Computational MRI

Parallel Imaging II: k-space methods

Friedrich-Alexander-Universität
Computational Imaging Lab

NYU

Exam registration

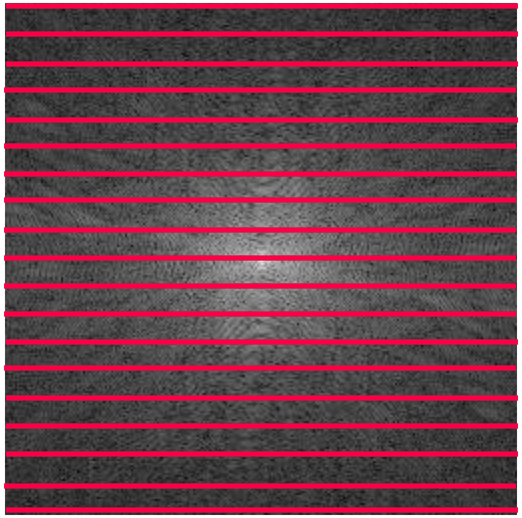
Exam details	EP	Type of examination	Date	Room	Attendee statistics
Computational magnetic resonance imaging 31091					ZU RT BEW
Computational magnetic resonance imaging Knoll, Florian (Prüfer/-in)	1	Variable	02/04/2025 10:15-11:15		21 0 0
Computational magnetic resonance imaging 31092					ZU RT BEW
Computational magnetic resonance imaging Knoll, Florian (Prüfer/-in)	1	Tutorial achievement	02/04/2025 10:15-11:15		12 1 0

Overview

- Review of parallel MRI
  - Image space: SENSE [1]
- What kind of k-space parallel MRI methods are available and how do they work?
  - SMASH [2]
  - GRAPPA [3]
- What are the advantages, disadvantages?

[1] Pruessmann MRM 1999
[2] Sodickson MRM 1997
[3] Griswold MRM 2002

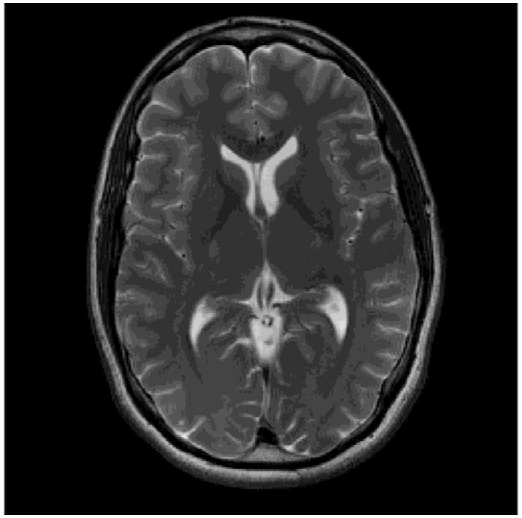
k-space undersampling



Δk_y

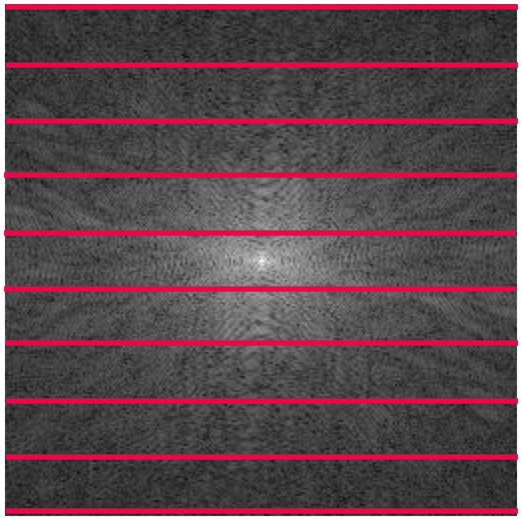
IFT

$\longrightarrow$



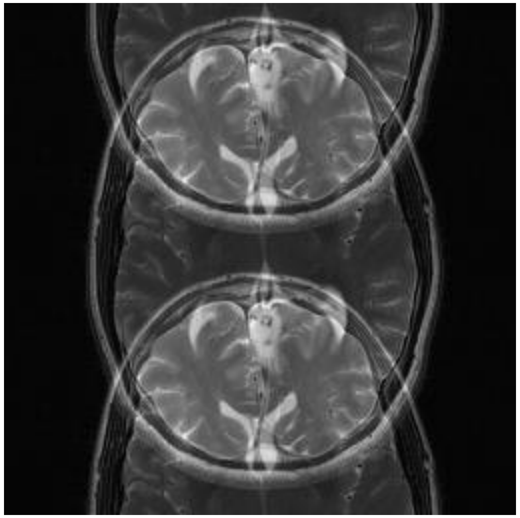
FOV

k-space undersampling



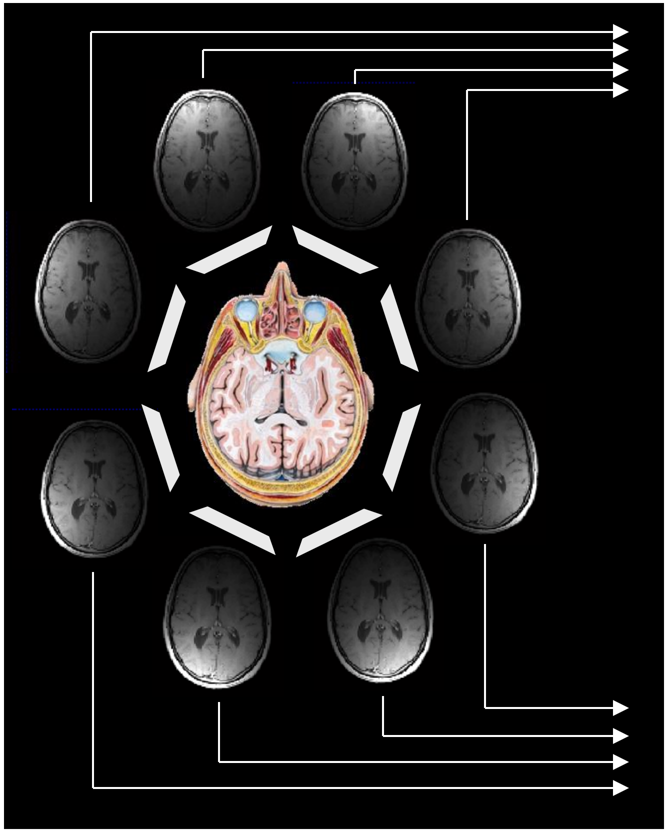
$2 \cdot \Delta k_y$

$\xrightarrow{\text{IFT}}$



$\frac{FOV}{2}$

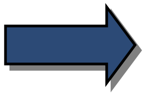
Spatial Information of Surface Coils



Array of multiple surface coils:

8 independent receiver channels

8 images with different spatial sensitivity information



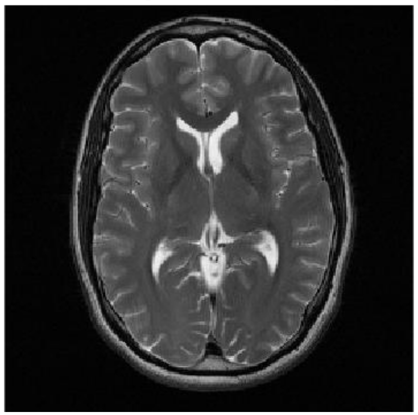
Use additional spatial information for faster imaging

Student question: Phase of reconstructed image

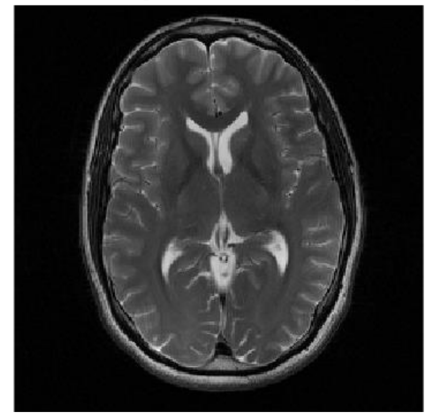
$$f(r) = \sqrt{\sum_{i=1}^{N_c} |m_i(r)|^2}$$

$$f(r) = \frac{\sum_{i=1}^{N_c} c_i^*(r) m_i(r)}{\sqrt{\sum_{i=1}^{N_c} |c_i(r)|^2}}$$

