Computational MRI

Parallel Imaging III

Non-Cartesian Imaging and Iterative Reconstruction

Friedrich-Alexander-Universität
Computational Imaging Lab

NYU

Overview

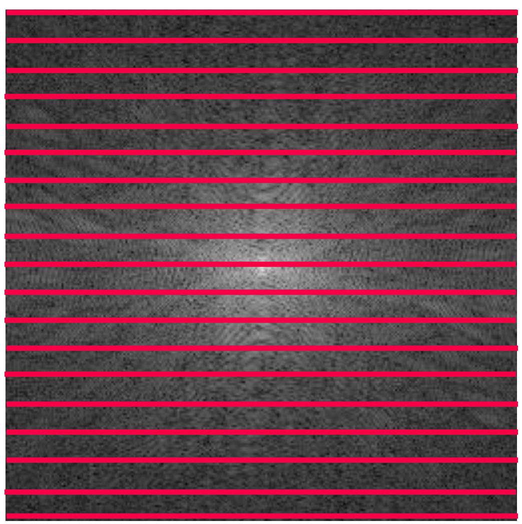
- Non-Cartesian parallel imaging
- Iterative algorithms:
  - Gradient descent
  - Conjugate gradient
- CG-SENSE image reconstruction

Magnetic Resonance in Medicine 46:638–651 (2001)

Advances in Sensitivity Encoding With Arbitrary *k*-Space Trajectories

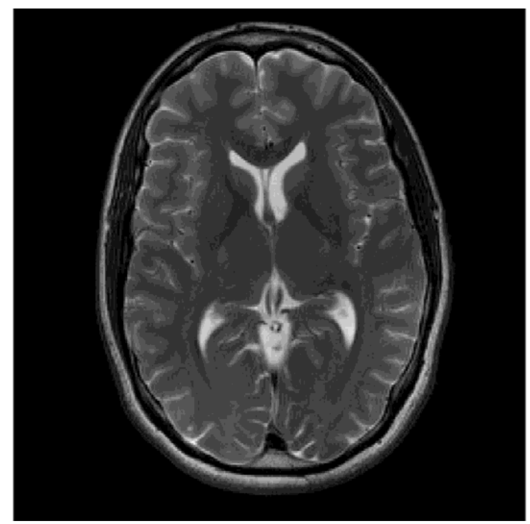
Klaas P. Pruessmann,[1] Markus Weiger,[1] Peter Börnert,[2] and Peter Boesiger[1*]

