Parallel imaging I: Image-domain methods

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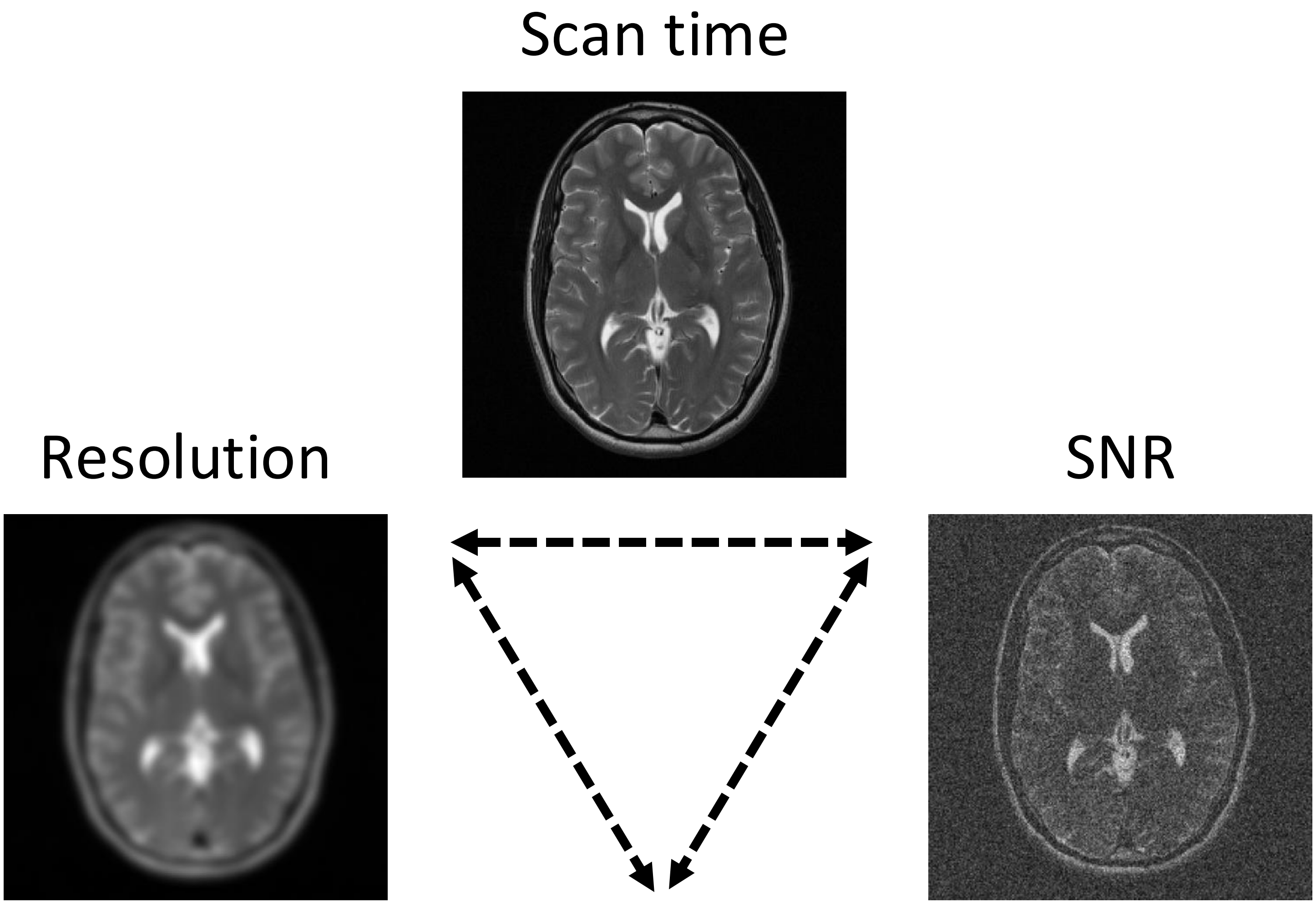
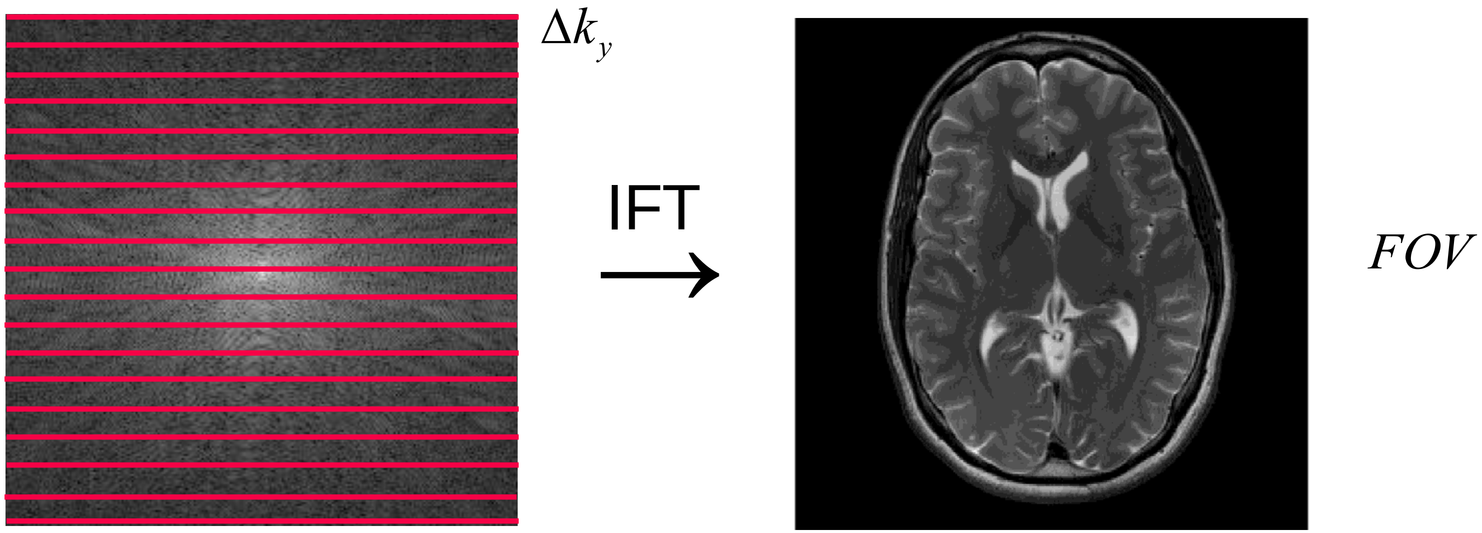
MR imaging constraints: The triangle of fate



