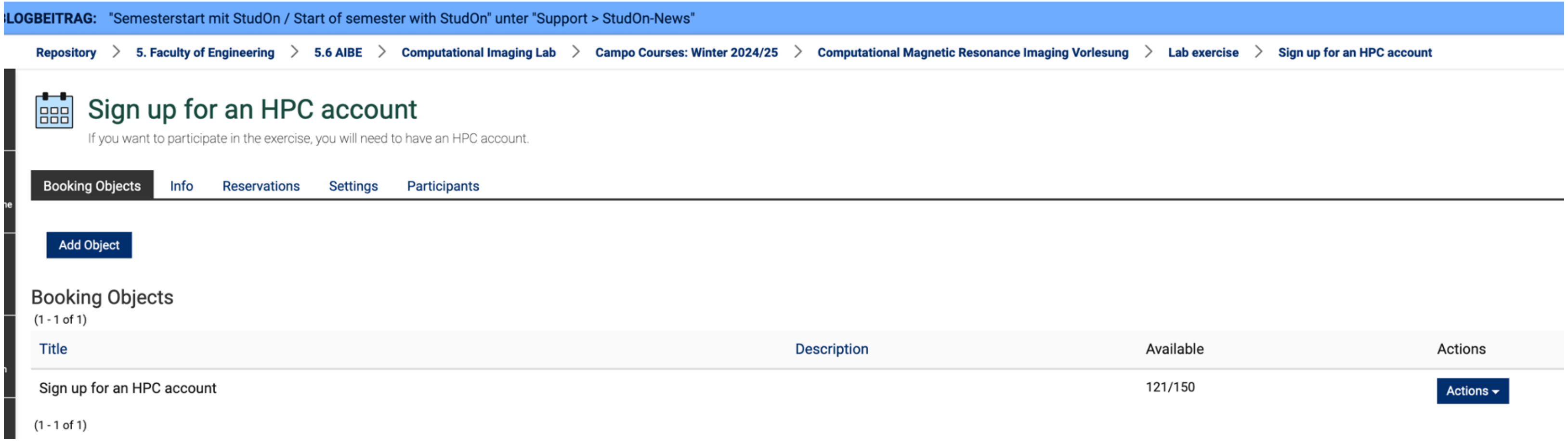
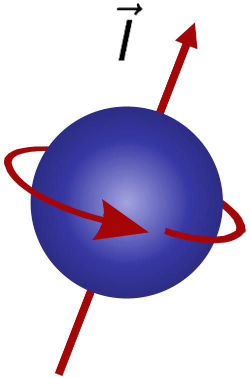
Lecture 1: Fundamentals of Magnetic Resonance Imaging

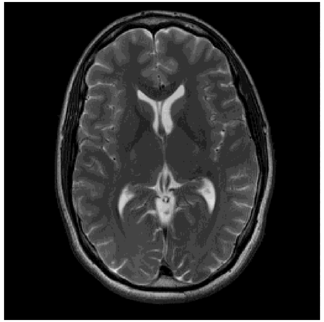
Friedrich-Alexander-Universität
Computational Imaging Lab

Administrative: Exercise registration poll



Outline

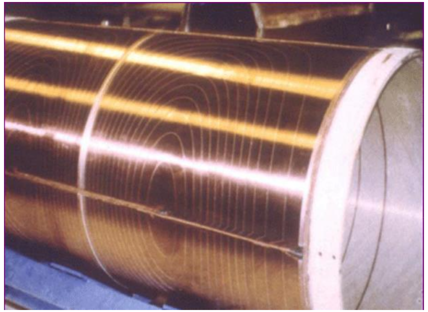




B_0

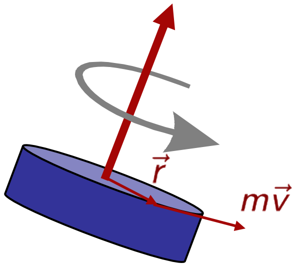
B_1

G

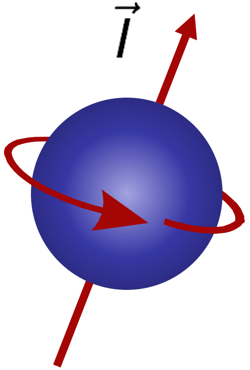


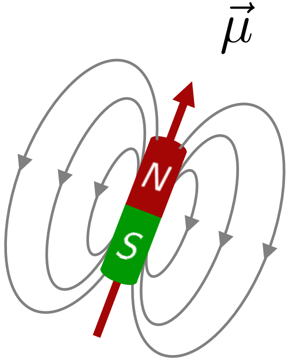
Nuclear spin

$$\vec{L} = \vec{r} \times (m\vec{v})$$



Angular momentum of nucleus





Magnetic dipole moment

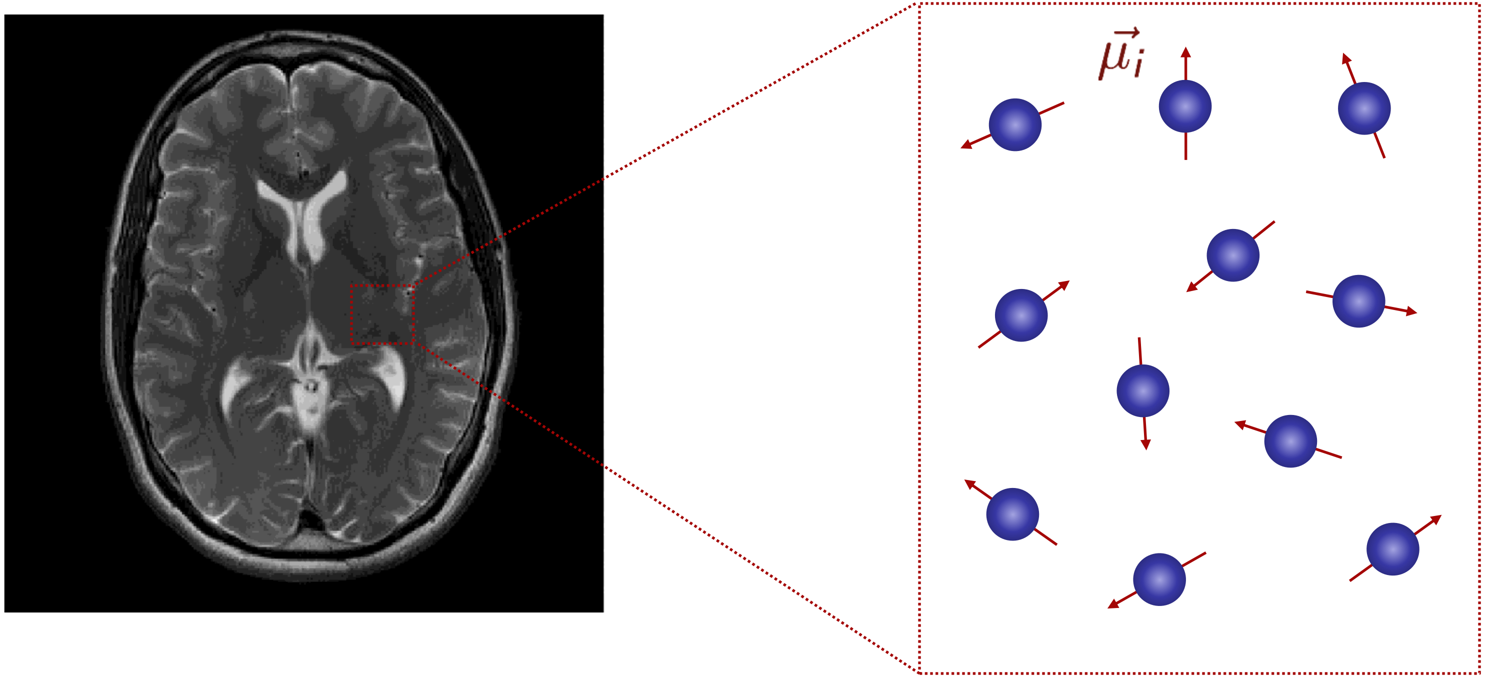
$$\vec{\mu} = \gamma \vec{I}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

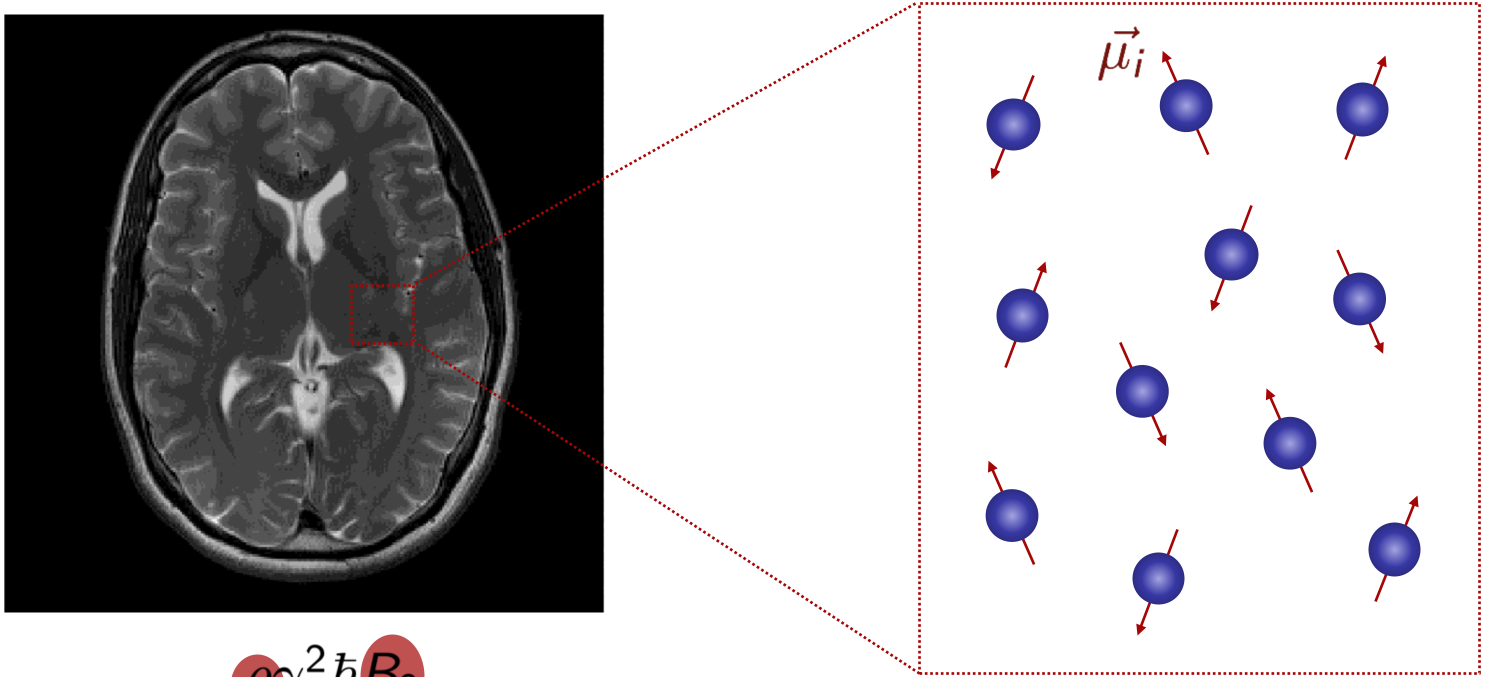
Gyromagnetic ratio

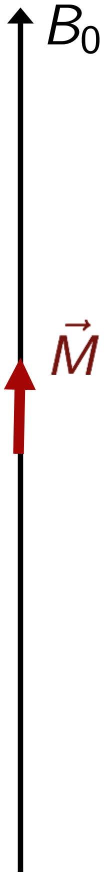
Element	(MHz/T)
^{1}H	**42.58**
^{3}He	-32.43
^{23}Na	11.26
^{31}P	17.24

Thermal motion: Random orientation



Interaction with B_0: Magnetization





$$M = \frac{\rho\gamma^2\hbar B_0}{4kT}$$

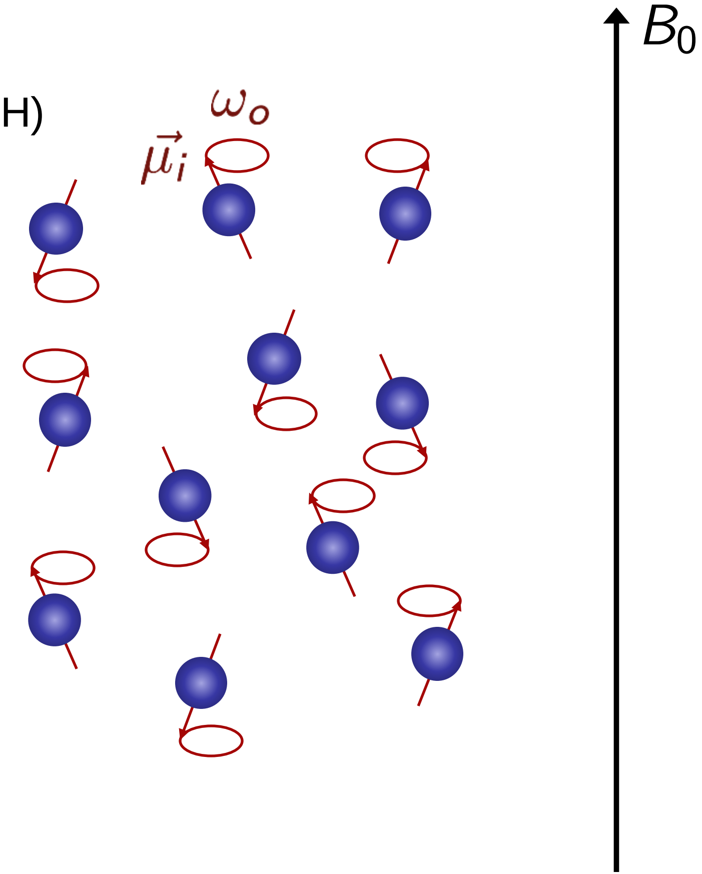
$$\vec{M} = \sum \mu_i$$

Interaction with B_0: Precession

$$\omega_0 = \gamma B_0$$ Larmor frequency ([1]H)

