appropriate fixed rates on a currency swap are 5.5% in dollars and 5.0% in Swiss francs.

*Solution:* Issue a \$20 million bond in dollars, paying interest at 5.5%. Enter into a currency swap on \$20 million, equivalent to SF25 million. The currency swap will involve the receipt of dollar interest at 5.5% and payment of Swiss franc interest at 5.0%. You will receive \$20 million at the start and pay back \$20 million at maturity. The annual cash flows will be as follows:

On dollar bond issued:	$- 0.055(\$20,000,000) =$	$- \$1,100,000$
On swap:		
Dollars	$+ 0.055(\$20,000,000) =$	$+ \$1,100,000$
Swiss francs	$- 0.05(SF25,000,000) =$	$- SF1,250,000$
Net		$- SF1,250,000$

In the next section, we look at swap strategies in the management of equity market risk.

## 4. STRATEGIES AND APPLICATIONS FOR MANAGING EQUITY MARKET RISK

Equity portfolio managers often want to realign the risk of their portfolios. Swaps can be used for this purpose. In the reading on risk management applications of forward and futures strategies, we covered equity swaps, which are swaps in which at least one set of payments is tied to the price of a stock or stock index. Equity swaps are ideal for use by equity managers to make changes to portfolios by synthetically buying and selling stock without making any trades in the actual stock. Of course, equity swaps have a defined expiration date and thus achieve their results only temporarily. To continue managing equity market risk, a swap would need to be renewed periodically and would be subject to whatever new conditions exist in the market on the renewal date.

### 4.1. Diversifying a Concentrated Portfolio

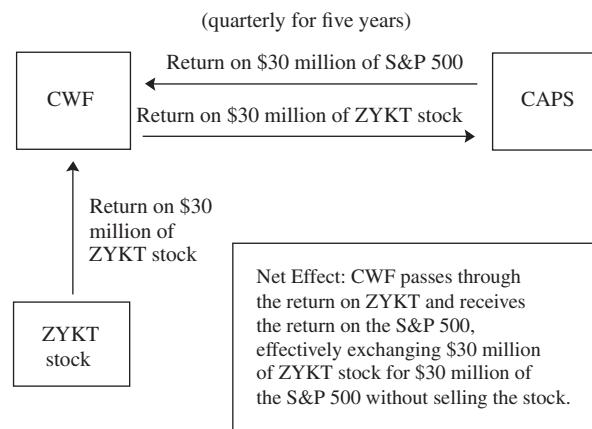
Diversification is one of the most important principles of sound investing. Some portfolios, however, are not very diversified. A failure to diversify can be due to investor ignorance or inattention, or it can arise through no fault of the investor. For example, a single large donation

to a charitable organization can result in a high degree of concentration of an endowment portfolio. The recipient could be constrained or at least feel constrained from selling the stock. Equity swaps can be used to achieve diversification without selling the stock, as we shall see in the following example.

Commonwealth Foundation (CWF) is a charitable organization with an endowment of \$50 million invested in diversified stock. Recently, Samuel Zykes, a wealthy member of the community, died and left CWF a large donation of stock in a company he founded called Zykes Technology (ZYKT). The stock is currently worth \$30 million. The overall endowment value is now at \$80 million, but the portfolio is highly undiversified, with more than a third of its value concentrated in one stock. CWF has considered selling the stock, but its development director believes that the Zykes family will possibly give more money to the foundation at a later date. If CWF sells the stock, the Zykes family may get the impression that the foundation does not want or appreciate the gift. Therefore, the foundation has concluded that it must hold onto the stock. The prospects for very limited growth in the portfolio through other sources, combined with the desire to attract further donations from the Zykes family, lead CWF to conclude that it cannot diversify the portfolio by traditional means anytime soon.

CWF's bank suggests that it consult with a swap dealer called Capital Swaps (CAPS). CAPS recommends an equity swap in which CWF would pay CAPS the return on the \$30 million of ZYKT stock, while CAPS would pay CWF the return on \$30 million of the S&P 500 Index, considered by all parties to be an acceptable proxy for a diversified portfolio. The payments will be made quarterly. CAPS mentions that technically the transaction would need an ending date. Anticipating the possibility of another transaction of this sort pending further donations by the Zykes family in about five years, the parties agree to set the maturity date of the swap at five years. The transaction entails no exchange of notional principal at the start or at the end of the life of the swap. Thus, CWF will maintain possession of the stock, including the voting rights. Exhibit 8 illustrates the structure of the transaction.

#### EXHIBIT 8 Diversifying a Concentrated Portfolio



So, CWF passes through the return on \$30 million of ZYKT stock and receives the return on the S&P 500. Both parties, however, must keep in mind a number of considerations. One is that a cash flow problem could arise for CWF, which must make cash payments each quarter equal to the return on the ZYKT stock. Though CWF will receive cash payments equal to the return on the S&P 500, CWF will have a net cash outflow if ZYKT outperforms the

S&P 500. In fact, it is quite possible that in some quarters, ZYKT will have a positive total return, necessitating a cash obligation, and the S&P 500 will have a negative total return. In that case, the cash payment that CAPS would ordinarily make to CWF for the S&P 500 return would actually be reversed: CWF would owe CAPS for the S&P 500. In short, CWF would owe on both legs of the swap. This possibility could pose a significant cash flow problem and might necessitate the actual sale of some ZYKT stock. The position would then be imbalanced because CWF would own less than \$30 million of ZYKT stock but still owe payments on \$30 million of ZYKT stock. Cash flow management can be a major difficulty in equity swaps.

### EXAMPLE 8

The manager of a charitable foundation's \$50 million stock portfolio is concerned about the portfolio's heavy concentration in one stock, Noble Petroleum (NBP). Specifically, the fund has \$20 million of this stock as a result of a recent donation to the fund. She is considering using an equity swap to reduce the exposure to NBP and allow the fund to invest indirectly in the Wilshire 5000 Index. The stock is currently selling for \$20 a share, and the fund owns 1 million shares. The manager is not quite ready to reduce all of the fund's exposure to NBP, so she decides to synthetically sell off one-quarter of the position. Explain how she would do this and identify some problems she might encounter.

*Solution:* To reduce her exposure on one quarter of her NBP holdings, the manager would have the fund enter into a swap to sell the total return on \$5 million of NBP stock, which is 250,000 shares. The fund will receive from the swap dealer the return on \$5 million invested in the Wilshire 5000.

The swap may result in cash flow problems, however, because the fund must pay out the return on 250,000 shares of NBP stock but does not want to sell that stock. If the return received on \$5 million invested in the Wilshire 5000 is significantly less than the return the fund pays, or if the return on the Wilshire is negative, the fund could have insufficient cash to make its payment. Then it might be forced to sell the stock, something it was trying to avoid in the first place.

Continuing with the example of ZYKT stock, what is the position of the dealer CAPS? It agrees to accept the return on ZYKT stock and pay the return on the S&P 500. This means it is long ZYKT and short the S&P 500. It is likely to hedge its position by buying the equivalent of the S&P 500 through either an index fund or an exchange-traded fund, and selling short ZYKT stock.<sup>22</sup> In fact, its short sale of the ZYKT stock is analogous to CWF selling the stock. CAPS effectively sells the stock for CWF. Also, CAPS is not likely to be able to sell all of the ZYKT stock at one time so it will probably do so over a period of a few days. CAPS may also have a cash flow problem on occasion. If it owes more on the S&P 500 payment than is due it on the ZYKT payment, CAPS may have to liquidate some S&P 500 stock.<sup>23</sup>

<sup>22</sup>Instead of buying or selling short stock, it could use any of a variety of derivative strategies in which it would benefit from a decrease in the price of ZYKT relative to the S&P 500.

<sup>23</sup>To make matters worse, if the S&P goes up and ZYKT goes down, it will owe both sets of payments.

In addition, to make a profit CAPS would probably either not pay quite the full return on the S&P 500 stock or require that CWF pay slightly more than the full return on the ZYKT stock.

We see that equity swaps can be used to diversify a concentrated portfolio. Next we turn to a situation in which equity swaps can be used to achieve diversification on an international scale.

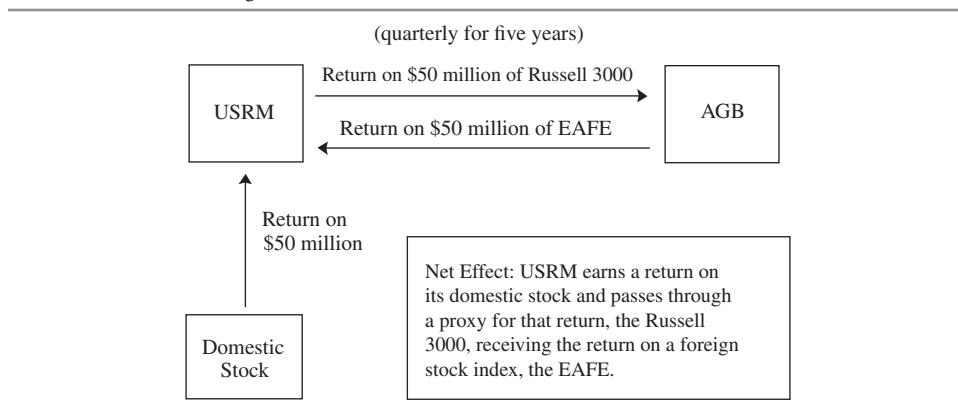
#### 4.2. Achieving International Diversification

The benefits of international diversification are well documented. The correlations between foreign markets and domestic markets generally lead to greater diversification and more efficient investing. Nonetheless, many investors have not taken the step of diversifying their portfolios across international boundaries. Here we shall take a look at a situation in which equity swaps can facilitate the transition from domestic to global diversification.

In this example, Underscore Retirement Management (USRM) is responsible for a \$500 million fund of retirement accounts in the United States. It has never diversified internationally, investing all of its funds in US stock. Representing US large-, medium- and small-cap stocks, the Russell 3000 Index is the portfolio's benchmark. USRM has decided that it needs to add non-US stocks to its portfolio. It would like to start by selling 10% of its US stock and putting the funds in non-US stock. Its advisor, American Global Bank (AGB), has suggested that an equity swap would be a better way to do this than to transact in the stock directly. AGB often deals in non-US stock and has subsidiaries and correspondent relationships in many countries to facilitate the transactions. It is capable of transacting in all stock at lower costs than its clients, and can pass on those savings through derivative transactions.

AGB suggests an equity swap with quarterly payments in which USRM would pay it the return on \$50 million of the Russell 3000 Index. USRM would presumably generate this return from the portfolio it holds. AGB would, in turn, pay USRM the return on \$50 million invested in the Morgan Stanley Capital International (MSCI) EAFE Index, which provides broad coverage of equity markets in Europe, Australasia, and the Far East. This transaction would result in USRM giving up some diversified domestic stock performance and receiving diversified international stock performance. Exhibit 9 illustrates the structure of the transaction.

EXHIBIT 9 Achieving International Diversification



USRM must also consider a number of additional factors. The points made in Section 4.1 regarding the possibility of negative cash flow are highly relevant here as well, and we shall not repeat them. In addition, USRM's domestic stock holding generates a return that will not match perfectly the return on the Russell 3000. This difference in returns, in which the performance of an index does not match the performance of a portfolio that is similar to the index, results in a tracking error. In an extreme case, the domestic stock may go down while the Russell index goes up, which could pose a serious cash flow problem for USRM. USRM may be able to quantify this problem and find that it can effectively manage it. Otherwise, this concern could be an important one for USRM to weigh against the benefits of doing this transaction, which are primarily the savings in transaction costs on the domestic side and on the foreign side. In addition, AGB has currency risk and market risk and passes on to USRM its costs of hedging that risk.

### EXAMPLE 9

A Canadian trust fund holds a portfolio of C\$300 million of Canadian domestic stock. The manager would like to sell off C\$100 million and invest the funds in a pan-European portfolio. The manager arranges to do so using an equity swap in which the domestic stock is represented by the Toronto 300 Composite and the European portfolio is represented by the Dow Jones Euro STOXX 50, an index of leading stocks in the eurozone. Explain how to structure such a swap, and describe how tracking error could potentially interfere with the success of the transaction.

*Solution:* The swap would specify the following transactions on a periodic basis for a specific number of years:

- receive return on DJ Euro STOXX 50,
- pay return on Toronto 300.

Tracking error here is the failure of the derivative cash flow to match precisely the cash flow from the underlying portfolio. In this case, tracking error means that the return actually earned on the domestic portfolio is not likely to perfectly match the Toronto 300 return. These returns are supposed to offset, but they are not likely to do so, certainly not with perfection. The return received on the DJ Euro STOXX 50 does not give rise to tracking error concerns. The index will simply represent the return on the investment in European stocks. If an actual investment in European stocks were made, it would likely differ from this return.

We see in this example that a company can use an equity swap to change its asset allocation. Indeed, an asset allocation change is the major use of equity swaps. In the next section, we shall see a company use equity swaps, combined with a similar swap based on a fixed-income instrument, to implement an asset allocation change. This fixed-income swap will be a slightly new and different instrument from what we have already seen.

### 4.3. Changing an Asset Allocation between Stocks and Bonds

Consider an investment management firm called Tactical Money Management (TMM). It is interested in changing the asset allocation on a \$200 million segment of its portfolio. This money is invested 75% in domestic stock and 25% in US government and corporate bonds. Within the stock sector, the funds are invested 60% in large cap, 30% in mid cap, and 10% in small cap. Within the bond sector, the funds are invested 80% in US government and 20% in investment-grade corporate bonds. TMM would like to change the overall allocation to 90% stock and 10% bonds. Within each class, TMM would also like to make some changes. Specifically, TMM would like to change the stock allocation to 65% large cap and 25% mid cap, leaving the small-cap allocation at 10%. It would like to change the bond allocation to 75% US government and 25% investment-grade corporate. TMM knows that these changes would entail a considerable amount of trading in stocks and bonds. Below we show the current position, the desired new position, and the necessary transactions to get from the current position to the new position:

Stock	Current (\$150 Million, 75%)	New (\$180 Million, 90%)	Transaction
Large cap	\$90 million (60%)	\$117 million (65%)	Buy \$27 million
Mid cap	\$45 million (30%)	\$45 million (25%)	None
Small cap	\$15 million (10%)	\$18 million (10%)	Buy \$3 million

Bonds	Current (\$50 Million, 25%)	New (\$20 Million, 10%)	Transaction
Government	\$40 million (80%)	\$15 million (75%)	Sell \$25 million
Corporate	\$10 million (20%)	\$5 million (25%)	Sell \$5 million

TMM decides it can execute a series of swaps that would enable it to change its position temporarily, but more easily and less expensively than by executing the transactions in stock and bonds. It engages a dealer, Dynamic Derivatives Inc. (DYDINC), to perform the swaps. The return on the large-cap sector is represented by the return on \$27 million invested in the S&P 500 (SP500) Index. Note that the mid-cap exposure of \$45 million does not change, so we do not need to incorporate a mid-cap index into the swap. The return on the small-cap sector is represented by the return on \$3 million invested in the S&P Small Cap 600 Index (SPSC). The return on the government bond sector is represented by the return on \$25 million invested in the Lehman Long Treasury Bond Index (LLTB), and the return on the corporate bond sector is represented by the return on \$5 million invested in the Merrill Lynch Corporate Bond Index (MLCB). Note that for the overall fixed-income sector, TMM will be reducing its exposure.