Cartesian subsampling



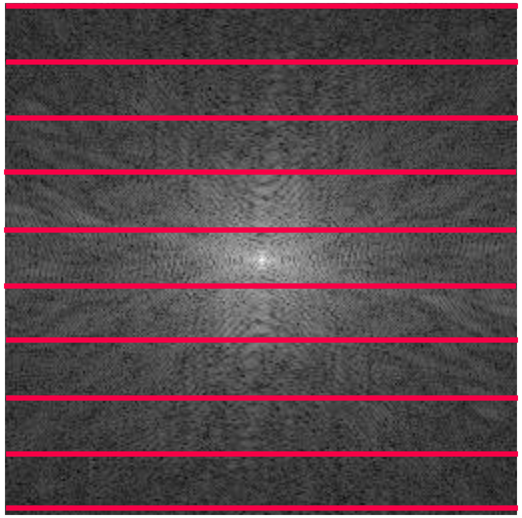
Δk_y

IFT $\longrightarrow$

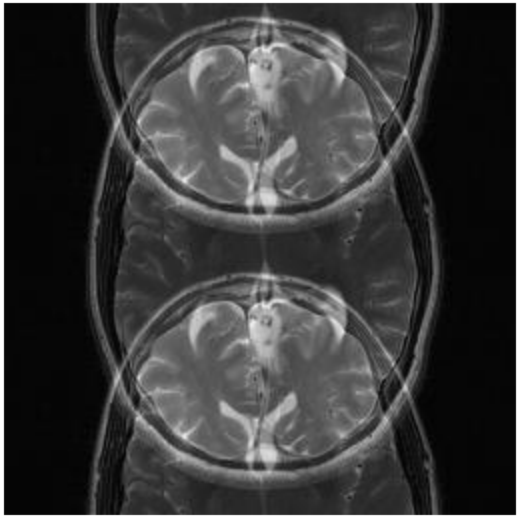


FOV

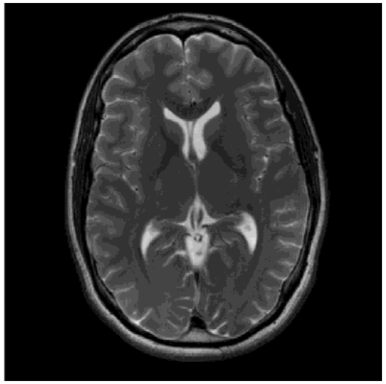
Cartesian subsampling



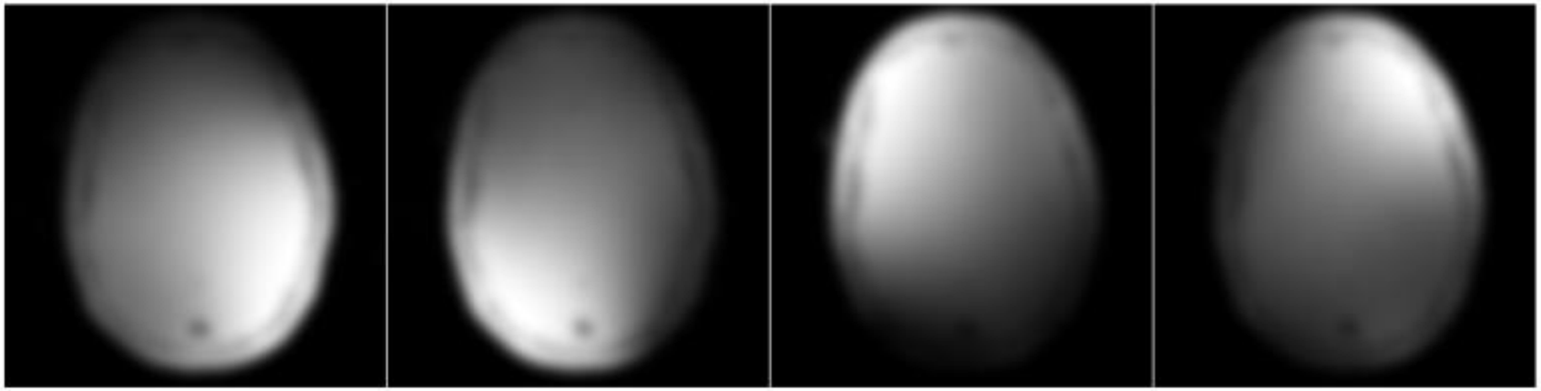
IFT $\longrightarrow$



Cartesian SENSE

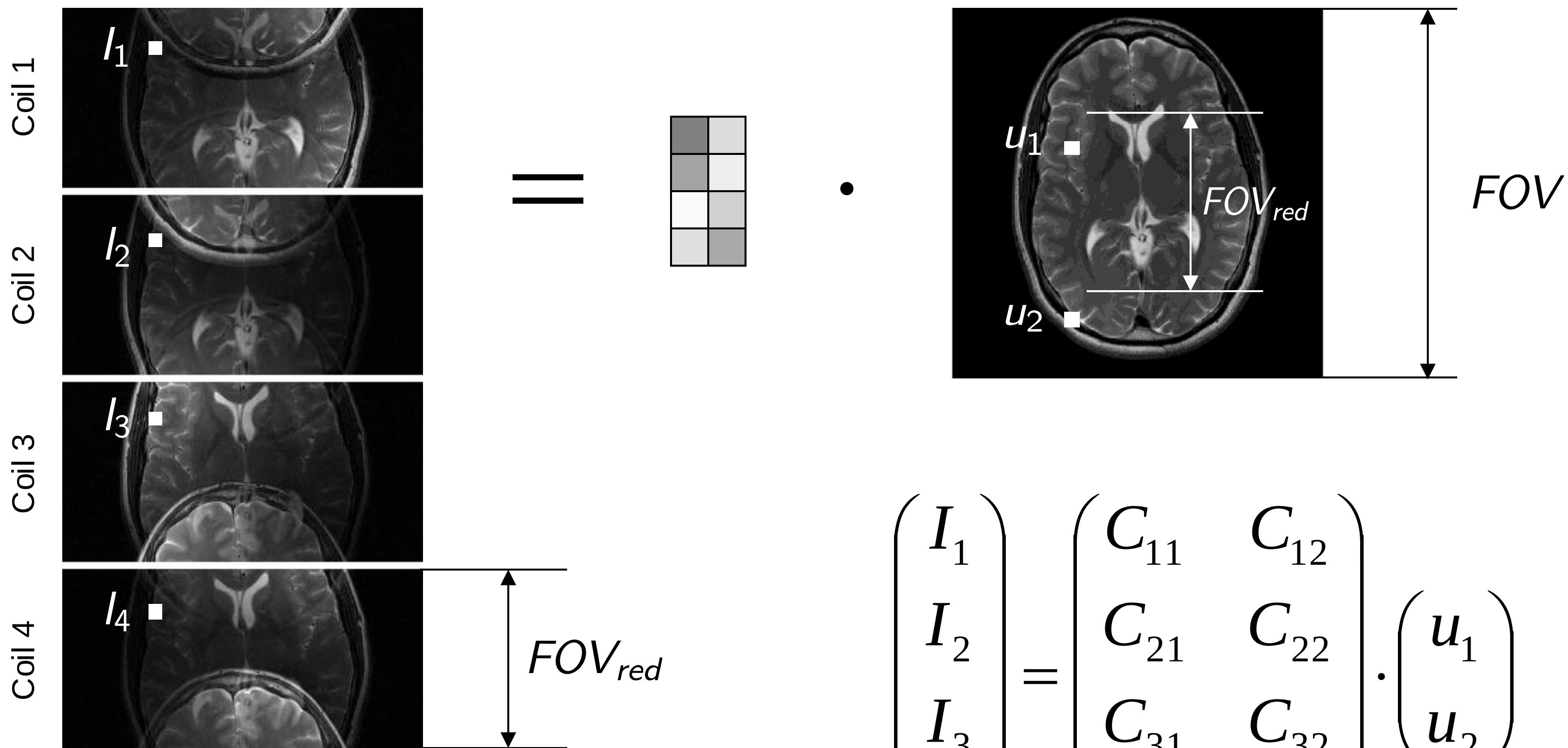


- T2 weighted brain scan
- 4-Channel receive coil
- Sensitivities are known



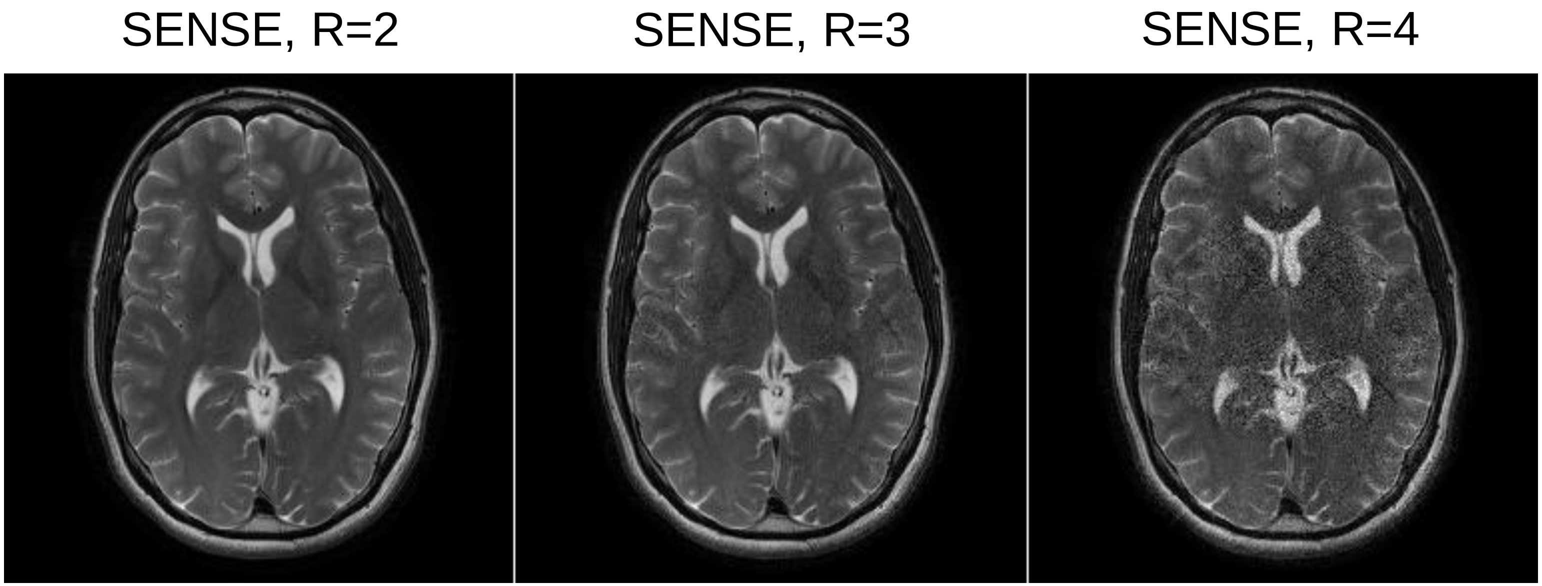
Pruessmann et al., MRM 42: 952-962 (1999)

Cartesian SENSE

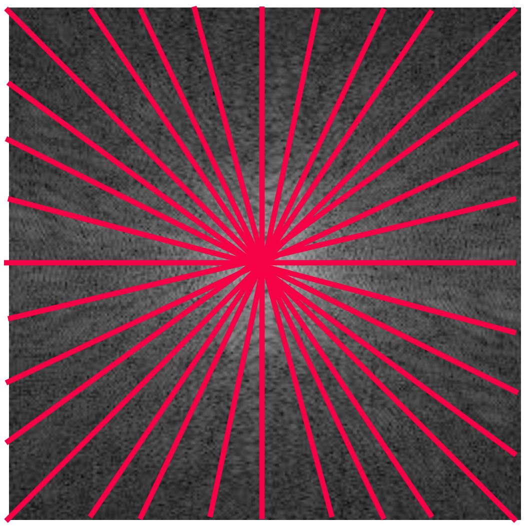


$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

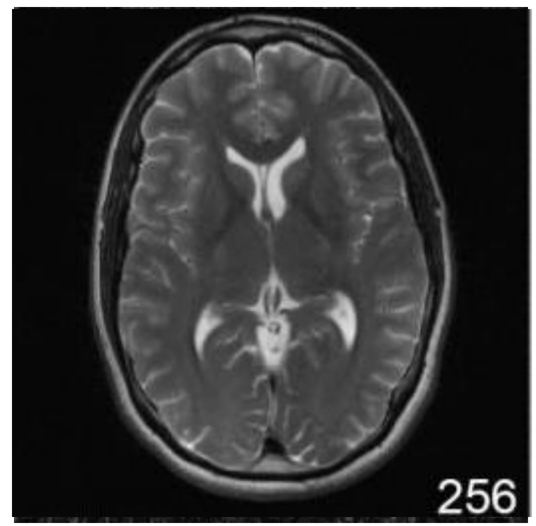
Parallel Imaging: SENSE



Radial Subsampling



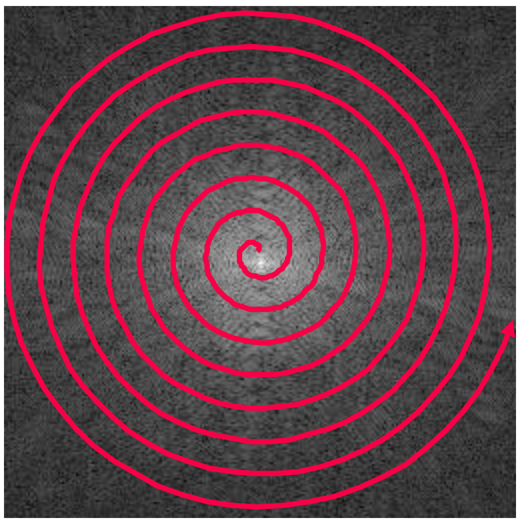
$$\xrightarrow{\mathcal{F}_{\Omega}^{-1}}$$



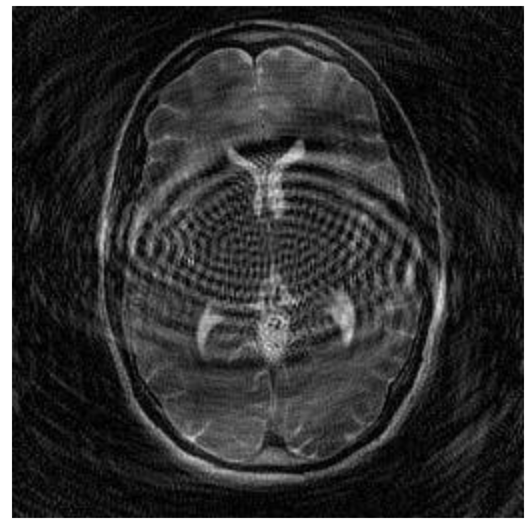
R=8

Different structure of aliasing

Spiral Subsampling



$$\xrightarrow{\mathcal{F}_{\Omega}^{-1}}$$



Different structure of aliasing

Building the encoding operator

$$g_k(k_x, k_y) = \int\int c_k(x,y)e^{-i(k_x x + k_y y)}u(x,y)dxdy$$

Discretization

$$g_k(k_x, k_y) = \sum\sum c_k(x,y)e^{-i(k_x x + k_y y)}u(x,y)$$

Forward operator,
encoding matrix

$$\vec{g} = \mathcal{F}_\Omega C\vec{u} = K\vec{u}$$

Image reconstruction as inverse problem

Encoding: $\vec{g} = K\vec{u}$ linear system of equations

Reconstruction:

