J - SUSHI

Q- What is Expectation of an event.

Ans → The expected value (or mean) of X, where X is a discreate random variable, is a weighted average of possible values that X can take, each value being according to probability of that event occuring

$$E(X) = S \times P(X=x)$$

Example :-

(1) Event → Roll a dice (1-6)

X → number appearing on dice

$P = 1/6$

$$E(X) = \sum p \times x$$

$$= 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6} = \frac{21}{6}$$

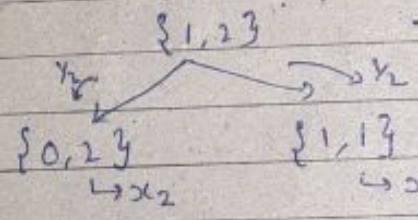
(ex-2) :- Event → Roll a dice till you get a (6)
X → No. of times the dice is rolled.//

$$E(X) = 1\times\frac{1}{6} + 2\times\left(\frac{5}{6}\times\frac{1}{6}\right) + 3\times\left(\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right) + \text{----} \quad \text{so on.}$$

$\frac{1}{6}$ ↓ 6 in first time; $\frac{5}{6}\times\frac{1}{6}$ ↓ 6 in 2nd time.; $\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$ ↙ 6 in 3rd time

Now ⟹ (ques)

Simply if we have state {1,2} initially
↳ X



$$\left\{ E(X) = 1 + \frac{1}{2}E(x_2) + \frac{1}{2}E(x_1) \right\}$$

⇒ Now as sushies in a plate can be $1 \le S \le 3$
we will consider count of different ~~sp~~ plates (sushies in plate) as states of DP.

x = Plates having only 1 sushi
y = " " 2 sushi , z = Plate with 3 sushies
0 = Plates with zero sushi left.

$$0 = N - (x+y+z)$$

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⇒ As we need to exahust all sushies :-

$dp(x,y,z)$ → expected no. of operations to reach $(0,0,0)$

$$dp(x,y,z) = 1 + p_0\, dp(x,y,z) + p_x\, dp(x-1,y,z) + p_y\, dp(x+1,y-1,z) + p_z\, dp(x,y+1,z-1)$$

(eaten sushi from plate ↑ having 1 sushi)

$$dp(x,y,z) = \frac{p_x\, dp(x-1,y,z) + p_y\, dp(x+1,y-1,z) + p_z\, dp(x,y+1,z-1)}{1-p_0}$$

we know $p_x = \frac{x}{N}$, $p_y = \frac{y}{N}$, $p_z = \frac{z}{N}$

$$p_0 = \frac{N-(x+y+z)}{N}$$

$$1-p_0 = \frac{N-N+(x+y+z)}{N} = \frac{x+y+z}{N}$$

$$dp(x,y,z) = \frac{x\cdot dp(x-1,y,z) + y\cdot dp(x,y-1,z) + z\cdot dp(x,y+1,z-1)}{(x+y+z)}$$

$dp(0,0,0) = 0$ ← base case