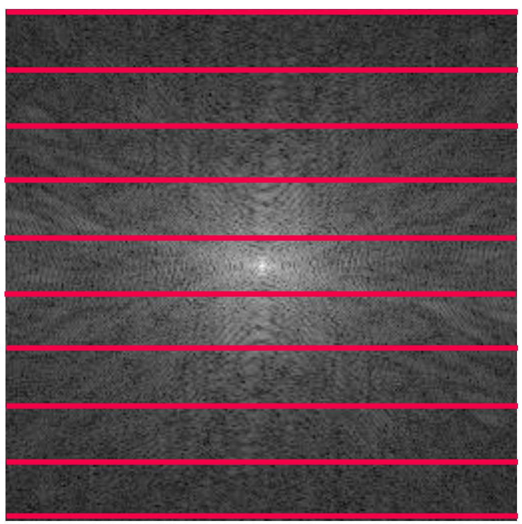
k-space undersampling

k-space undersampling



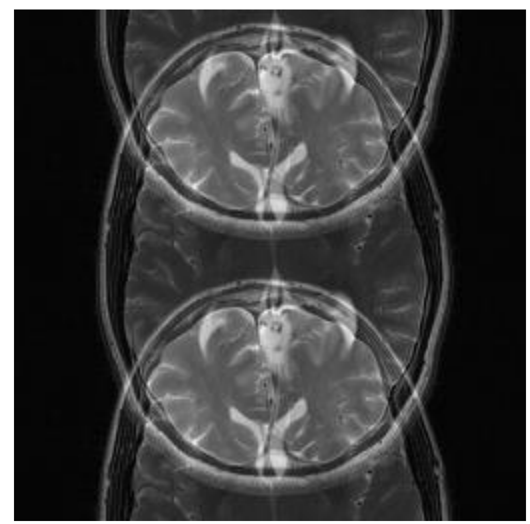
k-space undersampling

- Faster, no changes in gradient switching, but conventional Fourier reconstruction will result in aliasing artifacts



$2 \cdot \Delta k_y$

IFT $\longrightarrow$



$\frac{FOV}{2}$

Question: Can we undersample in the readout dimension?

k-space undersampling

- Reconstruction
  - Exploit data redundancies!

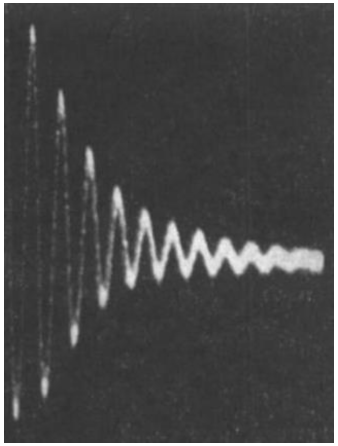
- Partial Fourier (constrained reconstruction), compressed sensing, machine learning,…
  - Image compressibility/sparsity
    (inherent redundancy)

- Parallel imaging
  - Multiple coils with different spatial sensitivities (real data redundancy)