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

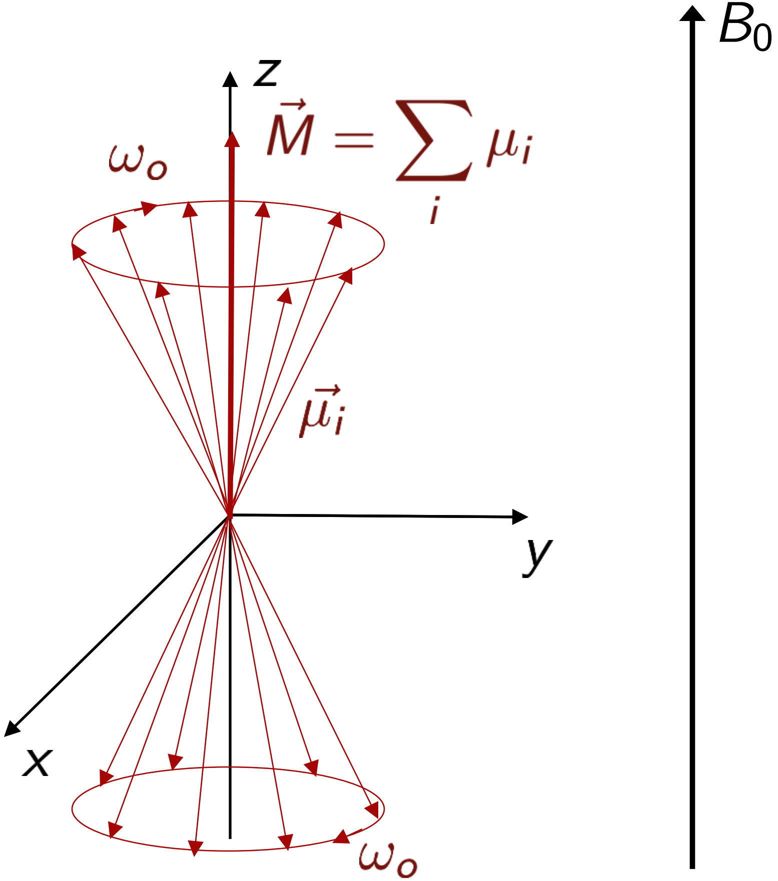
$$\frac{\gamma}{2\pi} = 42.6(MHz/T)$$



Angular momentum of top in gravitation field

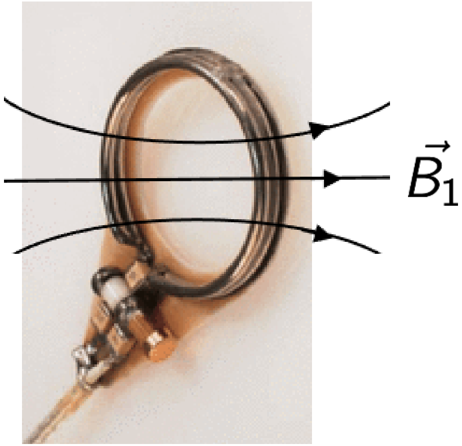
Nolan 2010

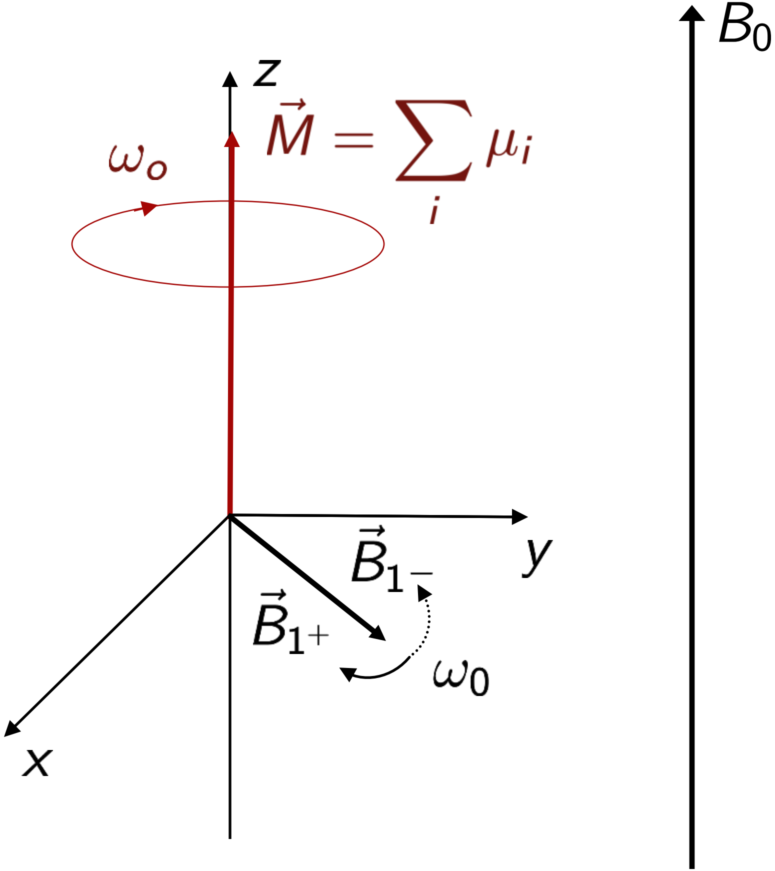
Interaction with B_0: Precession



Interaction with radiofrequency field B1

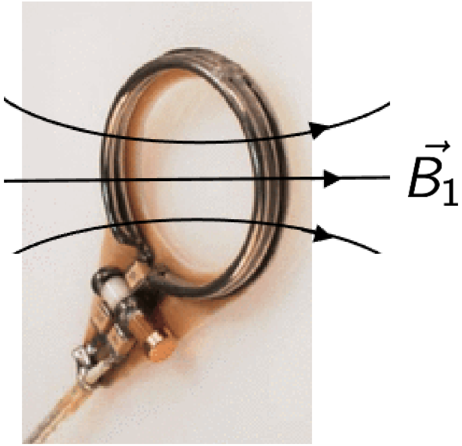
$$f_0 = \frac{\gamma}{2\pi} B_0$$



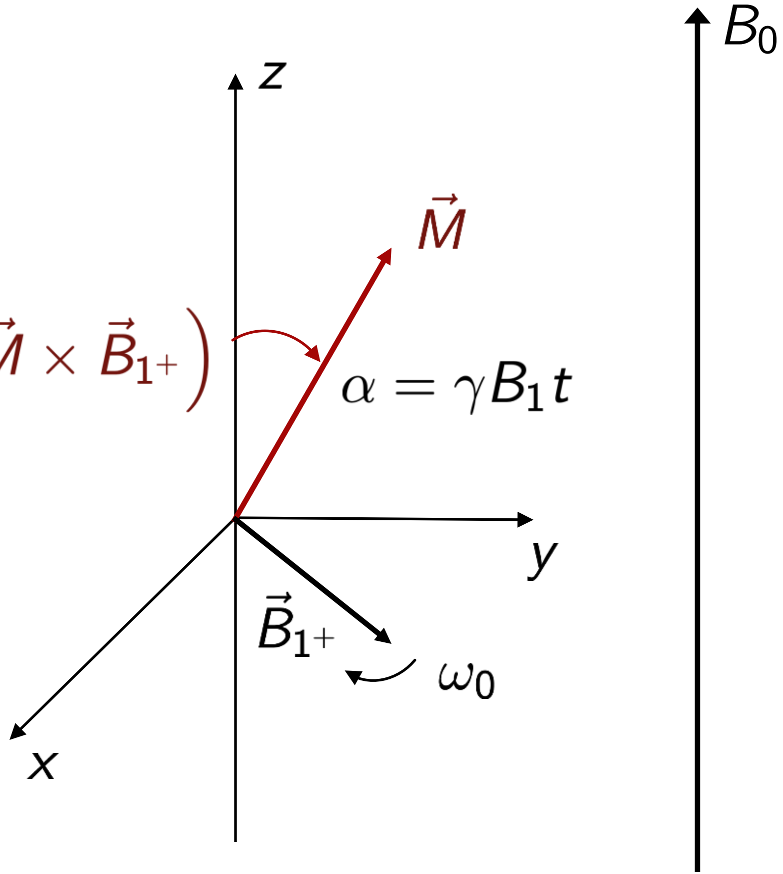


Interaction with radiofrequency field B1

$$f_0 = \frac{\gamma}{2\pi} B_0$$

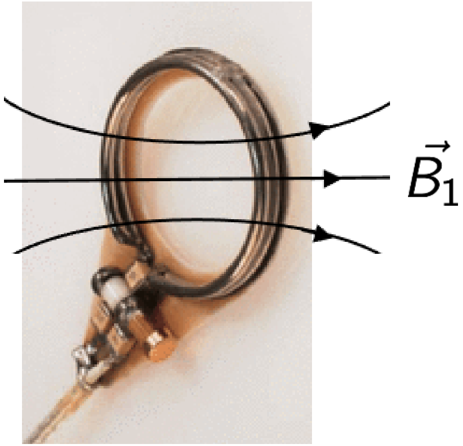


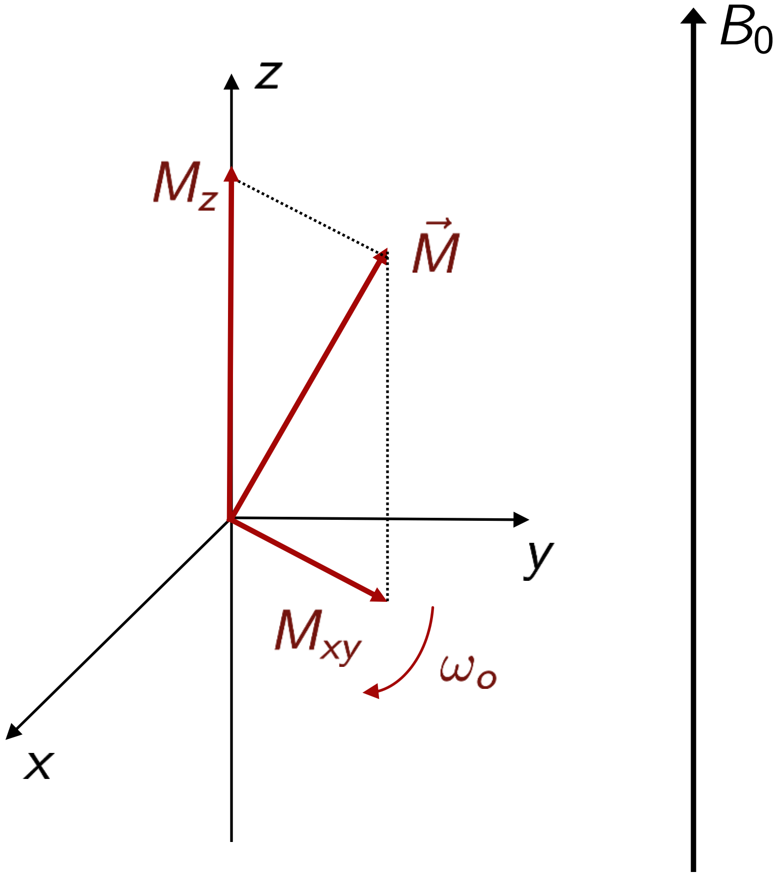
$$\frac{d\vec{M}}{dt} = \gamma \left(\vec{M} \times \vec{B}_{1^+} \right)$$