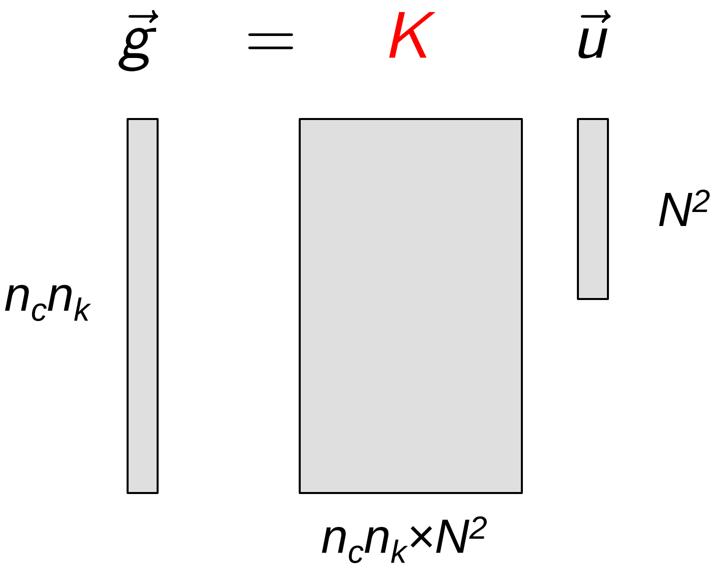
- Solution of the system of equations
- Inversion of the encoding matrix

$$\vec{u} = K^{-1}\vec{g}$$

Inversion of the encoding matrix

Direct methods: Pseudoinverse: $\vec{u} = (K^H K)^{-1} K^H \vec{g}$

Matrix K can become very large!



N=256
8 coils
256 radial spokes
256 samples per spoke
Dimensions of K: 528.384 x 65.536
139 GB of memory

Use iterative methods
Gradient descent
Conjugate Gradient algorithm

Magnus and Stiefel, Journal of Research of the National Bureau of Standards 1952
Pruessmann MRM 2001

Iterative reconstruction

Given:

- Forward Operator: K
- Measured k-space data: $\vec{g}$

Forward problem

$$K\vec{u} = \vec{g}$$

Iterative reconstruction

$$\hat{\vec{u}} = \arg\min_{\vec{u}} \left\| K\vec{u} - \vec{g} \right\|_2^2$$

Iterative optimization: Gradient descent

$$K\vec{u} = \vec{g}$$

$$\hat{\vec{u}} = \arg\min_{\vec{u}} \|K\vec{u} - \vec{g}\|_2^2$$

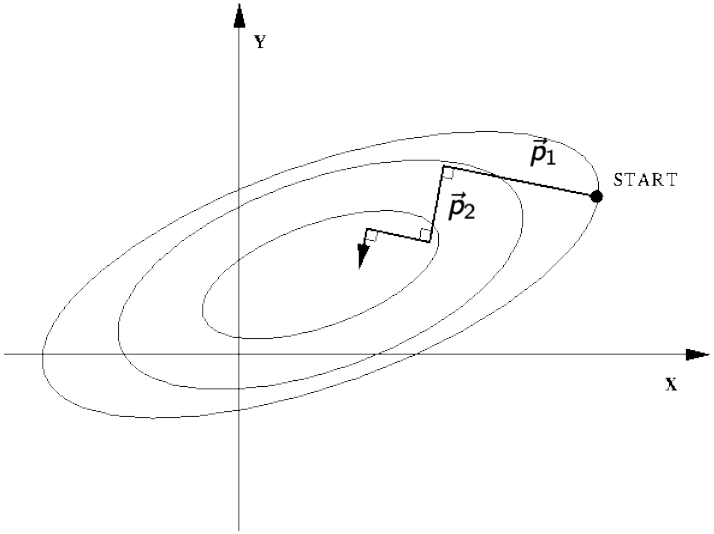
$$\vec{u}_{n+1} = \vec{u}_n - t\frac{\partial}{\partial \vec{u}}\|K\vec{u} - \vec{g}\|_2^2$$

$$\frac{\partial}{\partial \vec{u}}\|K\vec{u} - \vec{g}\|_2^2$$

$$\frac{\partial}{\partial \vec{u}}\|K\vec{u} - \vec{g}\|_2^2 = 2K^T(K\vec{u} - \vec{g})$$

Exercise :-)

Steepest descent



$$\vec{p}_k^T \vec{p}_{k+1} = 0$$

GD implementation (exercise)

$$K : g_l = \mathcal{F}(c_l \odot u)$$

$$K^T : u = \sum_l c_l^* \odot \mathcal{F}^T(g_l)$$

$$u_0 = 0$$

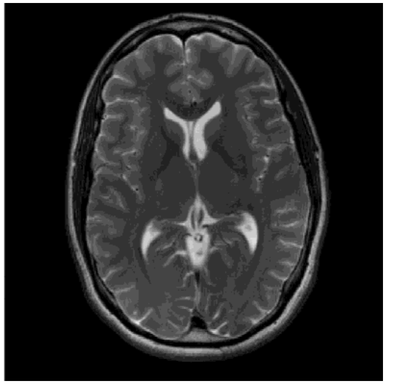
$$t > 0$$

$$for : n = 1 : maxit$$

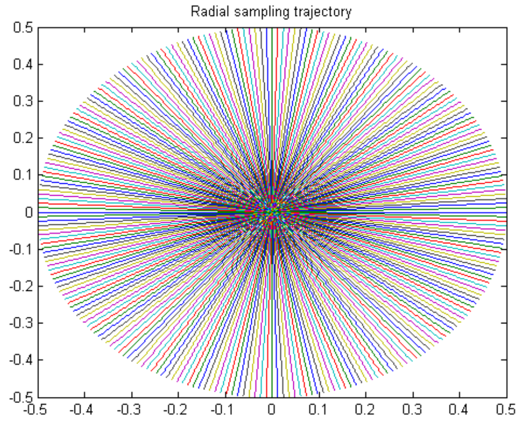
$$u_{n+1} = u_n - t(2K^T(Ku_n - g))$$

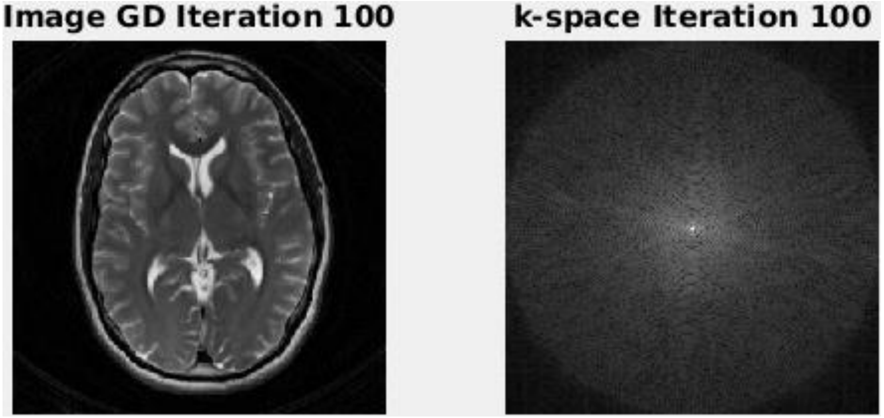
Exercise example

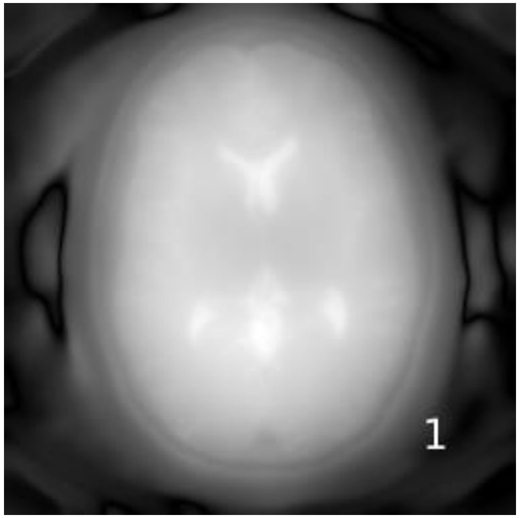
- T2 weighted brain scan
- 3T System
- 4 channel head coil
- TSE: Sequence parameters:
  - TR 5000ms
  - TE 99ms
  - Turbo Factor 10
  - Matrix Size 256x256
  - In Plane Resolution 0.9mmx0.9mm
  - Slice Thickness 4mm



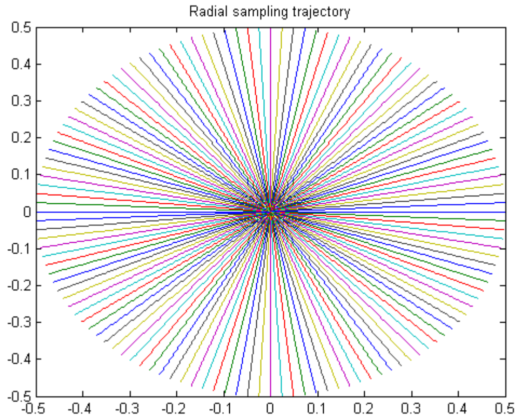
GD radial trajectory, 128 projections, R=2