TMM must decide the frequency of payments and the length of the swap. Equity swap payments tend to be set at quarterly intervals. Fixed-income payments in the form of coupon interest tend to occur semiannually. TMM could arrange for quarterly equity swap payments and semiannual fixed-income swap payments. It decides, however, to structure the swap to

have all payments occur on the same dates six months apart. The length of the swap should correspond to the period during which the firm wants this new allocation to hold. TMM decides on one year. Should it wish to extend this period, TMM would need to renegotiate the swap at expiration. Likewise, TMM could decide to unwind the position prior to one year, which it could do by executing a new swap with opposite payments for the remainder of the life of the original swap.

The equity swaps in this example involve receiving payments tied to the SP500 and the SPSC and making either fixed payments or floating payments tied to Libor. Let us start by assuming that the equity swap payments will be paired with Libor-based floating payments. For the fixed-income payments, however, TMM needs a slightly different type of swap—specifically, a fixed-income swap. This instrument is exactly like an equity swap, but instead of the payment being tied to a stock or stock index, it is tied to a bond or bond index. This type of swap is not the same as an interest rate swap, which involves payments tied to a floating rate such as Libor. Fixed-income swaps, like equity swaps, require the payment of the total return on a bond or bond index against some other index, such as Libor. They are very similar to equity swaps in many respects: The total return is not known until the end of the settlement period, and because the capital gain can be negative, it is possible for the overall payment to be negative. In contrast to equity swaps, however, fixed-income swaps are more dominated by the fixed payment of interest. For equities, the dividends are small, not fixed, and do not tend to dominate capital gains. Other than the amounts paid, however, fixed-income swaps are conceptually the same as equity swaps.<sup>24</sup>

The swaps are initially structured as follows:

*Equity swaps*

Receive return on SP500 on \$27 million

Pay Libor on \$27 million

Receive return on SPSC on \$3 million

Pay Libor on \$3 million

*Fixed-income swaps*

Receive Libor on \$25 million

Pay return on LLTB on \$25 million

Receive Libor on \$5 million

Pay return on MLCB on \$5 million

Note that the overall position involves no Libor payments. TMM pays Libor on \$27 million and on \$3 million from its equity swaps, and it receives Libor on \$25 million and on \$5 million from the fixed-income swaps. Therefore, the Libor payments can be eliminated. Furthermore, the equity and fixed-income swaps can be combined into a single swap with the following payments:

Receive return on SP500 on \$27 million

Receive return on SPSC on \$3 million

Pay return on LLTB on \$25 million

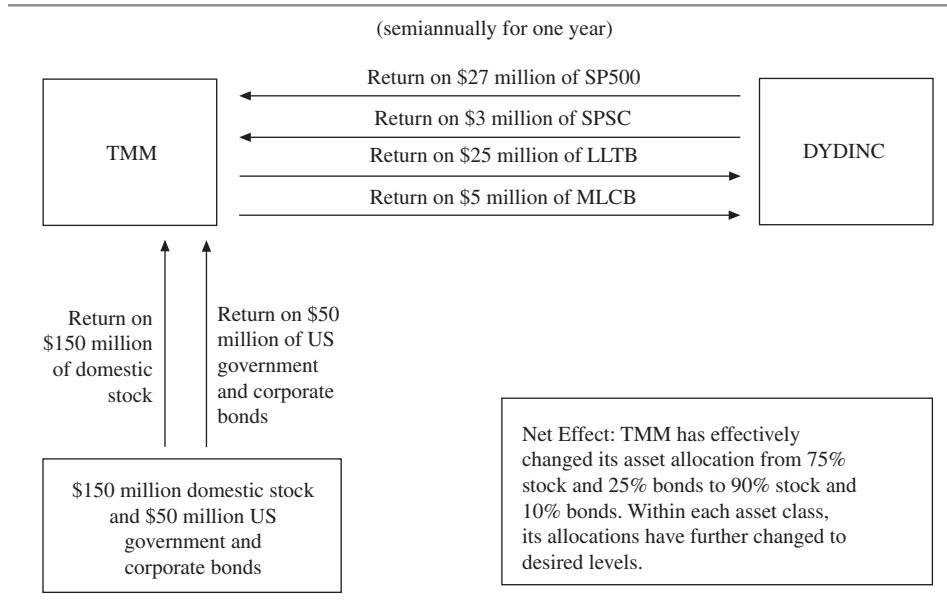
Pay return on MLCB on \$5 million

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<sup>24</sup>Fixed-income swaps, when referred to as total return swaps, are a form of a credit derivative.

This combined equity/fixed-income swap is a single transaction that accomplishes TMM's objective. Exhibit 10 illustrates the overall transaction.

**EXHIBIT 10** Changing an Asset Allocation



Of course, this transaction will not completely achieve TMM's goals. The performance of the various sectors of its equity and fixed-income portfolios are not likely to match perfectly the indices on which the swap payments are based. This problem is what we referred to previously as tracking error. In addition, TMM could encounter a cash flow problem if its fixed-income payments exceed its equity receipts and its portfolio does not generate enough cash to fund its net obligation. The actual stock and bond portfolio will generate cash only from dividends and interest. The capital gains on the stock and bond portfolio will not be received in cash unless a portion of the portfolio is liquidated. But avoiding liquidation of the portfolio is the very reason that TMM wants to use swaps.<sup>25</sup>

**EXAMPLE 10**

A \$30 million investment account of a bank trust fund is allocated one-third to stocks and two-thirds to bonds. The portfolio manager wants to change the overall allocation to 50% stock and 50% bonds, and the allocation within the stock fund from 70% domestic stock and 30% foreign stock to 60% domestic and 40% foreign. The bond allocation will remain entirely invested in domestic corporate issues. Explain how an equity swap could be used to implement this adjustment. You do not need to refer to specific stock indices.

<sup>25</sup>Even worse would be if its fixed-income payments were positive and its equity receipts were negative.

*Solution:* Currently the allocation is \$10 million stock and \$20 million bonds. Within the stock category, the current allocation is \$7 million domestic and \$3 million foreign. The desired allocation is \$15 million stock and \$15 million bonds. Thus, the allocation must change by moving \$5 million into stock and out of bonds. The desired stock allocation is \$9 million domestic and \$6 million foreign. The desired bond allocation is \$15 million, all domestic corporate.

To make the change with a swap, the manager must enter into a swap to receive the return on \$5 million based on a domestic equity index and pay the return on \$5 million based on a domestic corporate bond index. The \$5 million return based on a domestic equity index should be allocated such that \$2 million is based on domestic stock and \$3 million is based on foreign stock.

So far we have seen that an equity swap can be used to reduce or increase exposure to a stock or stock index. One type of investor that is highly exposed to the performance of a single stock is a corporate insider. In the following section, we examine a swap strategy that has been increasingly used in recent years to reduce such exposure.

#### 4.4. Reducing Insider Exposure

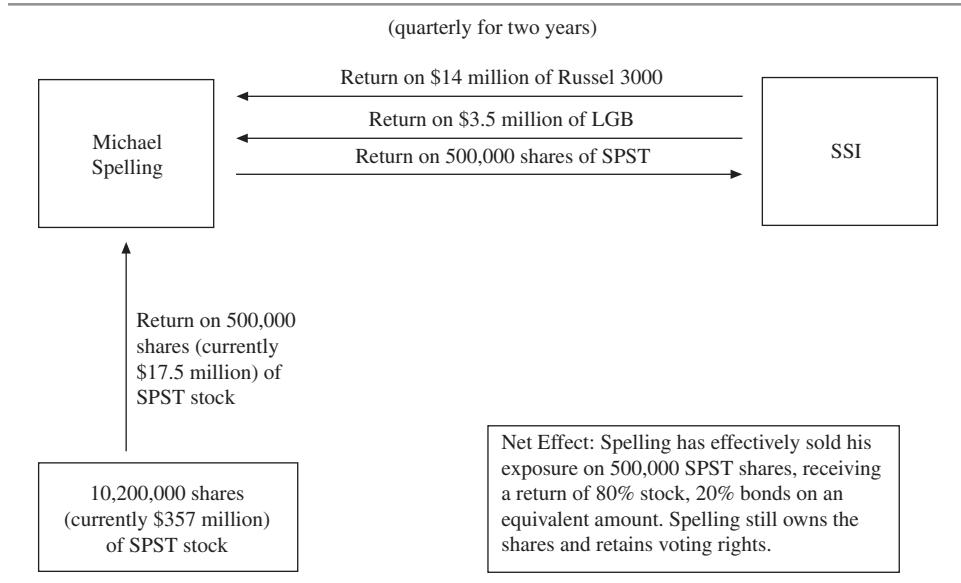
Michael Spelling is the founder and sole owner of a US-based company called Spelling Software and Technology (SPST). After founding the company about 10 years ago, Spelling took it public 2 years ago and retains significant ownership in the form of 10,200,000 shares, currently valued at \$35 a share, for a total value of \$357 million, which represents about 10% of the company. Spelling wants to retain this degree of control of the company, so he does not wish to sell any of his shares. He is concerned, however, that his personal wealth is nearly 100% exposed to the fortunes of a single company.

A swap dealer called Swap Solutions Inc. (SSI) approaches Spelling about a strategy that it has been using lately with much success. This transaction involves an equity swap whereby Spelling would pay the dealer the return on some of his shares in SPST and receive a diversified portfolio return. Spelling finds the idea intriguing and begins thinking about how he would like to structure the arrangement. He decides to initially base the transaction on 500,000 shares of stock, about 4.9% of his ownership. If he is satisfied with how the strategy works, he may later increase his commitment to the swap. At \$35 a share, this transaction has an exposure of \$17.5 million. Specifically, Spelling will pay the total return on 500,000 shares of SPST stock and receive a diversified portfolio return on \$17.5 million. He decides to split the diversified return into 80% stock and 20% bonds. The former will be represented by the return on \$14.0 million invested in the Russell 3000, and the latter will be represented by the return on \$3.5 million invested in the Lehman Brothers Government Bond Index (LGB). The payments will occur quarterly for two years, at which time Spelling will re-evaluate his position and may choose to extend the swap, terminate it, or change the allocation or other terms.

Exhibit 11 illustrates the structure of the swap. Spelling achieves his objectives, but he must consider some important issues in addition to the cash flow problem we have already mentioned. One is that under US law, this transaction is considered an insider sale and must be reported to the regulatory authorities. Thus, there is some additional paperwork. Shareholders

and potential investors may consider the sale a signal of bad prospects for the company. US tax laws also require that the synthetic sale of securities through equity swaps forces a termination of the holding period on the stock. Hence, this transaction has no tax advantages. Spelling will also want to consider the fact that he has sold off some of his exposure but retains control. Shareholders will surely resent the fact that Spelling controls 500,000 votes but does not have any exposure to this stock.<sup>26</sup> Of course, he still retains exposure to 9.7 million shares.

#### EXHIBIT 11 Reducing Insider Exposure



#### EXAMPLE 11

The CEO of a corporation owns 100 million shares of his company's stock, which is currently priced at €30 a share. Given the tremendous exposure of his personal wealth to this one company, he has decided to sell 10% of his position and invest the funds in a floating interest rate instrument. A derivatives dealer suggests that he do so using an equity swap. Explain how to structure such a swap.

*Solution:* The swap is structured so that the executive pays the return on 10 million shares, which is 10% of his holdings, of the company's stock and receives the return based on a floating interest rate, such as Libor, on a notional principal of €300 million.

<sup>26</sup>An interesting question in this regard is whether the shareholders would actually know that the executive had done such a transaction. Careful research is required to identify that executives have made these transactions.

Equity swaps of this sort can be a significant concern for financial analysts. Their possible use makes it difficult to determine if executives have the full exposure represented by the number of shares they own.

Equity swaps involving executives can also have significant agency cost implications. A company incurs agency costs when an executive does not act on behalf of shareholders. Consider the extreme case of an executive who owns more than 50% of a company but who reduces her equity exposure to zero with equity swaps. The executive retains full control of the company, although she has eliminated her equity exposure. This action could entail significant costs to outside shareholders, as the executive does not bear any of the costs of actions or expenditures that increase her personal welfare at the expense of the company. Of course, executives are unlikely to sell off all of their exposure, but the elimination of any exposure on shares still retained for control purposes raises significant questions about whether an executive would act in the best interests of the shareholders. The executive's incentive to perform well would certainly be reduced.

In Sections 2, 3, and 4, we examined the use of interest rate swaps, currency swaps, and equity swaps for managing risk. In the following section, we examine strategies involving the use of swaptions to manage risk.

## 5. STRATEGIES AND APPLICATIONS USING SWAPPTIONS

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A swaption is an option to enter into a swap. Although there are swaptions to enter into equity, currency, and commodity swaps, we will focus exclusively on swaptions to enter into interest rate swaps, which is by far the largest swaptions market. Let us briefly review swaptions.

First, recall that there are two types of swaptions, payer swaptions and receiver swaptions, which are analogous to puts and calls. A payer swaption is an option that allows the holder to enter into a swap as the fixed-rate payer, floating-rate receiver. A receiver swaption is an option that allows the holder to enter into a swap as the fixed-rate receiver, floating-rate payer. In both cases, the fixed rate is specified when the option starts. The buyer of a swaption pays a premium at the start of the contract and receives the right to enter into a swap. The counterparty is the seller of the swaption. The seller receives the premium at the start and grants the right to enter into the swap at the specified fixed rate to the buyer of the swaption. A swaption can be European style or American style, meaning that it can be exercised only at expiration (European) or at any time prior to expiration (American). We shall illustrate applications of both.

A swaption is based on an underlying swap. The underlying swap has a specific set of terms: the notional principal, the underlying interest rate, the time it expires, the specific dates on which the payments will be made, and how the interest is calculated. *All* of the terms of the underlying swap must be specified. Although an ordinary option on an asset has an exercise *price*, a swaption is more like an interest rate option in that it has an exercise *rate*. The exercise rate is the fixed rate at which the holder can enter into the swap as either a fixed-rate payer or fixed-rate receiver. When a swaption expires, the holder decides whether to exercise it based on the relationship of the then-current market rate on the underlying swap to the exercise rate on the swaption. A swaption can be exercised either by actually entering into the swap or by having the seller pay the buyer an equivalent amount of cash. The method used is determined by the parties when the contract is created.

For example, suppose the underlying swap is a three-year swap with semiannual payments with Libor as the underlying floating rate. Consider a payer swaption, which allows entry into this swap as the fixed-rate payer, with an exercise rate of 7%. At expiration, let us say that three-year, semiannual-pay Libor swaps have a fixed rate of 7.25%. If the holder exercises the swaption, it enters a swap, agreeing to pay a fixed rate of 7% and receive a floating rate

of Libor. If the holder has another position for which it might want to maintain the swap, it might simply hold the swap in place. If the holder does not want to maintain the swap, it can enter into a swap in the market, specifying the opposite set of payments—it can pay Libor and receive the market fixed rate of 7.25%. If this swap is done with a different counterparty than the swaption seller, then the two sets of Libor payments are made but are equivalent in amount. Then the payer swaption holder finds itself with a stream of cash flows consisting of 7% payments and 7.25% receipts, for a net overall position of an annuity of 0.25%, split into 0.125% twice a year, for three years. If this swap at the market rate of 7.25% is done with the swaption seller, the two parties are likely to agree to offset the Libor payments and have the swaption seller pay the holder the stream of payments of 0.125% twice a year. If the parties settle the contract in cash, the swaption seller pays the swaption holder the present value of a series of six semiannual payments of 0.125%.