Interaction with radiofrequency field B1

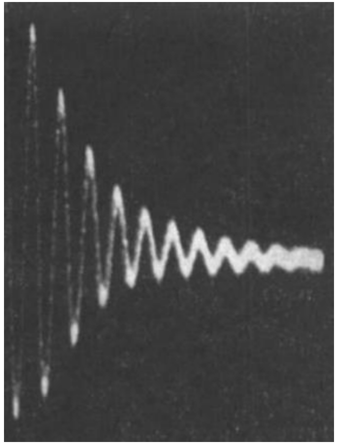
$$f_0 = \frac{\gamma}{2\pi} B_0$$



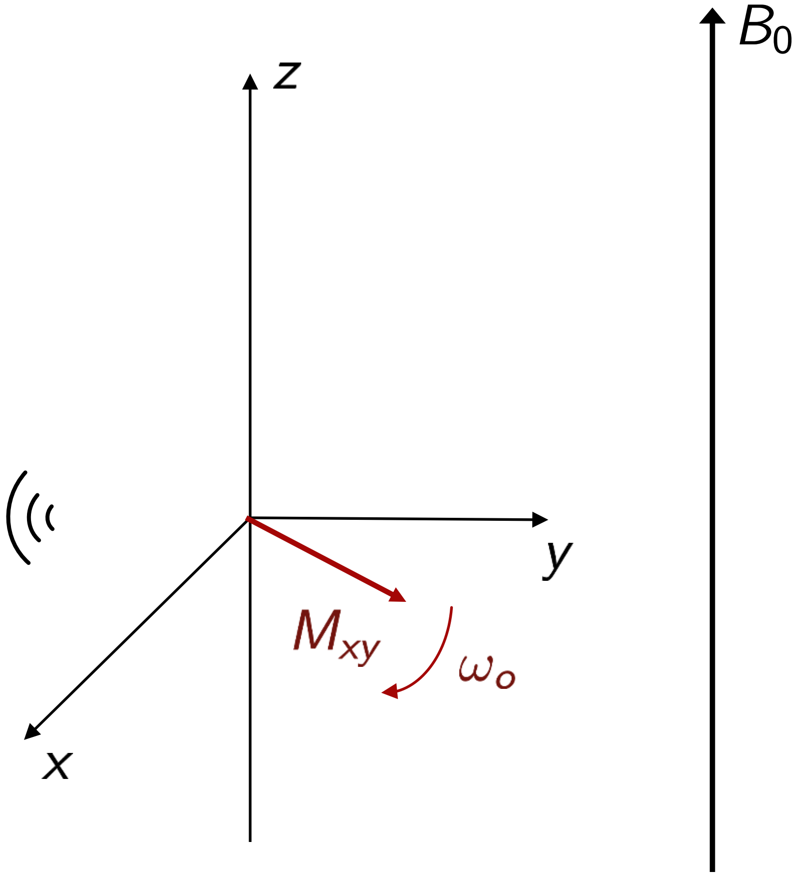


Signal reception

$$u_{ind} = -\frac{d\Phi}{dt}$$

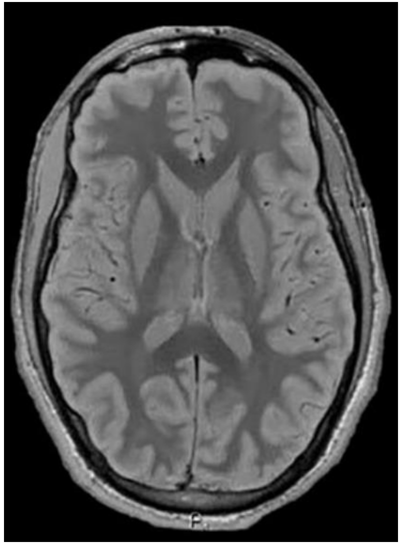


Hahn 1950

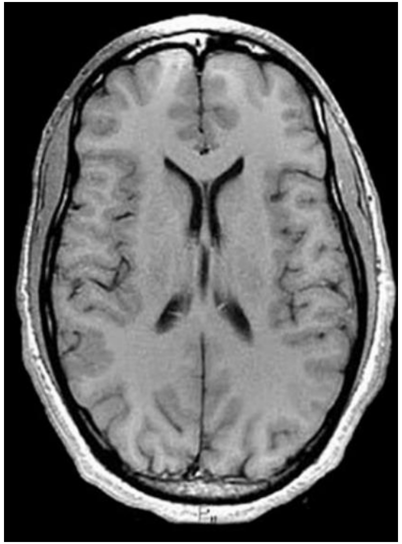


Contrast

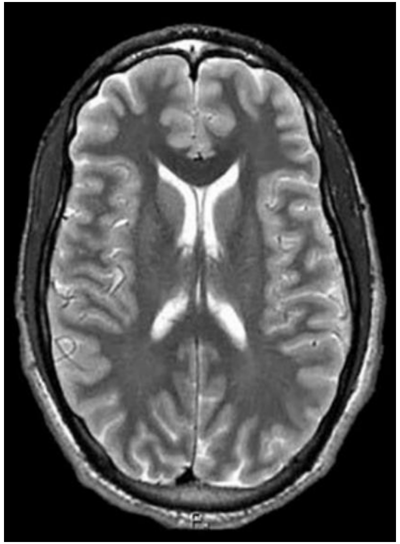
PD_w



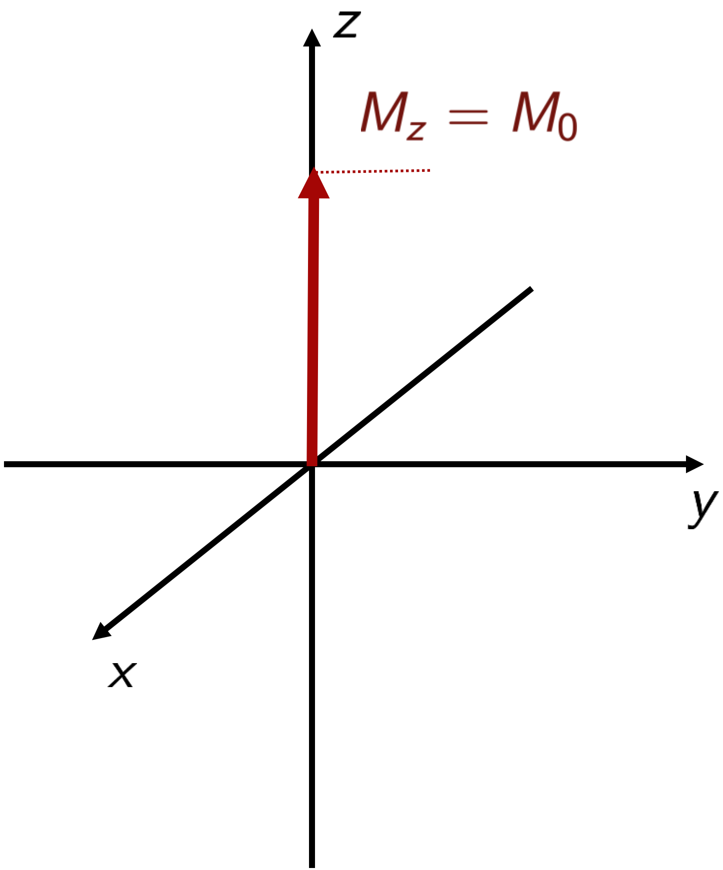
$T1_w$



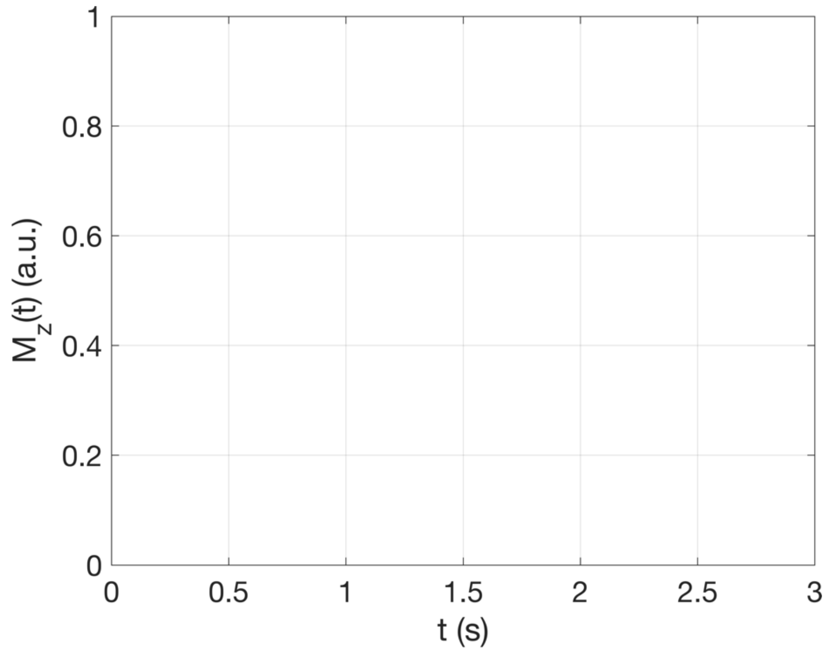
$T2_w$

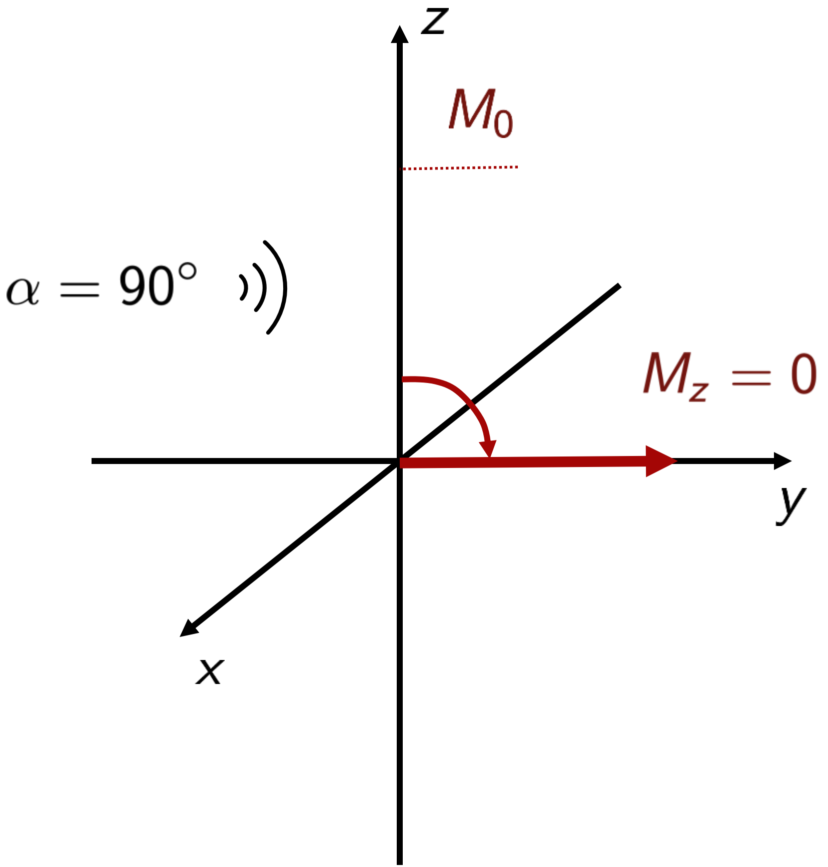


Longitudinal Relaxation

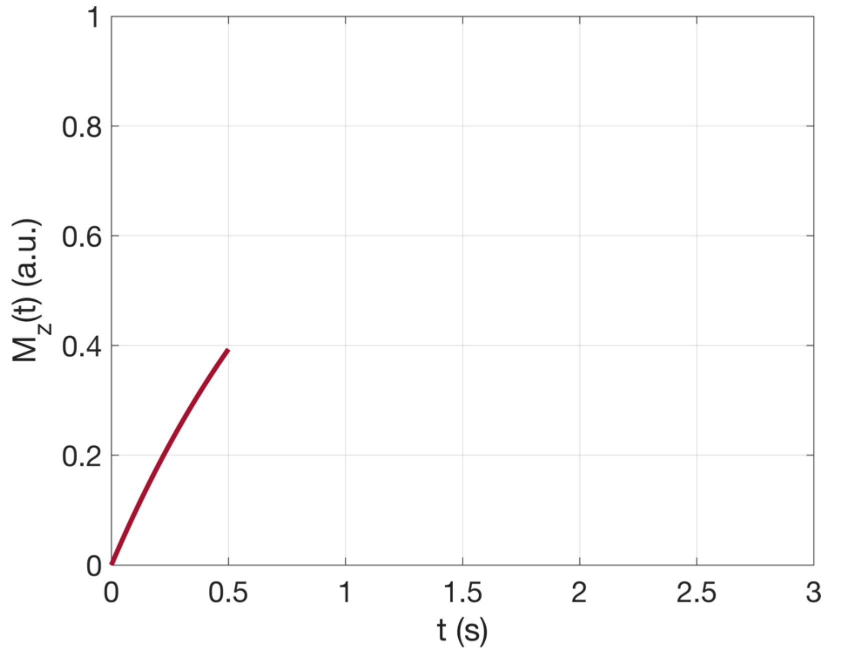


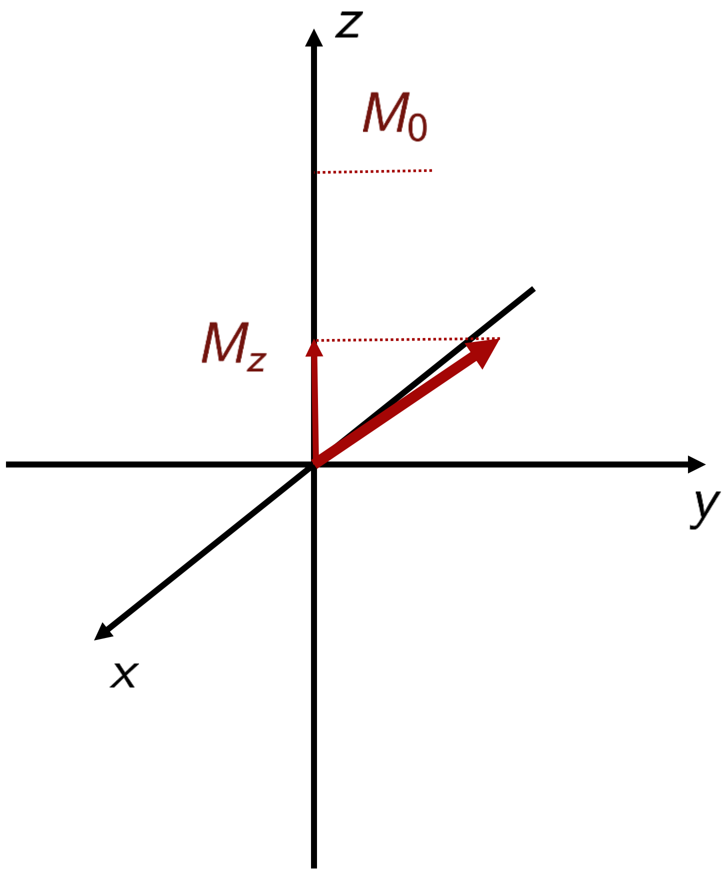
Longitudinal Relaxation



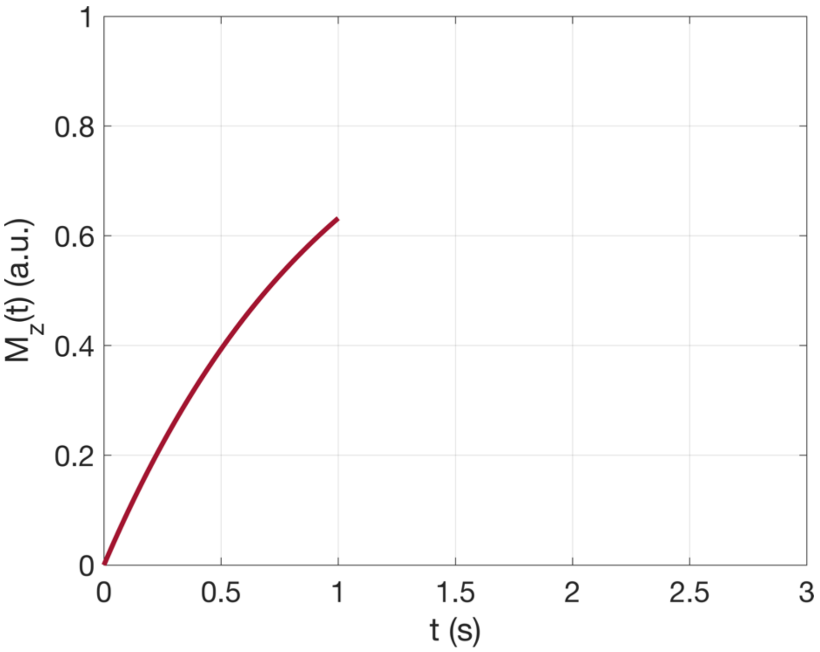


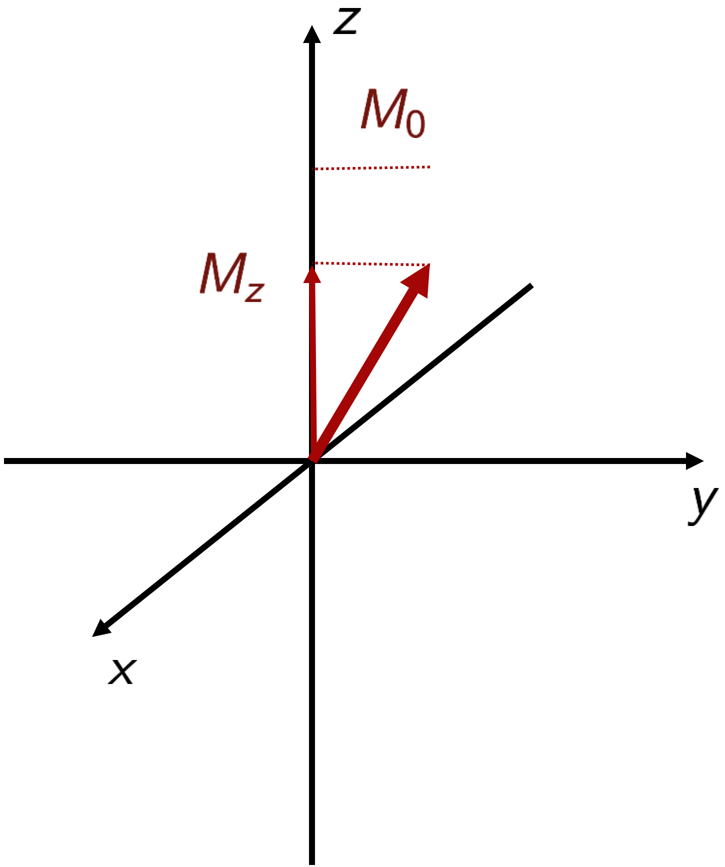
Longitudinal Relaxation





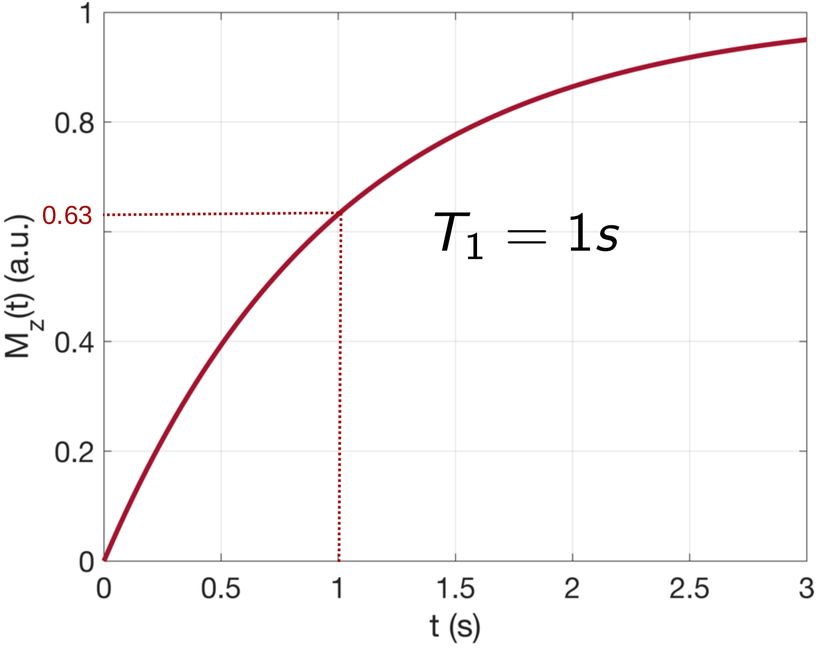
Longitudinal Relaxation

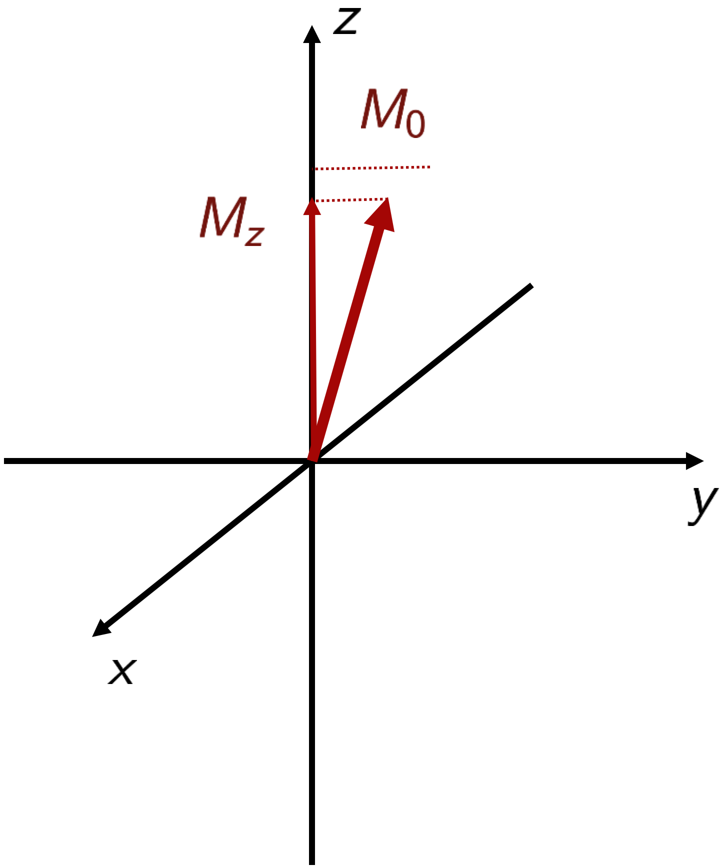




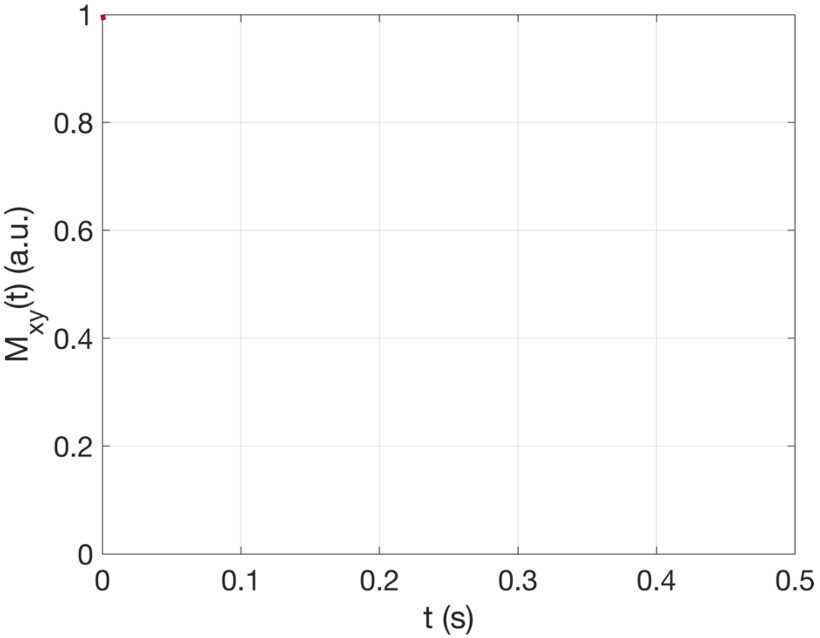
Longitudinal Relaxation

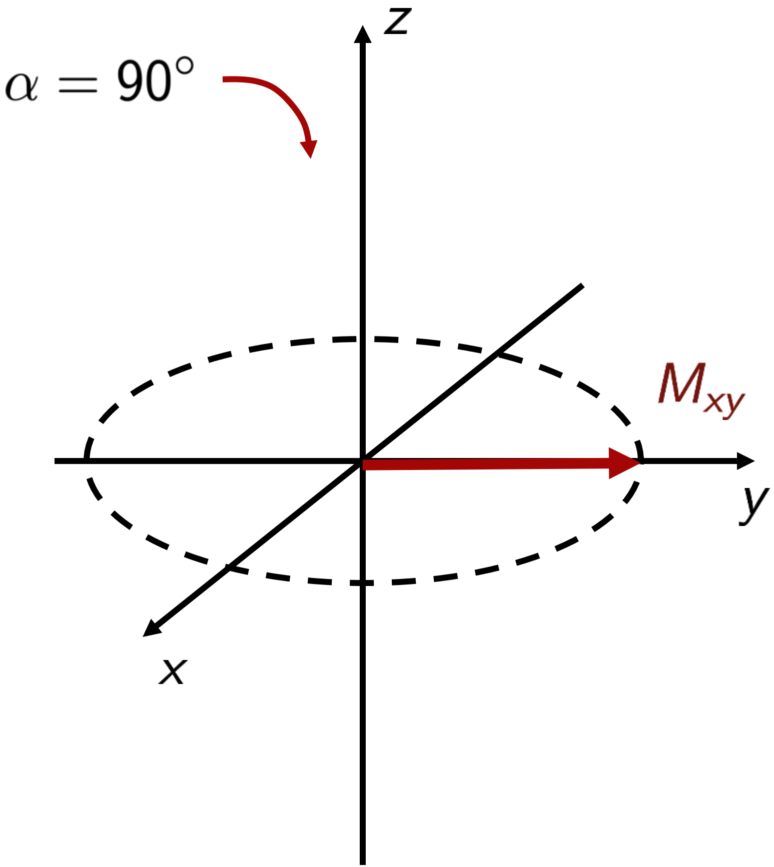
$$M_z = M_0 \left(1 - e^{-t/T_1}\right)$$



