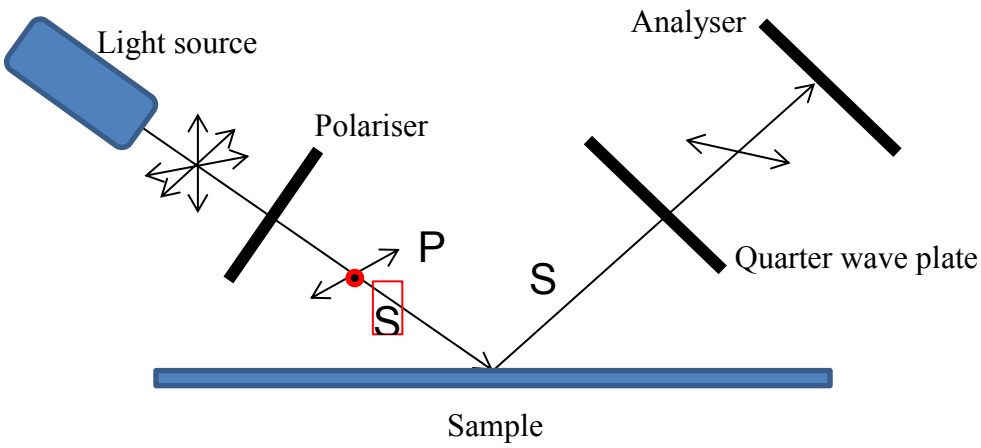

Fig. 1 Schematic Diagram of an Elipsometry

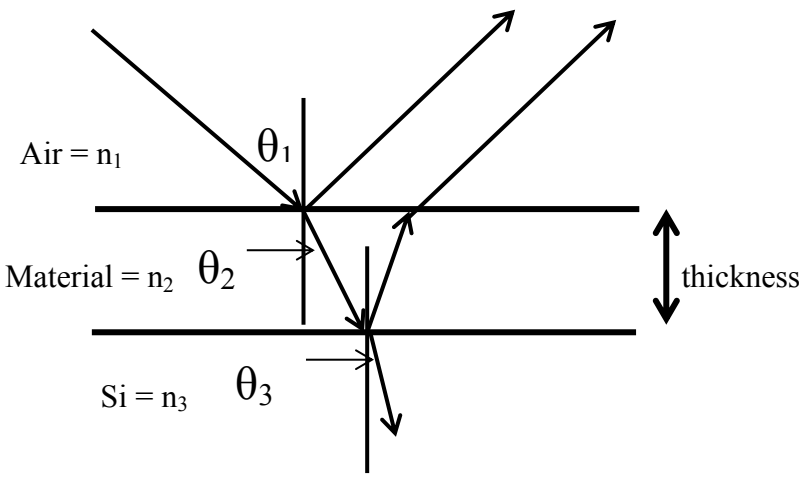

Fig. 2 Sample model

The schematic diagram of an Ellipsometry is shown in Fig.1. It has a visible light source that passes through a polariser to polarise the unpolarised light. The polarised light has p and s components. The p component is when the electric field is parallel to the plane of incidence and s component is when it is perpendicular. Once the light hits the sample, some of them reflect back and some of them transmit through. When the angle of incidence is close to Brewster angle, almost every p-component are transmitted thorough and whatever is reflected back from the sample contains s-component only as labelled in Fig. 1.

Once the light reflects from the sample, it becomes unpolarised. Therefore, quarter wave plate is needed to polarize the light again. The Ellipsometry only deals with the reflected portion of the light.

This code is designed to calculate Psi and Delta that is determined from Ellipsometry in order to calculate thickness and refractive index. Assuming the substrate is silicon (Si), let's say an air is medium 1, material is medium 2 and Si is medium 3. The pathway of the light is shown in Fig.2,

assuming that $n_1<n_2<n_3$, where n is the refractive index. In order to calculate the Psi and Delta, the Ellipsometry uses Cauchy equation that describes the relationship between refractive index and wavelength.

$$n(\lambda) = N_0 + \frac{N_1}{\lambda^2}$$

Where N_0 and N_1 are Cauchy constants, n is the refractive index and λ is the wavelength.

The first calculation and second calculation calculates the refractive index of your material and the substrate respectively. The wavelength is in the range of 400 to 800 nm.

The third calculation is finding reflection_12 in p-polarisation. The '12' represents medium 1 and 2.

$$r_{12}^{p} = \frac{n_2 \cos\emptyset_1 - n_1 \cos\emptyset_2}{n_2 \cos\emptyset_1 + n_1 \cos\emptyset_2}$$

The fourth calculation is finding reflection in s-polarisation in medium 1 and 2.

$$r_{12}^{s} = \frac{n_1 \cos\emptyset_1 - n_2 \cos\emptyset_2}{n_1 \cos\emptyset_1 + n_2 \cos\emptyset_2}$$

Similarly, the reflectance can be calculated in medium 2 and 3 in p and s polarisation as done in the calculation five and six respectively.

$$r_{23}^{p} = \frac{n_3 \cos\emptyset_2 - n_2 \cos\emptyset_3}{n_3 \cos\emptyset_2 + n_2 \cos\emptyset_3}$$

$$r_{23}^{S} = \frac{n_2 \cos\emptyset_2 - n_3 \cos\emptyset_3}{n_2 \cos\emptyset_2 + n_3 \cos\emptyset_3}$$

The thin film phase, β is calculated in calculation seven by the following equation:

$$\beta = 2\pi \frac{d}{\lambda} n_2 \cos\emptyset_2$$

The total reflectance in both p and s components can be calculated by the following equations that are named as calculation eighth and ninth respectively.

$$R^P = \frac{r_{12}^{p} + r_{23}^{p} \exp(-i2\beta)}{1 + r_{12}^{p} r_{23}^{p} \exp(-i2\beta)}$$

$$R^s = \frac{r_{12}^{s} + r_{23}^{s} \exp(-i2\beta)}{1 + r_{12}^{s} r_{23}^{s} \exp(-i2\beta)}$$

Once the total reflectance of both p and s components are calculated, ρ is calculated in calculation tenth by the following formula:

$$\rho = \frac{R^P}{R^S}$$

The Psi is then calculated in calculation eleventh by the following equation:

$$\tan\Psi = \mathrm{abs}(R^P / R^S)$$

The value of Psi here is in radians, which is converted into degree in calculation twelfth. Finally, in calculation thirteenth, Delta is calculated by the following equation:

$$\rho = \tan\Psi e^{i\Delta}$$

Same as Psi, Delta is also calculated in radians. So, in calculation fourteenth, these values are converted into degree.

In terms of drawing graph, I have made three graphs altogether. The first one is from the first calculation (result1 from the code) that shows the relationship between the refractive index of your material and the wavelength (400- 800 nm). The second graph is from the calculation twelfth that shows the relationship between Psi (degree) and wavelength (result12 from the code), and final graph is of calculation fourteenth (Delta in degree vs wavelength, result14 from the code).

Reference:

Tompkins, H. G. (1992). *A User's Guide to Ellipsometry.* Academic Press.