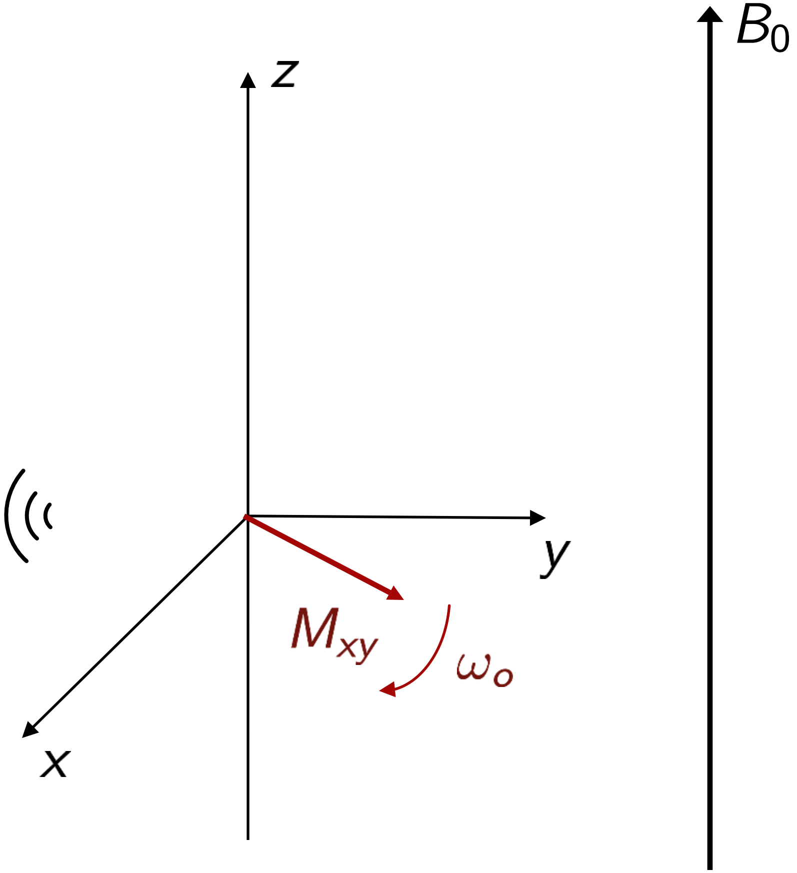
Multi-channel receive coils

Signal reception: MR receive coils

$$u_{ind} = -\frac{d\Phi}{dt}$$

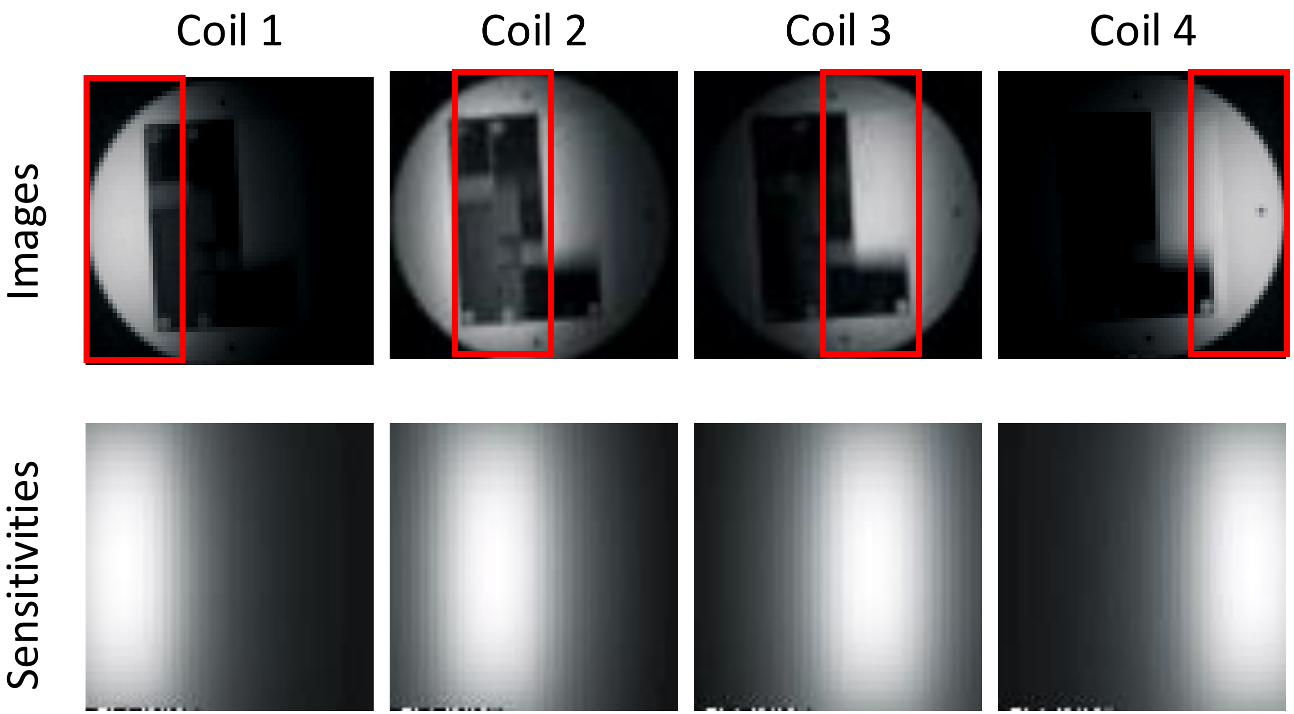


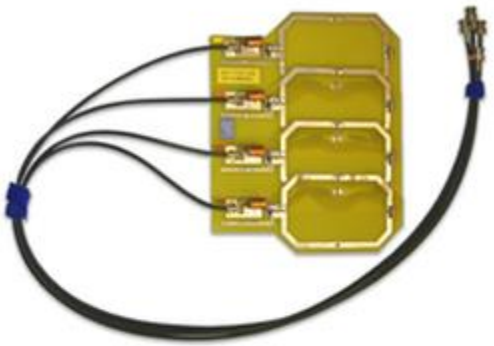
Hahn 1950



Multiple receiver coils

- Different spatial sensitivities

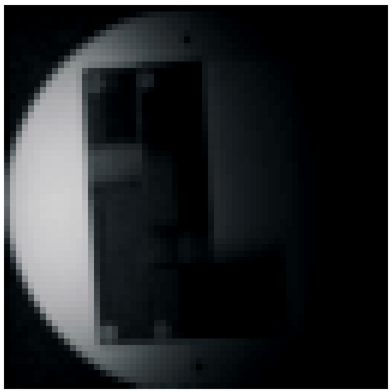




Multiple receiver coils

- Sensitivity-encoding equation

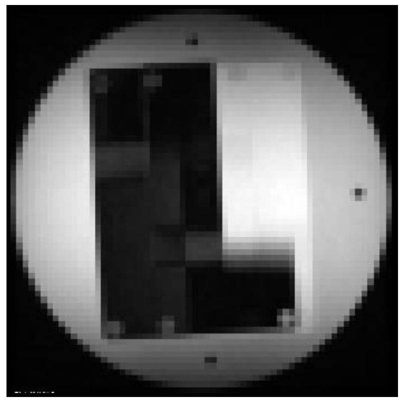
$m_i(r)$



Coil image

$=$

$f(r)$



Image

$\times$

$c_i(r)$

Coil sensitivity

$+\, n_i(r)$

Noise

Multiple receiver coils

- First used to improve SNR
  - What is the optimal coil combination?

- Matched-filter or least-squares combination

$$f(r) = \frac{\sum_{i=1}^{N_c} c_i^*(r) m_i(r)}{\sqrt{\sum_{i=1}^{N_c} |c_i(r)|^2}}$$

$m_i(r)$: single coil images
$c_i(r)$: coil sensitivities

Roemer FB et al. Magn Reson Med. 1990; 16(2):192-225.

Multiple receiver coils

- Matched-filter or least-squares combination
  - In matrix form (for each pixel)

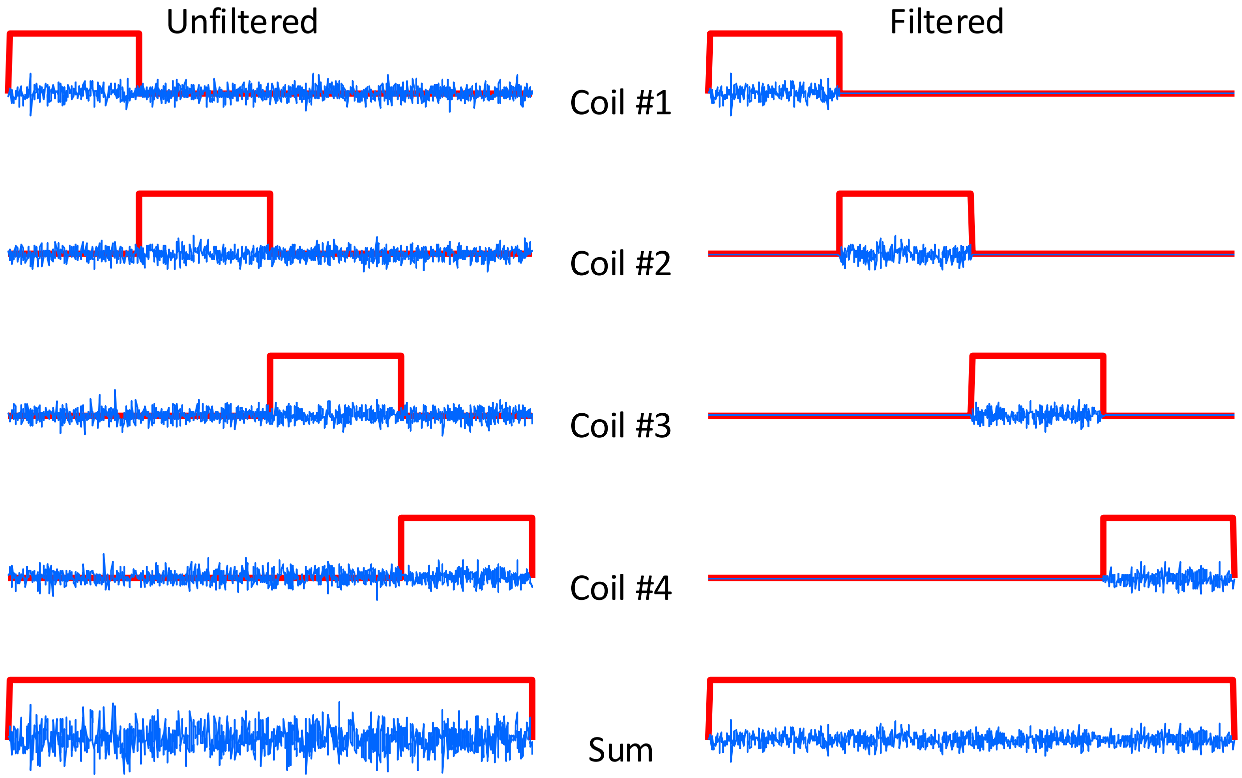
$$f = \left(\mathbf{C}^H \mathbf{C}\right)^{-1} \mathbf{C}^H \mathbf{m}$$

$$\mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_N \end{pmatrix}$$

Roemer FB et al. Magn Reson Med. 1990; 16(2):192-225.

