Import Libraries

In [2]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.stattools import adfuller
from statsmodels.tsa.arima.model import ARIMA
from statsmodels.tsa.statespace.sarimax import SARIMAX
import pmdarima as pm
from sklearn.model_selection import GridSearchCV
import warnings
warnings.filterwarnings('ignore')
```

Read and Prepare Data

In [5]:
```python
df = pd.read_csv("C:\\Users\\user\\OneDrive\\Desktop\\Project\\city_day.csv")
delhi = df[df['City'] == 'Delhi']
```

In [7]:
```python
delhi['Date'] = pd.to_datetime(delhi['Date'])
delhi.set_index('Date', inplace = True)
```

In [9]:
```python
delhi.head()
```

Out[9]:

Date	City	PM2.5	PM10	NO	NO2	NOx	NH3	CO	SO2	O3	Benzene	Tol
2015-01-01	Delhi	313.22	607.98	69.16	36.39	110.59	33.85	15.20	9.25	41.68	14.36	
2015-01-02	Delhi	186.18	269.55	62.09	32.87	88.14	31.83	9.54	6.65	29.97	10.55	
2015-01-03	Delhi	87.18	131.90	25.73	30.31	47.95	69.55	10.61	2.65	19.71	3.91	
2015-01-04	Delhi	151.84	241.84	25.01	36.91	48.62	130.36	11.54	4.63	25.36	4.26	
2015-01-05	Delhi	146.60	219.13	14.01	34.92	38.25	122.88	9.20	3.33	23.20	2.80	

Check for missing values

In [12]:
```python
pm25 = delhi['PM2.5']
pm25[pm25.isna()]
```

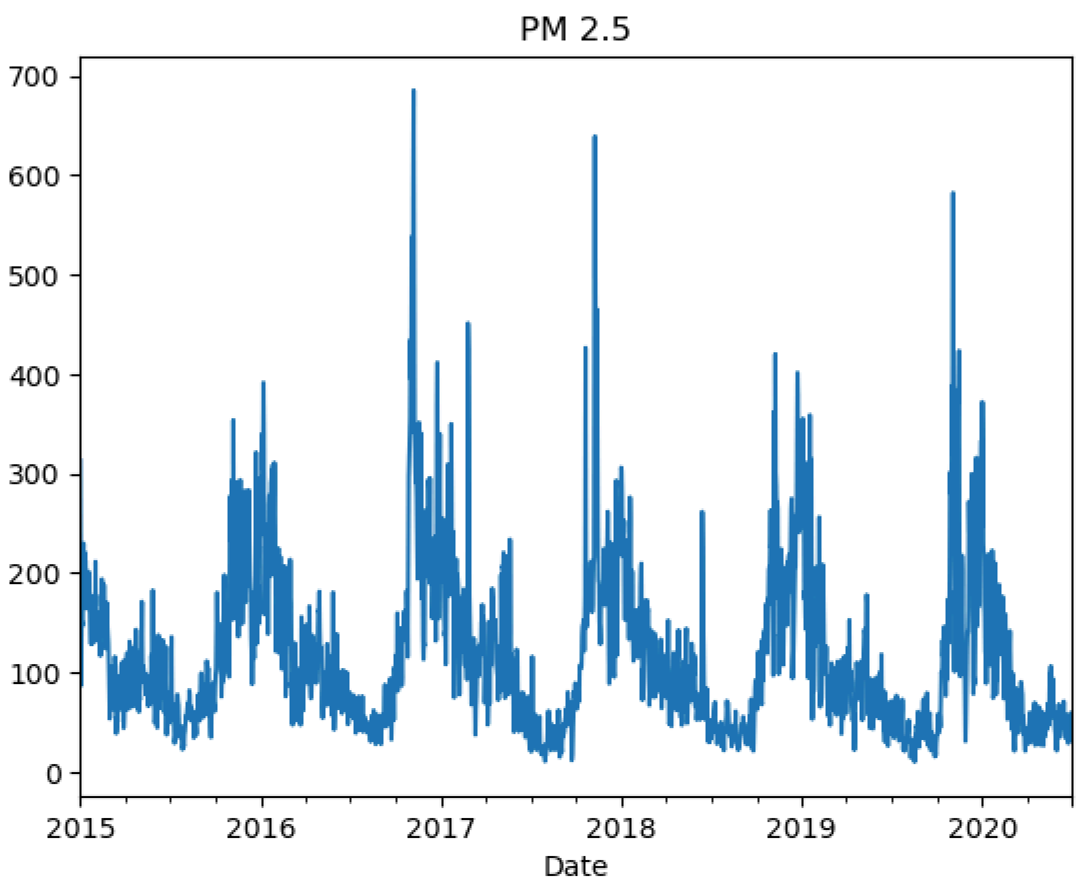
Out[12]:
```
Date
2017-08-12   NaN
2017-08-13   NaN
Name: PM2.5, dtype: float64
```

In [14]:
```
# filling the missing values with linear interpolation method
pm25 = pm25.interpolate(method = 'linear')
```

Plotting the Data

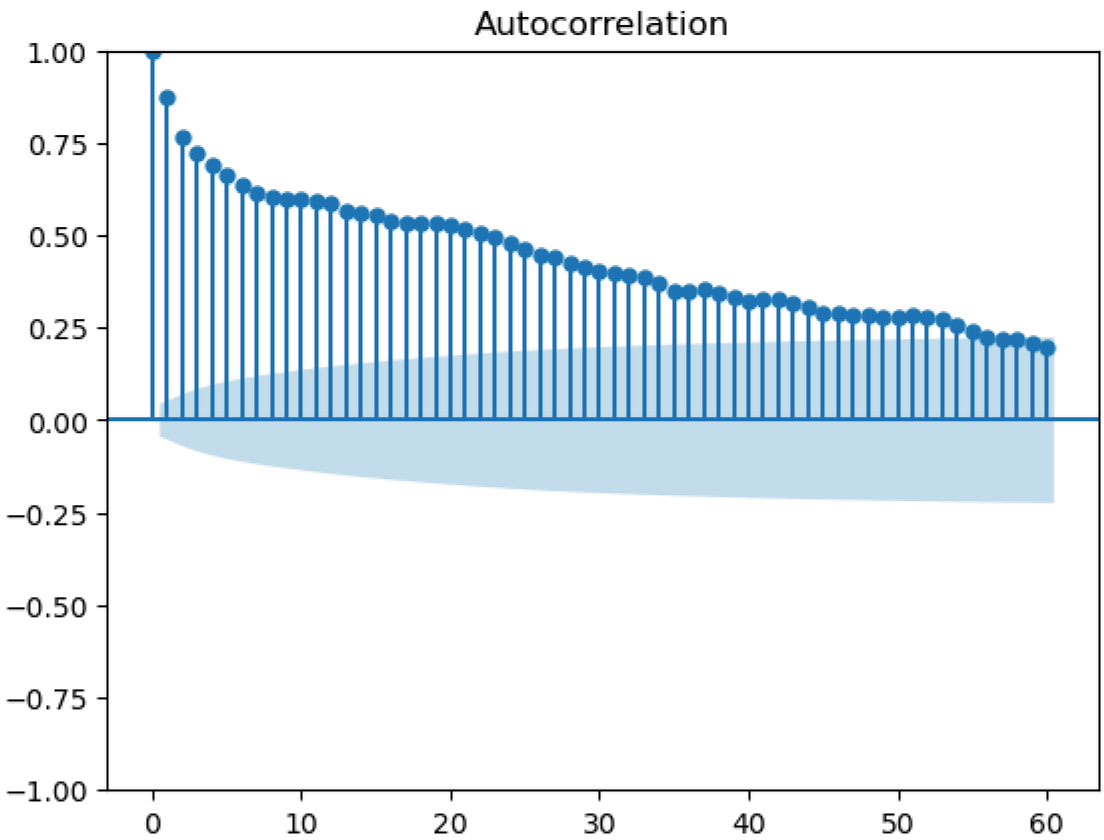
In [17]:
```
pm25.plot(title = 'PM 2.5')
```

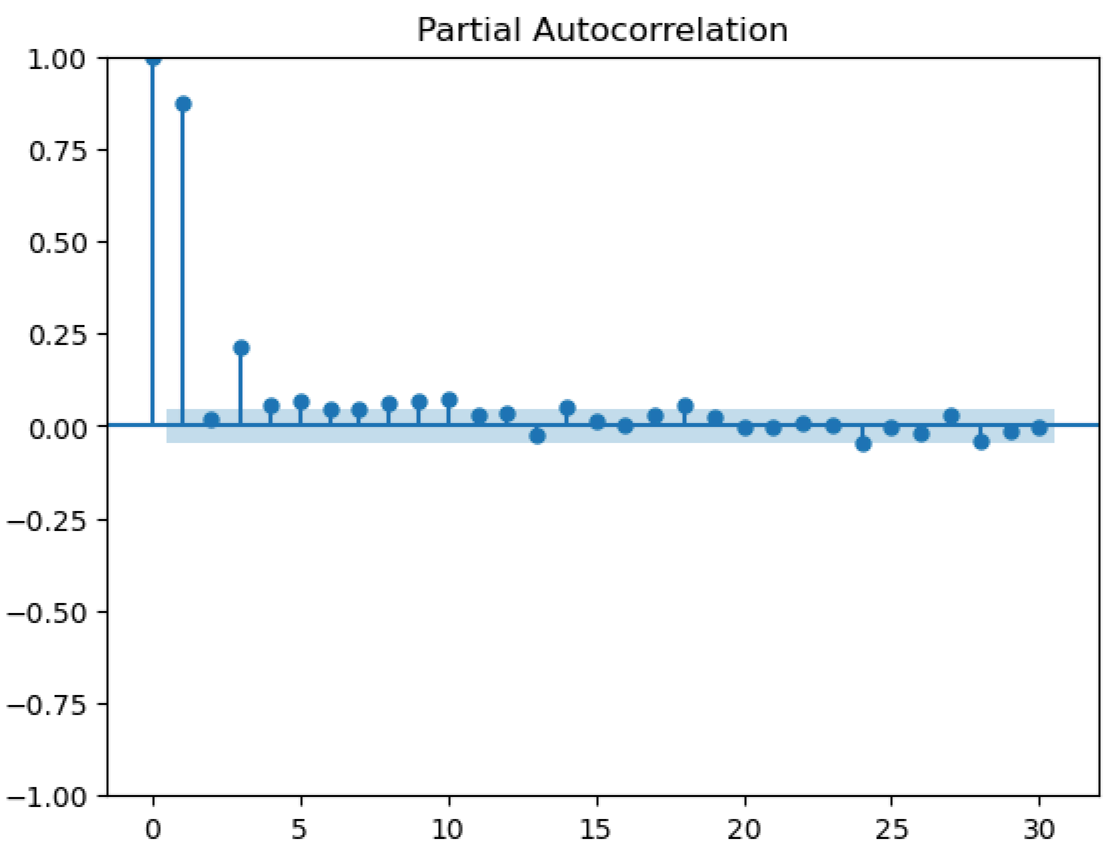
Out[17]:
```
<Axes: title={'center': 'PM 2.5'}, xlabel='Date'>
```



ACF and PACF Plot

In [20]:
```
pm25_acf = plot_acf(pm25, lags = 60)
pm25_pacf = plot_pacf(pm25, lags = 30)
```


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
10
20
30
40
50
60


Partial Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
5
10
15
20
25
30

From the above plots, it can be seen that ACF is gradually falling off with the PACF falling off completely after a few lags indicating an AR process. Possible orders for an AR model :- p = (1,3,5,10)

Train-Test Split

In [24]:
```
train_pm25 = pm25[pm25.index < '2019-01-01']
test_pm25 = pm25[pm25.index >= '2019-01-01']
```

Model Fitting

In [27]:
```
# Checking and comparing accuracy of the model with different possible orders

ps = [1,3,5,10]
for p in ps:
    model = ARIMA(train_pm25, order = (p,0,0)).fit()

    predicted = model.predict(start = '2019-01-01', end = '2019-03-01')