A swaption can also be viewed as an option on a coupon bond. Specifically, a payer swaption with exercise rate  $x$  in which the underlying is a swap with notional principal  $P$  and maturity of  $N$  years at the swaption expiration is equivalent to an at-the-money put option in which the underlying is an  $N$ -year bond at expiration with a coupon of  $x\%$ . Likewise, a receiver swaption is analogous to an at-the-money call option on a bond. These identities will be useful in understanding swaption strategies.

### 5.1. Using an Interest Rate Swaption in Anticipation of a Future Borrowing

We have illustrated extensively the use of swaps to convert fixed-rate loans to floating-rate loans and vice versa. We now consider a situation in which a company anticipates taking out a loan at a future date. The company expects that the bank will require the loan to be at a floating rate, but the company would prefer a fixed rate. It will use a swap to convert the payment pattern of the loan. A swaption will give it the flexibility to enter into the swap at an attractive rate.

In this section, we will use the notation  $FS(1,3)$  for the fixed rate on a swap established at time 1 and ending at time 3.

Benelux Chemicals (BCHEM) is a Brussels-based industrial company that often takes out floating-rate loans. In the course of planning, BCHEM finds that it must borrow €10 million in one year at the floating rate of Euribor, the rate on euros in Frankfurt, from the Antwerp National Bank (ANB). The loan will require semiannual payments for two years. BCHEM knows that it will swap the loan into a fixed-rate loan, using the going rate for two-year Euribor-based swaps at the time the loan is taken out. BCHEM is concerned that interest rates will rise before it takes out the loan. DTD, a Rotterdam derivatives dealer, approaches BCHEM with the idea of doing a European-style swaption. Specifically, for a cash payment up front of €127,500, BCHEM can obtain the right to enter into the swap in one year as a fixed-rate payer at a rate of 7%. BCHEM decides to go ahead with the deal; that is, it buys a 7% payer swaption.

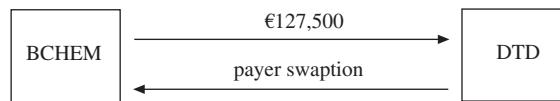
Exhibit 12 illustrates this transaction. In Panel A, BCHEM pays DTD €127,500 in cash and receives the payer swaption. In Panel B, we examine what happens starting when the swaption expires one year later. Note first that regardless of the outcome of the swaption, BCHEM will make floating interest payments of Euribor  $(180/360)\epsilon 10$  million on its loan.<sup>27</sup> In Part (i) of Panel B, we assume that at expiration of the swaption, the rate in the market on the underlying swap,  $FS(1,3)$ , is greater than the swaption exercise rate of 7%. In this case, the swaption is worth exercising.<sup>28</sup> BCHEM enters into the swap with DTD, thereby making payments of

<sup>27</sup>Again, recall that being a floating rate, Euribor is set at the beginning of the settlement period, and the payment is made at the end of that period.

$0.07(180/360)\text{€}10$  million and receiving payments of Euribor  $(180/360)\text{€}10$  million.<sup>29</sup> Both streams of floating payments at Euribor are made, but the payment from DTD exactly offsets the payment to ANB. BCHEM is left paying 7% fixed.

**EXHIBIT 12** Using a Swaption in Anticipation of a Future Borrowing

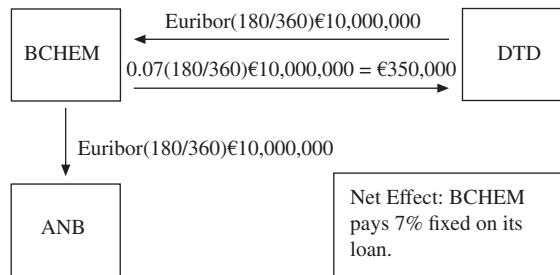
*A. Today*



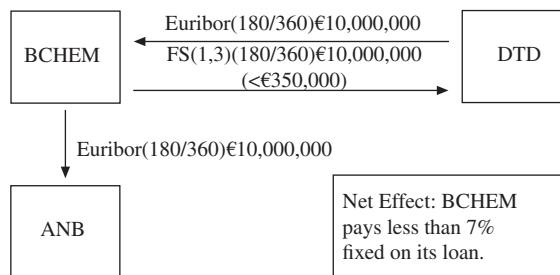
Net Effect: BCHEM pays €127,500 and receives a European-style payer swaption expiring in one year that enable it to enter into a two-year, semiannual-pay €10 million swap to pay fixed and receive Euribor.

*B. Starting One Year Later, Semiannually for Two Years*

(i) Rate on swap in market  $FS(1,3) > 7\%$ , swaption exercised



(ii) Rate on swap in market  $FS(1,3) \leq 7\%$ , swaption not exercised



In Part (ii) of Panel B, at expiration of the swaption, the rate in the market on the underlying swap,  $FS(1,3)$ , is less than or equal to the swaption exercise rate of 7%. The swaption, therefore, expires out-of-the-money. BCHEM still enters into a swap with DTD but does so

<sup>28</sup>To review, remember that at the swaption expiration in one year, which we denote as time 1, the underlying swap is a two-year swap. If the fixed rate on a two-year swap is higher than the rate at which the swaption holder can pay to enter a two-year swap, the swaption is in-the-money. Its value at that point is the present value of a stream of payments equal to the difference between the market fixed rate and the exercise rate on the swaption.

<sup>29</sup>The swap payments would, of course, be netted, but that fact does not affect the point we are making here.

at the market rate of  $FS(1,3)$ , which is less than 7% and the payments are less than €350,000. Of course, both sets of Euribor payments must be made on the loan.

Thus, BCHEM obtained the advantage of flexibility, the right to pay a fixed rate of 7% or less. Of course, this right does not come without a cost. BCHEM had to pay a premium of €127,500 for that right. Therefore, when the loan was taken out one year after the swaption was purchased, the €10 million received was effectively reduced by the €127,500 paid one year earlier plus one year's interest. Whether this premium would be worth paying depends on whether the swaption is correctly priced.<sup>30</sup> Whether this premium was worth it after the fact depends on how far the market rate ended above 7% at the time the loan was taken out.

### EXAMPLE 12

A company plans to take out a \$10 million floating-rate loan in two years. The loan will be for five years with annual payments at the rate of Libor. The company anticipates using a swap to convert the loan into a fixed-rate loan. It would like to purchase a swaption to give it the flexibility to enter into the swap at an attractive rate. The company can use a payer or a receiver swaption. Assume that the exercise rate would be 6.5%.

- A. Identify what type of swaption would achieve this goal and whether the company should buy or sell the swaption.
- B. Calculate the company's annual cash flows beginning two years from now for two cases: The fixed rate on a swap two years from now to terminate five years later,  $FS(2,7)$ , is 1) greater or 2) not greater than the exercise rate. Assume the company takes out the \$10 million floating-rate loan as planned.
- C. Suppose that when the company takes out the loan, it has changed its mind and prefers a floating-rate loan. Now assume that the swaption expires in-the-money. What would the company do, given that it now no longer wants to convert to a fixed-rate loan?

*Solution to A:* The company wants the option to enter into the swap as a fixed-rate payer, so the company would buy a payer swaption.

*Solution to B:* The outcomes based on the swap rate at swaption expiration, denoted as  $FS(2,7)$ , are as follows:

$$FS(2,7) > 6.5\%$$

Exercise the swaption, entering into a swap. The annual cash flows will be as follows:

Pay 0.065(\$10 million) = \$650,000 on swap

Receive L(\$10 million) on swap

Pay L(\$10 million) on loan

Net, pay \$650,000

<sup>30</sup>The basic idea behind swaption pricing is that a model would be used to obtain a fair price for the swaption, to which the market price of €127,500 would be compared.

$FS(2,7) \leq 6.5\%$

Do not exercise swaption; enter into swap at market rate. The annual cash flows will be as follows:

Pay  $FS(2,7)(\$10 \text{ million})$  on swap

Receive  $L(\$10 \text{ million})$  on swap

Pay  $L(\$10 \text{ million})$  on loan

Net, pay  $FS(2,7)(\$10 \text{ million})$

(Note: This is less than \$650,000)

*Solution to C:* In this situation, the company has changed its mind about converting the floating-rate loan to a fixed-rate loan. If the swaption expires out-of-the-money, the company will simply take out the floating-rate loan. If the swaption expires in-the-money, it has value and the company should not fail to exercise it. But exercising the swaption will initiate a swap to pay fixed and receive floating, which would leave the company in the net position of paying a fixed rate of 6.5% when it wants a floating-rate loan. The company would exercise the swaption and then enter into the opposite swap in the market, receiving a fixed rate of  $FS(2,7)$  and paying  $L$ . The net effect is that the company will pay 6.5%, receive  $FS(2,7)$ , which is more than 6.5%, and pay  $L$ . So in effect it will pay a floating-rate loan of less than Libor.

In this example, we showed how a swaption is used to create a swap. Similarly, a swaption can be used to terminate a swap.

## 5.2. Using an Interest Rate Swaption to Terminate a Swap

When a company enters a swap, it knows it may need to terminate the swap before the expiration day. It can do so by either entering an offsetting swap or buying a swaption.

As with any over-the-counter option, the holder of a swap can terminate the swap by entering into an identical swap from the opposite perspective at whatever rate exists in the market. Consider, for example, a Japanese company that enters into a five-year ¥800 million notional principal swap in which it pays a fixed rate and receives a floating rate; that is, it enters a pay-fixed swap. Two years later, the company wants to terminate the swap. It can do so by entering into a new swap with a notional principal of ¥800 million, a remaining life of three years, and with the company paying the floating rate and receiving the fixed rate. If it engages in this swap with a different counterparty than the counterparty of the original swap, then both swaps would remain in place, but the floating payments would be equivalent. The net effect would be that the company would make a stream of fixed payments at one rate and receive a stream of fixed payments at another rate. The rate that is greater depends on the course of interest rates since the time the original swap was put into place. If the new swap is done with the same counterparty as in the original swap, the two parties would likely agree to offset and eliminate both swaps. Then one party would be paying the other a lump sum of the present value of the difference between the two streams of fixed payments. If the company offsets the

swap with a new swap in this manner, it must accept the conditions in the market at the time it offsets the swap.

The second way of terminating a swap is for a company to buy a swaption before it wants to offset the swap. Suppose that when this Japanese company enters into a pay-fixed, receive-floating swap, it also purchases a receiver swaption that allows it to enter into an ¥800 million swap to receive fixed and pay floating with the same terms as the original swap. The swaption exercise rate is 8%. The company must pay cash up front for the swaption, but it then has the right to enter into a new swap to receive a fixed rate of 8% and pay the floating rate. We assume for maximum flexibility that the swaption is structured as an American-style option, allowing the company to exercise it at any time. We also assume that the swaption counterparty is the counterparty to the swap, so that if the swaption is exercised, the payments can be canceled and replaced by a lump sum payment.

Consider this example. Internet Marketing Solutions (IMS) takes out a \$20 million one-year loan with quarterly floating payments at Libor from a lender called Financial Solutions (FINSOLS). Fearing an increase in interest rates, IMS engages in a pay-fixed, receive-floating swap that converts the loan into a fixed-rate loan at 8%. IMS believes, however, that the interest rate outlook could change, and it would like the flexibility to terminate the swap, thereby returning to the status of a floating-rate payer. To give it this flexibility, IMS purchases an American-style receiver swaption for \$515,000. The swaption allows it to enter into a receive-fixed, pay-floating swap at a fixed rate of 8% at the swaption expiration. The swap and swaption counterparty is Wheatstone Dealer (WHD).

Exhibit 13 illustrates this transaction. In Panel A, IMS takes out the loan from FINSOLS, receiving \$20 million. It engages in the swap with WHD, thereby committing to pay fixed and receive Libor. There are no cash flows at the start of the swap contract, but IMS pays WHD \$515,000 for the swaption. Now let us move to the expiration of the swaption, at which time we shall assume that IMS is no longer concerned about rising interest rates and would like to return to the status of a floating-rate borrower. In Panel B(i), at the expiration of the swaption, the market swap rate is greater than or equal to 8%. This panel shows the cash flows if the loan plus swap (note that the loan is floating rate) is converted to a fixed rate using the market fixed rate because the swaption is out-of-the-money. IMS makes interest payments of  $\text{Libor}(90/360)\$20$  million to FINSOLS. IMS makes a swap payment of 8%, which is \$400,000, to WHD, which pays Libor.<sup>31</sup> Thus, to offset the effect of the pay-fixed swap, IMS is better off entering a new swap rather than exercising its swaption. IMS then enters into a swap to receive the market fixed rate, FS, which is greater than or equal to 8%, and pay Libor. IMS is, in effect, paying a floating rate less than Libor (or equal to Libor if the market swap rate is exactly 8%).<sup>32</sup>

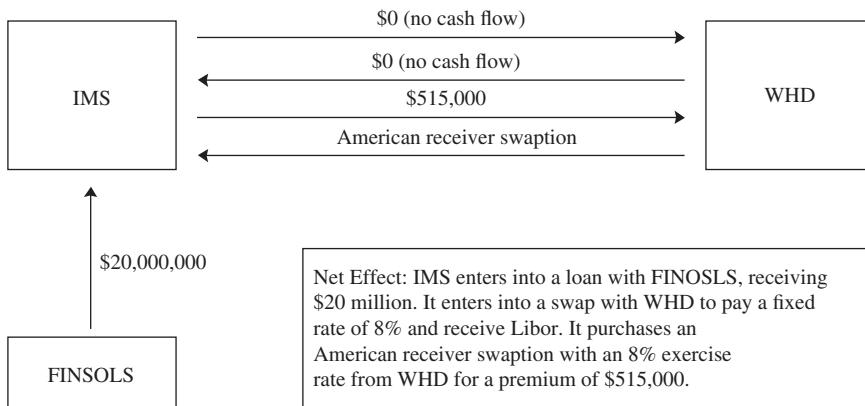
In Panel B(ii), the market swap rate is less than 8% and the loan is converted back to a floating-rate loan by exercising the swaption. IMS makes loan interest payments at Libor to FINSOLS and swap payment of 8% or \$400,000 to WHD, which pays LIBOR. Exercise of the swaption results in IMS entering into a swap to receive a fixed rate of 8% and pay a floating rate of Libor. The swap and swaption would probably be structured to offset and terminate both swaps. At the end of the transaction, the loan is paid off and there are no payments on the swap or swaption. If IMS wants to continue as a fixed-rate payer, the swaption would still be exercised if it is in-the-money but not if it is out-of-the-money.

<sup>31</sup>In practice, the two parties would net the difference and have one party pay the other.