Multiple receiver coils

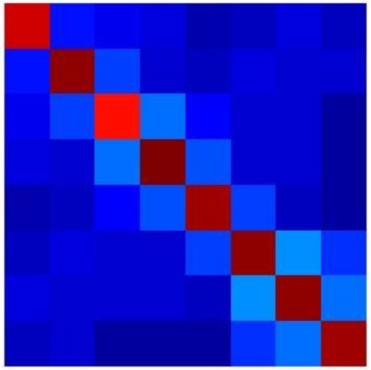
- Matched-filter or sensitivity-weighted combination
  - Effects on noise



Multiple receiver coils

- Noise signals from different coils are correlated

8-coil array



$\boldsymbol{\Psi}$: coil noise covariance matrix

- Least-squares combination using the covariance matrix

$$f = \left(\mathbf{C}^H \boldsymbol{\Psi}^{-1} \mathbf{C}\right)^{-1} \mathbf{C}^H \boldsymbol{\Psi}^{-1} \mathbf{m}$$

Multiple receiver coils

- Pre-whitening
  - Virtual coils with uncorrelated noise

$$\mathbf{m}_w = \mathbf{\Psi}^{-\frac{1}{2}}\mathbf{m}$$

$$\mathbf{C}_w = \mathbf{\Psi}^{-\frac{1}{2}}\mathbf{C}$$

- Solution

$$f = \left(\mathbf{C}_w{}^H\mathbf{C}_w\right)^{-1}\mathbf{C}_w^H\mathbf{m}_w$$

Multiple receiver coils

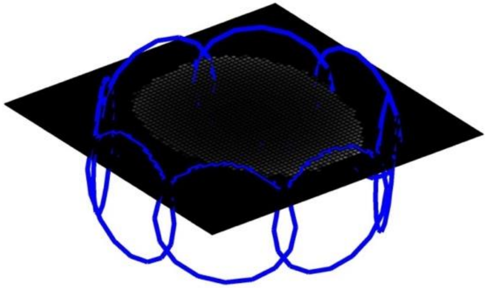
- Sum of squares
  - Approximation to the optimal combination
  - Images as coil sensitivities
  - SNR penalty of about 10%

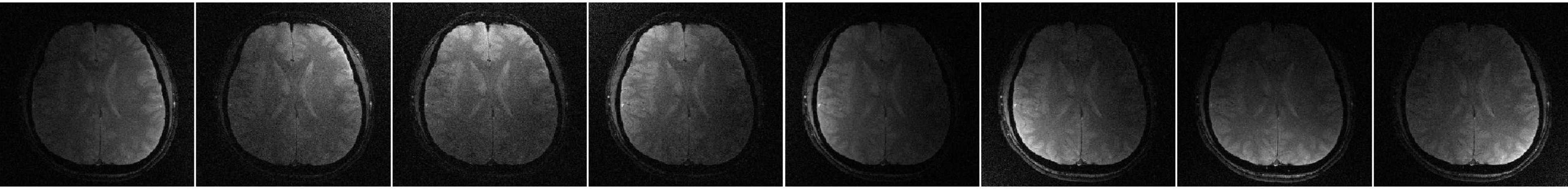
$$c_i(r) = m_i(r) \quad \Rightarrow \quad f(r) = \sqrt{\sum_{i=1}^{N_c} \left| m_i(r) \right|^2}$$

$$\mathbf{c} = \mathbf{m} \quad \Rightarrow \quad f = \sqrt{\mathbf{m}^H \mathbf{m}}$$

Signal combination example

- 8-coil circular array

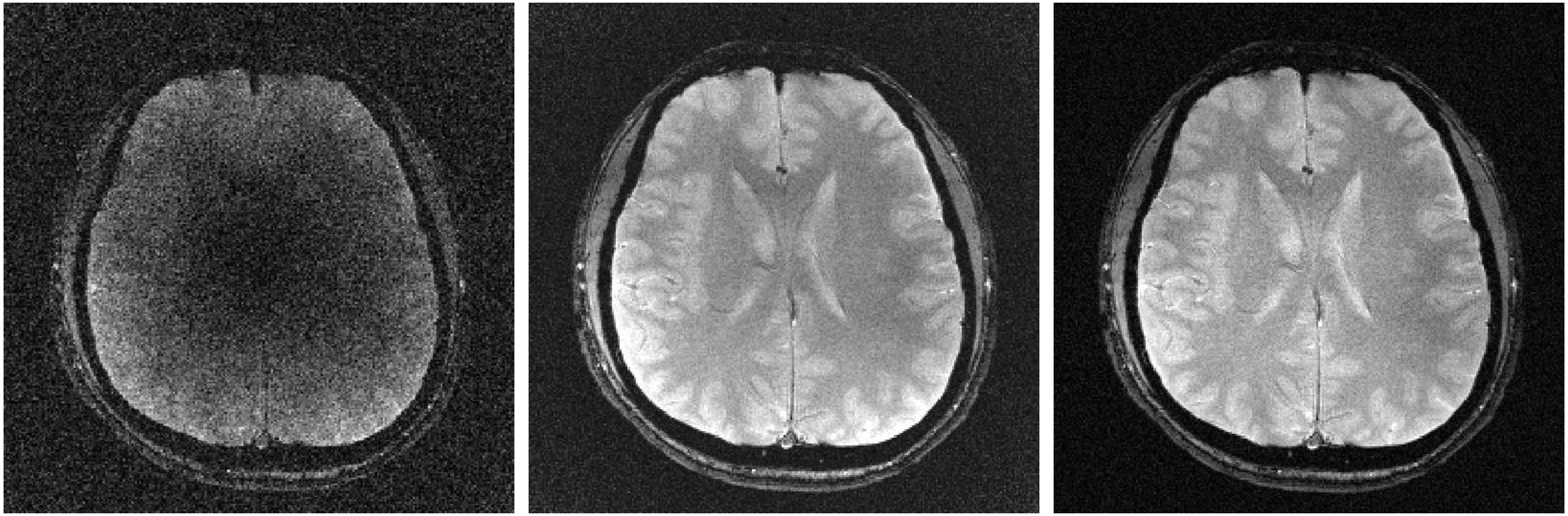




Complex sum

Sum of squares

Least-squares (matched-filter)



Signal combination example

- Matched-filter combination

Without Ψ

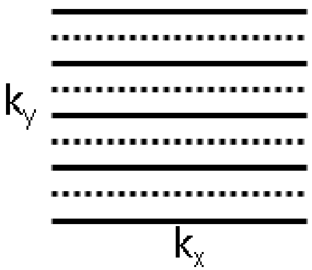
With Ψ

Parallel imaging

Parallel imaging

- Multiple coils enable acceleration of MRI data acquisition
  - Multiple coil data are redundant!

- Regular k-space undersampling

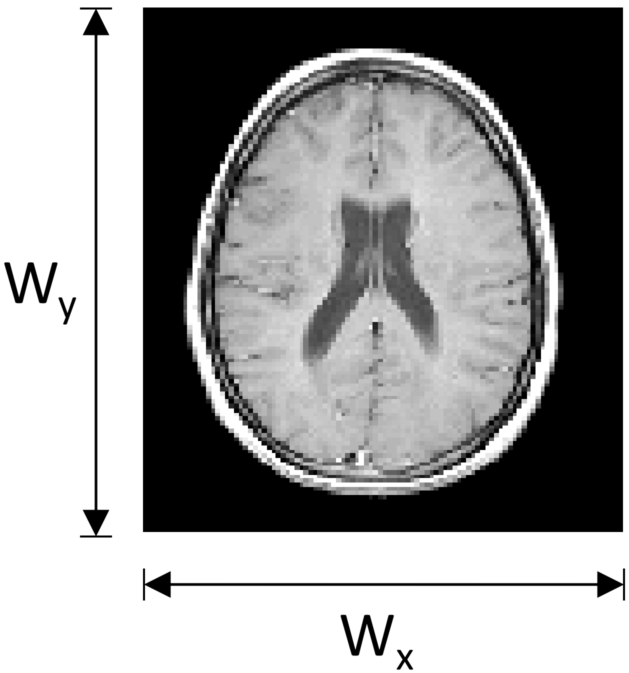


- Reconstruction using matrix inversion

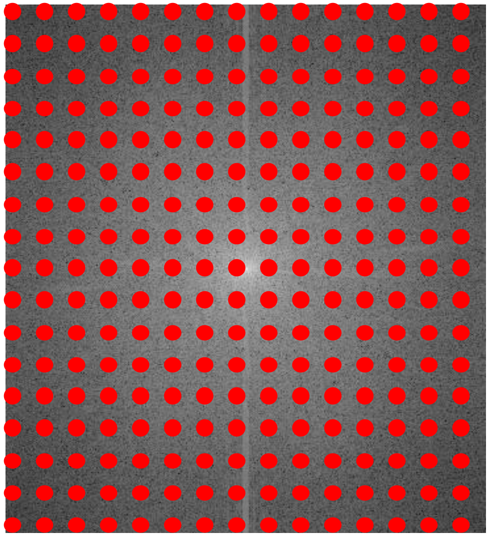
Sodickson DK, Manning WJ. Magn Reson Med. 1997; 38: 591-603
Pruessmann KP et al. Magn Reson Med 1999; 42: 952-962