    actual = test_pm25[test_pm25.index <= '2019-03-01']
    rmse = ((actual - predicted)**2).mean()**0.5
    mae = abs(actual - predicted).mean()
    print('p = {}, AIC = {}, BIC = {}, Log Livelihood = {}, RMSE = {}, MAE = {}'.fo
```

```
p = 1, AIC = 14954.348502174324, BIC = 14970.209131409576, Log Livelihood = -7474.17
4251087162, RMSE = 68.29758621142291, MAE = 52.344893415080676
p = 3, AIC = 14868.848904167926, BIC = 14895.283286226679, Log Livelihood = -7429.42
4452083963, RMSE = 63.390975332842594, MAE = 52.294141990986965
p = 5, AIC = 14857.232168108876, BIC = 14894.240302991131, Log Livelihood = -7421.61
6084054438, RMSE = 63.88501852143921, MAE = 54.28728557342916
p = 10, AIC = 14844.156755141354, BIC = 14907.599272082361, Log Livelihood = -7410.0
78377570677, RMSE = 75.86552866929638, MAE = 65.35324905353842
```

Comparing the above parameters of different models, we choose the AR Model of order 3

In [30]:
```
model = ARIMA(train_pm25, order = (3,0,0)).fit()
print(model.summary())
```

```
                                SARIMAX Results
==============================================================================
Dep. Variable:                  PM2.5   No. Observations:                 1461
Model:                 ARIMA(3, 0, 0)   Log Likelihood               -7429.424
Date:                Tue, 13 May 2025   AIC                          14868.849
Time:                        00:06:03   BIC                          14895.283
Sample:                    01-01-2015   HQIC                         14878.710
                         - 12-31-2018
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const        123.8733     15.203      8.148      0.000      94.075     153.671
ar.L1          0.8591      0.014     62.641      0.000       0.832       0.886
ar.L2         -0.1937      0.021     -9.198      0.000      -0.235      -0.152
ar.L3          0.2452      0.018     13.644      0.000       0.210       0.280
sigma2      1527.4262     25.673     59.495      0.000    1477.108    1577.745
===================================================================================
Ljung-Box (L1) (Q):                   0.30   Jarque-Bera (JB):              6847.40
Prob(Q):                              0.58   Prob(JB):                         0.00
Heteroskedasticity (H):               1.31   Skew:                             1.47
Prob(H) (two-sided):                  0.00   Kurtosis:                        13.19
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-ste
p).