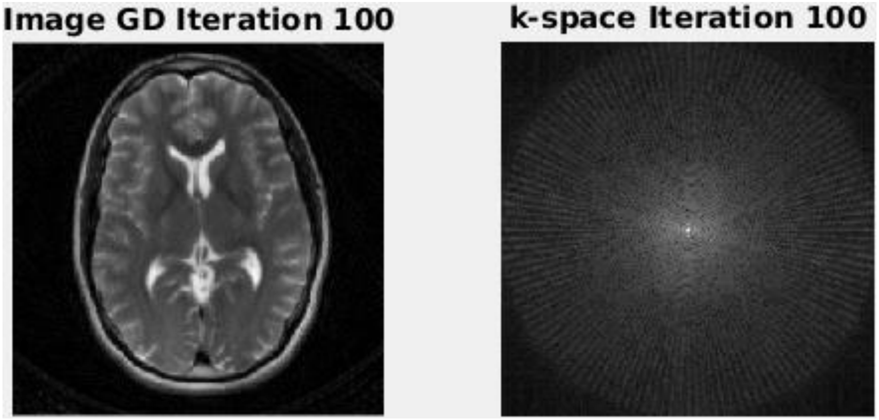


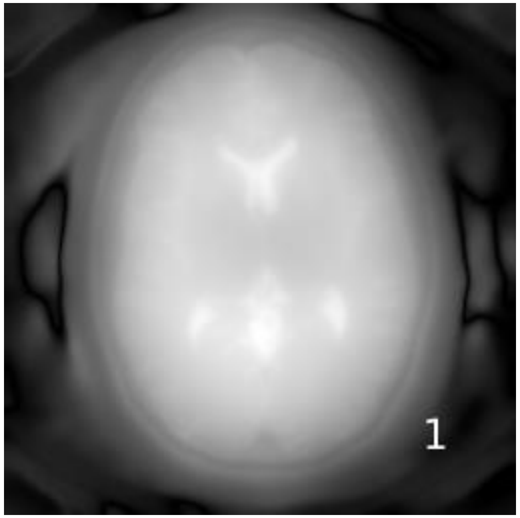




GD radial trajectory, 64 projections, R=4







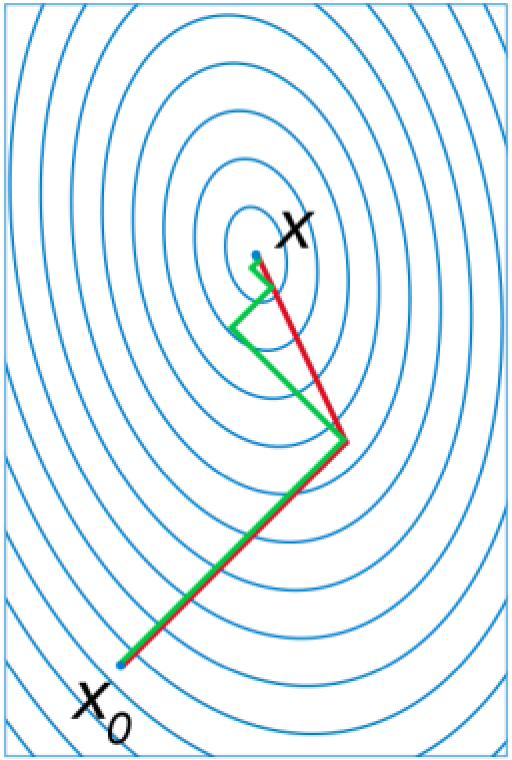
The conjugate gradient method

Minimize $f(\vec{u})$ in subspace spanned by search directions that are K orthogonal (conjugate):

$$\vec{p}_k^T K \vec{p}_{k+1} = 0$$

Converges to exact solution in n steps (n is matrix size)

Source: Wikipedia



n=2 quadratic form
Gradient descent (red)
Conjugate gradient minimization (green)

The conjugate gradient method

$$\vec{r}_k = \vec{g} - K\vec{u}_k$$

Residual

$$\vec{p}_k = \vec{r}_k - \sum_{i<k} \frac{\vec{p}_i^T K \vec{r}_k}{\vec{p}_i^T K \vec{p}_i} \vec{p}_i$$

CG search direction

$$\vec{u}_{k+1} = \vec{u}_k + \alpha \vec{p}_k$$

Update step

$$\alpha_k = \frac{\vec{p}_k^T \vec{r}_{k-1}}{\vec{p}_k^T K \vec{p}_k}$$

Step length

The conjugate gradient method

- Exact solution: Interpretation as direct method

- In practice: Used as iterative method
  - Unstable to perturbations, e.g. noise
  - Required tolerance usually achieved after *it*<<*n*

- Very easy to implement (≈20 lines of code)

- Implementations in Scipy, Matlab, Numerical Recipes etc.

scipy.sparse.linalg.cg

```
scipy.sparse.linalg.cg(A, b, x0=None, tol=1e-05, maxiter=None, M=None, callback=None, atol=None)
```

Use Conjugate Gradient iteration to solve `Ax = b`.

CG on the normal equations and preconditioning

$$K\vec{u} = \vec{g}$$

Requirement: *K* symmetric and positive definite

$$K^T K\vec{u} = K^T\vec{g}$$

CG on the normal equations

$$MK\vec{u} = M\vec{g}$$

Preconditioning: „Modify equation such that right hand side is approximate solution“

CG radial trajectory, 128 projections, R=2

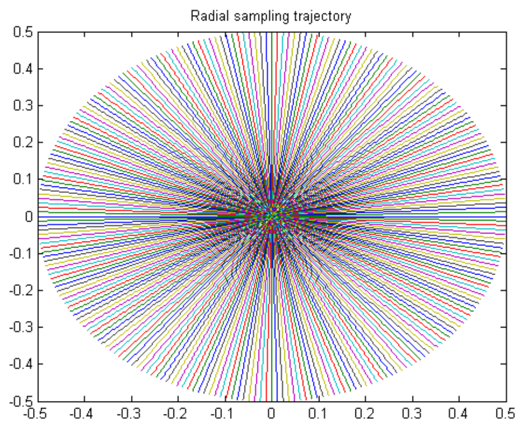
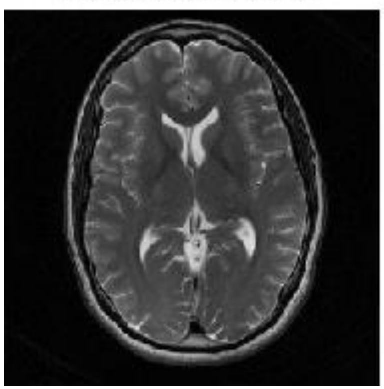
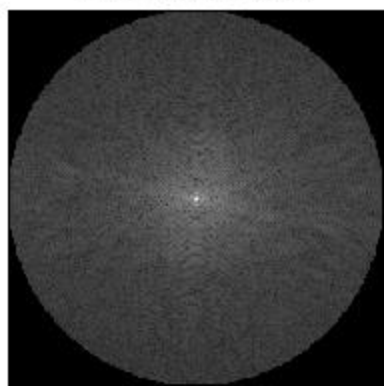
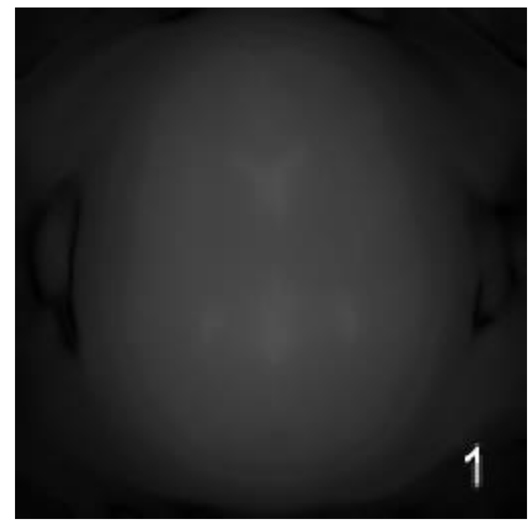