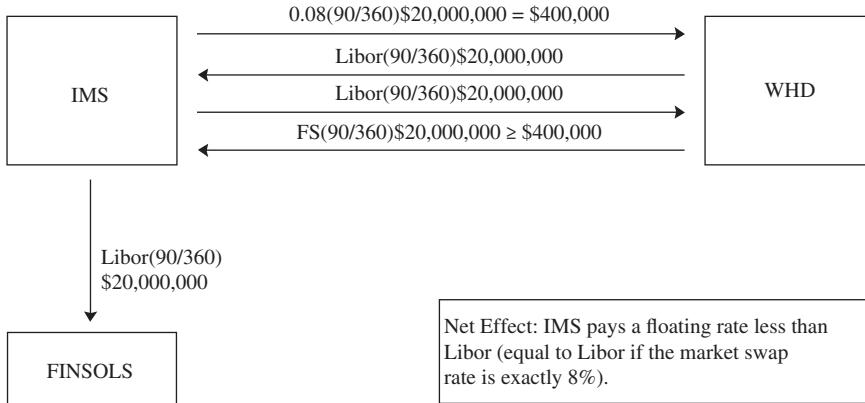
<sup>32</sup>In practice, IMS might choose to not enter into the swap at the market fixed rate and just carry the old swap to reduce the cost of the loan.

## EXHIBIT 13 Using an American-Style Swaption to Terminate a Swap

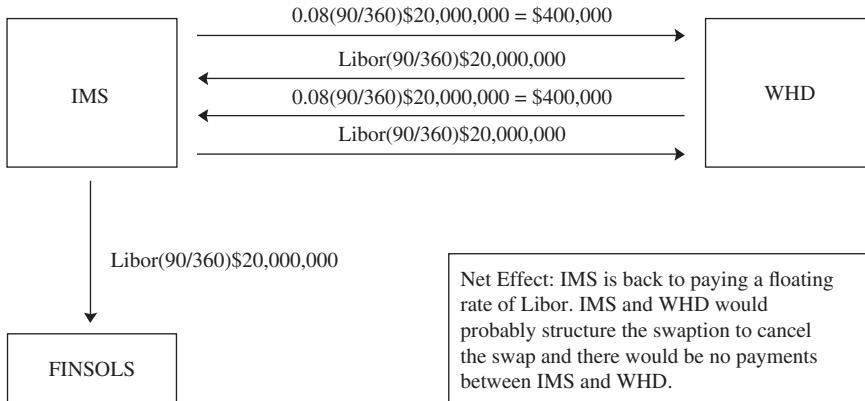
## A. Today



## B. During Life of Loan

(i) Swap rate  $\geq 8\%$ . Swaption not exercised.Enter into receive-fixed, pay-floating swap at market fixed rate ( $\geq 8\%$ ).(ii) Swap rate  $< 8\%$ . Swaption exercised.

Enter into receive-fixed, pay-floating swap at fixed rate of 8%.



We see that the swaption offers the holder the opportunity to terminate the swap at the exercise rate or better. Because the swaption is American style, a variety of complex issues are involved in the exercise decision, but let us focus on the moneyness and the holder's view of market conditions. If a borrower feels that rates will fall, it would then want to convert its pay-fixed position to a pay-floating position. If the market rate is more than the exercise rate, the borrower can do so by entering into a swap at the market rate. It can then receive more than the exercise rate, which more than offsets the rate it pays on the swap. The borrower would then effectively be paying less than Libor. If the rate in the market is less than the exercise rate, the borrower can exercise the swaption, thereby receiving the exercise rate to offset the rate it pays on the swap. Alternatively, it can choose to continue paying a floating rate but can still exercise the swaption if doing so is optimal.

As we previously described, swaptions are equivalent to options on bonds. A payer swaption is equivalent to a put option on a bond, and a receiver swaption is equivalent to a call option on a bond. The interest rate swaptions market is a very liquid one, and many companies use swaptions as substitutes for options on bonds. Any strategy that one might apply with options on bonds can be applied with swaptions. We shall not go over the myriad of such strategies, as they have been covered extensively in other literature. We shall, however, look at a particular one, in which a swaption can be used to substitute for a callable bond.

### EXAMPLE 13

A company is engaged in a two-year swap with quarterly payments. It is paying 6% fixed and receiving Libor. It would like the flexibility to terminate the swap at any time prior to the end of the two-year period.

- A. Identify the type of swaption that would achieve this objective.
- B. Consider a time  $t$  during this two-year life of the swaption in which it is being considered for exercise. Use a 7% exercise rate. The fixed rate in the market on a swap that would offset the existing swap is denoted as  $FS(t,2)$ . Examine the payoffs of the swaption based on whether  $FS(t,2)$  is 1) equal to or above 7% or 2) below 7%.

*Solution to A:* Because the company is paying a fixed rate and receiving a floating rate, it should enter into a swap to receive a fixed rate and pay a floating rate. It thus would want a receiver swaption. For maximum flexibility, it should structure the transaction as an American-style swaption.

*Solution to B:*  $FS(t,2) \geq 7\%$

The swaption is out-of-the-money and is not exercised. To terminate the existing swap, one would enter into a swap at the market rate. This swap would involve receiving the market rate  $FS(t,2)$ , which is at least 7%, and paying Libor. The Libor payments offset, and the net effect is a net positive cash flow of  $FS(t,2) - 6\%$ .

$FS(t,2) < 7\%$

Exercise the swaption, entering into a swap to receive 7% and pay Libor. The other swap involves paying 6% and receiving Libor. The Libors offset, leaving a net positive cash flow of  $7 - 6 = 1\%$ .

Note: It is not necessary that the net cash flow be positive. The positive net cash flow here is a result of choosing a 7% exercise rate, but a lower exercise rate could be chosen. The higher the exercise rate, the more expensive the receiver swaption.

### 5.3. Synthetically Removing (Adding) a Call Feature in Callable (Noncallable) Debt

A callable bond is a bond in which the issuer has the right to retire it early. The issuer has considerable flexibility to take advantage of declining interest rates. This feature is like a call option on the bond. As interest rates fall, bond prices rise. By calling the bond, the issuer essentially buys back the bond at predetermined terms, making it equivalent to exercising a call option to buy the bond. The issuer pays for this right by paying a higher coupon rate on the bond.

In some cases, the issuer of a callable bond may find that it no longer expects interest rates to fall sufficiently over the remaining life of the bond to justify calling the bond. Then it would feel that it is not likely to use the call feature, but it is still paying the higher coupon rate for the call feature. A swaption can be used to effectively sell the embedded call. This strategy involves synthetically removing the call from callable debt by selling a receiver swaption.<sup>33</sup> A receiver swaption (receive fixed) becomes more valuable as rates decline, thus balancing the short call. In effect, the call feature is sold for cash. Recall that a receiver swaption is like a call option on a bond. Because the issuer of the callable bond holds a call on the bond, it would need to sell a call to offset the call embedded in the debt. It can effectively do so by selling a receiver swaption. This swaption will not cancel the bond's call feature. Both options will be in force, but both options should behave identically. If the call feature is worth exercising, so should the swaption. Let us see how this strategy works.

#### 5.3.1. Synthetically Removing the Call from Callable Debt

Several years ago, Chemical Industries (CHEMIND) issued a callable \$20 million face value bond that pays a fixed rate of 8% interest semiannually. The bond now has five years until maturity. CHEMIND does not believe it is likely to call the bond for the next two years and would like to effectively eliminate the call feature during that time. To simplify the problem somewhat, we shall assume that the bond would be called only in exactly two years and not any time sooner. Thus, CHEMIND can manage this problem by selling a European swaption that would expire in two years.<sup>34</sup> Because the bond would have a three-year life when it is called, the swap underlying the swaption would be a three-year swap. It would also be a swap to receive fixed and pay floating, with payment dates aligned with the interest payment dates on the bond.

Let us suppose that the 8% rate CHEMIND is paying on the bond includes a credit spread of 2.5%, which should be viewed as a credit premium paid over the Libor par rate. CHEMIND is paying 2.5% for the credit risk it poses for the holder of the bond. On the receiver swaption it wants to sell, CHEMIND must set the exercise rate at  $8 - 2.5 = 5.5\%$ . Note that the credit spread is not part of the exercise rate. The swaption can be used to manage only the risk of interest rate changes driven by the term structure and not credit. We are assuming

<sup>33</sup>This strategy is sometimes referred to as *monetizing* a call.

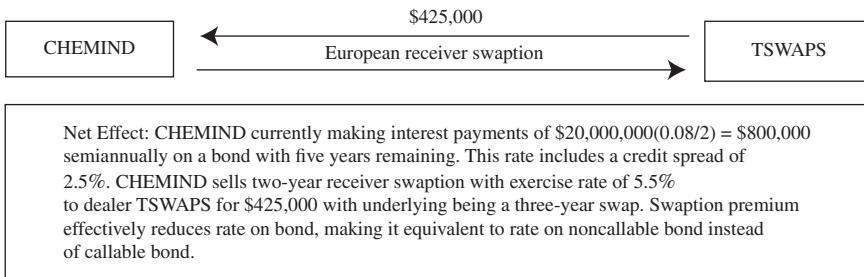
<sup>34</sup>CHEMIND might prefer an American swaption to give it the flexibility to exercise at any time, but we simplify the problem a little and use a European swaption.

no change in CHEMIND's credit risk. Hence, it will continue to pay the credit spread in the rate on the new bond that it issues if it calls the old bond.

The swaption dealer, Top Swaps (TSWAPS), prices the swaption at \$425,000. The strategy is illustrated in Exhibit 14.

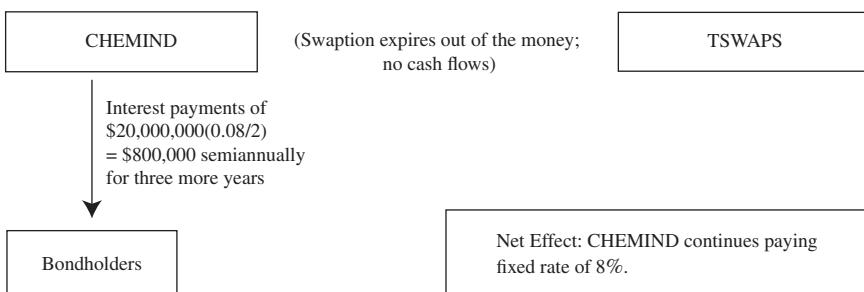
**EXHIBIT 14** Selling a Receiver Swaption to Offset the Call Feature on a Bond

*A. Today; During Life of the Bond*

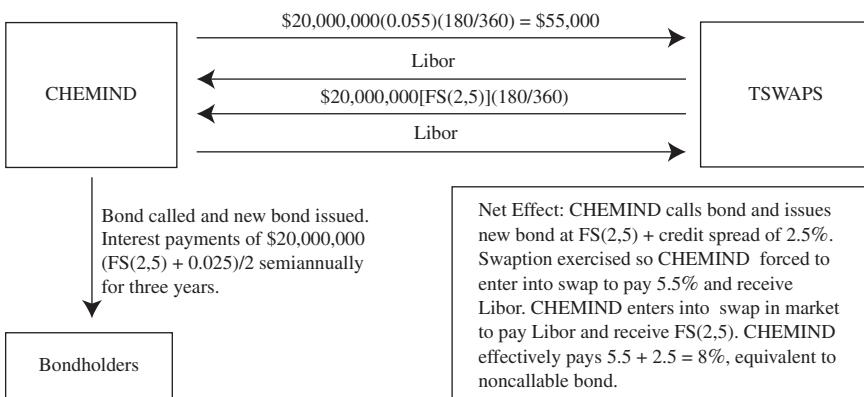


*B. At Expiration of the Swaption*

(i) If  $FS(2,5)$  (Market rate on underlying swap)  $\geq 5.5\%$



(ii) If  $FS(2,5)$  (Market rate on underlying swap)  $< 5.5\%$



Panel A shows that CHEMIND receives \$425,000 from selling the receiver swaption to dealer TSWAPS. This cash effectively reduces its remaining interest payments on the bond.

In Panel B, we see what happens at the swaption expiration in two years. Remember that the swaption is identical to a call option on the bond, so if the swaption is exercised, the call on the bond will be exercised at the same time. Let  $FS(2,5)$  be the fixed rate at the swaption expiration on a three-year swap. We first assume that  $FS(2,5)$  is greater than or equal to the exercise rate on the swaption of 5.5%. Because interest rates have not fallen below 5.5%, it is unprofitable to exercise the swaption or call the bond. CHEMIND continues making interest payments of 8% on \$20 million, which is \$800,000 semiannually for three more years. Panel B(i) illustrates this outcome.

In Panel B(ii), we let  $FS(2,5)$  be less than 5.5%. Then the swaption will be exercised and the bond will be called. To fund the bond call, a new bond will be issued at a rate of  $FS(2,5)$  plus the credit spread of 2.5%, which we assume has not changed. The swaption is exercised, so CHEMIND is obligated to enter into a swap to pay 5.5% and receive Libor. Now, however, CHEMIND is receiving Libor and making fixed payments to its bondholders and to TSWAPS. It can reverse the Libor flow by entering into a swap at the market rate of  $FS(2,5)$ . In other words, it enters into a new swap to receive  $FS(2,5)$  and pay Libor. Note from the figure that it receives Libor and pays Libor. These two flows would likely be canceled. CHEMIND makes fixed swap payments at a rate of 5.5% and receives fixed swap payments at a rate of  $FS(2,5)$ , which is 250 basis points (the credit spread) less than the rate on the new fixed-rate bond it has issued. These payments at the rate  $FS(2,5)$  offset all but the credit spread portion of the interest payments on its loan. CHEMIND then effectively pays a fixed rate of 5.5%, the swaption exercise rate, plus 2.5%, the credit spread. So, CHEMIND ends up paying 8%, the same as the rate on the original debt. The swaption has effectively converted the callable bond into a noncallable bond by removing the call feature from the bond. It hopes that this outcome, in which the bond is called and the swaption is exercised, does not occur, or it will regret having removed the call feature. Nonetheless, it received cash up front for the swaption and is paying a lower effective interest rate as it would had the bond been noncallable in the first place, so it must accept this risk.

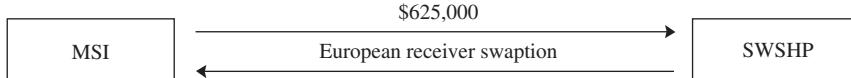
### 5.3.2. Synthetically Adding a Call to Noncallable Debt

If a swaption can undo a call feature, it can also add a call feature. Market Solutions, Inc. (MSI) has a \$40 million noncallable bond outstanding at a rate of 9% paid semiannually with three more years remaining. Anticipating the possibility of declining interest rates in about one year, MSI wishes this bond were callable. It can synthetically add the call feature by purchasing a receiver swaption. A receiver swaption is equivalent to a call option on a bond because the option to receive a fixed rate increases in value as rates decline. By purchasing the receiver swaption, it has in effect purchased an option on the bond.