```

Prediction

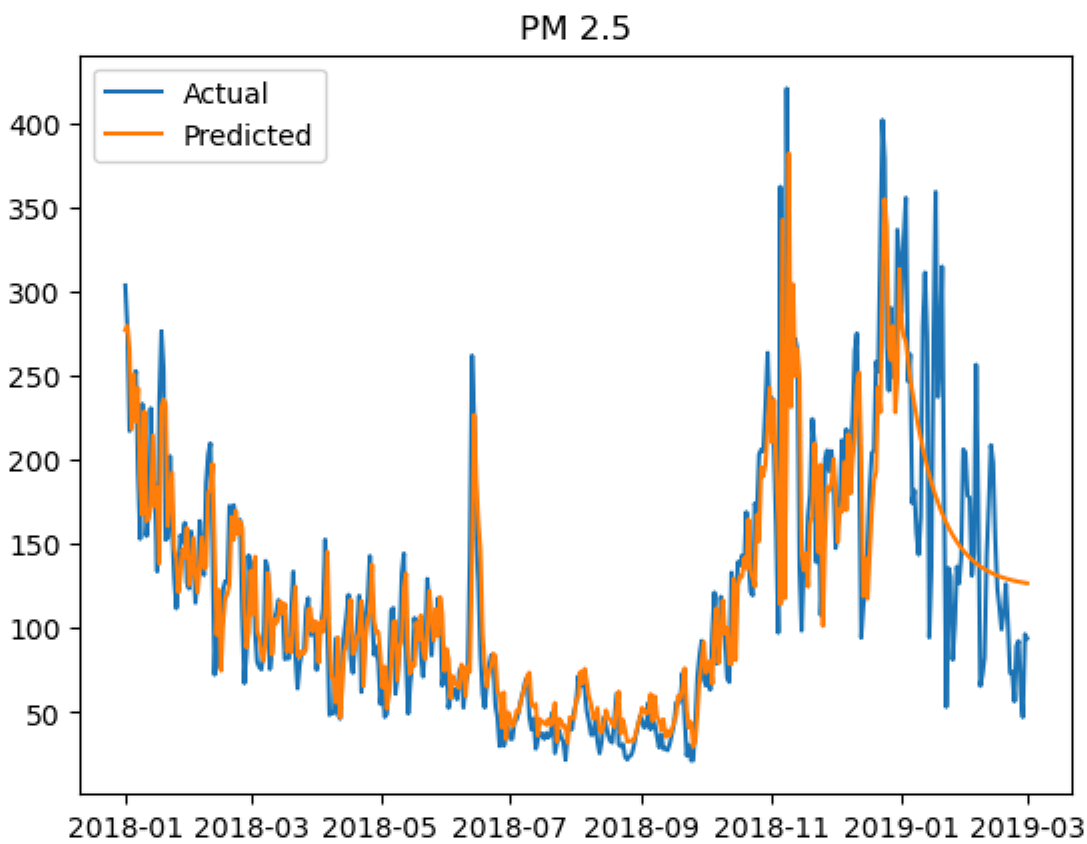
Actual vs Prediction Plot

In [34]:

```
plt.plot(pm25[(pm25.index >= '2018-01-01') & (pm25.index <= '2019-03-01')], label =
plt.plot(model.predict(start = '2018-01-01', end = '2019-03-01'), label = 'Predicte
plt.title('PM 2.5')
plt.legend()
```

Out[34]:

```
<matplotlib.legend.Legend at 0x1d082a7d520>
```



MAE and RMSE on test data

In [37]:
```
predicted = model.predict(start = '2019-01-01', end = '2019-03-01')
actual = test_pm25[test_pm25.index <= '2019-03-01']
mae = abs(actual - predicted).mean()
rmse = ((actual - predicted)**2).mean()**0.5
print('MAE = {}, RMSE = {}'.format(mae, rmse))
```

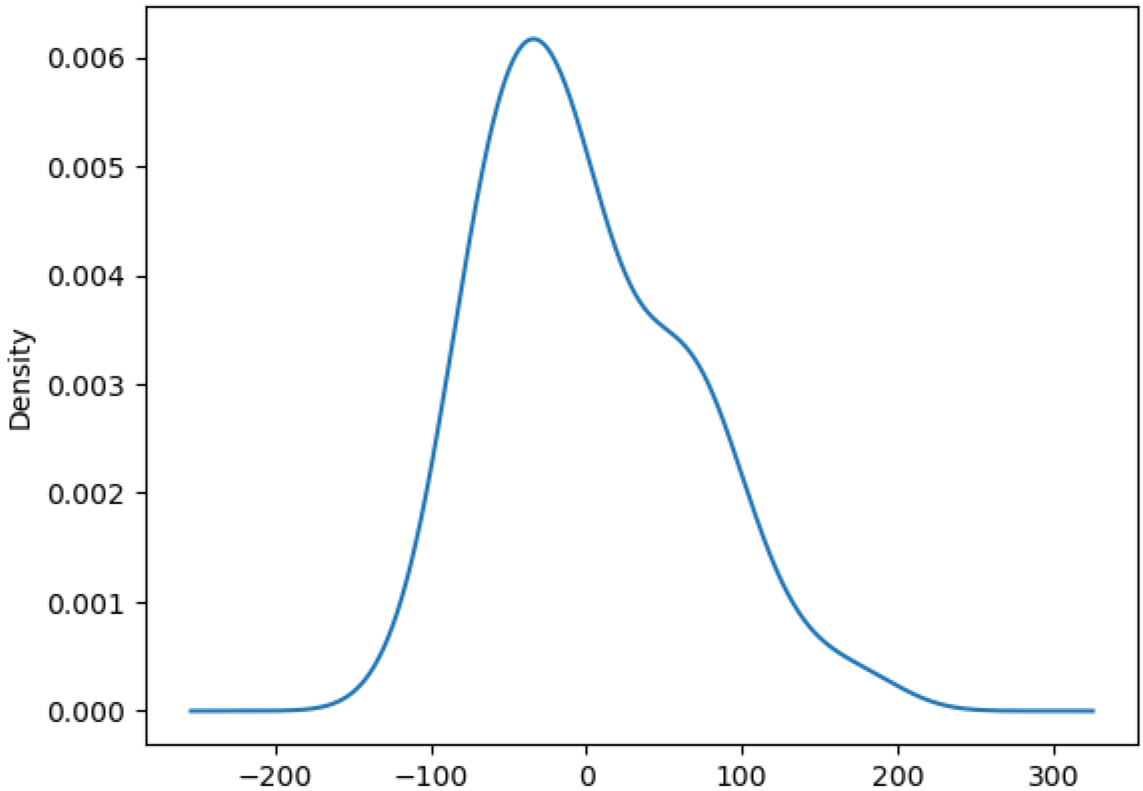
```
MAE = 52.294141990986965, RMSE = 63.390975332842594
```

Residuals

Density plot of Residuals

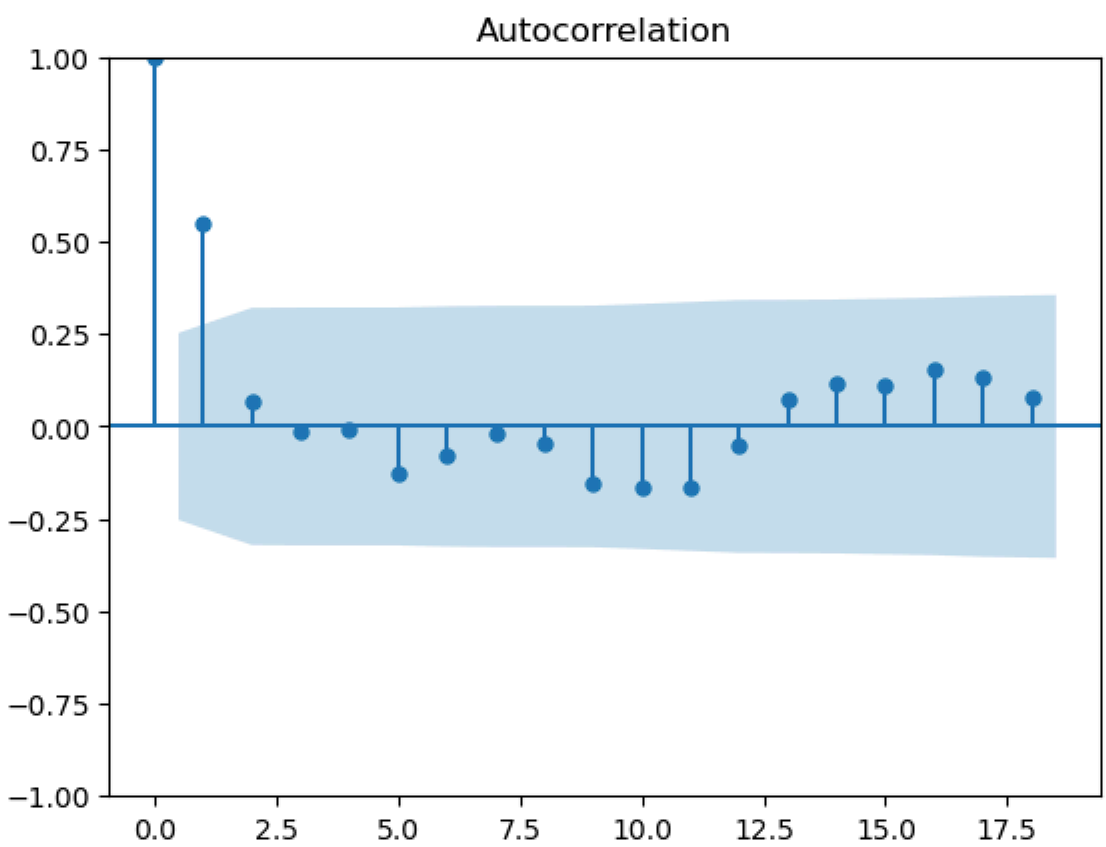
In [41]:
```
residuals = actual - predicted
residuals.plot(kind = 'kde')
```

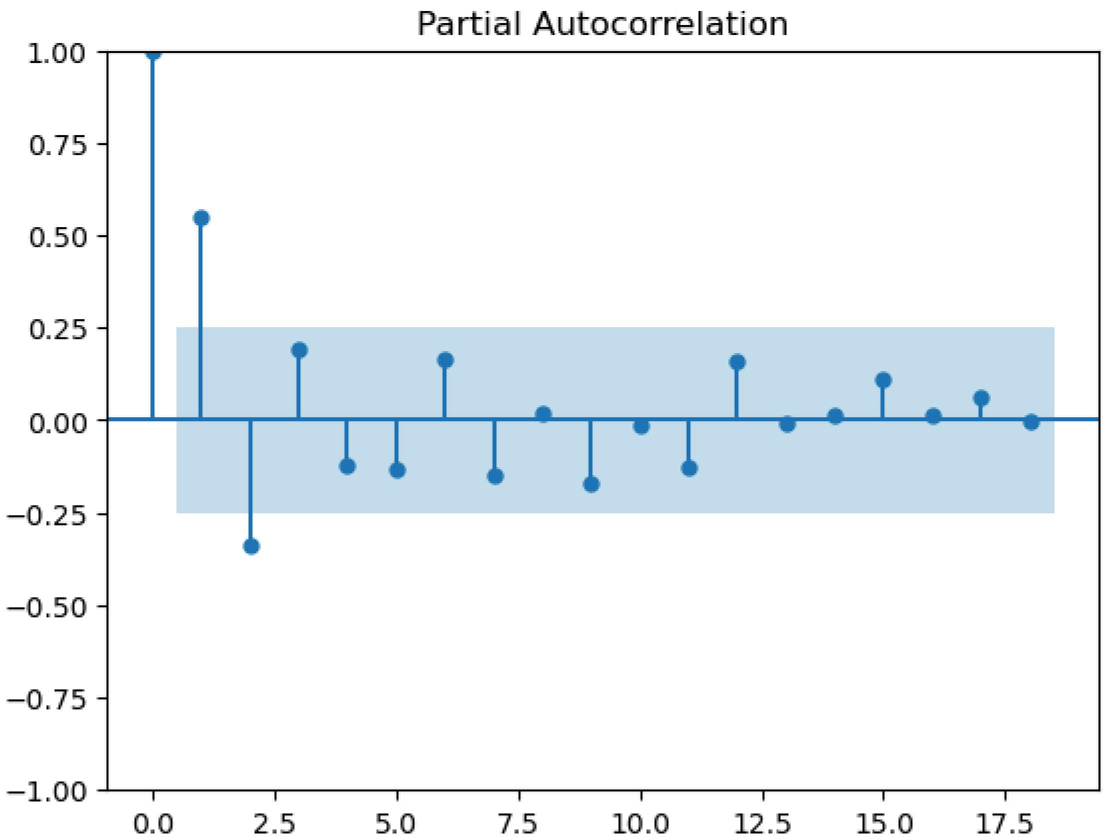
Out[41]: <Axes: ylabel='Density'>



ACF and PACF Plot of Residuals

In [44]:
```
res_acf = plot_acf(residuals)
res_pacf = plot_pacf(residuals)
