Circle Assignment

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Problem

Show that the tangents of circle drawn at the ends of diameter are parallel.

Solution

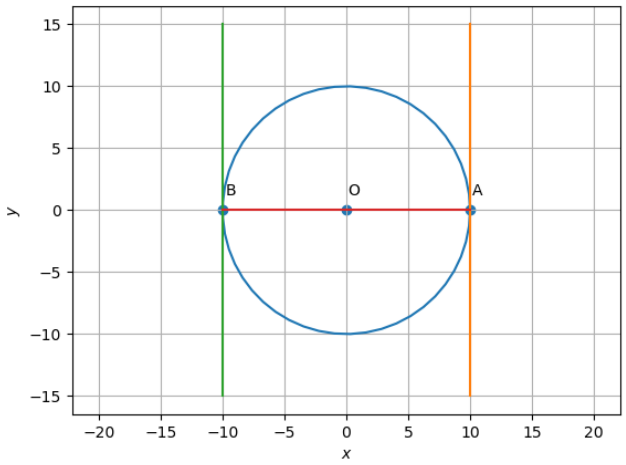

Figure 1: Circle with tangents at ends of it's diameter

Construction

Input taken for the construction of the Circle and the tangents is 'r' radius of the circle.

Symbol	Value	Description
r	10	circle radius
O	(0,0,0)	Center
A	$(rcos(\theta), rsin(\theta), 0)$	point A
B	$(-rcos(\theta), -rsin(\theta), 0)$	point B

Let us assume a circle with radius 'r' and center at origin.

$$\boldsymbol{x}^T\boldsymbol{V}\boldsymbol{x} + 2\boldsymbol{u}^T x + f = 0 \tag{1}$$

but, for a Circle

$$\boldsymbol{V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

and the center of the circle is at origin,

$$\boldsymbol{u}^T = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Here, the points $\boldsymbol{A}$ and $\boldsymbol{B}$ are the ends of diameter of the circle.
To prove that tangets at the ends of a diameter are parallel, we need to find the slope of tangents at the given points.

$$\mathbf{A} = \begin{pmatrix} r.cos\theta \\ r.sin\theta \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -r.cos\theta \\ -r.sin\theta \\ 0 \end{pmatrix} \tag{4}$$

We know that the cross product of two vectors results in a perpendicular vector, so we can find the tanget by cross multiplying radius vector and z-axisunit vector.

$$\mathbf{Z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{5}$$

So, the tangents at $\boldsymbol{A}$ and $\boldsymbol{B}$ are given by,

$$\boldsymbol{T_1} = \boldsymbol{A}X\boldsymbol{Z}, \boldsymbol{T_2} = \boldsymbol{B}X\boldsymbol{Z} \tag{6}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are the radius vectors towards the points A and B on the circle form the center of the

circle.
Then,

$$\mathbf{T_1} = \begin{pmatrix} r.sin\theta \\ -r.cos\theta \\ 0 \end{pmatrix} \mathbf{T_2} = \begin{pmatrix} -r.sin\theta \\ r.cos\theta \\ 0 \end{pmatrix} \tag{7}$$

Here, $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ are the directional vectors of tangents at points A and B, which are the end points of a diameter.

We know that, cross product of any two parallel vectors results zero.

If we cross multiply the vectors $\mathrm{T}_1 and \boldsymbol{T_2}$

$$\mathbf{T_1} X \mathbf{T_2} = \begin{pmatrix} r.sin\theta \\ -r.cos\theta \\ 0 \end{pmatrix} X \begin{pmatrix} -r.sin\theta \\ r.cos\theta \\ 0 \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

The cross product of the two tangents is equal to zero, so we can say that the tangents are parallel.