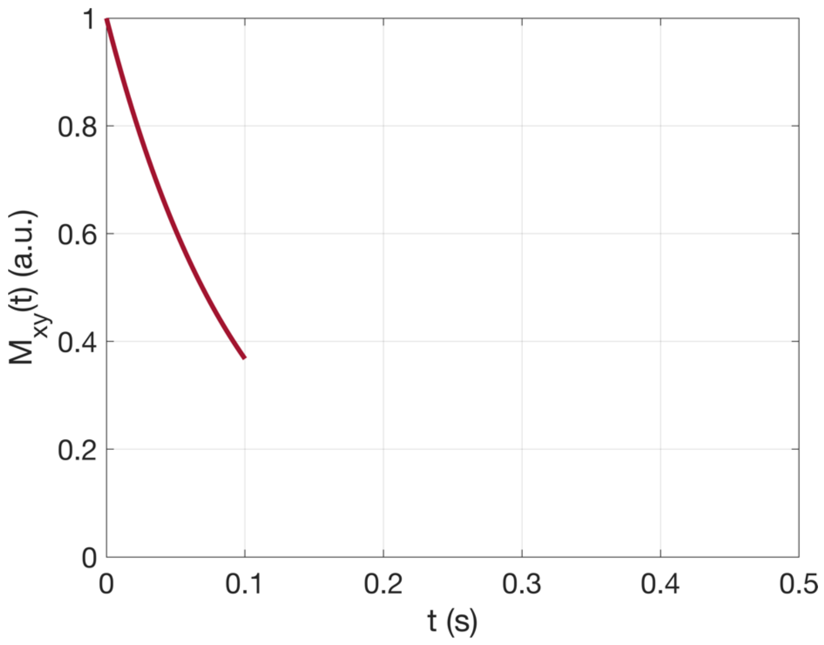


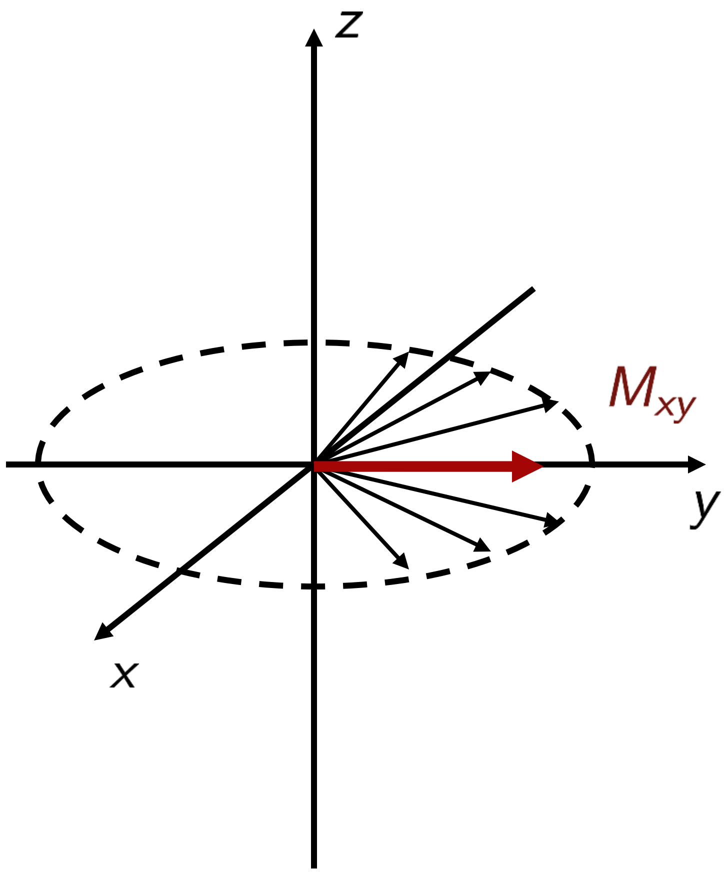
Transversal Relaxation



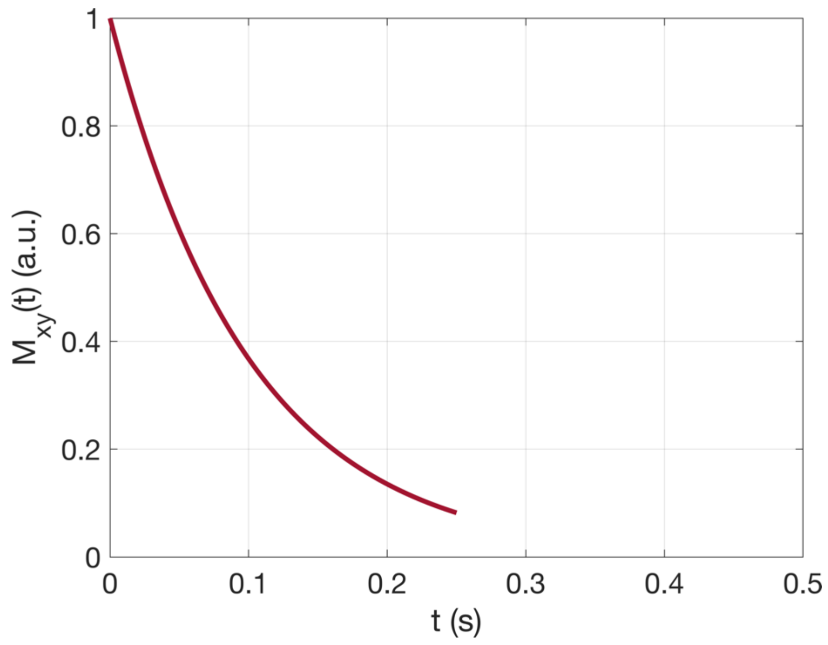


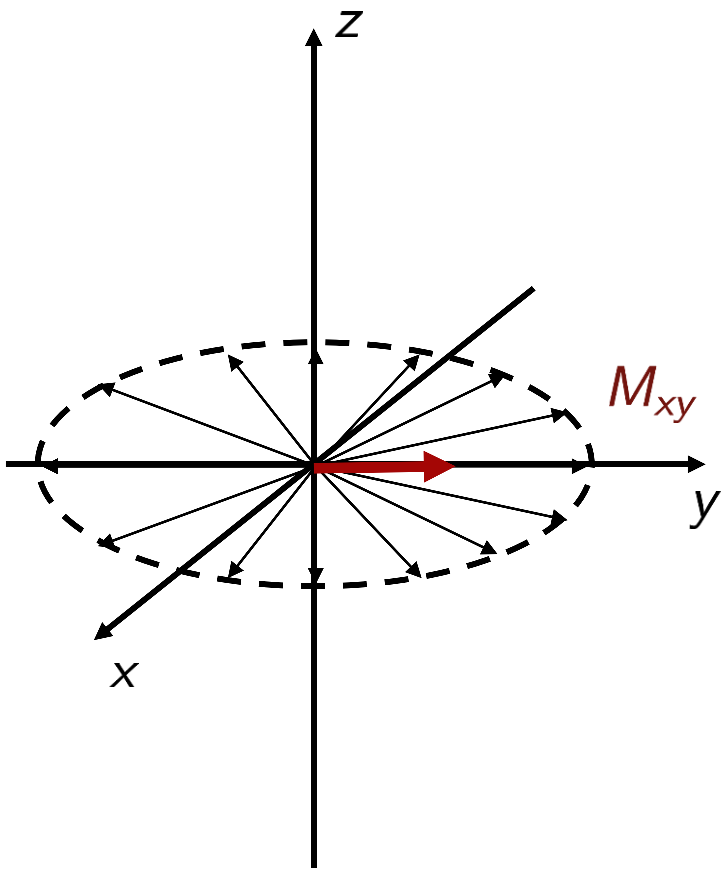
Transversal Relaxation





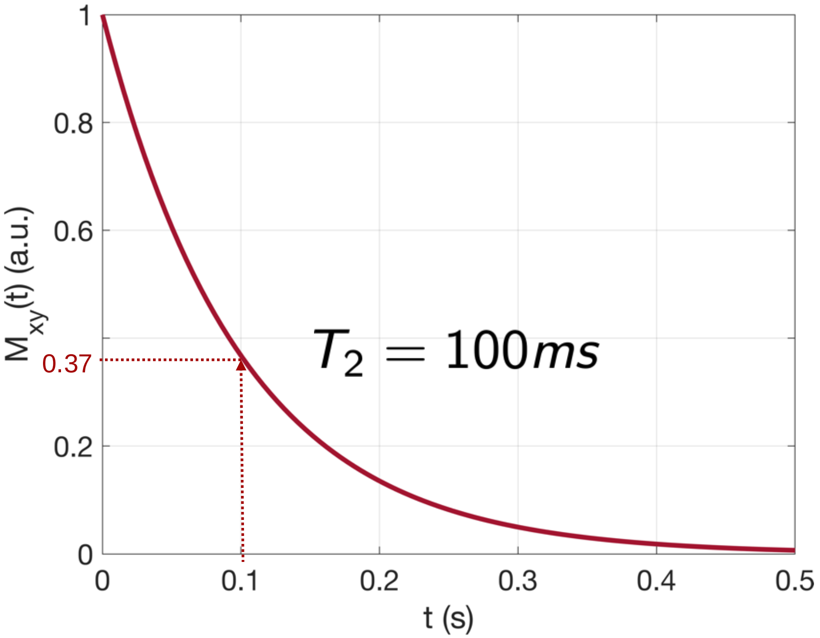
Transversal Relaxation

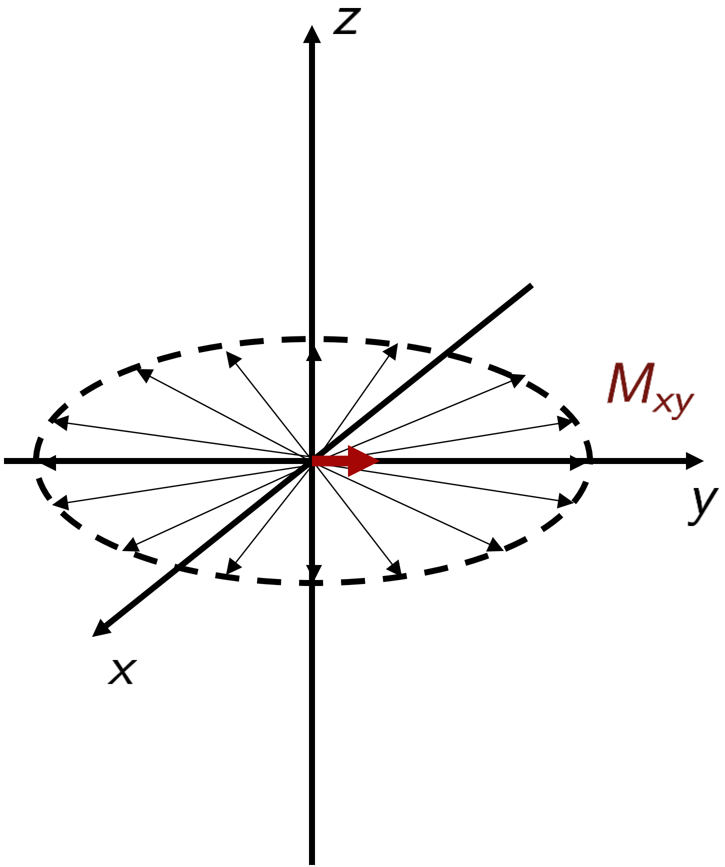




Transversal Relaxation

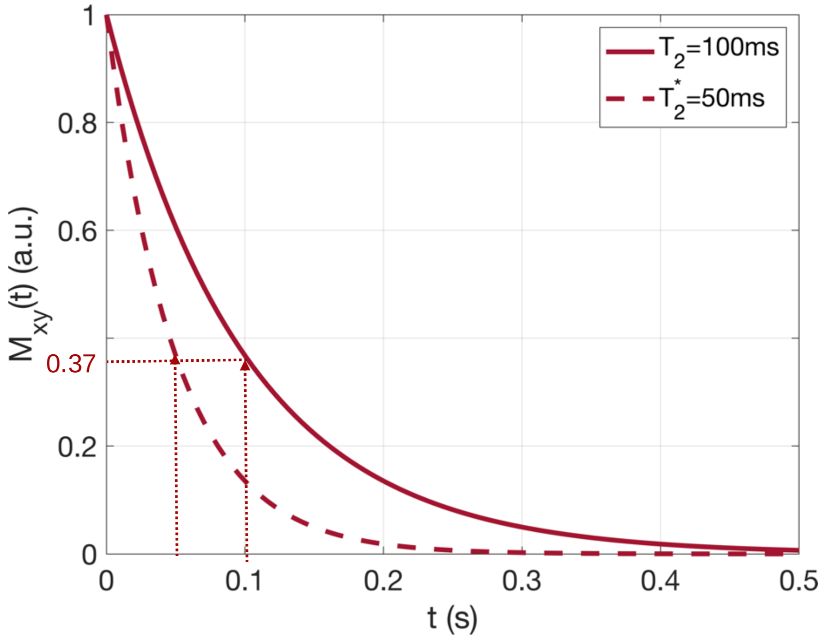
$$M_{xy} = M_0 e^{-t/T_2}$$





Transversal Relaxation

$$M_{xy} = M_0 e^{-t/T_2^*}$$



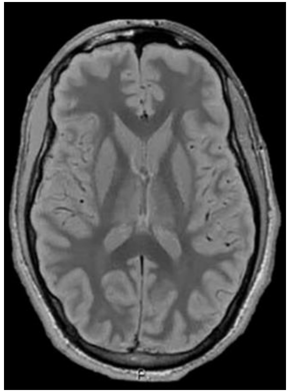
$$\frac{1}{T_2^*} = \frac{1}{T_2'} + \frac{1}{T_2}$$

T_2 Tissue property, irreversible

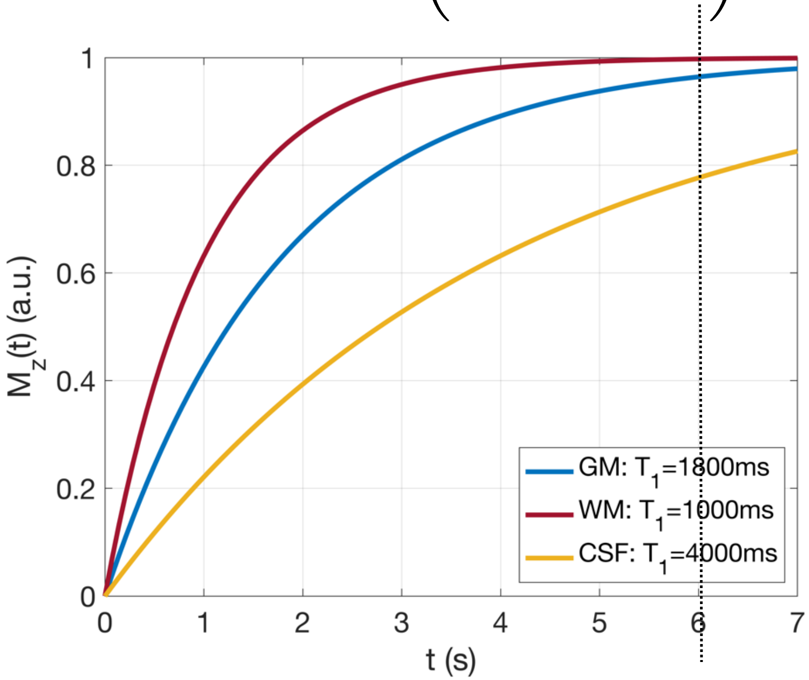
T_2' Field inhomogeneity (magnet, susceptibility), reversible