```





From the density plot it seems that the residuals follows a normal distribution but not too accurately indicating the model didn't quite capture the data. Further the ACF and PACF plot shows correlation between residuals suggesting the model missed some important patterns and trends and is too simple.

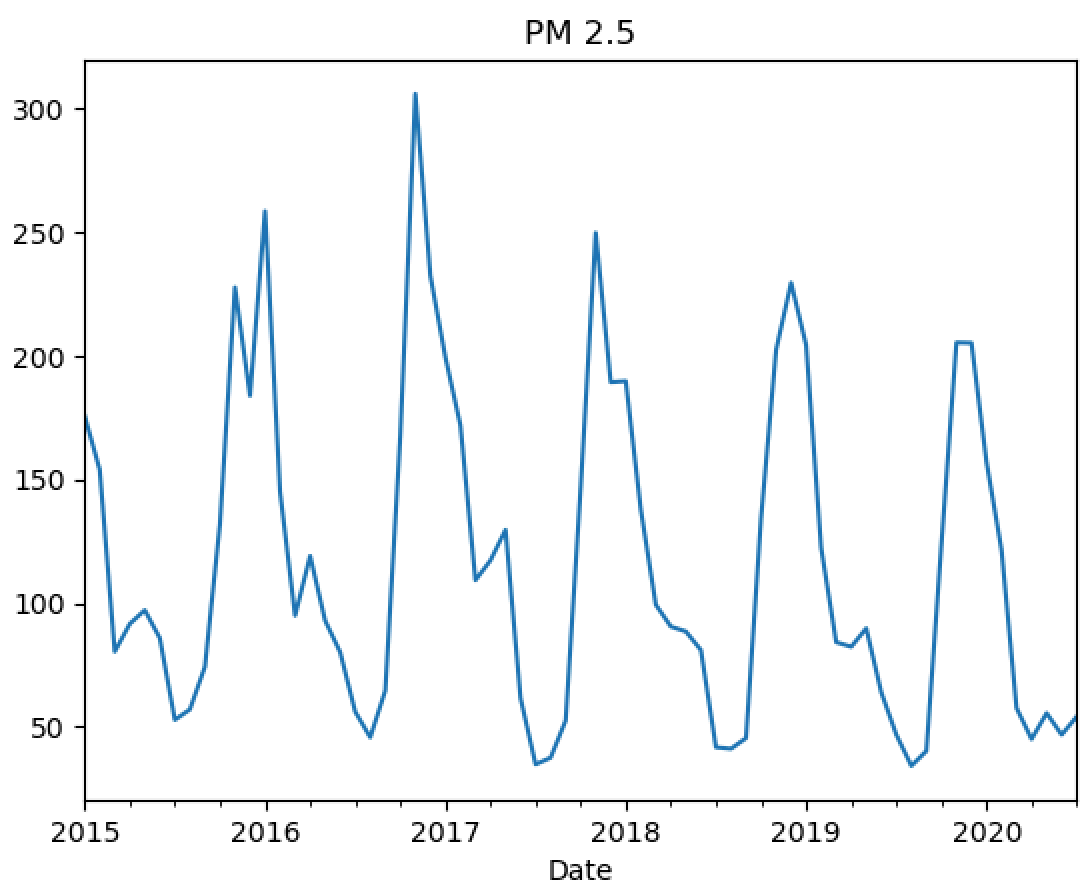
Converting the daily data into a monthly data

In [47]:
```
pm25m = delhi['PM2.5'].resample(rule = 'MS').mean()
pm25m
```

Out[47]:
```
Date
2015-01-01    175.690645
2015-02-01    153.920357
2015-03-01     80.338065
2015-04-01     91.562333
2015-05-01     97.109355
                 ...
2020-03-01     57.506452
2020-04-01     44.940000
2020-05-01     55.448710
2020-06-01     46.694667
2020-07-01     54.010000
Freq: MS, Name: PM2.5, Length: 67, dtype: float64
```

In [49]:
```
pm25m.plot(title = 'PM 2.5')
```

Out[49]:
```
<Axes: title={'center': 'PM 2.5'}, xlabel='Date'>
```



The data clearly doesn't show any trend but indeed has a visible seasonality. So, we use the **adfuller test** to check for stationarity of the series.