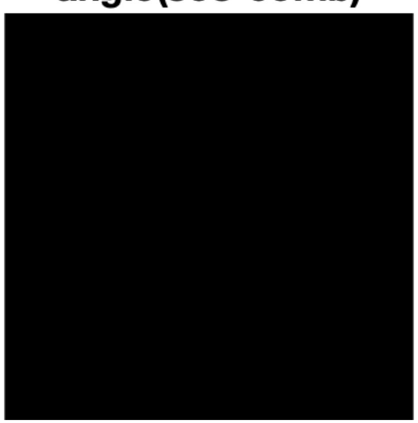
abs(sos-comb)



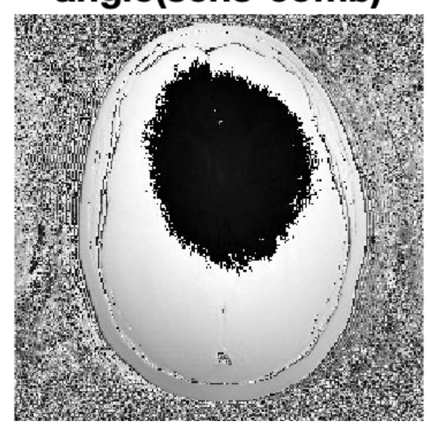
abs(sens-comb)



angle(sos-comb)

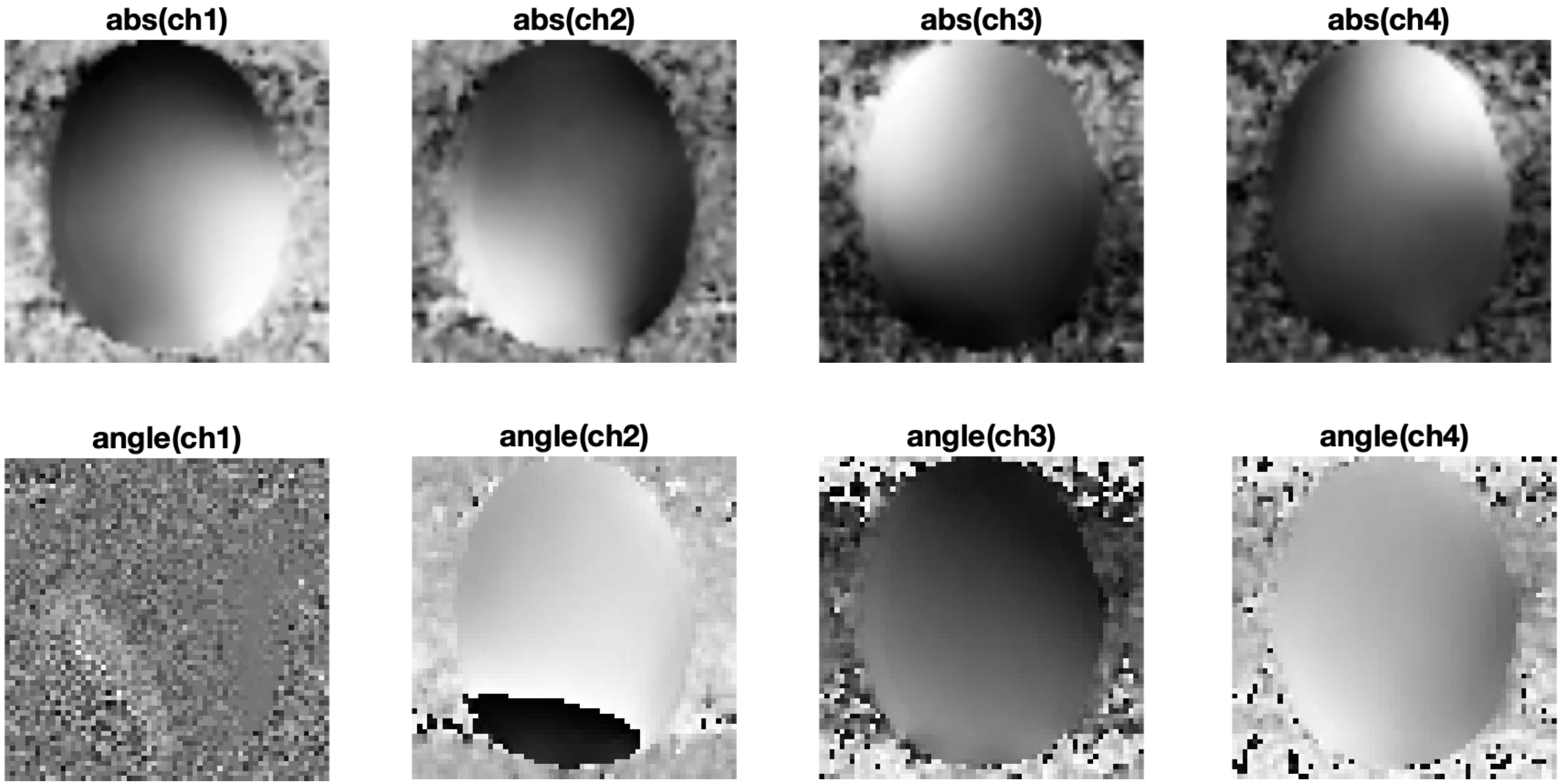


angle(sens-comb)



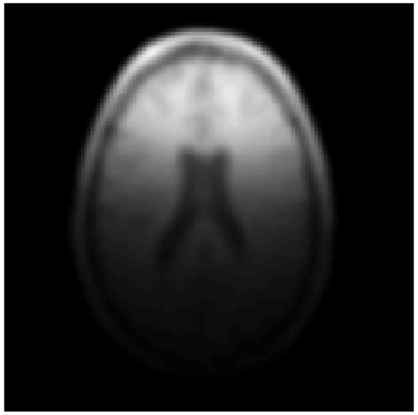
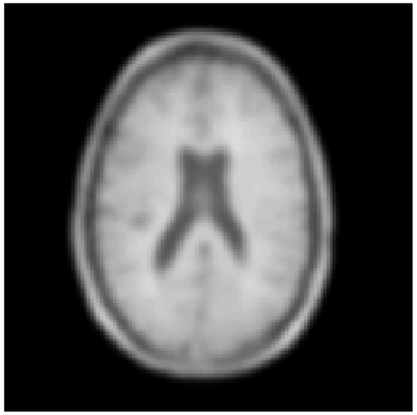
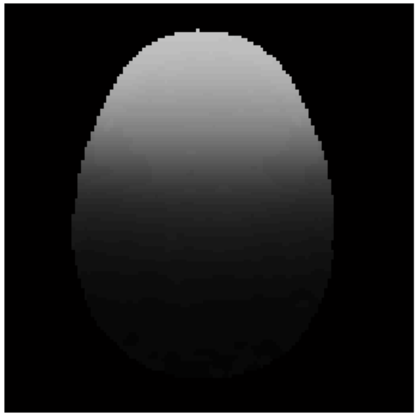
PC imaging: Need to separate phase of sensitivities and image (Bernstein MRM 1994)