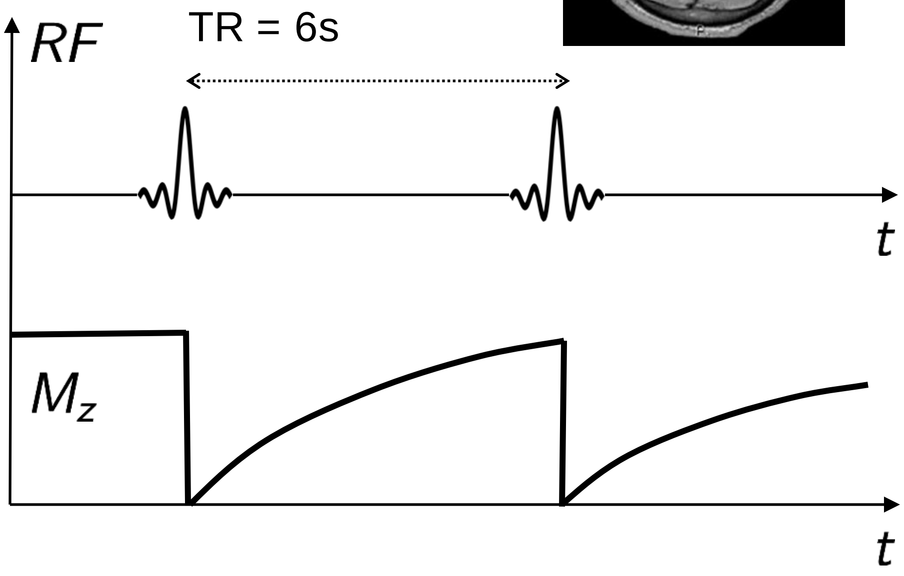
$T_2^* < T_2$

PD contrast



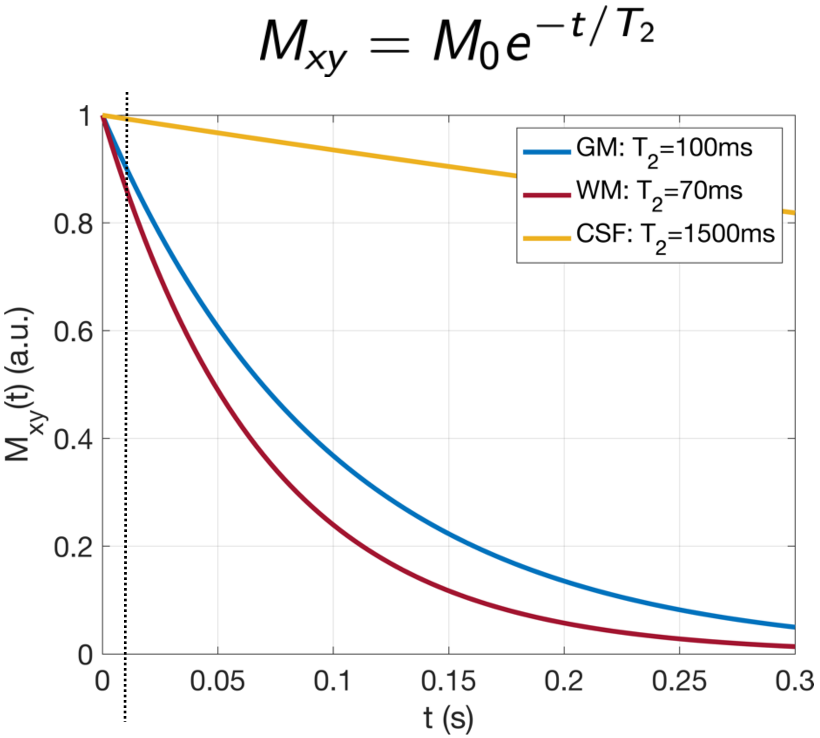
$$M_z = M_0\left(1 - e^{-t/T_1}\right)$$

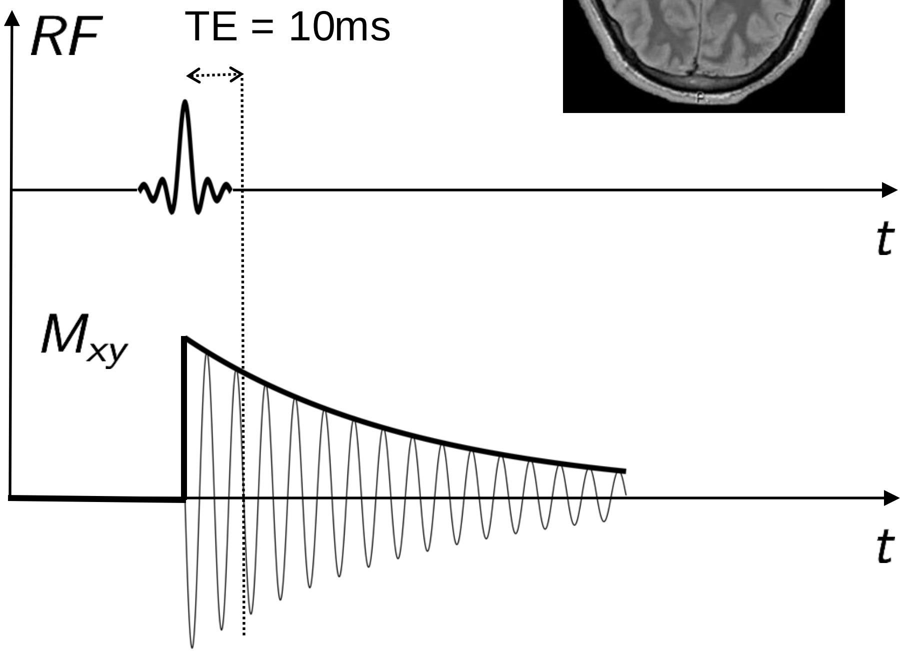


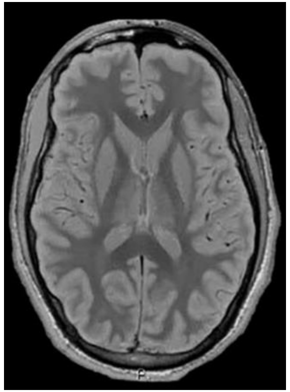


Stanisz 2005

PD contrast

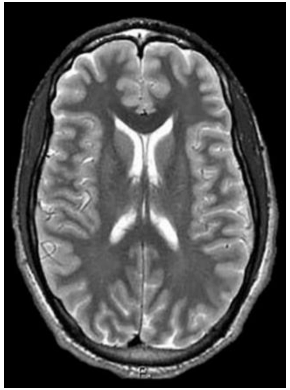




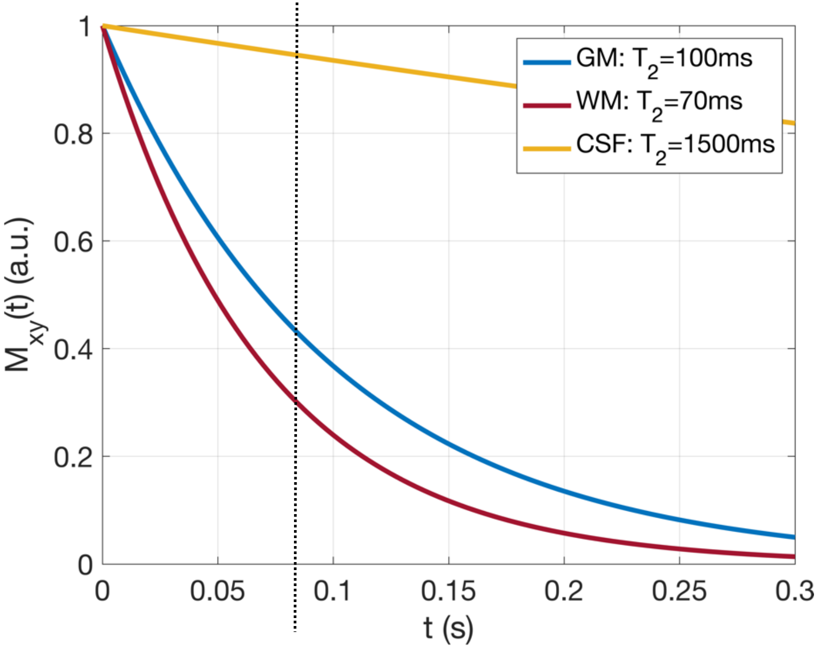


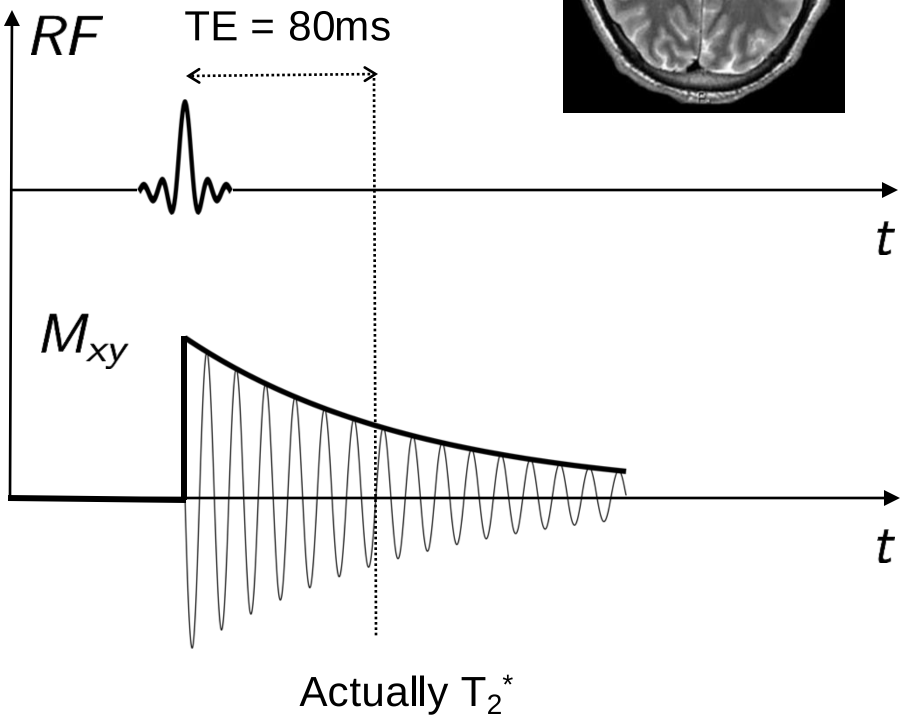
Stanisz 2005

T_2 contrast



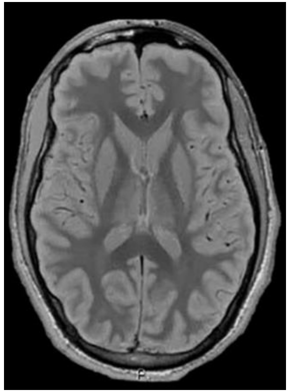
$$M_{xy} = M_0 e^{-t/T_2}$$



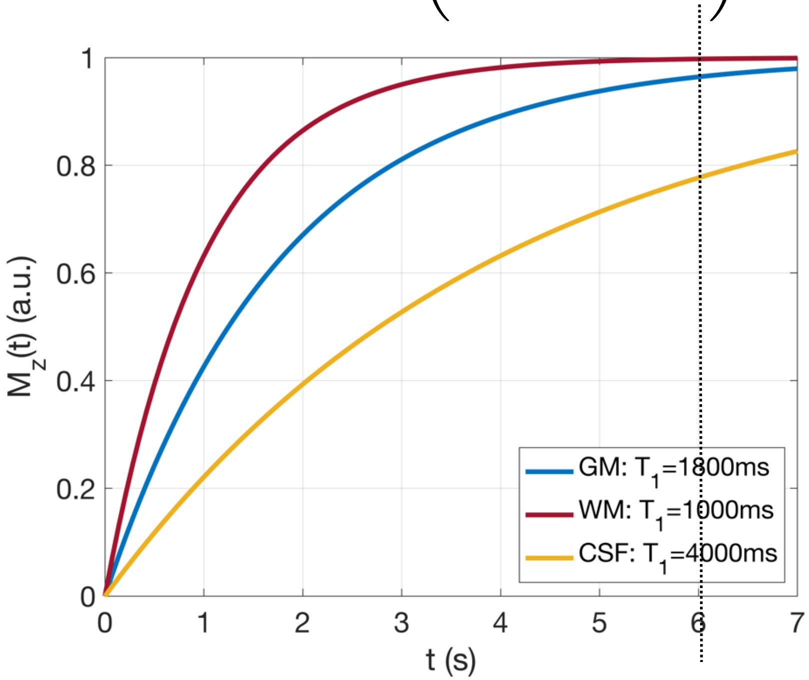


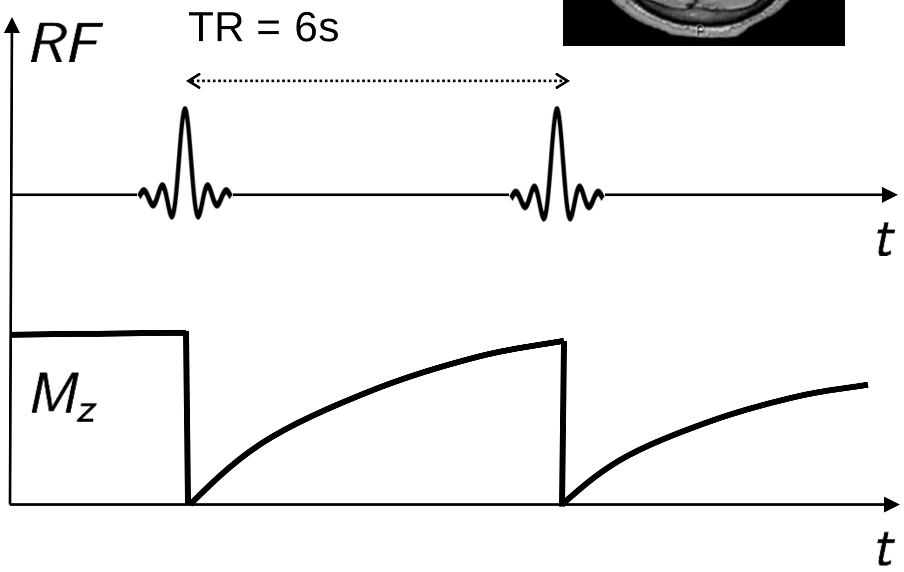
Stanisz 2005

PD contrast



$$M_z = M_0 \left(1 - e^{-t/T_1}\right)$$

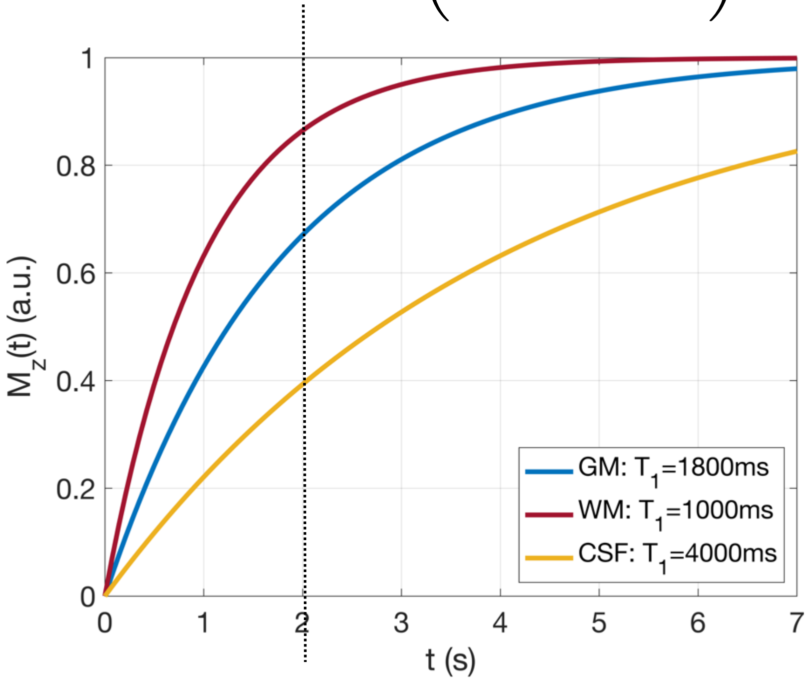


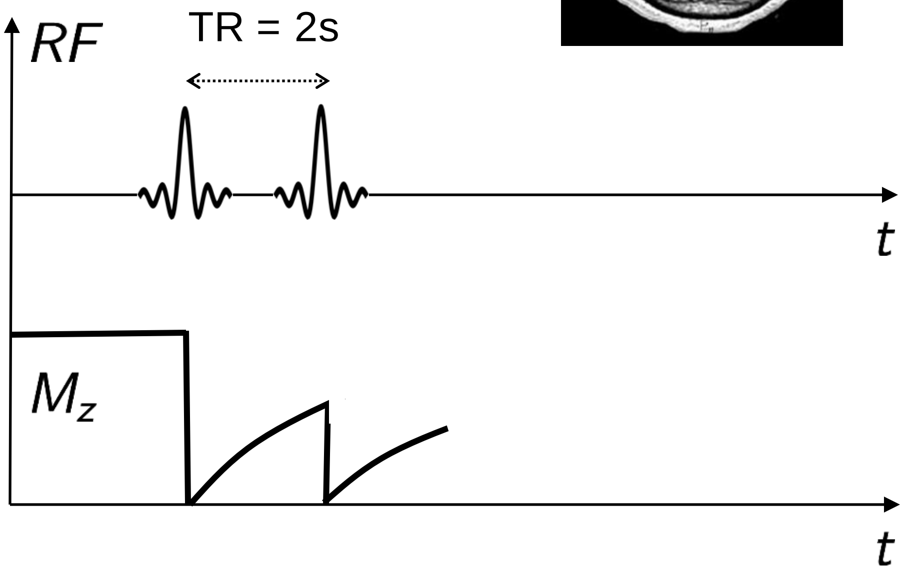


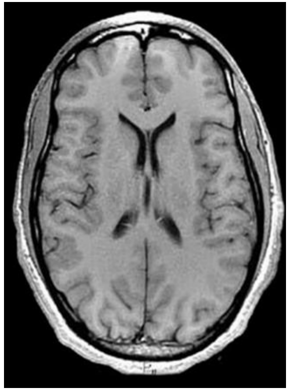
Stanisz 2005

T_1 contrast

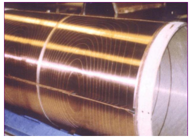
$$M_z = M_0\left(1 - e^{-t/T_1}\right)$$