To structure the receiver swaption properly, MSI notes that the interest rate it is paying on the bond includes a credit spread of 3% over the par bond rate from the Libor term structure. It should set the exercise rate on the swaption at  $9 - 3 = 6\%$ . The swaption will be on a two-year swap with payment dates coinciding with the interest payment dates on the bond. The notional principal will be the \$40 million face value on the bond. To simplify the problem, we assume a European swaption, meaning that the only time MSI will consider exercising the swaption or calling the bond will be in exactly one year, with the bond having two years to maturity at that time. The swaption will cost \$625,000, and the counterparty dealer will be Swap Shop (SWSHP). Exhibit 15 illustrates the transaction.

## EXHIBIT 15 Buying a Receiver Swaption to Add a Call Feature to a Bond

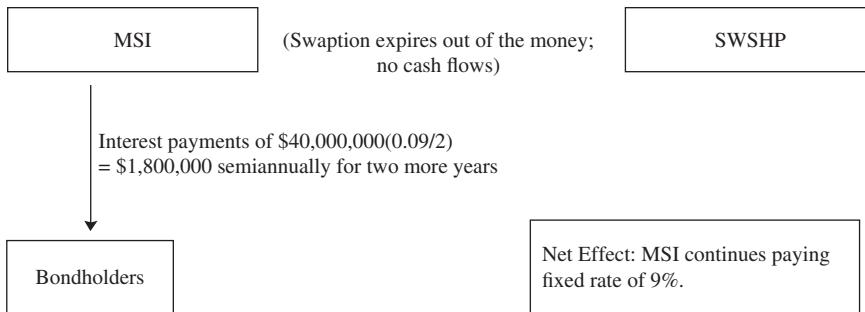
## A. Today



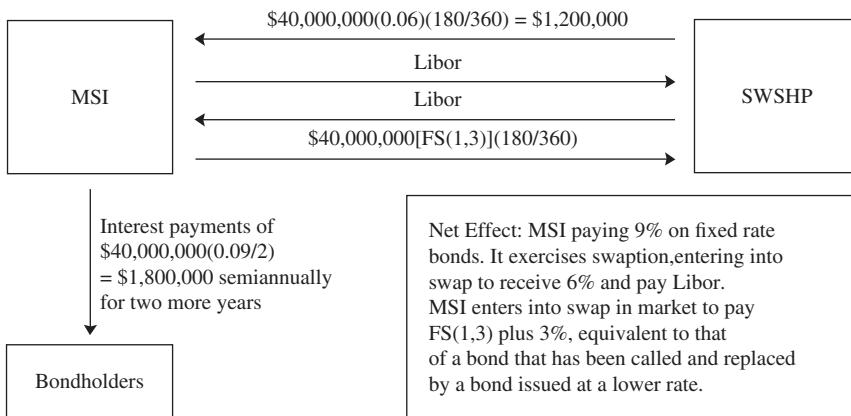
Net Effect: MSI currently making interest payments of  $\$40,000,000(0.09/2) = \$1,800,000$  semiannually on a bond with three years remaining. This rate includes a credit spread of 3%. MSI buys one-year receiver swaption with exercise rate of 6% from dealer Swap Shop for \$625,000 with underlying being two-year swap. Swaption premium effectively increases rate on bond, making it equivalent to rate on callable bond instead of noncallable bond.

## B. At Expiration of the Swaption

(i) If  $FS(1,3)$  (Market rate on underlying swap)  $\geq 6\%$



(ii) If  $FS(1,3)$  (Market rate on underlying swap)  $< 6\%$ .



In Panel A, we see MSI paying \$625,000 for the swaption. This cost effectively raises the interest rate MSI pays on the bond to that of a callable bond. Panel B(i) illustrates the case in which the fixed rate on the underlying swap,  $FS(1,3)$ , is greater than or equal to the exercise rate on the swaption at the swaption expiration. Remember that if market conditions are such that the swaption would be exercised, then the bond would be called. In this case, however, interest rates are not low enough to justify exercise of the swaption or calling of the bond. MSI will continue making its 9% interest payments on the bond.

In Panel B(ii), we let  $FS(1,3)$  be less than 6%. Then MSI will exercise the swaption, thereby entering into a swap to pay Libor and receive 6%. Note, however, that it is receiving a fixed rate of 6%, paying a fixed rate of 9%, and paying Libor. Here, this transaction is not equivalent to it having called the bond, because MSI makes floating payments. To offset the floating payments, it enters into a new swap in the market at the market rate of  $FS(1,3)$ . Specifically, it pays  $FS(1,3)$  and receives Libor. The two streams of Libor payments would offset and would probably be canceled, leaving an inflow of 6% and an outflow of  $FS(1,3)$  on the swaps and an outflow of 9% on the bond. The net effect would be an outflow of  $FS(1,3)$  plus 3%. Because  $FS(1,3)$  is below 6%, the overall rate paid is below 9%, thereby making this position similar to that of a bond that has been called, with a new bond issued in its place at a lower rate.

So we see that a swaption can be used to replicate the call feature on a callable bond. A swaption can synthetically add a call feature when it does not exist or offset a call feature that does exist. The cash paid or received from the swaption occurs all at once, but if allocated appropriately over time, it would be equivalent to the additional amount of interest that a borrower pays for the call feature. Of course, there are some tricky elements to making this strategy work. We have ignored taxes and transaction costs, which can affect exercise and call decisions. Also, when the swaption is held by another party, there is no guarantee that exercise will occur at the optimal time.

### EXAMPLE 14

A German company issues a five-year noncallable bond with a face value of €40 million. The bond pays a coupon annually of 10%, of which 3% is estimated to be a credit premium.

- The company would like to make the bond callable in exactly two years. Design a strategy using a European swaption that will achieve this goal. When the swaption expires, the fixed rate on the underlying swap will be denoted as  $FS(2,5)$ . Evaluate what happens when this rate is at least the exercise rate and also when it is less than the exercise rate.
- Reconsider the bond described above and assume it was actually issued as a callable bond with a 10% coupon. Construct a swaption strategy that will synthetically remove the call feature. As in Part A, let the swaption expire in two years and evaluate the outcomes.

*Solution to A:* To synthetically add the call feature to this bond, the company should purchase a receiver swaption. The exercise rate should be the coupon rate on the bond minus the credit premium:  $10 - 3 = 7\%$ . At the swaption expiration, we have the following outcomes:

$$FS(2,5) \geq 7\%$$

The swaption will not be exercised, and the bond will not be called. The company continues to pay 10% on its bond.

$$FS(2,5) < 7\%$$

The swaption is exercised.

Enter into swap

Receive 7%

Pay Libor

Enter into a new swap at the market rate.

Receive Libor

Pay FS(2,5)

Company continues to pay 10% on its bond

Net effect: Pay FS(2,5) + 10% - 7% = Pay FS(2,5) + 3% < 10%

The company has thus effectively issued a new bond at a lower rate. The option premium, however, effectively raised the coupon rate on the old bond to that of a callable bond.

*Solution to B:* To synthetically remove the call feature on this bond, the company should sell a receiver swaption. The exercise rate should be the coupon rate on the bond minus the credit premium:  $10 - 3 = 7\%$ . At the swaption expiration, we have the following outcomes:

$FS(2,5) \geq 7\%$

The swaption will not be exercised. The company continues to pay 10% on its bond.

$FS(2,5) < 7\%$

The swaption is exercised.

Enter into swap

Receive Libor

Pay 7%

Enter into a new swap at the market rate.

Receive FS(2,5)

Pay Libor

Bond called. Issue new bond at  $FS(2,5) + 3\%$

Net effect: Pay  $FS(2,5) + 3\% + 7\% - FS(2,5) = 10\%$

Therefore, if the company sells the receiver swaption, the bond's call option is offset and effectively removed. The option premium, received up front, effectively reduces the coupon rate on the outstanding bond to make it equivalent to that of a noncallable bond.

Finally, you may be wondering why a receiver swaption was used in these strategies. Why not a payer swaption? Remember that a call feature on a bond is a call option. To add or offset a call feature, we need to use an instrument equivalent to a call option. A receiver swaption is equivalent to a call option. A payer swaption is equivalent to a put option. Payer swaptions would be useful in situations involving put features. Putable bonds do exist but are not

particularly common. A putable bond allows the bondholder to sell the bond back, usually at par, to the issuer. Therefore, the option, which is a put, is held by the bondholder and sold by the bond issuer. If a bond is putable, the coupon rate on the bond would be lower. If the issuer of the bond wanted to synthetically add a put to an otherwise nonputable bond, it would sell a payer swaption. The premium received would effectively lower the coupon rate on the bond. If the issuer of a putable bond wanted to eliminate the put, it would buy a payer swaption. This would give it the right to exercise the swaption, which is a put on the bond, at the same time as the put feature would be exercised by the holder of the bond. Again, we note that put features are not common, and we shall not pursue this strategy here.

#### 5.4. A Note on Forward Swaps

There are also forward contracts on swaps. Called forward swaps, these instruments are commitments to enter into swaps. They do not require a cash payment at the start but force the parties to enter into a swap at a later date at terms, including the fixed rate, set at the start. Although we shall not examine forward swap strategies, note that the same strategies examined in this section can all be used with forward swaps.

### 6. CONCLUSIONS

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In the previous readings we saw how to use forwards, futures, options, and swaps in strategies. These instruments are designed to manage risk. Managing risk involves the buying and selling of risk, perhaps to increase the overall level of one's risk or perhaps to offset an existing risk. As we have seen, these instruments are highly leveraged. As you can imagine, proper use of derivatives requires a significant amount of expertise. More importantly, however, monitoring and control are essential ingredients for the proper use of derivatives. Managing risk is the primary justification for the use of derivatives.

### 7. SUMMARY

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- A floating-rate loan can be converted to a fixed-rate loan by entering into an interest rate swap to pay a fixed rate and receive a floating rate. The floating cash flows offset, leaving the borrower with a net fixed payment. Likewise, a fixed-rate loan can be converted to a floating-rate loan by entering into an interest rate swap to pay a floating rate and receive a fixed rate. The fixed cash flows offset, leaving the party paying a floating rate.
- To obtain the duration of an interest rate swap, consider the difference between the duration of a fixed-rate bond and the duration of a floating-rate bond. The latter is close to zero, leaving the duration of an interest rate swap close to that of a fixed-rate bond. If the party pays a fixed rate and receives a floating rate, the duration of the position is that of the equivalent floating-rate bond minus that of the equivalent fixed-rate bond.
- When a floating-rate loan is converted to a fixed-rate loan, the resulting duration is that of a fixed-rate loan. The duration of a fixed-rate loan is normally much higher than that of a floating-rate loan, which has a duration relatively close to zero. Compared with a floating-rate loan, however, a fixed-rate loan has stable cash flows, which reduce cash flow risk, but has a much greater duration, which increases market value risk.

- The notional principal on an interest rate swap added to a position to adjust its overall duration is determined by the existing duration of the portfolio, the duration of the swap, the desired duration, and the market value of the portfolio. A swap can be used to change the duration of the position without changing the market value.
- An interest rate swap can be used to manage the risk related to a structured note with a coupon at a multiple of a floating rate by adjusting the notional principal on the swap to reflect the coupon multiple for the structured note. The swap should be a receive-floating, pay-fixed swap.
- An interest rate swap can be used to manage the risk of the issuance of an inverse floating-rate note by paying the floating rate to the swap dealer. When interest rates rise (fall), the inverse floater payments decrease (increase), and this effect is passed on to the dealer, which in turn pays a fixed rate.
- A loan in one currency can be converted into a loan in another currency by entering into a currency swap in which it pays interest in one currency and receives interest in the currency in which it makes its loan interest payments. This strategy leaves the borrower paying interest in a different currency than the one in which the loan interest is paid. To offset the principal payment, the currency swap should provide for payment of the notional principal as well.
- Converting a loan in one currency into a loan in another using a currency swap can offer savings because a borrower can normally issue debt at a more attractive rate in its own currency. By entering into a swap with a dealer that can operate more efficiently in global markets, the borrower can effectively convert its domestic debt into foreign debt. In addition, by engaging in the currency swap rather than borrowing in the desired currency in the first place, the borrower takes on a small amount of credit risk that can generate savings if no default takes place.
- The party to a currency swap would make the payments be fixed or floating depending on whether a loan paired with the currency swap is made at a fixed or floating rate and whether the party wants to make payments at a fixed or floating rate. This decision is usually made based on the expected direction of interest rates.
- A series of foreign cash receipts can be combined with a currency swap with no notional principal payments to convert the receipts into domestic currency cash flows. The foreign interest payments on the currency swap must equal the amounts of the foreign cash flows.
- In a dual-currency bond, the interest is paid in one currency and the principal is paid in another. A borrower issuing a dual-currency bond can use the proceeds to buy a bond denominated in the currency of the principal repayment on the dual-currency bond. It can then enter into a currency swap with no notional principal payment, enabling it to fund the interest payments from the dual-currency bond in one currency and make interest payments in another currency.
- An equity swap can be used to provide diversification to a concentrated portfolio by having the party pay the return on the stock that makes up too large a portion of the portfolio and receive the return on a diversified market proxy.
- An equity swap can add international diversification to a domestic portfolio by having the party pay the return on a domestic market benchmark and receive the return on an international market benchmark.
- An equity swap can be used to change the allocation between stock and bond asset classes by having the party pay the return on the asset class in which it wants to reduce its exposure and receive the return on the asset class in which it wants to increase its exposure.

- A corporate insider can use an equity swap to reduce exposure to his company by paying the return on the company's stock and receiving the return on a diversified portfolio benchmark or a fixed- or floating-rate interest payment.
- There can be important implications if corporate insiders use equity swaps. Insiders can reduce their exposure without giving up their voting rights, which can lead to significant agency costs. Although it is clearly necessary for investors and analysts to gauge the exposure of corporate insiders, equity swaps can make this task more difficult.
- Equity swaps pose some difficulties not faced in interest rate and currency swaps. In particular, equity swaps can generate significant cash flow problems, resulting from the fact that equity returns can be negative, meaning that one party can be required to make both sides of payments. In addition, equity swaps can involve tracking error, in which the swap returns, which are pegged to an index, do not match the returns on the actual equity portfolio that is combined with the swap.
- A party would use an interest rate swaption if it anticipates taking out a loan at a future date and entering into a swap to convert the loan from floating rate to fixed rate or vice versa. The swaption gives the party the right to enter into the swap at a specific fixed rate or better. The cost of this flexibility is the swaption premium paid up front.
- An interest rate swaption can be used to provide a means of terminating a swap at a favorable rate. A party engaged in a swap can use a swap with the opposite cash flows to effectively terminate the position. By purchasing a swaption, the party can enter into this swap at a specific rate, established in advance, or take a better rate as given in the market.
- An interest rate receiver swaption is equivalent to a call option on a bond. A party that has issued a callable bond and believes it will not call the bond can sell an interest rate receiver swaption to offset the call feature. The swaption premium received at the start offsets the higher coupon paid for the call feature on the bond. If interest rates fall enough to trigger the bond being called, the swaption will also be exercised. The party must enter into the underlying swap and can enter into an opposite swap at the market rate. The net effect is that the party ends up paying the same rate it would have paid if it had not called the bond.
- A party that has issued a noncallable bond can synthetically add a call feature by purchasing an interest rate receiver swaption. The premium paid for the swaption effectively raises the coupon rate on the bond. If rates fall sufficiently, the receiver swaption is exercised and the party enters into the underlying swap. The party then enters into a swap in the market at the market rate. The net effect is that the party pays a lower fixed rate, as though the bond had been called.