Student question: Phase of coil sensitivity maps



Walsh MRM 2000
Uecker MRM 2014

Student question: Coil sensitivity estimation

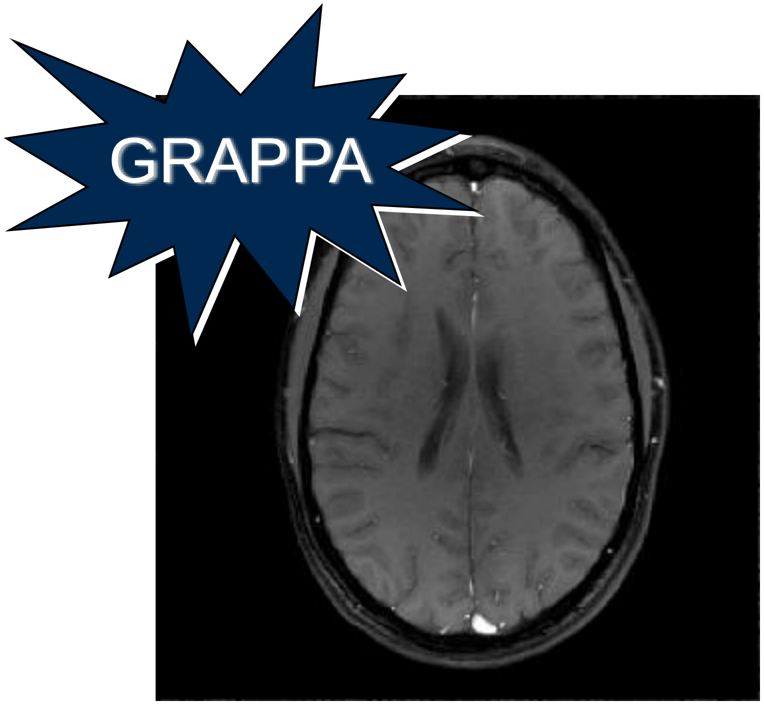
 %  = 

$m \cdot e^{i\Phi_m}$ $\qquad$ $ref \cdot e^{i\Phi_{ref}}$ $\qquad$ $c = \dfrac{m}{ref} \cdot e^{i(\Phi_m - \Phi_{ref})}$

$$
\begin{aligned}
c^* m &= \left(\frac{m}{ref} \cdot e^{i(\Phi_m - \Phi_{ref})}\right)^* \left(m \cdot e^{i\Phi_m}\right) \\
&= \left(\frac{m}{ref} \cdot e^{i(\Phi_{ref} - \Phi_m)}\right) \left(m \cdot e^{i\Phi_m}\right) \\
&= \frac{m^2}{ref} \cdot e^{i\Phi_{ref}}
\end{aligned}
$$

Pruessmann MRM 1999

Parallel MRI



K-Space

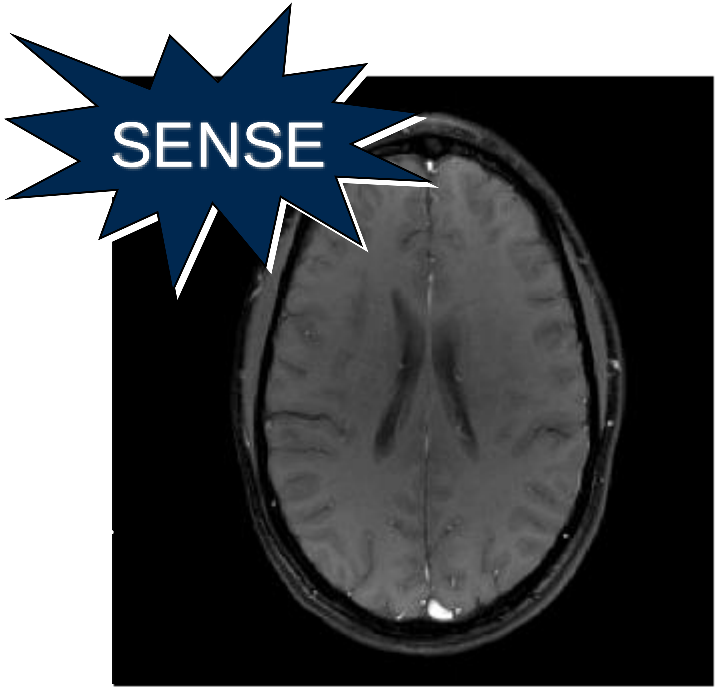


Image - Space

Revision: Fourier transform properties

- Linearity: $F\{as_1(r)+bs_2(r)\} = aS_1(k)+bS_2(k)$
- Shifting: $F\{s(r-r_0)\} = e^{-i2\pi k r_0} S(k)$
- Modulation: $F\{e^{i2\pi k_0 r} s(r)\} = S(k-k_0)$
- Conjugate symmetry: $s(r) real \Rightarrow S(-k) = S^*(k)$
- Scaling: $F\{s(ar)\} = \frac{1}{|a|} S\left(\frac{k}{a}\right)$

Revision: Fourier transform properties

- Parseval's formula: $\int s_1(r)s_2(r)\,dr = \int S_1(k)S_2(k)\,dk$

$$s_1 = s_2 = s \quad \Longrightarrow \quad \int |s(r)|^2\,dr = \int |S(k)|^2\,dk$$

- Convolution & multiplication

$$F\{s_1(r) * s_2(r)\} = S_1(k)S_2(k)$$

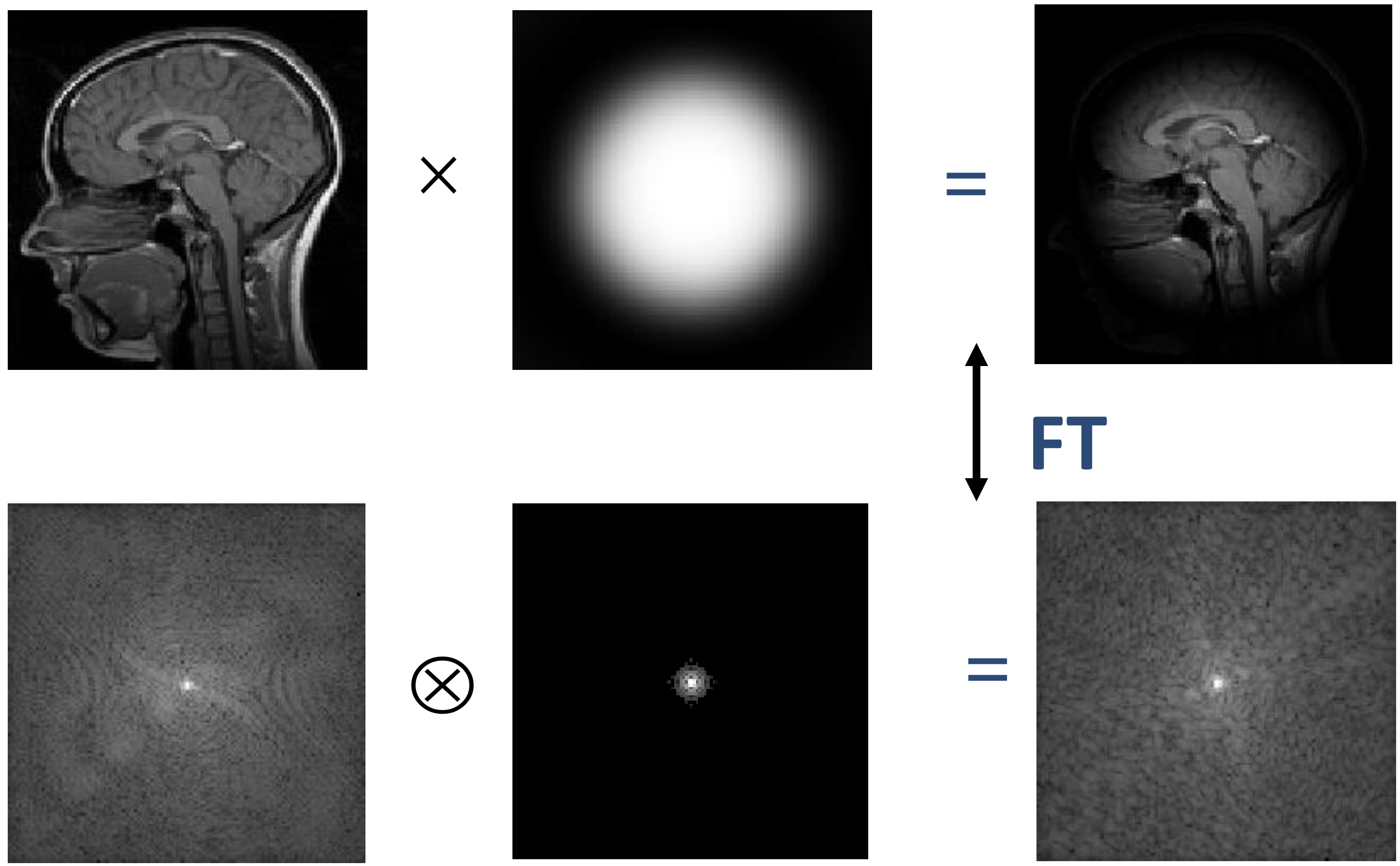
$$F\{s_1(r)s_2(r)\} = S_1(k) * S_2(k)$$

Revision: Image and k-space domains

object | coil sensitivity | coil image

Image Domain multiplication

k-space convolution



object k-space | coil sensitivity k-space | coil image k-space

k-space parallel imaging: SMASH

Simultaneous Acquisition of Spatial Harmonics (SMASH): Fast Imaging with Radiofrequency Coil Arrays