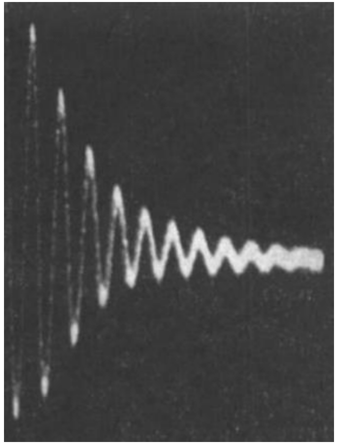




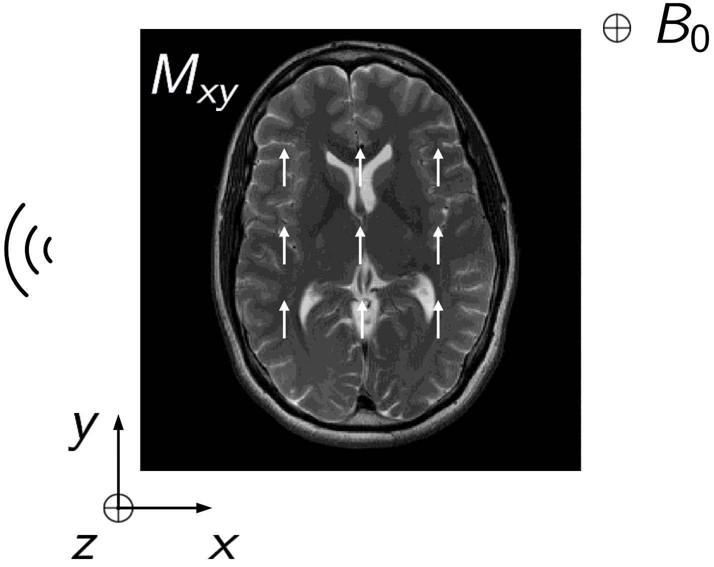
Stanisz 2005

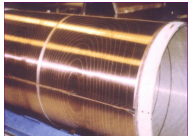


How do we get an image?