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## PROBLEMS

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1. A company has issued floating-rate notes with a maturity of one year, an interest rate of Libor plus 125 basis points, and total face value of \$50 million. The company now believes that interest rates will rise and wishes to protect itself by entering into an interest rate swap. A dealer provides a quote on a swap in which the company will pay a fixed rate 6.5% and receive Libor. Interest is paid quarterly, and the current Libor is 5%. Indicate how the company can use a swap to convert the debt to a fixed rate. Calculate the overall net payment (including the loan) by the company. Assume that all payments will be made on the basis of 90/360.
2. Assume that you manage a \$100 million bond portfolio with a duration of 1.5 years. You wish to increase the duration of the bond portfolio to 3.5 years by using a swap. Assume the duration of a fixed-rate bond is 75% of its maturity.
  - A. Discuss whether the swap you enter into should involve paying fixed, receiving floating or paying floating, receiving fixed.
  - B. Would you prefer a four-year swap with quarterly payments or a three-year swap with semiannual payments?
  - C. Determine the notional principal of the swap you would prefer.
3. A company issues a leveraged floating-rate note with a face value of \$5,000,000 that pays a coupon of 2.5 times Libor. The company plans to generate a profit by selling the notes, using the proceeds to purchase a bond with a fixed coupon rate of 7% a year, and hedging the risk by entering into an appropriate swap. A swap dealer provides a quote with a fixed rate of 6% and a floating rate of Libor. Discuss whether the company should enter into a swap involving paying fixed, receiving floating or paying floating, receiving fixed. Calculate the amount of the arbitrage profit the company can earn by entering into the appropriate swap. In your answer, indicate the cash flows generated at each step. Also explain what additional risk the company is taking on by doing the swap.
4. A US company needs to raise €100,000,000. It plans to raise this money by issuing dollar-denominated bonds and using a currency swap to convert the dollars to euros. The company expects interest rates in both the United States and the eurozone to fall.
  - A. Should the swap be structured with interest paid at a fixed or a floating rate?
  - B. Should the swap be structured with interest received at a fixed or a floating rate?
5. A company based in the United Kingdom has a German subsidiary. The subsidiary generates €15,000,000 a year, received in equivalent semiannual installments of €7,500,000. The British company wishes to convert the euro cash flows to pounds twice a year. It plans to engage in a currency swap in order to lock in the exchange rate at which it can convert the euros to pounds. The current exchange rate is €1.5/£. The fixed rate on a plain vanilla currency swap in pounds is 7.5% per year, and the fixed rate on a plain vanilla currency swap in euros is 6.5% per year.
  - A. Determine the notional principals in euros and pounds for a swap with semiannual payments that will help achieve the objective.
  - B. Determine the semiannual cash flows from this swap.
6. A portfolio has a total market value of \$105,000,000. The portfolio is allocated as follows: \$65,000,000 is invested in a broadly diversified portfolio of domestic stocks, and

\$40,000,000 is invested in the stock of the JK Corporation. The portfolio manager wishes to reduce exposure to JK stock by \$30,000,000. The manager plans to achieve this objective by entering into a three-year equity swap using the S&P 500. Assume that settlement is made at the end of each year. Also assume that after one year the return on JK stock is 4% and the return on the S&P 500 market index is -3%.

- A. Explain the structure of the equity swap.
  - B. Calculate the net cash flow for the swap at the end of one year.
7. The LKS Company is a US-based mutual fund company that manages a global portfolio 80% invested in domestic stocks and 20% invested in international stocks. The international component mimics the MSCI EAFE Index. The total market value of the portfolio is \$750,000,000. The fund manager wishes to reduce the allocation to domestic stocks to 70% and increase the international allocation to 30%. The manager plans to achieve this objective by entering into a two-year equity swap using the Russell 3000 and the EAFE Index. Assume that settlement is made at the end of the first year. Also assume that after one year, the return on the Russell 3000 market index is 5% and the return on the EAFE Index is 6%.
- A. Explain the structure of the equity swap.
  - B. Calculate the net cash flow for the swap at the end of one year.
8. A diversified portfolio with a market value of \$800,000,000 currently has the following allocations:

Equity	80%	\$640,000,000
Bonds	20%	\$160,000,000

The equity portion of the portfolio is allocated as follows:

US large-cap stocks	70%	\$448,000,000
International stocks	30%	\$192,000,000

The bond portion of the portfolio is allocated as follows:

US government bonds	80%	\$128,000,000
US corporate bonds	20%	\$32,000,000

The portfolio manager wishes to change the overall allocation of the portfolio to 75% equity and 25% bonds. Within the equity category, the new allocation is to be 75% US large cap and 25% international stocks. In the bond category, the new allocation is to be 75% US government bonds and 25% US corporate bonds. The manager wants to use four-year swaps to achieve the desired allocations, with settlements at the end of each year. Assume that the counterparty payments or receipts are tied to Libor. Use generic stock or bond indices where appropriate. Indicate how the manager can use swaps to achieve the desired allocations. Construct the most efficient overall swap, in which all equivalent but opposite Libor payments are consolidated.

9. A company plans to borrow \$20,000,000 in two years. The loan will be for three years and pay a floating interest rate of Libor with interest payments made every quarter. The company expects interest rates to rise in future years and thus is certain to swap the loan into a fixed-rate loan. In order to ensure that it can lock in an attractive rate, the company plans to purchase a payer swaption expiring in two years, with an exercise rate of 5% a year. The cost of the swaption is \$250,000, and the settlement dates coincide with the interest payment dates for the original loan. Assume Libor at the beginning of the settlement period is 6.5% a year.

- A. Calculate the net cash flows on the first settlement date if  $FS(2,5)$  is above the exercise rate.
- B. Calculate the net cash flows on the first settlement date if  $FS(2,5)$  is below the exercise rate.

**The following information relates to Questions 10–14 and is based on the readings on Risk Management Applications of Derivatives**

Catherine Gide is the risk management director of the Millau Corporation, a large, diversified, French multinational corporation with subsidiaries in Japan, the United States, and Switzerland. One of Gide's primary responsibilities is to manage Millau's currency exposure. She has the flexibility to take tactical positions in foreign exchange markets if these positions are justified by her research. Gide and her assistant, Albert Darc, are meeting to discuss how best to deal with Millau's currency exposure over the next 12 months.

Specifically, Gide is concerned about the following:

1. Millau has just sold a Japanese subsidiary for 65 billion yen (JPY65,000,000,000). Because of an impending tax law change, Gide wishes to wait six months before repatriating these funds. Gide plans to invest the sale proceeds in six-month Japanese government securities and hedge the currency risk by using forward contracts. Gide's research indicates that the yen will depreciate against the euro (EUR) over the next six months. Darc has gathered the exchange rate and interest rate information given in Exhibit 1. The day-count convention is 30/360.
2. Millau has a contract to deliver computerized machine tools to a US buyer in three months. A payment of 50 million US dollars (USD50,000,000) is due from the buyer at that time. Gide is concerned about the dollar weakening relative to the euro. She plans to use options to hedge this currency exposure. Specifically, Gide expects the US dollar to weaken to 1.2250USD/EUR in the next three months. Euro options quotations are given in Exhibit 2. All options are European-style and expire in three months.
3. Darc says to Gide:

"I believe the volatility of the USD/EUR exchange rate will soon increase by more than the market expects. We may be able to profit from this volatility increase by buying an equal number of at-the-money call and put options on the euro at the same strike price and expiration date."

4. Millau needs 100 million Swiss francs (CHF100,000,000) for a period of one year. Millau can issue at par a 2.8% one-year euro-denominated note with semiannual coupons and swap the proceeds into Swiss francs. The euro swap fixed rate is 2.3% and the Swiss franc swap fixed rate is 0.8%.

Darc tells Gide that he expects interest rates in both the euro currency zone and Switzerland to rise in the near future. Exchange rate and interest rate information is given in Exhibit 1.

## EXHIBIT 1 Exchange Rate and Interest Rate Information

Currency Exchange Rates	Spot	3-Month Forward	6-Month Forward	1-Year Forward
US dollars per euro (USD/EUR)	1.1930	1.1970	1.2030	1.2140
Japanese yen per euro (JPY/EUR)	133.83	133.14	132.46	131.13
Swiss francs per euro (CHF/EUR)	1.5540	1.5490	1.5440	1.5340
Annualized Risk-Free				
Interest Rates (%)	1 Month	3 Month	6 Month	1 Year
Euro area	2.110	2.120	2.130	2.150
United States	3.340	3.560	3.770	3.990
Japan	0.040	0.056	0.066	0.090
Switzerland	0.730	0.750	0.760	0.780

## EXHIBIT 2 Euro Options Quotations (Options Expire in 3 Months)

Strike (USD/EUR)	Calls on Euro (USD/EUR)	Puts on Euro (USD/EUR)
1.1800	0.0275	0.0125
1.1900	0.0216	0.0161
1.2000	0.0169	0.0211
1.2100	0.0127	0.0278

10. If Gide uses a six-month forward currency contract to convert the yen received from the sale of the Japanese subsidiary into euros, the total amount Millau will receive is *closest* to:
  - A. EUR490,714,000.
  - B. EUR490,876,000.
  - C. EUR491,038,000.
11. If Gide uses a six-month forward currency contract to convert the yen received from the sale of the Japanese subsidiary into euros, the annualized return in euros that Millau will realize is *closest* to:
  - A. 0.066%.
  - B. 2.130%.
  - C. 2.196%.
12. Darc's statement to Gide (in concern #3) about the option strategy to use in order to profit from a volatility increase of the euro/US dollar exchange rate is:
  - A. correct.
  - B. incorrect, because he is describing a strategy that benefits only from a weakening euro.
  - C. incorrect, because he is describing a strategy that benefits from low volatility in the exchange rate.

13. If Millau issues euro-denominated debt and enters into a fixed-rate currency swap (in concern #4), which of the following *best* describes transactions between Millau and the swap counterparty in six months? Millau pays the swap counterparty:
  - A. EUR740,026 and receives CHF400,000.
  - B. CHF400,000 and receives EUR740,026.
  - C. CHF800,000 and receives EUR900,901.
14. Based on Darc's interest rate expectations for the euro currency zone and Switzerland, Gide's *best* choice is to structure the currency swap so that Millau pays interest at a:
  - A. fixed rate and receives it at a fixed rate.
  - B. fixed rate and receives it at a floating rate.
  - C. floating rate and receives it at a floating rate.

**The following information relates to Questions 15–20 and is based on the readings on Risk Management Applications of Derivatives**

Hadley Elbridge, managing director for Humber Wealth Managers, LLC, is concerned about the risk level of a client's equity portfolio. The client, Pat Cassidy, has 60% of this portfolio invested in two equity positions: Hop Industries and Sure Securities. Cassidy refuses to sell his shares in either company, but has agreed to use option strategies to manage these concentrated equity positions. Elbridge recommends either a collar strategy or a protective put strategy on the Hop position, and a covered call strategy on the Sure position. The options available to construct the positions are shown in Exhibit 1.

**EXHIBIT 1** Equity Positions and Options Available

Stock	Shares	Stock Price	Options	Option Price
Hop	375,000	\$26.20	September 25.00 put	\$0.80
			September 27.50 call	\$0.65
Sure	300,000	\$34.00	September 32.50 put	\$0.85
			September 35.00 call	\$1.20

Cassidy makes the following comments:

- Comment #1 "The Hop protective put position provides a maximum per share loss of \$2.00 and a breakeven underlying price at expiration of \$27.00."
- Comment #2 "The Sure covered call position provides a maximum per share gain of \$2.20 and a breakeven underlying price at expiration of \$32.80."
- Comment #3 "The general shape of a profit-and-loss graph for the protective put closely resembles the general shape of the graph for another common option position."

Elbridge also investigates whether a privately negotiated equity swap could be used to reduce the risk of the Hop and Sure holdings. A swap dealer offers Elbridge the following:

- The dealer will receive the return on 250,000 shares of Hop and 200,000 shares of Sure from Cassidy.
- The dealer will pay Cassidy the return on an equivalent dollar amount on the Russell 3000 Index.

The dealer demonstrates the quarterly cash flows of this transaction under the assumptions that Hop is up 2%, Sure is up 4%, and the Russell 3000 is up 5% for the quarter.

The remaining 40% of Cassidy's equity portfolio is invested in a diversified portfolio of equities valued at \$13,350,000. Elbridge believes this portfolio is too risky, so he recommends lowering the beta of this portfolio from its current level of 1.20 to a target beta of 0.80. To accomplish this, he will use a two-month futures contract with a price (including multiplier) of \$275,000 and a beta of 0.97.

15. Disregarding the initial cost of the Hop collar strategy, the value per share of the strategy at expiration with the stock at \$26.90 is:

- A. \$26.05.
- B. \$26.20.
- C. \$26.90.

16. Cassidy's Comments #1 and #2 about the Hop protective put and Sure covered call positions, respectively, are:

<u>Protective Put</u>	<u>Covered Call</u>
A. Correct	Correct
B. Correct	Incorrect
C. Incorrect	Incorrect

17. The general shape of the profit-and-loss graph in Cassidy's Comment #3 is *most* similar to the general shape of the profit-and-loss graph for:

- A. buying a call.
- B. selling a call.
- C. buying a put.

18. If an options dealer takes the other side of the Sure option position, the dealer's initial option delta and hedging transaction, respectively, will be:

<u>Dealer's Initial Option Delta</u>	<u>Dealer's Hedging Transaction</u>
A. Negative	Buy the underlying
B. Positive	Buy the underlying
C. Positive	Sell the underlying

19. What is the payoff to Cassidy in the equity swap example?

- A. -\$269,500.
- B. \$264,500.
- C. \$269,500.

20. To achieve the target beta on Cassidy's diversified stock portfolio, Elbridge would sell the following number of futures contracts (rounded to the nearest whole contract):

- A. 13.
- B. 20.
- C. 27.

### Questions 21 through 26 relate to the Westfield Tool Company

The Westfield Machine Tool and Die Company (WMTC) is a US-based manufacturer of cutting tools that operates production plants in the United States and Spain. WMTC's CEO

has received an economic report forecasting that interest rates in the future will likely increase worldwide. He has asked WMTC's CFO, Yolanda Lopez, to examine ways by which different kinds of swaps could be used as a means of reducing the company's interest rate and currency risks.

Lopez has identified the following areas where swaps might be an attractive tool for managing risk:

- WMTC's employee pension plan portfolio
- WMTC's existing five-year bank loan
- Foreign exchange risk associated with cash flows repatriated from the operations in Spain
- New debt issue associated with upcoming expansion projects

Information regarding WMTC's pension plan portfolio is shown in Exhibit 1. Within the WMTC pension plan portfolio, the allocation within equities is heavily weighted towards the company's own stock. WMTC would like to retain these shares for corporate control purposes.

EXHIBIT 1 Pension Plan Portfolio of Westfield Machine  
Tool Company (in millions of US dollars)

Equities	
Diversified Equities	\$200
WMTC Common Stock	<u>\$400</u>
Equities Total	\$600
Fixed Income (Bonds)*	
Treasuries	\$200
Corporates	<u>\$300</u>
Fixed Income Total	\$500
Bond Portfolio Duration	6 years
Total Portfolio Value	\$1,100

\*All bonds are fixed rate, and pay interest semiannually and on the same date.