Image CG iteration 40



k-space iteration 40





CG radial trajectory, 64 projections, R=4

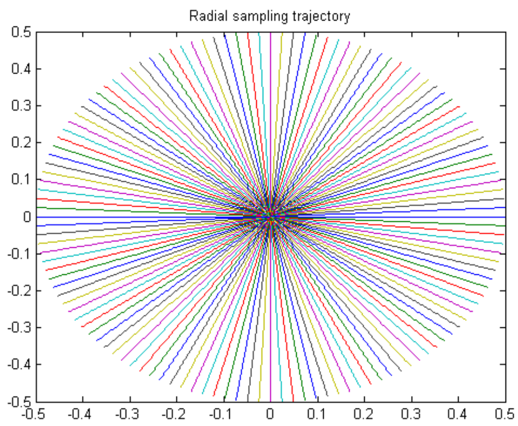
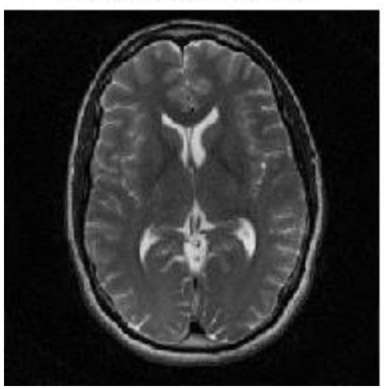
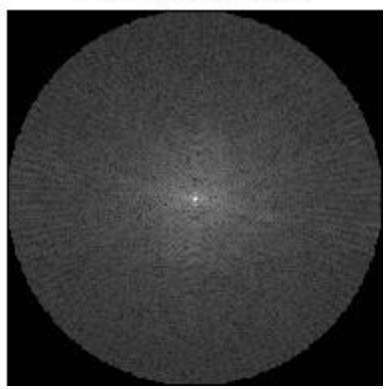
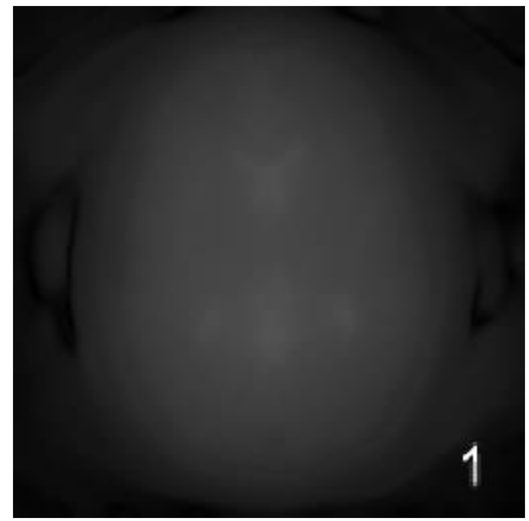


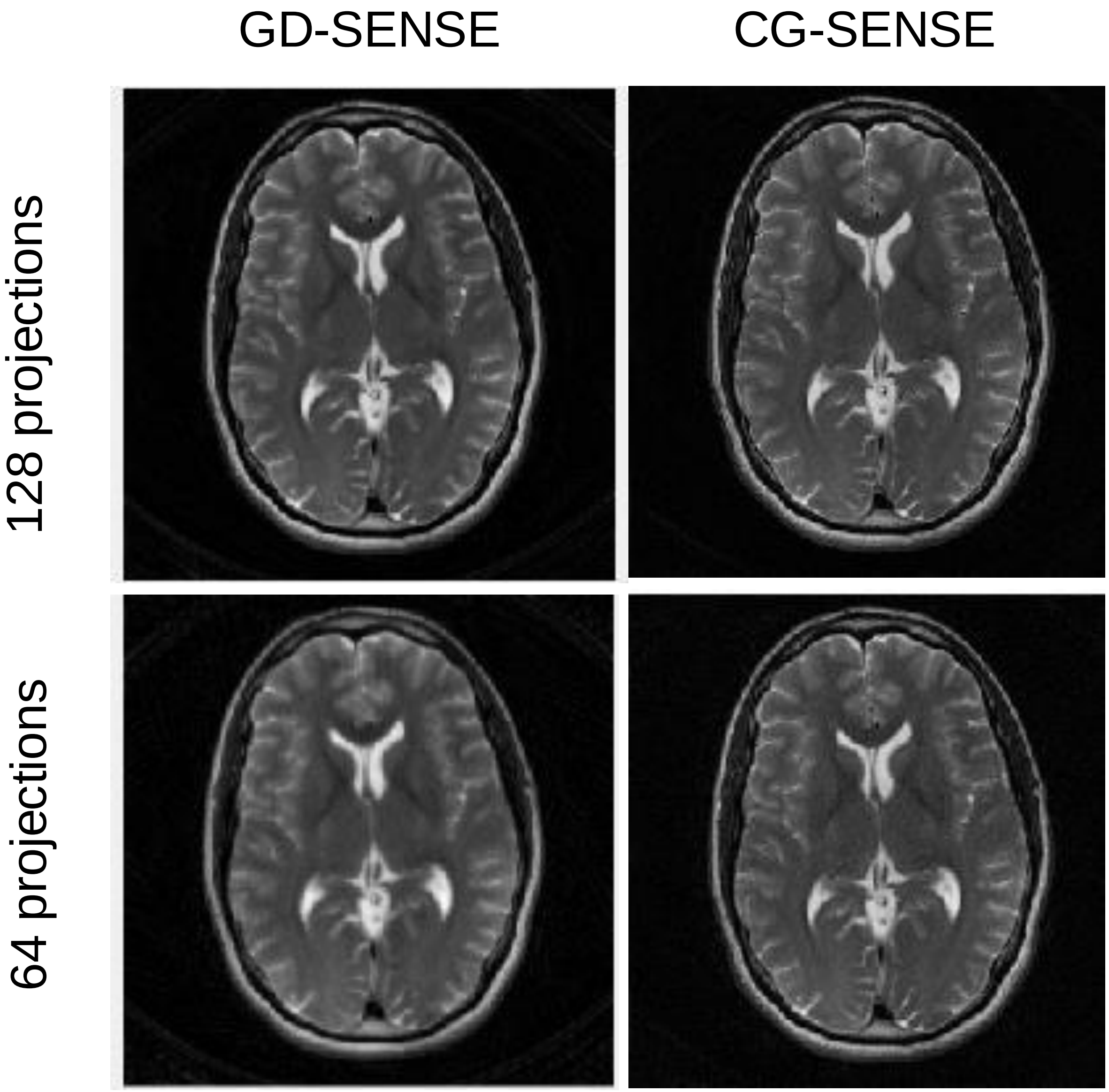
Image CG iteration 40



k-space iteration 40






GD-SENSE
CG-SENSE
128 projections
64 projections

CG radial trajectory, 32 projections, R=8

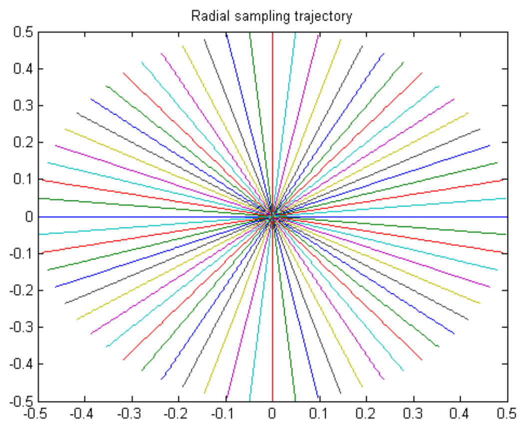
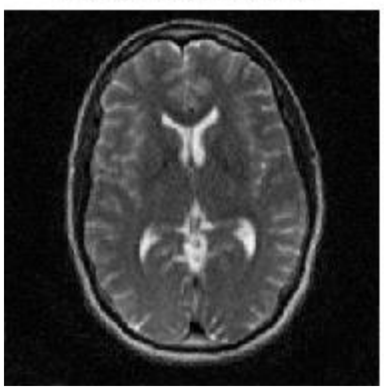
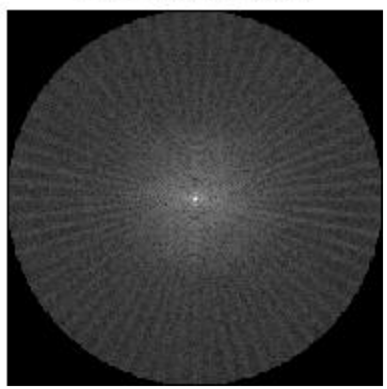
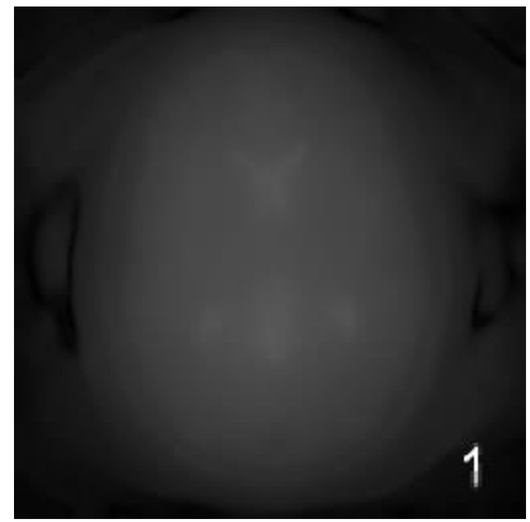