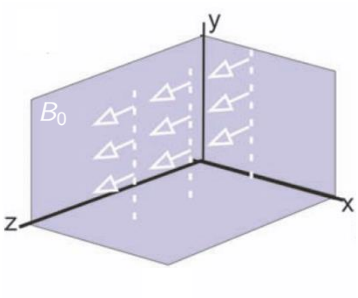


Hahn 1950

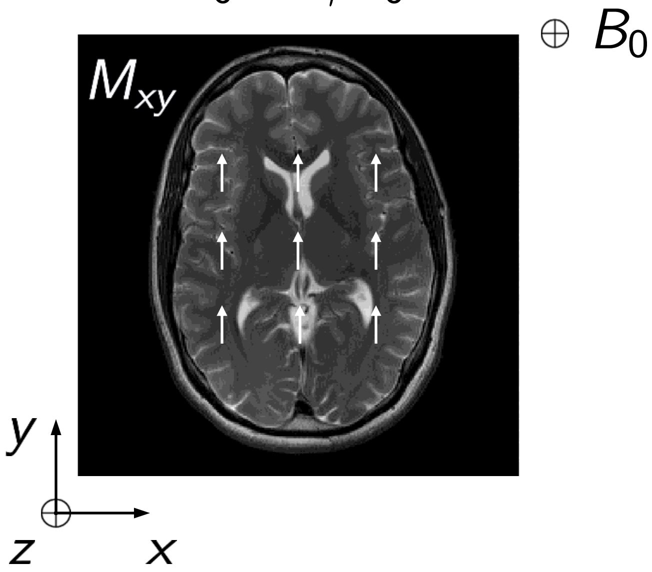




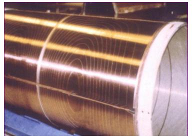
Interaction with gradient fields G



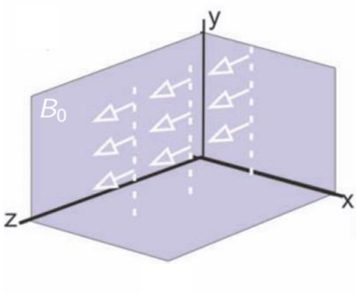
$$\omega_0 = \gamma B_0$$

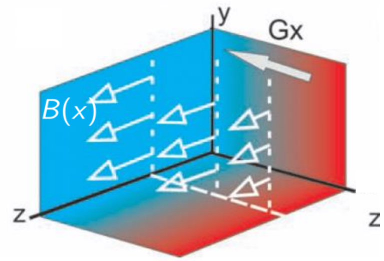


Plewes 2012



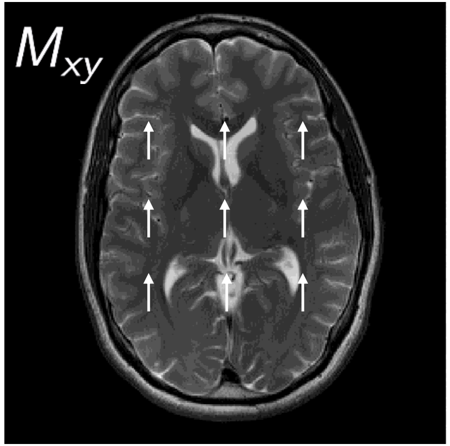
Interaction with gradient fields G

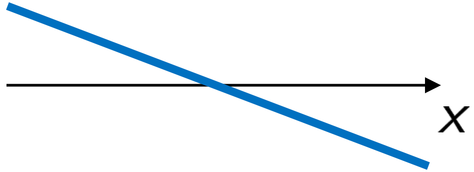




$$\omega(x) = \omega_0 + \gamma G_x x$$

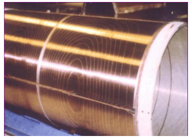
$\oplus$ B_0



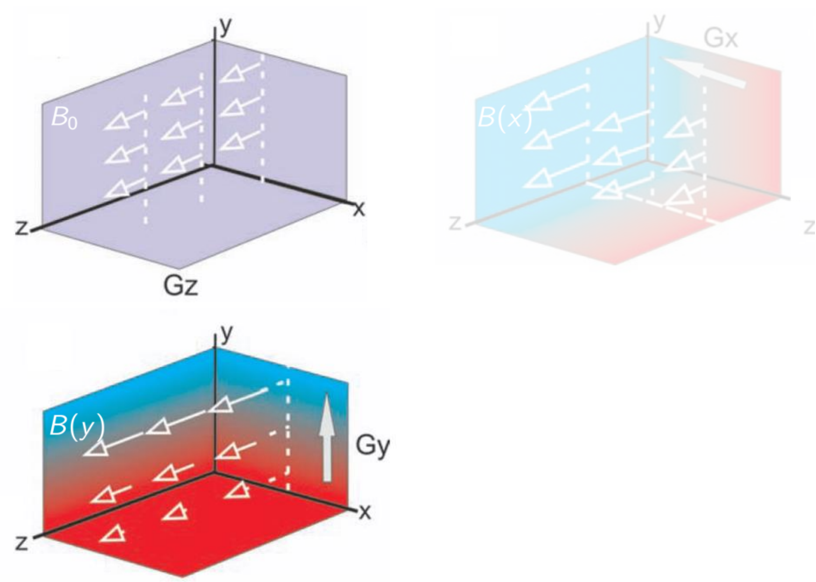


$$B(x) = B_0 + G_x x$$

Plewes 2012

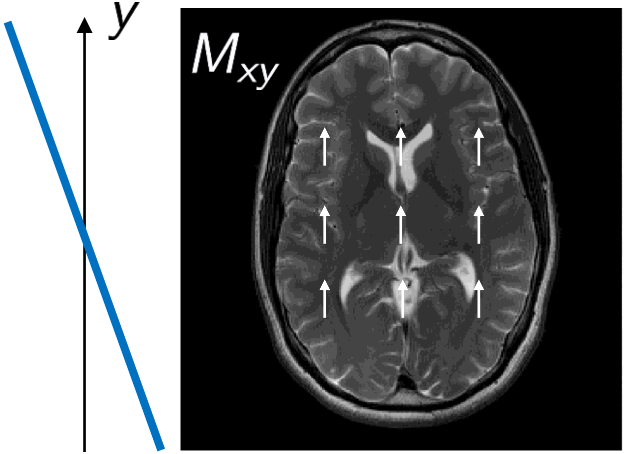


Interaction with gradient fields G



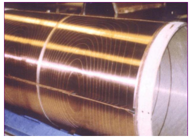
$$\omega(y) = \omega_0 + \gamma G_y y$$

$\oplus\ B_0$

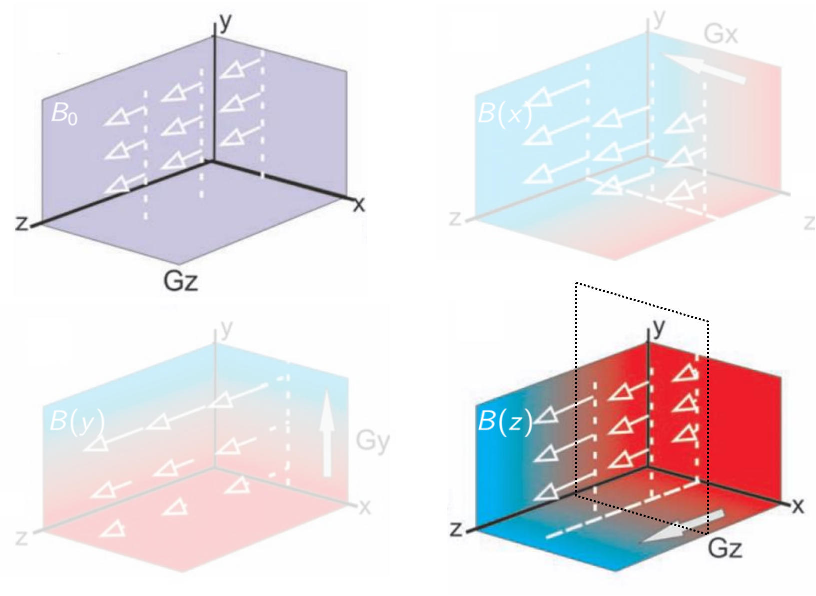


$$B(y) = B_0 + G_y y$$

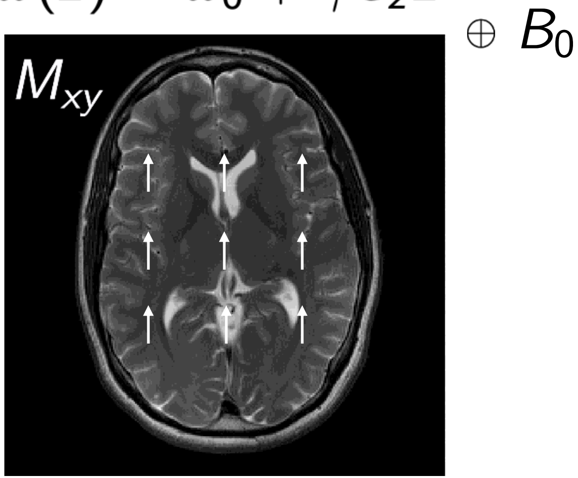
Plewes 2012



Interaction with gradient fields G

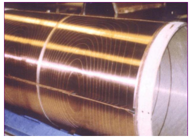


$$\omega(z) = \omega_0 + \gamma G_z z$$

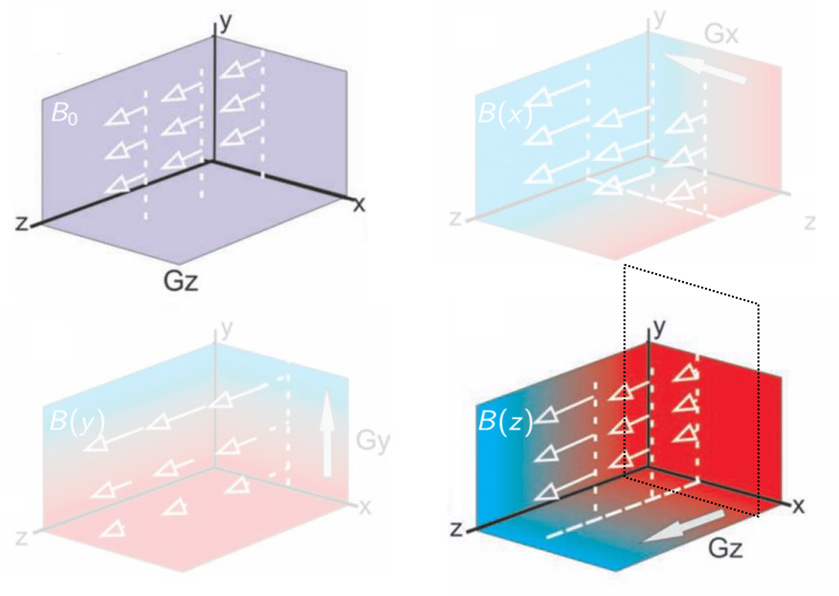


$$B(z) = B_0 + G_z z$$

Plewes 2012

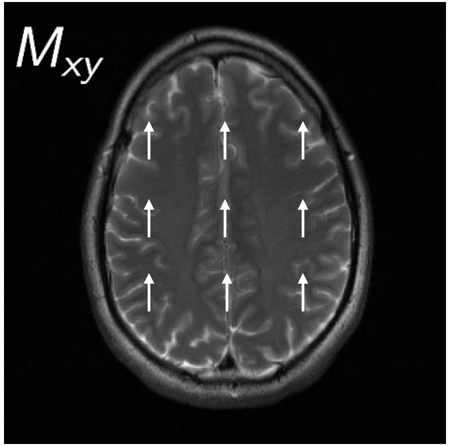


Interaction with gradient fields G



$$\omega(z) = \omega_0 + \gamma G_z z$$

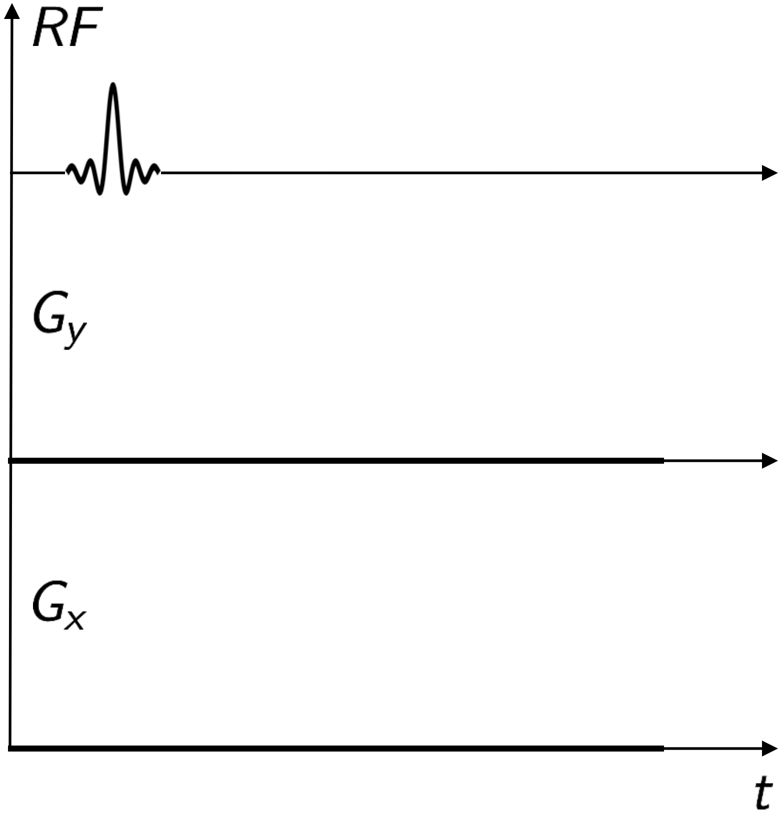
$\oplus$ B_0



$$B(z) = B_0 + G_z z$$

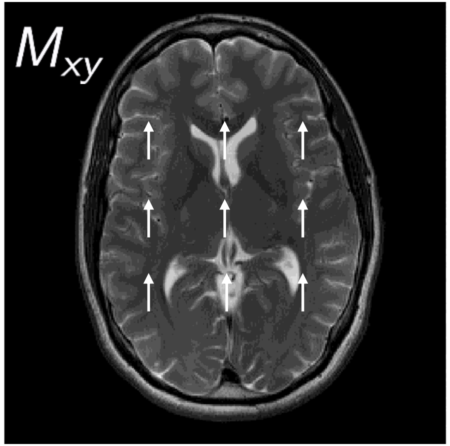
Plewes 2012

Pulse sequences

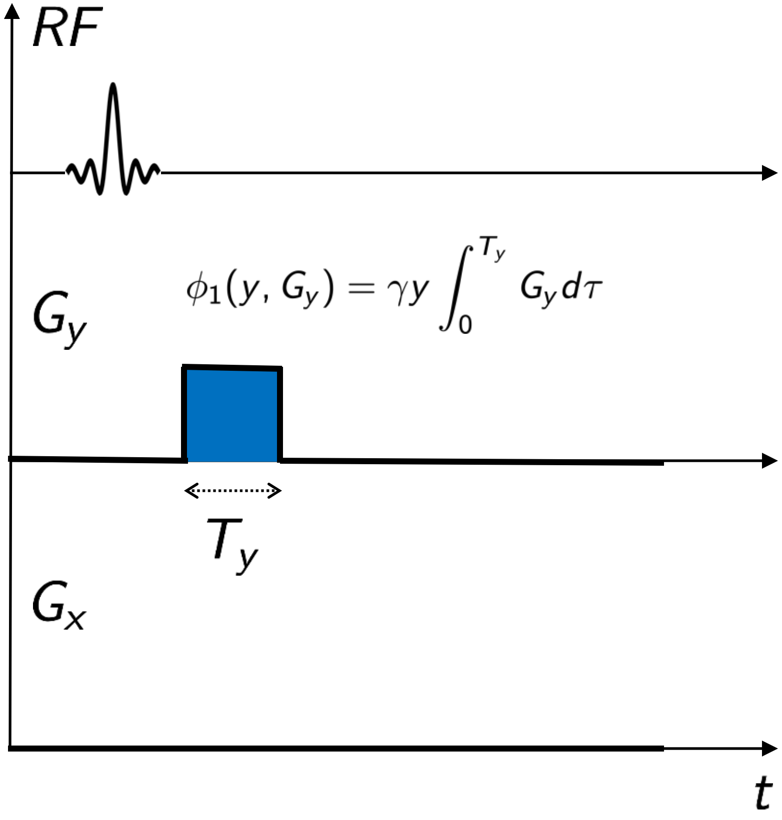


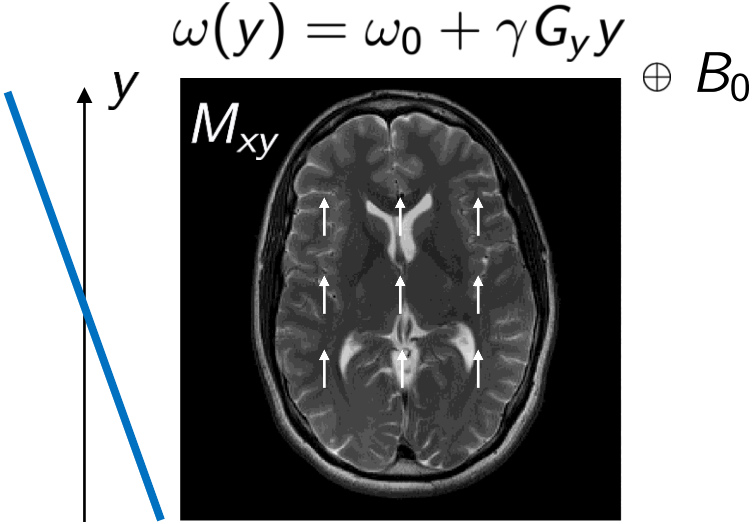
$$\omega_0 = \gamma B_0$$

$\oplus\ B_0$

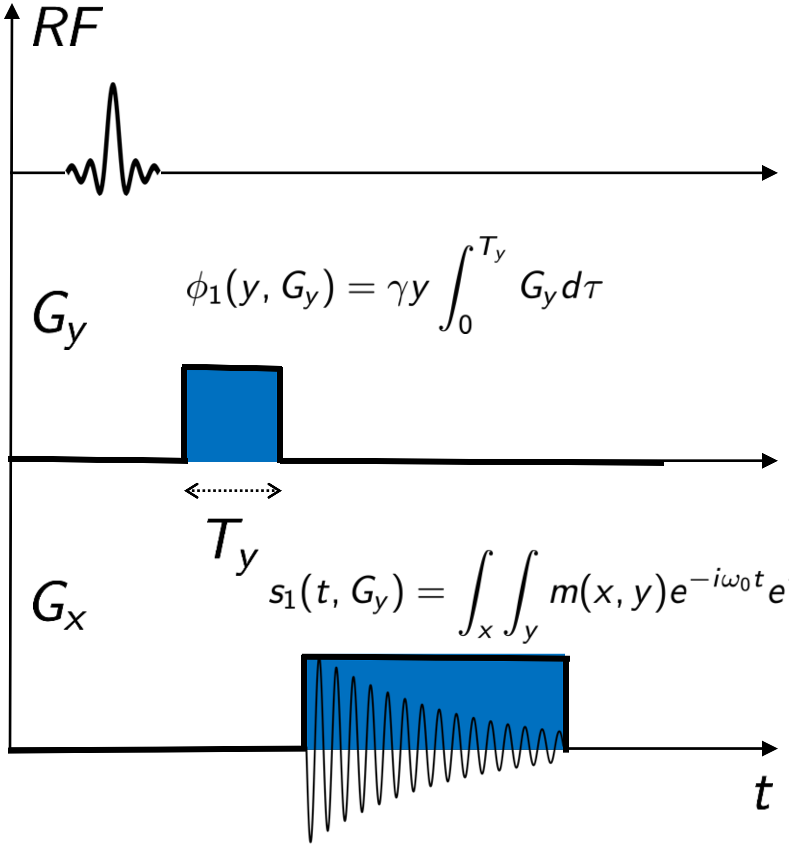


Pulse sequences



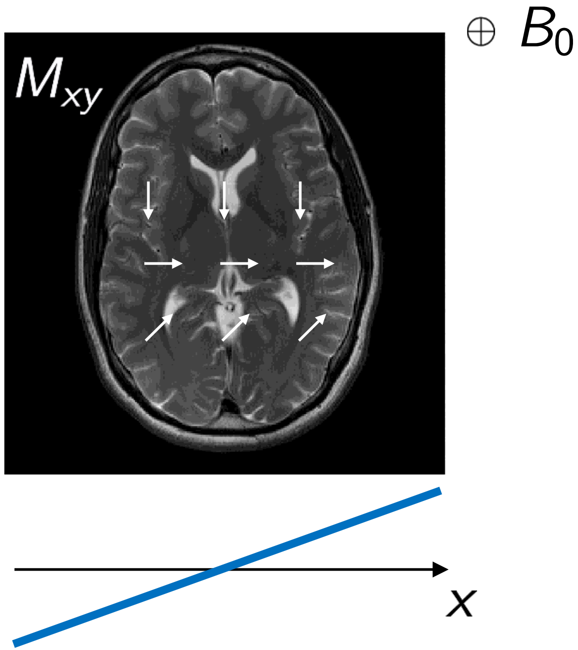


Pulse sequences

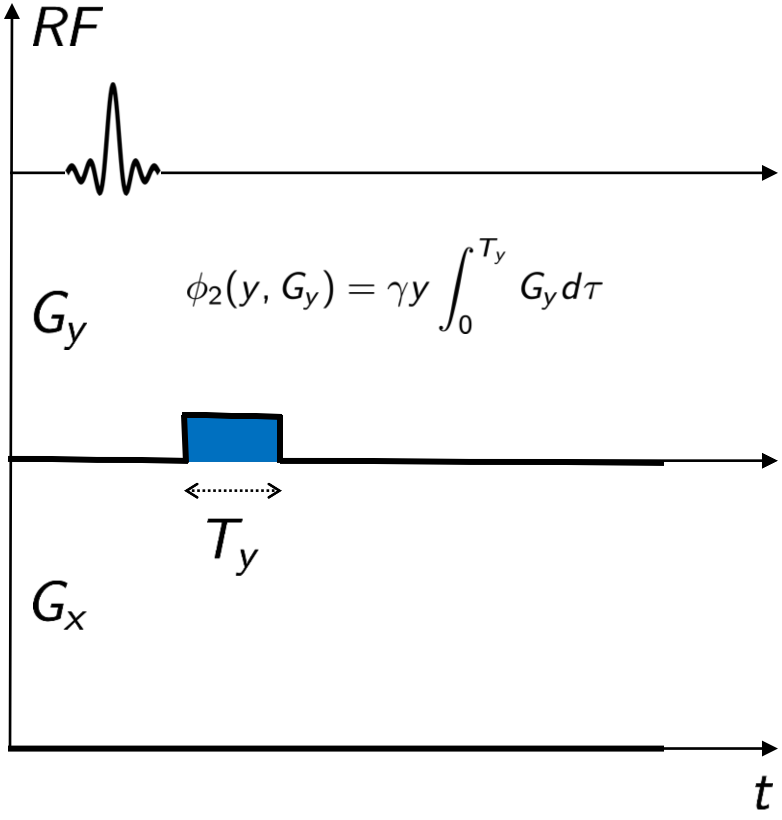


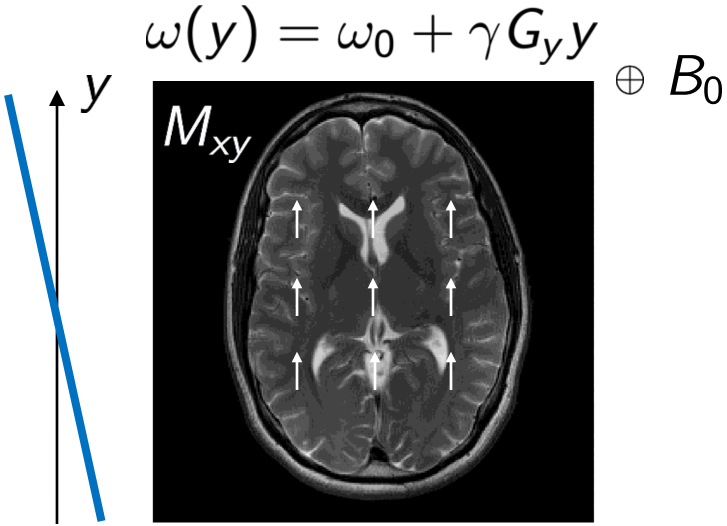
$$\phi_1(y, G_y) = \gamma y \int_0^{T_y} G_y d\tau$$

$$s_1(t, G_y) = \int_x \int_y m(x, y) e^{-i\omega_0 t} e^{-i\gamma G_x t} e^{-i\phi_1(y, G_y)} dxdy$$