Daniel K. Sodickson, Warren J. Manning

SiMultaneous Acquisition of Spatial Harmonics (SMASH) is a new fast-imaging technique that increases MR image acquisition speed by an integer factor over existing fast-imaging methods, without significant sacrifices in spatial resolution or signal-to-noise ratio. Image acquisition time is reduced by exploiting spatial information inherent in the geometry of a surface coil array to substitute for some of the phase encoding usually produced by magnetic field gradients. This allows for partially parallel image acquisitions using many of the existing fast-imaging sequences. Unlike the data combination algorithms of prior proposals for parallel imaging, SMASH reconstruction involves a small set of MR signal combinations prior to Fourier transformation, which can be advantageous for artifact handling and practical implementation. A twofold savings in image acquisition time is demonstrated here using commercial phased array coils on two different MR-imaging systems. Larger time savings factors can be expected for appropriate coil designs.

Key words: fast imaging; RF coil array; simultaneous acquisition; MR image reconstruction.

INTRODUCTION

The speed with which magnetic resonance images may be acquired has increased dramatically over the past decade. The improvements in speed may be traced to a combination of advances in technology and innovations in imaging strategy. Strong, fast-switching magnetic field gradients and fast electronics have allowed the intervals between data collections to be reduced significantly. Meanwhile, fast gradient-echo and spin-echo sequences have reduced image acquisition time by allowing greater portions of k-space to be sampled after each spin excitation. For example, echo-planar imaging (EPI) (1), fast low-angle shot (FLASH) (2), turbo spin echo (TSE) (3), spiral imaging (4, 5), and BURST (6, 7) sequences all allow very short intervals between acquisition of successive data points. One common feature of these fast imaging techniques, however, is that they all acquire data in a sequential fashion. Whether the k-space data matrix is filled in a rectangular raster pattern, a spiral pattern, a rapid series of line scans, or some other novel trajectory, it is always acquired one point and one line at a time.

MRM 38:591–603 (1997)

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Received October 9, 1996; revised February 4, 1997; accepted March 14, 1997.

0740-3194/97 $3.00

This report presents a fast-imaging technique that allows some fraction of signal data points to be acquired *in parallel*, rather than sequentially in time. Previously, several fast imaging schemes have been proposed using simultaneous data acquisition in multiple RF coils (8–13). The technique described here, dubbed simultaneous acquisition of spatial harmonics (SMASH), reduces image acquisition times by a multiplicative integer factor without a significant sacrifice in spatial resolution or signal-to-noise ratio (SNR), in effect by scanning many lines of k-space at a time. The SMASH procedure operates by using linear combinations of simultaneously acquired signals from multiple surface coils with different spatial sensitivities to generate multiple data sets with distinct offsets in k-space. The full k-space matrix may then be generated with only a fraction of the usual number of phase-encoding gradient steps. Consequently, the total image acquisition time may be reduced by the same fraction. A factor of two time savings has been implemented with standard equipment, although in principle, there is no limit to the number of lines that may be scanned simultaneously, provided coil arrays with sufficient numbers of components are constructed. Importantly, the SMASH technique may be combined with most existing fast-imaging sequences, thereby multiplying the intrinsic speed of these sequences.

THEORY

Phase Encoding by Amplitude Modulation

In the general case, the MR signal for a plane with spin density $\rho(x, y)$ and receiver coil sensitivity $C(x, y)$ may be written as

$$S(k_x, k_y) = \iint dx dy C(x, y)\rho(x, y)\exp\{-ik_x x - ik_y y\} \quad [1]$$

where $k_x = \gamma G_x t_x$ and $k_y = \gamma G_y t_y$ as usual, with γ the gyromagnetic ratio, G_x and G_y the magnitude of the x and y gradients, and t_x and t_y the times spent in the x and y gradients, respectively. Here, the spin excitation function as well as the effects of relaxation have been incorporated into the pulse-sequence-specific sensitivity function C. For regions of the sample in which the coil sensitivity is relatively homogeneous, $C(x, y) \approx 1$, and $S(k_x, k_y)$ is then equal to the spatial Fourier transform of the spin-density function. Inverse Fourier transformation with respect to k_x and k_y reconstructs the usual spin-density image $\rho(x, y)$.

It is well known that MR receiver coils, especially surface coils, do not have uniform sensitivity. Signals from different regions of the imaged volume produce different currents in an RF coil, with the spatial variation

Sodickson MRM 1997

Review: Spatial encoding in MRI

$$S(k_x, k_y) = \int_{\Omega_x}\int_{\Omega_y} \rho(x,y) \cdot e^{-ik_x x} \cdot e^{-ik_y y} \, dx\, y$$

$$k_x = \int_0^T G_x dt$$

$$k_y = \int_0^T G_y dt$$

Review: Spatial encoding in MRI

$$S(k_x, k_y) = \int_{\Omega_x}\int_{\Omega_y} \rho(x,y) \cdot e^{-ik_x x} \cdot e^{-ik_y y} \, dxy$$

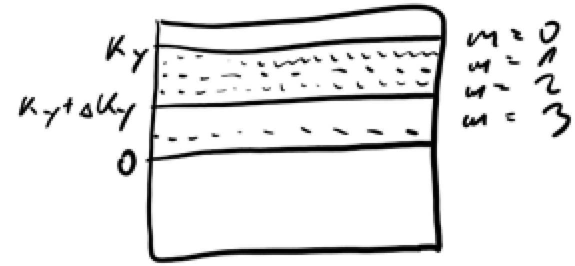
$$S(k_y) = \int_{\Omega_y} \rho(y) \cdot \underset{\text{Gradient}}{e^{-ik_y y}} \, dy$$

$$k_y = \int_0^T G_y \, dt$$

Review: Spatial encoding in MRI

$$S(K_x, K_y) = \int_{\Omega_x}\int_{\Omega_y} \rho(x,y)\cdot e^{-ik_x x}\cdot e^{-ik_y y}\, dx\, y$$

$$S(K_y) = \int_{\Omega_y} \rho(y)\cdot \underset{\text{Gradient}}{e}^{-iK_y y}\, dy$$



Review: Spatial encoding in MRI

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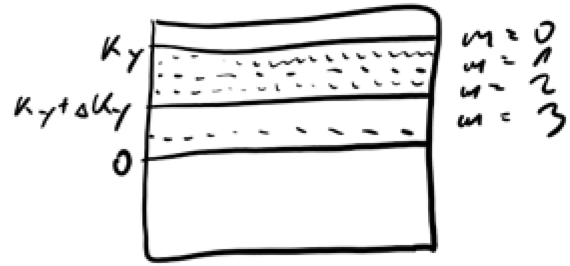
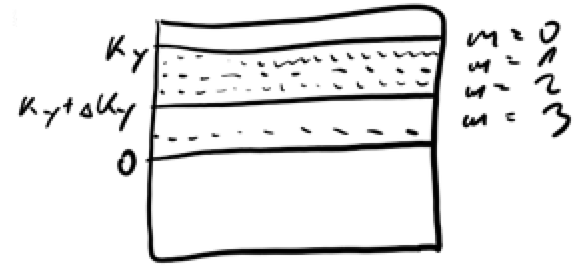


$$S(K_y + m\Delta K_y) = \int_{\Omega} \rho(y) \cdot e^{-ik_y y} \cdot e^{-im \cdot \Delta K_y y}\, dy$$