Image CG iteration 40

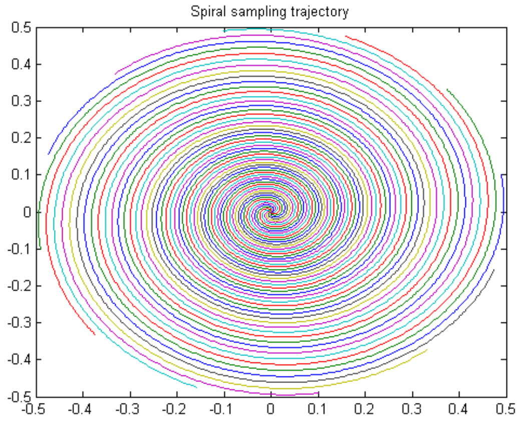


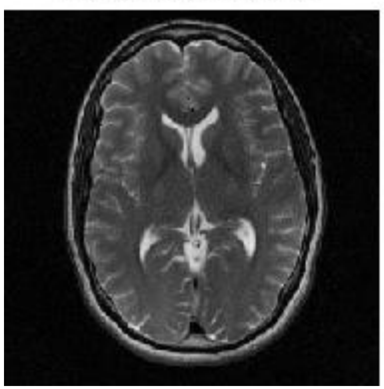
k-space iteration 40

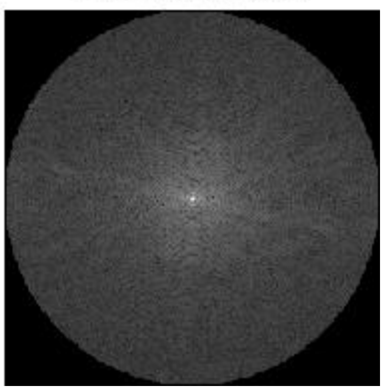


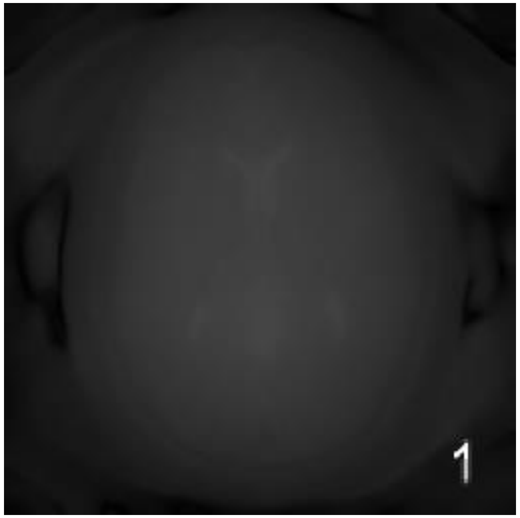


CG spiral trajectory, 12 interleaves, R≈4









CG spiral trajectory, 6 interleaves, R≈8

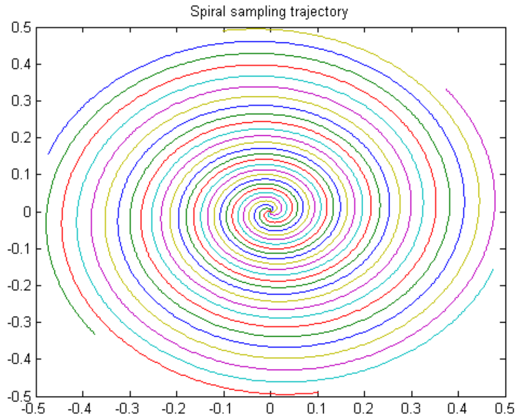
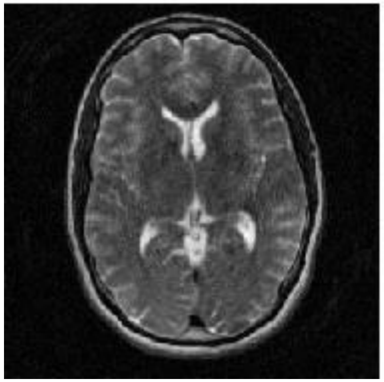
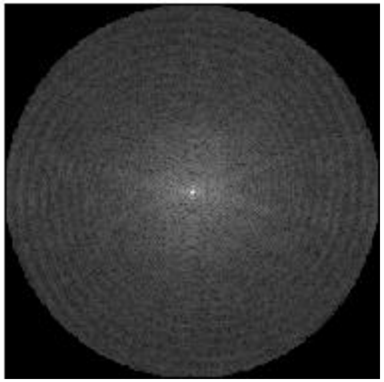
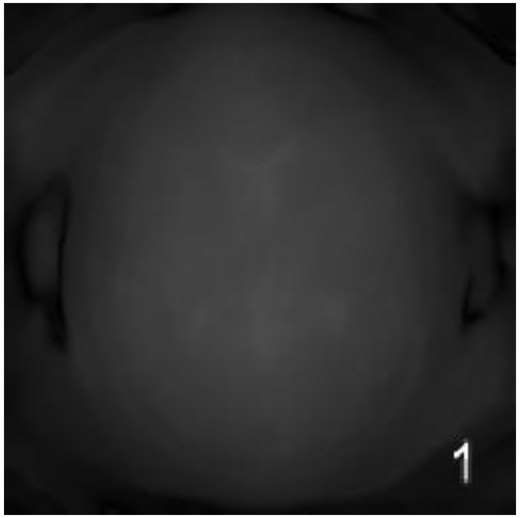


Image CG iteration 40

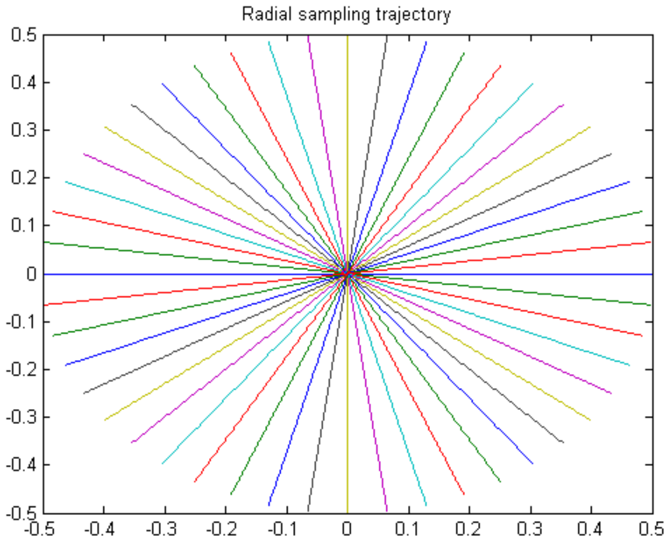


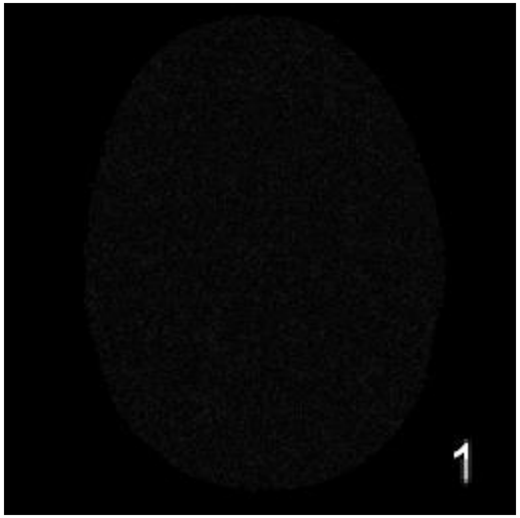
k-space iteration 40

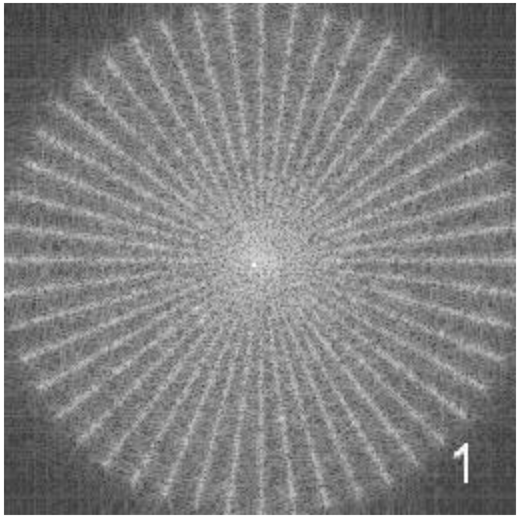




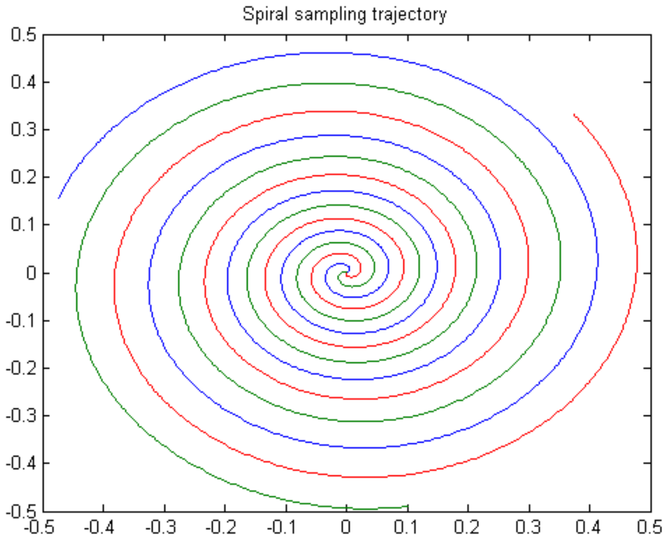
Noise propagation: Radial trajectory, 24 projections