Check for stationarity

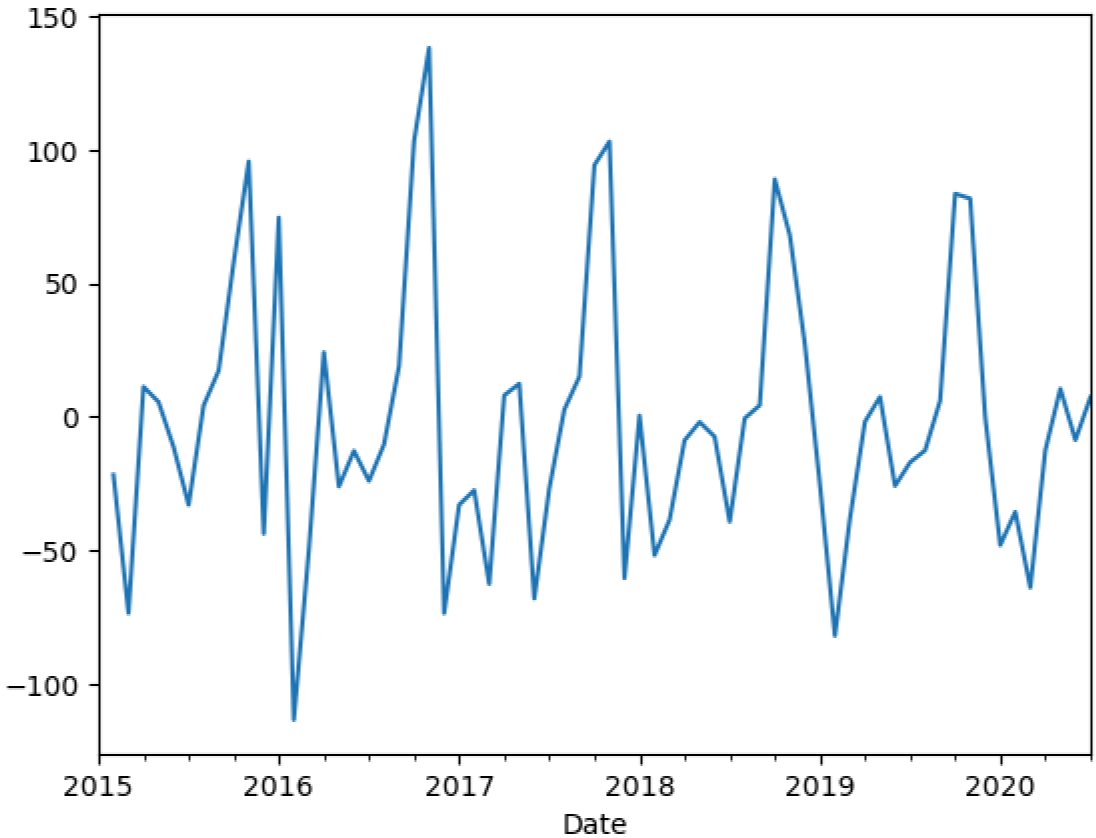
In [53]:
```
pm25m_result = adfuller(pm25m)
print('Test Statistic: {}'.format(pm25m_result[0]))
print('p value: {}'.format(pm25m_result[1]))
```

```
Test Statistic: 0.1051175256024009
p value: 0.966408759417222
```

The p value is 0.966 > 0.05, which indicates that there is very weak evidence to reject the null hypothesis and the series is not stationary. So we compute the first order differences of the data and check again for stationarity.

In [58]:
```
pm25m_diff1 = pm25m - pm25m.shift(1)
pm25m_diff1.plot()
```

Out[58]: <Axes: xlabel='Date'>



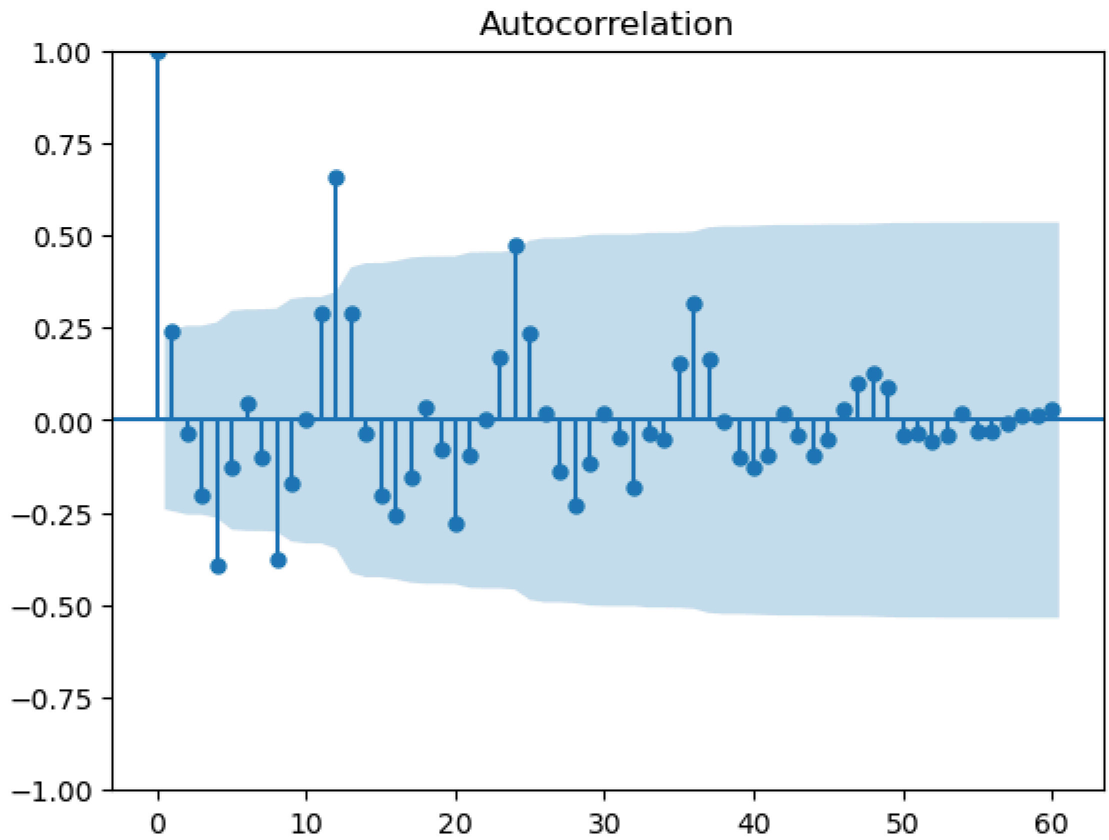
In [60]:
```
pm25m_result2 = adfuller(pm25m_diff1.dropna())
print('Test Statistic: {}'.format(pm25m_result2[0]))
print('p value: {}'.format(pm25m_result2[1]))
```

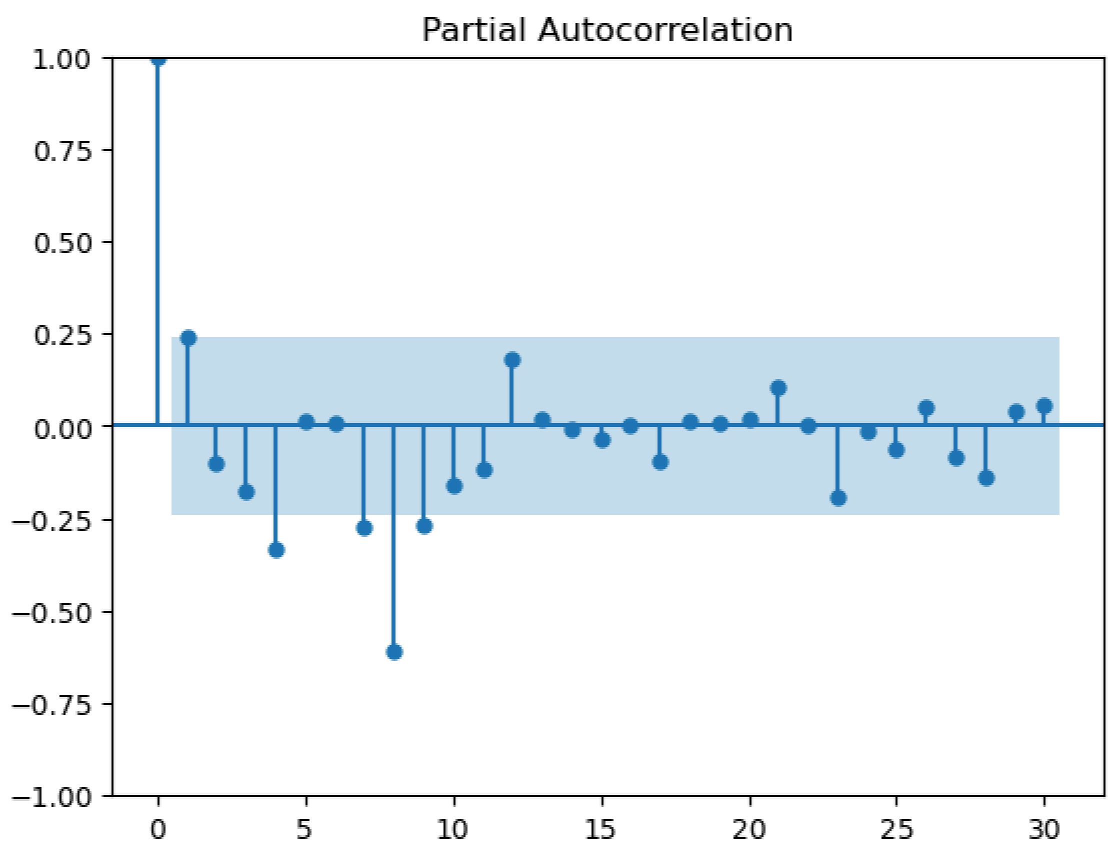
```
Test Statistic: -7.71642271845219
p value: 1.2248286849539635e-11
```

Here the p value is very very smaller than 0.05 indicating a very strong evidence against the null hypothesis. So we conclude that the first order differences are stationary.