Review: Spatial encoding in MRI

$$S(K_x, K_y) = \int_{\Omega_x}\int_{\Omega_y} \rho(x,y)\cdot e^{-ik_x x}\cdot e^{-ik_y y}\, dx\, y$$



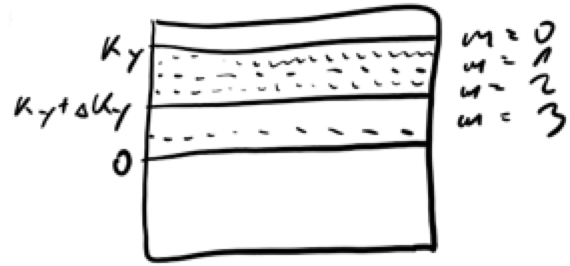
$$S(K_y) = \int_{\Omega_y} \rho(y)\cdot \underset{\text{Gradient}}{e}^{-iK_y y}\, dy$$

$$S(K_y + m\Delta K_y) = \int_{\Omega} \rho(y)\cdot e^{-ik_y y}\cdot e^{-im\cdot\Delta K_y y}\, dy$$

$$S(0\cdot\Delta K_y)\quad G_y\,\underline{\qquad\qquad}\quad e^{-i0\cdot\Delta K_y y}\quad\underline{\qquad\qquad}$$

Review: Spatial encoding in MRI

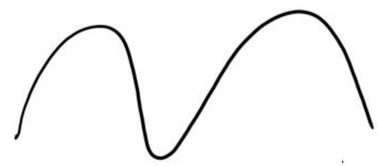
$$S(K_x, K_y) = \int_{\Omega_x}\int_{\Omega_y} \rho(x,y)\cdot e^{-ik_x x}\cdot e^{-ik_y y}\, dx\, y$$



$$S(K_y) = \int_{\Omega_y} \rho(y)\cdot \underset{\text{Gradient}}{e}^{-iK_y y}\, dy$$

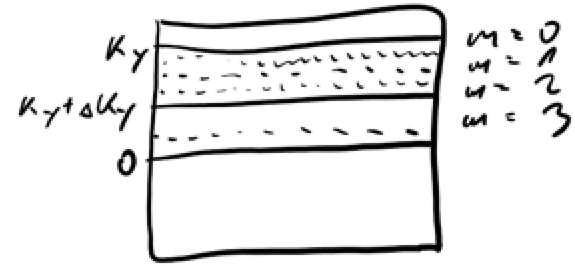
$$S(K_y + m\Delta K_y) = \int_{\Omega} \rho(y)\cdot e^{-ik_y y}\cdot e^{-im\cdot\Delta K_y y}\, dy$$

$S(0\cdot\Delta K_y)$ G_y  $e^{-i0\cdot\Delta K_y y}$ 

$S(1\cdot\Delta K_y)$ G_y  $e^{-i1\cdot\Delta K_y y}$ 

Review: Spatial encoding in MRI

$$S(k_x,k_y)=\int_{\Omega_x}\int_{\Omega_y}\rho(x,y)\cdot e^{-ik_x x}\cdot e^{-ik_y y}\,dx\,y$$



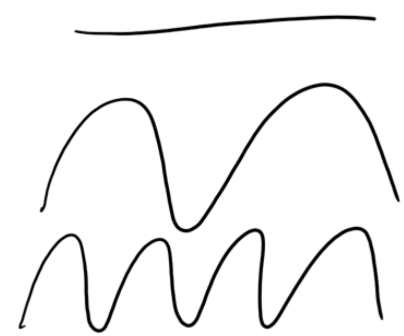
$$S(k_y)=\int_{\Omega_y}\rho(y)\cdot \underbrace{e^{-ik_y y}}_{\text{Gradient}}\,dy$$

$$S(k_y+m\Delta k_y)=\int_{\Omega}\rho(y)\cdot e^{-ik_y y}\cdot e^{-im\cdot\Delta k_y y}\,dy$$

$S(0\cdot\Delta k_y)$ G_y  $e^{-i\,0\cdot\Delta k_y y}$

$S(1\cdot\Delta k_y)$ G_y  $e^{-i\,1\cdot\Delta k_y y}$

$S(2\cdot\Delta k_y)$ G_y  $e^{-i\,2\cdot\Delta k_y y}$



Magnetic field gradients generate spatial harmonics

Multi-coil MRI acquisition

$$S_l(k_y + m\Delta k_y) = \int_{\Omega_y} C_l(y)\cdot \rho(y)\cdot e^{-ik_y y}\cdot e^{m-i\Delta k_y y}\, dy$$

Multi-coil MRI acquisition

$$S_l(k_y + m\Delta k_y) = \int_{\Omega_y} C_l(y) \cdot \rho(y) \cdot e^{-ik_y y} \cdot e^{m-i\Delta k_y y} \, dy$$

PI: $$S(k_y + m\Delta k_y) = \sum_{l=1}^{N_c} n_l \cdot S_l(k_y) \qquad m = 0, \dots, R-1$$

Multi-coil MRI acquisition

$$S_l(k_y + m\Delta k_y) = \int_{\Omega_y} C_l(y) \cdot \rho(y) \cdot e^{-ik_y y} \cdot e^{m-i\Delta k_y y} \, dy$$

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Multi-coil MRI acquisition

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$$S(k_y + m\Delta k_y) = \sum_{l=1}^{N_c} m_l \int_{\Omega_y} C_l(y) \cdot \rho(y) \cdot e^{-ik_y y}\, dy$$

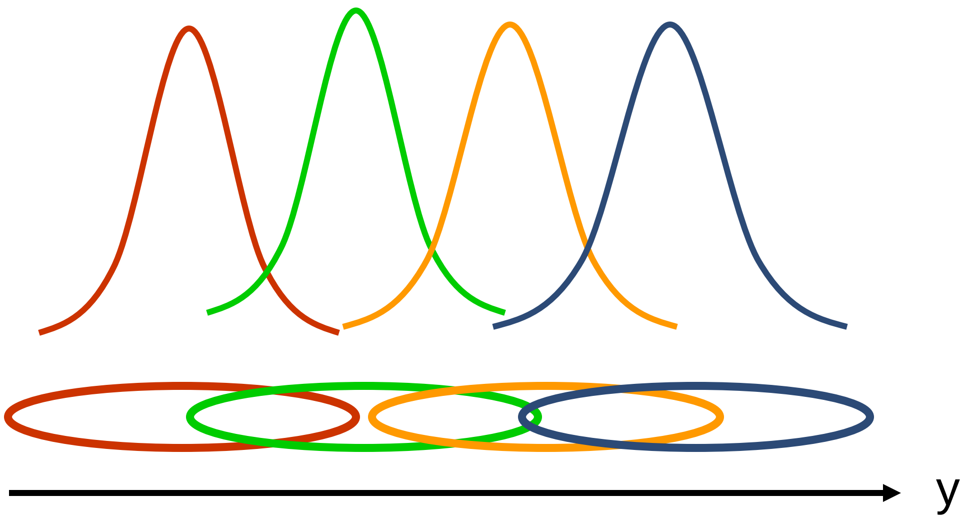
$$= \int_{\Omega_y} \sum m_l\, C_l(y) \cdot \rho(y) \cdot e^{-ik_y y}\, dy$$

SMASH

- Synthesize missing k-space lines from undersampled multi-coil data……

$$S(k_y + m\Delta k_y) = \sum_{l=1}^{N_c} w_l^{(m)} C_l(y)\rho(y)e^{-ik_y y}\,dy \qquad \sum_{l=1}^{N_c} w_l^{(m)} C_l(y) \approx e^{-im\Delta k_y y}$$

- …. if coil profiles can be combined to build spatial harmonics of order m!



