Set Theory

Definitions:

- an object either belongs or does not belong
- set-collection of things that are brought together because they obey a certain (well defined) rule
  - ex: numbers, people, shapes
- element- "thing" that belongs to a given set

Symbols:

- {...}-the set of ...
  - ex:{-3,-2,-1,0,1,2}, {integers between -3 and 3 inclusive}, {x|x is an integer amd |x| <4}
- ($\in$) symbol means element of
- set usually uppercase A,B and elements lowercase x,y
- U-universal set, all things under discussion
- {},($\varnothing$)-empty set, null set
- N-natural numbers, whole numbers starting at 1
- Z-integers
- R-real numbers

Set Operations:

- (A $\cap$ B)intersection (two sets overlap)
- (A $\cup$ B) union (elements in either)
- (A - B) or (A B) difference (elements in A but not B)
- (A') or(A^C) or complement (everything not in A is in A')
- cardinality (if A={lowercase letters of the alphabet}, | A| =26)
- P(A) powerset-set of all subsets (including empty) of A

  - if | A |=k then | P(A)| =2^k proof (for each element we can choose to include element or not)

- Cartesian Products
  - if we have n sets: A1 , A2, ..., An, then their Cartesian product is defined by: $A1A2...An$ = { (a1, a2, ..., an) | a1 ∈ A1, a2 ∈ A2, ..., an ∈ An) } and (a1, a2, ..., an) is called an ordered n-tuple.

Relationships:

- Equality = (same elements, repeats ignored)
- subsets (A⊆B) all elements of A are also elements of B
  - A ⊆ B and B ⊆ A, then A = B
  - proper subset (A ⊂ B) if B contains at least one element that isn't in A
- disjoint no elements in common

Foundational Rules of Set Theory:

- The Laws of Sets
  - Commutative Laws
    - ∩ B = B ∩ A
    - A ∪ B = B ∪ A
  - Associative Laws
    - (A ∩ B) ∩ C = A ∩ (B ∩ C)
    - (A ∪ B) ∪ C = A ∪ (B ∪ C)
  - Distributive Laws
    - A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
    - A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
  - Idempotent Laws
    - A ∩ A = A
    - A ∪ A = A
  - Identity Laws
    - A ∪ ∅ = A
    - A ∩ U = A
    - A ∪ U = U
    - A ∩ ∅ = ∅
  - Involution Law

        - $(A')' = A$
    - Complement Laws
        - $A \cup A' = U$
        - $A \cap A' = \varnothing$
        - $U' = \varnothing$
        - $\varnothing' = U$
    - De Morgan's Laws
        - $(A \cap B)' = A' \cup B'$
        - $(A \cup B)' = A' \cap B'$
        - proof
            - $(A \cup B)' \subseteq A' \cap B'$
            - $A' \cap B' \subseteq (A \cup B)'$

Sources:

- https://en.wikibooks.org/wiki/Discrete_Mathematics/Set_theory