Conic Assignment

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Problem

Find the equation of the circle with center (2,2) and passes through the point (4,5).
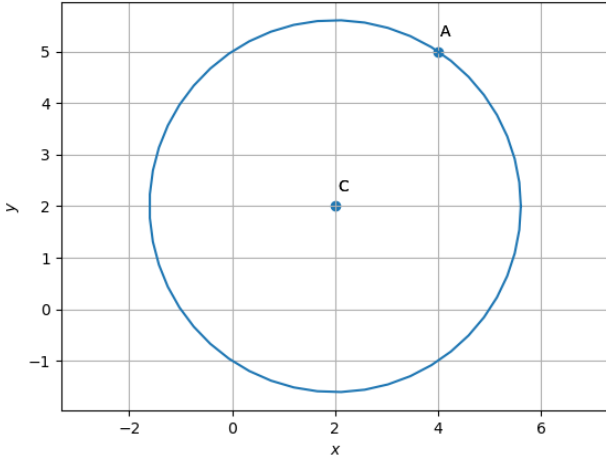
Solution

Figure 1: Circle with center (2,2) and passing through (4,5)

Construction

Input taken for the construction of the circle are it's center and point it is passing through.

Symbol	Value	Description
C	(2,2)	circle center
A	(4,5)	point on circle
r	$\|A - C\|$	circle radius

Let us assume a circle with radius 'r' and center at C which is passing through a point A.

Radius of the circle is the distance between center and any point on circle.

$$r = \|A - C\| \tag{1}$$
$$r = \sqrt{13} \tag{2}$$

The desired circle can be expressed as a conic with the parameters

$$\mathbf{V} = \mathbf{I} \tag{3}$$
$$\mathbf{u} = \begin{pmatrix} -2 & -2 \end{pmatrix} \tag{4}$$

the constant 'f' in the equation can be calculated as

$$f = \|\mathbf{C}\|^2 - \mathbf{r}^2 \tag{5}$$
$$f = \sqrt{2^2 + 2^2}^2 - 13 \tag{6}$$
$$f = -5 \tag{7}$$

The general equation of the circle as a conic is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{8}$$

by subtitution the considered parameters in the general form,

$$\mathbf{x}^T \mathbf{x} + 2 \begin{pmatrix} -2 & -2 \end{pmatrix} \mathbf{x} - 5 = 0 \tag{9}$$

this the desired equation of the circle with center at (2,2) and passes through (4,5).
We can verify the equation by substituting the point in the equation,

$$\mathbf{x} = \begin{pmatrix} 4 & 5 \end{pmatrix} \tag{10}$$

from the eqn 9

$$\begin{pmatrix} 4 & 5 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} -2 \\ -2 \end{pmatrix} \begin{pmatrix} 4 & 5 \end{pmatrix} - 5 = 0 \tag{11}$$
$$\implies 4^2 + 5^2 - 16 - 20 - 5 = 0 \tag{12}$$
$$\implies 0 = 0 \tag{13}$$
$$LHS = RHS \tag{14}$$

Hence, equation 9 is the required circle equation.