Sodickson MRM 1997

SMASH

- Synthesize missing k-space lines from undersampled multi-coil data……

$$S(k_y + m\Delta k_y) = \sum_{l=1}^{N_c} w_l^{(m)} C_l(y)\rho(y)e^{-ik_y y}\,dy \qquad \sum_{l=1}^{N_c} w_l^{(m)} C_l(y) \approx e^{-im\Delta k_y y}$$

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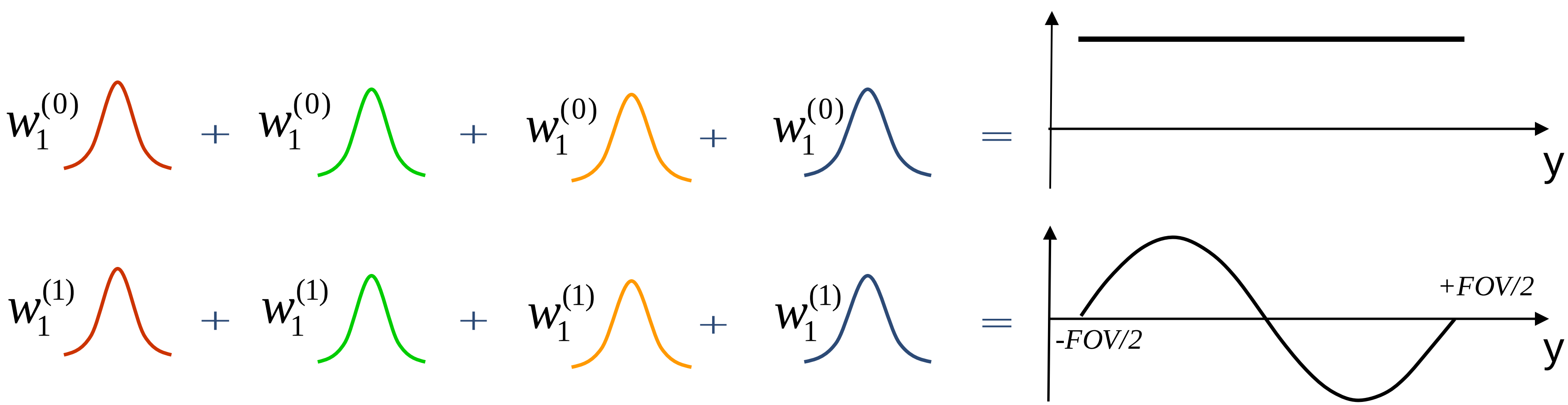
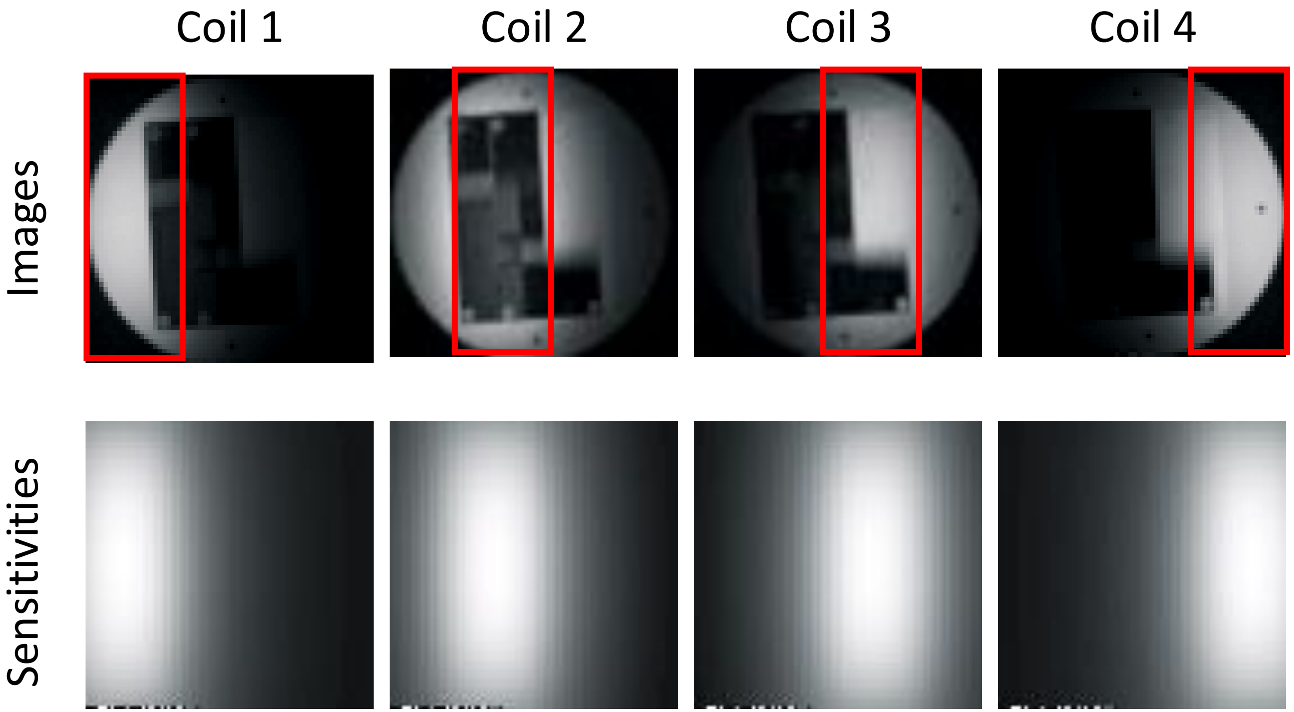
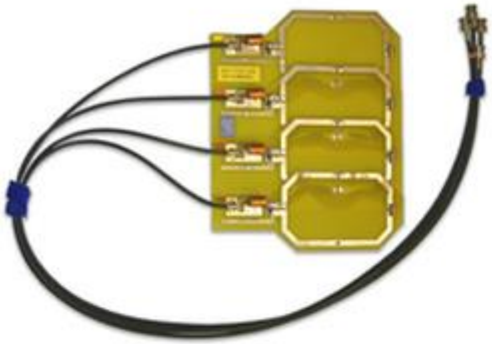


Sodickson MRM 1997

A coil array where SMASH works





Sodickson MRM 1997

GRAPPA: Review: Image and k-space domains

object

coil sensitivity

coil image

Image Domain multiplication

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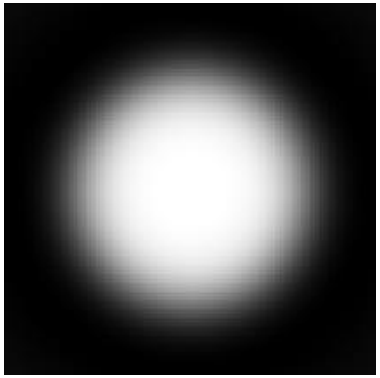

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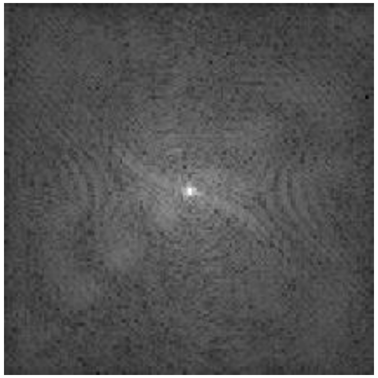
FT

k-space convolution



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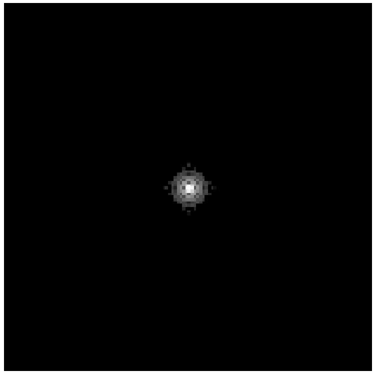

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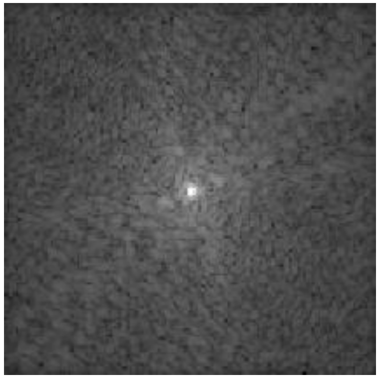