ACF and PACF plot of seasonal data

In [66]:
```
pm25m_diff1_acf = plot_acf(pm25m_diff1.dropna(), lags = 60)
pm25m_diff1_pacf = plot_pacf(pm25m_diff1.dropna(), lags = 30)
```



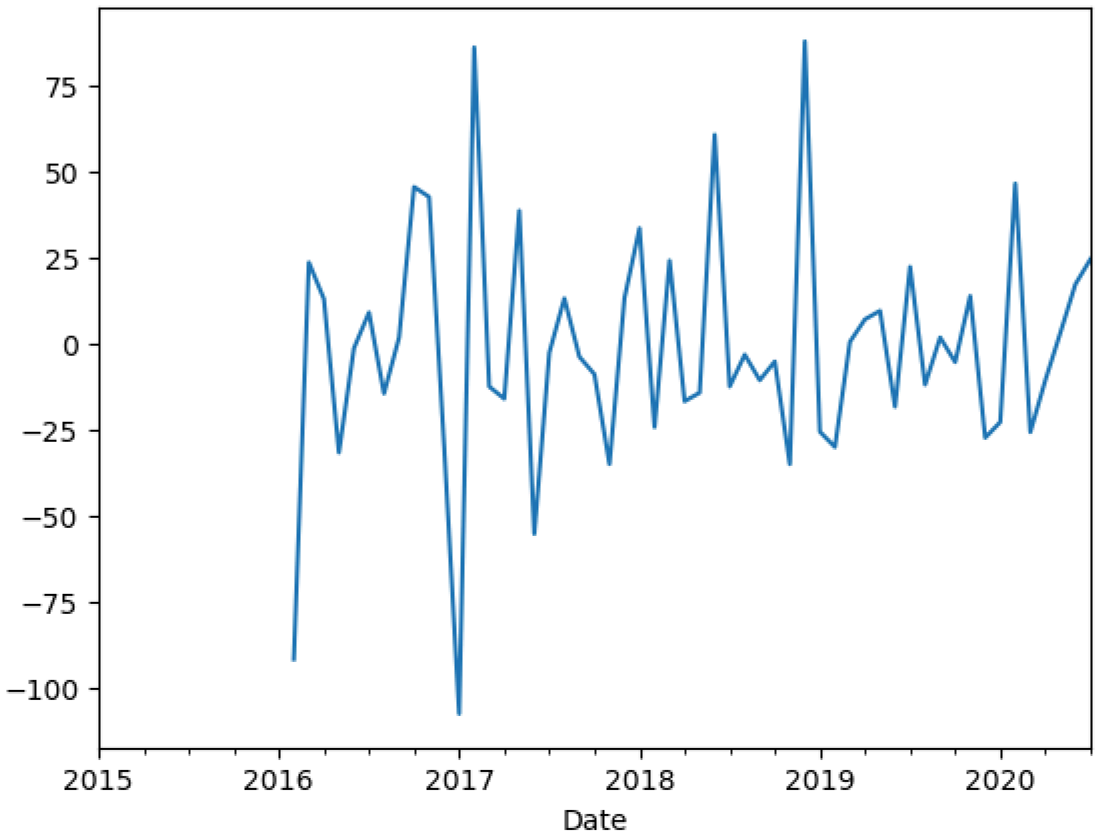


Seasonal difference

Now we take a difference of 12 steps to remove the seasonality from the the data

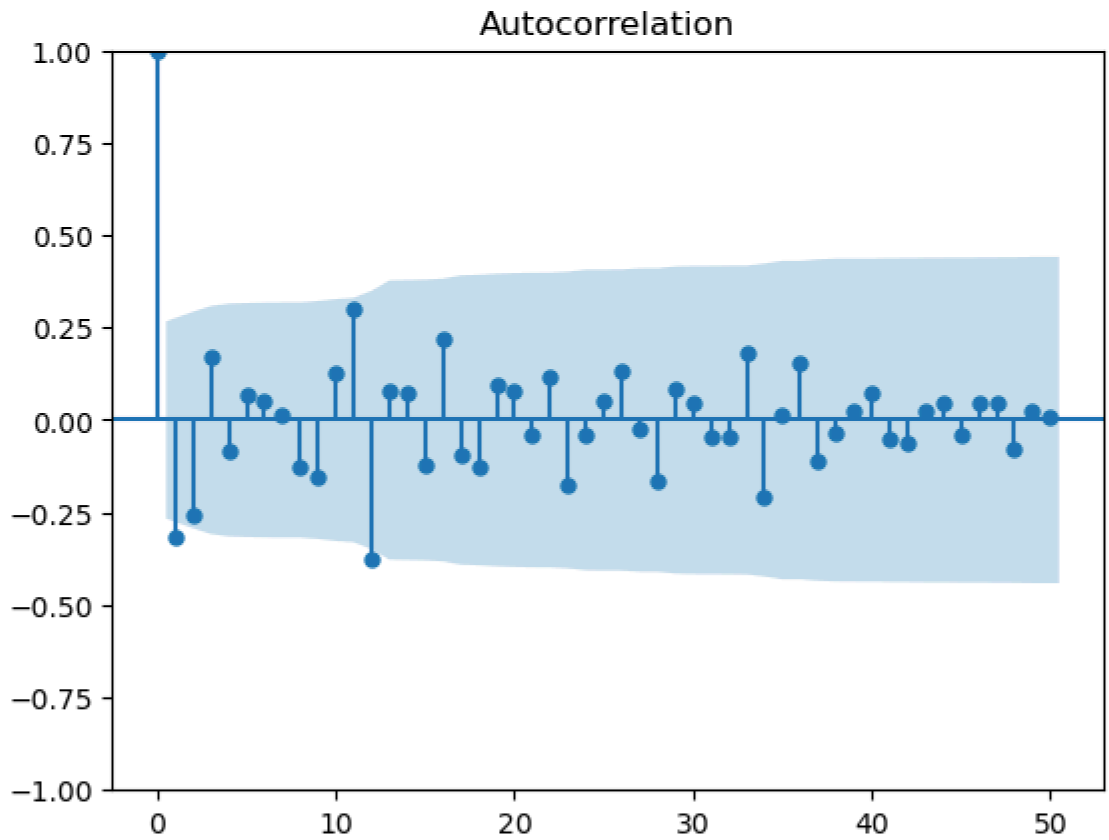
In [72]:
```
pm25m_sdiff = pm25m_diff1 - pm25m_diff1.shift(12)
pm25m_sdiff.plot()
```

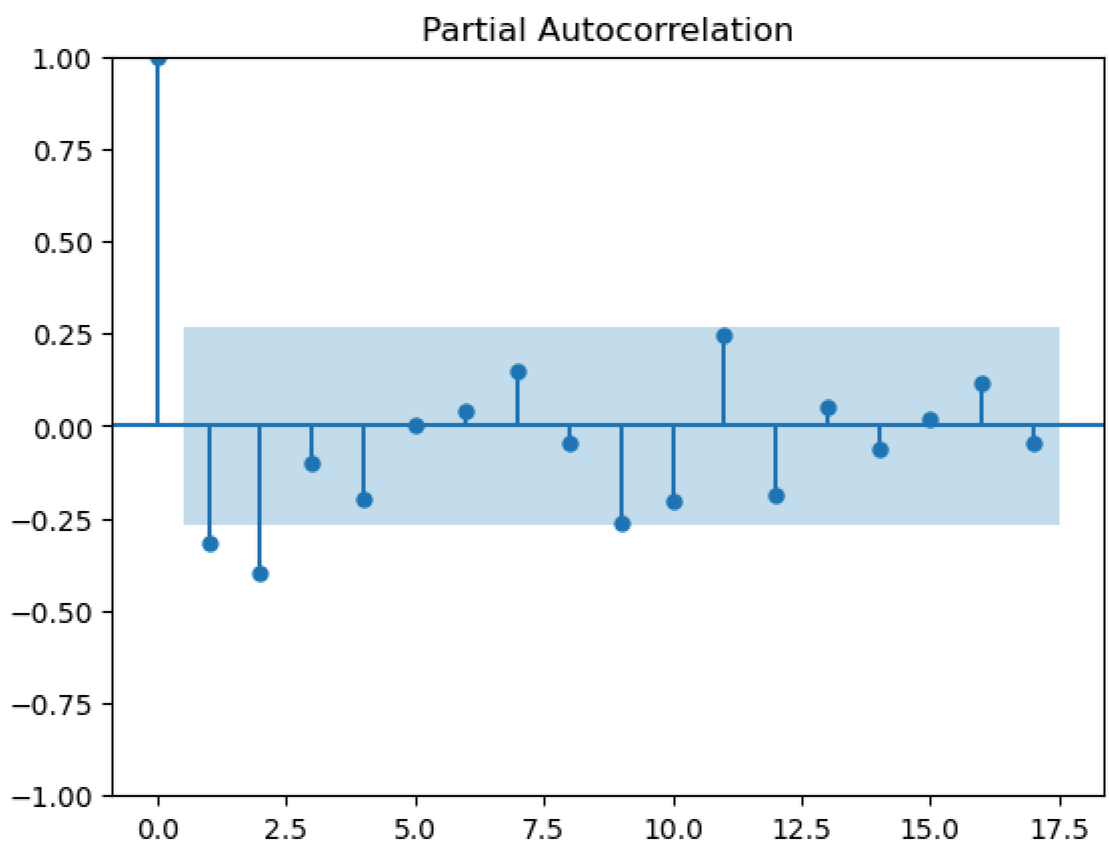
Out[72]: <Axes: xlabel='Date'>



ACF and PACF plot after seasonal differencing

In [77]:
```
pm25m_sdiff_acf = plot_acf(pm25m_sdiff.dropna(), lags = 50)
pm25m_sdiff_pacf = plot_pacf(pm25m_sdiff.dropna(), lags = 17)
```





From the ACF and PACF plot we should take values of p,q and P,Q as -

p = 1,2

q = 1,2

P = 0

Q = 0,1

Data Splitting

In [83]:
```
train_pm25m = pm25m[pm25m.index < '2019-01-01']
test_pm25m = pm25m[pm25m.index >= '2019-01-01']
```

Model Fitting

In [86]:
```
orders = [(1,1,1), (1,1,2), (2,1,1), (2,1,2)]
seasonal_orders = [(0,1,0,12), (0,1,1,12)]
```

In [88]:
```
for order in orders:
    for seasonal_order in seasonal_orders:
        model = SARIMAX(train_pm25m, order = order, seasonal_order = seasonal_order

        predicted = model.predict(start = '2019-01-01', end = test_pm25m.index[-1])
        actual = test_pm25m
        rmse = ((actual - predicted)**2).mean()**0.5
        mae = abs(actual - predicted).mean()
        print('{}{}: AIC = {}, BIC = {}, LLF = {}, MAE = {}, RMSE = {}'.format(orde
```

```
(1, 1, 1)(0, 1, 0, 12): AIC = 350.71441011921627, BIC = 355.38045430368453, LLF = -1
72.35720505960813, MAE = 13.783739308949135, RMSE = 16.83077378271882
(1, 1, 1)(0, 1, 1, 12): AIC = 340.6853581957597, BIC = 346.90675044171735, LLF = -16
6.34267909787985, MAE = 18.53866191616136, RMSE = 23.266918136208744
(1, 1, 2)(0, 1, 0, 12): AIC = 350.91267048183283, BIC = 357.1340627277905, LLF = -17
1.45633524091642, MAE = 14.557995963382115, RMSE = 19.955372913048063
(1, 1, 2)(0, 1, 1, 12): AIC = 342.5982920790851, BIC = 350.3750323865322, LLF = -16
6.29914603954256, MAE = 18.646966966577708, RMSE = 23.73927651224552
(2, 1, 1)(0, 1, 0, 12): AIC = 351.8736596983498, BIC = 358.09505194430744, LLF = -17