Recap: k-space sampling density and image FOV

Image



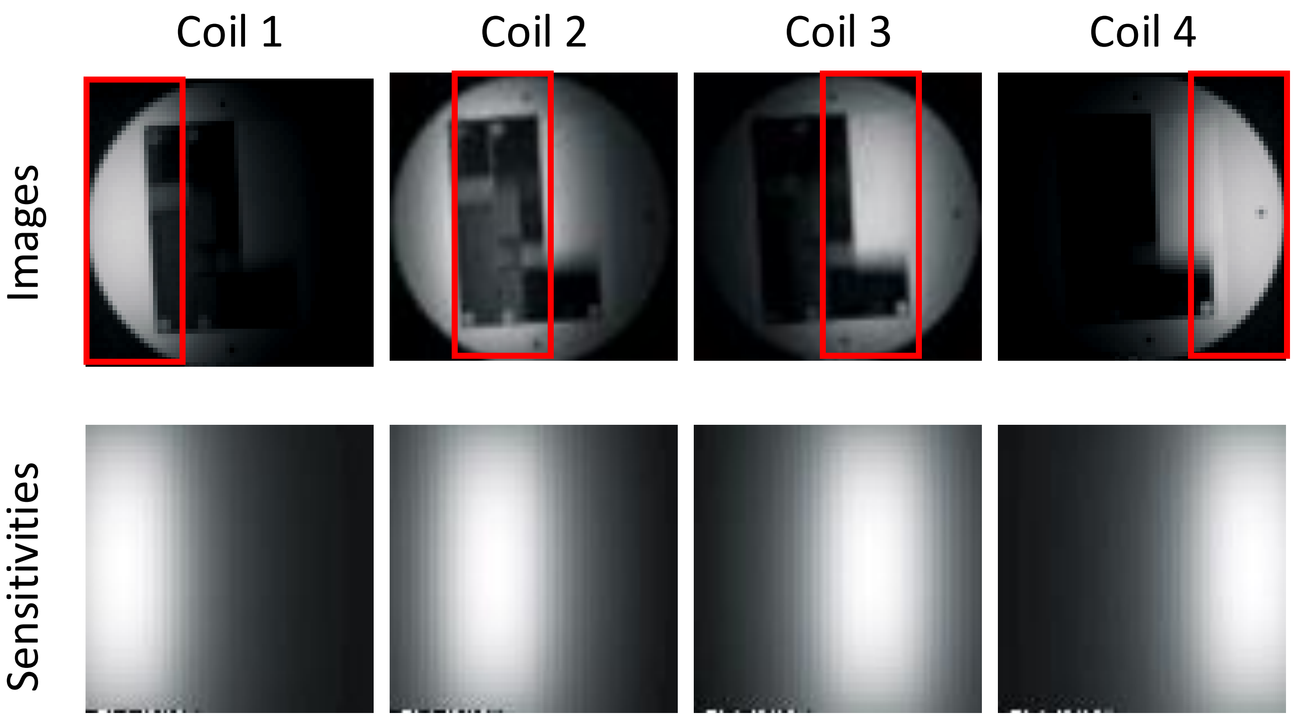
k-space

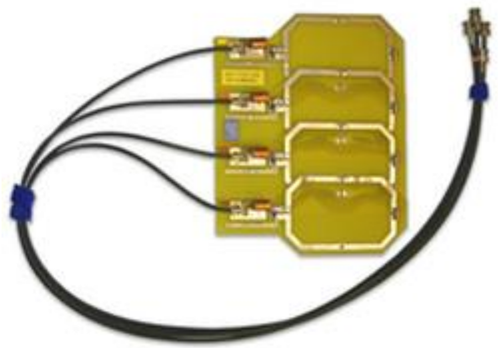


Nyquist rate:

$$\Delta k_x = \frac{1}{W_x}; \quad \Delta k_y = \frac{1}{W_y}$$

Spatial encoding of receive coils



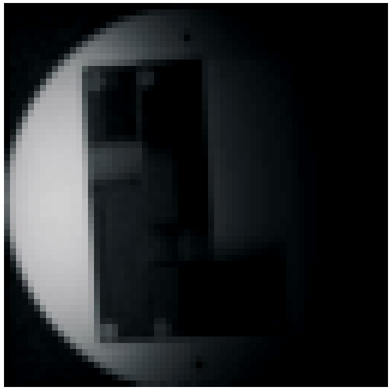


Coils also perform spatial encoding

- Pixels close to the coil are bright
- Pixels far from the coil are dark

Back to sensitivity-encoding equation

$m_i(r)$ $\quad$ $f(r)$ $\quad$ $c_i(r)$



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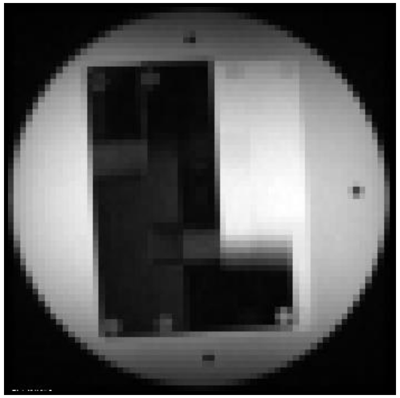
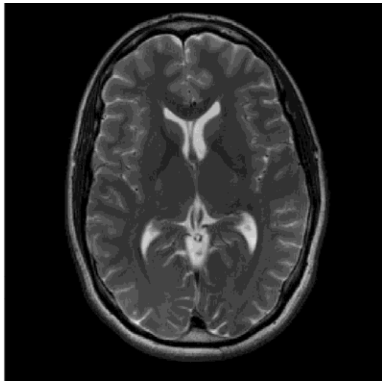