Lopez recommends the allocation to WMTC equity be reduced to 20% of the overall equity portfolio. Lopez determines that WMTC can achieve this reallocation objective by executing an equity swap that would enable it to alter the allocation more easily and less expensively than by executing transactions in the underlying securities. Furthermore, using the equity swap would allow WMTC to retain the company shares held in the WMTC pension portfolio.

Lopez also recommends that WMTC reduce the duration of the bond portfolio by 50%. She states that, in order to achieve this duration target, WMTC should use a 6-year interest rate swap with semiannual payments. Lopez estimates the duration of the swap's fixed payments to be 75% of the swap maturity.

Lopez is also concerned about WMTC's five-year variable rate loan given the forecast of rising interest rates. Additionally, Lopez would like to use a currency swap to lock in the exchange rate when WMTC repatriates Euro cash flows from Spain into US dollars over the next two years. Additional pertinent facts regarding WMTC's existing debt obligation and cash flows from Spain are provided in Exhibit 2.

## EXHIBIT 2 Relevant Debt and Cash Flow Information

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<b>Debt:</b>	Five-year variable rate loan. Principal amount: \$10,000,000. Rate: Libor + 200 basis points, paid semiannually, reset every six months. Loan rate was reset today at a Libor of 5%.
<b>Cash Flows:</b>	Estimated €12 million annually to be repatriated to US from operations in Spain, in equal semiannual installments. Current spot exchange rate: 1.4 USD/EUR

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To hedge the interest rate risk on the five-year variable rate loan, Lopez recommends that WMTC enter into a contract with Swap Traders International (STI), who offers an interest rate swap with a notional principal of \$10 million that provides a fixed rate of 6% in exchange for Libor, with semiannual payments.

To hedge the currency risk associated with the cash flows to be repatriated from its operations in Spain, Lopez recommends that WMTC enter into a currency swap with semiannual payments, where the fixed swap rate in Euros is 4.5%, and the fixed swap rate in US dollars is 5.00%.

WMTC also has some major expansion plans for its Spanish operations. In two years, Lopez expects that WMTC will need to raise €50 million. Lopez expects that WMTC will raise the funds using a floating interest rate loan at the prevailing Libor rate in 2 years with annual interest payments. Lopez is considering hedging the interest rate risk relating to the future borrowing, so she contacts STI, who offers a swaption expiring in 2 years with Libor as the underlying floating rate and an exercise rate of 6%.

21. Lopez will *most likely* achieve the pension plan's equity reallocation objective by entering into an equity swap whereby WMTC receives a return on:
  - A. \$320 million of the S&P 500 Index and pays a return on \$320 million of WMTC common stock.
  - B. \$280 million of the S&P 500 Index and pays a return on \$280 million of WMTC common stock.
  - C. \$280 million of WMTC common stock and pays a return on \$280 million of the S&P 500 Index.
22. To achieve the target duration for the pension plan's bond portfolio, WMTC should enter into an interest rate swap with a modified duration that is:
  - A. negative, requiring WMTC to make fixed-rate payments and receive floating-rate payments.
  - B. negative, requiring WMTC to make floating-rate payments and receive fixed-rate payments.
  - C. positive, requiring WMTC to make fixed-rate payments and receive floating-rate payments.
23. WMTC can achieve the bond portfolio duration target by using an interest rate swap with a notional principal *closest* to:
  - A. \$343 million.
  - B. \$353 million.
  - C. \$375 million.

24. If WMTC hedges the interest rate risk on the five-year variable rate loan by using the interest rate swap recommended by Lopez, the net interest payment at the first settlement date in six months would be *closest* to:
  - A. \$300,000.
  - B. \$400,000.
  - C. \$800,000.
25. If WMTC hedges the currency risk relating to the cash flows from its Spanish operations using the currency swap recommended by Lopez, WMTC would generate semiannual cash inflows from the swap *closest* to:
  - A. \$4.8 million.
  - B. \$8.4 million.
  - C. \$9.3 million.
26. If Lopez decides to use a swaption with STI to hedge the interest rate risk relating to the expansion loan, then Lopez should:
  - A. sell a payer swaption.
  - B. buy a payer swaption.
  - C. buy a receiver swaption.



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# GLOSSARY

**Accounting risk** The risk associated with accounting standards that vary from country to country or with any uncertainty about how certain transactions should be recorded.

**Active risk** A synonym for tracking risk.

**Actual extreme events** A type of scenario analysis used in stress testing. It involves evaluating how a portfolio would have performed given movements in interest rates, exchange rates, stock prices, or commodity prices at magnitudes such as occurred during past extreme market events (e.g., the stock market crash of October 1987).

**Advanced set** The reference interest rate is set at beginning of the settlement period.

**Advanced settled** An arrangement in which the settlement is made at the beginning of the settlement period.

**American-style** Said of an option contract that can be exercised at any time up to the option's expiration date.

**Arbitrage-free pricing** The overall process of pricing derivatives by arbitrage and risk neutrality. Also called the *principle of no arbitrage*.

**Arbitrage** 1) The simultaneous purchase of an undervalued asset or portfolio and sale of an overvalued but equivalent asset or portfolio, in order to obtain a riskless profit on the price differential. Taking advantage of a market inefficiency in a risk-free manner. 2) The condition in a financial market in which equivalent assets or combinations of assets sell for two different prices, creating an opportunity to profit at no risk with no commitment of money. In a well-functioning financial market, few arbitrage opportunities are possible. 3) A risk-free operation that earns an expected positive net profit but requires no net investment of money.

**At market** When a forward contract is established, the forward price is negotiated so that the market value of the forward contract on the initiation date is zero.

**At-the-money** Said of an option in which the underlying's price equals the exercise price.

**Back office** Administrative functions at an investment firm such as those pertaining to transaction processing, record keeping, and regulatory compliance.

**Backtesting** A method for gaining information about a model using past data. As used in reference to VaR, it is the process of comparing the number of violations of VaR thresholds over a time period with the figure implied by the user-selected probability level.

**Basis point value (BPV)** The change in the bond price for a 1 basis point change in yield. Also called *present value of a basis point* or *price value of a basis point (PVBP)*.

**Bear spread** A spread that becomes more valuable when the price of the underlying asset declines.

**Beta** A measure of the sensitivity of a given investment or portfolio to movements in the overall market.

**Bid-ask spread** The difference between the current bid price and the current ask price of a security.

**Binomial model** A model for pricing options in which the underlying price can move to only one of two possible new prices.

**Box spread** An option strategy that combines a bull spread and a bear spread having two different exercise prices, which produces a risk-free payoff of the difference in the exercise prices.

**Bull spread** A spread that becomes more valuable when the price of the underlying asset rises.

**Business risk** The equity risk that comes from the nature of the firm's operating activities.

**Butterfly spread** An option strategy that combines two bull or bear spreads and has three exercise prices.

- Calendar spread** A strategy in which an investor sells (or buys) a near-dated call and buys (or sells) a longer-dated one on the same underlying asset and with the same strike.
- Call option** An option that gives the holder the right to buy an underlying asset from another party at a fixed price over a specific period of time.
- Call** An option that gives the holder the right to buy an underlying asset from another party at a fixed price over a specific period of time.
- Cap** A combination of interest rate call options designed to hedge a borrower against rate increases on a floating-rate loan.
- Capital adequacy ratio** A measure of the adequacy of capital in relation to assets.
- Caplet** Each component call option in a cap.
- Carry arbitrage model** A no-arbitrage approach in which the underlying instrument is either bought or sold along with an opposite position in a forward contract.
- Carry benefits** Benefits that arise from owning certain underlyings; for example, dividends, foreign interest, and bond coupon payments.
- Carry costs** Costs that arise from owning certain underlyings. They are generally a function of the physical characteristics of the underlying asset and also the interest forgone on the funds tied up in the asset.
- Carry** The net of the costs and benefits of holding, storing, or “carrying” an asset.
- Cash flow at risk** A variation of VaR that measures the risk to a company’s cash flow, instead of its market value; the minimum cash flow loss expected to be exceeded with a given probability over a specified time period.
- Cash markets** See *spot markets*.
- Cash prices** See *spot prices*.
- Cash settled** A procedure used in certain derivative transactions that specifies that the long and short parties engage in the equivalent cash value of a delivery transaction.
- Cash-secured put** An option strategy involving the writing of a put option and simultaneously depositing an amount of money equal to the exercise price into a designated account.
- Cash-settled forwards** See *non-deliverable forwards*.
- Cheapest-to-deliver** A bond in which the amount received for delivering the bond is largest compared with the amount paid in the market for the bond.
- Clearing** The process by which the exchange verifies the execution of a transaction and records the participants’ identities.
- Closeout netting** In a bankruptcy, a process by which multiple obligations between two counterparties are consolidated into a single overall value owed by one of the counterparties to the other.
- Collar** An option position in which the investor is long shares of stock and then buys a put with an exercise price below the current underlying price and writes a call with an exercise price above the current underlying price.
- Collateralized bond obligations** A structured asset-backed security that is collateralized by a pool of bonds.
- Collateralized debt obligation** Generic term used to describe a security backed by a diversified pool of one or more debt obligations.
- Collateralized loan obligations** A structured asset-backed security that is collateralized by a pool of loans.
- Collateralized mortgage obligation** A security created through the securitization of a pool of mortgage-related products (mortgage pass-through securities or pools of loans).
- Contingent claims** Derivatives in which the payoffs occur if a specific event occurs; generally referred to as options.
- Contracts for differences** See *non-deliverable forwards*.
- Convenience yield** A non-monetary advantage of holding an asset.
- Convergence** The property of forward and futures contracts in which the derivative price becomes the spot price at expiration of the derivative.
- Convexity** A measure of how interest rate sensitivity changes with a change in interest rates.
- Corporate governance** The system of internal controls and procedures used to define and protect the rights and responsibilities of various stakeholders.

**Cost of carry** See *carry*.

**Covered call** An option strategy in which an investor who already owns the underlying asset sells a call option giving someone else the right to buy the asset at the exercise price.

**Covered interest rate parity** Relationship among the spot exchange rate, forward exchange rate, and the interest rates in two currencies that ensures that the return on a hedged (i.e., covered) foreign risk-free investment is the same as the return on a domestic risk-free investment. Also called *interest rate parity*.

**Credit default swap (CDS)** A type of credit derivative in which one party, the credit protection buyer who is seeking credit protection against a third party, makes a series of regularly scheduled payments to the other party, the credit protection seller. The seller makes no payments until a credit event occurs.

**Credit default swap** A swap used to transfer credit risk to another party. A protection buyer pays the protection seller in return for the right to receive a payment from the seller in the event of a specified credit event.

**Credit derivative** A contract in which one party has the right to claim a payment from another party in the event that a specific credit event occurs over the life of the contract.

**Credit risk** The risk of loss caused by a counterparty's or debtor's failure to make a timely payment or by the change in value of a financial instrument based on changes in default risk. Also called *default risk*.

**Credit spread forward** A forward contract used to transfer credit risk to another party; a forward contract on a yield spread.

**Credit spread option** An option based on the yield spread between two securities that is used to transfer credit risk.

**Credit VaR** A variation of VaR related to credit risk; it reflects the minimum loss due to credit exposure with a given probability during a period of time.

**Credit-linked note (CLN)** Fixed-income security in which the holder of the security has the right to withhold payment of the full amount due at maturity if a credit event occurs.

**Cross-default provision** A provision stipulating that if a borrower defaults on any outstanding credit obligations, the borrower is considered to be in default on all obligations.

**Current credit risk** The risk of credit-related events happening in the immediate future; it relates to the risk that a payment currently due will not be paid. Also called *jump-to-default risk*.

**Daily settlement** See *mark to market* and *marking to market*.

**Delta hedge** An option strategy in which a position in an asset is converted to a risk-free position with a position in a specific number of options. The number of options per unit of the underlying changes through time, and the position must be revised to maintain the hedge.

**Delta-normal method** A measure of VaR equivalent to the analytical method but that refers to the use of delta to estimate the option's price sensitivity.

**Delta** The relationship between the option price and the underlying price, which reflects the sensitivity of the price of the option to changes in the price of the underlying. Delta is a good approximation of how an option price will change for a small change in the stock.

**Derivatives** A financial instrument whose value depends on the value of some underlying asset or factor (e.g., a stock price, an interest rate, or exchange rate).

**Diversification effect** In reference to VaR across several portfolios (for example, across an entire firm), this effect equals the difference between the sum of the individual VaRs and total VaR.

**Downside deviation** A measure of volatility using only rate of return data points below the investor's minimum acceptable return.

**Duration** A measure of the approximate sensitivity of a security to a change in interest rates (i.e., a measure of interest rate risk).

**Earnings at risk (EAR)** A variation of VaR that reflects the risk of a company's earnings instead of its market value.

**Economic exposure** The risk associated with changes in the relative attractiveness of products and services offered for sale, arising out of the competitive effects of changes in exchange rates.

- Enhanced derivatives products companies** A type of subsidiary separate from an entity's other activities and not liable for the parent's debts. They are often used by derivatives dealers to control exposure to ratings downgrades. Also called *special purpose vehicles*.
- Enterprise risk management** An overall assessment of a company's risk position. A centralized approach to risk management sometimes called firmwide risk management.
- Equity swap** A swap transaction in which at least one cash flow is tied to the return on an equity portfolio position, often an equity index.
- ESG risk** The risk to a company's market valuation resulting from environmental, social, and governance factors.
- European-style** Said of an option contract that can only be exercised on the option's expiration date.
- Exercise price** The fixed price at which an option holder can buy or sell the underlying. Also called *strike price, striking price, or strike*.
- Exercise value** The value obtained if an option is exercised based on current conditions. Also known as *intrinsic value*.
- Expectations approach** A procedure for obtaining the value of an option derived from discounting at the risk-free rate its expected future payoff based on risk neutral probabilities.
- Factor push** A simple stress test that involves pushing prices and risk factors of an underlying model in the most disadvantageous way to estimate the impact of factor extremes on the portfolio's value.
- Fiduciary call** A combination of a European call and a risk-free bond that matures on the option expiration day and has a face value equal to the exercise price of the call.
- Financial risk** Risks derived from events in the external financial markets, such as changes in equity prices, interest rates, or currency exchange rates.
- Fixed-for-floating interest rate swap** An interest rate swap in which one party pays a fixed rate and the other pays a floating rate, with both sets of payments in the same currency. Also called *plain vanilla swap* or *vanilla swap*.
- Floor** A combination of interest rate options designed to provide protection against interest rate decreases.
- Floorlet** Each component put option in a floor.
- Forward commitments** Class of derivatives that provides the ability to lock in a price to transact in the future at a previously agreed-upon price.
- Forward contract** An agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a later date for a price established at the start of the contract.
- Forward price** The fixed price or rate at which the transaction scheduled to occur at the expiration of a forward contract will take place. This price is agreed to at the initiation date of the contract.
- Forward rate agreement** A forward contract calling for one party to make a fixed interest payment and the other to make an interest payment at a rate to be determined at the contract expiration.
- Forward value** The monetary value of an existing forward contract.
- Front office** The revenue generating functions at an investment firm such as those pertaining to trading and sales.
- Funding risk** The risk that liabilities funding long asset positions cannot be rolled over at reasonable cost.
- Futures contract** A variation of a forward contract that has essentially the same basic definition but with some additional features, such as a clearinghouse guarantee against credit losses, a daily settlement of gains and losses, and an organized electronic or floor trading facility.
- Futures price** The price at which the parties to a futures contract agree to exchange the underlying (or cash). In commodity markets, the price agreed on to deliver or receive a defined quantity (and often quality) of a commodity at a future date.
- Futures value** The monetary value of an existing futures contract.
- Gamma** A measure of how sensitive an option's delta is to a change in the underlying. The change in a given instrument's delta for a given small change in the underlying's value, holding everything else constant.

