Probability

Probability numbers between 0 and 1, 0 is impossible, 1 is certain

---

Interpretations:

- Frequentist-proportion of heads if we toss a coin many times
- Propensity-tendency of a coin to land heads
- Subjectivist-how strongly we believe that a coin will land heads

---

Notes:

- Probability vs statistics (prob-likelihood of certain events vs stat-observe results and determine probabilities from which they might have originated)
- Random isnt really randon just chaotic (underlying principles very complicated and tiny changes affect result, just really hard to predict)
- true randomness does exists-radioactive decay
- Quantum mechanics is the only known effect in nature that produces true randomness
- As we roll dice more and more often, the observed frequencies become closer and closer to the frequencies we predict using probability theory. This principle always applies in probability and is called the Law of large numbers.
- As we increase the number of dice rolled at once, we also see that the shape of the probability distribution changes from a triangular shape to a bell-shaped curve. This is known as the Central Limit Theorem.

---

sources:

- `https://en.wikipedia.org/wiki/Probability`

Combinatorics

Factorial:

- factorial (5*4*3*2*1=5!)
- 0!=1

r-s Principle:

- 5 pairs of pants to go with 2 shirts 5*2 options
- ordered pair-pair of 'things' arranged in a certain order

Permutations and combinations:

- Permutations (care about order)
  - place n objects in k positions
  - $$ {}^{n}P_{k} = \frac{n!}{(n-k)!} $$
- combinations (dont care about order)
  - divide by n! because compared to permutation n places(order) to place first choice, n-1 to place second...
  - $$ {}^{n}C_{k} = \frac{n!}{k!(n-k)!} = \binom{n}{k} $$

Bonomial Identity:

$$ (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} $$

$$ \binom{n}{0} = \binom{n}{n} = 1 \quad \text{for all integers} $$

sources:

- `http://world.mathigon.org/Combinatorics`

Set Theory

Definitions:

- an object either belongs or does not belong
- set-collection of things that are brought together because they obey a certain (well defined) rule
  - ex: numbers, people, shapes
- element- ”thing” that belongs to a given set

Symbols:

- {...}-the set of ...
  - ex:{-3,-2,-1,0,1,2}, {integers between -3 and 3 inclusive}, {x|x is an integer amd |x| <4}
- ($\in$) symbol means element of
- set usually uppercase A,B and elements lowercase x,y
- U-universal set, all things under discussion
- {},($\varnothing$)-empty set, null set
- N-natural numbers, whole numbers starting at 1
- Z-integers
- R-real numbers

Set Operations:

- (A $\cap$ B)intersection (two sets overlap)
- (A $\cup$ B) union (elements in either)
- (A - B) or (A B) difference (elements in A but not B)
- (A’) or(A^C) or complement (everything not in A is in A’)
- cardinality (if A={lowercase letters of the alphabet}, | A| =26)
- P(A) powerset-set of all subsets (including empty) of A

  - if | A |=k then | P(A)| =2^k proof (for each element we can choose to include element or not)

- Cartesian Products
  - if we have n sets: A1 , A2, ..., An, then their Cartesian product is defined by: $A1A2...An$ = { (a1, a2, ..., an) | a1 ∈ A1, a2 ∈ A2, ..., an ∈ An) } and (a1, a2, ..., an) is called an ordered n-tuple.

Relationships:

- Equality = (same elements, repeats ignored)
- subsets (A⊆B) all elements of A are also elements of B
  - A ⊆ B and B ⊆ A, then A = B
  - proper subset (A ⊂ B) if B contains at least one element that isn't in A
- disjoint no elements in common

Foundational Rules of Set Theory:

- The Laws of Sets
  - Commutative Laws
    - ∩ B = B ∩ A
    - A ∪ B = B ∪ A
  - Associative Laws
    - (A ∩ B) ∩ C = A ∩ (B ∩ C)
    - (A ∪ B) ∪ C = A ∪ (B ∪ C)
  - Distributive Laws
    - A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
    - A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
  - Idempotent Laws
    - A ∩ A = A
    - A ∪ A = A
  - Identity Laws
    - A ∪ ∅ = A
    - A ∩ U = A
    - A ∪ U = U
    - A ∩ ∅ = ∅
  - Involution Law

    - $(A')' = A$
  - Complement Laws
    - $A \cup A' = U$
    - $A \cap A' = \varnothing$
    - $U' = \varnothing$
    - $\varnothing' = U$
  - De Morgans Laws
    - $(A \cap B)' = A' \cup B'$
    - $(A \cup B)' = A' \cap B'$
    - proof
      - $(A \cup B)' \subseteq A' \cap B'$
      - $A' \cap B' \subseteq (A \cup B)'$

---

sources:

- `https://en.wikibooks.org/wiki/Discrete_Mathematics/Set_theory`