x

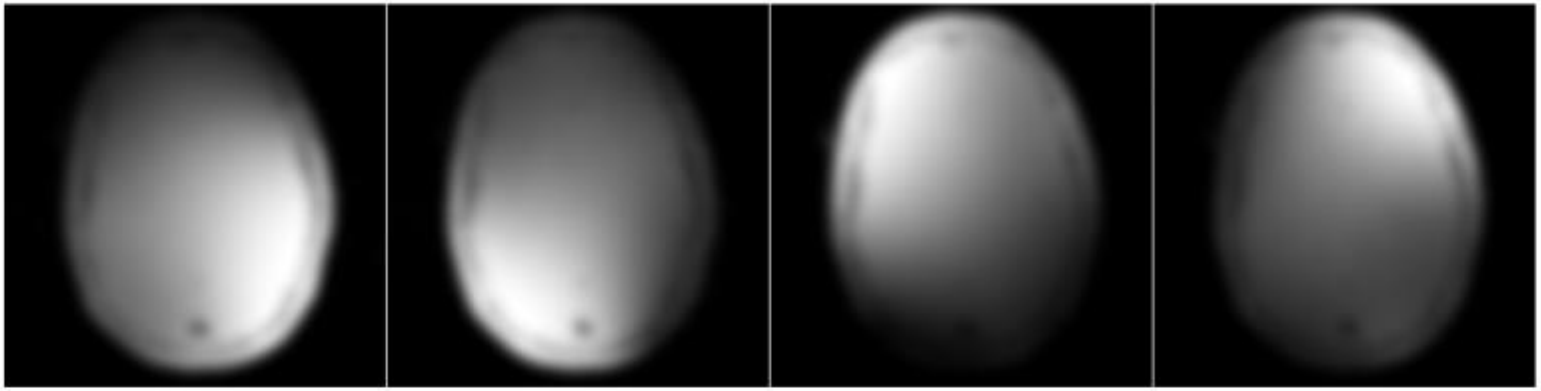
$$m_i(r) = f(r) \times c_i(r)$$

Pruessmann et al., MRM 42: 952-962 (1999)

Cartesian SENSE

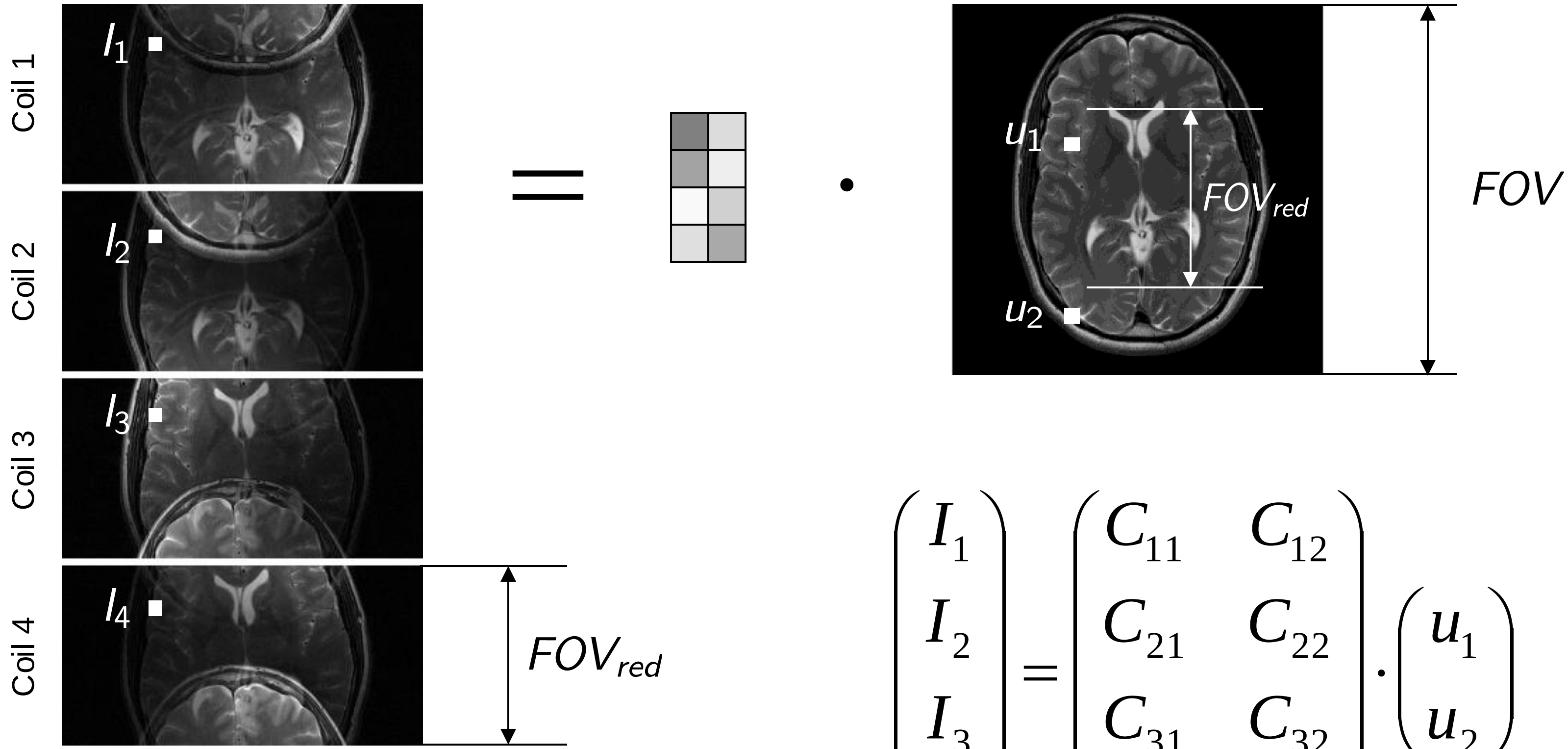


- T2 weighted brain scan
- 4-Channel receive coil
- Sensitivities are known



Pruessmann et al., MRM 42: 952-962 (1999)

Cartesian SENSE

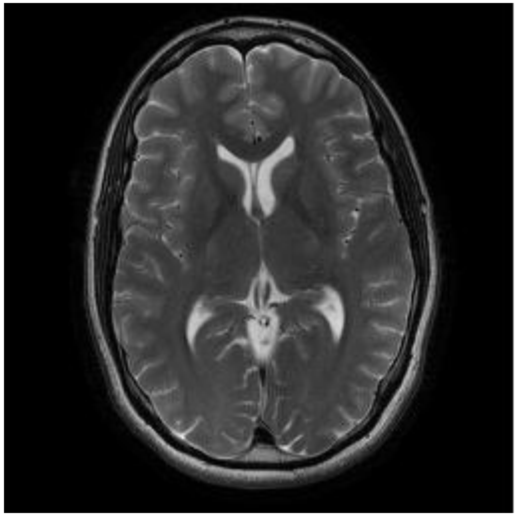


$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Parallel Imaging: SENSE

$$u = (C^H C)^{-1} C^H I$$

SENSE, R=2



SNR penalty in parallel imaging

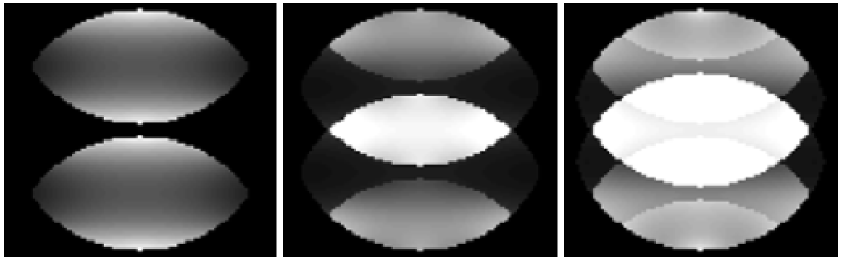
$$SNR_{acc} = \frac{SNR_{no-acc}}{g\sqrt{R}}$$