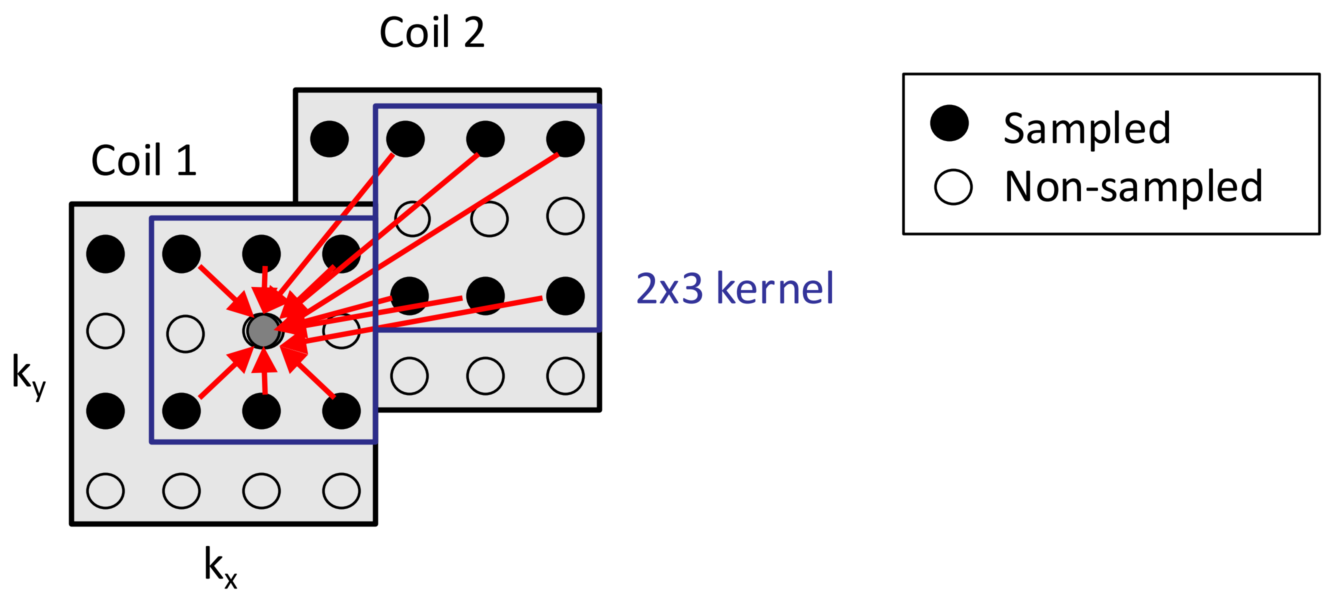
object k-space

coil sensitivity k-space

coil image k-space

GRAPPA: More general k-space fitting

- Coil-by-coil k-space reconstruction
- Linear combination of k-space neighbors from all coils



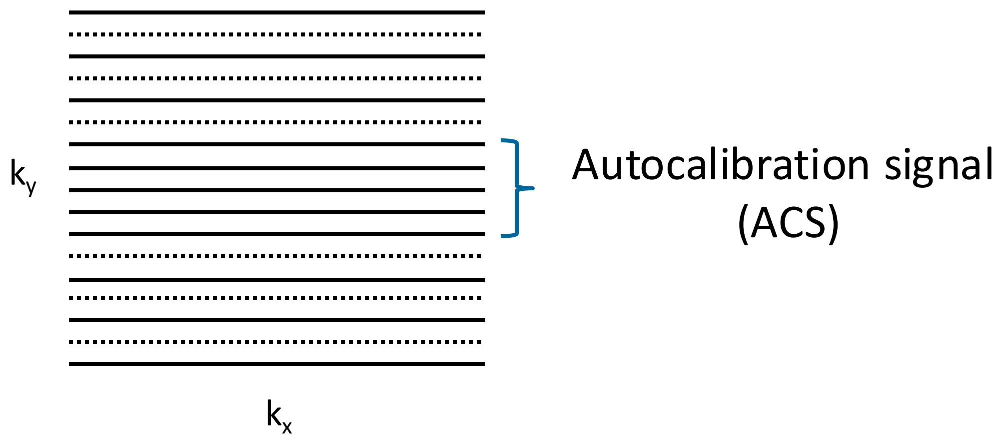
$$\mathbf{T} = \mathbf{S}\mathbf{w}$$

S: source matrix ($1 \times n_k n_c$)
T: target matrix ($1 \times n_c$)
w: weight matrix ($n_k n_c \times n_c$)

Griswold MRM 2002

GRAPPA: More general k-space fitting

- Reconstruction weights (GRAPPA kernel)
  - Fully-sampled k-space region (calibration)
    - Within the accelerated data (autocalibration)
    - Separate acquisition
  - Least-square fit using examples of target and source points



Griswold MRM 2002

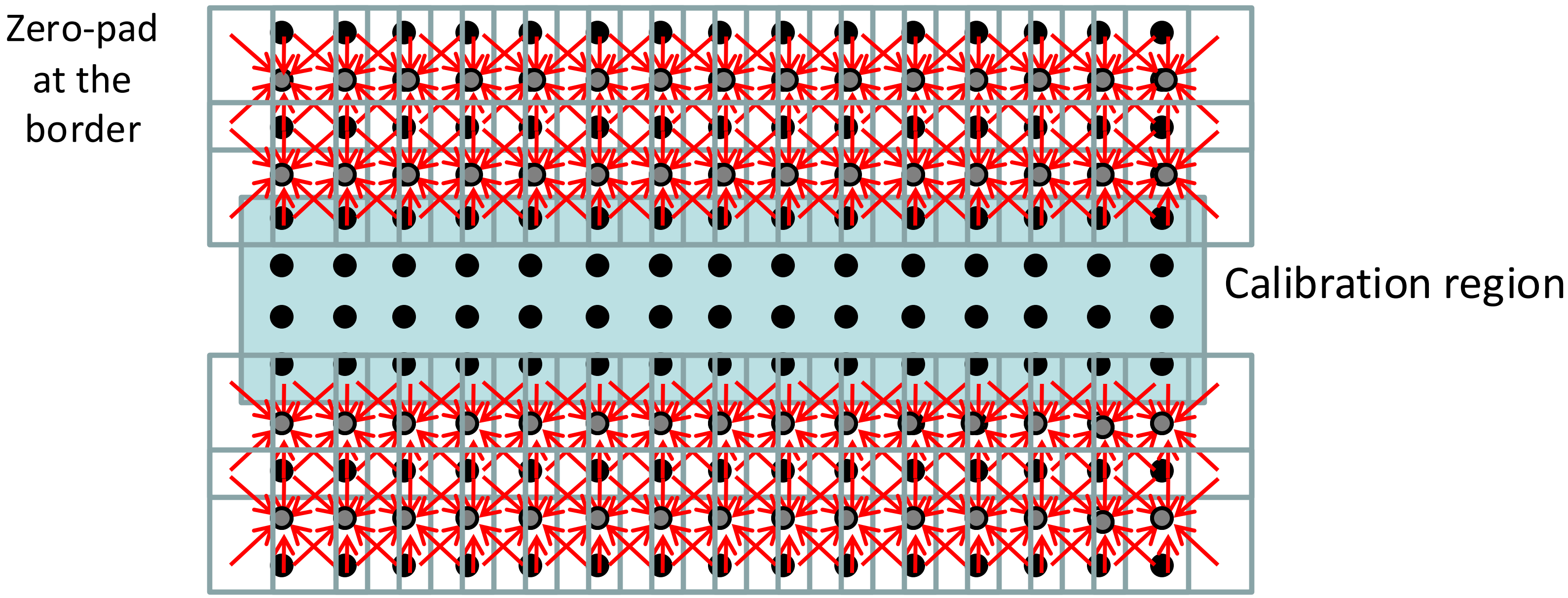
GRAPPA Algorithm

Compute GRAPPA weights from calibration data

Compute missing k-space data using GRAPPA weights

Reconstruct individual coil images

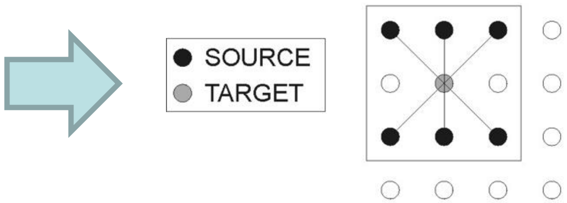
Combine coil images



GRAPPA example

Work through calibration step and reconstruction step

ACS: 4x4 matrix
Kernel size: 2x3
R_y=2
Coils n_c = 8



Calibration model: $\mathbf{T} = \mathbf{S}\mathbf{w}$

S: source matrix ($n_b \times n_k n_c$)
T: target matrix ($n_b \times n_c$)
w: weight matrix ($n_k n_c \times n_c$)