1.9368298491749, MAE = 14.680252170184067, RMSE = 19.12832515133312
(2, 1, 1)(0, 1, 1, 12): AIC = 342.33168907874636, BIC = 350.10842938619345, LLF = -1
66.16584453937318, MAE = 18.22831850608672, RMSE = 23.33102106996987
(2, 1, 2)(0, 1, 0, 12): AIC = 352.4793623427092, BIC = 360.25610265015627, LLF = -17
1.2396811713546, MAE = 16.482797881245638, RMSE = 21.668772340471925
(2, 1, 2)(0, 1, 1, 12): AIC = 344.1209674973802, BIC = 353.4530558663167, LLF = -16
6.0604837486901, MAE = 18.2116176921267, RMSE = 23.5236967590518
```

After checking all the metrices for fitiing and prediction accuracy we observe that the AIC/BIC doesn't change to much with varying orders same with the Log Likelihood but the MAE and RMSE are lower with the most simple order. So we choose the simpler model that is SARIMAX(1,1,1)(0,1,0,12)

In [91]:
```
pm25m_model = SARIMAX(train_pm25m, order = (1,1,1), seasonal_order = (0,1,0,12)).fi
print(pm25m_model.summary())
```

```
                                      SARIMAX Results
==========================================================================================
=======
Dep. Variable:                                  PM2.5   No. Observations:
48
Model:             SARIMAX(1, 1, 1)x(0, 1, [], 12)   Log Likelihood                     -
172.357
Date:                            Tue, 13 May 2025   AIC
350.714
Time:                                    00:21:19   BIC
355.380
Sample:                                01-01-2015   HQIC
352.325
                                     - 12-01-2018
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.0458      0.247      0.185      0.853      -0.439       0.531
ma.L1         -0.8296      0.173     -4.802      0.000      -1.168      -0.491
sigma2      1074.6356    294.051      3.655      0.000     498.306    1650.966
===================================================================================
Ljung-Box (L1) (Q):                   0.05   Jarque-Bera (JB):                 0.27
Prob(Q):                              0.83   Prob(JB):                         0.87
Heteroskedasticity (H):               0.39   Skew:                             0.03
Prob(H) (two-sided):                  0.12   Kurtosis:                         3.43
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-ste
p).