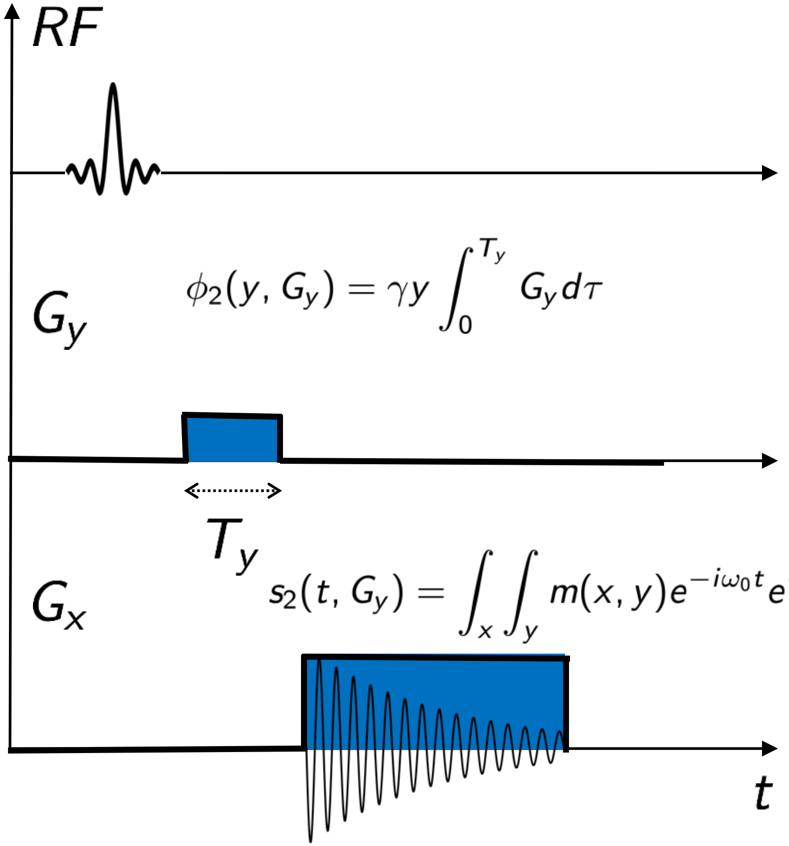


Pulse sequences



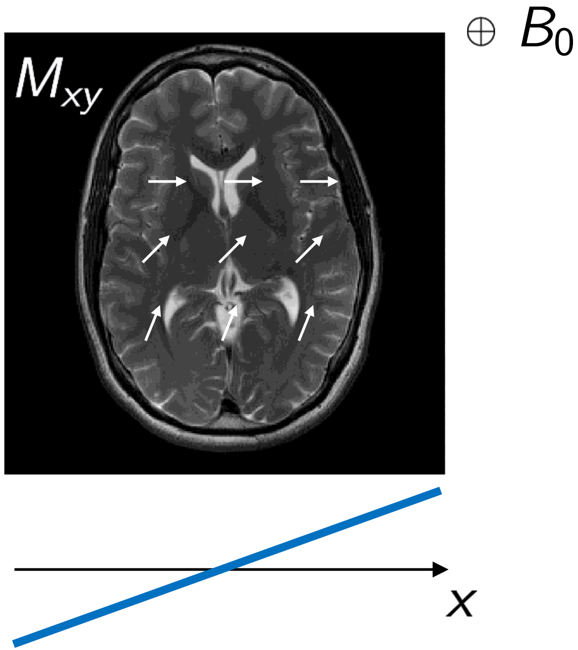


Pulse sequences

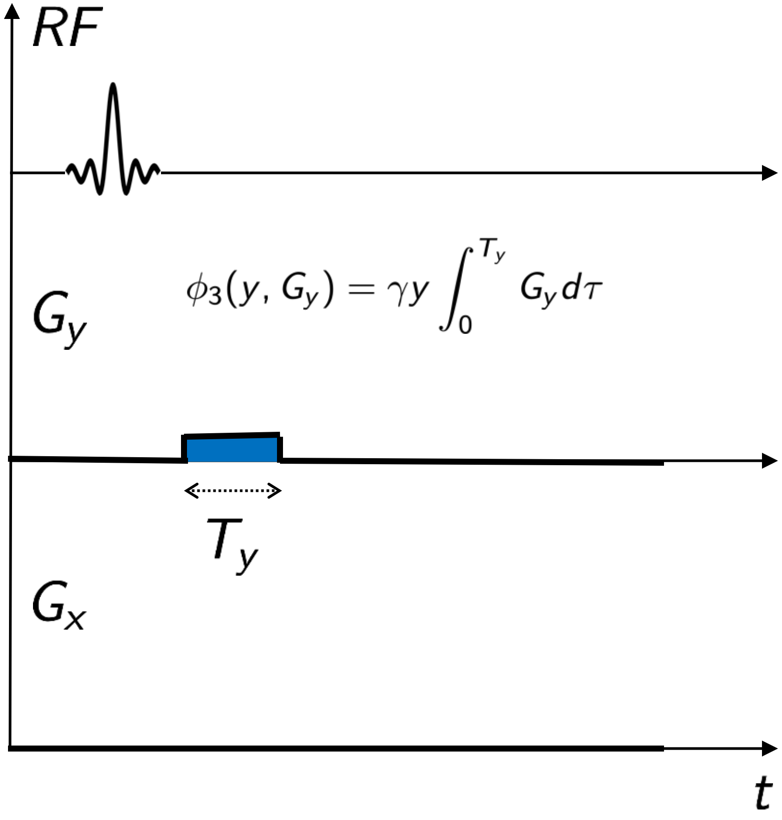


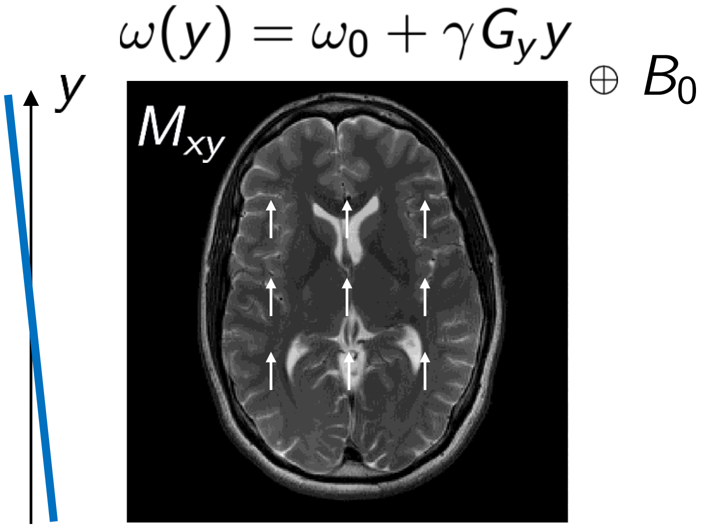
$$\phi_2(y, G_y) = \gamma y \int_0^{T_y} G_y \, d\tau$$

$$s_2(t, G_y) = \int_x \int_y m(x, y) e^{-i\omega_0 t} e^{-i\gamma G_x t} e^{-i\phi_2(y, G_y)} dxdy$$

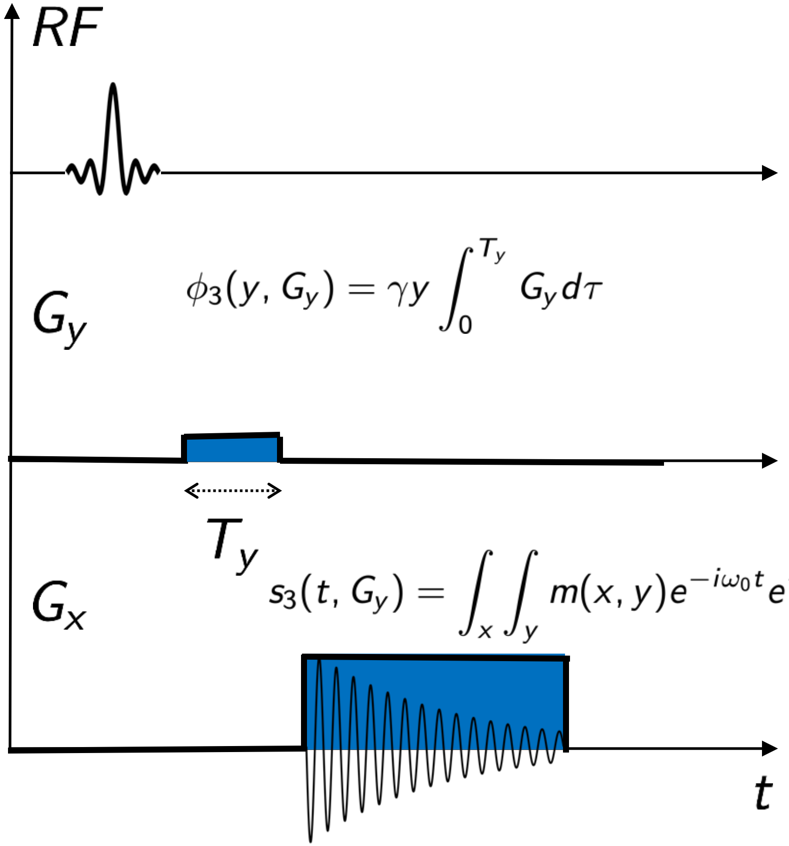


Pulse sequences



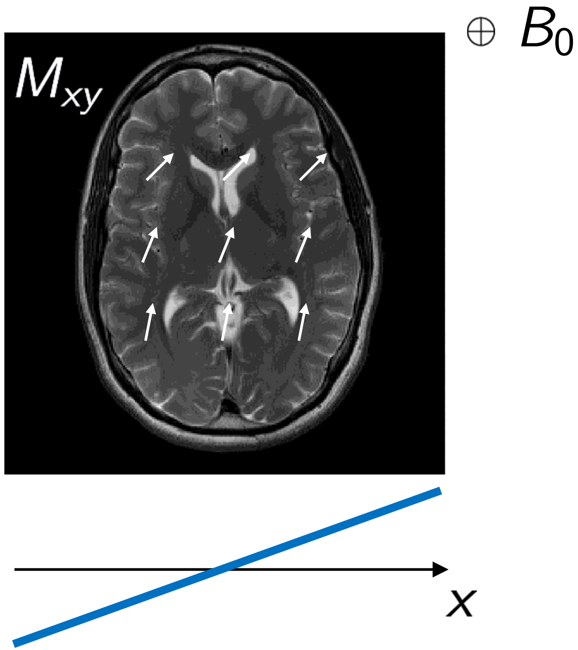


Pulse sequences

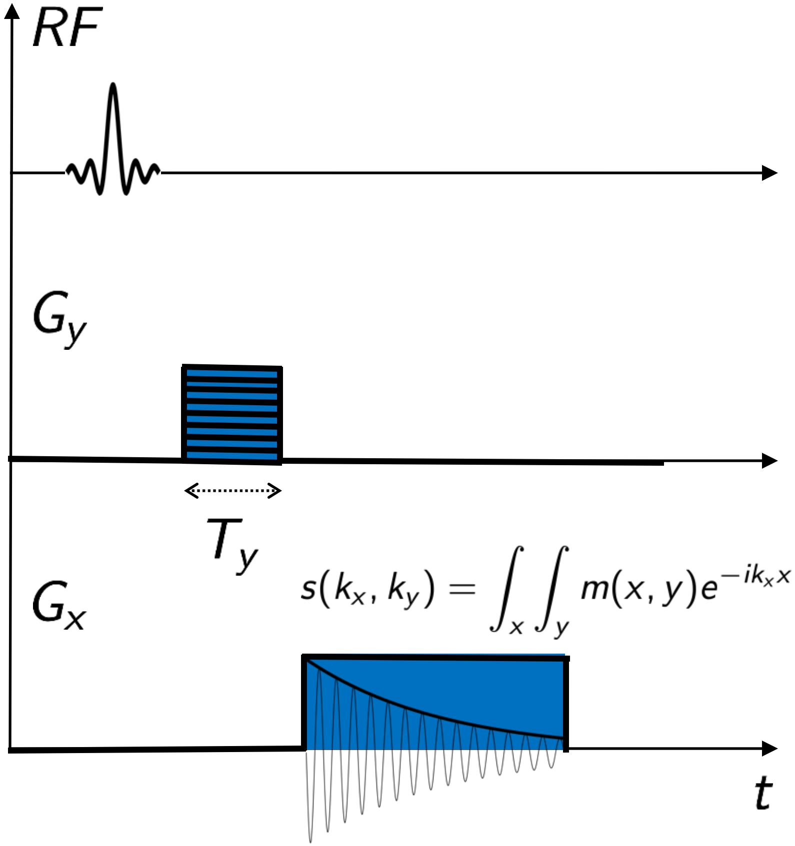


$$\phi_3(y, G_y) = \gamma y \int_0^{T_y} G_y d\tau$$

$$s_3(t, G_y) = \int_x \int_y m(x, y) e^{-i\omega_0 t} e^{-i\gamma G_x t} e^{-i\phi_3(y, G_y)} dxdy$$



Pulse sequences

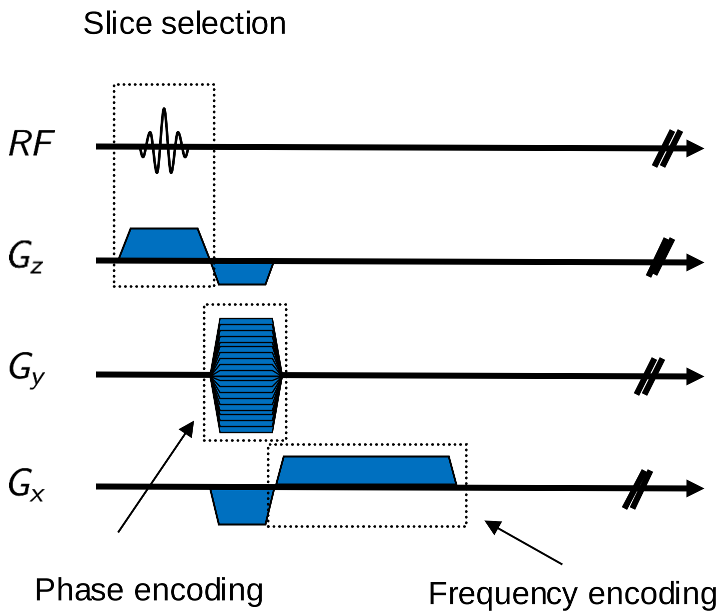


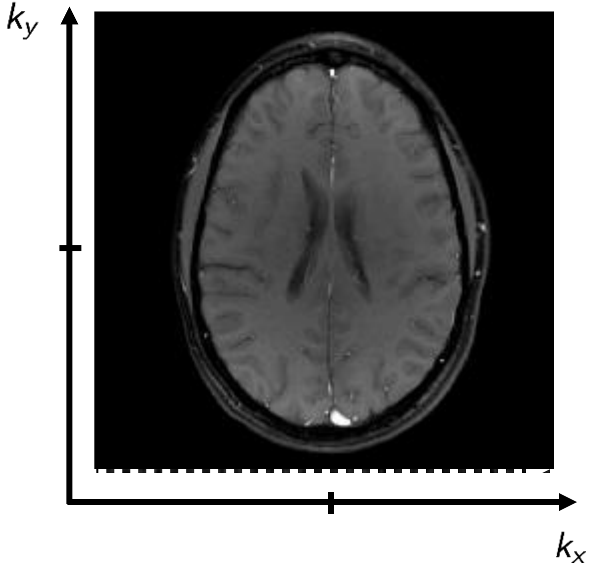
$$s(k_x, k_y) = \int_x \int_y m(x, y) e^{-ik_x x} e^{-ik_y y} dx dy$$

$$\phi(x, t) = \gamma x \int_0^t G_x d\tau \equiv k_x x$$

$$\phi(y, G_y) = \gamma y \int_0^{T_y} G_y d\tau \equiv k_y y$$

MR pulse sequence and k-space





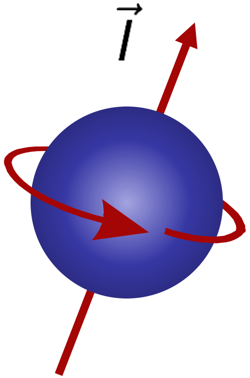
$$s(k_x, k_y) = \int_x \int_y m(x,y) e^{-ik_x x} e^{-ik_y y} dxdy$$

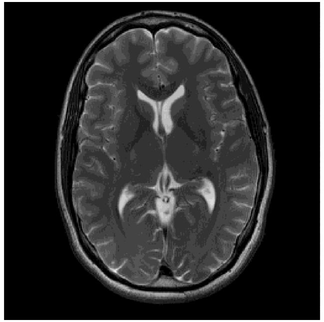
$$\phi(x,t) = \gamma x \int_0^t G_x d\tau \equiv k_x x$$

$$\phi(y, G_y) = \gamma y \int_0^{T_y} G_y d\tau \equiv k_y y$$

Nishimura p35

Summary

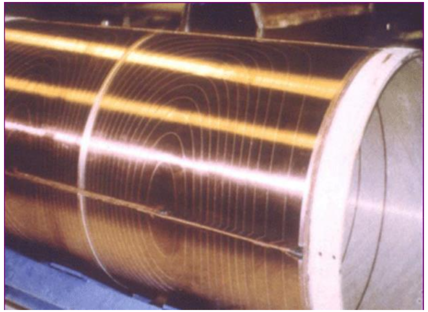




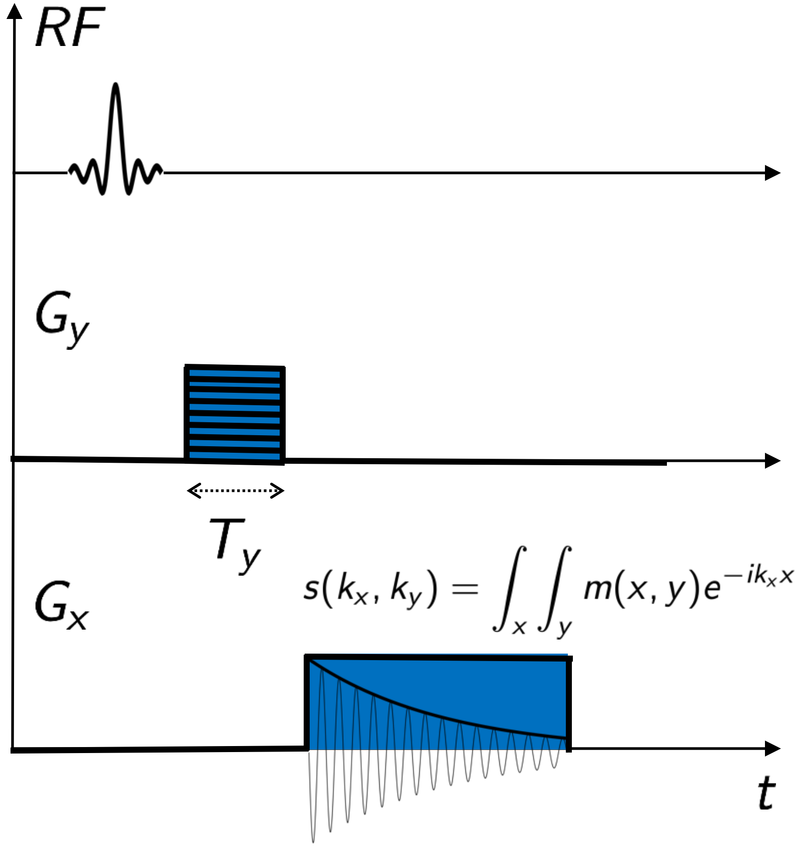
B_0

B_1

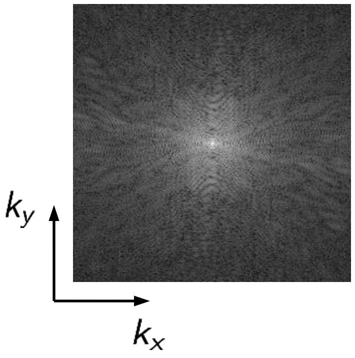
G



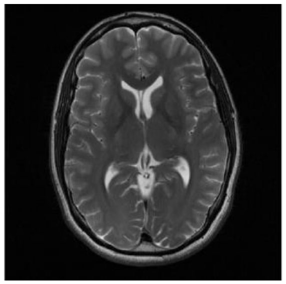
Next week



$$s(k_x, k_y) = \int_x \int_y m(x, y) e^{-ik_x x} e^{-ik_y y} \, dx dy$$



$$\xrightarrow{\mathcal{FT}^{-1}}$$



$$\phi(x, t) = \gamma x \int_0^t G_x d\tau \equiv k_x x$$

$$\phi(y, G_y) = \gamma y \int_0^{T_y} G_y d\tau \equiv k_y y$$

Outlook lab exercise