Invert to get the weights: $\mathbf{w} = (\mathbf{S}^{\mathrm{H}}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{H}}\mathbf{T}$

Find how many replica of the kernel you find in the ACS

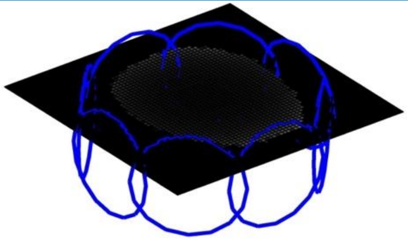
Pay attention to dimensions of matrices

Try to understand why the matrices have these dimensions

Bonus: Do you expect that this calibration step will work very well? Why/why not?

Reconstruction examples

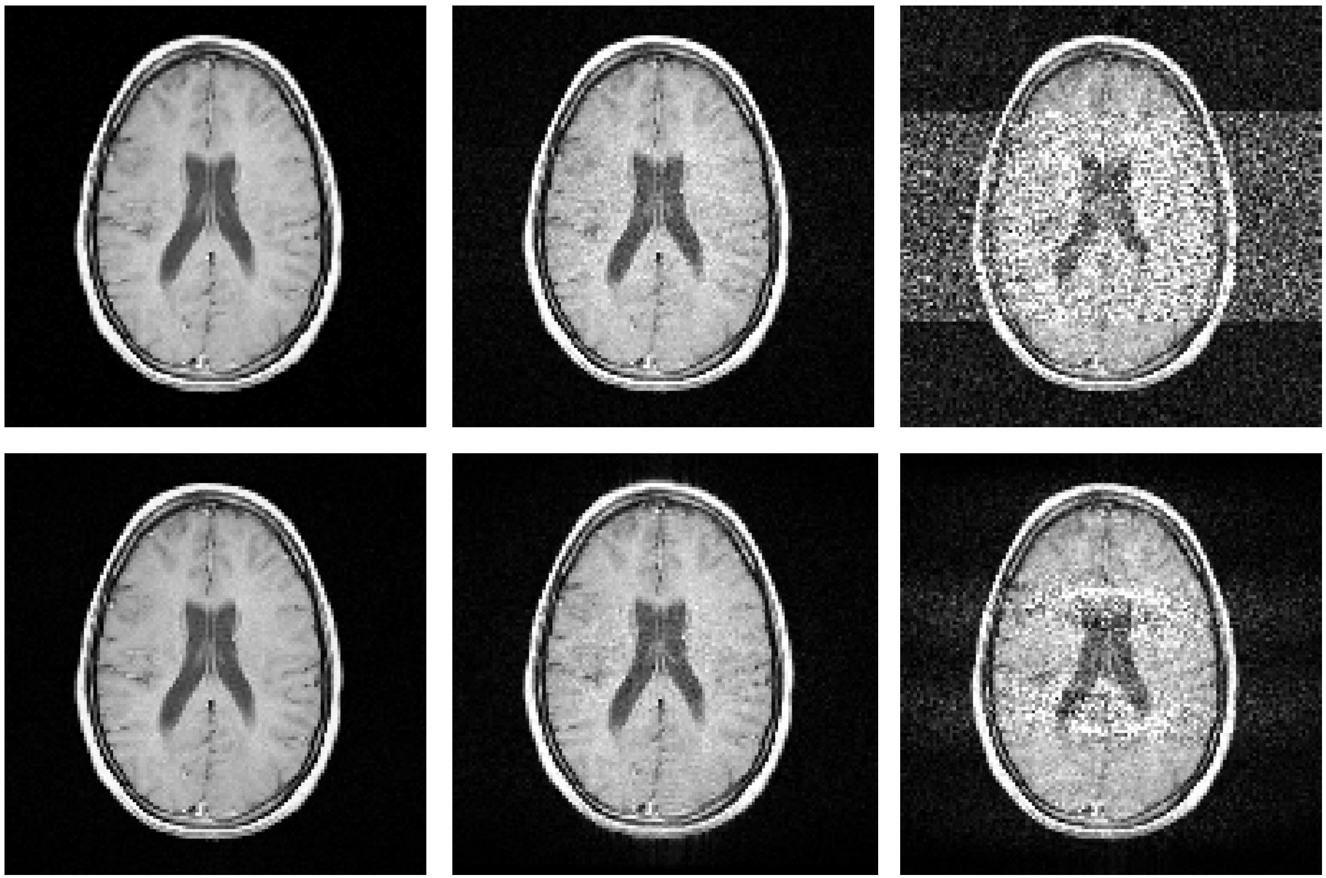
- Simulation of brain imaging acceleration
- 8-channel circular array coil



R=2 R=3 R=4

SENSE

GRAPPA



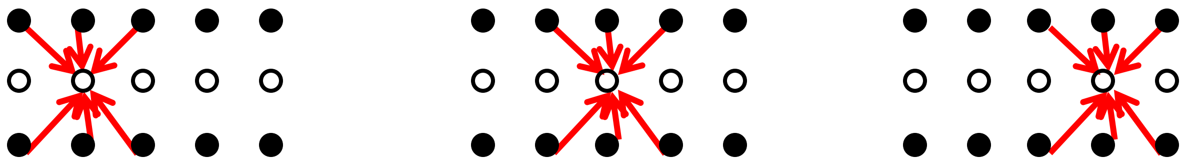
GRAPPA: More general k-space fitting

- Differences to SENSE
  - No need to estimate coil sensitivities
  - Small size of GRAPPA kernel in comparison to image size serves as implicit regularization
  - More robust than (unregularized) SENSE to inconsistencies between calibration and imaging data

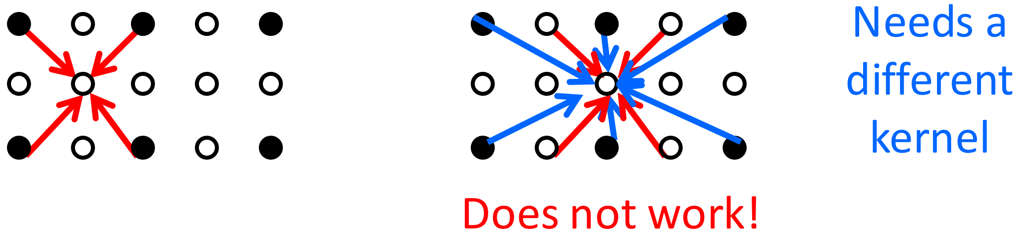
- Issues: Less flexible
  - Calibration region size
  - GRAPPA kernel size
  - Sampling geometry dependence

GRAPPA: More general k-space fitting

- Sampling geometry dependence
  - Simple for 1D acceleration (same weights work everywhere)



  - Harder for 2D acceleration (each geometry has its own weights)



  - Very cumbersome for irregular undersampling (non-Cartesian)

Summary

- Parallel MRI reconstruction in k-space
  - Coil-by-coil reconstruction
  - No need to estimate coil sensitivity maps
- GRAPPA algorithm
  - Unknown k-space points reconstructed as a linear combination of known k-space points
  - GRAPPA weights computed from calibration data
- Personal opinion: If you can, use SENSE. But GRAPPA is widely used, particularly on Siemens scanners so it is important to understand it