- g-factor: noise amplification due to ill-conditioning of the encoding matrix

$$g(r) = \sqrt{\left(\mathbf{E}^H \mathbf{\Psi}^{-1} \mathbf{E}\right)^{-1} \mathbf{E}^H \mathbf{\Psi}^{-1} \mathbf{E}_{(r)}}$$

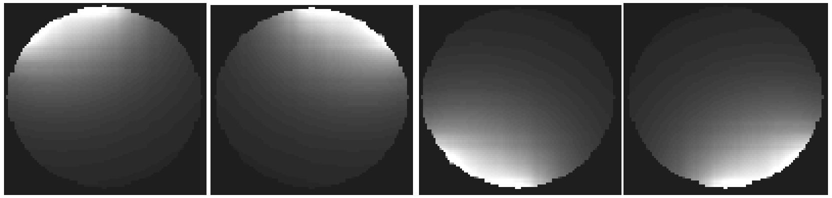
	R=2	R=3	R=4

g-factor



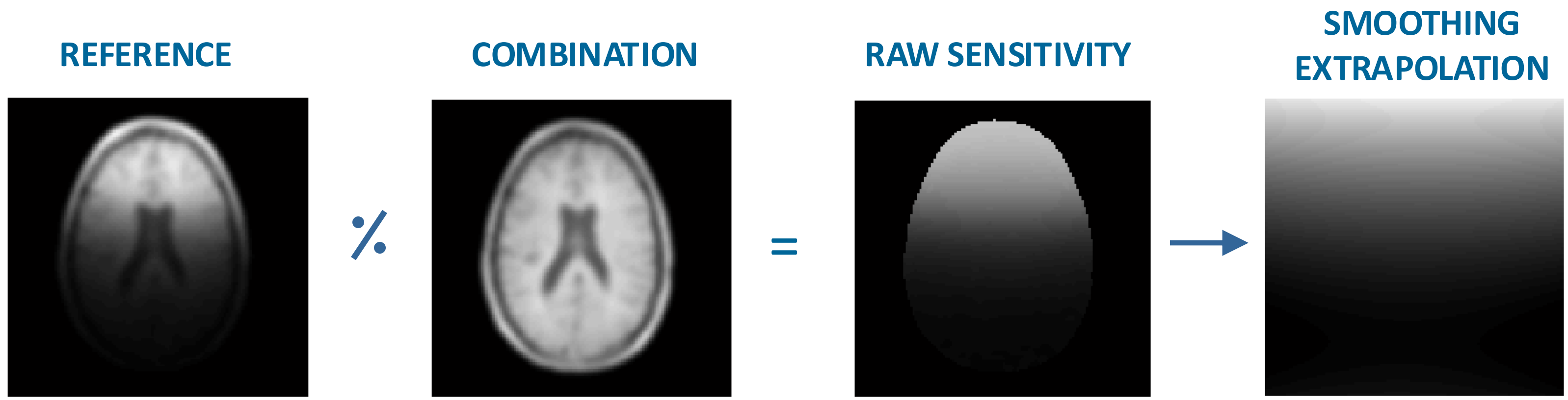
1<g<1.2 mean=1.07 max=1.2	1<g<3 mean=2.2 max=3.1	1<g<5 mean=5.4 max=8.6

Coil sensitivities



Coil sensitivity estimation for SENSE

- Estimation of pure coil sensitivities (Pruessmann et al. MRM 1999).
  - Separate low resolution image for each coil.



How to reduce noise amplification?

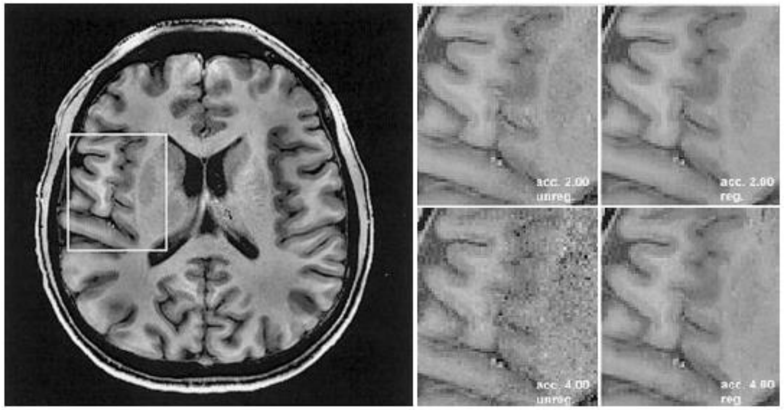
- Use more coils
- Improve coil array design
- Regularization of the inverse reconstruction
- 2D acceleration instead of 1D acceleration (3D imaging)

Regularization of the inverse reconstruction

- Constrain the inverse problem to reduce noise amplification and control numerical instabilities

- Method 1: Tikhonov regularization
  - Constrain the power of the solution

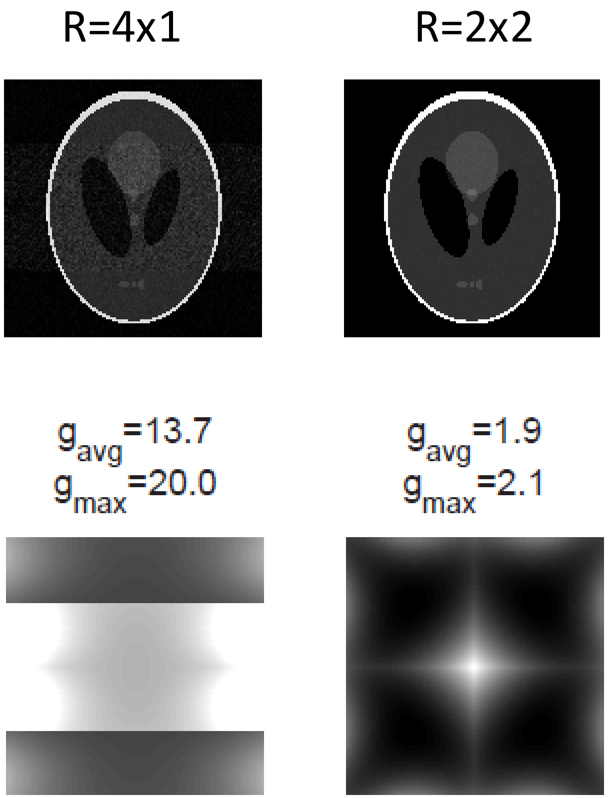
$$\hat{\mathbf{m}} = \min_{\mathbf{m}} \left\{ \left\| \mathbf{E}\mathbf{m} - \mathbf{s} \right\|_2^2 + \lambda \left\| \mathbf{m} \right\|_2 \right\} = \left(\mathbf{E}^H \mathbf{E} + \lambda \mathbf{I} \right)^{-1} \mathbf{E}^H \mathbf{s}$$