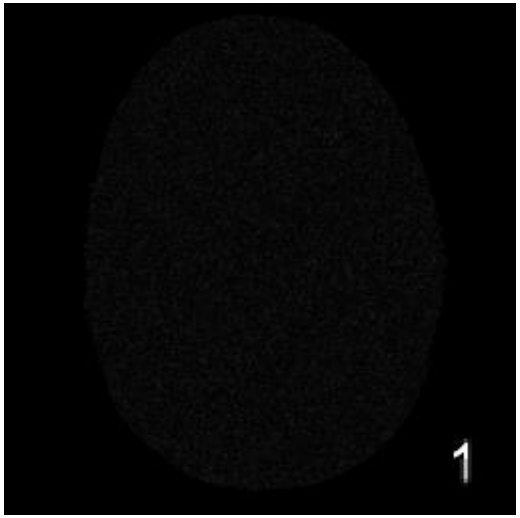


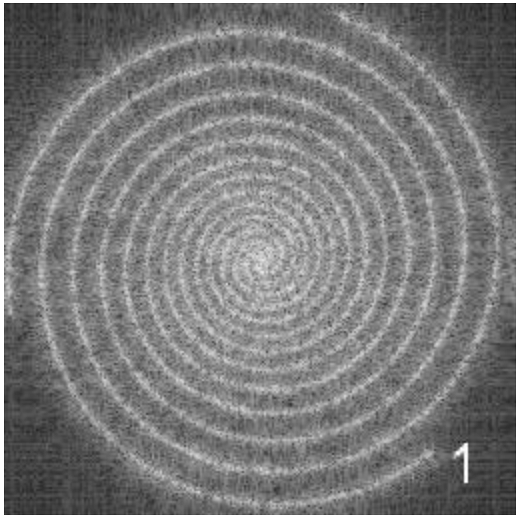




Noise propagation: Spiral trajectory, 3 interleaves







Noise propagation: Cartesian, R=4

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Can you reproduce this seminal MRM paper? Participate in the reproducible research study group challenge!

April 2, 2019

2832 0

If you believe Copy Link nding time on reproducibility makes SENSE, we encourage you to join the ISMRM reproducible research study group's 2019 challenge! As the challenge unfolds, so will sub-Nyquist aliasing artifacts. Have we dropped enough hints about the MRM paper selected to be reproduced?

Yes, you probably guessed right. The paper selected for the 2019 reproducibility challenge is:

Klaas P. Pruessmann, Markus Weiger, Peter Börnert, Peter Boesiger. *Advances in sensitivity encoding with arbitrary k-space trajectories.* Magn Reson Med. 2001 Oct;46(4):638-51.

The data

We provided two example datasets, brain (12 receive channels, 96 radial projections) and cardiac (34 receive channels, 55 radial projections), from a radial trajectory acquired with multi-channel coils. The data is provided in the h5 format, and we are following the conventions of the BART toolbox regarding array dimensions of the raw data [1, Readout, Spokes, Channels] and the trajectory [3, Readout, Spokes] where the first dimension encodes the k-space coordinate (for 2D acquisitions the third coordinate is always zero) and the unit of measurement is 1 / FOV.

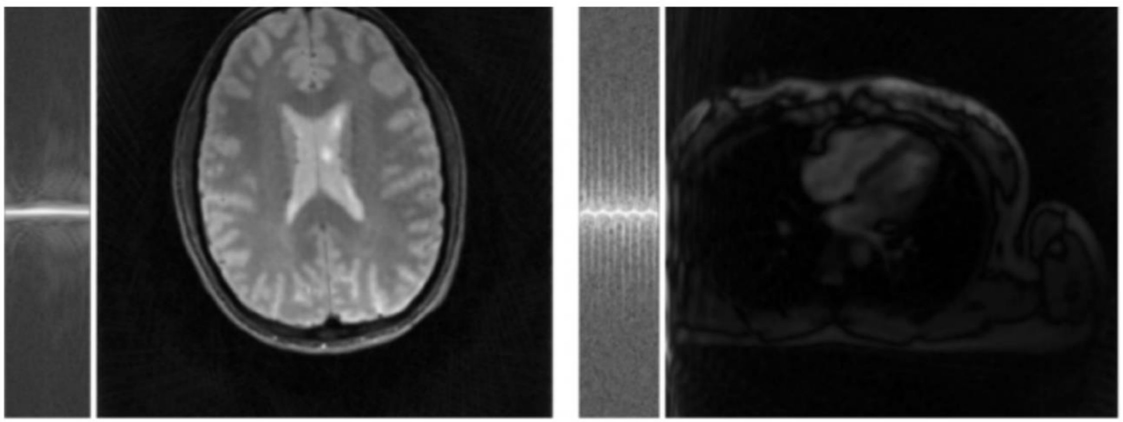


Figure 1: Raw k-space data from one coil and a gridding sum of squares example reconstruction of the provided brain (left) and cardiac (right) data

Brain data (5.3 MB): rawdata_brain_radial_6proj_12ch.h5: rawdata: [1, 512, 96, 12], trajectory: [1, 512, 96]

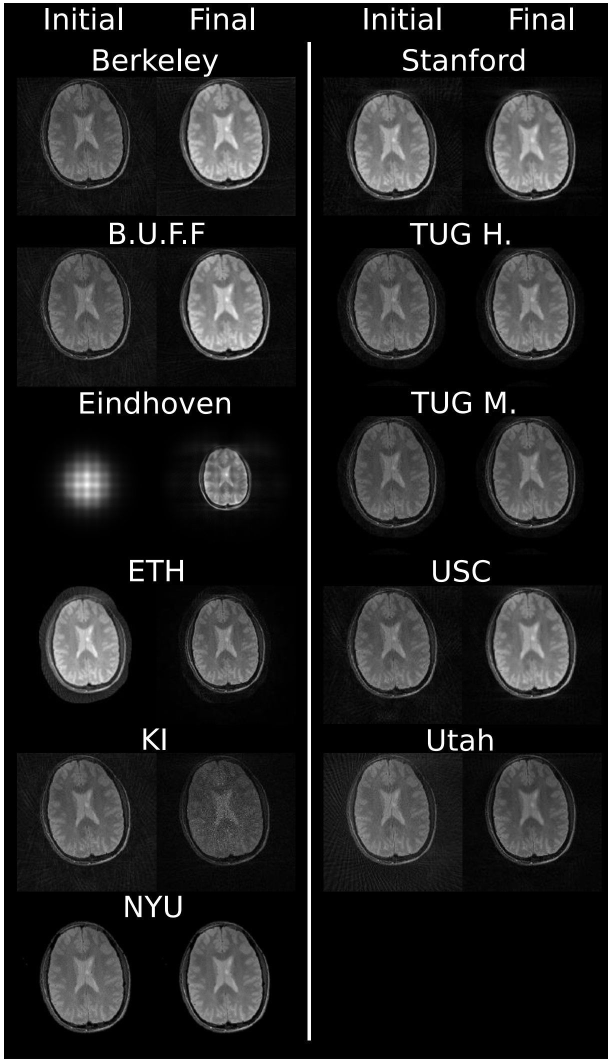
Cardiac data (5 MB): rawdata_heart_radial_55proj_34ch.h5: rawdata: [1, 320, 55, 34], trajectory: [1, 320, 55]

We also provided starter-scripts for MATLAB and Python which are able to read in and display the data. These scripts are also available from the Github repository.

Submissions

Collated list of submissions - May 2019:

Authors (or principal author)	Link	Info
Steven Baete (NYU)	https://bitbucket.org/sbaete/ismrm2019reprodcgsense	MATLAB, NUFFT (Fessler), gpuNUFFT (Schwarzl, Knoll)
Alexander Fyrdahl (Karolinska Institutet and Karolinska University Hospital, Sweden)	https://github.com/fyrdahl/rrsg_challenge	MATLAB, NUFFT (Fessler)
Kerstin Hammernik (Graz University of Technology)	https://github.com/khammernik/ISMRM2019_RRSG	Python, BART, primal-dual-toolbox, medutils
Seb Harrevelt (Eindhoven University of Technology)	https://github.com/zwep/ismrm19_challenge	Python, PyNUFFT (Lin),
Namgyun Lee (University of Southern California)	https://drive.google.com/file/d/10qD6K-sCEkNjpynRZTpLm8VBUPJFhJCt/view	MATLAB, custom MEX for gridding
Gilad Liberman (MGH)	https://github.com/giladddd/LinopScript	MATLAB, *Demo of linear-operator scripting for BART on challenge datasets*
Michael Loecher (Stanford)	https://github.com/mloecher/rrsg_challenge	Python, custom Cython for gridding
Oliver Maier (Graz University of Technology)	https://github.com/MaierOli2010/ISMRM_RRSG	Python, BART, requires GPU for use of GPyFFT
Franz Patzig, Lars Kasper, Thomas Ulrich, Maria Engel, Johanna Vannesjo, Markus Weiger, David Brunner, Bertram Wilm, Klaas Prüssmann (ETHZ)	https://github.com/mrtm-zurich/rrsg-arbitrary-sense	MATLAB, custom gridding in MATLAB
Ludger Starke (MDC-Berlin)	.../reproducibleResearch19_LudgerStarke.zip	MATLAB, BART
Ye Tian (UCAIR - University of Utah)	https://github.com/YeTianMRI/ISMRM-2019-reproducible	MATLAB, NUFFT (Fessler)
Ke Wang, Miki Lustig, Ekin Karasan, Suma Anand, Volert Roeloffs (Berkeley)	https://github.com/KeWang0622/rrsg_challenge_sigpy	Python, SigPy (Ong)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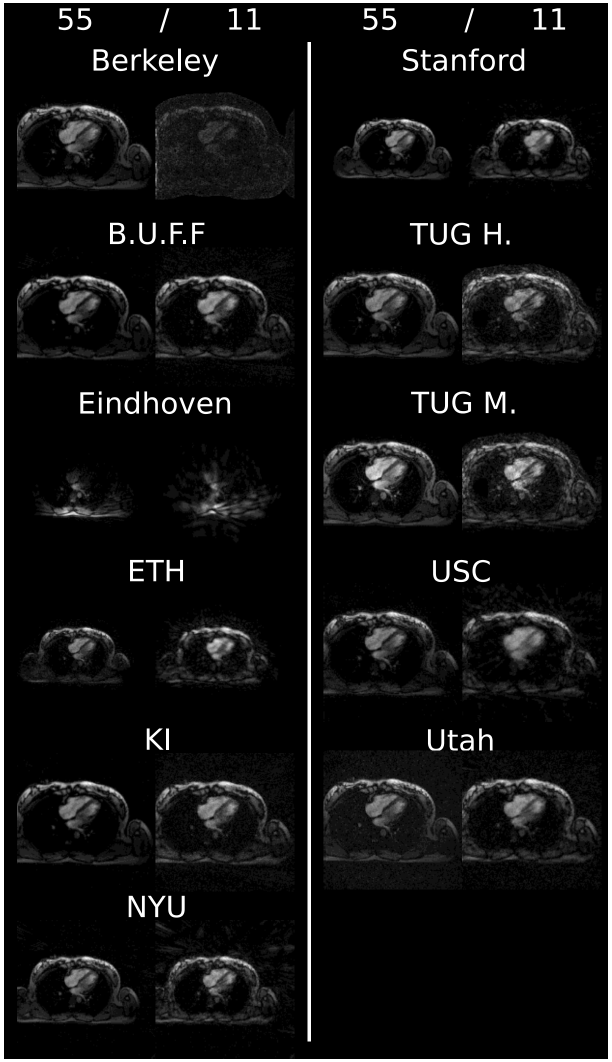
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REVIEW

Magnetic Resonance in Medicine

CG-SENSE revisited: Results from the first ISMRM reproducibility challenge

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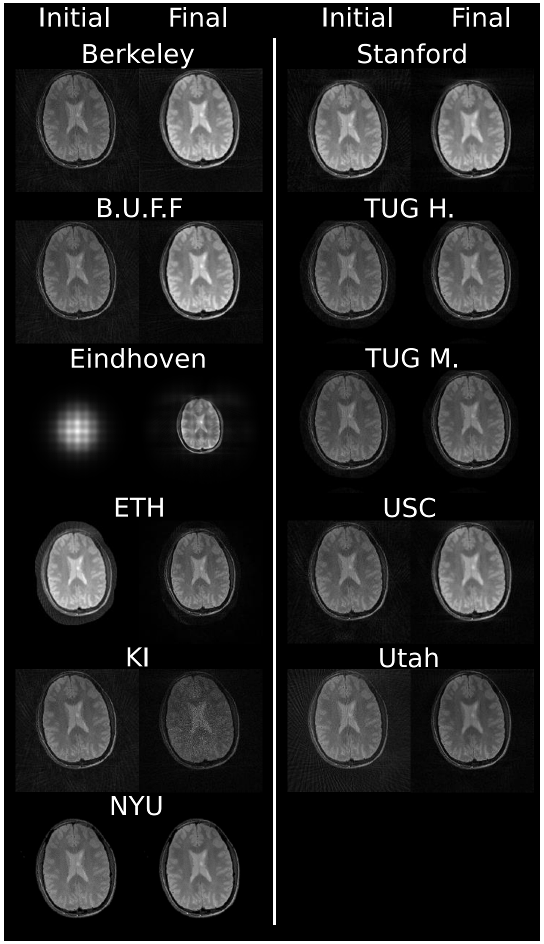
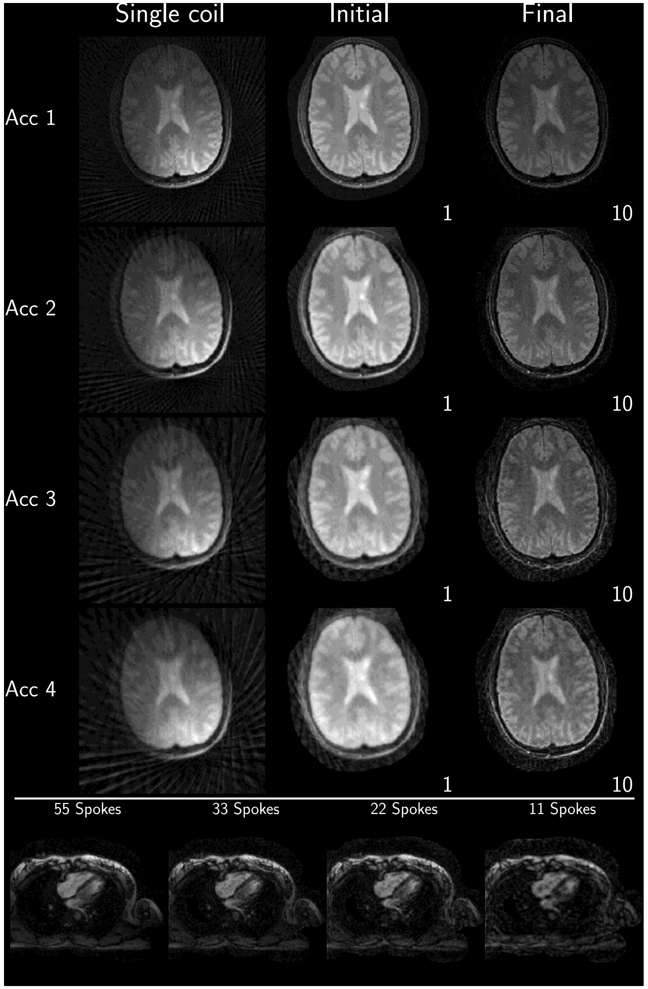
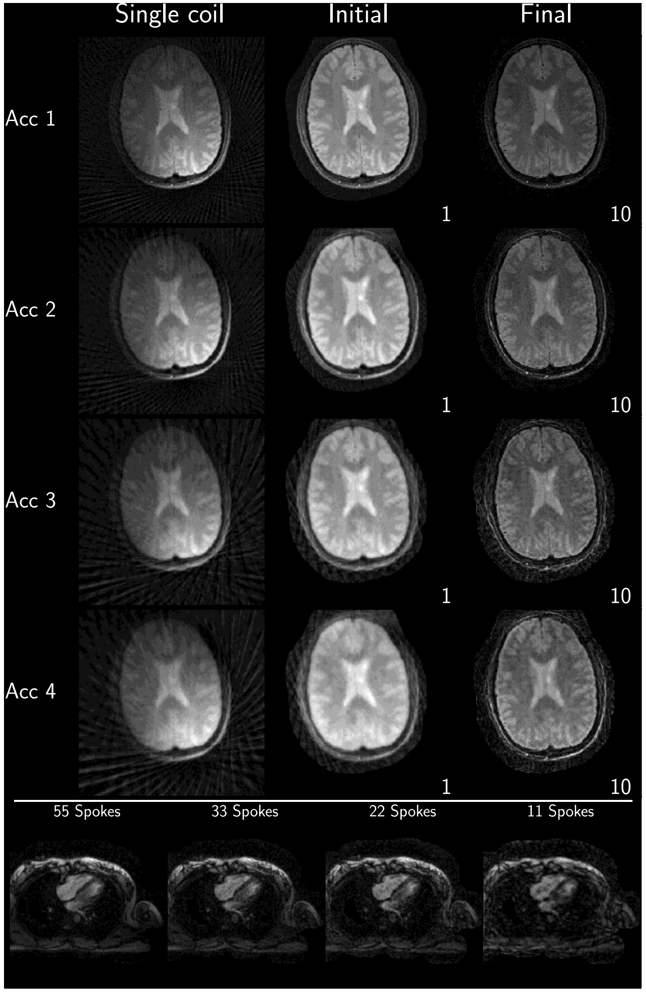


Consolidated Python implementation



Consolidated Matlab implementation



Summary

- Aliasing for non-Cartesian subsampling
- Iterative reconstruction: GD, and CG
- Convergence behavior
- Noise propagation
- Sidenote: Reproducibility