```

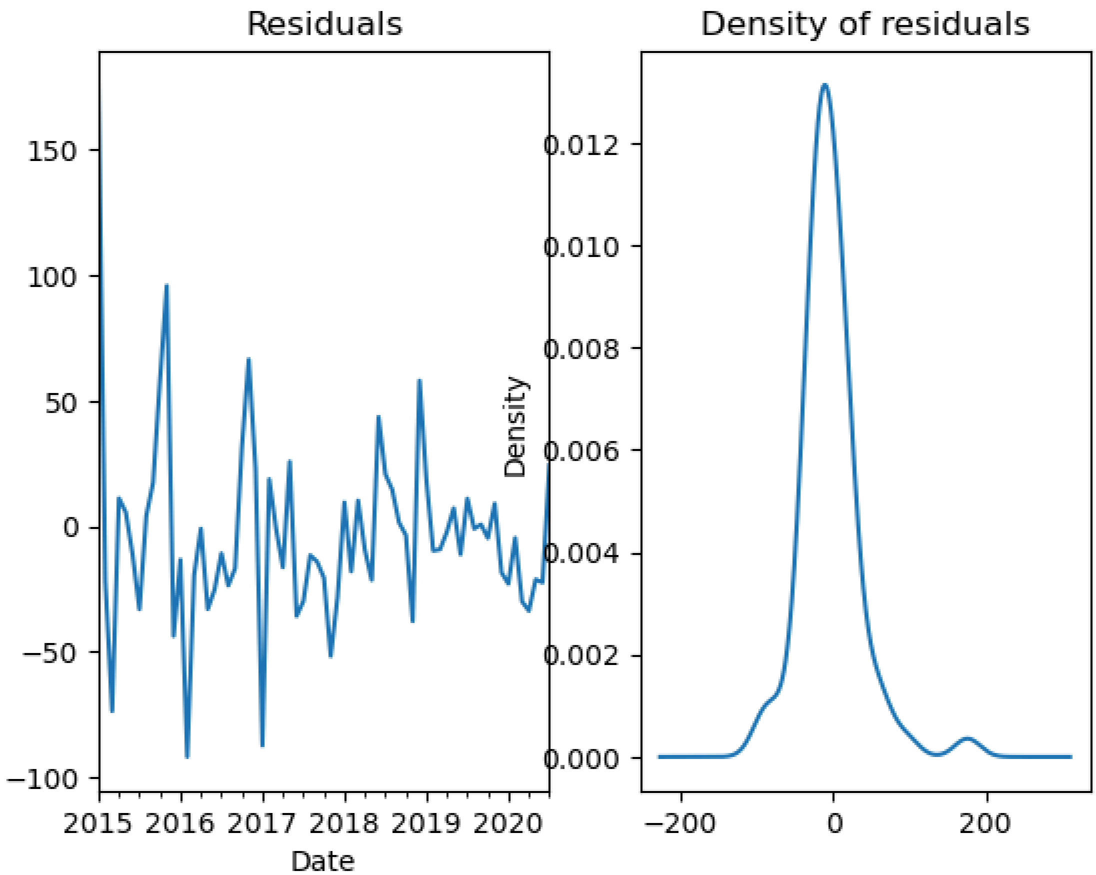
Residuals

In [94]:

```python
pm25m_res = pm25m - pm25m_model.predict(start = pm25m.index[0], end = pm25m.index[-
fig, ax = plt.subplots(1,2)
pm25m_res.plot(title = 'Residuals', ax = ax[0])
pm25m_res.plot(title = 'Density of residuals', kind = 'kde', ax = ax[1])
```

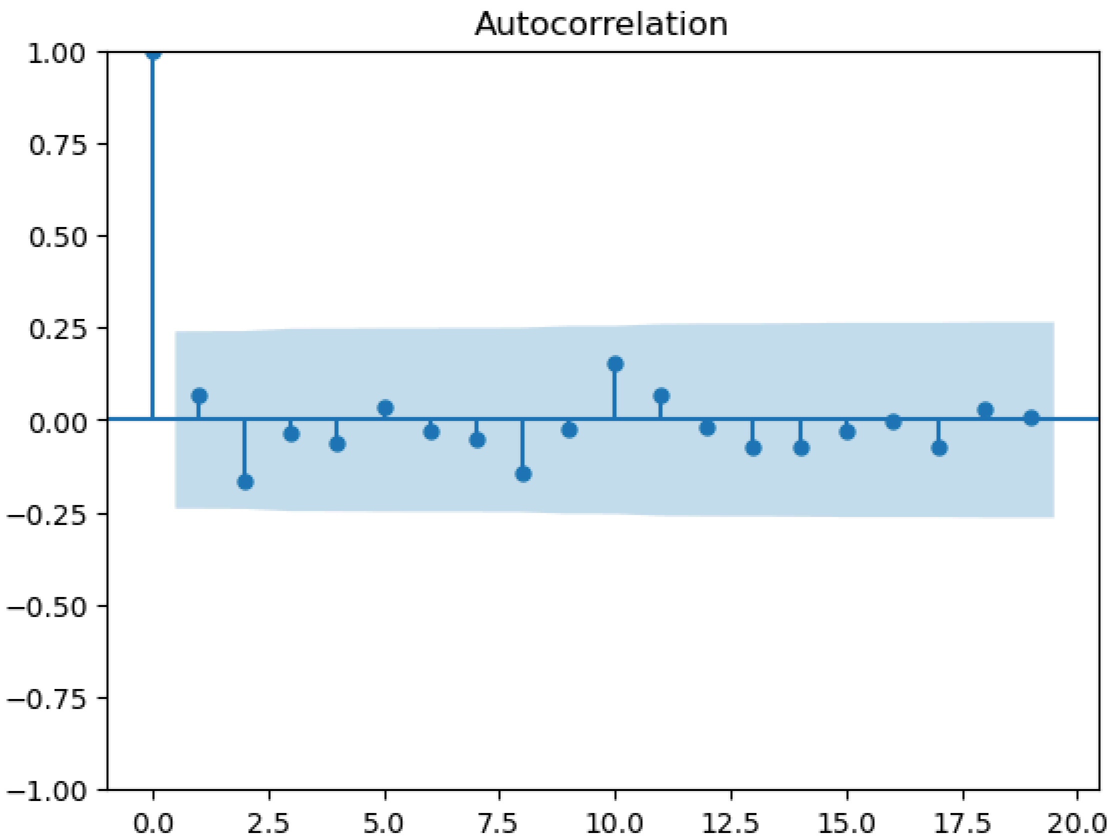
Out[94]:

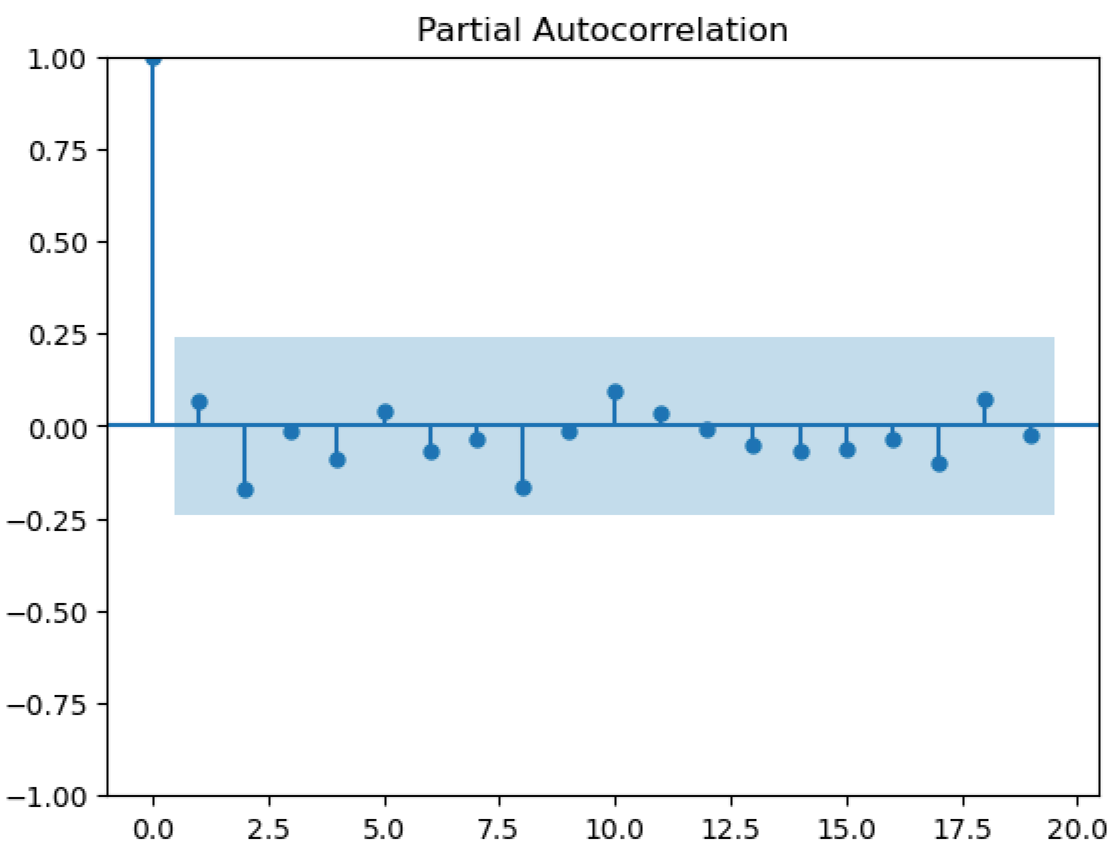
```
<Axes: title={'center': 'Density of residuals'}, ylabel='Density'>
```



ACF and PACF plot of residuals

In [97]:
```
pm25m_res_acf = plot_acf(pm25m_res)
pm25m_res_pacf = plot_pacf(pm25m_res)
```





The density plot of the residuals shows that the residuals are random and normaly distributed around 0, also from the ACF and PACF plot it is clear that the residuals are completely uncorrelated indicating that the assumptions of the model are not violated and our model has captured most of the components and patterns from the data.

Forecasting and Comparing

In [101…

```
plt.plot(pm25m, label = 'Actual')
plt.plot(pm25m_model.forecast(steps = 30), label = 'Predicted')
plt.title('PM 2.5')
plt.legend()
```

Out[101…

```
<matplotlib.legend.Legend at 0x1d0870a99a0>
```