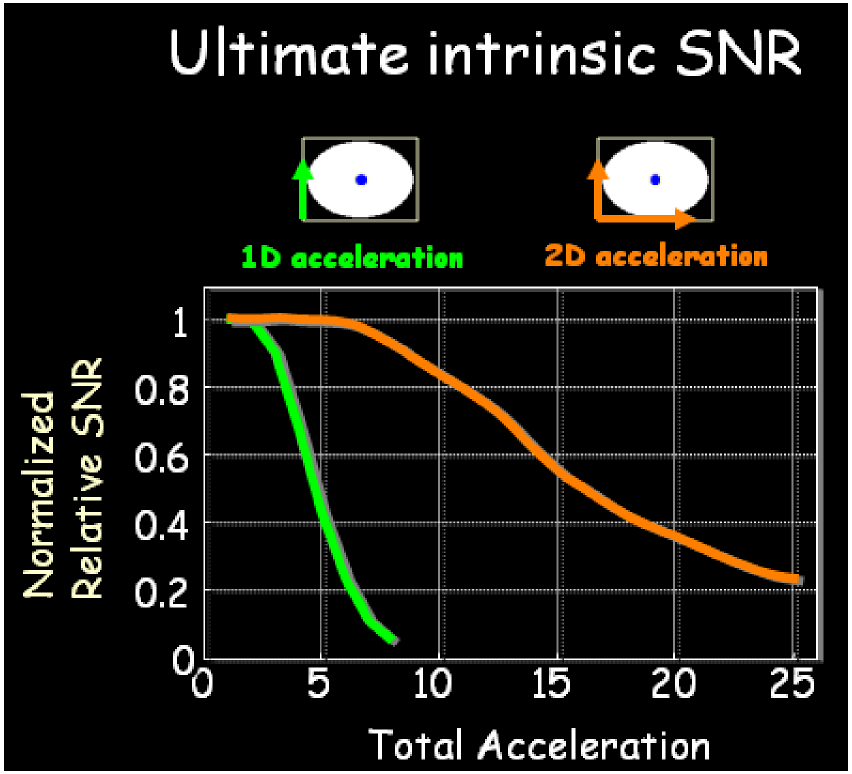


Lin FH et al, Magn Reson Med 2004; 51:559-567

2D acceleration Vs. 1D acceleration

- 2D acceleration reduces g-factor

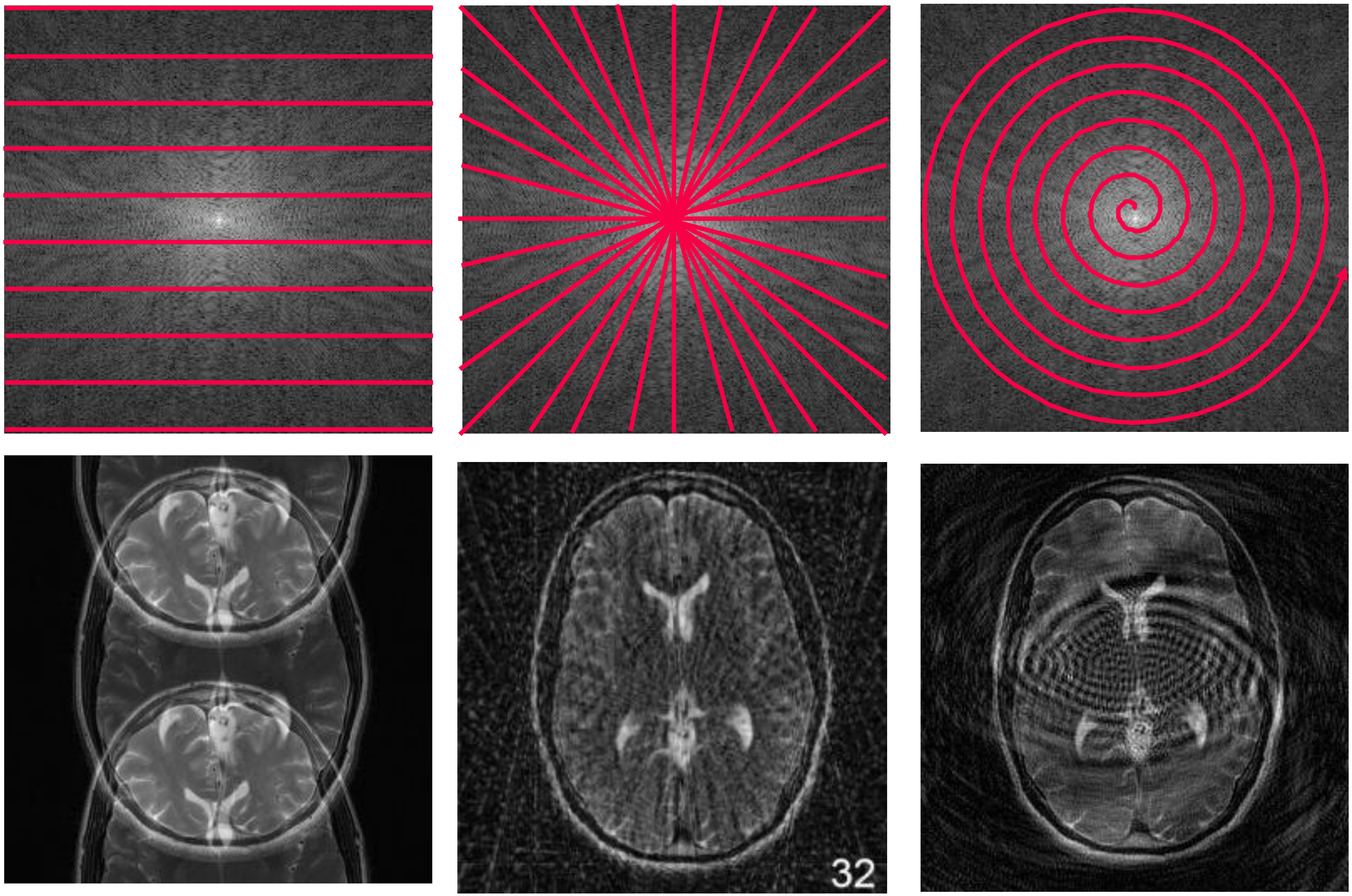




Ohliger MA et al. MRM 2003;50:1018-30

Non-Cartesian undersampling

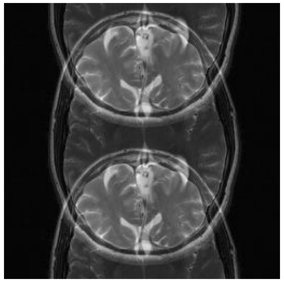
Cartesian Radial Spiral



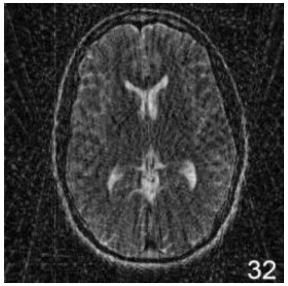
Non-Cartesian SENSE

- Decoupling is lost
- Each pixel is aliased with all other pixels
  - e.g. streaks in undersampled radial imaging
- Need to invert the full encoding equation
- Calls for an iterative algorithm
  - No explicit matrix inverse
  - Matrix-vector multiplications only

Cartesian



Radial



Summary

- Fast, rapid or accelerated MRI
  - k-space undersampling
  - Reconstruction is more challenging, but more fun
  - Exploit redundancies in the acquired data

Summary

- Parallel imaging
  - Exploit additional encoding provided by multiple receiver coils with different sensitivities
  - SNR penalty
  - SENSE (image-domain)
    - Unfolding images using coil sensitivities
    - Matrix inversion
  - SMASH, GRAPPA (k-space)
    - Next lecture