- Global custodian** An entity that effects trade settlement, safekeeping of assets, and the allocation of trades to individual custody accounts.
- Hedge portfolio** A hypothetical combination of the derivative and its underlying that eliminates risk.
- Hedging** A general strategy usually thought of as reducing, if not eliminating, risk.
- Historical simulation method** The application of historical price changes to the current portfolio.
- Hypothetical events** A type of scenario analysis used in stress testing that involves the evaluation of performance given events that have never happened in the markets or market outcomes to which we attach a small probability.
- Implied volatility** The standard deviation that causes an option pricing model to give the current option price.
- Implied yield** A measure of the yield on the underlying bond of a futures contract implied by pricing it as though the underlying will be delivered at the futures expiration.
- In the money** Options that, if exercised, would result in the value received being worth more than the payment required to exercise.
- Incremental VaR** A measure of the incremental effect of an asset on the VaR of a portfolio by measuring the difference between the portfolio's VaR while including a specified asset and the portfolio's VaR with that asset eliminated.
- Initial margin** The amount that must be deposited in a clearinghouse account when entering into a futures contract.
- Interest rate parity** See *covered interest rate parity*.
- Intrinsic value** See *exercise value*.
- Inverse floater** A floating-rate note or bond in which the coupon is adjusted to move opposite to a benchmark interest rate.
- Law of one price** The condition in a financial market in which two equivalent financial instruments or combinations of financial instruments can sell for only one price. Equivalent to the principle that no arbitrage opportunities are possible.
- Legal/contract risk** The possibility of loss arising from the legal system's failure to enforce a contract in which an enterprise has a financial stake; for example, if a contract is voided through litigation.
- Leveraged floating-rate note** (leveraged floater) A floating-rate note or bond in which the coupon is adjusted at a multiple of a benchmark interest rate.
- Limit down** A limit move in the futures market in which the price at which a transaction would be made is at or below the lower limit.
- Limit up** A limit move in the futures market in which the price at which a transaction would be made is at or above the upper limit.
- Liquidity risk** Any risk of economic loss because of the need to sell relatively less liquid assets to meet liquidity requirements; the risk that a financial instrument cannot be purchased or sold without a significant concession in price because of the market's potential inability to efficiently accommodate the desired trading size.
- Locked limit** A condition in the futures markets in which a transaction cannot take place because the price would be beyond the limits.
- Long** The buyer of a derivative contract. Also refers to the position of owning a derivative.
- Macaulay duration** The percentage change in price for a percentage change in yield. The term, named for one of the economists who first derived it, is used to distinguish the calculation from modified duration. (See also *modified duration*).
- Maintenance margin** The minimum amount that is required by a futures clearinghouse to maintain a margin account and to protect against default. Participants whose margin balances drop below the required maintenance margin must replenish their accounts.
- Margin bond** A cash deposit required by the clearinghouse from the participants to a contract to provide a credit guarantee. Also called a *performance bond*.
- Margin call** A request for the short to deposit additional funds to bring their balance up to the initial margin.
- Margin** The amount of money that a trader deposits in a margin account. The term is derived from the stock market practice in which an investor borrows a portion of the money required to purchase a

certain amount of stock. In futures markets, there is no borrowing so the margin is more of a down payment or performance bond.

**Mark to market** The revaluation of a financial asset or liability to its current market value or fair value.

**Market risk** The risk associated with interest rates, exchange rates, and equity prices.

**Marking to market** A procedure used primarily in futures markets in which the parties to a contract settle the amount owed daily. Also known as the *daily settlement*.

**Maximum loss optimization** A stress test in which we would try to optimize mathematically the risk variable that would produce the maximum loss.

**Model risk** The risk that a model is incorrect or misapplied; in investments, it often refers to valuation models.

**Modified duration** An adjustment of the duration for the level of the yield. Contrast with *Macaulay duration*.

**Moneyness** The relationship between the price of the underlying and an option's exercise price.

**No-arbitrage approach** A procedure for obtaining the value of an option based on the creation of a portfolio that replicates the payoffs of the option and deriving the option value from the value of the replicating portfolio.

**Non-deliverable forwards** Cash-settled forward contracts, used predominately with respect to foreign exchange forwards. Also called *contracts for differences*.

**Nonfinancial risk** Risks that arise from sources other than the external financial markets, such as changes in accounting rules, legal environment, or tax rates.

**Nonparametric** Involving minimal probability-distribution assumptions.

**Notional principal** An imputed principal amount.

**Open interest** The number of outstanding contracts in a clearinghouse at any given time. The open interest figure changes daily as some parties open up new positions, while other parties offset their old positions.

**Operational risk** The risk of loss from failures in a company's systems and procedures (for example, due to computer failures or human failures) or events completely outside of the control of organizations (which would include "acts of God" and terrorist actions).

**Option combination** An option strategy that typically uses both puts and calls, an example of which is the straddle, which involves buying one call and one put.

**Option premium** The amount of money a buyer pays and seller receives to engage in an option transaction.

**Option spread** The investor buys one call and writes another with a different exercise price or expiration or buys one put and writes another with a different exercise price or expiration.

**Option** A financial instrument that gives one party the right, but not the obligation, to buy or sell an underlying asset from or to another party at a fixed price over a specific period of time. Also referred to as *contingent claim* or *option contract*.

**Out of the money** Options that, if exercised, would require the payment of more money than the value received and therefore would not be currently exercised.

**Payment netting** A means of settling payments in which the amount owed by the first party to the second is netted with the amount owed by the second party to the first; only the net difference is paid.

**Performance bond** See *margin bond*.

**Performance netting risk** For entities that fund more than one strategy and have asymmetric incentive fee arrangements with the portfolio managers, the potential for loss in cases where the net performance of the group of managers generates insufficient fee revenue to fully cover contractual payout obligations to all portfolio managers with positive performance.

**Political risk** The risk of war, government collapse, political instability, expropriation, confiscation, or adverse changes in taxation. Also called *geopolitical risk*.

**Position delta** The overall delta of a position that contains some combination of assets and derivatives.

**Present value of a basis point (PVBP)** The change in the bond price for a 1 basis point change in yield. Also called *basis point value* (BPV).

- Price limits** Limits imposed by a futures exchange on the price change that can occur from one day to the next.
- Price value of a basis point (PVBP)** The change in the bond price for a 1 basis point change in yield. Also called *basis point value* (BPV).
- Principle of no arbitrage** See *arbitrage-free pricing*.
- Protective put** An option strategy in which a long position in an asset is combined with a long position in a put.
- Put option** An option that gives the holder the right to sell an underlying asset to another party at a fixed price over a specific period of time.
- Put-call parity** An equation expressing the equivalence (parity) of a portfolio of a call and a bond with a portfolio of a put and the underlying, which leads to the relationship between put and call prices.
- Put-call-forward parity** The relationship among puts, calls, and forward contracts.
- Put** An option that gives the holder the right to sell an underlying asset to another party at a fixed price over a specific period of time.
- Ratio spread** An option strategy in which a long position in a certain number of options is offset by a short position in a certain number of other options on the same underlying, resulting in a risk-free position.
- Regulatory risk** The risk associated with the uncertainty of how a transaction will be regulated or with the potential for regulations to change.
- Replication** The creation of an asset or portfolio from another asset, portfolio, and/or derivative.
- Reverse carry arbitrage** A strategy in involving the short sale of the underlying and an offsetting opposite position in the derivative.
- Rho** The change in a given derivative instrument for a given small change in the risk-free interest rate, holding everything else constant. Rho measures the sensitivity of the option to the risk-free interest rate.
- Risk budgeting** The establishment of objectives for individuals, groups, or divisions of an organization that takes into account the allocation of an acceptable level of risk.
- Risk exposure** A source of risk. Also, the state of being exposed or vulnerable to a risk.
- Risk management** The process of identifying the level of risk an entity wants, measuring the level of risk the entity currently has, taking actions that bring the actual level of risk to the desired level of risk, and monitoring the new actual level of risk so that it continues to be aligned with the desired level of risk.
- Risk tolerance** The capacity to accept risk; the level of risk an investor (or organization) is willing and able to bear.
- Risk-neutral pricing** Sometimes said of derivatives pricing, uses the fact that arbitrage opportunities guarantee that a risk-free portfolio consisting of the underlying and the derivative must earn the risk-free rate.
- Risk-neutral probabilities** Weights that are used to compute a binomial option price. They are the probabilities that would apply if a risk-neutral investor valued an option.
- Sandwich spread** An option strategy that is equivalent to a short butterfly spread.
- Settled in arrears** An arrangement in which the interest payment is made at the end of the settlement period.
- Settlement** The process that occurs after a trade is completed, the securities are passed to the buyer, and payment is received by the seller.
- Settlement netting risk** The risk that a liquidator of a counterparty in default could challenge a netting arrangement so that profitable transactions are realized for the benefit of creditors.
- Settlement price** The official price, designated by the clearinghouse, from which daily gains and losses will be determined and marked to market.
- Settlement risk** When settling a contract, the risk that one party could be in the process of paying the counterparty while the counterparty is declaring bankruptcy.
- Short** The seller of an asset or derivative contract. Also refers to the position of being short an asset or derivative contract.

**Sovereign risk** A form of credit risk in which the borrower is the government of a sovereign nation.

**Spot markets** Markets in which assets are traded for immediate delivery.

**Spot prices** The price of an asset for immediate delivery.

**Straddle** An option strategy involving the purchase of a put and a call on the same underlying with the same exercise price and expiration date. If the put and call are held long, it is a long straddle; if they are held short, it is a short straddle.

**Straight-through processing** Systems that simplify transaction processing through the minimization of manual and/or duplicative intervention in the process from trade placement to settlement.

**Strangle** A variation on a straddle in which the put and call have different exercise prices; if the put and call are held long, it is a long strangle; if they are held short, it is a short strangle.

**Strap** An option strategy involving the purchase of two calls and one put.

**Strip** An option strategy involving the purchase of two puts and one call.

**Structured note** A variation of a floating-rate note that has some type of unusual characteristic such as a leverage factor or in which the rate moves opposite to interest rates.

**Stylized scenario** A type of analysis often used in stress testing. It involves simulating the movement in at least one interest rate, exchange rate, stock price, or commodity price relevant to the portfolio.

**Surplus** The difference between the value of assets and the present value of liabilities. With respect to an insurance company, the net difference between the total assets and total liabilities (equivalent to policyholders' surplus for a mutual insurance company and stockholders' equity for a stock company).

**Swap contract** An agreement between two parties to exchange a series of future cash flows.

**Swap rate** The interest rate applicable to the pay-fixed-rate side of an interest rate swap.

**Synthetic long position** A combination of options (buying a call and writing a put) having the same expiration date and the same exercise price, which is approximately equivalent to a long position in the stock.

**Synthetic short position** A derivatives strategy that creates the same performance as a short position in the underlying.

**Tail value at risk** (or conditional tail expectation) The VaR plus the expected loss in excess of VaR, when such excess loss occurs.

**Tax risk** The uncertainty associated with tax laws.

**Theta** The change in a derivative instrument for a given small change in calendar time, holding everything else constant. Specifically, the theta calculation assumes nothing changes except calendar time. Theta also reflects the rate at which an option's time value decays.

**Time value decay** Said of an option when, at expiration, no time value remains and the option is worth only its exercise value.

**Time value** The difference between the market price of the option and its intrinsic value, determined by the uncertainty of the underlying over the remaining life of the option.

**Total return swap** A swap in which one party agrees to pay the total return on a security. Often used as a credit derivative, in which the underlying is a bond.

**Tracking risk** The standard deviation of the differences between a portfolio's returns and its benchmark's returns; a synonym of active risk. Also called *tracking error*.

**Trade settlement** Completion of a trade wherein purchased financial instruments are transferred to the buyer and the buyer transfers money to the seller.

**Transaction exposure** The risk associated with a foreign exchange rate on a specific business transaction such as a purchase or sale.

**Translation exposure** The risk associated with the conversion of foreign financial statements into domestic currency.

**Transparency** Said of something (e.g., a market) in which information is fully disclosed to the public and/or regulators.

**Twist** With respect to the yield curve, a movement in contrary directions of interest rates at two maturities; a nonparallel movement in the yield curve.

**Underlying** An asset that trades in a market in which buyers and sellers meet, decide on a price, and the seller then delivers the asset to the buyer and receives payment. The underlying is the asset or other derivative on which a particular derivative is based. The market for the underlying is also referred to as the *spot market*.

**Value at risk (VaR)** A probability-based measure of loss potential for a company, a fund, a portfolio, a transaction, or a strategy over a specified period of time.

**Vega** The change in a given derivative instrument for a given small change in volatility, holding everything else constant. A sensitivity measure for options that reflects the effect of volatility.

**Volatility** Represented by the Greek letter sigma ( $\sigma$ ), the standard deviation of price outcomes associated with an underlying asset.

**Worst-case scenario analysis** A stress test in which we examine the worst case that we actually expect to occur.

**Yield beta** A measure of the sensitivity of a bond's yield to a general measure of bond yields in the market that is used to refine the hedge ratio.

**Zero-cost collar** A transaction in which a position in the underlying is protected by buying a put and selling a call with the premium from the sale of the call offsetting the premium from the purchase of the put. It can also be used to protect a floating-rate borrower against interest rate increases with the premium on a long cap offsetting the premium on a short floor.



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**Robert Brooks**, PhD, CFA, is the Wallace D. Malone, Jr. endowed chair of Financial Management at The University of Alabama and president of Financial Risk Management, LLC, a financial risk management consulting firm focused on market risks. He has taught for over 30 years, primarily financial risk management classes, and is the author of over 75 articles appearing in the *Journal of Financial and Quantitative Analysis*, *Journal of Derivatives*, *Financial Analysts Journal*, and many others.

Brooks is the co-author of *An Introduction to Derivatives and Risk Management* (Seventh through Tenth Editions) with Don Chance and has authored several books including *Building Financial Risk Management Applications with C++*. He has testified in a subcommittee hearing of the US House of Representatives in Washington, D.C. as well as in a field hearing of the SEC in Birmingham, Alabama. He has consulted with major public utilities, energy companies, auditing firms, corporations, investment bankers, elected municipal officials, and commercial bankers regarding managing financial risks, derivatives valuation and software development. Further, he has testified in several court cases as well as conducting professional development seminars on various aspects of finance.

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